Exercise 1

Given, $x_{t+1} = x_t + \delta t\, \dot{x}_t + \omega_t^x$

$y_{t+1} = y_t + \delta t\, \dot{y}_t + \omega_t^y$

$\psi_{t+1} = \psi_t + \delta t\, \dot{\psi}_t + \omega_t^\psi$

& landmarks don't change.

$$\therefore\ x_{t+1} = \begin{bmatrix} x_{t+1} \\ y_{t+1} \\ \psi_{t+1} \\ m_x^1 \\ m_y^1 \\ \vdots \\ m_x^n \\ m_y^n \end{bmatrix} = \begin{bmatrix} x_t \\ y_t \\ \psi_t \\ m_x^1 \\ m_y^1 \\ \vdots \\ m_x^n \\ m_y^n \end{bmatrix} + \begin{bmatrix} \dot{x}_{t+1} \\ \dot{y}_{t+1} \\ \dot{\psi}_{t+1} \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix} \delta t + \begin{bmatrix} \omega_t^x \\ \omega_t^y \\ \omega_t^\psi \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$$\Rightarrow\ x_{t+1} = \begin{bmatrix} x_{t+1} \\ y_{t+1} \\ \vdots \\ m_x^n \\ m_y^n \end{bmatrix} = \begin{bmatrix} x_t \\ y_t \\ \psi_t \\ m_x^1 \\ m_y^1 \\ \vdots \\ m_x^n \\ m_y^n \end{bmatrix} + \delta t \begin{bmatrix} \dot{x}_{t+1}\cos\psi_t - \dot{y}_{t+1}\sin\psi_t \\ \dot{x}_{t+1}\sin\psi_t + \dot{y}_{t+1}\cos\psi_t \\ \dot{\psi}_{t+1} \\ 0 \\ \vdots \\ 0 \end{bmatrix} + \begin{bmatrix} \omega_t^x \\ \omega_t^y \\ \omega_t^\psi \\ 0 \\ 0 \end{bmatrix}$$

This is of the form

$$x_{t+1} = f(x_t, u_{t+1}) + \omega_t$$

$$u_t \cancel{\in} = [\mathring{x}_t \;\; \mathring{y}_t \;\; \mathring{\varphi}_t]^T$$

$$f(x_t, u_{t+1}) = \begin{bmatrix} x_t + (\mathring{x}_{t+1}\cos\varphi_{t+1} \cancel{+} - \mathring{y}_{t+1}\sin\varphi_{t+1})\delta t \\ y_t + (\mathring{x}_{t+1}\cos\varphi_t + \mathring{y}_{t+1}\sin\varphi_t)\delta t \\ \varphi_{t+1} + \mathring{\varphi}\delta t \\ m_x^1 \\ m_y^1 \\ \vdots \\ m_x^n \\ m_y^n \end{bmatrix}$$

From definition of kalman filter

$$F_t = \frac{\partial f}{\partial x}\Big|_{x_t, u_{t+1}} = \begin{bmatrix} 1 & 0 & (-\mathring{x}_{t+1}\sin\varphi_t - \mathring{y}_{t+1}\cos\varphi_t)\delta t & 0_{2n}^T \\ 0 & 1 & (\mathring{x}_{t+1}\cos\varphi_t - \mathring{y}_{t+1}\sin\varphi_t)\delta t & 0_{2n}^T \\ 0 & 0 & 1 & 0_{2n}^T \\ 0_{2n} & 0_{2n} & 0_{2n} & I_{2n} \end{bmatrix}$$

$$0_{2n} = \underset{1\times 2n}{[0 \; 0 \; \cdots \; 0]} \text{ 2n lines}$$

③

$$y_t = h(x_t) + v_t$$

$$h(\bar{x_t}) = \begin{bmatrix} \|m^1 - P_t\| \\ \vdots \\ \|m^n - P_t\| \\ \operatorname{atan2}(m_y^1 - y_t, m_x^1 - x_t) - \varphi_t \\ \vdots \\ \operatorname{atan2}(m_y^n - y_t ; m_x^n - x_t) - \varphi_t \end{bmatrix}$$

$$H_t = \left[\frac{\partial h}{\partial x}\right]_{x_t} = \begin{bmatrix} \frac{\partial h_1}{\partial x} & \frac{\partial h_1}{\partial y} & \frac{\partial h_1}{\partial \varphi} & \frac{\partial h_1}{\partial m_x^1} & \frac{\partial h_1}{\partial m_y^1} & \cdots & \frac{\partial h_1}{\partial m_x^n} & \frac{\partial h_1}{\partial m_y^n} \\ \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\ \frac{\partial h_{2n}}{\partial x} & \frac{\partial h_{2n}}{\partial y} & \frac{\partial h_{2n}}{\partial \varphi} & \frac{\partial h_{2n}}{\partial m_x^1} & \frac{\partial h_{2n}}{\partial m_y^1} & \cdots & \frac{\partial h_{2n}}{\partial m_x^n} & \frac{\partial h_{2n}}{\partial m_y^n} \end{bmatrix}$$

