P1 Problems

Exercise 1.

Given, $S_1 = \begin{bmatrix} y \\ \dot{y} \\ \psi \\ \dot{\psi} \end{bmatrix} \begin{matrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{matrix}$

$l_r = 1.39m$

$l_f = 1.55m$

$C_\alpha = 20000N$

$I_z = 25854 kgm^2$

$\mu = 0.019$

$\Delta t = 0.032$

$f_1 = \dot{y}$ $\quad \dot{S}_1 = \begin{bmatrix} \dot{y} \\ \ddot{y} \\ \dot{\psi} \\ \ddot{\psi} \end{bmatrix}$.

$$\dot{S}_1 = \begin{bmatrix} \dot{y} \\ -\dot{\psi}\dot{x} + \frac{2C_\alpha}{m}\left(\cos\delta\left(\delta - \frac{\dot{y} + I_p\dot{\psi}}{\dot{x}}\right) - \frac{\dot{y} - l_r\dot{\psi}}{\dot{x}}\right) \\ \dot{\psi} \\ \frac{2l_fC_\alpha}{I_z}\left(\delta - \frac{\dot{y} + l_f\dot{\psi}}{\dot{x}}\right) - \frac{2l_rC_\alpha}{I_z}\left(\frac{-\dot{y} - l_r\dot{\psi}}{\dot{x}}\right) \end{bmatrix}$$

At equilibrium pt,

$$\dot{y} = 0$$

$$\dot{y} = 0 \Rightarrow -\dot{\psi}\dot{x} + \frac{2C_\alpha}{m}\cos\delta\left(\delta - \frac{\dot{y}+l_f\dot{\psi}}{\dot{x}}\right) - \frac{\dot{y}-l_r\dot{\psi}}{\dot{x}} = 0$$

$$\dot{\psi} = 0 \Rightarrow \frac{2C_\alpha}{m}\delta\cos\delta = 0 \Rightarrow \delta = 0$$

(next relation)

$$\ddot{\psi} = 0 \Rightarrow \frac{2l_fC_\alpha}{I_z}\delta = 0 \Rightarrow \delta = 0$$

$$\left.\frac{\partial \dot{x}_i}{\partial x_j}\right|_{\text{at Eq}} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \frac{-4C_\alpha}{m\dot{x}} & 0 & \frac{2C_\alpha(l_r-l_f)}{m\dot{x}} - \dot{x} \\ 0 & 0 & 0 & 1 \\ 0 & \frac{2C_\alpha(l_r-l_f)}{I_z\dot{x}} & 0 & \frac{-2C_\alpha(l_f^2+l_r^2)}{I_z\dot{x}} \end{bmatrix}$$

$$\left.\frac{\partial \dot{y}}{\partial \delta}\right|_{\text{eqm}} = \frac{-2C_\alpha(\cos\delta + 1)}{m\dot{x}} = -\frac{4C_\alpha}{m\dot{x}}$$

$$\left.\frac{\partial \ddot{y}}{\partial \delta}\right|_{\text{steam}} = \frac{2C_a(\cos\delta - \delta\sin\delta)}{m} + \frac{2C_a(\dot{y} + l_f\dot{\psi})}{\dot{x}} \times \sin\delta$$

$$= \frac{2C_a}{m}$$

$$\left.\frac{\partial \ddot{y}}{\partial \dot{\psi}}\right|_{\text{steam}} = -\dot{x} + \frac{2C_a(l_r - l_f\cos\delta)}{m\dot{x}}$$

$$= \left| \frac{2C_a(l_r - l_f)}{m\dot{x}} \quad -\dot{x} \right|$$

$$\left.\frac{\partial \ddot{\psi}}{\partial \dot{\psi}}\right|_{\text{steam}} = -\frac{2l_fC_a}{I_z}\frac{l_f}{\dot{x}} + \frac{2l_rC_a}{I_z}\left(\frac{-l_r}{\dot{x}}\right)$$

