P1 Problems

Exercise 1

Given,

$$
\begin{array}{cc}
S_1 = \begin{bmatrix} y \\ \dot{y} \\ \psi \\ \dot{\psi} \end{bmatrix} & \begin{array}{c} f_1 \\ f_2 \\ f_3 \\ f_4 \end{array}
\end{array}
$$

$l_r = 1.39m$

$l_f = 1.55m$

$C_\alpha = 20000N$

$I_z = 25854 kgm^2$

$\rho = 0.019$

$\Delta t = 0.032$

$$
\cancel{f_1 = \dot{y}} \quad \dot{S}_1 = \begin{bmatrix} \dot{y} \\ \ddot{y} \\ \dot{\psi} \\ \ddot{\psi} \end{bmatrix}.
$$

$$
\dot{S}_1 = \begin{bmatrix} \dot{y} \\ -\dot{\psi}\dot{x} + \frac{2C_\alpha}{m}\left(\cos\delta\left(\delta - \frac{\dot{y} + l_f\dot{\psi}}{\dot{x}}\right) - \frac{\dot{y} - l_r\dot{\psi}}{\dot{x}}\right) \\ \dot{\psi} \\ \frac{2l_fC_\alpha}{I_z}\left(\delta - \frac{\dot{y} + l_f\dot{\psi}}{\dot{x}}\right) - \frac{2l_rC_\alpha}{I_z}\left(\frac{-\dot{y} - l_r\dot{\psi}}{\dot{x}}\right) \end{bmatrix}
$$

At equilibrium pt,

$$y = 0$$

$$\dot{y} = 0 \Rightarrow -\psi \dot{x} + \frac{2bC_\alpha}{m}\cos\delta\left(\delta - \frac{\dot{y}+l_f\dot{\psi}}{\dot{x}}\right) - \frac{\dot{y}-l_r\dot{\psi}}{\dot{x}} = 0$$

$$\dot{\psi} = 0 \Rightarrow \frac{2aC_\alpha}{m}\delta\cos\delta = 0 \Rightarrow \delta = 0$$

(next relation)

$$\ddot{\psi} = 0 \Rightarrow \frac{2l_f C_\alpha}{I_z}\delta = 0 \Rightarrow \delta = 0$$

$$\left.\frac{\partial \dot{s}_i}{\partial t}\right|_{\text{at Eq}} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \frac{-4C_\alpha}{m\dot{x}} & 0 & \frac{2C_\alpha(l_r-l_f)}{m\dot{x}} - \dot{x} \\ 0 & 0 & 0 & 1 \\ 0 & \frac{2C_\alpha(l_r-l_f)}{I_z\dot{x}} & 0 & \frac{-2C_\alpha(l_r^2+l_f^2)}{I_z\dot{x}} \end{bmatrix}$$

$$\left.\frac{\partial \dot{y}}{\partial \delta}\right|_{\text{eqm}} = \frac{-2C_\alpha(\cos\delta+1)}{m\dot{x}} = \frac{-4C_\alpha}{m\dot{x}}$$

$$\left.\frac{\partial \ddot{y}}{\partial \delta}\right|_{ateqm} = \frac{2C_a(\cos\delta - \delta\sin\delta)}{m} + \frac{2C_a(\dot{y} + l_f\dot{\psi})}{\dot{x}} \times \sin\delta$$

$$= \frac{2C_a}{m} + \cancel{0}$$

$$\left.\frac{\partial \ddot{y}}{\partial \dot{\psi}}\right|_{ateqm} = -\dot{x} + \frac{2C_a(l_r - l_f\cos\delta)}{m\dot{x}}$$

$$= \boxed{\frac{2C_a(l_r - l_f)}{m\dot{x}} - \dot{x}}$$

$$\left.\frac{\partial \ddot{\psi}}{\partial \dot{\psi}}\right|_{ateqm} = -\frac{2l_fC_a}{I_z}\frac{l_f}{\dot{x}} + \frac{2l_rC_a}{I_z}\left(-\frac{l_r}{\dot{x}}\right)$$

$$= \frac{-2C_a}{I_z\dot{x}}(l_r^2 + l_f^2)$$

$$\frac{\partial \ddot{\psi}}{\partial \dot{y}} = \frac{2l_fC_a}{\dot{x}I_z}(-1) + \frac{2l_rC_a}{I_z\dot{x}}(1).$$

