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GOVERNMENT OF TAMIL NADU

HIGHER SECONDARY FIRST YEAR

PHYSICS

VOLUME - I

A publication under Free Textbook Programme of Government of Tamil Nadu

Department of School Education

Untouchability is Inhuman and a Crime

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Government of Tamil Nadu

First Edition - 2018
Revised Edition - 2019, 2020, 2022
(Published Under New Syllabus)

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Content Creation

State Council of Educational
Research and Training
© SCERT 2018

Printing & Publishing

Tamil NaduTextbook and Educational
Services Corporation

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Assessment

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HOW TO USE THE BOOK

Scope of Physics	• Awareness on higher learning -courses, institutions and required competitive exams • Financial assistance possible to help students to climb academic ladder
Learning Objectives:	• Overview of the unit • Gives clarity on the goals and intention of the topics
DO YOU KNOW?	• Additional facts related to the topics covered to facilitate curiosity driven learning
Example problems	• To ensure understanding, problems/illustrations are given at every stage before advancing to next level
	• Visual representation of concepts with illustrations • Videos, animations, and tutorials
ICT	• To harness the digital skills to class room learning and experimenting
Summary	• Recap of salient points of the lesson
Concept Map	• Schematic outline of salient learning of the unit
Evaluation	• Evaluate students' understanding and get them acquainted with the application of physical concepts to numerical and conceptual questions
Books for Reference	• List of relevant books for further reading
Solved examples	• Numerical/conceptual questions are solved to enable students to tackle standard problems in mechanics in appendix 1
Competitive Exam corner	• Model Questions - To motivate students aspiring to take up competitive examinations such as NEET, JEE, Physics Olympiad, JIPMER etc
Appendix	• Additional information including the chronological development of physics is provided
Glossary	• Scientific terms frequently used with their Tamil equivalents

Physics learning - Correct method

- The correct way to learn is to understand the concept, express the same in the language of mathematics which are equations.
- Each equation conveys the meaning of a phenomena or relationship between various parameters in the equations. Such relationship can be diagrammatically expressed as graphs.
- This interlink should be clear in mind while going through the entire text.

Value addition of the book

- Mathematical topics such as vectors, differentiation and integration are essential to understand and express physical phenomena.
- Inclusion of these topics and usage of vector notation wherever necessary is the salient feature of this book.
- Becoming familiar with vector notations and basic mathematics for physics will solve a lot of difficulty currently faced by students pursuing higher education in engineering, technology and science disciplines.

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Scope of Physics - Higher Education

Exams

- JEE-Joint Entrance Examination
- Physics Olympiad Exam
- NEET- National Eligibility and Entrance Test
- NEST- National Entrance Screening Test
- AIEEE- All India Engineering Entrance Exam
- AIIMS- All India Institute of Medical Science (Entrance Examination)
- JIPMER- Jawaharlal institute of Postgraduate Medical Education and Research (Entrance Examination)
- KVPY- Kishore Vaigyanik Protsahan Yojana
- JAM- Joint Admission Test
- TIFR GS - Tata Institute of Fundamental Research Graduate School Admissions Examination
- JEST- Joint Entrance Screening Test
- NET- National Eligibility Test (CSIR and UGC)
- GATE-Graduate Aptitude Test in Engineering
- ICAR -AIEEA-Indian Council of Agricultural Research All India Entrance Examination

After completing +2

- B.Sc (Physics)
- Integrated M.Sc (Physics) (Central Universities)
- Integrated M.Sc (in Central Research Institutes through NEST and KVPY with stipend)
- B.Sc./B.S./B.Stat./B.Math./M.S. in Mathematics, Chemistry and Biology. (KVPY)
- B.E/B.Tech/ B.Arch (JEE, AIEEE in IITs and NITs) MBBS/ B.D.S/B.Pharm (NEET, JIPMER, AIIMS)
- B.Sc. (Agriculture) (ICAR -AIEEA)
- Dual Degree Program BS & MS (JEE, JEST in IITs and IISERs)
- B.Sc (Hospitality administration)
- B.Sc (Optoelectronics)
- B.Sc (Optometry)
- B.Tech (Optics and Optoelectronics)

After completing undergraduate course in Physics (B.Sc Physics)

- M.Sc. (Physics) (In Central and State Universities and Colleges)
- M.Sc. Physics (JAM in IISc ,IITs and NITs)
- M.Sc. (In State and Central Universities)
- Medical Physics
- Materials Science
- Energy
- Earth Sciences
- Space science
- Oceanography
- Remote sensing
- Electronics
- Photonics
- Optoelectronics
- Acoustics
- Applied electronics
- Astronomy and Astrophysics
- Nanoscience and Nanotechnology
- Biostatistics
- Bio informatics
- Vacuum sciences

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Opportunities after B.Sc. Physics

Jobs in Government Sector

- Indian Forest Services
- Scientist Job in ISRO, DRDO, CSIR labs
- Union Public Service Commission
- Staff selection commission
- Indian Defence services etc.
- Public sector Bank
- State PCS
- Grade III & Compiler Post
- Tax Assistant
- Statistical Investigator
- Combined Higher Secondary
- Combined Graduate Level Exam etc.

Financial assistance to pursue higher education

Scholarships for graduate and post graduate courses

- International Olympiad: for getting stipend for Higher Education in Science and Mathematics
- DST – INSPIRE Scholarships (for UG and PG)
- DST – INSPIRE Fellowships (for Ph.D)
- UGC National Fellowship (for Ph.D)
- Indira Gandhi Fellowship for Single girl child (for UG and PG)
- Moulana Azad Fellowship for minorities (for Ph.D)
- In addition various fellowships for SC/ST/PWD, OBC etc are available.
- Visit website of University Grants Commission (UGC) and Department of Science and Technology (DST)

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Institutes in india to pursue research in physics

After completing Post Graduate course in Physics (M.Sc Physics)

Topics of Research

- Quantum Physics and Quantum Optics
- Astrophysics, Astronomy
- String theory, Quantum gravity
- Mathematical Physics, Statistical Mechanics
- Quantum Field Theory
- Particle Physics and Quantum Thermodynamics
- Quantum information theory
- Condensed Matter Physics, Materials Science
- Electro magnetic Theory
- Black Holes, Cosmology
- Crystal Growth, Crystallography
- Spectroscopy, Atomic, Molecular and Optical Physics
- Nano Science and Nanotechnology
- Energy and Environment Studies
- Biophysics, Medical Physics
- Cryptography, Spintronics
- Optics and Photonics
- Meteorology and Atmospheric Science

Research Institutions in various areas of science

Name of the Institution	Website
Indian Institute of Science (IISc) Bangalore	www.iisc.ac.in
Raman Research Institute (RRI) Bangalore	www.rri.res.in
Institute of Mathematical Sciences (IMSc) Chennai	www.imsc.res.in
Indian Association for Cultivation of Science (IACS) Calcutta	www.iacs.res.in
Chennai Mathematical Institute (CMI) Chennai	www.cmi.ac.in
Tata Institute of Fundamental Research (TIFR) Mumbai	www.tifr.res.in
Bhaba Atomic Research centre (BARC) Mumbai	www.barc.gov.in
SN Bose centre Basic Natural science Calcutta	www.bose.res.in
Indian Institute of Space Science and Technology (IIST) Trivandrum	www.iist.ac.in
Physics Research Laboratory (PRL) Ahmedabad	www.prl.res.in
Indian Institute of Astrophysics (IIA) Bangalore	www.iiap.res.in
Institute of Physics (IOP) Bhubaneswar	www.iopb.res.in
Institute for Plasma Research (IPR) Gujarat	www.ipr.res.in
Inter university centre for Astronomy and Astrophysics (IUCAA) Pune	www.iucaa.in
Indira Gandhi centre for Atomic Research (IGCAR), Kalpakkam	www.igcar.gov.in
Hyderabad central university, Hyderabad	www.uohyd.ac.in
Delhi University, Delhi	www.du.ac.in
Mumbai University, Mumbai	www.mu.ac.in
SavithiribaiPhule Pune university, Pune	www.unipune.ac.in
National Institute of Science Education and Research (NISER), Bhubaneshwar	www.niser.ac.in
IISER Educational Institutions	www.iiseradmission.in
Indian Institute of Technology in various places (IIT's)	www.iitm.ac.in
National Institute of Technology (NITs)	www.nitt.edu
Jawaharlal Nehru University (JNU)	www.jnu.ac.in
Central Universities	www.ugc.ac.in
State Universities	https://www.ugc.ac.in
CSIR – Academy (National laboratories, Delhi, Hyderabad, Trivandrum, Chennai, Calcutta etc)	

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UNIT 1

NATURE OF PHYSICAL WORLD AND MEASUREMENT

"Education is not the learning of facts, but the training of the mind to think" – **Albert Einstein**

LEARNING OBJECTIVES

In this unit, the student is exposed to

- excitement generated by the discoveries in Physics
- an understanding of physical quantities of importance
- different system of units
- an understanding of errors and corrections in physics measurements
- the importance of significant figures
- usage of dimensions to check the homogeneity of physical quantities

KEZWC

1.1 SCIENCE—INTRODUCTION

The word **'science'** has its root in the Latin verb ***scientia***, meaning ***"to know"***. In Tamil language, it is **'அறிவியல்'** (Ariviyal) meaning **'knowing the truth'.** The human mind is always curious to know and understand different phenomena like the bright celestial objects in nature, cyclic changes in the seasons, occurrence of rainbow, etc. The inquisitive mind looks for meaningful patterns and relations in such phenomena. Today's modern science and technology is an offshoot of the understanding of nature. Science is the systematic organization of knowledge gained through observation, experimentation and logical reasoning. The knowledge of science dealing with non-living things is physical science (Physics and Chemistry), and that dealing with living things is biological science (Botany, Zoology etc.).

Curiosity-driven observations of natural happenings was the origin of science. The word **'science'** was coined only in the 19th century. Natural philosophy was the earlier name given to science, when ancient civilization knew and practised astronomy, chemistry, human physiology and agriculture. Oral communication was the mode of conveying knowledge when writing systems were not yet developed. One of the oldest forerunners of scientific advancements, from astronomy to medicine, were the Egyptians. Scientific and mathematical excellence in India dates back to prehistoric human activity in the Indus Valley Civilization (3300 – 1300 BC(BCE).

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According to part IV Article 51A (h) of Indian Constitution *"It shall be the duty of every citizen of India to develop scientific temper, humanism and spirit of inquiry and reform"*. This is the aim of our Science Education.

1.1.1 The Scientific Method

The scientific method is a step-by-step approach in studying natural phenomena and establishing laws which govern these phenomena. Any scientific method involves the following general features.

(i) Systematic observation
(ii) Controlled experimentation
(iii) Qualitative and quantitative reasoning
(iv) Mathematical modeling
(v) Prediction and verification or falsification of theories

Example

Consider a metalic rod being heated. When one end of the rod is heated, heat is felt at the other end. The following questions can be asked on this observation

a) What happens within the rod when it is heated?
b) How does the heat reach the other end?
c) Is this effect true for all materials?
d) If heat flows through the material, is it possible to visualize heat?

The process of finding the answers to these queries is scientific investigation.

The basic phenomenon of heat is discussed in unit 8.

DO YOU KNOW?

The name Physics was introduced by Aristotle in the year 350 BC

1.2 PHYSICS - INTRODUCTION

The word 'physics' is derived from the Greek word ***"Fusis"***, meaning nature. The study of nature and natural phenomena is dealt within physics. Hence physics is considered as the most basic of all sciences.

Unification and ***Reductionism*** are the two approaches in studying physics. Attempting to explain diverse physical phenomena with a few concepts and laws is unification. For example, Newton's universal law of gravitation (in unit 6) explains the motion of freely falling bodies towards the Earth, motion of planets around the Sun, motion of the Moon around the Earth, thus unifying the fundamental forces of nature.

An attempt to explain a macroscopic system in terms of its microscopic constituents is ***reductionism***. For example, thermodynamics (unit 8) was developed to explain macroscopic properties like temperature, entropy, etc., of bulk systems. The above properties have been interpreted in terms of the molecular constituents (microscopic) of the bulk system by kinetic theory (unit 9) and statistical mechanics.

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1.2.1 Branches of Physics

Physics as a fundamental science helps to uncover the laws of nature. The language of its expression is mathematics. In ancient times, humans lived with nature – their lifestyles were integrated with nature. They could understand the signals from the movement of the stars and other celestial bodies. They could determine the time to sow and reap by watching the sky. Thus, astronomy and mathematics were the first disciplines to be developed. The chronological development of various branches of physics is presented in Appendix A1.1. The various branches of physics are schematically shown in figure 1.1. The essential focus of different areas is given in Table 1.1.

Some of the fundamental concepts of basic areas of physics are discussed in higher secondary first year physics books volume 1 and 2. Mechanics is covered in unit 1 to 6. Unit 1 gives an idea of the development of physics along with discussion on basic elements such as measurement, units etc. Unit 2 gives the basic mathematics needed to express the impact of physical principles and their governing laws. The impact of forces acting on objects in terms of the fundamental laws of motion of Newton are very systematically covered in unit 3. Work and energy which are the basic parameters of investigation of the mechanical world are presented in unit 4. Unit 5 deals with the mechanics of rigid bodies (in contrast, objects are viewed as point objects in units

Figure 1.1 Branches of Physics

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Table 1.1 Branches of Physics

Classical Physics		**Refers to traditional physics that was recognized and developed before the beginning of the 20th century**
	Branch	**Major focus**
1.	Classical mechanics	The study of forces acting on bodies whether at rest or in motion
2.	Thermodynamics	The study of the relationship between heat and other forms of energy
3.	Optics	The study of light
4.	Electricity and magnetism	The study of electricity and magnetism and their mutual relationship
5.	Acoustics	The study of the production and propagation of sound waves
6.	Astrophysics	The branch of physics which deals with the study of the physics of astronomical bodies
7.	Relativity	One of the branches of theoretical physics which deals with the relationship between space, time and energy particularly with respect to objects moving in different ways .
Modern Physics		**Refers to the concepts in physics that have surfaced since the beginning of the 20th century.**
1.	*Quantum mechanics	The study of the discrete nature of phenomena at the atomic and subatomic levels
2.	Atomic physics	The branch of physics which deals with the structure and properties of the atom
3.	Nuclear physics	The branch of physics which deals with the structure, properties and reaction of the nuclei of atoms.
4.	Condensed matter physics	The study of the properties of condensed materials (solids, liquids and those intermediate between them and dense gas). It branches into various sub-divisions including developing fields such as nano science, photonics etc. It covers the basics of materials science, which aims at developing new material with better properties for promising applications.
5.	High energy physics	The study of the nature of the particles.

*Quantum mechanics is a broader approach; classical results can be reproduced in quantum mechanics also. Detailed explanation is beyond the scope of this book.

3 and 4). The basics of gravitation and its consequences are discussed in unit 6. Older branches of physics such as different properties of matter are discussed in unit 7. The impact of heat and investigations of its consequences are covered in units 8 and 9. Important features of oscillations and wave motion are covered in units 10 and 11.

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1.2.2 Scope and Excitement of Physics

Discoveries in physics are of two types; accidental discoveries and well-analysed research outcome in the laboratory based on intuitive thinking and prediction. For example, magnetism was accidentally observed but the reason for this strange behavior of magnets was later analysed theoretically. This analysis revealed the underlying phenomena of magnetism. With this knowledge, artificial magnets were prepared in the laboratories. Theoretical predictions are the most important contribution of physics to the developments in technology and medicine. For example, the famous equation of Albert Einstein, $E=mc^2$ was a theoretical prediction in 1905 and experimentally proved in 1932 by Cockcroft and Walton. Theoretical predictions aided with recent simulation and computation procedures are widely used to identify the most suited materials for robust applications. The pharmaceutical industry uses this technique very effectively to design new drugs. Biocompatible materials for organ replacement are predicted using quantum prescriptions of physics before fabrication. Thus, experiments and theory work hand in hand complimenting one another.

Physics has a huge scope as it covers a tremendous range of magnitude of various physical quantities (length, mass, time, energy etc). It deals with systems of very large magnitude as in astronomical phenomena as well as those with very small magnitude involving electrons and protons.

- Range of time scales: astronomical scales to microscopic scales, 10^{18}s to 10^{-22}s.
- Range of masses: from heavenly bodies to electron, 10^{55} kg (mass of known observable universe) to 10^{-31} kg (mass of an electron) [the actual mass of an electron is 9.11×10^{-31} kg].

The study of physics is not only educative but also exciting in many ways.

- A small number of basic concepts and laws can explain diverse physical phenomena.
- The most interesting part is the designing of useful devices based on the physical laws.

  For example i) use of robotics ii) journey to Moon and to nearby planets with controls from the ground iii) technological advances in health sciences etc.
- Carrying out new challenging experiments to unfold the secrets of nature and in verifying or falsifying the existing theories.
- Probing and understanding the science behind natural phenomena like the eclipse, and why one feels the heat when there is a fire? (or) What causes the wind, etc.

In today's world of technological advancement, the building block of all engineering and technical education is physics which is explained with the help of mathematical tools.

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1.3 PHYSICS IN RELATION TO TECHNOLOGY AND SOCIETY

Technology is the application of the principles of physics for practical purposes. The application of knowledge for practical purposes in various fields to invent and produce useful products or to solve problems is known as technology. Thus, physics and technology can both together impact our society directly or indirectly. For example,

i. Basic laws of electricity and magnetism led to the discovery of wireless communication technology which has shrunk the world with effective communication over large distances.
ii. The launching of satellite into space has revolutionized the concept of communication.
iii. Microelectronics, lasers, computers, superconductivity and nuclear energy have comprehensively changed the thinking and living style of human beings.

Physics being a fundamental science has played a vital role in the development of all other sciences. A few examples:

1. **Physics in relation to Chemistry:** In physics, we study the structure of atom, radioactivity, X-ray diffraction etc. Such studies have enabled researchers in chemistry to arrange elements in the periodic table on the basis of their atomic numbers. This has further helped to know the nature of valency, chemical bonding and to understand the complex chemical structures. Inter-disciplinary branches like Physical chemistry and Quantum chemistry play important roles here.
2. **Physics in relation to biology:** Biological studies are impossible without a microscope designed using physics principles. The invention of the electron microscope has made it possible to see even the structure of a cell. X-ray and neutron diffraction techniques have helped us to understand the structure of nucleic acids, which help to control vital life processes. X-rays are used for diagnostic purposes. Radio-isotopes are used in radiotherapy for the cure of cancer and other diseases. In recent years, biological processes are being studied from the physics point of view.
3. **Physics in relation to mathematics:** Physics is a quantitative science. It is most closely related to mathematics as a tool for its development.
4. **Physics in relation to astronomy:** Astronomical telescopes are used to study the motion of planets and other heavenly bodies in the sky. Radio telescopes have enabled the astronomers to observe distant points of the universe. Studies of the universe are done using physical principles.
5. **Physics in relation to geology:** Diffraction techniques help to study the crystal structure of various rocks. Radioactivity is used to estimate the age of rocks, fossils and the age of the Earth.
6. **Physics in relation to oceanography:** Oceanographers seek to understand the physical and chemical processes of the

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oceans. They measure parameters such as temperature, salinity, current speed, gas fluxes, chemical components.

7. **Physics in relation to psychology:** All psychological interactions can be derived from a physical process. The movements of neurotransmitters are governed by the physical properties of diffusion and molecular motion. The functioning of our brain is related to our underlying wave-particle dualism.

Nature teaches true science with physics as an efficient tool. Science and technology should be used in a balanced manner so that they do not become weapons to destroy nature which taught us science. Global warming and other negative impacts of technology need to be checked. Safe science with moderate and appropriate use of technology is the need of this century.

The scope and opportunities for higher education in physics and various fellowships offered is given in the beginning of the book.

1.4 MEASUREMENT

"When you can measure what you are speaking about and can express it in numbers, you know something about it; but when you cannot express it in numbers, your knowledge is of a meager and unsatisfactory kind" - **Lord Kelvin**

The comparison of any physical quantity with its standard unit is known as measurement.

Measurement is the basis of all scientific studies and experimentation. It plays an important role in our daily life. Physics is a quantitative science and physicists always deal with numbers which are the measurement of physical quantities.

1.4.1 Definition of Physical Quantity

Quantities that can be measured, and in terms of which, laws of physics are described are called physical quantities. Examples are length, mass, time, force, energy, etc.

1.4.2 Types of Physical Quantities

Physical quantities are classified into two types. They are fundamental and derived quantities.

Fundamental or base quantities are quantities which cannot be expressed in terms of any other physical quantities. These are length, mass, time, electric current, temperature, luminous intensity and amount of substance.

Quantities that can be expressed in terms of fundamental quantities are called derived quantities. For example, area, volume, velocity, acceleration, force, etc.

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1.4.3 Definition of Unit and its Types

The process of measurement is basically a process of comparison. To measure a quantity, we always compare it with some reference standard. For example, when we state that a rope is 10 meter long, it is to say that it is 10 times as long as an object whose length is defined as 1 metre. Such a standard is known as the unit of the quantity. Here 1 metre is the unit of the quantity 'length'.

An arbitrarily chosen standard of measurement of a quantity, which is accepted internationally is called unit of the quantity.

The units in which the fundamental quantities are measured are called *fundamental or base units* and the units of measurement of all other physical quantities, which can be obtained by a suitable multiplication or division of powers of fundamental units, are called *derived units*.

1.4.4 Different types of Measurement Systems

A complete set of units which is used to measure all kinds of fundamental and derived quantities is called a system of units. Here are the common system of units used in mechanics:

(a) **the f.p.s. system** is the British Engineering system of units, which uses ***foot***, ***pound*** and ***second*** as the three basic units for measuring length, mass and time respectively.

(b) **The c.g.s system** is the Gaussian system, which uses ***centimeter, gram*** and ***second*** as the three basic units for measuring length, mass and time respectively.

(c) **The m.k.s system** is based on ***metre, kilogram*** and ***second*** as the three basic units for measuring length, mass and time respectively.

The cgs, mks and SI are metric or decimal system of units. The fps system is not a metric system.

1.4.5 SI unit System

The system of units used by scientists and engineers around the world is commonly called ***the metric system*** but, since 1960, it has been known officially as the International System, or SI (the abbreviation for its French name, Système International). The SI with a standard scheme of symbols, units and abbreviations, were developed and recommended by the General Conference on Weights and Measures in 1971 for international usage in scientific, technical, industrial and commercial work. The advantages of the SI system are,

i) This system makes use of only one unit for one physical quantity, which means a rational system of units.

ii) In this system, all the derived units can be easily obtained from basic and supplementary units, which means it is a coherent system of units.

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iii) It is a metric system which means that multiples and submultiples can be expressed as powers of 10.

In SI, there are seven fundamental units as given in Table 1.2

Table 1.2 SI Base Quantities and Units

Base Quantity	SI Units		
	Unit	Symbol	Definition
Length	metre	m	One metre is the length of the path travelled by light in vacuum in 1/299,792,458 of a second (1983)
Mass	kilogram	kg	One kilogram is the mass of the prototype cylinder of platinum iridium alloy (whose height is equal to its diameter), preserved at the International Bureau of Weights and Measures at Serves, near Paris, France. (1901)
Time	second	s	One second is the duration of 9,192,631,770 periods of radiation corresponding to the transition between the two hyperfine levels of the ground state of Cesium-133 atom.(1967)
Electric current	ampere	A	One ampere is the constant current, which when maintained in each of the two straight parallel conductors of infinite length and negligible cross section, held one metre apart in vacuum shall produce a force per unit length of 2×10^{-7} N/m between them. (1948)
Temperature	kelvin	K	One kelvin is the fraction $\left(\frac{1}{273.16}\right)$ of the thermodynamic temperature of the triple point* of the water. (1967)
Amount of substance	mole	mol	One mole is the amount of substance which contains as many elementary entities as there are atoms in 0.012 kg of pure carbon-12. (1971)
Luminous intensity	candela	cd	One candela is the luminous intensity in a given direction, of a source that emits monochromatic radiation of frequency 5.4×10^{14} Hz and that has a radiant intensity of $\frac{1}{683}$ watt/steradian in that direction. (1979)

* Triple point of water is the temperature at which saturated vapour, pure water and melting ice are all in equilibrium. The triple point temperature of water is 273.16K

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Table 1.3 lists some of the derived quantities and their units.

Table 1.3 Derived Quantities and their Units

Physical quantity	Expression	Unit
Plane angle	arc / radius	rad
Solid angle	surface area/radius^2	sr
Area	length × breadth	m^2
Volume	area × height	m^3
Velocity	displacement / time	m s^{-1}
Acceleration	velocity / time	m s^{-2}
Angular velocity	angular displacement / time	rad s^{-1}
Angular acceleration	angular velocity / time	rad s^{-2}
Density	mass / volume	kg m^{-3}
Linear momentum	mass × velocity	kg m s^{-1}
Moment of inertia	mass × (distance)^2	kg m^2
Force	mass × acceleration	kg m s^{-2} or N
Pressure	force / area	N m^{-2} or Pa
Energy (work)	force × distance	N m or J
Power	Work / time	J s^{-1} or watt (W)
Impulse	force × time	N s
Surface tension	force / length	N m^{-1}
Moment of force (torque)	force × distance	N m
Electric charge	current × time	A s or C
Current density	current / area	A m^{-2}
Magnetic induction	force / (current × length)	N A^{-1} m^{-1} or tesla
Force constant	force / displacement	N m^{-1}
Plank's constant	energy of photon / frequency	J s
Specific heat (S)	heat energy / (mass × temperature)	J kg^{-1} K^{-1}
Boltzmann constant (k)	energy/temperature	J K^{-1}

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Note:
π radian = 180°

$$1 \text{ radian} = \frac{180°}{\pi} = \frac{180° \times 7}{22} = 57.27°$$

Also, 1° (degree of arc) = 60′ (minute of arc) and 1′ (minute of arc) = 60″ (seconds of arc)

Relations between radian, degree and minutes:

$$1° = \frac{\pi}{180} \text{ rad} = 1.744 \times 10^{-2} \text{ rad}$$

$$\therefore 1' = \frac{1°}{60} = \frac{1.744 \times 10^{-2}}{60} = 2.906 \times 10^{-4} rad$$
$$\approx 2.91 \times 10^{-4} rad$$

$$\therefore 1'' = \frac{1°}{3600} = \frac{1.744 \times 10^{-2}}{3600} = 4.844 \times 10^{-6} rad$$
$$\approx 4.84 \times 10^{-6} rad$$

The Radian (rad): *One radian is the angle subtended at the centre of a circle by an arc equal in length to the radius of the circle.*
The Steradian (sr): *One steradian is the solid angle subtended at the centre of a sphere, by that surface of the sphere, which is equal in area, to the square of radius of the sphere*

1.5 MEASUREMENT OF BASIC QUANTITIES

1.5.1 Measurement of length

The concept of length in physics is related to the concept of distance in everyday life. Length is defined as the distance between any two points in space. The SI unit of length is metre. The objects of our interest vary widely in sizes. For example, large objects like the galaxy, stars, Sun, Earth, Moon etc., and their distances constitute a ***macrocosm***. It refers to a large world, in which both objects and distances are large. On the contrary, objects like molecules, atoms, proton, neutron, electron, bacteria etc., and their distances constitute ***microcosm***, which means a small world in which both objects and distances are small-sized.

Distances ranging from 10^{-5} m to 10^{2} m can be measured by direct methods. For example, a metre scale can be used to measure the distance from 10^{-3} m to 1 m, vernier calipers up to 10^{-4} m, a screw gauge up to 10^{-5} m and so on. The atomic and astronomical distances cannot be measured by any of the above mentioned direct methods. Hence, to measure the very small and the very large distances, indirect methods have to be devised and used. In Table 1.4, a list of powers of 10 (both positive and negative powers) is given. Prefixes for each power are also mentioned. These prefixes are used along with units of length, and of mass.

The supplementary quantities of plane and solid angle were converted into Derived quantities in 1995 (GCWM)

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Table 1.4 Prefixes for Powers of Ten

Multiple	Prefix	Symbol	Sub multiple	Prefix	Symbol
10^{1}	deca	da	10^{-1}	deci	d
10^{2}	hecto	h	10^{-2}	centi	c
10^{3}	kilo	k	10^{-3}	milli	m
10^{6}	mega	M	10^{-6}	micro	μ
10^{9}	giga	G	10^{-9}	nano	n
10^{12}	tera	T	10^{-12}	pico	p
10^{15}	peta	P	10^{-15}	femto	f
10^{18}	exa	E	10^{-18}	atto	a
10^{21}	zetta	Z	10^{-21}	zepto	z
10^{24}	yotta	Y	10^{-24}	yocto	y

i) Measurement of small distances: screw gauge and vernier caliper

Screw gauge: The screw gauge is an instrument used for measuring accurately the dimensions of objects up to a maximum of about 50 mm. The principle of the instrument is the magnification of linear motion using the circular motion of a screw. The least count of the screw gauge is 0.01 mm

Vernier caliper: A vernier caliper is a versatile instrument for measuring the dimensions of an object namely diameter of a hole, or a depth of a hole. The least count of the vernier caliper is 0.01 cm

Figure 1.2 Screw gauge and vernier caliper with errors

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ii) Measurement of large distances

For measuring larger distances such as the height of a tree, distance of the Moon or a planet from the Earth, some special methods are adopted. Triangulation method, parallax method and radar method are used to determine very large distances.

Triangulation method for the height of an accessible object

Let AB = h be the height of the tree or tower to be measured. Let C be the point of observation at distance x from B. Place a range finder at C and measure the angle of elevation, $\angle ACB = \theta$ as shown in Figure 1.3.

From right angled triangle ABC,

$$\tan\theta = \frac{AB}{BC} = \frac{h}{x}$$

(or)

$$\text{height } h = x \tan\theta$$

Knowing the distance x, the height h can be determined.

Figure 1.3 Triangulation method

Range and order of lengths of various objects are listed in Table 1.5

EXAMPLE 1.1

From a point on the ground, the top of a tree is seen to have an angle of elevation 60°. The distance between the tree and a point is 50 m. Calculate the height of the tree?

Solution

$$\text{Angle } \theta = 60°$$

The distance between the tree and a point $x = 50$ m

Height of the tree (h) = ?

For triangulation method $\tan\theta = \frac{h}{x}$

$$h = x \tan\theta$$
$$= 50 \times \tan 60°$$
$$= 50 \times 1.732$$
$$h = 86.6 \text{ m}$$

The height of the tree is 86.6 m.

Parallax method

Very large distances, such as the distance of a planet or a star from the Earth can be measured by the parallax method. *Parallax is the name given to the apparent change in the position of an object with respect to the background, when the object is seen from two different positions*. The distance between the two positions (i.e., points of observation) is called the basis (b). Consider any object at the location O (Figure 1.4)

Let L and R represent the positions of the left and right eyes of the observer respectively.

The object (O) is viewed with the left eye (L) keeping the right eye closed and the same object (O) is viewed with the right eye (R) keeping the left eye closed.

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In Figure 1.4, LO and RO are the lines drawn from the positions of the left and right eyes to the object. These two lines make an angle θ at O. This angle θ is called the angle of parallax.

OL and OR are considered as the radii (x) of a circle. For astronomical calculation, the distance LR = b (basis) can be treated as an arc of this circle, then

$OL = OR = x$

as $LR = b$

$$\theta = \frac{b}{x}$$

Knowing b and θ, x can be calculated which is approximately the distance of the object from the observer.

If the object is the Moon or any near by star, then the angle θ will be too small due to the large astronomical distance and the place of observation. In this case, the two points of observation should be sufficiently spaced on the surface of the Earth.

Figure 1.4 Parallax method

Determination of distance of Moon from Earth

In Figure 1.5, C is the centre of the Earth. A and B are two diametrically opposite places on the surface of the Earth. From A and B, the parallaxes θ_1 and θ_2 respectively of Moon M with respect to some distant star are determined with the help of an astronomical telescope. Thus, the total parallax of the Moon subtended on Earth $\angle AMB = \theta_1 + \theta_2 = \theta$.

If θ is measured in radians, then $\theta = \frac{AB}{AM}$; AM≈MC (AM is approximately equal to MC)

$\theta = \frac{AB}{MC}$ or $MC = \frac{AB}{\theta}$; Knowing the values of AB and θ, we can calculate the distance MC of Moon from the Earth.

Figure 1.5 Parallax method: determination of distance of Moon from Earth

EXAMPLE 1.2

The Moon subtends an angle of 1° 55′ at the base line equal to the diameter of the Earth. What is the distance of the Moon from the Earth? (Radius of the Earth is $6.4 \times 10^6 m$)

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Solution

angle $\theta = 1° \, 55' = 115'$

$= (115 \times 60)'' \times (4.85 \times 10^{-6})$ rad

$= 3.34 \times 10^{-2}$ rad

since $1'' = 4.85 \times 10^{-6}$ rad

Radius of the Earth $= 6.4 \times 10^6 m$

From the Figure 1.5, AB is the diameter of the Earth (b)= $2 \times 6.4 \times 10^6 m$ Distance of the Moon from the Earth $x = ?$

$$x = \frac{b}{\theta} = \frac{2 \times 6.4 \times 10^6}{3.34 \times 10^{-2}}$$

$$x = 3.83 \times 10^8 \text{m}$$

RADAR method

The word RADAR stands for radio detection and ranging. A radar can be used to measure accurately the distance of a nearby planet such as Mars. In this method, radio waves are sent from transmitters which, after reflection from the planet, are detected by the receiver. By measuring, the time interval (t) between the instants the radio waves are sent and received, the distance of the planet can be determined as

Speed = distance travelled / time taken
(Speed is explained in unit 2)
Distance(d) = Speed of radio waves × time taken

$$d = \frac{v \times t}{2}$$

where v is the speed of the radio wave. As the time taken (t) is for the distance covered during the forward and backward path of the radio waves, it is divided by 2 to get the actual distance of the object. This method can also be used to determine the height, at which an aeroplane flies from the ground.

Figure 1.6 RADAR method

EXAMPLE 1.3

A RADAR signal is beamed towards a planet and its echo is received 7 minutes later. If the distance between the planet and the Earth is 6.3×10^{10} m. Calculate the speed of the signal?

Solution

The distance of the planet from the Earth $d = 6.3 \times 10^{10}$ m

Time t = 7 minutes = 7×60 s.
the speed of signal $v = ?$

The speed of signal

$$v = \frac{2d}{t} = \frac{2 \times 6.3 \times 10^{10}}{7 \times 60} = 3 \times 10^8 \, ms^{-1}$$

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Table 1.5 Range and Order of Lengths

Size of objects and distances	Length (m)
Distance to the boundary of observable universe	10^{26}
Distance to the Andromeda galaxy	10^{22}
Size of our galaxy	10^{21}
Distance from Earth to the nearest star (other than the Sun)	10^{16}
Average radius of Pluto's orbit	10^{12}
Distance of the Sun from the Earth	10^{11}
Distance of Moon from the Earth	10^{8}
Radius of the Earth	10^{7}
Height of the Mount Everest above sea level	10^{4}
Length of a football field	10^{2}
Thickness of a paper	10^{-4}
Diameter of a red blood cell	10^{-5}
Wavelength of light	10^{-7}
Length of typical virus	10^{-8}
Diameter of the hydrogen atom	10^{-10}
Size of atomic nucleus	10^{-14}
Diameter of a proton	10^{-15}

Some Common Practical Units

(i) Fermi = 1 fm = 10^{-15} m

(ii) 1 angstrom = 1 Å = 10^{-10} m

(iii) 1 nanometer = 1 nm = 10^{-9} m

(iv) 1 micron = 1μm = 10^{-6} m

(v) 1 Light year (Distance travelled by light in vacuum in one year) 1 Light Year = 9.467×10^{15} m

(vi) 1 astronomical unit (the mean distance of the Earth from the Sun) 1 AU = 1.496×10^{11} m

(vii) 1 parsec (Parallactic second) (Distance at which an arc of length 1 AU subtends an angle of 1 second of arc)

1 parsec = 3.08×10^{16} m =3.26 light year

Figure 1.7 The international 1 kg standard of mass, a platinum-iridium (9:1) cylinder 3.9 cm in height and diameter.

DO YOU KNOW?

Why is the cylinder used in defining kilogram made up of platinum-iridium alloy?

This is because the platinum-iridium alloy is least affected by environment and time.

Chandrasekhar Limit (CSL) is the largest practical unit of mass.

1 CSL = 1.4 times the mass of the Sun

The smallest practical unit of time is Shake.

1 Shake = 10^{-8} s

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1.5.2 Measurement of mass

Mass is a property of matter. It does not depend on temperature, pressure and location of the body in space. *Mass of a body is defined as the quantity of matter contained in a body*. The SI unit of mass is kilogram (kg). The masses of objects which we shall study in this course vary over a wide range. These may vary from a tiny mass of electron (9.11×10^{-31}kg) to the huge mass of the known universe ($=10^{55}$ kg). The order of masses of various objects is shown in Table 1.6.

Ordinarily, the mass of an object is determined in kilograms using a common balance like the one used in a grocery shop. For measuring larger masses like that of planets, stars etc., we make use of gravitational methods. For measurement of small masses of atomic/subatomic particles etc., we make use of a mass spectrograph.

Some of the weighing balances commonly used are common balance, spring balance, electronic balance, etc.