For $h_1 - h_n$,

$$\frac{\partial h_i}{\partial x} = \frac{x - m_x^i}{\sqrt{(x - m_x^i)^2 + (y - m_y^i)^2}} \qquad \frac{\partial h_i}{\partial y} = \frac{y - m_y^i}{\sqrt{(x - m_x^i)^2 + (y - m_y^i)^2}}$$

$$\frac{\partial h_i}{\partial \varphi} = 0; \quad \frac{\partial h_i}{\partial m_x^i} = \frac{m_x^i - x}{\sqrt{(m_x^i - x)^2 + (m_y^i - y)^2}}; \quad \frac{\partial h_i}{\partial m_y^i} = \frac{m_y^i - y}{\sqrt{(m_y^i - y)^2 + (m_x^i - x)^2}}$$

$$\frac{\partial h_i}{\partial m_y^j} = 0; \quad \frac{\partial h_i}{\partial x^j} = 0$$

for $i \neq j$ $\qquad$ $i \neq j$

For $h_{n+1} \rightarrow h_{2n}$).

$$\frac{\partial h_i}{\partial x} = \frac{m_y^i - y}{(m_x^i - x)^2 + (m_y^i - y)^2}; \quad \frac{\partial h_i}{\partial y} = \frac{x - m_x^i}{(m_x^i - x)^2 + (m_y^i - y)^2}$$

$$\frac{\partial h_i}{\partial \varphi} = -1; \quad \frac{\partial h_i}{\partial m_x^i} = \frac{y - m_y^i}{(m_x^i - x)^2 + (m_y^i - y)^2};$$

$$\frac{\partial h_i}{\partial m_y^i} = \frac{m_x^i - x}{(m_x^i - x)^2 + (m_y^i - y)^2}; \quad \frac{\partial h_i}{\partial m_x^{j \neq i}} = 0; \quad \frac{\partial h_i}{\partial m_y^{j \neq i}} = 0$$

$$
H = \begin{bmatrix}
\frac{x-m_x^1}{\sqrt{(x-m_x^1)^2+(y-m_y^1)^2}} & \frac{y-m_y^1}{\sqrt{(x-m_x^1)^2+(y-m_y)^2}} & 0 & \frac{m_x^1-x}{\sqrt{(m_x^1-x)^2+(m_y^1-y)^2}} & \frac{m_y^1-y}{\sqrt{(m_y^1-y)^2+(m_x^1-x)^2}} & 0 \cdots 0 \\
\vdots & \vdots & \vdots & \vdots & \ddots & \ddots \\
\frac{x-m_x^n}{\sqrt{(x-m_x^n)^2+(y-m_y^n)^2}} & \frac{y-m_y^n}{\sqrt{(x-m_x^n)^2+(y-m_y^n)^2}} & 0 \cdots 0 \cdots & & \frac{m_x^n-x}{\sqrt{(m_x^n-x)^2+(m_y^n-y)^2}} & \frac{m_y^n-y}{\sqrt{(m_y^n-y)^2+(m_x^n-x)^2}} \\
\frac{m_y^1-y}{(m_x^1-x)^2+(m_y^1-y)^2} & \frac{x-m_x^1}{(m_x^1-x)^2+(m_y^1-y)^2} & -1 & \frac{y-m_y^1}{(m_x^1-x)^2+(m_y^1-y)^2} & \frac{m_x^1-x}{(m_x^1-x)^2+(m_y^1-y)^2} & 0 \cdots 0 \\
\vdots & & & & \ddots & \ddots \\
\frac{m_y^1-y}{(m_x^1-x)^2+(m_y^1-y)^2} & \frac{x-m_x^n}{(m_x^n-x)^2+(m_y^n-y)^2} & -1 & 0 \cdots 0 & \frac{y-m_y^n}{(x-m_x^n)^2+(y-m_y^n)^2} & \frac{m_x^n-x}{(x-m_x^n)^2+(m_y-y)^2}
\end{bmatrix}
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