PI Problems

Exercise 1.

Given, $S_1 = \begin{array}{c|c} \begin{bmatrix} y \\ \dot{y} \\ \psi \\ \dot{\psi} \end{bmatrix} & \begin{array}{c} f_1 \\ f_2 \\ f_3 \\ f_4. \end{array} \end{array}$

$l_r = 139m$

$l_f = 1.55m$

$C_\alpha = 20000N$

$I_z = 25854 kgm^2$

$f = 0.019$

$\Delta t = 0.032$

$\cancel{f_1 = \dot{y}}$ $\dot{S}_1 = \begin{bmatrix} \dot{y} \\ \ddot{y} \\ \dot{\psi} \\ \ddot{\psi} \end{bmatrix}.$

$$\dot{S}_1 = \begin{bmatrix} \dot{y} \\ -\dot{\psi}\dot{x} + \frac{2C_\alpha}{m}\left(\cos\delta\left(\delta - \frac{\dot{y} + l_f\dot{\psi}}{\dot{x}}\right) - \frac{\dot{y} - l_r\dot{\psi}}{\dot{x}}\right) \\ \dot{\psi} \\ \frac{2l_fC_\alpha}{I_z}\left(\delta - \frac{\dot{y} + l_f\dot{\psi}}{\dot{x}}\right) - \frac{2l_rC_\alpha}{I_z}\left(\frac{-\dot{y} - l_r\dot{\psi}}{\dot{x}}\right) \end{bmatrix}$$

At equlibrium pt,

$$\dot{y} = 0$$

$$\ddot{y} = 0 \Rightarrow -\dot{\psi}\dot{x} + \frac{2bC_\alpha}{m}\cos\delta\left(\delta - \frac{\dot{y}+l_f\dot{\psi}}{\dot{x}}\right) - \frac{\dot{y} - l_r\dot{\psi}}{\dot{x}} = 0$$

$$\dot{\psi} = 0 \Rightarrow \frac{2bC_\alpha}{m}\delta\cos\delta = 0 \Rightarrow \delta = 0$$

(next relation)

$$\ddot{\psi} = 0 \Rightarrow \frac{2l_f C_\alpha}{I_z}\delta = 0 \Rightarrow \delta = 0$$

$$\left.\frac{\partial \dot{s}_1}{\partial t}\right|_{at\ Eq} = \begin{array}{c} \begin{array}{cccc} y & \dot{y} & \psi & \dot{\psi} \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \frac{-4C_\alpha}{m\dot{x}} & 0 & \frac{2C_\alpha(l_r-l_f)}{m\dot{x}} - \dot{x} \\ 0 & 0 & 0 & 1 \\ 0 & \frac{2C_\alpha(l_r-l_f)}{I_z\dot{x}} & 0 & \frac{-2C_\alpha(l_r^2+l_f^2)}{I_z\dot{x}} \end{bmatrix} \end{array}$$

$$\left.\frac{\partial \ddot{y}}{\partial \delta}\right|_{eqm} = \frac{-2C_\alpha(\cos\delta + 1)}{m\dot{x}} = \frac{-4C_\alpha}{m\dot{x}}$$

$$\left.\frac{\partial \ddot{y}}{\partial \delta}\right|_{\text{ategm}} = \frac{2\,C_a(\cos\delta - \delta\sin\delta)}{m} + \frac{2C_a\,(\dot{y}+l_f\dot{\psi})}{\dot{x}} \times \sin\delta$$

$$= \frac{2C_a}{m} + \cancel{0}$$

$$\left.\frac{\partial \ddot{y}}{\partial \dot{\psi}}\right|_{\text{ategm}} = -\dot{x} + \frac{2C_a\,(l_r - l_f\cos\delta)}{m\dot{x}}$$

$$= \boxed{\frac{2C_a(l_r - l_f)}{m\dot{x}} - \dot{x}}$$

$$\left.\frac{\partial \ddot{\psi}}{\partial \dot{\psi}}\right|_{\text{ategm.}} = -\frac{2l_f C_a}{I_z}\frac{l_f}{\dot{x}} + \frac{2l_r C_a}{I_z}\left(\frac{-l_r}{\dot{x}}\right)$$

$$= \frac{-2C_a}{I_z\dot{x}}\left(l_r^2 + l_f^2\right)$$

$$\frac{\partial \ddot{\psi}}{\partial \dot{y}} = \frac{2l_f C_a}{\dot{x}_0 I_z}(-1) + \frac{2l_r C_a}{I_z\dot{x}}(1).$$

$$= \frac{2C_a}{I_z\dot{x}}(l_r - l_f).$$

$$\partial\ddot{\psi}/\partial\delta = 2l_f C_a/I_z$$

$$\left.\frac{\partial u}{\partial f}\right|_{at\ Eq} = \begin{bmatrix} \frac{\partial \dot{y}}{\partial \delta} & \frac{\partial \dot{y}}{\partial f} \\ \frac{\partial \ddot{y}}{\partial \delta} & \frac{\partial \ddot{y}}{\partial f} \\ \frac{\partial \dot{\psi}}{\partial \delta} & \frac{\partial \dot{\psi}}{\partial f} \\ \frac{\partial \ddot{\psi}}{\partial \delta} & \frac{\partial \ddot{\psi}}{\partial f} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 2C_\alpha/m & 0 \\ 0 & 0 \\ 2l_f C_\alpha/I_z & 0 \end{bmatrix}$$

