Exercise 1

Given,
$$\frac{d}{dt}\begin{bmatrix} e_1 \\ \dot{e}_1 \\ e_2 \\ \dot{e}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \frac{-4C_\alpha}{m\dot{x}} & \frac{4C_\alpha}{m} & \frac{2C_\alpha(l_r-l_f)}{m\dot{x}} \\ 0 & 0 & 0 & 1 \\ 0 & \frac{-2C_\alpha(l_f-l_r)}{I_z\dot{x}} & \frac{2C_\alpha(l_f-l_s)}{I_z} & \end{bmatrix}$$

Given
$$\frac{d}{dt}\begin{bmatrix} e_1 \\ \dot{e}_1 \\ e_2 \\ \dot{e}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \frac{-4C_\alpha}{mV} & \frac{4C_\alpha}{m} & \frac{-2C_\alpha(l_f-l_r)}{mV} \\ 0 & 0 & 0 & 1 \\ 0 & \frac{-2C_\alpha(l_f-l_r)}{I_z V} & \frac{2C_\alpha(l_f-l_r)}{I_z} & -\frac{2C_\alpha(l_f^2+l_r^2)}{I_z V} \end{bmatrix}\begin{bmatrix} e_1 \\ \dot{e}_1 \\ e_2 \\ \dot{e}_2 \end{bmatrix}$$

$$+ \begin{bmatrix} 0 \\ \frac{2C_\alpha}{m} \\ 0 \\ \frac{2C_\alpha l_f}{I_z} \end{bmatrix}[\delta]$$

Here $V = \dot{x}$

Provided, $l_r = 1.39\,m$, $l_f = 1.55\,m$, $C_\alpha = 20000$

$I_z = 25854$ $\quad m = 1888.6\,kg$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \frac{-42.359}{V} & 42.359 & \frac{-3.388}{V} \\ 0 & 0 & 0 & 1 \\ 0 & \frac{-0.2475}{V} & 0.2475 & \frac{-6.7}{V} \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 21.1295 \\ 0 \\ 2.398 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$P = [B \quad AB \quad A^2B \quad A^3B)$$

$$= \begin{bmatrix} 0 & 21.179 & -\frac{90527}{v} & \frac{38418.39}{v^2} + 101 \\ 21.179 & -\frac{905.27}{v} & \frac{38418.39}{v^2} + 101.57 & \frac{-5207 - 1628607}{v^3} \\ 0 & 2.398 & -21.3/v & \frac{366}{v^2} + 0.59 \\ 2.398 & \frac{-21.308}{v} & 366/v^2 + 0.59 & \frac{-34.39v^2 - 11966}{v^3} \end{bmatrix}$$

Because of the complexity of calcution, P & Q are calculated in python for velo = [2, 5, 8] m/s

From the code, we can see for all the three velocity the system is both controllable & observable.

Q2. From the Real values of poles vs velocity plot, as the velocity increases we can see that for pole 1-3, the values start from negative to zero hence becoming less stable & for pole 4 the value increases from 0 to high positive value, which also shows the system becoming less stable.

But in the $\log_{10}(\sigma_1/\sigma_n)$ vs velocity plot we can see the log ratio approaches zero at high velocity, which shows the system becoming more & more controllable, as σ_i value increases & provides full rank