Table 1.6 Range of masses

Object	Order of mass (kg)
Electron	10^{-30}
Proton or Neutron	10^{-27}
Uranium atom	10^{-25}
Red blood corpuscle	10^{-14}
A cell	10^{-10}
Dust particle	10^{-9}
Raindrop	10^{-6}
Mosquito	10^{-5}
Grape	10^{-3}
Frog	10^{-1}
Human	10^{2}
Car	10^{3}
Ship	10^{5}
Moon	10^{23}
Earth	10^{25}
Sun	10^{30}
Milky way	10^{41}
Observable Universe	10^{55}

1.5.3 Measurement of Time intervals

> *"Time flows uniformly forward"*
> *– Sir Issac Newton*
>
> *"Time is what a clock reads"*
> *– Albert Einstein*

A clock is used to measure the time interval. An atomic standard of time, is based on the periodic vibration produced in a Cesium atom. Some of the clocks developed later are electric oscillators, electronic oscillators, solar clock, quartz crystal clock, atomic clock, decay of elementary particles, radioactive dating etc. The order of time intervals are tabulated in Table 1.7.

Figure 1.8 The atomic clock, which keeps time on the basis of radiation from cesium atoms is accurate to about three millionths of a second per year.

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Table 1.7 Order of Time Intervals

Event	Order of time interval (s)
Lifespan of the most unstable particle	10^{-24}
Time taken by light to cross a distance of nuclear size	10^{-22}
Period of X-rays	10^{-19}
Time period of electron in hydrogen atom	10^{-15}
Period of visible light waves	10^{-15}
Time taken by visible light to cross through a window pane	10^{-8}
Lifetime of an excited state of an atom	10^{-8}
Period of radio waves	10^{-6}
Time period of audible sound waves	10^{-3}
Wink of an eye	10^{-1}
Time interval between two successive heart beats	10^{0}
Travel time of light from Moon to Earth	10^{0}
Travel time of light from Sun to Earth	10^{2}
Halflife time of a free neutron	10^{3}
Time period of a satellite	10^{4}
Time period of rotation of Earth around its axis (one day)	10^{5}
Time period of revolution of Earth around the Sun (one year)	10^{7}
Average life of a human being	10^{9}
Age of Egyptian pyramids	10^{11}
Age of Universe	10^{17}

1.6 THEORY OF ERRORS

The foundation of all experimental science and technology is measurement. The result obtained from any measurement will contain some uncertainty. Such an uncertainty is termed ***error***. Any calculation made using the measured values will also have an error. It is not possible to make exact measurements in an experiment. In measurements, two different terms, accuracy and precision are used and need to be distinguished at this stage. Accuracy refers to how far we are from the true value, and precision refers to how well we measure.

DO YOU KNOW?

In India, the National Physical Laboratory (New Delhi) has the responsibility of maintenance and improvement of physical standards of length, mass, time, etc.

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1.6.1 Accuracy and Precision

Let us say, you know your true height is exactly 5′9″. You first measure your height with a yardstick and get the value 5′0″. Your measurement is hence not accurate. Now you measure your height with a laser yardstick and get 5′9″ as the value. Now your measurement is accurate. The true value is also called theoretical value. The level of accuracy required for each application varies greatly. Highly accurate data can be very difficult to produce and compile. For example, if you consistently measure your height as 5′0″ with a yard stick, your measurements are precise. The level of precision required for different applications vary to a great extent. Engineering projects such as road and utility construction require very precise information measured to the millimeter or one-tenth of an inch.

If a measurement is precise, that does not necessarily mean that it is accurate. However, if the measurement is consistently accurate, it is also precise.

For example, if the temperature outside a building is 40°C as measured by a weather thermometer and if the real outside temperature is 40°C, the thermometer is accurate. If the thermometer consistently registers this exact temperature in a row, the thermometer is precise.

Consider another example. Let the temperature of a refrigerator repeatedly measured by a thermometer be given as 10.4°C, 10.2°C, 10.3°C, 10.1°C, 10.2°C, 10.1°C, 10.1°C, 10.1°C. However, if the real temperature inside the refrigerator is 9°C, we say that the thermometer is not accurate (it is almost one degree off the true value), but since all the measured values are close to 10°C, hence it is precise.

A visual example:

Target shooting is an example which explains the difference between accuracy and precision. In Figure 1.9 (a), the shots are focused so as to reach the bull's eye (midpoint), but the arrows have reached only around this point. Hence the shots are not accurate and also not precise.

In Figure 1.9 (b), all the shots are close to each other but not at the central point. Hence the shots are said to be precise but not accurate. In Figure 1.9 (c), the shots are closer and also at the central point. Hence the shots are both precise and accurate.

Figure 1.9 Visual example of accuracy and precision

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A numerical example

The true value of a certain length is nearly 5.678 cm. In one experiment, using a measuring instrument of resolution 0.1 cm, the measured value is found to be 5.5 cm. In another experiment using a measuring instrument of greater resolution, say 0.01 cm, the length is found to be 5.38 cm. We find that the first measurement is more accurate as it is closer to the true value, but it has lesser precision. On the contrary, the second measurement is less accurate, but it is more precise.

1.6.2 Errors in Measurement

The uncertainty in a measurement is called an error. Random error, systematic error and gross error are the three possible errors.

i) **Systematic errors**

Systematic errors are reproducible inaccuracies that are consistently in the same direction. These occur often due to a problem that persists throughout the experiment. Systematic errors can be classified as follows

1) **Instrumental errors**

When an instrument is not calibrated properly at the time of manufacture, instrumental errors may arise. If a measurement is made with a meter scale whose end is worn out, the result obtained will have errors. These errors can be corrected by choosing the instrument carefully.

2) **Imperfections in experimental technique or procedure**

These errors arise due to the limitations in the experimental arrangement. As an example, while performing experiments with a calorimeter, if there is no proper insulation, there will be radiation losses. This results in errors and to overcome these, necessary correction has to be applied.

3) **Personal errors**

These errors are due to individuals performing the experiment, may be due to incorrect initial setting up of the experiment or carelessness of the individual making the observation due to improper precautions.

4) **Errors due to external causes**

The change in the external conditions during an experiment can cause error in measurement. For example, changes in temperature, humidity, or pressure during measurements may affect the result of the measurement.

5) **Least count error**

Least count is the smallest value that can be measured by the measuring instrument, and the error due to this measurement is least count error. The instrument's resolution hence is the cause of this error. Least count error can be reduced by using a high precision instrument for the measurement.

ii) **Random errors**

Random errors may arise due to random and unpredictable variations in experimental conditions like pressure, temperature, voltage supply etc. Errors may also be due to personal errors by the observer who performs the experiment. Random errors are sometimes called **"chance error".** When different readings are obtained by a person every time he

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repeats the experiment, personal error occurs. For example, consider the case of the thickness of a wire measured using a screw gauge. The readings taken may be different for different trials. In this case, a large number of measurements are made and then the arithmetic mean is taken.

If n number of trial readings are taken in an experiment, and the readings are a_1, a_2, a_3,..... a_n. The arithmetic mean is

$$a_m = \frac{a_1 + a_2 + a_3 + ...a_n}{n} \quad (1.1)$$

or

$$a_m = \frac{1}{n}\sum_{i=1}^{n} a_i \quad (1.2)$$

Usually this arithmetic mean is taken as the best possible true value of the quantity.

Certain procedures to be followed to minimize experimental errors, along with examples are shown in Table 1.8.

iii) Gross Error

The error caused due to the shear carelessness of an observer is called gross error.

For example

(i) Reading an instrument without setting it properly.

(ii) Taking observations in a wrong manner without bothering about the sources of errors and the precautions.

(iii) Recording wrong observations.

(iv) Using wrong values of the observations in calculations.

These errors can be minimized only when an observer is careful and mentally alert.

Table 1.8 Minimizing Experimental Error

Type of error	Example	How to minimize it
Random error	Suppose you measure the mass of a ring three times using the same balance and get slightly different values. 15.46 g, 15.42 g, 15.44 g	Take more data. Random errors can be evaluated through statistical analysis and can be reduced by averaging over a large number of observations.
Systematic error	Suppose the cloth tape measure that you use to measure the length of an object has been stretched out from years of use. (As a result all of the length measurements are not correct).	Systematic errors are difficult to detect and cannot be analysed statistically, because all of the data is in the same direction. (Either too high or too low)

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1.6.3 Error Analysis

i) Absolute Error

The magnitude of difference between the true value and the measured value of a quantity is called absolute error. If a_1, a_2, a_3, ...a_n are the measured values of any quantity 'a' in an experiment performed n times, then the arithmetic mean of these values is called the true value (a_m) of the quantity.

$$a_m = \frac{a_1 + a_2 + a_3 + ... + a_n}{n} \text{ or}$$

$$a_m = \frac{1}{n}\sum_{i=1}^{n} a_i$$

The absolute error in measured values is given by

$$|\Delta a_1| = |a_m - a_1|$$

$$|\Delta a_2| = |a_m - a_2|$$

.....

.....

$$|\Delta a_n| = |a_m - a_n|$$

ii) Mean Absolute error

The arithmetic mean of absolute errors in all the measurements is called the mean absolute error.

$$\Delta a_m = \frac{|\Delta a_1| + |\Delta a_2| + |\Delta a_3| + ... + |\Delta a_n|}{n}$$

$$\text{or} = \frac{1}{n}\sum_{i=1}^{n} |\Delta a_i|$$

If a_m is the true value and Δa_m is the mean absolute error then the magnitude of the quantity may lie between $a_m + \Delta a_m$ and $a_m - \Delta a_m$

iii) Relative error

The ratio of the mean absolute error to the mean value is called relative error. This is also called as fractional error. Thus

$$\text{Relative error} = \frac{\textit{Mean absolute error}}{\textit{Mean value}} = \frac{\Delta a_m}{a_m}$$

Relative error expresses how large the absolute error is compared to the total size of the object measured. For example, a driver's speedometer shows that his car is travelling at 60 km h^{-1} when it is actually moving at 62 km h^{-1}. Then absolute error of speedometer is 62-60 km h^{-1} = 2 km h^{-1} Relative error of the measurement is 2 km h^{-1} / 62 km h^{-1} = 0.032.

iv) Percentage error

The relative error expressed as a percentage is called percentage error.

$$\text{Percentage error} = \frac{\Delta a_m}{a_m} \times 100\%$$

A percentage error very close to zero means one is close to the targeted value, which is good and acceptable. It is always necessary to understand whether error is due to impression of equipment used or a mistake in the experimentation.

EXAMPLE 1.4

In a series of successive measurements in an experiment, the readings of the period of oscillation of a simple pendulum were found to be 2.63s, 2.56 s, 2.42s, 2.71s and

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2.80s. Calculate (i) the mean value of the period of oscillation (ii) the absolute error in each measurement (iii) the mean absolute error (iv) the relative error (v) the percentage error. Express the result in proper form.

Solution

$t_1 = 2.63\,s,\ t_2 = 2.56\,s,\ t_3 = 2.42\,s,$
$t_4 = 2.71\,s,\ t_5 = 2.80\,s$

(i) $$T_m = \frac{t_1 + t_2 + t_3 + t_4 + t_5}{5}$$
$$= \frac{2.63 + 2.56 + 2.42 + 2.71 + 2.80}{5}$$

$$T_m = \frac{13.12}{5} = 2.624s$$

$T_m = 2.62s$ (Rounded off to 2nd decimal place)

(ii) Absolute error $|\Delta T| = |T_m - t|$

$$|\Delta T_1| = |2.62 - 2.63| = +0.01s$$
$$|\Delta T_2| = |2.62 - 2.56| = +0.06s$$
$$|\Delta T_3| = |2.62 - 2.42| = +0.20s$$
$$|\Delta T_4| = |2.62 - 2.71| = +0.09s$$
$$|\Delta T_5| = |2.62 - 2.80| = +0.18s$$

(iii) Mean absolute error $= \dfrac{\Sigma|\Delta T_i|}{n}$

$$\Delta T_m = \frac{0.01 + 0.06 + 0.20 + 0.09 + 0.18}{5}$$

$\Delta T_m = \dfrac{0.54}{5} = 0.108s = 0.11s$ (Rounded off to 2nd decimal place)

(iv) Relative error:

$$S_T = \frac{\Delta T_m}{T_m} = \frac{0.11}{2.62} = 0.0\ 419$$

$$S_T = 0.04$$

(v) Percentage error in T = 0.04 × 100% = 4%

(vi) Time period of simple pendulum = T = (2.62 ± 0.11)s

1.6.4 Propagation of errors

A number of measured quantities may be involved in the final calculation of an experiment. Different types of instruments might have been used for taking readings. Then we may have to look at the errors in measuring various quantities, collectively.

The error in the final result depends on

(i) The errors in the individual measurements

(ii) On the nature of mathematical operations performed to get the final result. So we should know the rules to combine the errors.

The various possibilities of the propagation or combination of errors in different mathematical operations are discussed below:

(i) Error in the sum of two quantities

Let ΔA and ΔB be the absolute errors in the two quantities A and B respectively. Then,

Measured value of A = A ± ΔA

Measured value of B = B ± ΔB

Consider the sum, Z = A + B

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The error ΔZ in Z is then given by

$$Z \pm \Delta Z = (A \pm \Delta A) + (B \pm \Delta B)$$

$$= (A + B) \pm (\Delta A + \Delta B)$$

$$= Z \pm (\Delta A + \Delta B)$$

(or) $$\Delta Z = \Delta A + \Delta B \qquad (1.3)$$

The maximum possible error in the sum of two quantities is equal to the sum of the absolute errors in the individual quantities.

EXAMPLE 1.5

Two resistances $R_1 = (100 \pm 3)\ \Omega$, $R_2 = (150 \pm 2)\ \Omega$, are connected in series. What is their equivalent resistance?

Solution

$$R_1 = 100 \pm 3\Omega;\ R_2 = 150 \pm 2\Omega$$

Equivalent resistance R = ?

Equivalent resistance $R = R_1 + R_2$

$$= (100 \pm 3) + (150 \pm 2)$$

$$= (100 + 150) \pm (3 + 2)$$

$$R = (250 \pm 5)\ \Omega$$

(ii) Error in the difference of two quantities

Let ΔA and ΔB be the absolute errors in the two quantities, A and B, respectively. Then,

Measured value of $A = A \pm \Delta A$

Measured value of $B = B \pm \Delta B$

Consider the difference, $Z = A - B$

The error ΔZ in Z is then given by

$Z \pm \Delta Z = (A \pm \Delta A) - (B \pm \Delta B)$

$$= (A - B)\ ) \pm \Delta A \mp \Delta B$$

$$= Z \pm \Delta A \mp \Delta B$$

(or) $$\Delta Z = \Delta A + \Delta B \ldots\ldots\ldots\ldots\ldots\ldots (1.4)$$

The maximum error in difference of two quantities is equal to the sum of the absolute errors in the individual quantities.

EXAMPLE 1.6

The temperatures of two bodies measured by a thermometer are $t_1 = (20 + 0.5)°C$, $t_2 = (50 \pm 0.5)°C$. Calculate the temperature difference and the error therein.

Solution

$t_1 = (20 \pm 0.5)°C \qquad t_2 = (50 \pm 0.5)°C$

temperature difference t=?

$t = t_2 - t_1 = (50 \pm 0.5) - (20 \pm 0.5)°C$

(Using equation1.4)

$$= (50 - 20) \pm (0.5 + 0.5)$$

$$t = (30 \pm 1)°C$$

(iii) Error in the product of two quantities

Let ΔA and ΔB be the absolute errors in the two quantities A, and B, respectively. Consider the product $Z = AB$

The error ΔZ in Z is given by $Z \pm \Delta Z = (A \pm \Delta A)\ (B \pm \Delta B)$

$$= (AB) \pm (A\ \Delta B) \pm (B\ \Delta A) \pm (\Delta A\ .\ \Delta B)$$

Dividing L.H.S by Z and R.H.S by AB, we get,

$$1 \pm \frac{\Delta Z}{Z} = 1 \pm \frac{\Delta B}{B} \pm \frac{\Delta A}{A} \pm \frac{\Delta A}{A} \cdot \frac{\Delta B}{B}$$

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As ΔA /A, ΔB / B are both small quantities, their product term $\frac{\Delta A}{A}.\frac{\Delta B}{B}$ can be neglected. The maximum fractional error in Z is

$$\frac{\Delta Z}{Z} = \pm\left(\frac{\Delta A}{A} + \frac{\Delta B}{B}\right) \qquad (1.5)$$

The maximum fractional error in the product of two quantities is equal to the sum of the fractional errors in the individual quantities.

[Alternative method is given in Appendix A1.2]

EXAMPLE 1.7

The length and breadth of a rectangle are (5.7 ± 0.1) cm and (3.4 ± 0.2) cm respectively. Calculate the area of the rectangle with error limits.

Solutions

Length $\ell = (5.7 \pm 0.1)$ cm

Breadth b =(3.4 ± 0.2) cm

Area A with error limit = A ± ΔA = ?

Area A = $\ell \times b = 5.7 \times 3.4 = 19.38 = 19.4$ cm^2

$$\frac{\Delta A}{A} = \frac{\Delta l}{l} + \frac{\Delta b}{b}$$

$$\Delta A = \left(\frac{\Delta l}{l} + \frac{\Delta b}{b}\right)A$$

$$\Delta A = \left(\frac{0.1}{5.7} + \frac{0.2}{3.4}\right)19.4$$

$$= (0.0175+0.0588) \times 19.4$$

$$= 1.48 = 1.5$$

Area with error limit

A = (19.4±1.5) cm^2

(iv) Error in the division or quotient of two quantities

Let ΔA and ΔB be the absolute errors in the two quantities A and B respectively.

Consider the quotient, $Z = \frac{A}{B}$

The error ΔZ in Z is given by

$$Z \pm \Delta Z = \frac{A \pm \Delta A}{B \pm \Delta B} = \frac{A\left(1 \pm \frac{\Delta A}{A}\right)}{B\left(1 \pm \frac{\Delta B}{B}\right)}$$

$$= \frac{A}{B}\left(1 \pm \frac{\Delta A}{A}\right)\left(1 \pm \frac{\Delta B}{B}\right)^{-1}$$

or $Z \pm \Delta Z = Z\left(1 \pm \frac{\Delta A}{A}\right)\left(1 \mp \frac{\Delta B}{B}\right)$ [using $(1+x)^n \approx 1+nx$, when $x<<1$]

Dividing both sides by Z, we get,

$$1 \pm \frac{\Delta Z}{Z} = \left(1 \pm \frac{\Delta A}{A}\right)\left(1 \mp \frac{\Delta B}{B}\right)$$

$$= 1 \pm \frac{\Delta A}{A} \mp \frac{\Delta B}{B} \pm \frac{\Delta A}{A}.\frac{\Delta B}{B}$$

As the terms ΔA/A and ΔB/B are small, their product term can be neglected.

The maximum fractional error in Z is given by $\frac{\Delta Z}{Z} = \left(\frac{\Delta A}{A} + \frac{\Delta B}{B}\right)$ (1.6)

The maximum fractional error in the quotient of two quantities is equal to the sum of their individual fractional errors.

(Alternative method is given in Appendix A1.2)

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EXAMPLE 1.8

The voltage across a wire is (100 ± 5)V and the current passing through it is (10±0.2) A. Find the resistance of the wire.

Solution

Voltage V = (100±5)V

Current I = (10±0.2) A

Resistance R = ?

Then resistance R is given by Ohm's law,

$$R = \frac{V}{I}$$

$$= \frac{100}{10} = 10\Omega$$

$$\frac{\Delta R}{R} = \left(\frac{\Delta V}{V} + \frac{\Delta I}{I}\right)$$

$$\Delta R = \left(\frac{\Delta V}{V} + \frac{\Delta I}{I}\right)R$$

$$= \left(\frac{5}{100} + \frac{0.2}{10}\right)10$$

$$= (0.05 + 0.02)10$$

$$= 0.07 \times 10 = 0.7$$

The resistance R = (10 ± 0.7)Ω

(v) Error in the power of a quantity

Consider the nth power of A, $Z = A^n$

The error ΔZ in Z is given by

$$Z \pm \Delta Z = (A \pm \Delta A)^n = A^n\left(1 \pm \frac{\Delta A}{A}\right)^n$$

$$= Z\left(1 \pm n\frac{\Delta A}{A}\right)$$

We get $[(1+x)^n \approx 1+nx$, when $x<<1]$ neglecting remaining terms, Dividing both sides by Z

$$1 \pm \frac{\Delta Z}{Z} = 1 \pm n\frac{\Delta A}{A} \; or$$

$$\frac{\Delta Z}{Z} = n.\frac{\Delta A}{A} \qquad (1.7)$$

The fractional error in the nth power of a quantity is n times the fractional error in that quantity.

General rule: If $Z = \frac{A^p B^q}{C^r}$ Then maximum fractional error in Z is given by $\frac{\Delta Z}{Z} = p\frac{\Delta A}{A} + q\frac{\Delta B}{B} + r\frac{\Delta C}{C}$

The percentage error in Z is given by

$$\frac{\Delta Z}{Z} \times 100 = p\frac{\Delta A}{A} \times 100 + q\frac{\Delta B}{B} \times 100 + r\frac{\Delta C}{C} \times 100$$

EXAMPLE 1.9

A physical quantity *x* is given by $x = \frac{a^2b^3}{c\sqrt{d}}$. If the percentage errors of measurement in a, b, c and d are 4%, 2%, 3% and 1% respectively, then calculate the percentage error in the calculation of *x*. (NEET 2013)

Solution

Given $x = \frac{a^2b^3}{c\sqrt{d}}$

The percentage error in *x* is given by

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$$\frac{\Delta x}{x}\times 100 = 2\frac{\Delta a}{a}\times 100 + 3\frac{\Delta b}{b}\times 100 + \frac{\Delta c}{c}\times 100 + \frac{1}{2}\frac{\Delta d}{d}\times 100$$

$$= (2 \times 4\%) + (3 \times 2\%) + (1 \times 3\%) + (½ \times 1\%)$$

$$= 8\% + 6\% + 3\% + 0.5\%$$

The percentage error is $x = 17.5\%$

1.7 SIGNIFICANT FIGURES

1.7.1 Definition and Rules of Significant Figures

Suppose we ask three students to measure the length of a stick using metre scale (the least count for metre scale is 1 mm or 0.1 cm). So, the result of the measurement (length of stick) can be any of the following, 7.20 cm or 7.22 cm or 7.23 cm. Note that all the three students measured first two digits correctly (with confidence) but last digit varies from person to person. So, the number of meaningful digits is 3 which communicate both measurement (quantitative) and also the precision of the instrument used. Therefore, significant number or significant digit is 3. It is defined as the **number of meaningful digits which contain numbers that are known reliably and first uncertain number**.

Examples: The significant figure for the digit 121.23 is 5, significant figure for the digit 1.2 is 2, significant figure for the digit 0.123 is 3, significant digit for 0.1230 is 4, significant digit for 0.0123 is 3, significant digit for 1230 is 3, significant digit for 1230 (with decimal) is 4 and significant digit for 20000000 is 1 (because $20000000 = 2 \times 10^7$ has only one significant digit, that is, 2).

In physical measurement, if the length of an object is $l = 1230$ m, then significant digit for l is 4.

The rules for counting significant figures are given in Table 1.9.

EXAMPLE 1.10

State the number of significant figures in the following

i) 600800
ii) 400
iii) 0.007
iv) 5213.0
v) $2.65 \times 10^{24}\, m$
vi) 0.0006032

Solution: i) four ii) one iii) one iv) five v) three vi) four

1.7.2 Rounding Off

Calculators are widely used now-a-days to do calculations. The result given by a calculator has too many figures. In no case should the result have more significant figures than the figures involved in the data used for calculation. The result of calculation with numbers containing more than one uncertain digit should be rounded off. The rules for rounding off are shown in Table 1.10.

EXAMPLE 1.11

Round off the following numbers as indicated

i) 18.35 up to 3 digits
ii) 19.45 up to 3 digits
iii) 101.55×10^6 up to 4 digits
iv) 248337 up to digits 3 digits
v) 12.653 up to 3 digits.

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Table 1.9 Rules for counting significant figures

	Rule	Example
i)	All non-zero digits are significant	1342 has **four** significant figures
ii)	All zeros between two non zero digits are significant	2008 has **four** significant figures
iii)	All zeros to the right of a non-zero digit but to the left of a decimal point are significant.	30700. has **five** significant figures
iv)	For the number without a decimal point, the terminal or trailing zero(s) are not significant.	30700 has **three** significant figures
v)	If the number is less than 1, the zero (s) on the right of the decimal point but to left of the first non zero digit are not significant.	0.00345 has **three** significant figures
vi)	All zeros to the right of a decimal point and to the right of non-zero digit are significant.	40.00 has **four** significant figures and 0.030400 has **five** significant figures
vii)	The number of significant figures does not depend on the system of units used	1.53 cm, 0.0153 m, 0.0000153 km, all have **three** significant figures

Note 1: Multiplying or dividing factors, which are neither rounded numbers nor numbers representing measured values, are exact and they have infinite numbers of significant figures as per the situation. For example, circumference of circle S = 2πr, Here the factor 2 is exact number. It can be written as 2.0, 2.00 or 2.000 as required.

Note 2: The power of 10 is irrelevant to the determination of significant figures.

For example $x = 5.70$ m $= 5.70 \times 10^2$ cm $= 5.70 \times 10^3$ mm $= 5.70 \times 10^{-3}$ km.

In each case the number of significant figures is **three.**

Table 1.10 Rules for Rounding Off

Rule	Example
i) If the digit to be dropped is smaller than 5, then the preceding digit should be left unchanged.	i) 7.32 is rounded off to 7.3 ii) 8.94 is rounded off to 8.9
ii) If the digit to be dropped is greater than 5, then the preceding digit should be increased by 1	i) 17.26 is rounded off to 17.3 ii) 11.89 is rounded off to 11.9
iii) If the digit to be dropped is 5 followed by digits other than zero, then the preceding digit should be raised by 1	i) 7.352, on being rounded off to first decimal becomes 7.4 ii) 18.159 on being rounded off to first decimal, become 18.2
iv) If the digit to be dropped is 5 or 5 followed by zeros, then the preceding digit is not changed if it is even	i) 3.45 is rounded off to 3.4 ii) 8.250 is rounded off to 8.2
v) If the digit to be dropped is 5 or 5 followed by zeros, then the preceding digit is raised by 1 if it is odd	i) 3.35 is rounded off to 3.4 ii) 8.350 is rounded off to 8.4

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Solution

i) 18.4 ii) 19.4 iii) 101.6×10^6
iv) 248000 v) 12.7

1.7.3 Arithmetical Operations with Significant Figures

(i) Addition and subtraction

In addition and subtraction, the final result should retain as many decimal places as there are in the number with the smallest number of decimal places.

Example:

1. 3.1 + 1.780 + 2.046 = 6.926

   Here the least number of significant digits after the decimal is one. Hence the result will be 6.9.

2. 12.637 – 2.42 = 10.217

   Here the least number of significant digits after the decimal is two. Hence the result will be 10.22

(ii) Multiplication and Division

In multiplication or division, the final result should retain as many significant figures as there are in the original number with smallest number of significant figures.

Example:

1. $1.21 \times 36.72 = 44.4312 = 44.4$

   Here the least number of significant digits in the measured values is three. Hence the result when rounded off to three significant digits is 44.4

2. $36.72 \div 1.2 = 30.6 = 31$

   Here the least number of significant digits in the measured values is two. Hence the result when rounded off to significant digit becomes 31.

1.8 DIMENSIONAL ANALYSIS

1.8.1 Dimension of Physical Quantities

In mechanics, we deal with the physical quantities like mass, time, length, velocity, acceleration, etc. which can be expressed in terms of three independent base quantities such as M, L and T. So, the dimension of a physical quantity can be defined as 'any physical quantity which is expressed in terms of base quantities whose exponent (power) represents the dimension of the physical quantity'. The notation used to denote the dimension of a physical quantity is [(physical quantity within square bracket)]. For an example, [length] means dimension of length, [area] means dimension of area, etc. The dimension of length can be expressed in terms of base quantities as

$$[\text{length}] = M^0 LT^0 = L$$

Similarly, $[\text{area}] = M^0 L^2 T^0 = L^2$

Similarly, $[\text{volume}] = M^0 L^3 T^0 = L^3$

Note that in all the cases, the base quantity L is same but exponent (power) are different, which means dimensions are different. For a pure number, exponent of base quantity is zero. For example, consider the number 2, which has no dimension and can be expressed as

$$\Rightarrow [2] = M^0 L^0 T^0 \qquad \text{(dimensionless)}$$

Let us write down the dimensions of a few more physical quantities.

$$\text{Speed}, s = \frac{\text{distance}}{\text{time taken}} \Rightarrow [s] = \frac{L}{T} = LT^{-1}$$

$$\text{Velocity}, \vec{v} = \frac{\text{displacement}}{\text{time taken}} \Rightarrow [\vec{v}] = \frac{L}{T} = LT^{-1}$$

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Note that speed is a scalar quantity and velocity is a vector quantity (scalar and vector will be discussed in Unit 2) but both of them have the same dimensional formula.

$$\text{Acceleration}, \vec{a} = \frac{\text{velocity}}{\text{time taken}} \Rightarrow [\vec{a}] = \frac{\mathrm{LT}^{-1}}{\mathrm{T}} = \mathrm{LT}^{-2}$$

Acceleration is velocity per time.

Linear momentum or Momentum,

$$[\vec{p}] = m\vec{v} \Rightarrow [\vec{p}] = MLT^{-1}$$

$$\text{Force}, \vec{F} = m\vec{a} \Rightarrow [\vec{F}] = MLT^{-2} = \frac{\text{Momentum}}{\text{time}}$$

This is true for any kind of force. There are only four types of forces that exist in nature viz strong force, electromagnetic force, weak force and gravitational force. Further, frictional force, centripetal force, centrifugal force, all have the dimension MLT^{-2}.

$$\text{Impulse}, \vec{I} = \vec{F}t \Rightarrow [\vec{I}] = MLT^{-1}$$

$$= \text{dimension of momentum}$$

Angular momentum is the moment of linear momentum (discussed in unit 5).

$$\text{Angular Momentum}, \vec{\mathrm{L}} = \vec{r} \times \vec{p} \Rightarrow [\vec{\mathrm{L}}] = ML^2T^{-1}$$

Work done,

$$W = \vec{F} \cdot \vec{d} \Rightarrow [W] = ML^2T^{-2}$$

Kinetic energy

$$KE = \frac{1}{2}mv^2 \Rightarrow [KE] = \left[\frac{1}{2}\right][m][v^2]$$

Since, number $\frac{1}{2}$ is dimensionless, the dimension of kinetic energy $[KE] = [m][v^2] = ML^2T^{-2}$. Similarly, to get the dimension of potential energy, let us consider the gravitational potential energy, $PE = mgh \Rightarrow [PE] = [m][g][h]$, , where, m is the mass of the particle, g is the acceleration due to gravity and h is the height from the ground level. Hence, $[PE] = [m][g][h] = ML^2T^{-2}$. Thus, for any kind of energy (such as for internal energy, total energy etc), the dimension is

$$[Energy] = ML^2T^{-2}$$

The moment of force is known as torque, $\vec{\tau} = \vec{r} \times \vec{F} \Rightarrow [\vec{\tau}] = ML^2T^{-2}$ (Read the symbol τ as tau – Greek alphabet). Note that the dimension of torque and dimension of energy are identical but they are different physical quantities. Further one of them is a scalar (energy) and another one is a vector (torque). This means that the dimensionally same physical quantities need not be the same physical quantities.

Note

1. We may come across dimensions in different situations in physics, so we often confuse with the term 'dimension'. For instance, we come across terms like 'dimension of energy', 'motion in one dimension' and 'dimension of atom'. It should be kept in mind that dimension of physical quantity means expressing physical quantity in terms of exponent of the base quantity. Motion in one dimension, two dimensions and three dimensions implies that it gives dimension of space. Dimension of atom implies the size of the atom. So, simply writing dimension is meaningless. Hence, the meaning should be taken with the context we write.

2. All the trigonometric functions like $\sin\theta$, $\cos\theta$ etc. are dimensionless (θ is dimensionless), exponential function e^x and logarithm function $ln\ x$ are dimensionless (x must be dimensionless). Suppose we expand a function in series expansion (finite or infinite) which contain terms like, $x^o, x^1, x^2, \ldots$ then x must be dimensionless quantity.

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Table 1.11 Dimensional Formula

Physical quantity	Expression	Dimensional formula
Area (Rectangle)	length × breadth	$[L^2]$
Volume	Area × height	$[L^3]$
Density	mass / volume	$[ML^{-3}]$
Velocity	displacement/time	$[LT^{-1}]$
Acceleration	velocity / time	$[LT^{-2}]$
Momentum	mass × velocity	$[MLT^{-1}]$
Force	mass × acceleration	$[MLT^{-2}]$
Work	force × distance	$[ML^2T^{-2}]$
Power	work / time	$[ML^2T^{-3}]$
Energy	Work	$[ML^2T^{-2}]$
Impulse	force × time	$[MLT^{-1}]$
Radius of gyration	Distance	$[L]$
Pressure (or) stress	force / area	$[ML^{-1}T^{-2}]$
Surface tension	force / length	$[MT^{-2}]$
Frequency	1 / time period	$[T^{-1}]$
Moment of Inertia	mass × (distance)^2	$[ML^2]$
Moment of force (or torque)	force × distance	$[ML^2T^{-2}]$
Angular velocity	angular displacement / time	$[T^{-1}]$
Angular acceleration	angular velocity / time	$[T^{-2}]$
Angular momentum	linear momentum × distance	$[ML^2T^{-1}]$
Co-efficient of Elasticity	stress/strain	$[ML^{-1}T^{-2}]$
Co-efficient of viscosity	(force × distance) / (area × velocity)	$[ML^{-1}T^{-1}]$
Surface energy	work / area	$[MT^{-2}]$
Heat capacity	heat energy / temperature	$[ML^2T^{-2}K^{-1}]$
Charge	current × time	$[AT]$
Magnetic induction	force / (current × length)	$[MT^{-2}A^{-1}]$
Force constant	force / displacement	$[MT^{-2}]$
Gravitational constant	[force × (distance)^2] / (mass)^2	$[M^{-1}L^3T^{-2}]$
Planck's constant	energy / frequency	$[ML^2T^{-1}]$
Faraday constant	avogadro constant × elementary charge	$[AT\ mol^{-1}]$
Boltzmann constant	energy / temperature	$[ML^2\,T^{-2}\,K^{-1}]$

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1.8.2 Dimensional Quantities, Dimensionless Quantities, Principle of Homogeneity

On the basis of dimension, we can classify quantities into four categories.

(1) **Dimensional variables**

Physical quantities, which possess dimensions and have variable values are called dimensional variables. Examples are length, velocity, and acceleration etc.

(2) **Dimensionless variables**

Physical quantities which have no dimensions, but have variable values are called dimensionless variables. Examples are specific gravity, strain, refractive index etc.

(3) **Dimensional Constant**

Physical quantities which possess dimensions and have constant values are called dimensional constants. Examples are Gravitational constant, Planck's constant etc.

(4) **Dimensionless Constant**

Quantities which have constant values and also have no dimensions are called dimensionless constants. Examples are π, e (Euler's number), numbers etc.

Principle of homogeneity of dimensions

The principle of homogeneity of dimensions states that the dimensions of all the terms in a physical expression should be the same. For example, in the physical expression $v^2 = u^2 + 2as$, the dimensions of v^2, u^2 and 2 as are the same and equal to $[L^2T^{-2}]$.

1.8.3 Application and Limitations of the Method of Dimensional Analysis.

This method is used to

(i) Convert a physical quantity from one system of units to another.

(ii) Check the dimensional correctness of a given physical equation.

(iii) Establish relations among various physical quantities.

(i) To convert a physical quantity from one system of units to another

This is based on the fact that the product of the numerical values (n) and its corresponding unit (u) is a constant. i.e, n [u] = constant (or) $n_1[u_1] = n_2[u_2]$.

Consider a physical quantity which has dimension '*a*' in mass, '*b*' in length and '*c*' in time. If the fundamental units in one system are M_1, L_1 and T_1 and the other system are M_2, L_2 and T_2 respectively, then we can write, $n_1 [M_1^a L_1^b T_1^c] = n_2 [M_2^a L_2^b T_2^c]$

We have thus converted the numerical value of physical quantity from one system of units into the other system.

EXAMPLE 1.12

Convert 76 cm of mercury pressure into Nm^{-2} using the method of dimensions.

Solution

In cgs system 76 cm of mercury pressure = $76 \times 13.6 \times 980$ dyne cm^{-2}

The dimensional formula of pressure P is $[ML^{-1}T^{-2}]$

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$$P_1\left[M_1^{\,a}L_1^{\,b}T_1^{\,c}\right]=P_2\left[M_2^{\,a}L_2^{\,b}T_2^{\,c}\right]$$

We have $P_2=P_1\left[\frac{M_1}{M_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c$

$M_1 = 1g, M_2 = 1kg$

$L_1 = 1\ cm, L_2 = 1m$

$T_1 = 1\ s, T_2 = 1s$

As a =1, b= -1, and c = -2

Then

$$P_2=76\times13.6\times980\left[\frac{1g}{1kg}\right]^1\left[\frac{1cm}{1m}\right]^{-1}\left[\frac{1s}{1s}\right]^{-2}$$

$$=76\times13.6\times980\left[\frac{10^{-3}kg}{1kg}\right]^1\left[\frac{10^{-2}m}{1m}\right]^{-1}\left[\frac{1s}{1s}\right]^{-2}$$

$$=76\times13.6\times980\times[10^{-3}]\times10^2$$

$$P_2=1.01\times10^5\ Nm^{-2}$$

EXAMPLE 1.13

If the value of universal gravitational constant in SI is $6.6\text{x}10^{-11}\ Nm^2\ kg^{-2}$, then find its value in CGS System?