$$= \frac{-2C_a}{I_z\dot{x}}(l_r^2 + l_f^2)$$

$$\frac{\partial \ddot{\psi}}{\partial \dot{y}} = \frac{2l_fC_a}{\dot{x}_0 I_z}(-1) + \frac{2l_rC_a}{I_z\dot{x}}(1)$$

$$= \frac{2C_a}{I_z\dot{x}}(l_r - l_f)$$

$$\partial\dot{\psi}/\partial\delta = 2l_fC_a/I_z$$

$$\left.\frac{\partial f}{\partial u}\right|_{\delta_{f_0}} = \begin{bmatrix} \frac{\partial \dot{Y}}{\partial \dot{Y}} & \frac{\partial \dot{Y}}{\partial Y} \\ \frac{\partial \dot{Y}}{\partial \dot{Y}} & \frac{\partial \dot{Y}}{\partial Y} \\ \frac{\partial \dot{\psi}}{\partial \dot{\psi}} & \frac{\partial \dot{\psi}}{\partial \psi} \\ \frac{\partial \dot{\psi}}{\partial \dot{\psi}} & \frac{\partial \dot{\psi}}{\partial \psi} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 2C_f/m & 0 \\ 0 & 0 \\ 2l_f C_f/I_z & 0 \end{bmatrix}$$

$$\dot{S}_1 = \begin{bmatrix} \dot{Y} \\ \ddot{Y} \\ \dot{\psi} \\ \ddot{\psi} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \frac{-4C_f}{m\dot{x}} & 0 & \frac{2C_f(l_r-l_f)}{m\dot{x}} \\ 0 & 0 & 0 & 1 \\ 0 & \frac{2C_f(l_r-l_f)}{I_z\dot{x}} & 0 & \frac{-2C_f(l_f^2+l_r^2)}{I_z\dot{x}} \end{bmatrix} S_1 + \begin{bmatrix} 0 & 0 \\ 2C_f/m & 0 \\ 0 & 0 \\ \frac{2l_f C_f}{I_z} & 0 \end{bmatrix} u$$

(1)

$$\dot{S}_1 = A_1 S_1 + B_1 u.$$

~~$\dot{S}_1 = A_1$~~

$$A_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \frac{-4C_\alpha}{m\dot{x}} & 0 & \frac{2C_\alpha (l_r - l_f)}{m\dot{x}} \\ 0 & 0 & 0 & 1 \\ 0 & \frac{2C_\alpha (l_r - l_f)}{I_z \dot{x}} & 0 & \frac{-2C_\alpha (l_r^2 + l_f^2)}{I_z \dot{x}} \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 0 & 0 \\ 2C_\alpha/m & 0 \\ 0 & 0 \\ \frac{2C_\alpha l_f}{I_z} & 0 \end{bmatrix}$$

for $x < 0.5$, $\dot{y} = -\dot{\psi}\dot{x} + \frac{2C_\alpha}{m}(0 + 0)$

$$= -\dot{\psi}\dot{x}$$

$$\ddot{\psi} = 0.$$

$$\therefore \dot{S} = \begin{bmatrix} \dot{\Delta} \\ -\dot{\psi}\dot{x} \\ \dot{\psi} \\ 0 \\ \dot{x} \\ \dot{\psi}\dot{y} + \frac{1}{m}(f - fmg) \end{bmatrix} \begin{bmatrix} \Delta \\ \psi \\ \dot{\psi} \\ x \\ \dot{x} \end{bmatrix}$$

$$\left.\frac{\partial \dot{S}}{\partial t}\right|_{\text{ateqm}} = \left[\begin{array}{c|c|c|c|c|c} 0 & 1 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & -\dot{x} & 0 & -\dot{\psi} \\ \hline 0 & 0 & 0 & 1 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 1 \\ \hline 0 & \dot{\psi} & 0 & \dot{y} & 0 & 0 \end{array}\right]$$

$$\left.\frac{\partial \dot{s}}{\partial u}\right|_{\text{alt-eqm}} = \begin{array}{c|c} \multicolumn{1}{c}{s} & \multicolumn{1}{c}{F} \\ \hline \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} & \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1/m \end{bmatrix} \end{array}$$

