1.1 Given common plane Π. & let x_Π be the common point that is being projected into camera planes. at ~~x~~ p, q. We need to prove the existance of homography between p & q.

Since M_1 & M_2 are projection matrix to cameras C_1 & C_2 & plane Π.

$M_1 = K_1 [R_1 | t]$ $\quad$ $M_2 = K_2 [R_2 | t]$

$\det(R_1) \neq 0$ $\quad$ $\det(R_2) \neq 0$

① ~~Given the cameras are related by pure rotational constraints~~

so

$p = M_1 x_\Pi$ $\quad$ $q = M_2 x_\Pi$

since $\det(R) \neq 0$

Here M_1^{-1} & M_2^{-1} exists, so p & q can be related as.

$$p \equiv M_1 M_2^{-1} q. \Rightarrow H \propto M_1 M_2^{-1}$$

& it exists.

1.2 In this Question, its given that Camera is separated by pure rotation

X is a point in 3D space

$$X = \begin{bmatrix} x_1 \\ y_1 \\ z_1 \\ 1 \end{bmatrix}$$

given $\underset{(3\times1)}{X_2} = K_2 \underset{3\times3\ \ 3\times1}{[R \mid 0]} \begin{bmatrix} x_1 \\ y_1 \\ z_1 \\ 1 \end{bmatrix}$

Since the translation part is 0

$$\begin{bmatrix} x_2' \\ y_2' \\ z_2' \end{bmatrix} = \underset{(3\times1)}{X_2} = \underset{3\times3}{K_2[R]} \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} \quad \text{—} (1)$$

Similarly,

$$\begin{bmatrix} x_1' \\ y_1' \\ z_1' \end{bmatrix} = \underset{(3\times1)}{X_1} = \underset{3\times3}{K_1[I]} \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix}$$

(in homogenous form)

$$K_1^{-1} X_1 = \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} \quad \text{—— (2)}$$

From (1) & (2),

$$X_2 = K_2 [R] K_1^{-1} X_1.$$

$\therefore$ The homography matrix H is given by,

$$H \propto K_2 [R] K_1^{-1}.$$

$$H = \lambda K_2 [R] K_1^{-1}$$

$\lambda \to$ scalar

$\therefore$ Homography exists.

1.3. Given $x_1^i = H x_2^i$

which can be re-written,

$$\begin{bmatrix} x_1^i \\ y_1^i \\ 1 \end{bmatrix} = H \cdot \begin{bmatrix} x_2^i \\ y_2^i \\ 1 \end{bmatrix}$$

$$= \alpha \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} x_2^i \\ y_2^i \\ 1 \end{bmatrix}$$

a) There are 9 variables in h, normalizing the h with h_{33}, the h will have 8 degree of freedom.

b) Since there are 8 variables in h. we need 8 equation. One pair of equation gives 2 equation. Hence we ~~give~~ need 4 pair points.

Proof: Proof that one pair of points gives 2 equations.

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \alpha \begin{bmatrix} h_1 & h_2 & h_3 \\ h_4 & h_5 & h_6 \\ h_7 & h_8 & h_9 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Solving

$$x' = \alpha(h_1 x + h_2 y + h_3)$$
$$y' = \alpha(h_4 x + h_5 y + h_6)$$
$$1 = \alpha(h_7 x + h_8 y + h_9)$$

Dividing out unknown scale factor:

$$x'(h_7 x + h_8 y + h_9) = (h_1 x + h_2 y + h_3)$$
$$y'(h_7 x + h_8 y + h_9) = (h_4 x + h_5 y + h_6)$$

Rearranging we have 2 equations

Equation 1 $\rightarrow h_7 x x' + h_8 y x' + h_9 x' - h_1 x - h_2 y - h_3 = 0$

Equation 2 $\rightarrow h_7 x y' + h_8 y y' + h_9 y' - h_4 x - h_5 y - h_6 = 0$

c) Derive A_i.