Solution

Let G_{SI} be the gravitational constant in the SI system and G_{cgs} in the cgs system. Then

$$G_{SI}=6.6\times10^{-11}Nm^2kg^{-2}$$

$$G_{cgs}=?$$

$$n_2=n_1\left[\frac{M_1}{M_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c$$

$$G_{cgs}=G_{SI}\left[\frac{M_1}{M_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c$$

$M_1 = 1\ kg \quad L_1 = 1m \quad T_1 = 1s$

$M_2 = 1\ g \quad L_2 = 1\ cm \quad T_2 = 1s$

The dimensional formula for G is $M^{-1}L^3T^{-2}$

a = -1 b = 3 and c = -2

$$G_{cgs}=6.6\times10^{-11}\left[\frac{1kg}{1g}\right]^{-1}\left[\frac{1m}{1cm}\right]^3\left[\frac{1s}{1s}\right]^{-2}$$

$$=6.6\times10^{-11}\left[\frac{1kg}{10^{-3}kg}\right]^{-1}\left[\frac{1m}{10^{-2}m}\right]^3\left[\frac{1s}{1s}\right]^{-2}$$

$$=6.6\times10^{-11}\times10^{-3}\text{x}\ 10^6\times1$$

$$G_{cgs}=6.6\times10^{-8}\,dyne\ cm^2\,g^{-2}$$

(ii) To check the dimensional correctness of a given physical equation

Let us take the equation of motion

v = u + at

Apply dimensional formula on both sides

$[LT^{-1}] = [LT^{-1}] + [LT^{-2}]\ [T]$

$$[LT^{-1}] = [LT^{-1}] + [LT^{-1}]$$

(Quantities of same dimension only can be added)

We see that the dimensions of both sides are same. Hence the equation is dimensionally correct.

EXAMPLE 1.14

Check the correctness of the equation $\frac{1}{2}mv^2=mgh$ using dimensional analysis method.

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Solution

Dimensional formula for

$$\frac{1}{2}mv^2 = [M][LT^{-1}]^2 = [ML^2T^{-2}]$$

Dimensional formula for

$$mgh = [M][LT^{-2}][L] = [ML^2T^{-2}]$$

$$[ML^2T^{-2}] = [ML^2T^{-2}]$$

Both sides are dimensionally the same, hence the equations $\frac{1}{2}mv^2 = mgh$ is dimensionally correct.

(iii) To establish the relation among various physical quantities

If the physical quantity Q depends upon the quantities Q_1, Q_2 and Q_3 ie. Q is proportional to Q_1, Q_2 and Q_3.

Then,

$$Q \propto Q_1^{\,a}\, Q_2^{\,b}\, Q_3^{\,c}$$

$$Q = k\, Q_1^{\,a}\, Q_2^{\,b}\, Q_3^{\,c}$$

where k is a dimensionless constant. When the dimensional formula of Q, Q_1, Q_2 and Q_3 are substituted, then according to the principle of homogeneity, the powers of M, L, T are made equal on both sides of the equation. From this, we get the values of a, b, c

EXAMPLE 1.15

Obtain an expression for the time period T of a simple pendulum. The time period T depends on (i) mass '*m*' of the bob (ii) length '*l*' of the pendulum and (iii) acceleration due to gravity *g* at the place where the pendulum is suspended. (Constant k = 2π) i.e

Solution

$$T \propto m^a\, l^b\, g^c$$

$$T = k m^a\, l^b\, g^c$$

Here k is the dimensionless constant. Rewriting the above equation with dimensions

$$[T^1] = [M^a]\,[L^b]\,[LT^{-2}]^c$$

$$[M^0L^0T^1] = [M^a\, L^{b+c}\, T^{-2c}]$$

Comparing the powers of M, L and T on both sides, a=0, b+c=0, -2c=1

Solving for a,b and c a = 0, b = 1/2, and c = −1/2

From the above equation T = $km^0\, \ell^{1/2}\, g^{-1/2}$

$$T = k\left(\frac{\ell}{g}\right)^{1/2} = k\sqrt{\ell/g}$$

Experimentally k = 2π, hence $T = 2\pi\sqrt{\ell/g}$

Limitations of Dimensional analysis

1. This method gives no information about the dimensionless constants in the formula like 1, 2, ……..π, e (Euler number), etc.
2. This method cannot decide whether the given quantity is a vector or a scalar.
3. This method is not suitable to derive relations involving trigonometric, exponential and logarithmic functions.
4. It cannot be applied to an equation involving more than three physical quantities.
5. It can only check on whether a physical relation is dimensionally correct but not the correctness of the relation. For example using dimensional analysis, $s = ut + \frac{1}{3}at^2$ is dimensionally correct whereas the correct relation is $s = ut + \frac{1}{2}at^2$.

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SUMMARY

- Physics is an experimental science in which measurements made must be expressed in units.
- All physical quantities have a magnitude (size) and a unit.
- The SI unit of length, mass, time, temperature, electric current, amount of substance and luminous intensity are metre, kilogram, second, kelvin, ampere, mole and candela respectively.
- Units of all mechanical, electrical, magnetic and thermal quantities are derived in terms of these base units.
- Screw gauge, vernier caliper methods are available for the measurement of length in the case of small distances.
- Parallax, RADAR methods are available for the measurement of length in the case of long distances.
- The uncertainty in a measurement is called error. The accuracy of a measurement is a measure of how close the measured value is to the true value of the quantity. Every accurate measurement is precise but every precise measurement need not be accurate.
- When two or more quantities are added or subtracted, the result can be as precise as the least of the individual precisions. When the quantities are multiplied or divided, the result has the same number of significant figures as the quantity with the smallest number of significant figures.
- Dimensional analysis is used to perform quick check on the validity of equations. Whenever the quantities are added, subtracted or equated, they must have the same dimension. A dimensionally correct equation may not be a true equation but every true equation is necessarily dimensionally correct.

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CONCEPT MAP

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EVALUATION

I. Multiple Choice Questions

1. One of the combinations from the fundamental physical constants is $\frac{hc}{G}$. The unit of this expression is

a) kg^2 b) m^3

c) s^{-1} d) m

2. If the error in the measurement of radius is 2%, then the error in the determination of volume of the sphere will be

a) 8% b) 2%

c) 4% d) 6%

3. If the length and time period of an oscillating pendulum have errors of 1% and 3% respectively then the error in measurement of acceleration due to gravity is

a) 4%

b) 5%

c) 6%

d) 7%

4. The length of a body is measured as 3.51 m, if the accuracy is 0.01m, then the percentage error in the measurement is

a) 351% b) 1%

c) 0.28% d) 0.035%

5. Which of the following has the highest number of significant figures?

a) 0.007 m^2

b) 2.64×10^{24} kg

c) 0.0006032 m^2

d) 6.3200 J

6. If π = 3.14, then the value of π^2 is

a) 9.8596

b) 9.860

c) 9.86

d) 9.9

7. Round of the following number 19.95 into three significant figures.

a) 19.9 b) 20.0

c) 20.1 d) 19.5

8. Which of the following pairs of physical quantities have same dimension?

a) force and power

b) torque and energy

c) torque and power

d) force and torque

9. The dimensional formula of Planck's constant h is [JEE Main, NEET]

a) $[ML^2T^{-1}]$ b) $[ML^2T^{-3}]$

c) $[MLT^{-1}]$ d) $[ML^3T^{-3}]$

10. The velocity of a particle v at an instant t is given by $v = at + bt^2$. The dimensions of b is

a) [L] b) $[LT^{-1}]$

c) $[LT^{-2}]$ d) $[LT^{-3}]$

11. The dimensional formula for gravitational constant G is

a) $[ML^3T^{-2}]$ b) $[M^{-1}L^3T^{-2}]$

c) $[M^{-1}L^{-3}T^{-2}]$ d) $[ML^{-3}T^2]$

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12. The density of a material in CGS system of units is 4 g cm^{-3} . In a system of units in which unit of length is 10 cm and unit of mass is 100 g, then the value of density of material will be

a) 0.04 b) 0.4

c) 40 d) 400

13. If the force is proportional to square of velocity, then the dimension of proportionality constant is [JEE-2000]

a) $[MLT^0]$ b) $[MLT^{-1}]$

c) $[ML^{-2}T]$ d) $[ML^{-1}T^0]$

14. The dimension of $(\mu_0\varepsilon_0)^{-\frac{1}{2}}$ is [Main AIPMT 2011]

(a) length (b) time

(c) velocity (d) force

15. Planck's constant (h), speed of light in vacuum (c) and Newton's gravitational constant (G) are taken as three fundamental constants. Which of the following combinations of these has the dimension of length? [NEET 2016 (phase II)]

(a) $\frac{\sqrt{hG}}{c^{\frac{3}{2}}}$ (b) $\frac{\sqrt{hG}}{c^{\frac{5}{2}}}$

(c) $\sqrt{\frac{hc}{G}}$ (d) $\sqrt{\frac{Gc}{h^{\frac{3}{2}}}}$

Answers:

1) a	2) d	3) d	4) c
5) d	6) c	7) b	8) b
9) a	10) d	11) b	12) c
13) d	14) c	15) a	

II. Short Answer Questions

1. Briefly explain the types of physical quantities.
2. How will you measure the diameter of the Moon using parallax method?
3. Write the rules for determining significant figures.
4. What are the limitations of dimensional analysis?
5. Define precision and accuracy. Explain with one example.

III. Long Answer Questions

1. i) Explain the use of screw gauge and vernier caliper in measuring smaller distances.

   ii) Write a note on triangulation method and radar method to measure larger distances.
2. Explain in detail the various types of errors.
3. What do you mean by propagation of errors? Explain the propagation of errors in addition and multiplication.
4. Write short notes on the following.

   a) Unit

   b) Rounding - off

   c) Dimensionless quantities
5. Explain the principle of homogeneity of dimensions. Give example.

IV. Exercises

1. In a submarine equipped with sonar, the time delay between the generation of a pulse and its echo after reflection from an enemy submarine is observed to be 80 s. If the speed of sound in water is 1460 ms^{-1}. What is the distance of enemy submarine? Ans: (58.40 km)

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2. The radius of the circle is 3.12 m. Calculate the area of the circle with regard to significant figures. Ans: (30.6 m^2)
3. Assuming that the frequency γ of a vibrating string may depend upon i) applied force (F) ii) length (ℓ) iii) mass per unit length (m), prove that $\gamma \alpha \frac{1}{l}\sqrt{\frac{F}{m}}$ using dimensional analysis. (related to JIPMER 2001)
4. Jupiter is at a distance of 824.7 million km from the Earth. Its angular diameter is measured to be 35.72″. Calculate the diameter of Jupiter. Ans: (1.428×10^5 km)
5. The measurement value of length of a simple pendulum is 20 cm known with 2 mm accuracy. The time for 50 oscillations was measured to be 40 s within 1 s resolution. Calculate the percentage accuracy in the determination of acceleration due to gravity 'g' from the above measurement. Ans: (6%)

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ICT CORNER

Screw Gauge and Vernier Caliper

Measure with Pleasure

STEPS:

- Get into/Access the application with the help of the link given below or the given QR code.
- To measure the given object's diameter / thickness, place the object as it should be fitted between the screw and anvil. The object can be fitted properly by adjusting the screw.
- After clicking the Answer button, You will get a box showing Your Measurement Result. Input your measured value in it and click Submit button. There you can verify your measurement if it is right or wrong.
- You can measure the diameter of various objects by clicking Next button.

STEPS:

- Use the given URL to access 'Vernier Caliper' simulation page. Click 'Play Button' to launch the simulation.
- Select the unit and set the 'Zero Error' from the dropdown above the scale.
- Click and drag the secondary scale and place the blue coloured object in between them. Find the measurement and enter it in the answer box above the scale.
- Change the size of the object by clicking and dragging the edge of the blue object and practice the measurement using Vernier Caliper.

Step1

Step2

Step3

Step4

Step1

Step2

Step3

Step4

Screw Gauge stimulation URL:

https://play.google.com/store/apps/details?id=com.priantos.screwgaugegames&hl=en

Vernier caliper stimulation URL:

http://iwant2study.org/ospsg/index.php/interactive-resources/physics/01-measurements/5-vernier-caliper#faqnoanchor

* Pictures are indicative only.

* If browser requires, allow **Flash Player** or **Java Script** to load the page.

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UNIT 2

KINEMATICS

All the laws of nature are written in the language of mathematics– Galileo

Learning Objectives

In this unit, the student is exposed to

- different types of motions (linear, rotational and oscillatory)
- the necessity for reference frames to explain the motion of objects
- the meaning of vectors, scalars and their properties
- the role of scalar and vector products in physics
- the basics of differentiation and integration
- the notions of displacement and distance and their variation with time
- the notions of speed, velocity, acceleration and their graphs.
- the notion of relative velocity
- kinematic equations of motion for constant acceleration
- analysis of various types of motion of objects under gravitational force
- radians and degrees
- uniform circular motion, centripetal acceleration and centripetal force.

2.1 INTRODUCTION

Physics is basically an experimental science and rests on two pillars–Experiments and mathematics. Two thousand three hundred years ago the Greek librarian Eratosthenes measured the radius of the Earth. The size of the atom was measured much later, only in the beginning of the 20th century. The central aspect in physics is motion. Motion is found at all levels–from microscopic level (within the atom) to macroscopic and galactic level (planetary system and beyond). In short the entire Universe is governed by various types of motion. The study of various types of motion is expressed using the language of mathematics.

How do objects move? How fast or slow do they move? For example, when ten athletes run in a race, all of them do not run in the same manner. Their performance cannot be qualitatively recorded by usage of words like 'fastest', 'faster', 'average', 'slower' or 'slowest'. It has to be quantified. Quantifying means assigning numbers to each athlete's motion. Comparing these numbers, one can analyse how fast or slow each athlete runs when compared to others. In this unit, the basic mathematics needed for analyzing motion in terms of its direction and magnitude is covered.

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Kinematics is the branch of mechanics which deals with the motion of objects without taking force into account. The Greek word ***"kinema"*** means ***"motion".***

2.2 CONCEPT OF REST AND MOTION

The concept of rest and motion can be well understood by the following elucidation (Figure 2.1). A person sitting in a moving bus is at rest with respect to a fellow passenger but is in motion with respect to a person outside the bus. The concepts of rest and motion have meaning only with respect to some reference frame. To understand rest or motion we need a convenient fixed reference frame.

Figure 2.1 Frame of Reference

Frame of Reference:

If we imagine a coordinate system and the position of an object is described relative to it, then such a coordinate system is called frame of reference.

At any given instant of time, the frame of reference with respect to which the position of the object is described in terms of position coordinates (x, y, z) (i.e., distances of the given position of an object along the x, y, and z–axes.) is called **"Cartesian coordinate system"** as shown in Figure 2.2

Figure 2.2 Cartesian coordinate system

It is to be noted that if the *x, y* and *z* axes are drawn in anticlockwise direction then the coordinate system is called as "right–handed Cartesian coordinate system". Though other coordinate systems do exist, in physics we conventionally follow the right–handed coordinate system as shown in Figure 2.3.

Figure 2.3 Right handed coordinate system

The following Figure 2.4 illustrates the difference between left and right handed coordinate systems.

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Figure 2.4 Right and left handed coordinate systems

Point mass

To explain the motion of an object which has finite mass, the concept of "point mass" is required and is very useful. Let the mass of any object be assumed to be concentrated at a point. Then this idealized mass is called "point mass". It has no internal structure like shape and size. Mathematically a point mass has finite mass with zero dimension. Even though in reality a point mass does not exist, it often simplifies our calculations. It is to be noted that the term "point mass" is a relative term. It has meaning only with respect to a reference frame and with respect to the kind of motion that we analyse.

Examples

- To analyse the motion of Earth with respect to Sun, Earth can be treated as a point mass. This is because the distance between the Sun and Earth is very large compared to the size of the Earth.
- If we throw an irregular object like a small stone in the air, to analyse its motion it is simpler to consider the stone as a point mass as it moves in space. The size of the stone is very much smaller than the distance through which it travels.

Types of motion

In our day-to-day life the following kinds of motion are observed:

a) **Linear motion**
An object is said to be in linear motion if it moves in a straight line.

Examples

- An athlete running on a straight track
- A particle falling vertically downwards to the Earth.

b) **Circular motion**
Circular motion is defined as a motion described by an object traversing a circular path.

Examples

- The whirling motion of a stone attached to a string

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- The motion of a satellite around the Earth

These two circular motions are shown in Figure 2.5

Figure 2.5 Examples of circular motion

c) Rotational motion

If any object moves in a rotational motion about an axis, the motion is called 'rotation'. During rotation every point in the object transverses a circular path about an axis, (except the points located on the axis).

Examples

- Rotation of a disc about an axis through its centre
- Spinning of the Earth about its own axis.

These two rotational motions are shown in Figure 2.6.

Figure 2.6 Examples of Rotational motion

d) Vibratory motion

If an object or particle executes a *to–and–fro* motion about a fixed point, it is said to be in vibratory motion. This is sometimes also called oscillatory motion.

Examples

- Vibration of a string on a guitar
- Movement of a swing

These motions are shown in Figure 2.7

Figure 2.7 Examples of Vibratory motion

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Other types of motion like elliptical motion and helical motion are also possible.

Motion in One, Two and Three Dimensions

Let the position of a particle in space be expressed in terms of rectangular coordinates x, y and z. When these coordinates change with time, then the particle is said to be in motion. However, it is not necessary that all the three coordinates should together change with time. Even if one or two coordinates change with time, the particle is said to be in motion. Then we have the following classification.

(i) Motion in one dimension

One dimensional motion is the motion of a particle moving along a straight line.

This motion is sometimes known as rectilinear or linear motion.

In this motion, only one of the three rectangular coordinates specifying the position of the object changes with time.

For example, if a car moves from position A to position B along *x*–direction, as shown in Figure 2.8, then a variation in x–coordinate alone is noticed.

Figure 2.8 Motion of a particle along one dimension

Examples

- Motion of a train along a straight railway track.
- An object falling freely under gravity close to Earth.

(ii) Motion in two dimensions

If a particle is moving along a curved path in a plane, then it is said to be in two dimensional motion.

In this motion, two of the three rectangular coordinates specifying the position of object change with time.

For instance, when a particle is moving in the $y - z$ plane, x does not vary, but y and z vary as shown in Figure 2.9

Figure 2.9 Motion of a particle along two dimensions

Examples

- Motion of a coin on a carrom board.
- An insect crawling over the floor of a room.

(iii) Motion in three dimensions

A particle moving in usual three dimensional space has three dimensional motion.

In this motion, all the three coordinates specifying the position of an object change with respect to time. When a particle moves in three dimensions, all the three coordinates x, y and z will vary.

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Examples

- A bird flying in the sky.
- Random motion of a gas molecule.
- Flying of a kite on a windy day.

2.3 ELEMENTARY CONCEPTS OF VECTOR ALGEBRA

In physics, some quantities possess only magnitude and some quantities possess both magnitude and direction. To understand these physical quantities, it is very important to know the properties of vectors and scalars.

Scalar

It is a property which can be described only by magnitude. In physics a number of quantities can be described by scalars.

Examples

Distance, mass, temperature, speed and energy.

Vector

It is a quantity which is described by both magnitude and direction. Geometrically a vector is a directed line segment which is shown in Figure 2.10. In physics certain quantities can be described only by vectors.

Figure 2.10 Geometrical representation of a vector

Examples

Force, velocity, displacement, position vector, acceleration, linear momentum and angular momentum.

2.3.1 Magnitude of a Vector

The length of a vector is called magnitude of the vector. It is always a positive quantity. Sometimes the magnitude of a vector is also called 'norm' of the vector. For a vector $\vec{A}$, the magnitude or norm is denoted by $|\vec{A}|$ or simply 'A' (Figure 2.11).

Figure 2.11 Magnitude of a vector

2.3.2 Different types of Vectors

1. **Equal vectors:** Two vectors $\vec{A}$ and $\vec{B}$ are said to be equal when they have equal magnitude and same direction and represent the same physical quantity (Figure 2.12.).

Figure 2.12 Geometrical representation of equal vectors

(a) **Collinear vectors**: Collinear vectors are those which act along the same line. The angle between them can be 0° or 180°.

(i) **Parallel Vectors**: If two vectors $\vec{A}$ and $\vec{B}$ act in the same direction along the same line or on parallel lines, then the angle between them is 0^0 (Figure 2.13).

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Figure 2.13 Geometrical representation of parallel vectors

(ii) **Anti–parallel vectors:** Two vectors $\vec{A}$ and $\vec{B}$ are said to be anti–parallel when they are in opposite directions along the same line or on parallel lines. Then the angle between them is 180^0 (Figure 2.14).

Figure 2.14 Geometrical representation of anti–parallel vectors.

2. **Unit vector**: A vector divided by its magnitude is a unit vector. The unit vector for $\vec{A}$ is denoted by $\hat{A}$ (read as A cap or A hat). It has a magnitude equal to unity or one.

$$\text{Since, } \hat{A} = \frac{\vec{A}}{A} \text{ we can write } \vec{A} = A\hat{A}$$

Thus, we can say that the unit vector specifies only the direction of the vector quantity.

3. **Orthogonal unit vectors**: Let $\hat{i}, \hat{j}$ *and* $\hat{k}$ be three unit vectors which specify the directions along positive x–axis, positive y–axis and positive z–axis respectively. These three unit vectors are directed perpendicular to each other, the angle between any two of them is 90°. $\hat{i}, \hat{j}$ and $\hat{k}$ are examples of orthogonal vectors. Two vectors which are perpendicular to each other are called orthogonal vectors as is shown in the Figure 2.15

Figure 2.15 Orthogonal unit vectors

2.3.3 Addition of Vectors

Since vectors have both magnitude and direction they cannot be added by the method of ordinary algebra. Thus, vectors can be added geometrically or analytically using certain rules called 'vector algebra'. In order to find the sum (resultant) of two vectors, which are inclined to each other, we use (i) Triangular law of addition method or (ii) Parallelogram law of vectors.

Triangular Law of addition method

Let us consider two vectors $\vec{A}$ and $\vec{B}$ as shown in Figure 2.16.

Figure 2.16 Head and tail of vectors

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To find the resultant of the two vectors we apply the triangular law of addition as follows:

Represent the vectors $\vec{A}$ and $\vec{B}$ by the two adjacent sides of a triangle taken in the same order. Then the resultant is given by the third side of the triangle taken in the reverse order as shown in Figure 2.17.

Figure 2.17 Triangle law of addition

To explain further, the head of the first vector $\vec{A}$ is connected to the tail of the second vector $\vec{B}$. Let θ be the angle between $\vec{A}$ and $\vec{B}$. Then $\vec{R}$ is the resultant vector connecting the tail of the first vector $\vec{A}$ to the head of the second vector $\vec{B}$. The magnitude of $\vec{R}$ (resultant) is given geometrically by the length of $\vec{R}$ (OQ) and the direction of the resultant vector is the angle between $\vec{R}$ and $\vec{A}$. Thus we write $\vec{R} = \vec{A} + \vec{B}$.

$$\overrightarrow{OQ} = \overrightarrow{OP} + \overrightarrow{PQ}$$

(1) Magnitude of resultant vector

The magnitude and angle of the resultant vector are determined as follows.

From Figure 2.18, consider the triangle ABN, which is obtained by extending the side OA to ON. ABN is a right angled triangle.

Figure 2.18 Resultant vector and its direction by triangle law of addition.

From Figure 2.18

$$\cos\theta = \frac{AN}{B} \therefore AN = B\cos\theta \text{ and}$$

$$\sin\theta = \frac{BN}{B} \quad \therefore BN = B\sin\theta$$

For ΔOBN, we have $OB^2 = ON^2 + BN^2$

$$\Rightarrow R^2 = (A + B\cos\theta)^2 + (B\ \sin\theta)^2$$

$$\Rightarrow R^2 = A^2 + B^2\cos^2\theta + 2AB\cos\theta + B^2\sin^2\theta$$

$$\Rightarrow R^2 = A^2 + B^2\left(\cos^2\theta + \sin^2\theta\right) + 2AB\cos\theta$$

$$\Rightarrow R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

which is the magnitude of the resultant of $\vec{A}$ and $\vec{B}$

(2) Direction of resultant vectors: If θ is the angle between $\vec{A}$ and $\vec{B}$, then

$$\left|\vec{A} + \vec{B}\right| = \sqrt{A^2 + B^2 + 2AB\cos\theta} \quad (2.1)$$

If $\vec{R}$ makes an angle α with $\vec{A}$, then in ΔOBN,

$$\tan\alpha = \frac{BN}{ON} = \frac{BN}{OA + AN}$$

$$\tan\alpha = \frac{B\sin\theta}{A + B\cos\theta}$$

$$\Rightarrow \alpha = \tan^{-1}\left(\frac{B\sin\theta}{A + B\cos\theta}\right)$$

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EXAMPLE 2.1

Two vectors $\vec{A}$ and $\vec{B}$ of magnitude 5 units and 7 units respectively make an angle 60° with each other as shown below. Find the magnitude of the resultant vector and its direction with respect to the vector $\vec{A}$.

Solution

By following the law of triangular addition, the resultant vector is given by

$$\vec{R} = \vec{A} + \vec{B}$$

as illustrated below

The magnitude of the resultant vector $\vec{R}$ is given by

$$R = |\vec{R}| = \sqrt{5^2 + 7^2 + 2 \times 5 \times 7 \cos 60^\circ}$$

$$R = \sqrt{25 + 49 + \frac{70 \times 1}{2}} = \sqrt{109}\, units$$

The angle α between $\vec{R}$ and $\vec{A}$ is given by

$$\tan\alpha = \frac{B \sin\theta}{A + B\cos\theta} \qquad (2.2)$$

$$\tan\alpha = \frac{7 \times \sin 60^\circ}{5 + 7\cos 60^\circ} = \frac{7\sqrt{3}}{10+7} = \frac{7\sqrt{3}}{17} \cong 0.713$$

$$\therefore \alpha \cong 35^\circ$$

Note Another method to determine the resultant and angle of resultant of two vectors is the Parallelogram Law of vector addition method. It is given in appendix 2.1

2.3.4 Subtraction of vectors

Since vectors have both magnitude and direction two vectors cannot be subtracted from each other by the method of ordinary algebra. Thus, this subtraction can be done either geometrically or analytically. We shall now discuss subtraction of two vectors geometrically using the Figure 2.19

For two non-zero vectors $\vec{A}$ and $\vec{B}$ which are inclined to each other at an angle θ, the difference $\vec{A} - \vec{B}$ is obtained as follows. First obtain $-\vec{B}$ as in Figure 2.19. The angle between $\vec{A}$ and $-\vec{B}$ is 180–θ.

Figure 2.19 Subtraction of vectors

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The difference $\vec{A}-\vec{B}$ is the same as the resultant of $\vec{A}$ and $-\vec{B}$.

We can write $\vec{A}-\vec{B}=\vec{A}+\left(-\vec{B}\right)$ and using the equation (2.1), we have

$$\left|\vec{A}-\vec{B}\right|=\sqrt{A^2+B^2+2AB\cos(180-\theta)} \quad (2.3)$$

Since, $\cos\left(180-\theta\right)=-\cos\theta$, we get

$$\Rightarrow\left|\vec{A}-\vec{B}\right|=\sqrt{A^2+B^2-2AB\cos\theta} \quad (2.4)$$

Again from the Figure 2.19, and using an equation similar to equation (2.2) we have

$$\tan\alpha_2=\frac{B\sin\left(180^\circ-\theta\right)}{A+B\cos\left(180^\circ-\theta\right)} \quad (2.5)$$

But $\sin\left(180^\circ-\theta\right)=\sin\theta$ hence we get

$$\Rightarrow\tan\alpha_2=\frac{B\sin\theta}{A-B\cos\theta} \quad (2.6)$$

Thus the difference $\vec{A}-\vec{B}$ is a vector with magnitude and direction given by equations 2.4 and 2.6 respectively.

EXAMPLE 2.2

Two vectors $\vec{A}$ and $\vec{B}$ of magnitude 5 units and 7 units make an angle 60° with each other. Find the magnitude of the difference vector $\vec{A}-\vec{B}$ and its direction with respect to the vector $\vec{A}$.

Solution

Using the equation (2.4),

$$\left|\vec{A}-\vec{B}\right|=\sqrt{5^2+7^2-2\times5\times7\cos60^\circ}$$

$$=\sqrt{25+49-35}=\sqrt{39}\ units$$

The angle that $\vec{A}-\vec{B}$ makes with the vector $\vec{A}$ is given by

$$\tan\alpha_2=\frac{7\sin60^\circ}{5-7\cos60^\circ}=\frac{7\sqrt{3}}{10-7}=\frac{7}{\sqrt{3}}=4.041$$

$$\alpha_2=\tan^{-1}\left(4.041\right)\cong76^\circ$$

2.4 COMPONENTS OF A VECTOR

In the Cartesian coordinate system any vector $\vec{A}$ can be resolved into three components along *x*, *y* and *z* directions. This is shown in Figure 2.20.

Consider a 3–dimensional coordinate system. With respect to this a vector can be written in component form as

$$\vec{A}=A_x\hat{i}+A_y\hat{j}+A_z\hat{k}$$

Figure 2.20 Components of a vector in 2 dimensions and 3 dimensions

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Here A_x is the x–component of $\vec{A}$, A_y is the y–component of $\vec{A}$ and A_z is the z component of $\vec{A}$.

In a 2–dimensional Cartesian coordinate system (which is shown in the Figure 2.20) the vector $\vec{A}$ is given by

$$\vec{A} = A_x\hat{i} + A_y\hat{j}$$

If $\vec{A}$ makes an angle θ with x axis, and A_x and A_y are the components of $\vec{A}$ along x–axis and y–axis respectively, then as shown in Figure 2.21,

$$A_x = A\cos\theta,\ \ A_y = A\sin\theta$$

where 'A' is the magnitude (length) of the vector $\vec{A}$, $A = \sqrt{A_x^2 + A_y^2}$

Figure 2.21 Resolution of a vector

EXAMPLE 2.3

What are the unit vectors along the negative x–direction, negative y–direction, and negative z– direction?

Solution

The unit vectors along the negative directions can be shown as in the following figure.

Then we have:

The unit vector along the negative x direction = $-\hat{i}$

The unit vector along the negative y direction = $-\hat{j}$

The unit vector along the negative z direction = $-\hat{k}$.

2.4.1 Vector addition using components

In the previous section we have learnt about addition and subtraction of two vectors using geometric methods. But once we choose a coordinate system, the addition and subtraction of vectors becomes much easier to perform.

The two vectors $\vec{A}$ and $\vec{B}$ in a Cartesian coordinate system can be expressed as

$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$
$$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$$

Then the addition of two vectors is equivalent to adding their corresponding x, y and z components.

$$\vec{A} + \vec{B} = (A_x + B_x)\hat{i} + (A_y + B_y)\hat{j} + (A_z + B_z)\hat{k}$$

Similarly the subtraction of two vectors is equivalent to subtracting the corresponding x, y and z components.

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$$\vec{A}-\vec{B}=\left(A_x-B_x\right)\hat{i}+\left(A_y-B_y\right)\hat{j}+\left(A_z-B_z\right)\hat{k}$$

The above rules form an analytical way of adding and subtracting two vectors.

EXAMPLE 2.4

Two vectors $\vec{A}$ and $\vec{B}$ are given in the component form as $\vec{A}=5\hat{i}+7\hat{j}-4\hat{k}$ and $\vec{B}=6\hat{i}+3\hat{j}+2\hat{k}$. Find $\vec{A}+\vec{B}$, $\vec{B}+\vec{A}$, $\vec{A}-\vec{B}$, $\vec{B}-\vec{A}$

Solution

$$\vec{A}+\vec{B}=(5\hat{i}+7\hat{j}-4\hat{k})+(6\hat{i}+3\hat{j}+2\hat{k})$$
$$=11\hat{i}+10\hat{j}-2\hat{k}$$
$$\vec{B}+\vec{A}=(6\hat{i}+3\hat{j}+2\hat{k})+(5\hat{i}+7\hat{j}-4\hat{k})$$
$$=(6+5)\hat{i}+(3+7)\hat{j}+(2-4)\hat{k}$$
$$=11\hat{i}+10\hat{j}-2\hat{k}$$
$$\vec{A}-\vec{B}=(5\hat{i}+7\hat{j}-4\hat{k})-(6\hat{i}+3\hat{j}+2\hat{k})$$
$$=-\hat{i}+4\hat{j}-6\hat{k}$$
$$\vec{B}-\vec{A}=\hat{i}-4\hat{j}+6\hat{k}$$

Note that the vectors $\vec{A}+\vec{B}$ and $\vec{B}+\vec{A}$ are same and the vectors $\vec{A}-\vec{B}$ and $\vec{B}-\vec{A}$ are opposite to each other.

Note

The addition of two vectors using components depends on the choice of the coordinate system. But the geometric way of adding and subtracting two vectors is independent of the coordinate system used.

2.5 MULTIPLICATION OF VECTOR BY A SCALAR

A vector $\vec{A}$ multiplied by a scalar λ results in another vector, $\lambda\vec{A}$. If λ is a positive number then $\lambda\vec{A}$ is also in the direction of $\vec{A}$. If λ is a negative number, $\lambda\vec{A}$ is in the opposite direction to the vector $\vec{A}$.

EXAMPLE 2.5

Given the vector $\vec{A}=2\hat{i}+3\hat{j}$, what is 3 $\vec{A}$?

Solution

$$3\vec{A}=3\left(2\hat{i}+3\hat{j}\right)=6\hat{i}+9\hat{j}$$

The vector $3\vec{A}$ is in the same direction as vector $\vec{A}$.

EXAMPLE 2.6

A vector $\vec{A}$ is given as in the following Figure. Find $4\vec{A}$ and $-4\vec{A}$

Solution

In physics, certain vector quantities can be defined as a scalar times another vector quantity.

For example

1) Force $\vec{F}=m\vec{a}$. Here mass 'm' is a scalar, and $\vec{a}$ is the acceleration. Since 'm' is always a positive scalar, the direction

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of force is always in the direction of acceleration.

2) Linear momentum $\vec{P} = m\vec{v}$. Here $\vec{v}$ is the velocity. The direction of linear momentum is also in the direction of velocity.

3) Force $\vec{F} = q\vec{E}$, Here the electric charge 'q' is a scalar, and $\vec{E}$ is the electric field. Since charge can be positive or negative, the direction of force $\vec{F}$ is correspondingly either in the direction of $\vec{E}$ or opposite to the direction of $\vec{E}$.

2.5.1 Scalar Product of Two Vectors

Definition

The scalar product (or dot product) of two vectors is defined as the product of the magnitudes of both the vectors and the cosine of the angle between them.

Thus if there are two vectors $\vec{A}$ and $\vec{B}$ having an angle θ between them, then their scalar product is defined as $\vec{A}\cdot\vec{B} = AB\cos\theta$. Here, A and B are magnitudes of $\vec{A}$ and $\vec{B}$.

Properties

(i) The product quantity $\vec{A}\cdot\vec{B}$ is always a scalar. It is positive if the angle between the vectors is acute (i.e., θ < 90°) and negative if the angle between them is obtuse (i.e. 90°<θ< 180°).

(ii) The scalar product is commutative, i.e. $\vec{A}.\vec{B} = \vec{B}.\vec{A}$

(iii) The vectors obey distributive law i.e. $\vec{A}.\left(\vec{B}+\vec{C}\right) = \vec{A}.\vec{B} + \vec{A}.\vec{C}$

(iv) The angle between the vectors

$$\theta = \cos^{-1}\left[\frac{\vec{A}\cdot\vec{B}}{AB}\right]$$

(v) The scalar product of two vectors will be maximum when cos θ = 1, i.e. θ = 0°, i.e., when the vectors are parallel;

$$(\vec{A}\cdot\vec{B})_{max} = AB$$

(vi) The scalar product of two vectors will be minimum, when cos θ = –1, i.e. θ = 180°

$(\vec{A}\cdot\vec{B})_{min} = -AB$, when the vectors are anti-parallel.

(vii) If two vectors $\vec{A}$ and $\vec{B}$ are perpendicular to each other then their scalar product $\vec{A}\cdot\vec{B} = 0$, because cos 90°=0. Then the vectors $\vec{A}$ and $\vec{B}$ are said to be mutually orthogonal.

(viii) The scalar product of a vector with itself is termed as self–dot product and is given by $\left(\vec{A}\right)^2 = \vec{A}\cdot\vec{A} = AA\cos\theta = A^2$. Here angle $\theta = 0°$

The magnitude or norm of the vector $\vec{A}$ is $\left|\vec{A}\right| = A = \sqrt{\vec{A}\cdot\vec{A}}$

(ix) In case of a unit vector $\hat{n}$

$\hat{n}\cdot\hat{n} = 1 \times 1 \times \cos 0 = 1$. For example, $\hat{i}.\hat{i} = \hat{j}.\hat{j} = \hat{k}.\hat{k} = 1$

(x) In the case of orthogonal unit vectors $\hat{i}$, $\hat{j}$ and $\hat{k}$,

$$\hat{i}.\hat{j} = \hat{j}.\hat{k} = \hat{k}.\hat{i} = 1\cdot 1\cos 90° = 0$$

(xi) In terms of components the scalar product of $\vec{A}$ and $\vec{B}$ can be written as

$$\vec{A}\cdot\vec{B} = (A_x\hat{i} + A_y\hat{j} + A_z\hat{k})\cdot\left(B_x\hat{i} + B_y\hat{j} + B_z\hat{k}\right)$$

$= A_xB_x + A_yB_y + A_zB_z$, with all other terms zero.