$\therefore$ For $\dot{\chi} < 0.5$, $\dot{s} = As + Bu$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\dot{x} & 0 & -\dot{\psi} \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & \dot{\psi} & 0 & \dot{y} & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1/m \end{bmatrix}$$

For System 2,

$$\dot{S}_2 = \begin{bmatrix} \dot{x} \\ \ddot{x} \end{bmatrix}, \quad u = \begin{bmatrix} \delta \\ F \end{bmatrix}, \quad S_2 = \begin{bmatrix} x \\ \dot{x} \end{bmatrix}$$

At equilibrium, $\dot{x} = 0$, $\ddot{x} = 0$;

$$\dot{S}_2 = \begin{bmatrix} \dot{x} \\ \ddot{\psi}\, j + \frac{1}{m}(F - fmg) \end{bmatrix}$$

$$\left.\frac{\partial \dot{S}_2}{\partial t}\right|_{\text{ateqm}} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$\left.\frac{\partial \dot{S}_2}{\partial t}\right|_{\text{ateqm}} = \begin{matrix} \partial u/\partial \delta & \partial u/\partial f \end{matrix} \begin{bmatrix} 0 & 0 \\ 0 & 1/m \end{bmatrix}$$

$$\dot{S}_2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 1/m \end{bmatrix} \begin{bmatrix} \delta \\ F \end{bmatrix}$$

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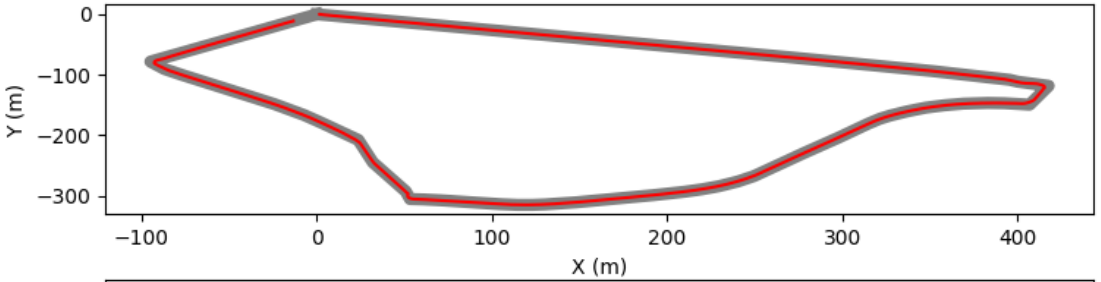
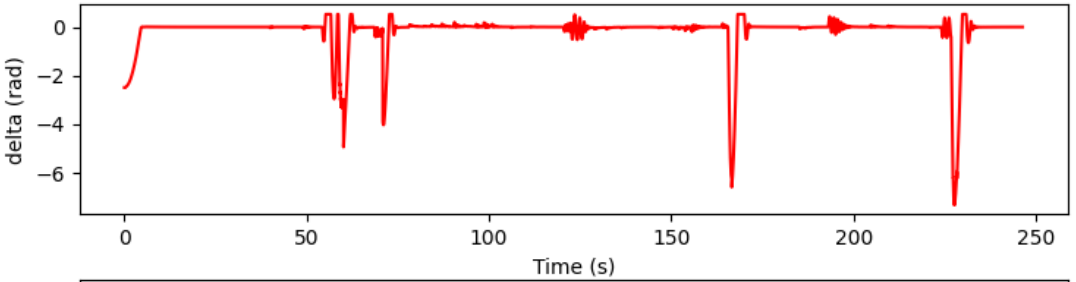
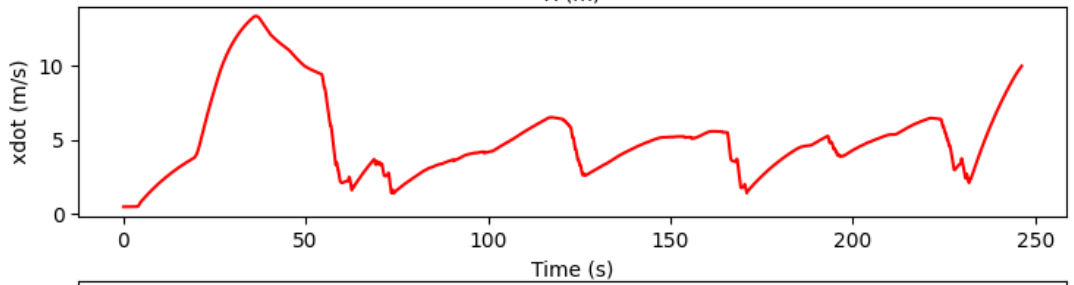
For m = ~~2000~~ 1888.6 kg.