Given equations,

$$h_7 x x' + h_8 y x' + h_9 x' - h_1 x - h_2 y - h_3 = 0$$

$$h_7 x y' + h_8 y y' + h_9 y' - h_4 x - h_5 y - h_6 = 0$$

In matrix form, $A_i h = 0$

$$A_i = \begin{bmatrix} -x & -y & -1 & 0 & 0 & 0 & xx' & yx' & x' \\ 0 & 0 & 0 & -x & -y & -1 & xy' & yy' & y' \end{bmatrix}$$

Rewriting

$$h = \begin{bmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \\ h_5 \\ h_6 \\ h_7 \\ h_8 \\ 1 \end{bmatrix}$$

$\Rightarrow$ For 4 pair of points ~~the~~ A_i equation can be rewritten as.

For 4 pair of points A_i can be rewritten as.

$$A_i h = \underbrace{\begin{bmatrix} -x_1 & -y_1 & -1 & 0 & 0 & 0 & -x_1x_1' & -y_1x_1' & x_1' \\ 0 & 0 & 0 & -x_1 & -y_1 & -1 & -x_1y_1' & y_1y_1' & y_1' \\ -x_2 & -y_2 & -1 & 0 & 0 & 0 & x_2x_2' & y_2x_2' & x_2' \\ 0 & 0 & 0 & -x_2 & -y_2 & -1 & -x_2y_2' & y_2y_2' & y_2' \\ -x_3 & -y_3 & -1 & 0 & 0 & 0 & x_3x_3' & y_3x_3' & x_3' \\ 0 & 0 & 0 & -x_3 & -y_3 & -1 & -x_3y_3' & y_3y_3' & y_3' \\ -x_4 & -y_4 & -1 & 0 & 0 & 0 & x_4x_4' & y_4x_4' & x_4' \\ 0 & 0 & 0 & -x_4 & -y_4 & -1 & -x_4y_4' & y_4y_4' & y_4' \end{bmatrix}}_{A_i} \underbrace{\begin{bmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \\ h_5 \\ h_6 \\ h_7 \\ h_8 \\ 1 \end{bmatrix}}_{h}$$

1.4. Since K, the intrinsic parameter is constant

H^{a} corresponds to R. & H^2

corresponds to R^2

Given,

$$R = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R^2 = \begin{bmatrix} \cos^2\theta - \sin^2\theta & -2\cos\theta\sin\theta & 0 \\ \cancel{\cos^2\theta - \sin^2\theta} \; 2\sin\theta\cos\theta & (\cos^2\theta - \sin^2\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos 2\theta & -\sin 2\theta & 0 \\ \sin 2\theta & \cos 2\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\therefore$ H^2 corresponds to 2θ.

1.6.

Equation of line in 3D:

$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{b} = \lambda \text{ (const)}.$$

$$x = a\lambda + x_1$$
$$y = b\lambda + y_1$$
$$z = c\lambda + z_1$$

In matrix form,

$$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \lambda \begin{bmatrix} a \\ b \\ c \end{bmatrix} + \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} = \begin{bmatrix} \lambda a + x_1 \\ \lambda b + y_1 \\ \lambda c + z_1 \end{bmatrix}$$

writing the 3D coordinate into projective space we have. $\left[x = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = PX \right]$

$$x = \begin{bmatrix} x/z \\ y/z \\ 1 \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \lambda a + x_1 / \lambda c + z_1 \\ \lambda b + y_1 / \lambda c + z_1 \\ 1 \end{bmatrix}$$

$$\therefore \quad \frac{x^2}{\lambda a + x_1} = \frac{1}{\lambda c + z_1} = \frac{y^2}{\lambda b + y_1}.$$

$$(\lambda b + y_1)x^2 - (\lambda a + x_1)y^2 = 0$$

↳ This is still the equation of a line

Hence lines are preserved in projective space. P.Q