The magnitude of vector $\left|\vec{A}\right|$ is given by

$$\left|\vec{A}\right| = A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

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EXAMPLE 2.7

Given two vectors $\vec{A} = 2\hat{i} + 4\hat{j} + 5\hat{k}$ and $\vec{B} = \hat{i} + 3\hat{j} + 6\hat{k}$, Find the product $\vec{A}.\vec{B}$, and the magnitudes of $\vec{A}$ and $\vec{B}$. What is the angle between them?

Solution

$$\vec{A}.\vec{B} = 2 + 12 + 30 = 44$$

$$\text{Magnitude A} = \sqrt{4+16+25} = \sqrt{45} \text{ units}$$

$$\text{Magnitude B} = \sqrt{1+9+36} = \sqrt{46} \text{ units}$$

The angle between the two vectors is given by

$$\theta = \cos^{-1}\left(\frac{\vec{A}.\vec{B}}{AB}\right)$$

$$= \cos^{-1}\left(\frac{44}{\sqrt{45\times 46}}\right) = \cos^{-1}\left(\frac{44}{45.49}\right)$$

$$= \cos^{-1}(0.967)$$

$$\therefore \theta \cong 15^\circ$$

EXAMPLE 2.8

Check whether the following vectors are orthogonal.

i) $\vec{A} = 2\hat{i} + 3\hat{j}$ and $\vec{B} = 4\hat{i} - 5\hat{j}$

ii) $\vec{C} = 5\hat{i} + 2\hat{j}$ and $\vec{D} = 2\hat{i} - 5\hat{j}$

Solution

$$\vec{A}.\vec{B} = 8 - 15 = -7 \neq 0$$

Hence $\vec{A}$ and $\vec{B}$ are not orthogonal to each other.

$$\vec{C}.\vec{D} = 10 - 10 = 0$$

Hence, $\vec{C}$ and $\vec{D}$ are orthogonal to each other.

It is also possible to geometrically show that the vectors $\vec{C}$ *and* $\vec{D}$ are orthogonal to each other. This is shown in the following Figure.

In physics, the work done by a force $\vec{F}$ to move an object through a small displacement $d\vec{r}$ is defined as,

$$W = \vec{F}.d\vec{r}$$

$$W = F\ dr\ \cos\theta$$

The work done is basically a scalar product between the force vector and the displacement vector. Apart from work done, there are other physical quantities which are also defined through scalar products.

Note In uniform circular motion, the centripetal force is perpendicular to the displacement. Hence, work done by this force is zero.

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2.5.2 The Vector Product of Two Vectors

Definition

The vector product or cross product of two vectors is defined as another vector having a magnitude equal to the product of the magnitudes of two vectors and the sine of the angle between them. The direction of the product vector is perpendicular to the plane containing the two vectors, in accordance with the right hand screw rule or right hand thumb rule (Figure 2.22).

Thus, if $\vec{A}$ and $\vec{B}$ are two vectors, then their vector product is written as $\vec{A}\times\vec{B}$ which is a vector $\vec{C}$ defined by

$$\vec{C} = \vec{A}\times\vec{B} = (AB\sin\theta)\hat{n}$$

The direction $\hat{n}$ of $\vec{A}\times\vec{B}$, i.e., $\vec{C}$ is perpendicular to the plane containing the vectors $\vec{A}$ and $\vec{B}$ and is in the sense of advancement of a right handed screw rotated from $\vec{A}$ (first vector) to $\vec{B}$ (second vector) through the smaller angle between them. Thus, if a right-handed screw whose axis is perpendicular to the plane formed by $\vec{A}$ and $\vec{B}$, is rotated from $\vec{A}$ to $\vec{B}$ through the smaller angle between them, then the direction of advancement of the screw gives the direction of $\vec{A}\times\vec{B}$ i.e. $\vec{C}$ which is illustrated in Figure 2.22.

Note According to Right Hand Rule, if the curvature of the fingers of the right hand represents the sense of rotation of the object, then the thumb, held perpendicular to the curvature of the fingers, represents the direction of the resultant $\vec{C}$.

VECTOR PRODUCT ("CROSS" PRODUCT)

The vector product of $\vec{A}$ and $\vec{B}$, written as $\vec{A}\times\vec{B}$, produces a third vector $\vec{C}$

$$\vec{C} = \vec{A}\times\vec{B} = |\vec{A}||\vec{B}|\sin\theta\,\hat{n} \qquad -\vec{C} = \vec{B}\times\vec{A}$$

$$\vec{A}\times\vec{B} = -(\vec{B}\times\vec{A})$$

Figure 2.22 Vector product of two vectors

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Properties of vector (cross) product.

(i) The vector product of any two vectors is always another vector whose direction is perpendicular to the plane containing these two vectors, i.e., orthogonal to both the vectors $\vec{A}$ and $\vec{B}$, even though the vectors $\vec{A}$ and $\vec{B}$ may or may not be mutually orthogonal.

(ii) The vector product of two vectors is not commutative, i.e., $\vec{A}\times\vec{B} \neq \vec{B}\times\vec{A}$ But., $\vec{A}\times\vec{B} = -[\vec{B}\times\vec{A}]$

Here it is worthwhile to note that $|\vec{A}\times\vec{B}| = |\vec{B}\times\vec{A}| = AB\sin\theta$ i.e., in the case of the product vectors $\vec{A}\times\vec{B}$ and $\vec{B}\times\vec{A}$, the magnitudes are equal but directions are opposite to each other.

(iii) The vector product of two vectors will have maximum magnitude when $\sin\theta = 1$, i.e., $\theta = 90°$ i.e., when the vectors $\vec{A}$ and $\vec{B}$ are orthogonal to each other.

$$\left(\vec{A}\times\vec{B}\right)_{max} = AB\hat{n}$$

(iv) The vector product of two non–zero vectors will be minimum when $|\sin\theta| = 0$, $i.e., \theta = 0°$ or $180°$

$$\left(\vec{A}\times\vec{B}\right)_{min} = 0$$

i.e., the vector product of two non–zero vectors vanishes, if the vectors are either parallel or antiparallel.

(v) The self–cross product, i.e., product of a vector with itself is the null vector

$$\vec{A}\times\vec{A} = AA\sin 0°\hat{n} = \vec{0}.$$

In physics the null vector $\vec{0}$ is simply denoted as zero.

(vi) The self–vector products of unit vectors are thus zero.

$$\hat{i}\times\hat{i} = \hat{j}\times\hat{j} = \hat{k}\times\hat{k} = \vec{0}$$

(vii) In the case of orthogonal unit vectors, $\hat{i}, \hat{j}, \hat{k}$, in accordance with the right hand screw rule:

$$\hat{i}\times\hat{j} = \hat{k},\ \hat{j}\times\hat{k} = \hat{i}\ \ and\ \ \hat{k}\times\hat{i} = \hat{j}$$

Also, since the cross product is not commutative,

$$\hat{j}\times\hat{i} = -\hat{k}, \hat{k}\times\hat{j} = -\hat{i}$$
$$and\ \hat{i}\times\hat{k} = -\hat{j}$$

(viii) In terms of components, the vector product of two vectors $\vec{A}$ and $\vec{B}$ is

$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

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$$= \hat{i}\left(A_y B_z - A_z B_y\right) + \hat{j}\left(A_z B_x - A_x B_z\right) + \hat{k}\left(A_x B_y - A_y B_x\right)$$

Note that in the $\hat{j}^{th}$ component the order of multiplication is different than $\hat{i}^{th}$ and $\hat{k}^{th}$ components.

(ix) If two vectors $\vec{A}$ *and* $\vec{B}$ form adjacent sides in a parallelogram, then the magnitude of $\vec{A}\times\vec{B}$ will give the area of the parallelogram as represented graphically in Figure 2.23.

Figure 2.23 Area of parallelogram

(x) Since we can divide a parallelogram into two equal triangles as shown in the Figure 2.24, the area of a triangle with $\vec{A}$ *and* $\vec{B}$ as sides is $\frac{1}{2}\left|\vec{A}\times\vec{B}\right|$. This is shown in the Figure 2.24. (This fact will be used when we study Kepler's laws in unit 6)

Figure 2.24 Area of triangle

A number of quantities used in Physics are defined through vector products. Particularly physical quantities representing rotational effects like torque, angular momentum, are defined through vector products.

Examples

(i) Torque $\vec{\tau} = \vec{r}\times\vec{F}$. where $\vec{F}$ is Force and $\vec{r}$ is position vector of a particle

(ii) Angular momentum $\vec{L} = \vec{r}\times\vec{p}$ where $\vec{p}$ is the linear momentum

(iii) Linear Velocity $\vec{v} = \vec{\omega}\times\vec{r}$ where $\vec{\omega}$ is angular velocity

EXAMPLE 2.9

Two vectors are given as $\vec{r} = 2\hat{i} + 3\hat{j} + 5\hat{k}$ and $\vec{F} = 3\hat{i} - 2\hat{j} + 4\hat{k}$. Find the resultant vector $\vec{\tau} = \vec{r}\times\vec{F}$

Solution

$$\vec{\tau} = \vec{r}\times\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 5 \\ 3 & -2 & 4 \end{vmatrix}$$

$$\vec{\tau} = (12 - (-10)\hat{i} + (15-8)\hat{j} + (-4-9)\hat{k}$$

$$\vec{\tau} = 22\hat{i} + 7\hat{j} - 13\hat{k}$$

2.5.3 Properties of the components of vectors

If two vectors $\vec{A}$ and $\vec{B}$ are equal, then their individual components are also equal.

Let $\vec{A} = \vec{B}$

Then $A_x\hat{i} + A_y\hat{j} + A_z\hat{k} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$

i.e., $A_x = B_x$, $A_y = B_y$, $A_z = B_z$

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EXAMPLE 2.10

Compare the components for the following vector equations

a) $\vec{F} = m\vec{a}$ Here m is positive number

b) $\vec{p} = 0$

Solution

Case (a):

$$\vec{F} = m\vec{a}$$

$$F_x\hat{i} + F_y\hat{j} + F_z\hat{k} = ma_x\hat{i} + ma_y\hat{j} + ma_z\hat{k}$$

By comparing the components, we get

$$F_x = ma_x, F_y = ma_y, F_z = ma_z$$

This implies that one vector equation is equivalent to three scalar equations.

Case (b)

$$\vec{p} = 0$$

$$p_x\hat{i} + p_y\hat{j} + p_z\hat{k} = 0\hat{i} + 0\hat{j} + 0\hat{k}$$

By comparing the components, we get

$$p_x = 0, \mathrm{p_y} = 0, \mathrm{p_z} = 0$$

EXAMPLE 2.11

Determine the value of the T from the given vector equation.

$$5\hat{j} - T\hat{j} = 6\hat{j} + 3T\hat{j}$$

Solution

By comparing the components both sides, we can write

$$5 - 6 = 3T + T$$

$$-1 = 4T$$

$$T = -\frac{1}{4}$$

EXAMPLE 2.12

Compare the components of vector equation $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = \vec{F}_4$

Solution

We can resolve all the vectors in x, y and z components with respect to Cartesian coordinate system.

Once we resolve the components we can separately equate the x components on both sides, y components on both sides, and z components on both the sides of the equation, we then get

$$F_{1x} + F_{2x} + F_{3x} = F_{4x}$$

$$F_{1y} + F_{2y} + F_{3y} = F_{4y}$$

$$F_{1z} + F_{2z} + F_{3z} = F_{4z}$$

2.6 POSITION VECTOR

It is a vector which denotes the position of a particle at any instant of time, with respect to some reference frame or coordinate system.

The position vector $\vec{r}$ of the particle at a point P is given by

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$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

where x, y and z are components of $\vec{r}$, Figure 2.25 shows the position vector $\vec{r}$.

Figure 2.25 Position vector in Cartesian coordinate system

EXAMPLE 2.13

Determine the position vectors for the following particles which are located at points P, Q, R, S.

Solution

The position vector for the point P is

$$\vec{r}_P = 3\hat{i}$$

The position vector for the point Q is

$$\vec{r}_Q = 5\hat{i} + 4\hat{j}$$

The position vector for the point R is

$$\vec{r}_R = -2\hat{i}$$

The position vector for the point S is

$$\vec{r}_s = 3\hat{i} - 6\hat{j}$$

EXAMPLE 2.14

A person initially at rest starts to walk 2 m towards north, then 1 m towards east, then 5 m towards south and then 3 m towards west. What is the position vector of the person at the end of the trip?

Solution

As shown in the Figure, the positive x axis is taken as east direction, positive y direction is taken as north.

After the trip, the person reaches the point P whose position vector given by

$$\vec{r} = -2\hat{i} - 3\hat{j}$$

The displacement direction is south west.

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2.7 DISTANCE AND DISPLACEMENT

Distance is the actual path length travelled by an object in the given interval of time during the motion. It is a positive scalar quantity.

Displacement is the difference between the final and initial positions of the object in a given interval of time. It can also be defined as the shortest distance between these two positions of the object and its direction is from the initial to final position of the object, during the given interval of time. It is a vector quantity. Figure 2.26 illustrates the difference between displacement and distance.

Figure 2.26 Distance and displacement

EXAMPLE 2.15

Assume your school is located 2 km away from your home. In the morning you are going to school and in the evening you come back home. In this entire trip what is the distance travelled and the displacement covered?

Solution

The displacement covered is zero. It is because your initial and final positions are the same.

But the distance travelled is 4 km.

EXAMPLE 2.16

An athlete covers 3 rounds on a circular track of radius 50 m. Calculate the total distance and displacement travelled by him.

Solution

The total distance the athlete covered =3x circumference of track

$$\text{Distance} = 3 \times 2\pi \times 50\ m$$
$$= 300\pi\ m \quad \text{(or)}$$
$$\text{Distance} \approx 300 \text{ x } 3.14 \approx 942\ m$$

The displacement is zero, since the athlete reaches the same point A after three rounds from where he started.

2.7.1 Displacement Vector in Cartesian Coordinate System

In terms of position vector, the displacement vector is given as follows. Let us consider a particle moving from a point P_1 having position vector $\vec{r}_1 = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$ to a

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point P_2 where its position vector is $\vec{r}_2 = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$.

The displacement vector is given by

$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$$
$$= (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}.$$

This displacement is also shown in Figure 2.27.

Figure 2.27 **Displacement vector**

EXAMPLE 2.17

Calculate the displacement vector for a particle moving from a point P to Q as shown below. Calculate the magnitude of displacement.

Solution

The displacement vector $\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$, with

$$\vec{r}_1 = \hat{i} + \hat{j} \text{ and } \vec{r}_2 = 4\hat{i} + 2\hat{j}$$
$$\therefore \Delta\vec{r} = \vec{r}_2 - \vec{r}_1 = \left(4\hat{i} + 2\hat{j}\right) - \left(\hat{i} + \hat{j}\right)$$
$$= (4-1)\hat{i} + (2-1)\hat{j}$$
$$\therefore \Delta\vec{r} = 3\hat{i} + \hat{j}$$

The magnitude of the displacement vector $\Delta r = \sqrt{3^2 + 1^2} = \sqrt{10}$ unit.

Note

(1) The Distance travelled by an object in motion in a given time is never negative or zero, it is always positive.

(2) The displacement of an object, in a given time can be positive, zero or negative.

(3) The displacement of an object can be equal or less than the distance travelled but never greater than distance travelled.

(4) The distance covered by an object between two positions can have many values, but the displacement between them has only one value (in magnitude).

2.8 DIFFERENTIAL CALCULUS

The Concept of a function

1) Any physical quantity is represented by a "function" in mathematics. Take the example of temperature T. We know that the temperature of the surroundings is changing throughout the day. It increases till noon and decreases in the evening. At any

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time "t" the temperature T has a unique value. Mathematically this variation can be represented by the notation 'T(t)' and it should be called "temperature as a function of time". It implies that if the value of 't' is given, then the function "T(t)" will give the value of the temperature at that time 't'. Similarly, the position of a bus in motion along the x direction can be represented by x(t) and this is called 'x' as a function of time'. Here 'x' denotes the x coordinate.

Example

Consider a function $f(x) = x^2$. Sometimes it is also represented as $y = x^2$. Here y is called the dependent variable and x is called independent variable. It means as x changes, y also changes. Once a physical quantity is represented by a function, one can study the variation of the function over time or over the independent variable on which the quantity depends. Calculus is the branch of mathematics used to analyse the change of any quantity.

If a function is represented by y = f(x), then dy/dx represents the derivative of y with respect to x. Mathematically this represents the variation of y with respect to change in x, for various continuous values of x.

Mathematically the derivative dy/dx is defined as follows

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{y(x+\Delta x) - y(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$$

$\frac{dy}{dx}$ represents the limit that the quantity $\frac{\Delta y}{\Delta x}$ attains, as Δx tends to zero.

Graphically this is represented as shown in Figure 2.28.

Figure 2.28 Derivative of a function

EXAMPLE 2.18

Consider the function $y = x^2$. Calculate the derivative $\frac{dy}{dx}$ using the concept of limit, at the point $x = 2$.

Solution

Let us take two points given by

$x_1 = 2$ and $x_2 = 3$, then $y_1 = 4$ and $y_2 = 9$

Here $\Delta x = 1$ and $\Delta y = 5$

Then

$$\frac{\Delta y}{\Delta x} = \frac{9-4}{3-2} = 5$$

If we take $x_1 = 2$ and $x_2 = 2.5$, then $y_1 = 4$ and $y_2 = (2.5)^2 = 6.25$

Here $\Delta x = 0.5 = \frac{1}{2}$ and $\Delta y = 2.25$

Then

$$\frac{\Delta y}{\Delta x} = \frac{6.25-4}{0.5} = 4.5$$

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If we take $x_1 = 2$ and $x_2 = 2.25$, then $y_1 = 4$ and $y_2 = 5.0625$

Here $\Delta x = 0.25 = \frac{1}{4}$, $\Delta y = 1.0625$

$$\frac{\Delta y}{\Delta x} = \frac{5.0625 - 4}{0.25} = \frac{(5.0625 - 4)}{\frac{1}{4}}$$
$$= 4(5.0625 - 4) = 4.25$$

If we take $x_1 = 2$ and $x_2 = 2.1$, then $y_1 = 4$ and $y_2 = 4.41$

Here $\Delta x = 0.1 = \frac{1}{10}$ and

$$\frac{\Delta y}{\Delta x} = \frac{(4.41 - 4)}{\frac{1}{10}} = 10(4.41 - 4) = 4.1$$

These results are tabulated as shown below:

x_1	x_2	Δx	y_1	y_2	$\frac{\Delta y}{\Delta x}$
2	2.25	0.25	4	5.0625	4.25
2	2.1	0.1	4	4.41	4.1
2	2.01	0.01	4	4.0401	4.01
2	2.001	0.001	4	4.004001	4.001
2	2.0001	0.0001	4	4.00040001	4.0001

From the above table, the following inferences can be made.

- As Δx tends to zero, $\frac{\Delta y}{\Delta x}$ approaches the limit given by the number 4.
- At a point x = 2, the derivative $\frac{dy}{dx} = 4$.
- It should also be mentioned here that $\Delta x \to 0$ does not mean that $\Delta x = 0$.

This is because, if we substitute $\Delta x = 0$, $\frac{\Delta y}{\Delta x}$ becomes indeterminate.

In general, we can obtain the derivative of the function $y = x^2$, as follows:

$$\frac{\Delta y}{\Delta x} = \frac{(x + \Delta x)^2 - x^2}{\Delta x} = \frac{x^2 + 2x\Delta x + \Delta x^2 - x^2}{\Delta x}$$
$$= \frac{2x\Delta x + \Delta x^2}{\Delta x} = 2x + \Delta x$$
$$\frac{dy}{dx} = \lim_{\Delta x \to 0} 2x + \Delta x = 2x$$

The table below shows the derivatives of some common functions used in physics

Function	Derivative
$y = x$	$dy/dx = 1$
$y = x^2$	$dy/dx = 2x$
$y = x^3$	$dy/dx = 3x^2$
$y = x^n$	$dy/dx = nx^{n-1}$
$y = \sin x$	$dy/dx = \cos x$
$y = \cos x$	$dy/dx = -\sin x$
$y = \text{constant}$	$dy/dx = 0$
$y = AB$	$\frac{dy}{dx} = A\left(\frac{dB}{dx}\right) + \left(\frac{dA}{dx}\right)B$

In physics, velocity, speed and acceleration are all derivatives with respect to time 't'. This will be dealt with in the next section.

EXAMPLE 2.19

Find the derivative with respect to t, of the function $x = A_0 + A_1 t + A_2 t^2$ where A_0, A_1 and A_2 are constants.

Solution

Note that here the independent variable is 't' and the dependent variable is 'x'

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The required derivative is $dx/dt = 0 + A_1 + 2A_2t$

The second derivative is $d^2x/dt^2 = 2A_2$

2.9 INTEGRAL CALCULUS

Integration is an area finding process. For certain geometric shapes we can directly find the area. But for irregular shapes the process of integration is used. Consider for example the areas of a rectangle and an irregularly shaped curve, as shown in Figure 2.29.

The area of the rectangle is simply given by A = length × breadth = (b–a) c

Figure 2.29 Area of rectangular and irregular shape

But to find the area of the irregular shaped curve given by f(x), we divide the area into rectangular strips as shown in the Figure 2.30.

The area under the curve is approximately equal to sum of areas of each rectangular strip.

This is given by $A \approx f(a)\Delta x + f(x_1)\Delta x + f(x_2)\Delta x + f(x_3)\Delta x$.

Figure 2.30 Area under the curve using rectangular strip

Where $f(a)$ is the value of the function $f(x)$ at x = a, $f(x_1)$ is the value of $f(x)$ for $x = x_1$ and so on.

As we increase the number of strips, the area evaluated becomes more accurate. If the area under the curve is divided into N strips, the area under the curve is given by

$$A = \sum_{n=1}^{N} f(x_n)\Delta x$$

As the number of strips goes to infinity, $N \to \infty$, the sum becomes an integral,

$$A = \int_a^b f(x)\,dx$$

(Note: As $N \to \infty$, $\Delta x \to 0$)

The integration will give the total area under the curve f (x). This is shown in Figure 2.31.

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$$\text{Area} = \int_a^b f(x)dx = \lim_{\Delta x \to 0} \sum_{i=1}^{N} f(x_i)\Delta x$$

Figure 2.31 Relation between summation and integration

Examples

In physics the work done by a force F(x) on an object to move it from point a to point b in one dimension is given by

$$W = \int_a^b F(x)\,dx$$

(No scalar products is required here, since motion here is in one dimension)

1) The work done is the area under the force displacement graph as shown in Figure 2.32

Figure 2.32 Work done by the force

2) The impulse given by the force in an interval of time is calculated between the interval from time t = 0 to time t = t_1 as

$$\text{Impulse } I = \int_0^{t_1} F dt$$

The impulse is the area under the force function F (t) - t graph as shown in Figure 2.33.

Figure 2.33 Impulse of a force

Average velocity

Consider a particle located initially at point P having position vector $\vec{r}_1$. In a time interval Δt the particle is moved to the point Q having position vector $\vec{r}_2$. The displacement vector is $\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$. This is shown in Figure 2.34.

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The average velocity is defined as ratio of the displacement vector to the corresponding time interval

$$\vec{v}_{avg} = \frac{\Delta\vec{r}}{\Delta t}.$$

It is a vector quantity. The direction of average velocity is in the direction of the displacement vector ($\Delta\vec{r}$).

This is also shown in Figure 2.34.

Figure 2.34 Average velocity

Average speed

The average speed is defined as the ratio of total path length travelled by the particle in a time interval.

Average speed = total path length / total time

EXAMPLE 2.20

Consider an object travelling in a semi-circular path from point O to point P in 5 second, as shown in the Figure given below. Calculate the average velocity and average speed.

Solution

Average velocity $\vec{v}_{avg} = \frac{\vec{r}_P - \vec{r}_O}{\Delta t}$

Here $\Delta t = 5\ s$

$\vec{r}_O = 0, \vec{r}_P = 10\hat{i}$

$$\vec{v}_{avg} = \frac{10\hat{i}\,cm}{5\,s} = 2\hat{i}\,cm\,s^{-1}.$$

The average velocity is in the positive x direction.

The average speed = total path length / time taken (the path is semi-circular)

$$= \frac{5\pi\,cm}{5\,s} = \pi\,cm\,s^{-1} \approx 3.14\ cm\,s^{-1}$$

Note that the average speed is greater than the magnitude of the average velocity.

Instantaneous velocity or velocity

The instantaneous velocity at an instant t or simply 'velocity' at an instant t is defined as limiting value of the average velocity as $\Delta t \to 0$, evaluated at time t.

In other words, velocity is equal to rate of change of position vector with respect to time. Velocity is a vector quantity.

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta\vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$

In component form, this velocity is

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{d}{dt}\left(x\hat{i} + y\hat{j} + z\hat{k}\right)$$
$$= \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k}.$$

Here $\frac{dx}{dt} = v_x = x-$ component of velocity

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$$\frac{dy}{dt} = v_y = y - \text{component of velocity}$$

$$\frac{dz}{dt} = v_z = z - \text{component of velocity}$$

The magnitude of velocity v is called speed and is given by

$$v = \sqrt{v_x^2 + v_y^2 + v_z^2}.$$

Speed is always a positive scalar. The unit of speed is also meter per second.

EXAMPLE 2.21

The position vector of a particle is given $\vec{r} = 2t\,\hat{i} + 3\,t^2\,\hat{j} - 5\,\hat{k}$.

a) Calculate the velocity and speed of the particle at any instant t

b) Calculate the velocity and speed of the particle at time $t = 2$ s

Solution

The velocity $\vec{v} = \frac{d\vec{r}}{dt} = 2\hat{i} + 6t\hat{j}$

The speed $v(t) = \sqrt{2^2 + (6t)^2}\ ms^{-1}$

The velocity of the particle at $t = 2$ s

$$\vec{v}(2sec) = 2\hat{i} + 12\hat{j}$$

The speed of the particle at $t = 2$ s

$$v(2\,s) = \sqrt{2^2 + 12^2} = \sqrt{4 + 144}$$
$$= \sqrt{148} \approx 12.16\,m\,s^{-1}$$

Note that the particle has velocity components along x and y direction. Along the z direction the position has constant value (–5) which is independent of time. Hence there is no z-component for the velocity.

EXAMPLE 2.22

The velocity of three particles A, B, C are given below. Which particle travels at the greatest speed?

$$\vec{v_A} = 3\hat{i} - 5\hat{j} + 2\hat{k}$$
$$\vec{v_B} = \hat{i} + 2\hat{j} + 3\hat{k}$$
$$\vec{v_C} = 5\hat{i} + 3\hat{j} + 4\hat{k}$$

Solution

We know that speed is the magnitude of the velocity vector. Hence,

$$\text{Speed of A} = \left|\vec{v_A}\right| = \sqrt{(3)^2 + (-5)^2 + (2)^2}$$
$$= \sqrt{9 + 25 + 4} = \sqrt{38}\ \text{m s}^{-1}$$

$$\text{Speed of B} = \left|\vec{v_B}\right| = \sqrt{(1)^2 + (2)^2 + (3)^2}$$
$$= \sqrt{1 + 4 + 9} = \sqrt{14}\ \text{m s}^{-1}$$

$$\text{Speed of C} = \left|\vec{v_C}\right| = \sqrt{(5)^2 + (3)^2 + (4)^2}$$
$$= \sqrt{25 + 9 + 16} = \sqrt{50}\ \text{m s}^{-1}$$

The particle C has the greatest speed.

$$\sqrt{50} > \sqrt{38} > \sqrt{14}$$

EXAMPLE 2.23

Two cars are travelling with respective velocities $\vec{v}_1 = 10\,m\,s^{-1}$ along east and $\vec{v}_2 = 10\,m\,s^{-1}$ along west. What are the speeds of the cars?

Solution

Both cars have the same magnitude of velocity. This implies that both cars travel at the same speed even though they have

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velocities in different directions. Speed will not give the direction of motion.

Speedometer

Momentum

The linear momentum or simply momentum of a particle is defined as product of mass with velocity. It is denoted as '$\vec{p}$'. Momentum is also a vector quantity.

$$\vec{p} = m\vec{v}.$$

The direction of momentum is also in the direction of velocity, and the magnitude of momentum is equal to product of mass and speed of the particle.

$$p = mv$$

In component form the momentum can be written as

$$p_x\hat{i} + p_y\hat{j} + p_z\hat{k} = mv_x\hat{i} + mv_y\hat{j} + mv_z\hat{k}$$

Here $p_x = x$ component of momentum and is equal to mv_x

$p_y = y$ component of momentum and is equal to mv_y

$p_z = z$ component of momentum and is equal to mv_z

The momentum of the particle plays a very important role in Newton's laws. The physical significance of momentum can be well understood by the following example.

Consider a butterfly and a stone, both moving towards you with the same velocity 5 m s^{-1}. If both hit your body, the effects will not be the same. The effects not only depend upon the velocity, but also on the mass. The stone has greater mass compared to the butterfly. The momentum of the stone is thus greater than the momentum of the butterfly. It is the momentum which plays a major role in explaining the 'state' of motion of the object.

The unit of the momentum is kg m s^{-1}

EXAMPLE 2.24

Consider two masses of 10 g and 1 kg moving with the same speed 10 m s^{-1}. Calculate the magnitude of the momentum.

Solution

We use $p = mv$

For the mass of 10 g, m = 0.01 kg

$$p = 0.01 \times 10 = 0.1 \text{ kg m s}^{-1}$$

For the mass of 1 kg

$$p = 1 \times 10 = 10 \text{ kg m s}^{-1}$$

Thus even though both the masses have the same speed, the momentum of the heavier mass is 100 times greater than that of the lighter mass.

2.10 MOTION ALONG ONE DIMENSION

2.10.1 Average velocity

If a particle moves in one dimension, say for example along the x direction, then

$$\text{The average velocity} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}.$$

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The average velocity is also a vector quantity. But in one dimension we have only two directions (positive and negative x direction), hence we use positive and negative signs to denote the direction.

The instantaneous velocity or velocity is defined as $v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$

Graphically the slope of the position-time graph will give the velocity of the particle. At the same time, if velocity time graph is given, the distance and displacement are determined by calculating the area under the curve. This is explained below.

We know that velocity is given by $\frac{dx}{dt} = v$.

Therefore, we can write $dx = vdt$

By integrating both sides, we get $\int_{x_1}^{x_2} dx = \int_{t_1}^{t_2} v\, dt$.

As already seen, integration is equivalent to area under the given curve. So the term $\int_{t_1}^{t_2} v\, dt$ represents the area under the curve v as a function of time.

Since the left hand side of the integration represents the displacement travelled by the particle from time t_1 to t_2, the area under the velocity time graph will give the displacement of the particle. If the area is negative, it means that displacement is negative, so the particle has travelled in the negative direction. This is shown in the Figure 2.35 below.

EXAMPLE 2.25

A particle moves along the *x-axis* in such a way that its coordinates x varies with time 't' according to the equation $x = 2 - 5t + 6t^2$. What is the initial velocity of the particle?

Solution

$$x = 2 - 5t + 6t^2$$

$$\text{Velocity, } v = \frac{dx}{dt} = \frac{d}{dt}(2 - 5t + 6t^2)$$

$$\text{or } v = -5 + 12t$$

For initial velocity, $t = 0$

$\therefore$ Initial velocity $= -5\text{ms}^{-1}$

The negative sign implies that at t = 0 the velocity of the particle is along negative x direction.

Average speed = total path length / total time period

Figure 2.35 Displacement in the velocity–time graph

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2.10.2 Relative Velocity in One and Two Dimensional Motion

When two objects A and B are moving with different velocities, then the velocity of one object A with respect to another object B is called relative velocity of object A with respect to B.

Case 1

Consider two objects A and B moving with uniform velocities V_A and V_B, as shown, along straight tracks in the same direction $\vec{V}_A$, $\vec{V}_B$ with respect to ground.

The relative velocity of object A with respect to object B is $\vec{V}_{AB} = \vec{V}_A - \vec{V}_B$

The relative velocity of object B with respect to object A is $\vec{V}_{BA} = \vec{V}_B - \vec{V}_A$

Thus, if two objects are moving in the same direction, the magnitude of relative velocity of one object with respect to another is equal to the difference in magnitude of two velocities.

EXAMPLE 2.26

Suppose two cars A and B are moving with uniform velocities with respect to ground along parallel tracks and in the same direction. Let the velocities of A and B be 35 km h^{-1} due east and 40 km h^{-1} due east respectively. What is the relative velocity of car B with respect to A?

Solution

The relative velocity of B with respect to A, $\vec{v}_{BA} = \vec{v}_B - \vec{v}_A = 5$ km h^{-1} due east

Similarly, the relative velocity of A with respect to B i.e., $\vec{v}_{AB} = \vec{v}_A - \vec{v}_B = 5$ km h^{-1} due west.

To a passenger in the car A, the car B will appear to be moving east with a velocity 5 km h^{-1}. To a passenger in car B, the car A will appear to move westwards with a velocity of 5 km h^{-1}

Case 2

Consider two objects A and B moving with uniform velocities V_A and V_B along the same straight tracks but opposite in direction

The relative velocity of object A *with respect to* object B is

$$\vec{V}_{AB} = \vec{V}_A - (-\vec{V}_B) = \vec{V}_A + \vec{V}_B$$

The relative velocity of object B with respect to object A is

$$\vec{V}_{BA} = -\vec{V}_B - \vec{V}_A = -(\vec{V}_A + \vec{V}_B)$$

Thus, if two objects are moving in opposite directions, the magnitude of relative velocity of one object with respect to other is equal to the sum of magnitude of their velocities.

Case 3

Consider the velocities $\vec{v}_A$ and $\vec{v}_B$ at an angle θ between their directions.

The relative velocity of A with respect to B, $\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$

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Then, the magnitude and direction of $\vec{v}_{AB}$ is given by $v_{AB} = \sqrt{v_A^2 + v_B^2 - 2\, v_A v_B \cos\theta}$

and $\tan\beta = \dfrac{v_B \sin\theta}{v_A - v_B \cos\theta}$ (Here β is angle between $\vec{v}_{AB}$ and $\vec{v}_B$)

(i) When θ = 0°, the bodies move along parallel straight lines in the same direction,

We have $v_{AB} = (v_A - v_B)$ in the direction of $\vec{v}_A$. Obviously $v_{BA} = (v_B - v_A)$ in the direction of $\vec{v}_B$.

(ii) When θ = 180°, the bodies move along parallel straight lines in opposite directions,

We have $v_{AB} = (v_A + v_B)$ in the direction of $\vec{v}_A$.

Similarly, $v_{BA} = (v_B + v_A)$ in the direction of $\vec{v}_B$.

(iii) If the two bodies are moving at right angles to each other, then θ = 90°. The magnitude of the relative velocity of A with respect to B $= v_{AB} = \sqrt{v_A^2 + v_B^2}$.

(iv) Consider a person moving horizontally with velocity $\vec{V}_M$. Let rain fall vertically with velocity $\vec{V}_R$. An umbrella is held to avoid the rain. Then the relative velocity of the rain with respect to the person is, **(Figure 2.36)**

$$\vec{V}_{RM} = \vec{V}_R - \vec{V}_M$$

which has magnitude

$$V_{RM} = \sqrt{V_R^2 + V_M^2}$$

and direction $\theta = \tan^{-1}\left(\dfrac{V_M}{V_R}\right)$ with the vertical as shown in Figure. 2.36

In order to save himself from the rain, he should hold an umbrella at an angle θ with the vertical.

EXAMPLE 2.27

Suppose two trains A and B are moving with uniform velocities along parallel tracks but in opposite directions. Let the velocity of train A be 40 km h^{-1} due east and that of train B be 40 km h^{-1} due west. Calculate the relative velocities of the trains

Solution

Relative velocity of A with respect to B, v_{AB} = 80 km h^{-1} due east

Thus to a passenger in train B, the train A will appear to move east with a velocity of 80 km h^{-1}

Figure 2.36 Angle of umbrella with respect to rain

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The relative velocity of B with respect to A, $V_{BA} = 80$ km h^{-1} due west

To a passenger in train A, the train B will appear to move westwards with a velocity of 80 km h^{-1}

EXAMPLE 2.28

Consider two trains A and B moving along parallel tracks with the same velocity in the same direction. Let the velocity of each train be 50 km h^{-1} due east. Calculate the relative velocities of the trains.

Solution

Relative velocity of B with respect to A,

$v_{BA} = v_B - v_A$

$= 50 \text{ km h}^{-1} + (-50) \text{ km h}^{-1}$

$= 0 \text{ km h}^{-1}$

Similarly, relative velocity of A with respect to B i.e., v_{AB} is also zero.

Thus each train will appear to be at rest with respect to the other.

EXAMPLE 2.29

How long will a boy sitting near the window of a train travelling at 36 km h^{-1} see a train passing by in the opposite direction with a speed of 18 km h^{-1}. The length of the slow-moving train is 90 m.

Solution

The relative velocity of the slow-moving train with respect to the boy is = (36 + 18) km h^{-1} = 54 km h^{-1} = $54 \times \frac{5}{18}$ m s^{-1} = 15 m s^{-1}

Since the boy will watch the full length of the other train, to find the time taken to watch the full train:

We have, $15 = \frac{90}{t}$

or $t = \frac{90}{15} = 6 \text{ s}$

EXAMPLE 2.30

A swimmer's speed in the direction of flow of a river is 12 km h^{-1}. Against the direction of flow of the river the swimmer's speed is 6 km h^{-1}. Calculate the swimmer's speed in still water and the velocity of the river flow.