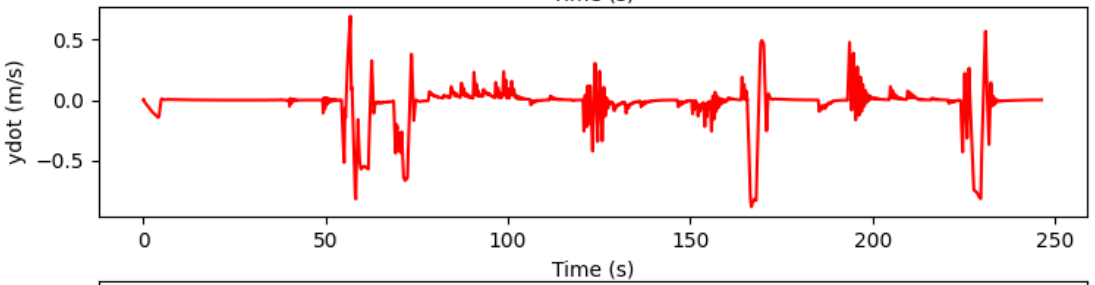
$$\dot{S}_2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 5.29\times10^{-4} \end{bmatrix} \begin{bmatrix} \delta \\ F \end{bmatrix}$$

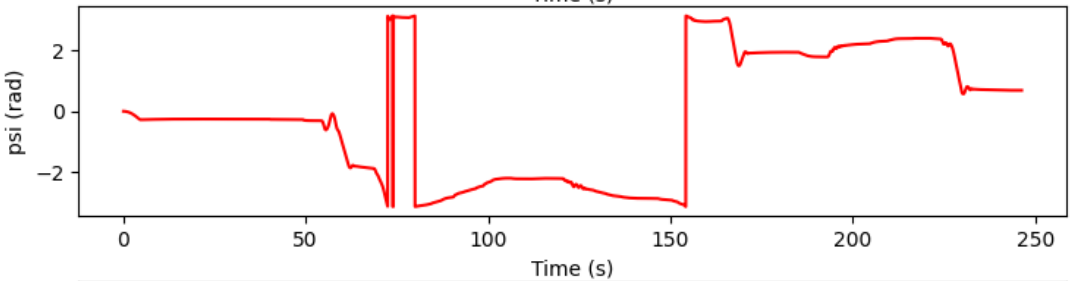
S_2 u

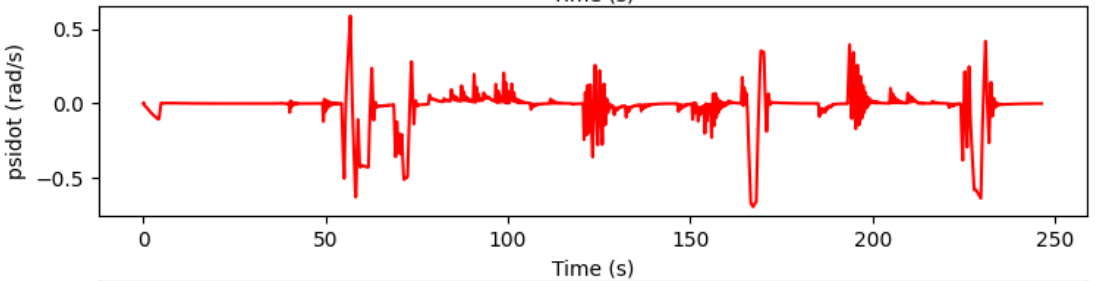