$$\dot{S}_1 = \begin{bmatrix} \dot{y} \\ \ddot{y} \\ \dot{\psi} \\ \ddot{\psi} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \frac{-4C_\alpha}{m\dot{x}} & 0 & \frac{2C_\alpha(l_r-l_f)}{m\dot{x}} \\ 0 & 0 & 0 & 1 \\ 0 & \frac{2C_\alpha(l_r-l_f)}{I_z\dot{x}} & 0 & \frac{-2C_\alpha(l_r^2+l_f^2)}{I_z\dot{x}} \end{bmatrix} S_1 + \begin{bmatrix} 0 & 0 \\ 2C_\alpha/m & 0 \\ 0 & 0 \\ \frac{2l_f C_\alpha}{I_z} & 0 \end{bmatrix} u \qquad (1)$$

$$\therefore \quad \dot{s}_1 = A_1 s_1 + B_1 u.$$

$$\cancel{\dot{s}_1 = A_1}$$

$$A_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \dfrac{-4C_\alpha}{m\dot{x}} & 0 & \dfrac{2C_\alpha(l_r - l_f)}{m\dot{x}} - \dot{x} \\ 0 & 0 & 0 & 1 \\ 0 & \dfrac{2C_\alpha(l_r - l_f)}{I_z \dot{x}} & 0 & \dfrac{-2C_\alpha(l_r^2 + l_f^2)}{I_z \dot{x}} \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 0 & 0 \\ 2C_\alpha/m & 0 \\ 0 & 0 \\ \dfrac{2C_\alpha l_f}{I_z} & 0 \end{bmatrix}$$

for $x < 0.5$, $y = -\dot{\psi}\dot{x} + \dfrac{2C_\alpha}{m}(0 + 0)$.

$$= -\dot{\psi}\dot{x}$$

$$\ddot{\psi} = 0.$$

$$\therefore \quad \dot{S} = \begin{bmatrix} \dot{y} \\ -\dot{\psi}\dot{x} \\ \dot{\psi} \\ 0 \\ \dot{x} \\ \dot{\psi}\dot{y} + \frac{1}{m}(f - fmg) \end{bmatrix} \quad \cancel{\begin{bmatrix} y \\ \dot{y} \\ \psi \\ \dot{\psi} \\ x \\ \dot{x} \end{bmatrix}}$$

$$\left.\frac{\partial \dot{S}}{\partial t}\right|_{\text{at eqm}} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\dot{x} & 0 & -\dot{\psi} \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & \dot{\psi} & 0 & \dot{y} & 0 & 0 \end{bmatrix}$$

$$\left.\frac{\partial \dot{s}}{\partial u}\right|_{at\ eqm} = \begin{array}{cc} \delta & F \end{array}$$
$$\left.\frac{\partial \dot{s}}{\partial u}\right|_{at\ eqm} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1/m \end{bmatrix}$$

For $\dot{x} < 0.5$, $\dot{s} = As + Bu$.

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\dot{x} & 0 & -\dot{\psi} \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & \dot{\psi} & 0 & \dot{y} & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1/m \end{bmatrix}$$

For system 2,

$$\overset{\circ}{S_2} = \begin{bmatrix} \dot{x} \\ \ddot{x} \end{bmatrix}, \quad u = \begin{bmatrix} \delta \\ F \end{bmatrix}, \quad S_2 = \begin{bmatrix} x \\ \dot{x} \end{bmatrix}$$

At equilibrium,

$$\dot{x} = 0, \quad \ddot{x} = 0; \quad \overset{\circ}{S_2} = \begin{bmatrix} \dot{x} \\ \dot{\varphi}\dot{y} + \frac{1}{m}(F - fmg) \end{bmatrix}$$

$$\left.\frac{\partial \overset{\circ}{S_2}}{\partial t}\right|_{at\ eqm} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}.$$

$$\left.\frac{\partial \overset{\circ}{S_2} u}{\partial t}\right|_{at\ eqm} = \begin{array}{cc} \partial u/\partial \delta & \partial u/\partial f \\ \end{array} \begin{bmatrix} 0 & 0 \\ 0 & 1/m \end{bmatrix}.$$

$$\dot{S}_2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 1/m \end{bmatrix} \begin{bmatrix} \delta \\ F \end{bmatrix}$$

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For $m =$ ~~20000~~ 1888.6 kg.

$$\dot{S}_2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \underbrace{\begin{bmatrix} x \\ \dot{x} \end{bmatrix}}_{S_2} + \begin{bmatrix} 0 & 0 \\ 0 & 5.29 \times 10^{-4} \end{bmatrix} \underbrace{\begin{bmatrix} \delta \\ F \end{bmatrix}}_{u}$$