Solution

Let v_s and v_r, represent the velocities of the swimmer and river respectively with respect to ground.

$$v_s + v_r = 12 \quad (1)$$

and

$$v_s - v_r = 6 \quad (2)$$

Adding the both equations (1) and (2)

$2v_s = 12 + 6 = 18$ km h^{-1} or

$v_s = 9$ km h^{-1}

From Equation (1),

$9 + v_r = 12$ or

$v_r = 3$ km h^{-1}

When the river flow and swimmer move in the same direction, the net velocity of swimmer is 12 km h^{-1}.

Accelerated Motion

During non-uniform motion of an object, the velocity of the object changes from instant to instant i.e., the velocity of the object is no more constant but changes

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with time. Such a motion is said to be an accelerated motion.

i) In accelerated motion, if the change in velocity of an object per unit time is same (constant) then the object is said to be moving with uniformly accelerated motion.
ii) On the other hand, if the change in velocity per unit time is different at different times, then the object is said to be moving with non-uniform accelerated motion.

Average acceleration

If an object changes its velocity from $\vec{v}_1$ to $\vec{v}_2$ in a time interval $\Delta t = t_2 - t_1$, then the average acceleration is defined as the ratio of change in velocity over the time interval $\Delta t = t_2 - t_1$

$$\vec{a}_{avg} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} = \frac{\Delta \vec{v}}{\Delta t}$$

Average acceleration is a vector quantity in the same direction as the vector $\Delta \vec{v}$.

Instantaneous acceleration

Usually, the average acceleration will give the change in velocity only over the entire time interval. It will not give value of the acceleration at any instant time t.

Instantaneous acceleration or acceleration of a particle at time 't' is given by the ratio of change in velocity over Δt, as Δt approaches zero.

$$\text{Acceleration } \vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$$

In other words, the acceleration of the particle at an instant t is equal to rate of change of velocity.

(i) Acceleration is a vector quantity. Its SI unit is ms^{-2} and its dimensional formula is $M^0L^1T^{-2}$
(ii) Acceleration is positive if its velocity is increasing, and is negative if the velocity is decreasing. The negative acceleration is called retardation or deceleration.

In terms of components, we can write

$$\vec{a} = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j} + \frac{dv_z}{dt}\hat{k} = \frac{d\vec{v}}{dt}$$

Thus $a_x = \frac{dv_x}{dt}$, $a_y = \frac{dv_y}{dt}$, $a_z = \frac{dv_z}{dt}$ are the components of instantaneous acceleration.

Since each component of velocity is the derivative of the corresponding coordinate, we can express the components a_x, a_y, and a_z, as

$$a_x = \frac{d^2x}{dt^2}, \quad a_y = \frac{d^2y}{dt^2}, \quad a_z = \frac{d^2z}{dt^2}$$

Then the acceleration vector $\vec{a}$ itself is

$$\vec{a} = \frac{d^2x}{dt^2}\hat{i} + \frac{d^2y}{dt^2}\hat{j} + \frac{d^2z}{dt^2}\hat{k} = \frac{d^2\vec{r}}{dt^2}$$

Thus acceleration is the second derivative of position vector with respect to time.

Graphically the acceleration is the slope in the velocity-time graph. At the same time if the acceleration-time graph is given, then the velocity can be found from the area under the acceleration-time graph.

From $\frac{dv}{dt} = a$, we have $dv = a\,dt$; hence

$$v = \int_{t_1}^{t_2} a\,dt$$

For an initial time t_1 and final time t_2

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EXAMPLE 2.31

A velocity–time graph is given for a particle moving in x direction, as below

a) Describe the motion qualitatively in the interval 0 to 55 s.

b) Find the distance and displacement travelled from 0 s to 40 s.

c) Find the acceleration at t = 5 s and at t = 20 s

Solution

(a) **From O to A: (0 s to 10 s)**

At t = 0 s the particle has zero velocity. At t > 0, particle has positive velocity and moves in the positive x direction. From 0 s to 10 s the slope $(\frac{dv}{dt})$ is positive, implying the particle is accelerating. Thus the velocity increases during this time interval.

From A to B: (10 s to 15 s)

From 10 s to 15 s the velocity stays constant at 60 m s^{-1}. The acceleration is 0 during this period. But the particle continues to travel in the positive *x*-direction.

From B to C : (15 s to 30 s)

From the 15 s to 30 s the slope is negative, implying the velocity is decreasing. But the particle is moving in the positive x direction. At t = 30 s the velocity becomes zero, and the particle comes to rest momentarily at t = 30 s.

From C to D: (30 s to 40 s)

From 30 s to 40 s the velocity is negative. It implies that the particle starts to move in the negative x direction. The magnitude of velocity increases to a maximum 40 m s^{-1}

From D to E: (40 s to 55 s)

From 40 s to 55 s the velocity is still negative, but starts increasing from –40 m s^{-1} At t = 55 s the velocity of the particle is zero and particle comes to rest.

(b) The total area under the curve from 0 s to 40 s will give the displacement. Here the area from O to C represents motion along positive x–direction and the area under the graph from C to D represents the particle's motion along negative x–direction.

The displacement travelled by the particle from 0 s to 10 s = $\frac{1}{2}\times 10\times 60 = 300\,m$

The displacement travelled from 10 s to 15 s = 60 × 5 = 300 m

The displacement travelled from 15 s to 30 s $= \frac{1}{2}\times 15\times 60 = 450\,m$

The displacement travelled from 30 s to 40 s $= \frac{1}{2}\times 10\times(-40) = -200\,m.$

Here the negative sign implies that the particle travels 200 m in the negative x direction.

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The total displacement from 0 s to 40 s is given by

$$300\text{ m} + 300\text{ m} + 450\text{ m} - 200\text{ m} = +850\text{ m}.$$

Thus the particle's net displacement is along the positive x-direction.

The total distance travelled by the particle from 0 s to 40 s = 300 + 300 + 450 + 200 = 1250 m.

(c) The acceleration is given by the slope in the velocity-time graph. In the first 10 seconds the velocity has constant slope (constant acceleration). It implies that the acceleration a is from $v_1 = 0$ to $v_2 = 60\,\text{m s}^{-1}$.

Hence $a = \dfrac{v_2 - v_1}{t_2 - t_1}$ gives

$$a = \frac{60-0}{10-0} = 6\,m\,s^{-2}$$

Next, the particle has constant negative slope from 15 s to 30 s. In this case $v_2 = 0$ and $v_1 = 60\,m\,s^{-1}$. Thus the acceleration at t = 20 s is given by $a = \dfrac{0-60}{30-15} = -4\,m\,s^{-2}$. Here the negative sign implies that the particle has negative acceleration.

EXAMPLE 2.32

If the position vector of the particle is given by $\vec{r} = 3t^2\hat{i} + 5t\hat{j} + 4\hat{k}$, Find the

a) The velocity of the particle at t = 3 s

b) Speed of the particle at t = 3 s

c) acceleration of the particle at time t = 3 s

Solution

(a) The velocity $\vec{v} = \dfrac{d\vec{r}}{dt} = \dfrac{dx}{dt}\hat{i} + \dfrac{dy}{dt}\hat{j} + \dfrac{dz}{dt}\hat{k}$

We obtain, $\vec{v}(t) = 6t\hat{i} + 5\hat{j}$

The velocity has only two components $v_x = 6t$, depending on time t and $v_y = 5$ which is independent of time.

The velocity at t = 3 s is $\vec{v}(3) = 18\hat{i} + 5\hat{j}$

(b) The speed at t = 3 s is $v = \sqrt{18^2 + 5^2} = \sqrt{349} \approx 18.68\,m\,s^{-1}$

(c) The acceleration $\vec{a}$ is, $\vec{a} = \dfrac{d^2\vec{r}}{dt^2} = 6\hat{i}$

The acceleration has only the x-component. Note that acceleration here is independent of t, which means $\vec{a}$ is constant. Even at t = 3 s it has same value $\vec{a} = 6\hat{i}$. The velocity is non-uniform, but the acceleration is uniform (constant) in this case.

EXAMPLE 2.33

An object is thrown vertically downward. What is the acceleration experienced by the object?

Solution

We know that when the object falls towards the Earth, it experiences acceleration due to gravity g = 9.8 m s^{-2} downward. We can choose the coordinate system as shown in the figure.

The acceleration is along the negative y direction.

$$\vec{a} = g\left(-\hat{j}\right) = -g\hat{j}$$

Note For convenience, sometimes we take the downward direction as positive Y-axis. As a vertically falling body accelerates downwards, g is taken as positive in this direction. (a = g)

2.10.3 Equations of Uniformly Accelerated Motion by Calculus Method

Consider an object moving in a straight line with uniform or constant acceleration 'a'.

Let u be the velocity of the object at time t = 0, and v be velocity of the body at a later time t.

Velocity - time relation

(i) The acceleration of the body at any instant is given by the first derivative of the velocity with respect to time,

$$a = \frac{dv}{dt} \text{ or } dv = a\,dt$$

Integrating both sides with the condition that as time changes from 0 to t, the velocity changes from u to v. For the constant acceleration,

$$\int_u^v dv = \int_0^t a\,dt = a\int_0^t dt \Rightarrow [v]_u^v = a[t]_0^t$$

$$v - u = at \quad (or) \quad v = u + at \qquad \rightarrow (2.7)$$

Note If *a* is dependent on time *t*, *a* cannot be brought outside the time integral.

Displacement – time relation

(ii) The velocity of the body is given by the first derivative of the displacement with respect to time.

$$v = \frac{ds}{dt} \; or \; ds = vdt$$

and since $v = u + at$,

We get $ds = \left(u + at\right)dt$

Assume that initially at time t = 0, the particle started from the origin. At a later time t, the particle displacement is s. Further assuming that acceleration is time-independent, we have

$$\int_0^s ds = \int_0^t u\,dt + \int_o^t at\,dt \;\; (or) \;\; s = ut + \frac{1}{2}at^2 \quad (2.8)$$

Velocity – displacement relation

(iii) The acceleration is given by the first derivative of velocity with respect to time.

$$a = \frac{dv}{dt} = \frac{dv}{ds}\frac{ds}{dt} = \frac{dv}{ds}v$$

[since ds/dt = v] where s is displacement traversed.

This is rewritten as $a = \frac{1}{2}\frac{d}{ds}(v^2)$

$$\text{or } ds = \frac{1}{2a}d(v^2)$$

Integrating the above equation, using the fact when the velocity changes from u to v, displacement changes from 0 to s, we get

$$\int_0^s ds = \int_u^v \frac{1}{2a}d\left(v^2\right)$$

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$$\therefore s = \frac{1}{2a}\left(v^2 - u^2\right)$$

$$\therefore v^2 = u^2 + 2as \qquad (2.9)$$

We can also derive the displacement s in terms of initial velocity u and final velocity v.

From the equation (2.7) we can write,

$$at = v - u$$

Substitute this in equation (2.8), we get

$$s = ut + \frac{1}{2}(v-u)t$$

$$s = \frac{(u+v)t}{2} \qquad (2.10)$$

The equations (2.7), (2.8), (2.9) and (2.10) are called kinematic equations of motion, and have a wide variety of practical applications.

Kinematic equations

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$

$$s = \frac{(u+v)t}{2}$$

It is to be noted that all these kinematic equations are valid only if the motion is in a straight line with constant acceleration. For circular motion and oscillatory motion these equations are not applicable.

Equations of motion under gravity

A practical example of a straight line motion with constant acceleration is the motion of an object near the surface of the Earth. We know that near the surface of the Earth, the acceleration due to gravity 'g' is constant. All straight line motions under this acceleration can be well understood using the kinematic equations given earlier.

Case (i): A body falling from a height h

Figure 2.37 An object in free fall

Consider an object of mass m falling from a height h. Assume there is no air resistance. For convenience, let us choose the downward direction as positive y-axis as shown in the Figure 2.37. The object experiences acceleration 'g' due to gravity which is constant near the surface of the Earth. We can use kinematic equations to explain its motion. We have

The acceleration $\vec{a} = g\hat{j}$

By comparing the components, we get

$$a_x = 0, a_z = 0, a_y = g$$

Let us take for simplicity, $a_y = a = g$

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If the particle is thrown with initial velocity 'u' downward which is in negative y axis, then velocity and position at of the particle any time t is given by

$$v = u + gt \qquad (2.11)$$

$$y = ut + \frac{1}{2}gt^2 \qquad (2.12)$$

The square of the speed of the particle when it is at a distance y from the hill-top, is

$$v^2 = u^2 + 2gy \qquad (2.13)$$

Suppose the particle starts from rest.

Then u = 0

Then the velocity v, the position of the particle and v^2 at any time t are given by (for a point y from the hill-top)

$$v = gt \qquad (2.14)$$

$$y = \frac{1}{2}gt^2 \qquad (2.15)$$

$$v^2 = 2gy \qquad (2.16)$$

The time (t = T) taken by the particle to reach the ground (for which y = h), is given by using equation (2.15),

$$h = \frac{1}{2}gT^2 \qquad (2.17)$$

$$T = \sqrt{\frac{2h}{g}} \qquad (2.18)$$

The equation (2.18) implies that greater the height(h), particle takes more time(T) to reach the ground. For lesser height(h), it takes lesser time to reach the ground.

The speed of the particle when it reaches the ground (y = h) can be found using equation (2.16), we get

$$v_{ground} = \sqrt{2gh} \qquad (2.19)$$

The above equation implies that the body falling from greater height(h) will have higher velocity when it reaches the ground.

The motion of a body falling towards the Earth from a small altitude (h << R), purely under the force of gravity is called free fall. (Here R is radius of the Earth)

EXAMPLE 2.34

An iron ball and a feather are both falling from a height of 10 m.

a) What are the time taken by the iron ball and feather to reach the ground?
b) What are the velocities of iron ball and feather when they reach the ground? (Ignore air resistance and take g = 10 m s^{-2})

Solution

Since kinematic equations are independent of mass of the object, according to equation (2.8) the time taken by both iron ball and feather to reach the ground are the same. This is given by

$$T = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2\times 10}{10}} = \sqrt{2}\,s \approx 1.414\,s$$

Thus, both feather and iron ball reach ground at the same time.

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By following equation (2.19) both iron ball and feather reach the Earth with the same speed. It is given by

$$v = \sqrt{2gh} = \sqrt{2 \times 10 \times 10}$$
$$= \sqrt{200}\, m\, s^{-1} \approx 14.14\, m\, s^{-1}$$

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Galileo concluded that in vaccum all objects fall with the same acceleration g and reach the ground at the same time.

https://www.youtube.com/watch?v=E43-CfukEgs&t=5s

EXAMPLE 2.35

Is it possible to measure the depth of a well using kinematic equations?

Consider a well without water, of some depth d. Take a small object (for example lemon) and a stopwatch. When you drop the lemon, start the stop watch. As soon as the lemon touches the bottom of the well, stop the watch. Note the time taken by the lemon to reach the bottom and denote the time as t.

Since the initial velocity of lemon $u = 0$ and the acceleration due to gravity g is constant over the well, we can use the equations of motion for constant acceleration.

$$s = ut + \frac{1}{2}at^2$$

Since $u = 0, s = d, a = g$ (Since we choose the y axis downwards), Then

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$$d = \frac{1}{2} g t^2$$

Substituting $g = 9.8$ m s^{-2} we get the depth of the well.

To estimate the error in our calculation we can use another method to measure the depth of the well. Take a long rope and hang the rope inside the well till it touches the bottom. Measure the length of the rope which is the correct depth of the well ($d_{correct}$). Then

$$error = d_{correct} - d$$

$$relative\ error = \frac{d_{correct} - d}{d_{correct}}$$

$$percentage\ of\ relative\ error = \frac{d_{correct} - d}{d_{correct}} \times 100$$

What would be the reason for an error, if any?

Repeat the experiment for different masses and compare the result with $d_{correct}$ every time.

Note If there is water in the well, this method can be used to measure depth of the well till the surface of the water.

Case (ii): A body thrown vertically upwards

Consider an object of mass m thrown vertically upwards with an initial velocity u. Let us neglect the air friction. In this case we choose the vertical direction as positive y axis as shown in the Figure 2.38, then the acceleration a = –g (neglect air friction) and g points towards the negative y axis. The kinematic equations for this motion are,

Figure 2.38. An object thrown vertically

The velocity and position of the object at any time t are,

$$v = u - gt \qquad (2.20)$$

$$s = ut - \frac{1}{2} g t^2 \qquad (2.21)$$

The velocity of the object at any position y (from the point where the object is thrown) is

$$v^2 = u^2 - 2gy \qquad (2.22)$$

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EXAMPLE 2.36

A train was moving at the rate of 54 km h^{-1} when brakes were applied. It came to rest within a distance of 225 m. Calculate the retardation produced in the train.

Solution

The final velocity of the particle $v = 0$

The initial velocity of the particle

$$u = 54 \times \frac{5}{18} m s^{-1} = 15\ m s^{-1}$$

$$S = 225\ m$$

Retardation is always against the velocity of the particle.

$$v^2 = u^2 - 2aS$$

$$0 = (15)^2 - 2a\,(225)$$

$$450\ a = 225$$

$$a = \frac{225}{450} m s^{-2} = 0.5\, m s^{-2}$$

Hence, retardation = 0.5 $m\ s^{-2}$

2.11 PROJECTILE MOTION

2.11.1 Introduction

When an object is thrown in the air with some initial velocity (NOT just upwards), and then allowed to move under the action of gravity alone, the object is known as a projectile. The path followed by the particle is called its **trajectory.**

Examples of projectile are

1. An object dropped from window of a moving train.
2. A bullet fired from a rifle.
3. A ball thrown in any direction.
4. A javelin or shot put thrown by an athlete.
5. A jet of water issuing from a hole near the bottom of a water tank.

It is found that a projectile moves under the combined effect of two velocities.

i) A uniform velocity in the horizontal direction, which will not change provided there is no air resistance.

ii) **A uniformly changing velocity (i.e., increasing or decreasing) in the vertical direction.**

There are two types of projectile motion:

(i) Projectile given an initial velocity in the horizontal direction (horizontal projection)

(ii) Projectile given an initial velocity at an angle to the horizontal (angular projection)

To study the motion of a projectile, let us assume that,

i) Air resistance is neglected.

ii) The effect due to rotation of Earth and curvature of Earth is negligible.

iii) The acceleration due to gravity is constant in magnitude and direction at all points of the motion of the projectile.

2.11.2 Projectile in horizontal projection

Consider a projectile, say a ball, thrown horizontally with an initial velocity $\vec{u}$ from the top of a tower of height h (Figure 2.39).

As the ball moves, it covers a horizontal distance due to its uniform horizontal

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Figure 2.39 Horizontal Projection.

velocity u, and a vertical downward distance because of constant acceleration due to gravity g. Thus, under the combined effect the ball moves along the path OPA. The motion is in a 2-dimensional plane. Let the ball take time t to reach the ground at point A, Then the horizontal distance travelled by the ball is x (t) = x, and the vertical distance travelled is y (t) = y

We can apply the kinematic equations along the x direction and y direction separately. Since this is two-dimensional motion, the velocity will have both horizontal component u_x and vertical component u_y.

Motion along horizontal direction

The particle has zero acceleration along x direction. So, the initial velocity u_x remains constant throughout the motion.

The distance traveled by the projectile at a time t is given by the equation $x = u_x t + \frac{1}{2}at^2$. Since a = 0 along x direction, we have

$$x = u_x t \qquad (2.23)$$

Motion along downward direction

Here u_y = 0 (initial velocity has no downward component), a = g (we choose the +ve y-axis in downward direction), and distance y at time t

$\therefore$ From equation, $y = u_y t + \frac{1}{2}at^2$, we get

$$y = \frac{1}{2}gt^2 \qquad (2.24)$$

Substituting the value of t from equation (2.23) in equation (2.24) we have

$$y = \frac{1}{2}g\frac{x^2}{u_x^{\ 2}} = \left(\frac{g}{2u_x^{\ 2}}\right)x^2$$

$$y = Kx^2 \qquad (2.25)$$

where $K = \frac{g}{2u_x^2}$ is constant

Equation (2.25) is the equation of a parabola. Thus, the path followed by the projectile is a parabola (curve OPA in the Figure 2.39).

(1) Time of Flight: *The time taken for the projectile to complete its trajectory or time taken by the projectile to hit the ground is called time of flight.*

Consider the example of a tower and projectile. Let h be the height of a tower. Let T be the time taken by the projectile to hit the ground, after being thrown horizontally from the tower.

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We know that $s_y = u_y t + \frac{1}{2}at^2$ for vertical motion. Here $s_y = h$, $t = T$, $u_y = 0$ (i.e., no initial vertical velocity). Then

$$h = \frac{1}{2}gT^2 \quad \text{or} \quad T = \sqrt{\frac{2h}{g}}$$

Thus, the time of flight for projectile motion depends on the height of the tower, but is independent of the horizontal velocity of projection. If one ball falls vertically and another ball is projected horizontally with some velocity, both the balls will reach the bottom at the same time. This is illustrated in the Figure 2.40.

Figure 2.40 Vertical distance covered by the two particles is same in equal intervals.

(2) Horizontal range: The horizontal distance covered by the projectile from the foot of the tower to the point where the projectile hits the ground is called *horizontal range*. For horizontal motion, we have

$$s_x = u_x t + \frac{1}{2}at^2.$$

Here, s_x = R (range), $u_x = u$, a = 0 (no horizontal acceleration) T is time of flight. Then horizontal range = uT.

Since the time of flight $T = \sqrt{\frac{2h}{g}}$, we substitute this and we get the horizontal range of the particle as $R = u\sqrt{\frac{2h}{g}}$

The above equation implies that the range R is directly proportional to the initial velocity u and inversely proportional to acceleration due to gravity g.

(3) Resultant Velocity (Velocity of projectile at any time): At any instant t, the projectile has velocity components along both x-axis and y-axis. The resultant of these two components gives the velocity of the projectile at that instant t, as shown in Figure 2.41.

Figure 2.41. Velocity resolved into two components

The velocity component at any t along horizontal (x-axis) is $v_x = u_x + a_x t$

Since, $u_x = u$, $a_x = 0$, we get

$$v_x = u \qquad \rightarrow (2.26)$$

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The component of velocity along vertical direction (y-axis) is $v_y = u_y + a_y t$

Since, $u_y = 0$, $a_y = g$, we get

$$v_y = gt \qquad \rightarrow (2.27)$$

Hence the velocity of the particle at any instant is

$$\vec{v} = u\hat{i} + gt\hat{j}$$

The speed of the particle at any instant t is given by

$$\therefore \quad v = \sqrt{v_x^2 + v_y^2}$$
$$v = \sqrt{u^2 + g^2t^2}$$

(4) Speed of the projectile when it hits the ground: When the projectile hits the ground after initially thrown horizontally from the top of tower of height h, the time of flight is

$$T = \sqrt{\frac{2h}{g}}$$

The horizontal component velocity of the projectile remains the same i.e $v_x = u$

The vertical component velocity of the projectile at time T is

$$v_y = gT = g\sqrt{\frac{2h}{g}} = \sqrt{2gh}$$

The speed of the particle when it reaches the ground is

$$v = \sqrt{u^2 + 2gh}$$

2.11.3 Projectile under an angular projection

This projectile motion takes place when the initial velocity is not horizontal, but at some angle with the vertical, as shown in Figure 2.42.

(Oblique projectile)

Examples:

- Water ejected out of a hose pipe held obliquely.
- Cannon fired in a battle ground.

(a) Water is ejected out of a pipe held obliquely

(b) Graphical representation of angular projection

Figure 2.42. Projectile motion

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Consider an object thrown with initial velocity $\vec{u}$ at an angle θ with the horizontal. Refer Figures 2.42 and 2.43.

Then,

$$\vec{u} = u_x\hat{i} + u_y\hat{j}$$

where $u_x = u\cos\theta$ is the horizontal component and $u_y = usin\theta$ the vertical component of velocity.

Since the acceleration due to gravity is in the direction opposite to the direction of vertical component u_y, this component will gradually reduce to zero at the maximum height of the projectile. At this maximum height, the same gravitational force will push the projectile to move downward and fall to the ground. There is no acceleration along the x direction throughout the motion. So, the horizontal component of the velocity (u_x = u cosθ) remains the same till the object reaches the ground.

Hence after the time t, the velocity along horizontal motion $v_x = u_x + a_x t = u_x = u\cos\theta$

The horizontal distance travelled by projectile in time t is $s_x = u_x t + \frac{1}{2}a_x t^2$

Here, $s_x = x$, $u_x = u\ cos\theta$, $a_x = 0$

Figure 2.43. Initial velocity resolved into components

$$\text{Thus, } x = u\cos\theta.t \text{ or } t = \frac{x}{u\cos\theta} \quad (2.28)$$

Next, for the vertical motion $v_y = u_y + a_y t$

Here $u_y = u\ sin\theta$, $a_y = -g$ (acceleration due to gravity acts opposite to the motion). Thus

$$\text{Thus, } v_y = usin\theta - gt \quad (2.29)$$

The vertical distance travelled by the projectile in the same time t is $s_y = u_y t + \frac{1}{2}a_y t^2$

Here, $s_y = y$, $u_y = u\ sin\theta$, $a_y = -g$. Then

$$y = u\ \sin\theta\, t - \frac{1}{2}gt^2 \quad (2.30)$$

Substitute the value of t from equation (2.28) in equation (2.30), we have

$$y = u\sin\theta\frac{x}{u\cos\theta} - \frac{1}{2}g\frac{x^2}{u^2\cos^2\theta}$$

$$y = x\tan\theta - \frac{1}{2}g\frac{x^2}{u^2\cos^2\theta} \quad (2.31)$$

Thus the path followed by the projectile is an inverted parabola.

Maximum height (h_{max})

The maximum vertical distance travelled by the projectile during its journey is called maximum height. This is determined as follows:

For the vertical part of the motion,

$$v_y^2 = u_y^2 + 2a_y s$$

Here, $u_y = u\ sin\theta$, $a_y = -g$, $s = h_{max}$, and at the maximum height $v_y = 0$

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Hence,

$$0 = u^2 \sin^2\theta - 2gh_{max}$$

$$\text{Or } h_{max} = \frac{u^2 \sin^2\theta}{2g} \qquad (2.32)$$

Time of flight (T_f)

The total time taken by the projectile from the point of projection till it hits the horizontal plane is called time of flight.

This time of flight is the time taken by the projectile to go from point O to B via point A (Figure 2.43)

We know that $s_y = u_y t + \frac{1}{2} a_y t^2$

Here, $s_y = y = 0$ (net displacement in y-direction is zero), $u_y = u \sin\theta$, $a_y = -g$, $t = T_f$ Then

$$0 = u \sin\theta \, T_f - \frac{1}{2} g T_f^2$$

$$T_f = 2u \frac{\sin\theta}{g}$$

Horizontal range (R)

The maximum horizontal distance between the point of projection and the point on the horizontal plane where the projectile hits the ground is called horizontal range (R). This is found easily since the horizontal component of initial velocity remains the same. We can write

Range R = Horizontal component of velocity x time of flight = $u\cos\theta \times T_f$

$$R = u\cos\theta \times \frac{2u\sin\theta}{g} = \frac{2u^2 \sin\theta\cos\theta}{g}$$

$$\therefore R = \frac{u^2 \sin 2\theta}{g} \qquad (2.33)$$

The horizontal range directly depends on the initial speed (u) and the sine of angle of projection (θ). It inversely depends on acceleration due to gravity 'g'

For a given initial speed u, the maximum possible range is reached when $\sin 2\theta$ is maximum, $\sin 2\theta = 1$. This implies $2\theta = \pi/2$

$$\text{or} \qquad \theta = \frac{\pi}{4}$$

This means that if the particle is projected at 45 degrees with respect to horizontal, it attains maximum range, given by.

$$R_{max} = \frac{u^2}{g} \qquad (2.34)$$

Projectile motion !

In Tamil Nadu there is an interesting traditional game 'kitti pull'. When the 'pull' is hit by the kitti, the path followed by the pull is 'parabolic'.

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EXAMPLE 2.37

Suppose an object is thrown with initial speed 10 m s^{-1} at an angle $\pi/4$ with the horizontal, what is the range covered? Suppose the same object is thrown similarly in the Moon, will there be any change in the range? If yes, what is the change? (The acceleration due to gravity in the Moon $g_{moon} = \frac{1}{6}g$)

Solution

In projectile motion, the range of particle is given by,

$$R = \frac{u^2 \sin 2\theta}{g}$$

$$\theta = \pi/4 \quad u = v_0 = 10 m\ s^{-1}$$

$$\therefore R_{earth} = \frac{(10)^2 \sin \pi/2}{9.8} = 100/9.8$$

$$R_{earth} = 10.20\ m \text{ (Approximately 10 m)}$$

If the same object is thrown in the Moon, the range will increase because in the Moon, the acceleration due to gravity is smaller than g on Earth,

$$g_{moon} = \frac{g}{6}$$

$$R_{moon} = \frac{u^2 \sin 2\theta}{g_{moon}} = \frac{v_0^{\ 2} \sin 2\theta}{g/6}$$

$$\therefore R_{moon} = 6R_{earth}$$

$$R_{moon} = 6 \times 10.20 = 61.20\ m$$

(Approximately 60 m)

The range attained on the Moon is approximately six times that on Earth.

EXAMPLE 2.38

In the cricket game, a batsman strikes the ball such that it moves with the speed 30 m s^{-1} at an angle 30^0 with the horizontal as shown in the figure. The boundary line of the cricket ground is located at a distance of 75 m from the batsman? Will the ball go for a six? (Neglect the air resistance and take acceleration due to gravity g = 10 m s^{-2}).

Solution

The motion of the cricket ball in air is essentially a projectile motion. As we have already seen, the range (horizontal distance) of the projectile motion is given by

$$R = \frac{u^2 \sin 2\theta}{g}$$

The initial speed $u = 30\, m\ s^{-1}$

The projection angle $\theta = 30°$

The horizontal distance travelled by the cricket ball

$$R = \frac{(30)^2 \times sin60°}{10} = \frac{900 \times \frac{\sqrt{3}}{2}}{10} = 77.94\,m$$

This distance is greater than the distance of the boundary line. Hence the ball will cross this line and go for a six.

2.11.4 Introduction to Degrees and Radians

In measuring angles, there are several possible units used, but the most common units are degrees and radians. Radians are used in measuring area, volume, and circumference of circles and surface area of spheres.

Radian describes the planar angle subtended by a circular arc at the centre of a circle. It is defined as the length of the arc divided by the radius of the arc. One radian is the angle subtended at the centre of a circle by an arc that is equal in length to the radius of the circle. This is shown in the Figure 2.44.

Figure 2.44 One radian (shown in yellow color)

Degree is the unit of measurement which is used to determine the size of an angle. When an angle goes all the way around in a circle, the total angle covered is equivalent to 360°. Thus, a circle has 360°. In terms of radians, the full circle has 2π radian.

Hence we write $360° = 2\pi$ *radians*

or 1 radians $= \frac{180}{\pi}$ degrees

which means 1 rad ≅ 57.27°

EXAMPLE 2.39

Calculate the angle θ subtended by the two adjacent wooden spokes of a bullock cart wheel is shown in the figure. Express the angle in both radian and degree.

Solution

The full wheel subtends 2π radians at the centre of the wheel. The wheel is divided into 12 parts (arcs).

So one part subtends an angle $\theta = \frac{2\pi}{12} = \frac{\pi}{6}$ radian at the centre

Since, $\pi\ rad = 180°$, $\frac{\pi}{6}$ radian is equal to 30 degree.

∴ The angle subtended by two adjacent wooden spokes is 30 degree at the centre.

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Note The number π plays a very important role in both physics and mathematics. It is an irrational number which cannot be expressed in finite decimal form. However, we can approximate π as ≈ 3.14 or $\frac{22}{7}$. But we should not forget that 3.14 or $\frac{22}{7}$ is only an approximation to π and not equal to π.

2.11.5 Angular displacement

Consider a particle revolving around a point O in a circle of radius r (Figure 2.45). Let the position of the particle at time t = 0 be A and after time t, its position is B.

Figure 2.45 Angular displacement

Then,

The angle described by the particle about the axis of rotation (or centre O) in a given time is called angular displacement.

i.e., angular displacement = $\angle AOB = \theta$

The unit of angular displacement is radian.

The angular displacement (θ) in radian is related to arc length S (AB) and radius r as

$$\theta = \frac{S}{r}, \quad \text{or} \quad S = r\theta$$

Angular velocity ($\vec{\omega}$)

The rate of change of angular displacement is called angular velocity.

If θ is the angular displacement in time t, then the angular velocity ω is

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

The unit of angular velocity is radian per second (rad s^{-1}). The direction of angular velocity is along the axis of rotation following the right hand rule. This is shown in Figure 2.46.

Figure 2.46 Direction of angular velocity

i) Angular acceleration (α)

The rate of change of angular velocity is called angular acceleration.

$$\vec{\alpha} = \frac{d\vec{\omega}}{dt}$$

The angular acceleration is also a vector quantity which need not be in the same direction as angular velocity.

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Tangential acceleration

Consider an object moving along a circle of radius r. In a time Δt, the object travels an arc distance Δs as shown in Figure 2.47. The corresponding angle subtended is $\Delta\theta$

Figure 2.47 Circular motion

The Δs can be written in terms of $\Delta\theta$ as,

$$\Delta s = r\Delta\theta \qquad (2.35)$$

In a time Δt, we have

$$\frac{\Delta s}{\Delta t} = r\frac{\Delta\theta}{\Delta t} \qquad (2.36)$$

In the limit $\Delta t \to 0$, the above equation becomes

$$\frac{ds}{dt} = r\omega \qquad (2.37)$$

Here $\frac{ds}{dt}$ is linear speed (v) which is tangential to the circle and ω is angular speed. So equation (2.37) becomes

$$v = r\omega \qquad (2.38)$$

which gives the relation between linear speed and angular speed.

Note The direction of linear velocity $\vec{v}$ is tangential to the circle, whereas the direction of angular velocity $\vec{\omega}$ is along the axis of rotation. The radius is also represented as a vector $\vec{r}$ directed radially from the centre of the circle.

Equation (2.38) is true only for circular motion. In general the relation between linear and angular velocity is given by

$$\vec{v} = \vec{\omega} \times \vec{r} \qquad (2.39)$$

For circular motion equation (2.39) reduces to equation (2.38) since $\vec{\omega}$ and $\vec{r}$ are perpendicular to each other.

Differentiating the equation (2.38) with respect to time, we get (since r is constant)

$$\frac{dv}{dt} = \frac{rd\omega}{dt} = r\alpha \qquad (2.40)$$

Here $\frac{dv}{dt}$ is the tangential acceleration and is denoted as a_t. $\frac{d\omega}{dt}$ is the angular acceleration α. Then eqn. (2.39) becomes

$$a_t = r\alpha \qquad (2.41)$$

The tangential acceleration a_t experienced by an object in circular motion is shown in Figure 2.48.

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Figure 2.48 Tangential acceleration

Note that the tangential acceleration is in the direction of linear velocity.

2.11.6 Circular Motion

When a point object is moving on a circular path with a constant speed, it covers equal distances on the circumference of the circle in equal intervals of time. Then the object is said to be in uniform circular motion. This is shown in Figure 2.49.

Figure 2.49 Uniform circular motion

In uniform circular motion, the velocity is always changing but speed remains the same. Physically it implies that magnitude of velocity vector remains constant and only the direction changes continuously.

If the velocity changes in both speed and direction during the circular motion, we get non uniform circular motion.

Centripetal acceleration

As seen already, in uniform circular motion the velocity vector turns continuously without changing its magnitude (speed), as shown in Figure 2.50.

Figure 2.50 Velocity in uniform circular motion

Note that the length of the velocity vector (blue) is not changed during the motion, implying that the speed remains constant. Even though the velocity is tangential at every point in the circle, the acceleration is acting towards the centre of the circle. This is called centripetal acceleration. It always points towards the centre of the circle. This is shown in the Figure 2.51.

Figure 2.51 Centripetal acceleration

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The centripetal acceleration is derived from a simple geometrical relationship between position and velocity vectors (Figure 2.48 or Figure 2.52)

Figure 2.52 Geometrical relationship between the postion and velocity vectors

Let the directions of position and velocity vectors shift through the same angle θ in a small interval of time Δt, as shown in Figure 2.52. For uniform circular motion, $r = |\vec{r}_1| = |\vec{r}_2|$ and $v = |\vec{v}_1| = |\vec{v}_2|$. If the particle moves from position vector $\vec{r}_1$ to $\vec{r}_2$, the displacement is given by $\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$ and the change in velocity from $\vec{v}_1$ to $\vec{v}_2$ is given by $\Delta\vec{v} = \vec{v}_2 - \vec{v}_1$. The magnitudes of the displacement Δr and of Δv satisfy the following relation

$$\frac{\Delta r}{r} = -\frac{\Delta v}{v} = \theta$$

Here the negative sign implies that Δv points radially inward, towards the centre of the circle.

$$\Delta v = -v\left(\frac{\Delta r}{r}\right)$$

Then, $$a = \frac{\Delta v}{\Delta t} = -\frac{v}{r}\left(\frac{\Delta r}{\Delta t}\right) = -\frac{v^2}{r}$$

For uniform circular motion $v = \omega r$, where ω is the angular velocity of the particle about the centre. Then the centripetal acceleration can be written as

$$a = -\omega^2 r$$

Note The magnitude of centripetal acceleration is constant for uniform circular motion. But centripetal acceleration is not constant, but takes different directions during the motion.