Y (m)
X (m)

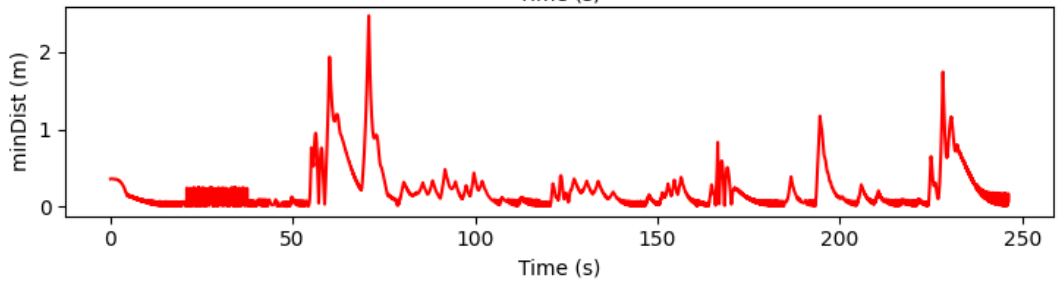

delta (rad)
Time (s)

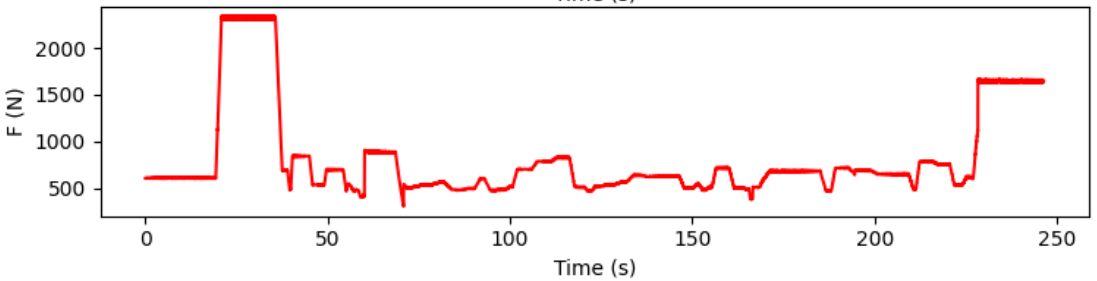

xdot (m/s)
Time (s)


ydot (m/s)
Time (s)


psi (rad)
Time (s)


psidot (rad/s)
Time (s)


minDist (m)
Time (s)


F (N)
Time (s)