$$= \frac{2C_a}{I_z\dot{x}}(l_r - l_f).$$

$$\partial\ddot{\psi}/\partial\delta = 2l_fC_a/I_z$$

$$\left.\frac{\partial u}{\partial f}\right|_{\text{at Eq}} = \begin{bmatrix} \frac{\partial \dot{y}}{\partial \delta} & \frac{\partial \dot{y}}{\partial f} \\ \frac{\partial \ddot{y}}{\partial \delta} & \frac{\partial \ddot{y}}{\partial f} \\ \frac{\partial \dot{\psi}}{\partial \delta} & \frac{\partial \dot{\psi}}{\partial f} \\ \frac{\partial \ddot{\psi}}{\partial \delta} & \frac{\partial \ddot{\psi}}{\partial f} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 2C_\alpha/M & 0 \\ 0 & 0 \\ 2l_f C_\alpha/I_z & 0 \end{bmatrix}$$

$$\dot{S}_1 = \begin{bmatrix} \dot{y} \\ \ddot{y} \\ \dot{\psi} \\ \ddot{\psi} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \frac{-4C_\alpha}{m\dot{x}} & 0 & \frac{2C_\alpha(l_r-l_f)}{m\dot{x}} \\ 0 & 0 & 0 & 1 \\ 0 & \frac{2C_\alpha(l_r-l_f)}{I_z\dot{x}} & 0 & \frac{-2C_\alpha(l_r^2+l_f^2)}{I_z\dot{x}} \end{bmatrix} S_1 + \begin{bmatrix} 0 & 0 \\ 2C_\alpha/M & 0 \\ 0 & 0 \\ \frac{2l_f C_\alpha}{I_z} & 0 \end{bmatrix} u. \tag{1}$$

$$\therefore \quad \dot{S}_1 = A_1 S_1 + B_1 u$$

~~$\dot{S}_1 = A_1$~~

$$A_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \frac{-4C_\alpha}{m\dot{x}} & 0 & \frac{2C_\alpha(l_r - l_f)}{m\dot{x}} - \dot{x} \\ 0 & 0 & 0 & 1 \\ 0 & \frac{2C_\alpha(l_r - l_f)}{I_z \dot{x}} & 0 & \frac{-2C_\alpha(l_r^2 + l_f^2)}{I_z \dot{x}} \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 0 & 0 \\ 2C_\alpha/m & 0 \\ 0 & 0 \\ \frac{2C_\alpha l_f}{I_z} & 0 \end{bmatrix}$$

For system 2,

$$\dot{S_2} = \begin{bmatrix} \dot{x} \\ \ddot{x} \end{bmatrix}, \quad u = \begin{bmatrix} \delta \\ F \end{bmatrix}, \quad S_2 = \begin{bmatrix} x \\ \dot{x} \end{bmatrix}$$

At equilibrium,

$$\dot{x} = 0, \quad \ddot{x} = 0 ; \quad \dot{S_2} = \begin{bmatrix} \dot{x} \\ \dot{\varphi}\dot{y} + \frac{1}{m}(F - fmg) \end{bmatrix}$$

$$\left.\frac{\partial \dot{S_2}}{\partial t}\right|_{at\,eqm} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$\left.\frac{\partial \dot{u}}{\partial t}\right|_{at\,eqm} = \begin{array}{c} \begin{array}{cc} \partial u/\partial \delta & \partial u/\partial f \end{array} \\ \begin{bmatrix} 0 & 0 \\ 0 & 1/m \end{bmatrix} \end{array}$$

$$\dot{S}_2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 1/m \end{bmatrix} \begin{bmatrix} \delta \\ F \end{bmatrix} \quad (2)$$

For $m =$ ~~20000~~ 1888.6 kg.

$$\dot{S}_2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \underbrace{\begin{bmatrix} x \\ \dot{x} \end{bmatrix}}_{S_2} + \begin{bmatrix} 0 & 0 \\ 0 & 5.29\times10^{-4} \end{bmatrix} \underbrace{\begin{bmatrix} \delta \\ F \end{bmatrix}}_{u}$$