Non uniform circular motion

If the speed of the object in circular motion is not constant, then we have non-uniform circular motion. For example, when the bob attached to a string moves in vertical circle, the speed of the bob is not the same at all time. Whenever the speed is not same in circular motion, the particle will have both centripetal and tangential acceleration as shown in the Figure 2.53.

Figure 2.53 Resultant acceleration (a_R) in non uniform circular motion

The resultant acceleration is obtained by vector sum of centripetal and tangential acceleration.

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Since centripetal acceleration is $\frac{v^2}{r}$, the magnitude of this resultant acceleration is given by $a_R = \sqrt{a_t^2 + \left(\frac{v^2}{r}\right)^2}$.

This resultant acceleration makes an angle θ with the radius vector as shown in Figure 2.53.

This angle is given by $\tan\theta = \frac{a_t}{\left(v^2/r\right)}$.

EXAMPLE 2.40

A particle moves in a circle of radius 10 m. Its linear speed is given by $v = 3t$ where t is in second and v is in m s^{-1}.

(a) Find the centripetal and tangential acceleration at t = 2 s.

(b) Calculate the angle between the resultant acceleration and the radius vector.

Solution

The linear speed at t = 2 s

$$v = 3t = 6\,m\,s^{-1}$$

The centripetal acceleration at t = 2 s is

$$a_c = \frac{v^2}{r} = \frac{(6)^2}{10} = 3.6\,m\,s^{-2}$$

The tangential acceleration is $a_t = \frac{dv}{dt} = 3\,m\,s^{-2}$

The angle between the radius vector with resultant acceleration is given by

$$\tan\theta = \frac{a_t}{a_c} = \frac{3}{3.6} = 0.833$$

$$\theta = \tan^{-1}(0.833) = 0.69 \text{ radian}$$

In terms of degree $\theta = 0.69 \times 57.27° \approx 40°$

Are we moving with the same speed both day and night with respect to Sun?

Earth orbits the Sun in an elliptical orbit. Let us specify the velocity of the centre of Earth with respect to Sun as $\vec{v}_c$. This $\vec{v}_c$ is due to the elliptical motion of the Earth around the Sun. We know that at the same time Earth is also spinning on its own axis. Due to this spinning, all objects on the surface of the Earth undergo circular motion with velocity $(\vec{v}_s)$ with respect to the axis of rotation of the Earth. At night both $\vec{v}_c$ and $\vec{v}_s$ are either in the same direction or at an acute angle with each other. So, the velocity of an object on the surface of Earth with respect to Sun at night time is $\vec{v}_{night} = \vec{v}_c + \vec{v}_s$. During the day $\vec{v}_c$ and $\vec{v}_s$ are either in opposite directions or at an obtuse angle with each other. So, the velocity of the object with respect to Sun at day time $\vec{v}_{day} = \vec{v}_c - \vec{v}_s$. From this, we can conclude that any object on the surface of the Earth travels faster with respect to Sun during night than during day time. This happens due to the rotation of the Earth.

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Kinematic Equations of circular motion

If an object is in circular motion with constant angular acceleration α, we can derive kinematic equations for this motion, analogous to those for linear motion.

Let us consider a particle executing circular motion with initial angular velocity ω_0. After a time interval t it attains a final angular velocity ω. During this time, it covers an angular displacement θ. Because of the change in angular velocity there is an angular acceleration α.

The kinematic equations for circular motion are easily written by following the kinematic equations for linear motion in section 2.4.3

The linear displacement (s) is replaced by the angular displacement (θ).

The velocity (v) is replaced by angular velocity (ω).

The acceleration (a) is replaced by angular acceleration (α).

The initial velocity (u) is replaced by the initial angular velocity (ω_0).

By following this convention, kinematic equations for circular motion are as in the table given below.

Kinematic equations for linear motion	Kinematic equations for angular motion
$v = u + at$	$\omega = \omega_0 + \alpha t$
$s = ut + \frac{1}{2}at^2$	$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$
$v^2 = u^2 + 2as$	$\omega^2 = \omega_0^2 + 2\alpha\theta$
$s = \frac{(v+u)t}{2}$	$\theta = \frac{(\omega_0 + \omega)t}{2}$

Note

Kinematic equations for linear motion are applicable for only constant acceleration. Similarly kinematic equations for angular motion are applicable to only constant angular acceleration.

EXAMPLE 2.41

A particle is in circular motion with an acceleration $\alpha = 0.2\, rad\, s^{-2}$.

(a) What is the angular displacement made by the particle after 5 s?

(b) What is the angular velocity at t = 5 s?. Assume the initial angular velocity is zero.

Solution

Since the initial angular velocity is zero $(\omega_0 = 0)$.

The angular displacement made by the particle is given by

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\theta = \frac{1}{2} \times 2 \times 10^{-1} \times 25 = 2.5\ rad$$

In terms of degree

$$\theta = 2.5 \times 57.27° \approx 143°$$

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SUMMARY

- A state of rest or of motion is defined with respect to a frame of reference.
- In Physics, we conventionally follow a right handed Cartesian coordinate system to explain the motion of objects
- To explain linear motion the concept of point mass is used.
- A vector is a quantity which has both magnitude and direction. A scalar has only magnitude.
- The length of a vector is called magnitude or norm of the vector
- In a Cartesian coordinate system the unit vectors are orthogonal to each other.
- Vectors can be added using either the triangular law of addition or the parallelogram law of addition.
- Any vector can be resolved into three components with respect to a Cartesian coordinate system
- The magnitude or norm of a vector $\vec{A}$ is given by $\left|\vec{A}\right| = A = \sqrt{A_x^2 + A_y^2 + A_z^2}$
- If two vectors are equal, then their corresponding individual components should be separately equal.
- The position vector of a particle with respect to a Cartesian coordinate system is given by $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$
- The scalar product of two vectors is defined by $\vec{A}.\vec{B} = AB\cos\theta$ (θ *is the angle between* $\vec{A}$ *and* $\vec{B}$)
- The vector product of two vectors is defined by $\vec{A} \times \vec{B} = \left(AB\sin\theta\right)\hat{n}$. The direction of $\hat{n}$ can be found using right hand thumb rule or right hand cork screw rule.
- In physics, scalar and vector products are used to describe various concepts.
- Distance is the total path length travelled by the particle and displacement is the difference between final and initial positions. Distance is a scalar quantity and displacement is a vector
- Average velocity is defined as $\vec{v}_{avg} = \dfrac{\Delta\vec{r}}{\Delta t}$ and instantaneous velocity is defined as $\vec{v} = \lim\limits_{\Delta t \to 0} \dfrac{\Delta\vec{r}}{\Delta t} = \dfrac{d\vec{r}}{dt}$. Both average velocity and instantaneous velocity are vector quantities.
- Momentum is defined as $\vec{p} = m\vec{v}$.

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SUMMARY (*cont*)

- The average acceleration is defined as $\vec{a}_{avg} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} = \frac{\Delta\vec{v}}{\Delta t}$ and instantaneous acceleration is defined as $\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$
- For constant acceleration, kinematic equations can be used to analyse motion of a particle.
- In projectile motion for which acceleration is uniform, the path traced by the particle is a parabola.
- The maximum height and range of the particle in projectile motion depend inversely on acceleration due to gravity g.
- The angular displacement of the particle is defined by $\theta = \frac{s}{r}$ and angular velocity $\vec{\omega} = \frac{d\vec{\theta}}{dt}$
- The relation between the linear velocity and angular velocity is given by $\vec{v} = \vec{\omega} \times \vec{r}$
- The centripetal acceleration is given by $a_c = -\frac{v^2}{r}$ or $-\omega^2 r$ and is always directed towards the centre of the circle.

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CONCEPT MAP

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EVALUATION

I. Multi Choice Question

1. Which one of the following Cartesian coordinate systems is not followed in physics?

2. Identify the unit vector in the following.

(a) $\hat{i}+\hat{j}$ (b) $\frac{\hat{i}}{\sqrt{2}}$

(c) $\hat{k}-\frac{\hat{j}}{\sqrt{2}}$ (d) $\frac{\hat{i}+\hat{j}}{\sqrt{2}}$

3. Which one of the following physical quantities cannot be represented by a scalar?

(a) Mass

(b) length

(c) momentum

(d) magnitude of acceleration

4. Two objects of masses m_1 and m_2 fall from the heights h_1 and h_2 respectively. The ratio of the magnitude of their momenta when they hit the ground is (AIPMT 2012)

(a) $\sqrt{\frac{h_1}{h_2}}$ (b) $\sqrt{\frac{m_1h_1}{m_2h_2}}$

(c) $\frac{m_1}{m_2}\sqrt{\frac{h_1}{h_2}}$ (d) $\frac{m_1}{m_2}$

5. If a particle has negative velocity and negative acceleration, its speed

(a) increases (b) decreases

(c) remains same (d) zero

6. If the velocity is $\vec{v}=2\hat{i}+t^2\hat{j}-9\vec{k}$, then the magnitude of acceleration at t = 0.5 s is

(a) 1m s^{-2} (b) 2 m s^{-2}

(c) zero (d) -1 m s^{-2}

7. If an object is dropped from the top of a building and it reaches the ground at t = 4 s, then the height of the building is (ignoring air resistance) (g = 9.8 ms^{-2})

(a) 77.3 m (b) 78.4 m

(c) 80.5 m (d) 79.2 m

8. A ball is projected vertically upwards with a velocity v. It comes back to ground in time t. Which v-t graph shows the motion correctly? (NSEP 00-01)

9. If one object is dropped vertically downward and another object is thrown horizontally from the same height, then the ratio of vertical distance covered by both objects at any instant t is

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(a) 1 (b) 2

(c) 4 (d) 0.5

10. A ball is dropped from some height towards the ground. Which one of the following represents the correct motion of the ball?

11. If a particle executes uniform circular motion in the xy plane in clock wise direction, then the angular velocity is in

(a) +y direction

(b) +z direction

(c) -z direction

(d) -x direction

12. If a particle executes uniform circular motion, choose the correct statement (NEET 2016)

(a) The velocity and speed are constant.

(b) The acceleration and speed are constant.

(c) The velocity and acceleration are constant.

(d) The speed and magnitude of acceleration are constant.

13. If an object is thrown vertically up with the initial speed u from the ground, then the time taken by the object to return back to ground is

(a) $\frac{u^2}{2g}$ (b) $\frac{u^2}{g}$

(c) $\frac{u}{2g}$ (d) $\frac{2u}{g}$

14. Two objects are projected at angles 30° and 60° respectively with respect to the horizontal direction. The range of two objects are denoted as R_{30^0} and R_{60^0}. Choose the correct relation from the following

(a) $R_{30^0} = R_{60^0}$

(b) $R_{30^0} = 4R_{60^0}$

(c) $R_{30^0} = \frac{R_{60^0}}{2}$

(d) $R_{30^0} = 2\,R_{60^0}$

15. An object is dropped in an unknown planet from height 50 m, it reaches the ground in 2 s. The acceleration due to gravity in this unknown planet is

(a) g = 20 m s^{-2} (b) g = 25 m s^{-2}

(c) g = 15 m s^{-2} (d) g = 30 m s^{-2}

Answers

1) d	**2)** d	**3)** c	**4)** c	**5)** a
6) a	**7)** b	**8)** c	**9)** a	**10)** a
11) c	**12)** d	**13)** d	**14)** a	**15)** b

II. Short Answer Questions

1. Explain what is meant by Cartesian coordinate system?

2. Define a vector. Give examples

3. Define a scalar. Give examples

4. Write a short note on the scalar product between two vectors.

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5. Write a short note on vector product between two vectors.
6. How do you deduce that two vectors are perpendicular?
7. Define displacement and distance.
8. Define velocity and speed.
9. Define acceleration.
10. What is the difference between velocity and average velocity.
11. Define a radian.
12. Define angular displacement and angular velocity.
13. What is non uniform circular motion?
14. Write down the kinematic equations for angular motion.
15. Write down the expression for angle made by resultant acceleration and radius vector in the non uniform circular motion.

III. Long Answer Questions

1. Explain in detail the triangle law of addition.
2. Discuss the properties of scalar and vector products.
3. Derive the kinematic equations of motion for constant acceleration.
4. Derive the equations of motion for a particle (a) falling vertically (b) projected vertically
5. Derive the equation of motion, range and maximum height reached by the particle thrown at an oblique angle θ with respect to the horizontal direction.
6. Derive the expression for centripetal acceleration.
7. Derive the expression for total acceleration in the non uniform circular motion.

IV. Exercises

1. The position vectors particle has length 1m and makes 30° with the x-axis. What are the lengths of the x and y components of the position vector?

   [Ans: $l_x = \frac{\sqrt{3}}{2}, l_y = 0.5$]

2. A particle has its position moved from $\vec{r}_1 = 3\hat{i} + 4\hat{j}$ to $\vec{r}_2 = \hat{i} + 2\hat{j}$. Calculate the displacement vector $(\Delta\vec{r})$ and draw the $\vec{r}_1$, $\vec{r}_2$ and $\Delta\vec{r}$ vector in a two dimensional Cartesian coordinate system.

   [Ans: $\Delta\vec{r} = -2\hat{i} - 2\hat{j}$]

3. Calculate the average velocity of the particle whose position vector changes from $\vec{r}_1 = 5\hat{i} + 6\hat{j}$ to $\vec{r}_2 = 2\hat{i} + 3\hat{j}$ in a time 5 second.

   [Ans: $\vec{v}_{avg} = -\frac{3}{5}(\hat{i} + \hat{j})$]

4. Convert the vector $\vec{r} = 3\hat{i} + 2\hat{j}$ into a unit vector.

   [Ans: $\hat{r} = \frac{(3\hat{i} + 2\hat{j})}{\sqrt{13}}$]

5. What are the resultants of the vector product of two given vectors given by

   $\vec{A} = 4\hat{i} - 2\hat{j} + \hat{k}$ and $\vec{B} = 5\hat{i} + 3\hat{j} - 4\hat{k}$?

   [Ans: $5\hat{i} + 21\hat{j} + 22\hat{k}$

6. An object at an angle such that the horizontal range is 4 times of the

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maximum height. What is the angle of projection of the object?

[Ans: $\theta = 45°$]

7. The following graphs represent velocity – time graph. Identify what kind of motion a particle undergoes in each graph.

[Ans: (a) $\vec{a} = constant$ (b) $\vec{v} = constant$ (c) $\vec{a} = constant$ but greater than first graph (d) $\vec{a}$ *is variable*]

8. The following velocity–time graph represents a particle moving in the positive x–direction. Analyse its motion from 0 to 7 s. Calculate the displacement covered and distance travelled by the particle from 0 to 2 s.

[Ans: distance=1.75 m, displacement = -1.25 m]

9. A particle is projected at an angle of θ with respect to the horizontal direction. Match the following for the above motion.

(a) v_x – decreases and increases

(b) v_y – remains constant

(c) Acceleration – varies

(d) Position vector – remains downward

[Ans: $v_x = remains\ constant$, $v_y = decreases\ and\ increases$, $a = remains\ downward$, $r = varies$]

10. A water fountain on the ground sprinkles water all around it. If the speed of the water coming out of the fountain is v. Calculate the total area around the fountain that gets wet.

[Ans: Area = $\frac{\pi v^4}{g^2}$]

11. The following table gives the range of a particle when thrown on different planets. All the particles are thrown at the same angle with the horizontal and with the same initial speed. Arrange the planets in ascending order according to their acceleration due to gravity, (g value).

Planet	Range
Jupiter	50 m
Earth	75 m
Mars	90 m
Mercury	95 m

[Ans: $g_{jupiter}$ is greater, $g_{mercury}$ is smaller]

12. The resultant of two vectors A and B is perpendicular to vector A and its magnitude is equal to half of the magnitude of vector B. Then the angle between A and B is

a) 30° b) 45°

c) 150° d) 120°

[Ans: $\theta = 150°$]

13. Compare the components for the following vector equations

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a) $T\hat{j} - mg\hat{j} = ma\hat{j}$ b) $\vec{T} + \vec{F} = \vec{A} + \vec{B}$

c) $\vec{T} - \vec{F} = \vec{A} - \vec{B}$ d) $T\hat{j} + mg\hat{j} = ma\hat{j}$

[Ans: (a) $T - mg = ma$
(b) $T_x + F_x = A_x + B_x$ etc.]

14. Calculate the area of the triangle for which two of its sides are given by the vectors $\vec{A} = 5\hat{i} - 3\hat{j}$, $\vec{B} = 4\hat{i} + 6\hat{j}$

[Ans: Area = 21 squared units]

15. If Earth completes one revolution in 24 hours, what is the angular displacement made by Earth in one hour. Express your answer in both radian and degree.

[Ans: $\theta = 15° \, or \, \frac{\pi}{12}$]

16. A object is thrown with initial speed 5 m s^{-1} with an angle of projection 30°. What is the height and range reached by the particle?

[Ans: height = 0.318 m Range =2.21 m]

17. A foot-ball player hits the ball with speed 20 m s^{-1} with angle 30° with respect to horizontal direction as shown in the figure. The goal post is at distance of 40 m from him. Find out whether ball reaches the goal post?

[Ans: Ball will not reach the goal post. The range =35.3 m]

18. If an object is thrown horizontally with an initial speed 10 m s^{-1} from the top of a building of height 100 m. what is the horizontal distance covered by the particle?

[Ans: R= 45 m]

19. An object is executing uniform circular motion with an angular speed of $\frac{\pi}{12}$ radian per second. At t = 0 the object starts at an angle θ = 0 What is the angular displacement of the particle after 4 s?

[Ans: 60°]

20. Consider the x–axis as representing east, the y–axis as north and z–axis as vertically upwards. Give the vector representing each of the following points.

a) 5 m north east and 2 m up

b) 4 m south east and 3 m up

c) 2 m north west and 4 m up

[Ans: (a) $\frac{5(\hat{i}+\hat{j})}{\sqrt{2}} + 2\hat{k}$ (b) $\frac{4(\hat{i}-\hat{j})}{\sqrt{2}} + 3\hat{k}$
(c) $(-\hat{i}+\hat{j})\sqrt{2} + 4\hat{k}$]

21. The Moon is orbiting the Earth approximately once in 27 days, what is the angle transversed by the Moon per day?

[Ans: 13°3']

22. An object of mass m has angular acceleration $\alpha = 0.2 \, rad \, s^{-2}$. What is the angular displacement covered by the object after 3 second? (Assume that the object started with angle zero with zero angular velocity).

[Ans: 0.9 rad or 51°]

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ICT CORNER

Projectile motion

HIT THE TARGET

Through this activity you will be able to understand the velocity variation and Different angles of projection and range

STEPS:

- Type the given URL (or) Scan the QR Code. You can see "Projectile Motion" PhET simulation page. Click 'Intro' to initiate the activity.
- Click the red coloured shoot button. Blast a ball out of a cannon, and challenge yourself to hit the target.
- Drag 'up & down' button to change the height of the cylinder. Click left and right button to change the speed of the cannon ball.
- Drag the target box and fix the target to measure time, range and height. Drag the Meter tape to measure the length from cannon. On the right side top, mark in the corresponding boxes to know the velocity vectors and acceleration vectors.

Step1

Step2

Step3

Step4

Timeline Project's URL:

https://phet.colorado.edu/sims/html/projectile-motion/latest/projectile-motion_en.html

* Pictures are indicative only.

* If browser requires, allow **Flash Player** or **Java Script** to load the page.

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UNIT 3

LAWS OF MOTION

"In the beginning there was a mechanics" – Von Laue

Learning Objectives

In this unit, the student is exposed to

- Newton's laws
- logical connection between laws of Newton
- free body diagram and related problems
- law of conservation of momentum
- role of frictional forces
- centripetal and centrifugal forces
- origin of centrifugal force

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3.1 INTRODUCTION

Each and every object in the universe interacts with every other object. The cool breeze interacts with the tree. The tree interacts with the Earth. In fact, all species interact with nature. But, what is the difference between a human's interaction with nature and that of an animal's. Human's interaction has one extra quality. We not only interact with nature but also try to understand and explain natural phenomena scientifically.

In the history of mankind, the most curiosity driven scientific question asked was about motion of objects–'How things move?' and 'Why things move?' Surprisingly, these simple questions have paved the way for development from early civilization to the modern technological era of the 21st century.

Objects move because something pushes or pulls them. For example, if a book is at rest, it will not move unless a force is applied on it. In other words, to move an object a force must be applied on it. About 2500 years ago, the famous philosopher, Aristotle, said that '*Force causes motion*'. This statement is based on common sense. But any scientific answer cannot be based on common sense. It must be endorsed with quantitative experimental proof.

In the 15th century, Galileo challenged Aristotle's idea by doing a series of experiments. He said force is not required to maintain motion.

Galileo demonstrated his own idea using the following simple experiment. When a ball rolls from the top of an inclined plane to its bottom, after reaching the ground it moves some distance and continues

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to move on to another inclined plane of same angle of inclination as shown in the Figure 3.1(a). By increasing the smoothness of both the inclined planes, the ball reach almost the same height(h) from where it was released (L1) in the second plane (L2) (Figure 3.1(b)). The motion of the ball is then observed by varying the angle of inclination of the second plane keeping the same smoothness. If the angle of inclination is reduced, the ball travels longer distance in the second plane to reach the same height (Figure 3.1 (c)). When the angle of inclination is made zero, the ball moves forever in the horizontal direction (Figure 3.1(d)). If the Aristotelian idea were true, the ball would not have moved in the second plane even if its smoothness is made maximum since no force acted on it in the horizontal direction. From this simple experiment, Galileo proved that force is not required to maintain motion. An object can be in motion even without a force acting on it.

Figure 3.1 Galileo's experiment with the second plane (a) at same inclination angle as the first (b) with increased smoothness (c) with reduced angle of inclination (d) with zero angle of inclination

In essence, Aristotle coupled the motion with force while Galileo decoupled the motion and force.

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3.2 NEWTON'S LAWS

Newton analysed the views of Galileo, and other scientist like Kepler and Copernicus on motion and provided much deeper insights in the form of three laws.

3.2.1 Newton's First Law

Every object continues to be in the state of rest or of uniform motion (constant velocity) unless there is external force acting on it.

This inability of objects to move on its own or change its state of motion is called inertia. Inertia means resistance to change its state. Depending on the circumstances, there can be three types of inertia.

1. **Inertia of rest:** When a stationary bus starts to move, the passengers experience a sudden backward push. Due to inertia, the body (of a passenger) will try to

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Figure 3.2 Passengers experience a backward push due to inertia of rest

continue in the state of rest, while the bus moves forward. This appears as a backward push as shown in Figure 3.2. *The inability of an object to change its state of rest is called inertia of rest.*

2. **Inertia of motion:** When the bus is in motion, and if the brake is applied suddenly, passengers move forward and hit against the front seat. In this case, the bus comes to a stop, while the body (of a passenger) continues to move forward due to the property of inertia as shown in Figure 3.3. *The inability of an object to change its state of uniform speed (constant speed) on its own is called inertia of motion.*
3. **Inertia of direction:** When a stone attached to a string is in whirling motion, and if the string is cut suddenly, the stone will not continue to move in circular motion but moves tangential to the circle as illustrated in Figure 3.4. This is because the body cannot change its direction of motion without any force acting on it. *The inability of an object to change its direction of motion on its own is called inertia of direction.*

Figure 3.3 Passengers experience a forward push due to inertia of motion

When we say that an object is at rest or in motion with constant velocity, it has a meaning only if it is specified with respect to some reference frames. In physics, any motion has to be stated with respect to a reference frame. It is to be noted that Newton's first law is valid only in certain special reference frames called inertial frames. In fact, Newton's first law defines an inertial frame.

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Figure 3.4 A stone moves tangential to circle due to inertia of direction

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Inertial Frames

If an object is free from all forces, then it moves with constant velocity or remains at rest when seen from inertial frames. Thus, there exists some special set of frames in which, if an object experiences no force, it moves with constant velocity or remains at rest. But how do we know whether an object is experiencing a force or not? All the objects in the Earth experience Earth's gravitational force. In the ideal case, if an object is in deep space (very far away from any other object), then Newton's first law will be certainly valid. Such deep space can be treated as an inertial frame. But practically it is not possible to reach such deep space and verify Newton's first law.

For all practical purposes, we can treat Earth as an inertial frame because an object on the table in the laboratory appears to be at rest always. This object never picks up acceleration in the horizontal direction since no force acts on it in the horizontal direction. So the laboratory can be taken as an inertial frame for all physics experiments and calculations. For making these conclusions, we analyse only the horizontal motion of the object as there is no horizontal force that acts on it. We should not analyse the motion in vertical direction as the two forces (gravitational force in the downward direction and normal force in upward direction) that act on it makes the net force is zero in vertical direction. Newton's first law deals with the motion of objects in the absence of any force and not the motion under zero net force. Suppose a train is moving with constant velocity with respect to an inertial frame, then an object at rest in the inertial frame (outside the train) appears to move with constant velocity with respect to the train (viewed from within the train). So the train can be treated as an inertial frame. All inertial frames are moving with constant velocity relative to each other. If an object appears to be at rest in one inertial frame, it may appear to move with constant velocity with respect to another inertial frame. For example, in Figure 3.5, the car is moving with uniform velocity v with respect to a person standing (at rest) on the ground. As the car is moving with constant velocity with respect to the person at rest on the ground, both frames (with respect to the car and to the ground) are inertial frames.

Figure 3.5 The person and vehicle are inertial frames

Suppose an object remains at rest on a smooth table kept inside the train, and if the train suddenly accelerates (which we may not sense), the object appears to accelerate backwards even without any force acting on it. It is a clear violation of Newton's first law as the object gets accelerated without being acted upon by a force. It implies that the train is not an inertial frame when it is accelerated. For example, Figure 3.6 shows that car 2 is a non-inertial frame since it moves with acceleration $\vec{a}$ with respect to the ground.

Figure 3.6 Car 2 is a non-inertial frame

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These kinds of accelerated frames are called non-inertial frames. A rotating frame is also a non inertial frame since rotation requires acceleration. In this sense, Earth is not really an inertial frame since it has self-rotation and orbital motion. But these rotational effects of Earth can be ignored for the motion involved in our day-to-day life. For example, when an object is thrown, or the time period of a simple pendulum is measured in the physics laboratory, the Earth's self-rotation has very negligible effect on it. In this sense, Earth can be treated as an inertial frame. But at the same time, to analyse the motion of satellites and wind patterns around the Earth, we cannot treat Earth as an inertial frame since its self-rotation has a strong influence on wind patterns and satellite motion.

3.2.2 Newton's Second Law

This law states that

The force acting on an object is equal to the rate of change of its momentum

$$\vec{F} = \frac{d\vec{p}}{dt} \qquad (3.1)$$

In simple words, whenever the momentum of the body changes, there must be a force acting on it. The momentum of the object is defined as $\vec{p} = m\vec{v}$. In most cases, the mass of the object remains constant during the motion. In such cases, the above equation gets modified into a simpler form

$$\vec{F} = \frac{d(m\vec{v})}{dt} = m\frac{d\vec{v}}{dt} = m\vec{a}.$$

$$\vec{F} = m\vec{a}. \qquad (3.2)$$

The above equation conveys the fact that if there is an acceleration $\vec{a}$ on the body, then there must be a force acting on it. This implies that if there is a change in velocity, then there must be a force acting on the body. The force and acceleration are always in the same direction. Newton's second law was a paradigm shift from Aristotle's idea of motion. According to Newton, the force need not cause the motion but only a change in motion. It is to be noted that *Newton's second law is valid only in inertial frames*. In non-inertial frames Newton's second law cannot be used in this form. It requires some modification.

In the SI system of units, the unit of force is measured in newtons and it is denoted by symbol 'N'.

One Newton is defined as the force which acts on 1 kg of mass to give an acceleration 1 m s^{-2} in the direction of the force.

Aristotle vs. Newton's approach on sliding object

Newton's second law gives the correct explanation for the experiment on the inclined plane that was discussed in section 3.1. In normal cases, where friction is not negligible, once the object reaches the bottom of the inclined plane (Figure 3.1), it travels some distance and stops. Note that it stops because there is a frictional force acting in the direction opposite to its velocity. It is this frictional force that reduces the velocity of the object to zero and brings it to rest. As per Aristotle's idea, as soon as the body reaches the bottom of the plane, it can travel only a small distance and stops because there is no force acting on the object. Essentially, he did not consider the frictional force acting on the object.

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Figure 3.7 Aristotle, Galileo and Newton's approach

3.2.3 Newton's Third Law

Consider Figure 3.8(a) whenever an object 1 exerts a force on the object 2 ($\vec{F}_{21}$), then object 2 must also exert equal and opposite force on the object 1 ($\vec{F}_{12}$). These forces must lie along the line joining the two objects.

$$\vec{F}_{12} = -\vec{F}_{21}$$

Newton's third law assures that the forces occur as equal and opposite pairs. An isolated force or a single force cannot exist in nature. *Newton's third law states that for every action there is an equal and opposite reaction*. Here, action and reaction pair of forces do not act on the same body but on two different bodies. Any one of the forces can be called as an action force and the other the reaction force. Newton's third law is valid in both inertial and non-inertial frames.

These action-reaction forces are not cause and effect forces. It means that when the object 1 exerts force on the object 2, the object 2 exerts equal and opposite force on the body 1 at the same instant.

(a) (b) (c)

Figure 3.8 Demonstration of Newton's third law (a) Hammer and the nail (b) Ball bouncing off the wall (c) Walking on the floor with friction

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ACTIVITY

Verification of Newton's third law

Attach two spring balances as shown in the figure. Fix one end with rigid support and leave the other end free, which can be pulled with the hand.

Pull one end with some force and note the reading on both the balances.

Repeat the exercise a number of times.

Note The reading in the spring balance A is due to the force given by spring balance B. The reading in the spring balance B is due to the reaction force given by spring balance A. Note that according to Newton's third law, both readings (force) are equal.

3.2.4 Discussion on Newton's Laws

1. Newton's laws are vector laws. The equation $\vec{F} = m\vec{a}$ is a vector equation and essentially it is equivalent to three scalar equations. In Cartesian coordinates, this equation can be written as $F_x\hat{i} + F_y\hat{j} + F_z\hat{k} = ma_x\hat{i} + ma_y\hat{j} + ma_z\hat{k}$. By comparing both sides, the three scalar equations are

   $F_x = ma_x$ The acceleration along the x direction depends only on the component of force acting along the x-direction.

   $F_y = ma_y$ The acceleration along the y direction depends only on the component of force acting along the y-direction.

   $F_z = ma_z$ The acceleration along the z direction depends only on the component of force acting along the z-direction.

   From the above equations, we can infer that the force acting along y direction cannot alter the acceleration along x direction. In the same way, F_z cannot affect a_y and a_x. This understanding is essential for solving problems.

2. The acceleration experienced by the body at time t depends on the force which acts on the body at that instant of time. It does not depend on the force which acted on the body before the time t. This can be expressed as

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$$\vec{F}(t) = m\vec{a}(t)$$

Acceleration of the object does not depend on the previous history of the force. For example, when a spin bowler or a fast bowler throws the ball to the batsman, once the ball leaves the hand of the bowler, it experiences only gravitational force and air frictional force. The acceleration of the ball is independent of how the ball was bowled (with a lower or a higher speed).

3. In general, the direction of a force may be different from the direction of motion. Though in some cases, the object may move in the same direction as the direction of the force, it is not always true. A few examples are given below.

Case 1: Force and motion in the same direction

When an apple falls towards the Earth, the direction of motion (direction of velocity) of the apple and that of force are in the same downward direction as shown in the Figure 3.9 (a).

Figure 3.9 (a) Force and motion in the same direction

Case 2: Force and motion not in the same direction

The Moon experiences a force towards the Earth. But it actually moves in elliptical orbit. In this case, the direction of the force is different from the direction of motion as shown in Figure 3.9 (b).

Figure 3.9 (b) Moon orbiting in elliptical orbit around the Earth

Case 3: Force and motion in opposite direction

If an object is thrown vertically upward, the direction of motion is upward, but gravitational force is downward as shown in the Figure 3.9 (c).

Figure 3.9 (c) Force and direction of motion are in opposite directions

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Case 4: Zero net force, but there is motion

When a raindrop gets detached from the cloud it experiences both downward gravitational force and upward air drag force. As it descends towards the Earth, the upward air drag force increases and after a certain time, the upward air drag force cancels the downward gravity. From then on the raindrop moves at constant velocity till it touches the surface of the Earth. Hence the raindrop comes with zero net force, therefore with zero acceleration but with non-zero terminal velocity. It is shown in the Figure 3.9 (d).

Figure 3.9 (d) Zero net force and non zero terminal velocity

4. If multiple forces $\vec{F}_1, \vec{F}_2, \vec{F}_3 \ldots . \vec{F}_n$ act on the same body, then the total force $(\vec{F}_{net})$ is equivalent to the vectorial sum of the individual forces. Their net force provides the acceleration.

$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \ldots + \vec{F}_n$$

Figure 3.10 Vector addition of forces

Newton's second law for this case is

$$\vec{F}_{net} = m\vec{a}$$

In this case the direction of acceleration is in the direction of net force.

Example

Bow and arrow

Figure 3.11 Bow and arrow – Net force is on the arrow

5. Newton's second law can also be written in the following form.

Since the acceleration is the second derivative of position vector of the body $\left(\vec{a} = \frac{d^2\vec{r}}{dt^2}\right)$, the force on the body is

$$\vec{F} = m\frac{d^2\vec{r}}{dt^2}.$$

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From this expression, we can infer that Newton's second law is basically a second order ordinary differential equation and whenever the second derivative of position vector is not zero, there must be a force acting on the body.

6. If no force acts on the body then Newton's second law, $m\frac{d\vec{v}}{dt} = 0$.

   It implies that $\vec{v} = constant$. It is essentially Newton's first law. It implies that the second law is consistent with the first law. However, it should not be thought of as the reduction of second law to the first when no force acts on the object. Newton's first and second laws are independent laws. They can internally be consistent with each other but cannot be derived from each other.

7. Newton's second law is cause and effect relation. Force is the cause and acceleration is the effect. Conventionally, the effect should be written on the left and cause on the right hand side of the equation. So the correct way of writing Newton's second law is $m\vec{a} = \vec{F}$ or $\frac{d\vec{p}}{dt} = \vec{F}$

3.3 APPLICATION OF NEWTON'S LAWS

3.3.1 Free Body Diagram

Free body diagram is a simple tool to analyse the motion of the object using Newton's laws.

The following systematic steps are followed for developing the free body diagram:

1. Identify the forces acting on the object.
2. Represent the object as a point.
3. Draw the vectors representing the forces acting on the object.

When we draw the free body diagram for an object or a system, the forces exerted by the object should not be included in the free body diagram.

EXAMPLE 3.1

A book of mass m is at rest on the table. (1) What are the forces acting on the book? (2) What are the forces exerted by the book? (3) Draw the free body diagram for the book.

Solution

(1) There are two forces acting on the book.

(i) Gravitational force (mg) acting downwards on the book

(ii) Normal contact force (N) exerted by the surface of the table on the book. It acts upwards as shown in the figure.

Note In the free body diagram, as the magnitudes of the normal force and the gravitational force are same, the lengths of both these vectors are also same.

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(2) According to Newton's third law, there are two reaction forces exerted by the book.

(i) The book exerts an equal and opposite force (mg) on the Earth which acts upwards.

(ii) The book exerts a force which is equal and opposite to normal force on the surface of the table (N) acting downwards.

Note

It is to be emphasized that while applying Newton's third law it is wrong to conclude that the book on the table is at rest due to the downward gravitational force exerted by the Earth and the equal and opposite reacting normal force exerted by the table on the book. Action and reaction forces never act on the same body.

(3) The free body diagram of the book is shown in the figure.

EXAMPLE 3.2

If two objects of masses 2.5 kg and 100 kg experience the same force 5 N, what is the acceleration experienced by each of them?

Solution

From Newton's second law (in magnitude form), F = ma

For the object of mass 2.5 kg, the acceleration is $a = \frac{F}{m} = \frac{5}{2.5} = 2\ m\ s^{-2}$

For the object of mass 100 kg, the acceleration is $a = \frac{F}{m} = \frac{5}{100} = 0.05\ m\ s^{-2}$

Note

Even though the force applied on both the objects is the same, acceleration experienced by each object differs. The acceleration is inversely proportional to mass. For the same force, the heavier mass experiences lesser acceleration and the lighter mass experiences greater acceleration.

When an apple falls, it experiences Earth's gravitational force. According to Newton's third law, the apple exerts equal and opposite force on the Earth. Even though both the apple and Earth experience the same force, their acceleration is different. The mass of Earth is enormous compared to that of an apple. So an apple experiences larger acceleration and the Earth experiences almost negligible acceleration. Due to the negligible acceleration, Earth appears to be stationary when an apple falls.

EXAMPLE 3.3

Which is the greatest force among the three force $\vec{F}_1, \vec{F}_2, \vec{F}_3$ shown below

Solution

Force is a vector and magnitude of the vector is represented by the length of the vector. Here $\vec{F}_1$ has greater length compared to other two. So $\vec{F}_1$ is largest of the three.

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EXAMPLE 3.4

Apply Newton's second law to a mango hanging from a tree. (Mass of the mango is 400 gm)

Solution

Note: Before applying Newton's laws, the following steps have to be followed:

1) Choose a suitable inertial coordinate system to analyse the problem. For most of the cases we can take Earth as an inertial coordinate system.
2) Identify the system to which Newton's laws need to be applied. The system can be a single object or more than one object.
3) Draw the free body diagram.
4) Once the forces acting on the system are identified, and the free body diagram is drawn, apply Newton's second law. In the left hand side of the equation, write the forces acting on the system in vector notation and equate it to the right hand side of equation which is the product of mass and acceleration. Here, acceleration should also be in vector notation.
5) If acceleration is given, the force can be calculated. If the force is given, acceleration can be calculated.