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Simulation View

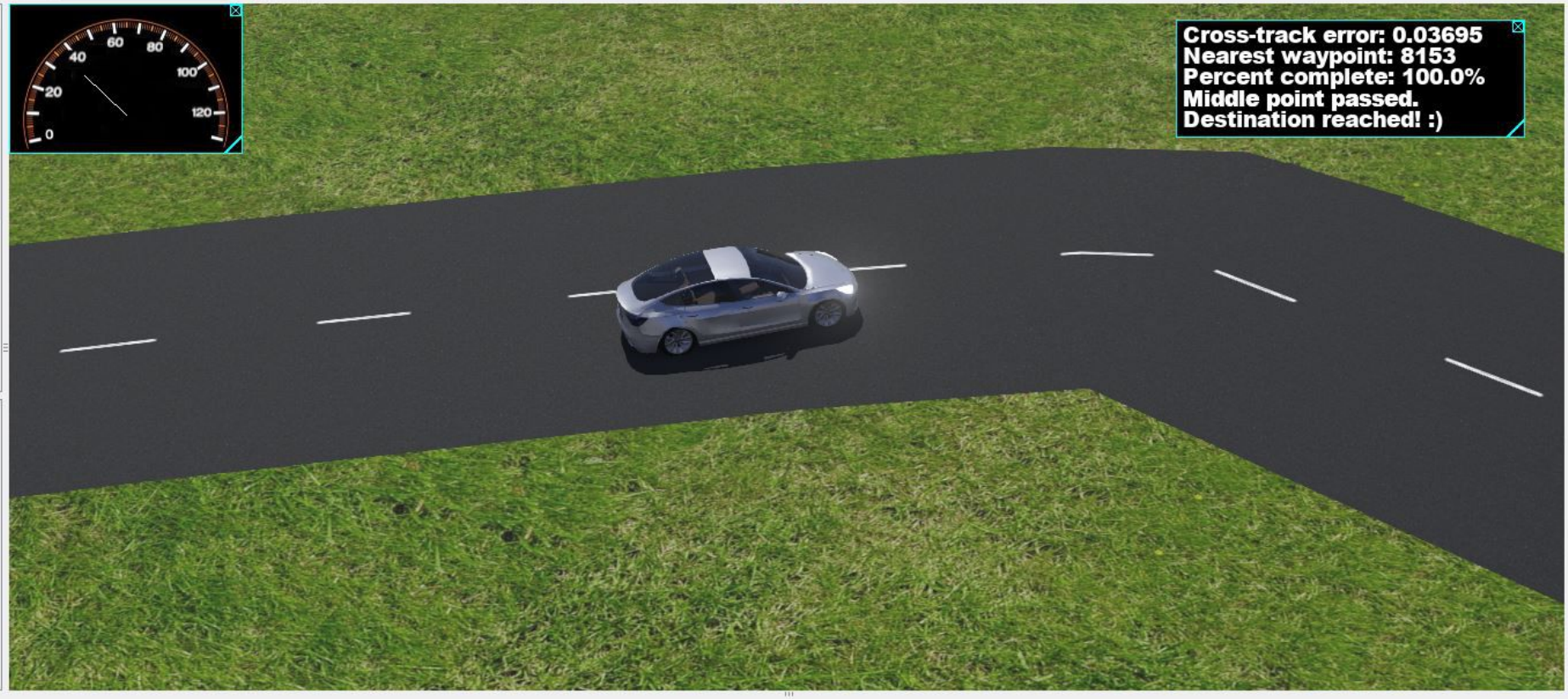
45:25:19:968 - 197x

- WorldInfo
- Viewpoint
- TexturedBackground
- TexturedBackgroundLight
- DEF TESLA Robot
- Floor
- Road
- Slide
- Swing
- Forest
- SimpleBuilding
- SimpleBuilding
- SimpleBuilding
- SimpleBuilding

Selection: Floor (Solid)

Node Position Velocity

DEF:



Console - All

```
avgMinDist:  0.21366685779408418
INFO: 'main' controller exited successfully.
INFO: main: Starting controller: python.exe -u main.py
DEPRECATION: Robot.getDisplay is deprecated, please use Robot.getDevice instead.
DEPRECATION: Robot.getDisplay is deprecated, please use Robot.getDevice instead.
Evaluating...
Score for completing the loop: 30.0/30.0
Score for average distance: 30.0/30.0
Score for maximum distance: 30.0/30.0
Your time is  246.304
Your total score is : 100.0/100.0
total steps:  246304
maxMinDist:  2.4689970053321137
avgMinDist:  0.21366685779408418
```