By following the above steps:

We fix the inertial coordinate system on the ground as shown in the figure.

The forces acting on the mango are

i) Gravitational force exerted by the Earth on the mango acting downward along negative y axis

ii) Tension (in the cord attached to the mango) acts upward along positive y axis.

The free body diagram for the mango is shown in the figure

$$\vec{F}_g = mg\left(-\hat{j}\right) = -mg\hat{j}$$

Here, mg is the magnitude of the gravitational force and $\left(-\hat{j}\right)$ represents the unit vector in negative y direction

$$\vec{T} = T\hat{j}$$

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Here T is the magnitude of the tension force and $(\hat{j})$ represents the unit vector in positive y direction

$$\vec{F}_{net} = \vec{F}_g + \vec{T} = -mg\hat{j} + T\hat{j} = (T - mg)\hat{j}$$

From Newton's second law $\vec{F}_{net} = m\vec{a}$

Since the mango is at rest with respect to us (inertial coordinate system) the acceleration is zero ($\vec{a} = 0$).

So $\vec{F}_{net} = m\vec{a} = 0$

$$(T - mg)\hat{j} = 0$$

By comparing the components on both sides of the above equation, we get $T - mg = 0$

So the tension force acting on the mango is given by $T = mg$

Mass of the mango m = 400g and $g = 9.8\ m\ s^{-2}$

Tension acting on the mango is $T = 0.4 \times 9.8 = 3.92$ N

EXAMPLE 3.5

A person rides a bike with a constant velocity $\vec{v}$ with respect to ground and another biker accelerates with acceleration $\vec{a}$ with respect to ground. Who can apply Newton's second law with respect to a stationary observer on the ground?

Solution

Second biker cannot apply Newton's second law, because he is moving with acceleration $\vec{a}$ with respect to Earth (he is not in inertial frame). But the first biker can apply Newton's second law because he is moving at constant velocity with respect to Earth (he is in inertial frame).

EXAMPLE 3.6

The position vector of a particle is given by $\vec{r} = 3t\hat{i} + 5t^2\hat{j} + 7\hat{k}$. Find the direction in which the particle experiences net force?

Solution

Velocity of the particle,

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{d}{dt}(3t)\hat{i} + \frac{d}{dt}(5t^2)\hat{j} + \frac{d}{dt}(7)\hat{k}$$

$$\frac{d\vec{r}}{dt} = 3\hat{i} + 10t\hat{j}$$

Acceleration of the particle

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2} = 10\hat{j}$$

Here, the particle has acceleration only along positive y direction. According to Newton's second law, net force must also act along positive y direction. In addition, the particle has constant velocity in positive x direction and no velocity in z direction. Hence, there are no net force along x or z direction.

EXAMPLE 3.7

Consider a bob attached to a string, hanging from a stand. It oscillates as shown in the figure.

a) Identify the forces that act on the bob?
b) What is the acceleration experienced by the bob?

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Solution

Two forces act on the bob.

(i) Gravitational force (mg) acting downwards

(ii) Tension (T) exerted by the string on the bob, whose position determines the direction of T as shown in figure.

The bob is moving in a circular arc as shown in the above figure. Hence it has centripetal acceleration. At a point A and C, the bob comes to rest momentarily and then its velocity increases when it moves towards point B. Hence, there is a tangential acceleration along the arc. The gravitational force can be resolved into two components (mg cosθ, mg sinθ) as shown below

Note Note that the bob does not move in the direction of the resultant force. At the points A and C, tension T = mg cosθ. At all other points, tension T is greater than mg cosθ, since it has non zero centripetal acceleration. At point B, the resultant force acts upward along the string. It is an example of a non uniform circular motion because the bob has both the centripetal and tangential accelerations.

EXAMPLE 3.8

The velocity of a particle moving in a plane is given by the following diagram. Find out the direction of force acting on the particle?

Solution

The velocity of the particle is $\vec{v} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$. As shown in the figure, the particle is moving in the xy plane, there is no motion in the z direction. So velocity in the z direction is zero ($v_z = 0$). The velocity of the particle has x component (v_x) and y component (v_y). From figure, as time increases from t = 0 sec to t = 3 sec, the length of the vector in y direction is changing (increasing). It means y component of velocity (v_y) is increasing with respect to time. According to Newton's second law, if velocity changes with respect to time then there must be acceleration. In this case, the particle has acceleration in the y direction since the y component of velocity changes. So the particle experiences force in the y direction. The length of the vector in x direction does not change. It means that the particle has constant velocity in the x direction. So no force or zero net force acts in the x direction.

EXAMPLE 3.9

Apply Newton's second law for an object at rest on Earth and analyse the result.

Solution

The object is at rest with respect to Earth (inertial coordinate system). There are two forces that act on the object.

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i) Gravity acting downward (negative y-direction)
ii) Normal force by the surface of the Earth acting upward (positive y-direction)

The free body diagram for this object is

$$\vec{F}_g = -mg\hat{j}$$
$$\vec{N} = N\hat{j}$$

Net force $\vec{F}_{net} = -mg\hat{j} + N\hat{j}$

But there is no acceleration on the ball. So $\vec{a} = 0$. By applying Newton's second law ($\vec{F}_{net} = m\vec{a}$)

Since $\vec{a} = 0, \vec{F}_{net} = -mg\hat{j} + N\hat{j}$

$$(-mg + N)\hat{j} = 0$$

By comparing the components on both sides of the equation, we get

$$-mg + N = 0$$
$$N = mg$$

We can conclude that if the object is at rest, the magnitude of normal force is exactly equal to the magnitude of gravity.

EXAMPLE 3.10

A particle of mass 2 kg experiences two forces, $\vec{F}_1 = 5\hat{i} + 8\hat{j} + 7\hat{k}$ and $\vec{F}_2 = 3\hat{i} - 4\hat{j} + 3\hat{k}$. What is the acceleration of the particle?

Solution

We use Newton's second law, $\vec{F}_{net} = m\vec{a}$ where $\vec{F}_{net} = \vec{F}_1 + \vec{F}_2$. From the above equations the acceleration is $\vec{a} = \frac{\vec{F}_{net}}{m}$, where

$$\vec{F}_{net} = (5+3)\hat{i} + (8-4)\hat{j} + (7+3)\hat{k}$$
$$\vec{F}_{net} = 8\hat{i} + 4\hat{j} + 10\hat{k}$$
$$\vec{a} = \left(\frac{8}{2}\right)\hat{i} + \left(\frac{4}{2}\right)\hat{j} + \left(\frac{10}{2}\right)\hat{k}$$
$$\vec{a} = 4\hat{i} + 2\hat{j} + 5\hat{k}$$

EXAMPLE 3.11

Identify the forces acting on blocks A, B and C shown in the figure.

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Solution

Forces on block A:

(i) Downward gravitational force exerted by the Earth ($m_A g$)

(ii) Upward normal force exerted by block B (N_B)

The free body diagram for block A is as shown in the following picture.

Force on block A

Forces on block B :

(i) Downward gravitational force exerted by Earth ($m_B g$)

(ii) Downward force exerted by block A (N_A)

(iii) Upward normal force exerted by block C (N_C)

Force on block B

Forces onblock C:

(i) Downward gravitational force exerted by Earth ($m_C g$)

(ii) Downward force exerted by block B (N_B)

(iii) Upward force exerted by the table (N_{table})

Force on block C

EXAMPLE 3.12

Consider a horse attached to the cart which is initially at rest. If the horse starts walking forward, the cart also accelerates in the forward direction. If the horse pulls the cart with force F_h in forward direction, then according to Newton's third law, the cart also pulls the horse by equivalent opposite force $F_c = F_h$ in backward direction. Then total force on 'cart+horse' is zero. Why is it then the 'cart+horse' accelerates and moves forward?

Solution

This paradox arises due to wrong application of Newton's second and third laws. Before applying Newton's laws, we should decide 'what is the system?'. Once we identify the 'system', then it is possible to identify all the forces acting on the system. We should not consider the force exerted by the system. If there is an unbalanced force acting on the system, then it should have acceleration in the direction of the resultant force. By following these steps we will analyse the horse and cart motion.

If we decide on the cart+horse as a 'system', then we should not consider the force exerted by the horse on the cart or the force exerted by cart on the horse. Both are internal forces acting on each other. According to Newton's third law, total internal force acting on the system is zero and it cannot accelerate the system. The acceleration of the system is caused by some external force. In this case, the force exerted by the road on the system is the external force acting on the system. It is wrong to conclude that the total force acting on the system (cart+horse) is zero without including all the forces acting on the system. The road is pushing the horse

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and cart forward with acceleration. As there is an external force acting on the system, Newton's second law has to be applied and not Newton's third law.

The following figures illustrates this.

If we consider the horse as the 'system', then there are three forces acting on the horse.

(i) Downward gravitational force ($m_h g$)
(ii) Force exerted by the road (F_r)
(iii) Backward force exerted by the cart (F_c)

It is shown in the following figure.

F_r – Force exerted by the road on the horse

F_c – Force exerted by the cart on the horse

$F_r^{\perp}$ – Perpendicular component of $F_r = N$

$F_r^{\parallel}$ – Parallel component of F_r which is reason for forward movement

The force exerted by the road can be resolved into parallel and perpendicular components. The perpendicular component balances the downward gravitational force. There is parallel component along the forward direction. It is greater than the backward force (F_c). So there is net force along the forward direction which causes the forward movement of the horse.

If we take the cart as the system, then there are three forces acting on the cart.

(i) Downward gravitational force ($m_c g$)
(ii) Force exerted by the road (F_r')
(iii) Force exerted by the horse (F_h)

It is shown in the figure

Force on the cart

The force exerted by the road ($\vec{F_r'}$) can be resolved into parallel and perpendicular components. The perpendicular component cancels the downward gravity ($m_c g$). Parallel component acts backwards and the force exerted by the horse ($\vec{F_h}$) acts forward. Force ($\vec{F_h}$) is greater than the parallel component acting in the opposite direction. So there is an overall unbalanced force in the forward direction which causes the cart to accelerate forward.

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If we take the cart+horse as a system, then there are two forces acting on the system.

(i) Downward gravitational force $(m_h + m_c)g$

(ii) The force exerted by the road (F_r) on the system.

It is shown in the following figure.

(iii) In this case the force exerted by the road (F_r) on the system (cart+horse) is resolved in to parallel and perpendicular components. The perpendicular component is the normal force which cancels the downward gravitational force $(m_h + m_c)g$. The parallel component of the force is not balanced, hence the system (cart+horse) accelerates and moves forward due to this force.

EXAMPLE 3.13

The position of the particle is represented by $y = ut - \frac{1}{2}gt^2$.

a) What is the force acting on the particle?

b) What is the momentum of the particle?

Solution

To find the force, we need to find the acceleration experienced by the particle.

The acceleration is given by $a = \frac{d^2y}{dt^2}$

$$\text{(or)} \quad a = \frac{dv}{dt}$$

Here

v = velocity of the particle in y direction

$$v = \frac{dy}{dt} = u - gt$$

The momentum of the particle = mv = m (u-gt).

$$a = \frac{dv}{dt} = -g$$

The force acting on the object is given by $F = ma = -mg$

The negative sign implies that the force is acting on the negative y direction. This is exactly the force that acts on the object in projectile motion.

3.3.2 Particle Moving in an Inclined Plane

When an object of mass m slides on a frictionless surface inclined at an angle θ as shown in the Figure 3.12, the forces acting on it decides the

a) acceleration of the object

b) speed of the object when it reaches the bottom

The force acting on the object is

(i) Downward gravitational force (mg)

(ii) Normal force perpendicular to inclined surface (N)

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Figure 3.12 Object moving in an inclined plane

To draw the free body diagram, the block is assumed to be a point mass (Figure 3.13 (a)). Since the motion is on the inclined surface, we have to choose the coordinate system parallel to the inclined surface as shown in Figure 3.13 (b).

The gravitational force mg is resolved in to parallel component $mg\sin\theta$ along the inclined plane and perpendicular component $mg\cos\theta$ perpendicular to the inclined surface (Figure 3.13 (b)).

Note that the angle made by the gravitational force (mg) with the perpendicular to the surface is equal to the angle of inclination θ as shown in Figure 3.13 (c).

There is no motion(acceleration) along the y axis. Applying Newton's second law in the y direction

$$-mg\cos\theta\hat{j} + N\hat{j} = 0\left(No\ acceleration\right)$$

By comparing the components on both sides, $N - mg\cos\theta = 0$

$$N = mg\cos\theta$$

The magnitude of normal force (N) exerted by the surface is equivalent to $mg\ cos\theta$.

The object slides (with an acceleration) along the x direction. Applying Newton's second law in the x direction

$$mg\sin\theta\hat{i} = ma\hat{i}$$

By comparing the components on both sides, we can equate

$$mg\sin\theta = ma$$

The acceleration of the sliding object is

$$a = g\ sin\theta$$

Figure 3.13 (a) Free body diagram, (b) mg resolved into parallel and perpendicular components (c) The angle θ_2 is equal to θ

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Note that the acceleration depends on the angle of inclination θ. If the angle θ is 90 degree, the block will move vertically with acceleration $a = g$.

Newton's kinematic equation is used to find the speed of the object when it reaches the bottom. The acceleration is constant throughout the motion.

$$v^2 = u^2 + 2as \text{ along the x direction} \quad (3.3)$$

The acceleration a is equal to g sinθ. The initial speed (u) is equal to zero as it starts from rest. Here s is the length of the inclined surface.

The speed (v) when it reaches the bottom is (using equation (3.3))

$$v = \sqrt{2sg\ sin\theta} \quad (3.4)$$

Note Here we choose the coordinate system along the inclined plane. Even if we choose the coordinate system parallel to the horizontal surface, we will get the same result. But the mathematics will be quite complicated. Choosing a suitable inertial coordinate system for the given problem is very important.

3.3.3 Two Bodies in Contact on a Horizontal Surface

Consider two blocks of masses m_1 and m_2 ($m_1 > m_2$) kept in contact with each other on a smooth, horizontal frictionless surface as shown in Figure 3.14.

Figure 3.14 (a) Two blocks of masses m_1 and m_2 ($m_1 > m_2$) kept in contact with each other on a smooth, horizontal frictionless surface

By the application of a horizontal force F, both the blocks are set into motion with acceleration 'a' simultaneously in the direction of the force F.

To find the acceleration $\vec{a}$, Newton's second law has to be applied to the system (combined mass $m = m_1 + m_2$)

$$\vec{F} = m\vec{a}$$

If we choose the motion of the two masses along the positive x direction,

$$F\hat{i} = ma\hat{i}$$

By comparing components on both sides of the above equation

$$F = ma \quad \text{where } m = m_1 + m_2$$

The acceleration of the system is given by

$$\therefore a = \frac{F}{m_1 + m_2} \quad (3.5)$$

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The force exerted by the block m_1 on m_2 due to its motion is called force of contact ($\vec{f}_{21}$). According to Newton's third law, the block m_2 will exert an equivalent opposite reaction force ($\vec{f}_{12}$) on block m_1.

Figure 3.14 (b) shows the free body diagram of block m_1.

Figure 3.14 (b) Free body diagram of block of mass m_1

$$\therefore F\hat{i} - f_{12}\hat{i} = m_1 a\hat{i}$$

By comparing the components on both sides of the above equation, we get

$$F - f_{12} = m_1 a$$

$$f_{12} = F - m_1 a \qquad (3.6)$$

Substituting the value of acceleration from equation (3.5) in (3.6) we get

$$f_{12} = F - m_1\left(\frac{F}{m_1 + m_2}\right)$$

$$f_{12} = F\left[1 - \frac{m_1}{m_1 + m_2}\right]$$

$$f_{12} = \frac{Fm_2}{m_1 + m_2} \qquad (3.7)$$

Equation (3.7) shows that the magnitude of contact force depends on mass m_2 which provides the reaction force. Note that this force is acting along the negative x direction.

In vector notation, the reaction force on mass m_1 is given by $\vec{f}_{12} = -\dfrac{Fm_2}{m_1 + m_2}\hat{i}$

For mass m_2 there is only one force acting on it in the x direction and it is denoted by $\vec{f}_{21}$. This force is exerted by mass m_1. The free body diagram for mass m_2 is shown in Figure 3.14 (c).

Figure 3.14 (c) Free body diagram of block of mass m_2

Applying Newton's second law for mass m_2 $\qquad f_{21}\hat{i} = m_2 a\hat{i}$

By comparing the components on both sides of the above equation

$$f_{21} = m_2 a \qquad (3.8)$$

Substituting for acceleration from equation (3.5) in equation (3.8), we get $f_{21} = \dfrac{Fm_2}{m_1 + m_2}$

In this case the magnitude of the contact force is

$f_{21} = \dfrac{Fm_2}{m_1 + m_2}$ The direction of this force is along the positive x direction.

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In vector notation, the force acting on mass m_2 exerted by mass m_1 is $\vec{f}_{21} = \frac{Fm_2}{m_1 + m_2}\hat{i}$

Note $\vec{f}_{12} = -\vec{f}_{21}$ which confirms Newton's third law.

3.3.4 Motion of Connected Bodies

When objects are connected by strings and a force F is applied either vertically or horizontally or along an inclined plane, it produces a tension T in the string, which affects the acceleration to an extent. Let us discuss various cases for the same.

Case 1: Vertical motion

Consider two blocks of masses m_1 and m_2 ($m_1 > m_2$) connected by a light and inextensible string that passes over a pulley as shown in Figure 3.15.

Figure 3.15 Two blocks connected by a string over a pulley

Let the tension in the string be T and acceleration a. When the system is released, both the blocks start moving, m_2 vertically upward and m_1 downward with same acceleration a. The gravitational force m_1g on mass m_1 is used in lifting the mass m_2.

The upward direction is chosen as y direction. The free body diagrams of both masses are shown in Figure 3.16.

Figure 3.16 Free body diagrams of masses m_1 and m_2

Applying Newton's second law for mass m_2

$$T\hat{j} - m_2 g\hat{j} = m_2 a\hat{j}$$

The left hand side of the above equation is the total force that acts on m_2 and the right hand side is the product of mass and acceleration of m_2 in y direction.

By comparing the components on both sides, we get

$$T - m_2 g = m_2 a \qquad (3.9)$$

Similarly, applying Newton's second law for mass m_1

$$T\hat{j} - m_1 g\hat{j} = -m_1 a\hat{j}$$

As mass m_1 moves downward $(-\hat{j})$, its acceleration is along $(-\hat{j})$

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By comparing the components on both sides, we get

$$T - m_1 g = -m_1 a$$

$$m_1 g - T = m_1 a \qquad (3.10)$$

Adding equations (3.9) and (3.10), we get

$$m_1 g - m_2 g = m_1 a + m_2 a$$

$$(m_1 - m_2) g = (m_1 + m_2) a \qquad (3.11)$$

From equation (3.11), the acceleration of both the masses is

$$a = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) g \qquad (3.12)$$

If both the masses are equal (m_1=m_2), from equation (3.12)

$$a = 0$$

This shows that if the masses are equal, there is no acceleration and the system as a whole will be at rest.

To find the tension acting on the string, substitute the acceleration from the equation (3.12) into the equation (3.9).

$$T - m_2 g = m_2 \left(\frac{m_1 - m_2}{m_1 + m_2}\right) g$$

$$T = m_2 g + m_2 \left(\frac{m_1 - m_2}{m_1 + m_2}\right) g \qquad (3.13)$$

By taking $m_2 g$ common in the RHS of equation (3.13)

$$T = m_2 g \left(1 + \frac{m_1 - m_2}{m_1 + m_2}\right)$$

$$T = m_2 g \left(\frac{m_1 + m_2 + m_1 - m_2}{m_1 + m_2}\right)$$

$$T = \left(\frac{2 m_1 m_2}{m_1 + m_2}\right) g$$

Equation (3.12) gives only magnitude of acceleration.

For mass m_1, the acceleration vector is given by $\vec{a} = -\left(\frac{m_1 - m_2}{m_1 + m_2}\right) g \hat{j}$

For mass m_2, the acceleration vector is given by $\vec{a} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) g \hat{j}$

Case 2: Horizontal motion

In this case, mass m_2 is kept on a horizontal table and mass m_1 is hanging through a small pulley as shown in Figure 3.17. Assume that there is no friction on the surface.

Figure 3.17 Blocks in horizontal motion

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As both the blocks are connected to the unstretchable string, if m_1 moves with an acceleration a downward then m_2 also moves with the same acceleration a horizontally.

The forces acting on mass m_2 are

(i) Downward gravitational force (m_2g)
(ii) Upward normal force (N) exerted by the surface
(iii) Horizontal tension (T) exerted by the string

The forces acting on mass m_1 are

(i) Downward gravitational force (m_1g)
(ii) Tension (T) acting upwards

The free body diagrams for both the masses is shown in Figure 3.18.

Figure 3.18 Free body diagrams of masses m_1 and m_2

Applying Newton's second law for m_1

$$T\hat{j} - m_1g\hat{j} = -m_1a\hat{j} \text{ (along y direction)}$$

By comparing the components on both sides of the above equation,

$$T - m_1g = -m_1a \qquad (3.14)$$

Applying Newton's second law for m_2

$$T\hat{i} = m_2a\hat{i} \text{ (along x direction)}$$

By comparing the components on both sides of above equation,

$$T = m_2a \qquad (3.15)$$

There is no acceleration along y direction for m_2.

$$N\hat{j} - m_2g\hat{j} = 0$$

By comparing the components on both sides of the above equation

$$N - m_2g = 0$$

$$N = m_2g \qquad (3.16)$$

By substituting equation (3.15) in equation (3.14), we can find the tension T

$$m_2a - m_1g = -m_1a$$

$$m_2a + m_1a = m_1g$$

$$a = \frac{m_1}{m_1 + m_2}g \qquad (3.17)$$

Tension in the string can be obtained by substituting equation (3.17) in equation (3.15)

$$T = \frac{m_1m_2}{m_1 + m_2}g \qquad (3.18)$$

Comparing motion in both cases, it is clear that the tension in the string for horizontal motion is half of the tension for vertical motion for same set of masses and strings.

This result has an important application in industries. The ropes used in conveyor belts (horizontal motion) work for longer duration than those of cranes and lifts (vertical motion).

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3.3.5 Concurrent Forces and Lami's Theorem

A collection of forces is said to be concurrent, if the lines of forces act at a common point. Figure 3.19 illustrates concurrent forces.

Concurrent forces need not be in the same plane. If they are in the same plane, they are concurrent as well as coplanar forces.

Figure 3.19 Concurrent forces

3.4 LAMI'S THEOREM

If a system of three concurrent and coplanar forces is in equilibrium, then Lami's theorem states that the magnitude of each force of the system is proportional to sine of the angle between the other two forces. The constant of proportionality is same for all three forces.

Let us consider three coplanar and concurrent forces $\vec{F}_1, \vec{F}_2$ and $\vec{F}_3$ which act at a common point O as shown in Figure 3.20. If the point is at equilibrium, then according to Lami's theorem

Figure 3.20 Three coplanar and concurrent forces $\vec{F}_1, \vec{F}_2$ and $\vec{F}_3$ acting at O

$$|\vec{F}_1| \propto \sin\alpha$$

$$|\vec{F}_2| \propto \sin\beta$$

$$|\vec{F}_3| \propto \sin\gamma$$

Therefore, $$\frac{|\vec{F}_1|}{\sin\alpha} = \frac{|\vec{F}_2|}{\sin\beta} = \frac{|\vec{F}_3|}{\sin\gamma} \quad (3.19)$$

Lami's theorem is useful to analyse the forces acting on objects which are in static equilibrium.

Application of Lami's Theorem

EXAMPLE 3.14

A baby is playing in a swing which is hanging with the help of two identical chains is at rest. Identify the forces acting on the baby. Apply Lami's theorem and find out the tension acting on the chain.

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Solution

The baby and the chains are modeled as a particle hung by two strings as shown in the figure. There are three forces acting on the baby.

i) Downward gravitational force along negative y direction (mg)
ii) Tension (T) along the two strings

These three forces are coplanar as well as concurrent as shown in the following figure.

By using Lami's theorem

$$\frac{T}{\sin(180-\theta)} = \frac{T}{\sin(180-\theta)} = \frac{mg}{\sin(2\theta)}$$

Since $\sin(180-\theta) = \sin\theta$ and $\sin(2\theta) = 2\sin\theta\cos\theta$

We get $$\frac{T}{\sin\theta} = \frac{mg}{2\sin\theta\cos\theta}$$

From this, the tension on each string is $T = \dfrac{mg}{2\cos\theta}$.

Note

When $\theta = 0°$, the strings are vertical and the tension on each string is $T = \dfrac{mg}{2}$

3.5 LAW OF CONSERVATION OF TOTAL LINEAR MOMENTUM

In nature, conservation laws play a very important role. The dynamics of motion of bodies can be analysed very effectively using conservation laws. There are three conservation laws in mechanics. Conservation of total energy, conservation of total linear momentum, and conservation of angular momentum. By combining Newton's second and third laws, we can derive the law of conservation of total linear momentum.

When two particles interact with each other, they exert equal and opposite forces on each other. The particle 1 exerts force $\vec{F}_{21}$ on particle 2 and particle 2 exerts an exactly equal and opposite force $\vec{F}_{12}$ on particle 1, according to Newton's third law.

$$\vec{F}_{21} = -\vec{F}_{12} \qquad (3.20)$$

In terms of momentum of particles, the force on each particle (Newton's second law) can be written as

$$\vec{F}_{12} = \frac{d\vec{p}_1}{dt} \quad \text{and} \quad \vec{F}_{21} = \frac{d\vec{p}_2}{dt}. \qquad (3.21)$$

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Here $\vec{p}_1$ is the momentum of particle 1 which changes due to the force $\vec{F}_{12}$ exerted by particle 2. Further $\vec{p}_2$ is the momentum of particle 2. This changes due to $\vec{F}_{21}$ exerted by particle 1.

Substitute equation (3.21) in equation (3.20)

$$\frac{d\vec{p}_1}{dt} = -\frac{d\vec{p}_2}{dt} \qquad (3.22)$$

$$\frac{d\vec{p}_1}{dt} + \frac{d\vec{p}_2}{dt} = 0 \qquad (3.23)$$

$$\frac{d}{dt}\left(\vec{p}_1 + \vec{p}_2\right) = 0$$

It implies that $\vec{p}_1 + \vec{p}_2 =$ constant vector (always).

$\vec{p}_1 + \vec{p}_2$ is the total linear momentum of the two particles ($\vec{p}_{tot} = \vec{p}_1 + \vec{p}_2$). It is also called as total linear momentum of the system. Here, the two particles constitute the system. From this result, the law of conservation of linear momentum can be stated as follows.

If there are no external forces acting on the system, then the total linear momentum of the system ($\vec{p}_{tot}$) is always a constant vector. In otherwords, the total linear momentum of the system is conserved in time. Here the word 'conserve' means that $\vec{p}_1$ and $\vec{p}_2$ can vary,in such a way that $\vec{p}_1 + \vec{p}_2$ is a constant vector.

The forces $\vec{F}_{12}$ and $\vec{F}_{21}$ are called the internal forces of the system, because they act only between the two particles. There is no external force acting on the two particles from outside. In such a case the total linear momentum of the system is a constant vector or is conserved.

EXAMPLE 3.15

Identify the internal and external forces acting on the following systems.

a) Earth alone as a system
b) Earth and Sun as a system
c) Our body as a system while walking
d) Our body + Earth as a system

Solution

a) Earth alone as a system

Earth orbits the Sun due to gravitational attraction of the Sun. If we consider Earth as a system, then Sun's gravitational force is an external force. If we take the Moon into account, it also exerts an external force on Earth.

b) (Earth + Sun) as a system

In this case, there are two internal forces which form an action and reaction pair- the gravitational force exerted by the Sun on Earth and gravitational force exerted by the Earth on the Sun.

c) Our body as a system

While walking, we exert a force on the Earth and Earth exerts an equal and opposite force on our body. If our body alone is considered as a system, then

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the force exerted by the Earth on our body is external.

Our body as a system

d) (Our body + Earth) as a system

In this case, there are two internal forces present in the system. One is the force exerted by our body on the Earth and the other is the equal and opposite force exerted by the Earth on our body.

Our body + Earth as a system

Meaning of law of conservation of momentum

1) The Law of conservation of linear momentum is a vector law. It implies that both the magnitude and direction of total linear momentum are constant. In some cases, this total momentum can also be zero.
2) To analyse the motion of a particle, we can either use Newton's second law or the law of conservation of linear momentum. Newton's second law requires us to specify the forces involved in the process. This is difficult to specify in real situations. But conservation of linear momentum does not require any force involved in the process. It is covenient and hence important.

For example, when two particles collide, the forces exerted by these two particles on each other is difficult to specify. But it is easier to apply conservation of linear momentum during the collision process.

Collision of two particles

Examples

- Consider the firing of a gun. Here the system is Gun+bullet. Initially the gun and bullet are at rest, hence the total linear momentum of the system is zero. Let $\vec{p}_1$ be the momentum of the bullet and $\vec{p}_2$ the momentum of the gun before firing. Since initially both are at rest,

$$\vec{p}_1 = 0, \vec{p}_2 = 0.$$

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Total momentum before firing the gun is zero, $\vec{p}_1 + \vec{p}_2 = 0$.

According to the law of conservation of linear momentum, total linear momemtum has to be zero after the firing also.

When the gun is fired, a force is exerted by the gun on the bullet in forward direction. Now the momentum of the bullet changes from $\vec{p}_1$ to $\vec{p}_1'$. To conserve the total linear momentum of the system, the momentum of the gun must also change from $\vec{p}_2$ to $\vec{p}_2'$. Due to the conservation of linear momentum, $\vec{p}_1' + \vec{p}_2' = 0$. It implies that $\vec{p}_1' = -\vec{p}_2'$, the momentum of the gun is exactly equal, but in the opposite direction to the momentum of the bullet. This is the reason after firing, the gun suddenly moves backward with the momentum $\left(-\vec{p}_2'\right)$. It is called 'recoil momemtum'. This is an example of conservation of total linear momentum.

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- Consider two particles. One is at rest and the other moves towards the first particle (which is at rest). They collide and after collison move in some arbitrary directions. In this case, before collision, the total linear momentum of the system is equal to the initial linear momentum of the moving particle. According to conservation of momentum, the total linear momentum after collision also has to be in the forward direction. The following figure explains this.

A more accurate calculation is covered in section 4.4. It is to be noted that the total momentum vector before and after collison points in the same direction. This simply means that the total linear momentum is constant before and after the collision. At the time of collision, each particle exerts a force on the other. As the two particles are considered as a system, these forces are only internal, and the total linear momentum cannot be altered by internal forces.

3.5.1 Impulse

If a very large force acts on an object for a very short duration, then the force is called impulsive force or impulse.

If a force (F) acts on the object in a very short interval of time (Δt), from Newton's second law in magnitude form

$$F dt = dp$$

Integrating over time from an initial time t_i to a final time t_f, we get

$$\int_i^f dp = \int_{t_i}^{t_f} F\, dt$$

$$p_f - p_i = \int_{t_i}^{t_f} F\, dt$$

p_i = initial momentum of the object at time t_i

p_f = final momentum of the object at time t_f

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$p_f - p_i = \Delta p =$ change in momentum of the object during the time interval $t_f - t_i = \Delta t$.

The integral $\int_{t_i}^{t_f} F\,dt = J$ is called the impulse and it is equal to change in momentum of the object.

If the force is constant over the time interval, then

$$\int_{t_i}^{t_f} F\,dt = F\int_{t_i}^{t_f} dt = F\left(t_f - t_i\right) = F\Delta t$$

$$F\,\Delta t = \Delta p \qquad (3.24)$$

Equation (3.24) is called the 'impulse-momentum equation'.

For a constant force, the impulse is denoted as $J = F\Delta t$ and it is also equal to change in momentum (Δp) of the object over the time interval Δt.

Impulse is a vector quantity and its unit is Ns.

The average force acted on the object over the short interval of time is defined by

$$F_{avg} = \frac{\Delta p}{\Delta t} \qquad (3.25)$$

From equation (3.25), the average force that act on the object is greater if Δt is smaller. Whenever the momentum of the body changes very quickly, the average force becomes larger.

The impulse can also be written in terms of the average force. Since Δp is change in momentum of the object and is equal to impulse (J), we have

$$J = F_{avg}\Delta t \qquad (3.26)$$

The graphical representation of constant force impulse and variable force impulse is given in Figure 3.21.

Figure 3.21 Constant force impulse and variable force impulse

Illustration

1. When a cricket player catches the ball, he pulls his hands gradually in the direction of the ball's motion. Why?

   If he stops his hands soon after catching the ball, the ball comes to rest very quickly. It means that the momentum of the ball is brought to rest very quickly. So the average force acting

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on the body will be very large. Due to this large average force, the hands will get hurt. To avoid getting hurt, the player brings the ball to rest slowly.

2. When a car meets with an accident, its momentum reduces drastically in a very short time. This is very dangerous for the passengers inside the car since they will experience a large force. To prevent this fatal shock, cars are designed with air bags in such a way that when the car meets with an accident, the momentum of the passengers will reduce slowly so that the average force acting on them will be smaller.

3. The shock absorbers in two wheelers play the same role as airbags in the car. When there is a bump on the road, a sudden force is transferred to the vehicle. The shock absorber prolongs the period of transfer of force on to the body of the rider. Vehicles without shock absorbers will harm the body due to this reason.
4. Jumping on a concrete cemented floor is more dangerous than jumping on the sand. Sand brings the body to rest slowly than the concrete floor, so that the average force experienced by the body will be lesser.

KLURW

Impulse

If an egg is thrown, can you catch the egg safely without breaking it? How?

EXAMPLE 3.16

An object of mass 10 kg moving with a speed of 15 $m\ s^{-1}$ hits the wall and comes to rest within

a) 0.03 second
b) 10 second

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Calculate the impulse and average force acting on the object in both the cases.

Solution

Initial momentum of the object $p_i = 10 \times 15 = 150\ kgm\ s^{-1}$

Final momentum of the object $p_f = 0$

$$\Delta p = 150 - 0 = 150\ kg\ ms^{-1}$$

(a) Impulse $J = \Delta p = 150\,N\,s.$

(b) Impulse $J = \Delta p = 150\,N\,s$

(a) Average force $F_{avg} = \dfrac{\Delta p}{\Delta t} = \dfrac{150}{0.03} = 5000\,N$

(b) Average force $F_{avg} = \dfrac{150}{10} = 15\,N$

We see that, impulse is the same in both cases, but the average force is different.

3.6 FRICTION

3.6.1 Introduction

If a very gentle force in the horizontal direction is given to an object at rest on the table, it does not move. It is because of the opposing force exerted by the surface on the object which resists its motion. This force is called the *frictional force which always opposes the relative motion between an object and the surface where it is placed.* If the force applied is increased, the object moves after a certain limit.

Relative motion: when a force parallel to the surface is applied on the object, the force tries to move the object with respect to the surface. This 'relative motion' is opposed by the surface by exerting a frictional force on the object in a direction opposite to applied force. Frictional force always acts on the object parallel to the surface on which the object is placed. There are two kinds of friction namely 1) Static friction and 2) Kinetic friction.

Figure 3.22 Frictional force

3.6.2 Static Friction ($\vec{f}_s$)

Static friction is the force which opposes the initiation of motion of an object on the surface. When the object is at rest on the surface, only two forces act on it. They are the downward gravitational force and upward normal force. The resultant of these two forces on the object is zero. As a result the object is at rest as shown in Figure 3.23.

If some external force F_{ext} is applied on the object parallel to the surface on which the object is at rest, the surface exerts

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exactly an equal and opposite force on the object to resist its motion and tries to keep the object at rest. It implies that external force and frictional force are exactly equal and opposite. Therefore, no motion parallel to the surface takes place. But if the external force is increased above a particular limit, the surface cannot provide sufficient opposing frictional force to balance the external force on the object. Then the object starts to slide. This is the maximal static friction that can be exerted by the surface. Experimentally, it is found that the magnitude of static frictional force f_s satisfies the following empirical relation.

$$0 \le f_s \le \mu_s N, \qquad (3.27)$$

where μ_s is the coefficient of static friction. It depends on the nature of the surfaces in contact. N is normal force exerted by the surface on the body and sometimes it is equal to mg. But it need not be equal to *mg* always.

Equation (3.27) implies that the force of static friction can take any value from zero to $\mu_s N$.

If the object is at rest and no external force is applied on the object, the static friction acting on the object is zero ($f_s = 0$).

If the object is at rest, and there is an external force applied parallel to the surface, then the force of static friction acting on the object is exactly equal to the external force applied on the object ($f_s = F_{ext}$). But still the static friction f_s is less than $\mu_s N$.

When object begins to slide, the static friction (f_s) acting on the object attains maximum,

The static and kinetic frictions (which we discuss later) depend on the normal force acting on the object. If the object is pressed hard on the surface then the normal force acting on the object will increase. As a consequence it is more difficult to move the object. This is shown in Figure 3.23 (a) and (b). The static friction does not depend upon the area of contact.

Fig 3.23 Static friction and kinetic friction (a) Easier to move (b) Harder to move

EXAMPLE 3.17

Consider an object of mass 2 kg resting on the floor. The coefficient of static friction between the object and the floor is $\mu_s = 0.8$. What force must be applied on the object to move it?

Solution

Since the object is at rest, the gravitational force experienced by an object is balanced by normal force exerted by floor.

$$N = mg$$

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The maximum static frictional force $f_s^{max} = \mu_s N = \mu_s mg$

$$f_s^{max} = 0.8 \times 2 \times 9.8 = 15.68\,N$$

Therefore to move the object the external force should be greater than maximum static friction.

$$F_{ext} > 15.68\,N$$

EXAMPLE 3.18

Consider an object of mass 50 kg at rest on the floor. A Force of 5 N is applied on the object but it does not move. What is the frictional force that acts on the object?

Solution

When the object is at rest, the external force and the static frictional force are equal and opposite.

The magnitudes of these two forces are equal, $f_s = F_{ext}$

Therefore, the static frictional force acting on the object is

$$f_s = 5N.$$

The direction of this frictional force is opposite to the direction of F_{ext}.

EXAMPLE 3.19

Two bodies of masses 7 kg and 5 kg are connected by a light string passing over a smooth pulley at the edge of the table as shown in the figure. The coefficient of static friction between the surfaces (body and table) is 0.9. Will the mass m_1 = 7 kg on the surface move? If not what value of m_2 should be used so that mass 7 kg begins to slide on the table?

Solution

As shown in the figure, there are four forces acting on the mass m_1

a) Downward gravitational force along the negative y-axis (m_1g)
b) Upward normal force along the positive y axis (N)
c) Tension force due to mass m_2 along the positive x axis
d) Frictional force along the negative x axis

Since the mass m_1 has no vertical motion, $m_1g = N$

Free body diagram for mass m 1

To determine whether the mass m_1 moves on the surface, calculate the maximum static friction exerted by the table on the mass m_1. If the tension on the mass m_1 is equal to or greater than this maximum static friction, the object will move.

$$f_s^{max} = \mu_s N = \mu_s m_1 g$$

$$f_s^{max} = 0.9 \times 7 \times 9.8 = 61.74\ N$$

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The tension $T = m_2 g = 5 \times 9.8 = 49\ N$

$$T < f_s^{max}$$

The tension acting on the mass m_1 is less than the maximum static friction. So the mass m_1 will not move.

To move the mass m_1, $T > f_s^{max}$ where $T = m_2 g$

$$m_2 = \frac{\mu_s m_1 g}{g} = \mu_s m_1$$

$$m_2 = 0.9 \times 7 = 6.3\ kg$$

If the mass m_2 is greater than 6.3 kg then the mass m_1 will begin to slide. Note that if there is no friction on the surface, the mass m_1 will move even when m_2 is just 1 kg.

The values of coefficient of static friction for pairs of materials are presented in Table 3.1. Note that the ice and ice pair have very low coefficient of static friction. This means a block of ice can move easily over another block of ice.

Table 3.1 Coefficient of Static Friction for a Pair of Materials

Material	Coefficient of Static Friction
Glass and glass	1.0
Ice and ice	0.10
Steel and steel	0.75
Wood and wood	0.35
Rubber tyre and dry concrete road	1.0
Rubber tyre and wet road	0.7

3.6.3 Kinetic Friction

If the external force acting on the object is greater than maximum static friction, the objects begin to slide. When an object slides, the surface exerts a frictional force called **kinetic friction** $\vec{f}_k$ (also called sliding friction or dynamic friction). To move an object at constant velocity we must apply a force which is equal in magnitude and opposite to the direction of kinetic friction.

Figure 3.24 Kinetic friction

Experimentally it was found that the magnitude of kinetic friction satisfies the relation

$$f_k = \mu_k N \qquad (3.28)$$

where μ_k is the coefficient of kinetic friction and N the normal force exerted by the surface on the object,

and $\mu_k < \mu_s$

This implies that starting of a motion is more difficult than maintaining it. The salient features of static and kinetic friction are given in Table 3.2.

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Table 3.2 Salient Features of Static and Kinetic Friction

Static friction	Kinetic friction
It opposes the starting of motion	It opposes the relative motion of the object with respect to the surface
Independent of surface area of contact	Independent of surface area of contact
μ_s depends on the nature of materials in mutual contact	μ_k depends on nature of materials and temperature of the surface
Depends on the magnitude of applied force	Independent of magnitude of applied force
It can take values from zero to $\mu_s N$	It can never be zero and always equals to $\mu_k N$ whatever be the speed (true $v < 10$ ms^{-1})
$f_s^{max} > f_k$	It is less than maximum value of static friction
$\mu_s > \mu_k$	Coefficient of kinetic friction is less than coefficient of static friction

The variation of both static and kinetic frictional forces with external applied force is graphically shown in Figure 3.25.

Figure 3.25 Variation of static and kinetic frictional forces with external applied force

The Figure 3.25 shows that static friction increases linearly with external applied force till it reaches the maximum. If the object begins to move then the kinetic friction is slightly lesser than the maximum static friction. Note that the kinetic friction is constant and it is independent of applied force.

Note The relation $f_s = \mu_s N$ is not a vector relation. This is because the normal force N and f_s are not in the same direction even though f_s is equal to μ_s times the normal force. This is also true in the case of kinetic friction.

3.6.4 To Move an Object - Push or pull? Which is easier?

When a body is pushed at an arbitrary angle θ $\left(0 \text{ to } \frac{\pi}{2}\right)$, the applied force F can be resolved into two components as F sinθ parallel to the surface and F cosθ perpendicular to the surface as shown in Figure 3.26. The total downward force acting on the body is mg + Fcosθ. It implies that the normal force acting on the body increases. Since there is no acceleration along the vertical direction the normal force N is equal to

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$$N_{push} = mg + F\cos\theta \qquad (3.29)$$

As a result the maximal static friction also increases and is equal to

$$f_s^{max} = \mu_s N_{push} = \mu_s \left(mg + F\cos\theta\right) \qquad (3.30)$$

Equation (3.30) shows that a greater force needs to be applied to push the object into motion.

Figure 3.26 An object is pushed at an angle θ

When an object is pulled at an angle θ, the applied force is resolved into two components as shown in Figure 3.27. The total downward force acting on the object is

$$N_{pull} = mg - F\cos\theta \qquad (3.31)$$

Figure 3.27 An object is pulled at an angle θ

Equation (3.31) shows that the normal force is less than N_{push}. From equations (3.29) and (3.31), it is easier to pull an object than to push to make it move.

3.6.5 Angle of Friction

The angle of friction is defined as the angle between the normal force (N) and the resultant force (R) of normal force and maximum friction force (f_s^{max})

Figure 3.28 Angle of Friction

In Figure 3.28 the resultant force is $R = \sqrt{\left(f_s^{max}\right)^2 + N^2}$

$$\tan\theta = \frac{f_s^{max}}{N} \qquad (3.32)$$

But from the frictional relation, the object begins to slide when $f_s^{max} = \mu_s N$

$$\text{or when } \frac{f_s^{max}}{N} = \mu_s \qquad (3.33)$$

From equations (3.32) and (3.33) the coefficient of static friction is

$$\mu_s = \tan\theta \qquad (3.34)$$

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The coefficient of static friction is equal to tangent of the angle of friction

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3.6.6 Angle of Repose

Consider an inclined plane on which an object is placed, as shown in Figure 3.29. Let the angle which this plane makes with the horizontal be θ. For small angles of θ, the object may not slide down. As θ is increased, for a particular value of θ, the object begins to slide down. This value is called angle of repose. Hence, the angle of repose is the angle of inclined plane with the horizontal such that an object placed on it begins to slide.

Figure 3.29 Angle of repose

Let us consider the various forces in action here. The gravitational force mg is resolved into components parallel ($mg\sin\theta$) and perpendicular ($mg\cos\theta$) to the inclined plane.

The component of force parallel to the inclined plane (mg sinθ) tries to move the object down.

The component of force perpendicular to the inclined plane (mg cosθ) is balanced by the Normal force (N).

$$N = mg\cos\theta$$

When the object just begins to move, the static friction attains its maximum value

$$f_s = f_s^{max} = \mu_s N = \mu_s mg\cos\theta \qquad (3.35)$$

This friction also satisfies the relation

$$f_s^{max} = mg\sin\theta \qquad (3.36)$$

Dividing equations (3.35) and (3.36), we get

$$\mu_s = \sin\theta / \cos\theta$$

From the definition of angle of friction, we also know that

$$\tan\theta = \mu_s, \qquad (3.37)$$

in which θ is the angle of friction.

Thus the angle of repose is the same as angle of friction. But the difference is that the angle of repose refers to inclined surfaces and the angle of friction is applicable to any type of surface.

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EXAMPLE 3.20

A block of mass m slides down the plane inclined at an angle 60° with an acceleration $\frac{g}{2}$. Find the coefficient of kinetic friction?

Solution

Kinetic friction comes to play as the block is moving on the surface.

The forces acting on the mass are the normal force perpendicular to surface, downward gravitational force and kinetic friction f_k along the surface.

Along the x-direction

$$mg\sin\theta - f_k = \text{ma}$$

But a = g/2

$$mg\sin 60° - f_k = \text{mg}/2$$

$$\frac{\sqrt{3}}{2}\text{mg} - f_k = \text{mg}/2$$

$$f_k = mg\left(\frac{\sqrt{3}}{2} - \frac{1}{2}\right)$$

$$f_K = \left(\frac{\sqrt{3}-1}{2}\right)\text{mg}$$

There is no motion along the y-direction as normal force is exactly balanced by the mg $\cos\theta$.

$$\text{mg}\cos\theta = \text{N} = \text{mg}/2$$

$$f_K = \mu_K \text{N} = \mu_K \text{ mg}/2$$

$$\mu_K = \frac{\left(\frac{\sqrt{3}-1}{2}\right)mg}{\frac{mg}{2}}$$

$$\mu_K = \sqrt{3} - 1$$

3.6.7 Application of Angle of Repose

1. Antlions make sand traps in such a way that when an insect enters the edge of the trap, it starts to slide towards the bottom where the antilon hide itself. The angle of inclination of sand trap is made to be equal to angle of repose. It is shown in the Figure 3.30.

Figure 3.30 Sand trap of antlions

2. Children are fond of playing on sliding board (Figure 3.31). Sliding will be easier

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when the angle of inclination of the board is greater than the angle of repose. At the same time if inclination angle is much larger than the angle of repose, the slider will reach the bottom at greater speed and get hurt.

Figure 3.31 Sliding board

ACTIVITY

Measuring the coefficients of friction

Take a hard bound note book and a coin. Keep the coin on the note book. The note book cover has to be in an inclined position as shown in the figure. Slowly increase the angle of inclination of the cover with respect to rest of the pages. When the angle of inclination reaches the angle of repose, the parallel component of gravitational force (mg sinθ) to book surface becomes equal to the frictional force and the coin begins to slide down. Measure the angle of inclination and take the tangent of this angle. It gives the coefficient of static friction between the surface of the cover and coin. The same can be repeated with other objects such as an eraser in

order to observe that the coefficient of static friction differs from case to case.

Note

At the point of sliding $\tan\theta_s = \mu_s$

To measure the coefficient of kinetic friction, reduce the inclination of the book after it starts sliding, such that the coin/eraser moves with uniform velocity. Now measure the angle from which coefficient of kinetic friction can be calculated as

$$\mu_K = \tan\theta_k$$

Observe that $\theta_K < \theta_S$

3.6.8 Rolling Friction

The invention of the wheel plays a crucial role in human civilization. One of the important applications is suitcases with rolling on coasters. Rolling wheels makes it easier than carrying luggage. When an object moves on a surface, essentially it is sliding on it. But wheels move on the surface through rolling motion. In rolling motion when a wheel moves on a surface, the point of contact with surface is always at rest. Since the point of contact is at rest, there is no relative motion between the wheel and surface. Hence the frictional force is very less. At the same time if an object moves

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without a wheel, there is a relative motion between the object and the surface. As a result frictional force is larger. This makes it difficult to move the object. The Figure 3.32 shows the difference between rolling and kinetic friction.

Figure 3.32 Rolling and kinetic friction

Ideally in pure rolling, motion of the point of contact with the surface should be at rest, but in practice it is not so. Due to the elastic nature of the surface at the point of contact there will be some deformation on the object at this point on the wheel or surface as shown in Figure 3.33. Due to this deformation, there will be minimal friction between wheel and surface. It is called 'rolling friction'. In fact, 'rolling friction' is much smaller than kinetic friction.

Figure 3.33 Rolling friction

3.6.9 Methods to Reduce Friction

Frictional force has both positive and negative effects. In some cases it is absolutely necessary. Walking is possible because of frictional force. Vehicles (bicycle, car) can move because of the frictional force between the tyre and the road. In the braking system, kinetic friction plays a major role. As we have already seen, the frictional force comes into effect whenever there is relative motion between two surfaces. In big machines used in industries, relative motion between different parts of the machine produce unwanted heat which reduces its efficiency. To reduce this kinetic friction lubricants are used as shown in Figure 3.34.

Figure 3.34 Reducing kinetic friction using lubricant

Ball bearings provides another effective way to reduce the kinetic friction (Figure 3.35) in machines. If ball bearings are fixed between two surfaces, during the relative motion only the rolling friction comes to effect and not kinetic friction. As we have seen earlier, the rolling friction is much smaller than kinetic

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friction; hence the machines are protected from wear and tear over the years.

Figure 3.35 Reducing kinetic friction using ball bearing

During the time of Newton and Galileo, frictional force was considered as one of the natural forces like gravitational force. But in the twentieth century, the understanding on atoms, electron and protons has changed the perspective. The frictional force is actually the electromagnetic force between the atoms on the two surfaces. Even well polished surfaces have irregularities on the surface at the microscopic level as seen in the Figure 3.36.

Points to Ponder

When you walk on the tiled floor where water is spilled, you are likely to slip. Why?

Figure 3.36 Irregularities on the surface at the microscopic level

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Frictional force in the motion of a bicycle

When a bicycle moves in the forward direction, what is the direction of frictional force in the rear and front wheels?

When we pedal a bicycle, we try to push the surface backward and the velocity of point of contact in the rear wheel is backwards. So, the frictional force pushes the rear wheel to move forward. But as the front wheel is connected with a rigid support to the back wheel, the forward motion of back wheel pushes the front wheel in the forward direction. So, the frictional forces act backward. Remember both frictional forces correspond to only static friction and not kinetic friction. If the wheel slips then kinetic friction comes into effect. In addition to static friction, the rolling friction also acts on both wheels in the backward direction.

EXAMPLE 3.21

Consider an object moving on a horizontal surface with a constant velocity. Some external force is applied on the object to keep the object moving with a constant velocity. What is the net force acting on the object?

Solution

If an object moves with constant velocity, then it has no acceleration. According to Newton's second law there is no net force acting on the object. The external force is balanced by the kinetic friction.

Note It is not that 'no force acts on the object'. In fact there are two forces acting on the object. Only the net force acting on the object is zero.

3.7 DYNAMICS OF CIRCULAR MOTION

In the previous sections we have studied how to analyse linear motion using Newton's laws. It is also important to know how to apply Newton's laws to circular motion, since circular motion is one of the very common types of motion that we come across in our daily life. A particle can be in linear motion with or without any external force. But when circular motion occurs there must necessarily be some force acting on the object. There is no Newton's first law for circular motion. In other words without a force, circular motion cannot occur in nature. A force can change the velocity of a particle in three different ways.

1. The magnitude of the velocity can be changed without changing the direction of the velocity. In this case the particle will move in the same direction but with acceleration.

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Examples

Particle falling down vertically, bike moving in a straight road with acceleration.

2. The direction of motion alone can be changed without changing the magnitude (speed). If this happens continuously then we call it 'uniform circular motion'.
3. Both the direction and magnitude (speed) of velocity can be changed. If this happens non circular motion occurs. For example oscillation of a swing or simple pendulum, elliptical motion of planets around the Sun.

In this section we will deal with uniform circular motion and non-uniform circular motion.

3.7.1 Centripetal force

If a particle is in uniform circular motion, there must be centripetal acceleration towards the centre of the circle. If there is acceleration then there must be some force acting on it with respect to an inertial frame. This force is called centripetal force.

As we have seen in chapter 2, the centripetal acceleration of a particle in the circular motion is given by $a = \frac{v^2}{r}$ and it acts towards centre of the circle. According to Newton's second law, the centripetal force is given by

$$F_{cp} = m a_{cp} = \frac{mv^2}{r}$$

The word Centripetal force means centre seeking force.

In vector notation $\vec{F}_{cp} = -\frac{mv^2}{r}\hat{r}$

For uniform circular motion $\vec{F}_{cp} = -m\omega^2 r\,\hat{r}$

The direction $-\hat{r}$ points towards the centre of the circle which is the direction of centripetal force as shown in Figure 3.38.

Figure 3.38 Centripetal force

It should be noted that 'centripetal force' is not other forces like gravitational force or spring force. It can be said as 'force towards centre'. The origin of the centripetal force can be gravitational force, tension in the string, frictional force, Coulomb force etc. Any of these forces can act as a centripetal force.

1. In the case of whirling motion of a stone tied to a string, the centripetal force on the particle is provided by the tensional force on the string. In circular motion in an amusement park, the centripetal force is provided by the tension in the iron ropes.
2. In motion of satellites around the Earth, the centripetal force is given by Earth's gravitational force on the satellites. Newton's second law for satellite motion is

$$F = \text{earth's gravitational force} = \frac{mv^2}{r}$$

Where r - distance of the planet from the centre of the Earth.

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Figure 3.39 Whirling motion of objects

m-mass of the satellite

v-speed of the satellite

3. When a car is moving on a circular track the centripetal force is given by the frictional force between the road and the tyres.

Figure 3.40 Car in the circular track

Newton's second law for this case is

$$\text{Frictional force} = \frac{mv^2}{r}$$

m-mass of the car

v-speed of the car

r-radius of curvature of track

Even when the car moves on a curved track, the car experiences the centripetal force which is provided by frictional force between the surface and the tyre of the car. This is shown in the Figure 3.41.

Figure 3.41 Centripetal force due to frictional force between the road and tyre

4. When the planets orbit around the Sun, they experience centripetal force towards the centre of the Sun. Here gravitational force of the Sun acts as centripetal force on the planets as shown in Figure 3.42

Figure 3.42 Centripetal force on the orbiting planet due Sun's gravity

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Newton's second law for this motion

Gravitational force of Sun on the planet $= \frac{mv^2}{r}$

EXAMPLE 3.22

If a stone of mass 0.25 kg tied to a string executes uniform circular motion with a speed of 2 m s^{-1} of radius 3 m, what is the magnitude of tensional force acting on the stone?

Solution: $F_{cp} = \frac{mv^2}{r}$

$$F_{cp} = \frac{\frac{1}{4}\times(2)^2}{3} = 0.333 \text{ N.}$$

EXAMPLE 3.23

The Moon orbits the Earth once in 27.3 days in an almost circular orbit. Calculate the centripetal acceleration experienced by the Moon? (Radius of the Earth is 6.4×10^6 m)

Solution

The centripetal acceleration is given by $a = \frac{v^2}{r}$. This expression explicitly depends on Moon's speed which is non trivial. We can work with the formula

$$\omega^2 R_m = a_m$$

a_m is centripetal acceleration of the Moon due to Earth's gravity.

ω is angular velocity.

R_m is the distance between Earth and the Moon, which is 60 times the radius of the Earth.

$$R_m = 60R = 60\times 6.4\times 10^6 = 384\times 10^6\ m$$

As we know the angular velocity $\omega = \frac{2\pi}{T}$ and T = 27.3 days = 27.3 × 24 × 60 × 60 second = 2.358×10^6 sec

By substituting these values in the formula for acceleration

$$a_m = \frac{(4\pi^2)(384\times 10^6)}{(2.358\times 10^6)^2} = 0.00272 \text{ m s}^{-2}$$

The centripetal acceleration of Moon towards the Earth is 0.00272 m s^{-2}

Note This result was calculated by Newton himself. In unit 6 we will use this result.

3.7.2 Vehicle on a leveled circular road

When a vehicle travels in a curved path, there must be a centripetal force acting on it. This centripetal force is provided by the frictional force between tyre and surface of the road. Consider a vehicle of mass 'm' moving at a speed 'v' in the circular track of radius 'r'. There are three forces acting on the vehicle when it moves as shown in the Figure 3.43

1. Gravitational force (mg) acting downwards
2. Normal force (N) acting upwards
3. Frictional force (F_s) acting horizontally inwards along the road

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Figure 3.43 Forces acting on the vehicle on a leveled circular road

Suppose the road is horizontal then the normal force and gravitational force are exactly equal and opposite. The centripetal force is provided by the force of static friction F_s between the tyre and surface of the road which acts towards the centre of the circular track,

$$\frac{mv^2}{r} = F_s$$

As we have already seen in the previous section, the static friction can increase from zero to a maximum value

$$F_s \le \mu_s mg.$$

There are two conditions possible:

a) If $\frac{mv^2}{r} \le \mu_s mg$, or $\mu_s \ge \frac{v^2}{rg}$ or $\sqrt{\mu_s rg} \ge v$ (Safe turn)

The static friction would be able to provide necessary centripetal force to bend the car on the road. So the coefficient of static friction between the tyre and the surface of the road determines what maximum speed the car can have for safe turn.

b) If $\frac{mv^2}{r} > \mu_s mg$, or $\mu_s < \frac{v^2}{rg}$ (skid)

If the static friction is not able to provide enough centripetal force to turn, the vehicle will start to skid.

EXAMPLE 3.24

Consider a circular leveled road of radius 10 m having coefficient of static friction 0.81. Three cars (A, B and C) are travelling with speed 7 m s^{-1}, 8 m s^{-1} and 10 ms^{-1} respectively. Which car will skid when it moves in the circular level road? (g =10 m s^{-2})

Solution

From the safe turn condition the speed of the vehicle (v) must be less than or equal to $\sqrt{\mu_s rg}$

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$$v \le \sqrt{\mu_s rg}$$

$$\sqrt{\mu_s rg} = \sqrt{0.81 \times 10 \times 10} = 9\, m\, s^{-1}$$

For Car C, $\sqrt{\mu_s rg}$ is less than v

The speed of car A, B and C are 7 m s^{-1}, 8 m s^{-1} and 10 m s^{-1} respectively. The cars A and B will have safe turns. But the car C has speed 10 m s^{-1} while it turns which exceeds the safe turning speed. Hence, the car C will skid.

3.7.3 Banking of Tracks

In a leveled circular road, skidding mainly depends on the coefficient of static friction μ_s The coefficient of static friction depends on the nature of the surface which has a maximum limiting value. To avoid this problem, usually the outer edge of the road is slightly raised compared to inner edge as shown in the Figure 3.44. This is called banking of roads or tracks. This introduces an inclination, and the angle is called banking angle.

Figure 3.44 Outer edge of the road is slightly raised to avoid skidding

Let the surface of the road make angle θ with horizontal surface. Then the normal force makes the same angle θ with the vertical. When the car takes a turn, there are two forces acting on the car:

a) Gravitational force mg (downwards)

b) Normal force N (perpendicular to surface)

We can resolve the normal force into two components. $N\cos\theta$ and $N\sin\theta$ as shown in Figure 3.46. The component $N\cos\theta$ balances the downward gravitational force 'mg' and component $N\sin\theta$ will provide the necessary centripetal acceleration. By using Newton second law

$$N\cos\theta = mg$$

$$N\sin\theta = \frac{mv^2}{r}$$

By dividing the equations we get $\tan\theta = \dfrac{v^2}{rg}$

$$v = \sqrt{rg\tan\theta}$$

The banking angle θ and radius of curvature of the road or track determines the safe speed of the car at the turning. If the speed of car exceeds this safe speed, then it starts to skid outward but frictional force comes into effect and provides an additional centripetal force to prevent the outward skidding. At the same time, if the speed of the car is little lesser than safe speed, it starts to skid inward and frictional force comes into effect, which reduces centripetal force to prevent inward skidding. However if the speed of the vehicle is sufficiently greater than the correct speed, then frictional force cannot stop the car from skidding.

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EXAMPLE 3.25

Consider a circular road of radius 20 meter banked at an angle of 15 degree. With what speed a car has to move on the turn so that it will have safe turn?

Solution

$$v = \sqrt{(rg\tan\theta)} = \sqrt{20 \times 9.8 \times \tan 15^\circ}$$

$$= \sqrt{20 \times 9.8 \times 0.26} = 7.1 \text{ m s}^{-1}$$

The safe speed for the car on this road is 7.1 m s^{-1}

3.7.4 Centrifugal Force

Circular motion can be analysed from two different frames of reference. One is the inertial frame (which is either at rest or in uniform motion) where Newton's laws are obeyed. The other is the rotating frame of reference which is a non-inertial frame of reference as it is accelerating. When we examine the circular motion from these frames of reference the situations are entirely different. To use Newton's first and second laws in the rotational frame of reference, we need to include a pseudo force called 'centrifugal force'. This 'centrifugal force' appears to act on the object with respect to rotating frames. To understand the concept of centrifugal force, we can take a specific case and discuss as done below.

Consider the case of a whirling motion of a stone tied to a string. Assume that the stone has angular velocity ω in the inertial frame (at rest). If the motion of the stone is observed from a frame which is also rotating along with the stone with same angular velocity ω then, the stone appears to be at rest. This implies that in addition to the inward centripetal force $-m\omega^2 r$ there must be an equal and opposite force that acts on the stone outward with value $+m\omega^2 r$. So the total force acting on the stone in a rotating frame is equal to zero ($-m\omega^2 r + m\omega^2 r = 0$). This outward force $+m\omega^2 r$ is called the centrifugal force. The word 'centrifugal' means 'flee from centre'. Note that the 'centrifugal force' appears to act on the particle, only when we analyse the motion from a rotating frame. With respect to an inertial frame there is only centripetal force which is given by the tension in the string. For this reason centrifugal force is called as a 'pseudo force'. A pseudo force has no origin. It arises due to the non inertial nature of the frame considered. When circular motion problems are solved from a rotating frame of reference, while drawing free body diagram of a particle, the centrifugal force should necessarily be included as shown in the Figure 3.45.

3.7.5 Effects of Centrifugal Force

Although centrifugal force is a pseudo force, its effects are real. When a car takes a turn in a curved road, person inside the car feels an outward force which pushes the person away. This outward force is also called centrifugal force. If there is sufficient friction between the person and the seat, it will prevent the person from moving outwards. When a car moving in a straight line suddenly takes a turn, the objects not fixed to the car try to continue in linear motion due to their inertia of direction. While observing this motion from an inertial frame, it appears as a straight line as shown in Figure 3.46. But, when it is observed from the rotating frame it appears to move outwards.

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Figure 3.45 Free body diagram of a particle including the centrifugal force

Figure 3.46 Effects of centrifugal force

A person standing on a rotating platform feels an outward centrifugal force and is likely to be pushed away from the platform. Many a time the frictional force between the platform and the person is not sufficient to overcome outward push. To avoid this, usually the outer edge of the platform is little inclined upwards which exerts a normal force on the person which prevents the person from falling as illustrated in Figures 3.47.

Caution!

It is dangerous to stand near the open door (or) steps while travelling in the bus. When the bus takes a sudden turn in a curved road, due to centrifugal force the person is pushed away from the bus. Even though centrifugal force is a pseudo force, its effects are real.

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Figure 3.47 Outward centrifugal force in rotating platform

3.7.6 Centrifugal Force due to Rotation of the Earth

Even though Earth is treated as an inertial frame, it is actually not so. Earth spins about its own axis with an angular velocity ω. Any object on the surface of Earth (rotational frame) experiences a centrifugal force. The centrifugal force appears to act exactly in opposite direction from the axis of rotation. It is shown in the Figure 3.48.

The centrifugal force on a man standing on the surface of the Earth is $F_{cf} = m\omega^2 r$

where r is perpendicular distance of the man from the axis of rotation. By using right angle triangle as shown in the Figure 3.48, the distance $r = R\cos\theta$

Here R = radius of the Earth

and θ = latitude of the Earth where the man is standing.

Figure 3. 48 Centrifugal force acting on a man on the surface of Earth

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EXAMPLE 3.26

Calculate the centrifugal force experienced by a man of 60 kg standing at Chennai? (Given: Latitude of Chennai is $13°$

Solution

The centrifugal force is given by $F_c = m\omega^2 R\cos\theta$

The angular velocity (ω) of Earth $= \frac{2\pi}{T}$, where T is time period of the Earth (24 hours)

$$\omega = \frac{2\pi}{24\times60\times60} = \frac{2\pi}{86400}$$
$$= 7.268\times10^{-5}\,radsec^{-1}$$

The radius of the Earth R = 6400 Km = 6400×10^3 m

Latitude of Chennai $= 13°$

$$F_{cf} = 60\times\left(7.268\times10^{-5}\right)^2\times6400\times10^3$$
$$\times\cos\left(13°\right) = 1.9678\,N$$

A 60 kg man experiences centrifugal force of approximately 2 Newton. But due to Earth's gravity a man of 60 kg experiences a force = mg = 60 × 9.8 = 588N. This force is very much larger than the centrifugal force.

3.7.7 Centripetal Force Versus Centrifugal Force

Salient features of centripetal and centrifugal forces are compared in Table 3.4.

Table 3.4 Salient Features of Centripetal and Centrifugal Forces

Centripetal force	Centrifugal force
It is a real force which is exerted on the body by the external agencies like gravitational force, tension in the string, normal force etc.	It is a pseudo force or fictitious force which cannot arise from gravitational force, tension force, normal force etc.
Acts in both inertial and non-inertial frames	Acts only in rotating frames (non-inertial frame)
It acts towards the axis of rotation or centre of the circle in circular motion	It acts outwards from the axis of rotation or radially outwards from the centre of the circular motion
$\lvert F_{cp}\rvert = m\omega^2 r = \frac{mv^2}{r}$	$\lvert F_{cf}\rvert = m\omega^2 r = \frac{mv^2}{r}$
Real force and has real effects	Pseudo force but has real effects
Origin of centripetal force is interaction between two objects.	Origin of centrifugal force is inertia. It does not arise from interaction. In an inertial frame the object's inertial motion appears as centrifugal force in the rotating frame.
In inertial frames centripetal force has to be included when free body diagrams are drawn.	In inertial frames there is no centrifugal force. In rotating frames, both centripetal and centrifugal force have to be included when free body diagrams are drawn.

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SUMMARY

- Aristotle's idea of motion: To maintain motion, a force is required
- Galileo's idea of motion: To maintain motion, a force is not required
- Mass is a measure of inertia of the body
- Newton's first law states that under no external force, the object continues its state of motion or state of rest.
- Newton's second law states that to change the momentum of the body, external force is required

  Mathematically it is defined as $\vec{F} = \frac{d\vec{p}}{dt}$
- Both Newton's first and second laws are valid only in inertial frames
- Inertial frame is the one in which if there is no force on the object, the object moves at constant velocity.
- Newton's third law states that for every force there is an equivalent and opposite force and such a pair of forces is called action and reaction pair.
- To draw a free body diagram for an object,
  - Isolate the object from other objects and identify the forces acting on it
  - The force exerted by that object should not be taken into account
  - Draw the direction of each force with relative magnitude
  - Apply Newton's second law in each direction
- If no net external force acts on a collection of particles (system), then the total momentum of the collection of particles (system) is a constant vector.
- Internal forces acting in the system cannot change the total momentum of the system.
- Lami's theorem states that if an object is in equilibrium under the concurrent forces, then the ratio of each force with the sine of corresponding opposite angle is same.
- An impulse acting on a body is equal to the change in momentum of the body. Whenever a force acts on the object for a very short time, it is difficult to calculate the force. But impulse can be calculated.
- Static friction is the force which always opposes the movement of the object from rest. It can take values from zero to $\mu_s N$. If an external force is greater than $\mu_s N$ then object begins to move.
- If the object begins to move, kinetic friction comes into effect. To move an object with constant velocity, the external force must be applied to overcome the kinetic friction. The kinetic friction is $\mu_k N$.
- Rolling friction is much smaller than static and kinetic friction. This is the reason that to move an object roller coaster is fixed in the bottom of the object. Example: Rolling suitcase

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SUMMARY (*cont*)

- The origin of friction is electromagnetic interaction between the atoms of two surfaces which are touching each other.
- Whenever there is a motion along a curve, there must be a centripetal force that acts towards the centre of the curve. In uniform circular motion the centripetal force acts at the centre of the circle.
- The centripetal force is not a separate natural force. Any natural force can behave as centripetal force. In planetary motion, Sun's gravitational force acts as centripetal force. In the whirling motion of a stone attached to a string, the centripetal force is given by the string. When Moon orbits the Earth, it experiences Earth's gravitational force as centripetal force.
- Centrifugal force arises whenever the motion is analysed from rotating frame. It is a pseudo force. The inertial motion of the object appears as centrifugal force in the rotating frame.
- The magnitude of centrifugal and centripetal force is $m\omega^2 r$. But centripetal force acts towards centre of the circular motion and centrifugal force appears to acts in the opposite direction to centripetal force.

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CONCEPT MAP

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EVALUATION

I. Multiple Choice Questions

1. When a car takes a sudden left turn in the curved road, passengers are pushed towards the right due to

(a) inertia of direction

(b) inertia of motion

(c) inertia of rest

(d) absence of inertia

2. An object of mass m held against a vertical wall by applying horizontal force F as shown in the figure. The minimum value of the force F is

(IIT JEE 1994)

(a) Less than mg

(b) Equal to mg

(c) Greater than mg

(d) Cannot determine

3. A vehicle is moving along the positive x direction, if sudden brake is applied, then

(a) frictional force acting on the vehicle is along negative x direction

(b) frictional force acting on the vehicle is along positive x direction

(c) no frictional force acts on the vehicle

(d) frictional force acts in downward direction

4. A book is at rest on the table which exerts a normal force on the book. If this force is considered as reaction force, what is the action force according to Newton's third law?

(a) Gravitational force exerted by Earth on the book

(b) Gravitational force exerted by the book on Earth

(c) Normal force exerted by the book on the table

(d) None of the above

5. Two masses m_1 and m_2 are experiencing the same force where $m_1 < m_2$. The ratio of their acceleration $\frac{a_1}{a_2}$ is

(a) 1

(b) less than 1

(c) greater than 1

(d) all the three cases

6. Choose appropriate free body diagram for the particle experiencing net acceleration along negative y direction. (Each arrow mark represents the force acting on the system).

a)

b)

c)

d)

7. A particle of mass m sliding on the smooth double inclined plane (shown in figure) will experience

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(a) greater acceleration along the path AB

(b) greater acceleration along the path AC

(c) same acceleration in both the paths

(d) no acceleration in both the paths.

8. Two blocks of masses m and 2m are placed on a smooth horizontal surface as shown. In the first case only a force F_1 is applied from the left. Later only a force F_2 is applied from the right. If the force acting at the interface of the two blocks in the two cases is same, then $F_1 : F_2$ is

(Physics Olympiad 2016)

(a) 1:1 (b) 1:2

(c) 2:1 (d) 1:3

9. Force acting on the particle moving with constant speed is

(a) always zero

(b) need not be zero

(c) always non zero

(d) cannot be concluded

10. An object of mass m begins to move on the plane inclined at an angle θ. The coefficient of static friction of inclined surface is μ_s. The maximum static friction experienced by the mass is

(a) mg

(b) $\mu_s$mg

(c) $\mu_s$mg sinθ

(d) μ_s mg cos θ

11. When the object is moving at constant velocity on the rough surface,

(a) net force on the object is zero

(b) no force acts on the object

(c) only external force acts on the object

(d) only kinetic friction acts on the object

12. When an object is at rest on the inclined rough surface,

(a) static and kinetic frictions acting on the object is zero

(b) static friction is zero but kinetic friction is not zero

(c) static friction is not zero and kinetic friction is zero

(d) static and kinetic frictions are not zero

13. The centrifugal force appears to exist

(a) only in inertial frames

(b) only in rotating frames

(c) in any accelerated frame

(d) both in inertial and non-inertial frames

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14. Choose the correct statement from the following
    (a) Centrifugal and centripetal forces are action reaction pairs
    (b) Centripetal forces is a natural force
    (c) Centrifugal force arises from gravitational force
    (d) Centripetal force acts towards the centre and centrifugal force appears to act away from the centre in a circular motion

15. If a person moving from pole to equator, the centrifugal force acting on him
    (a) increases
    (b) decreases
    (c) remains the same
    (d) increases and then decreases

Answers

1) a	**2)** c	**3)** a	**4)** c	**5)** c
6) c	**7)** b	**8)** c	**9)** b	**10)** d
11) a	**12)** c	**13)** b	**14)** d	**15)** a

II. Short Answer Questions

1. Explain the concept of inertia. Write two examples each for inertia of motion, inertia of rest and inertia of direction.
2. State Newton's second law.
3. Define one newton.
4. Show that impulse is the change of momentum.
5. Using free body diagram, show that it is easy to pull an object than to push it.
6. Explain various types of friction. Suggest a few methods to reduce friction.
7. What is the meaning by 'pseudo force'?
8. State the empirical laws of static and kinetic friction.
9. State Newton's third law.
10. What are inertial frames?
11. Under what condition will a car skid on a leveled circular road?

III. Long Answer Questions

1. Prove the law of conservation of linear momentum. Use it to find the recoil velocity of a gun when a bullet is fired from it.
2. What are concurrent forces? State Lami's theorem.
3. Explain the motion of blocks connected by a string in i) Vertical motion ii) Horizontal motion.
4. Briefly explain the origin of friction. Show that in an inclined plane, angle of friction is equal to angle of repose.
5. State Newton's three laws and discuss their significance.
6. Explain the similarities and differences of centripetal and centrifugal forces.
7. Briefly explain 'centrifugal force' with suitable examples.
8. Briefly explain 'rolling friction'.
9. Describe the method of measuring angle of repose.
10. Explain the need for banking of tracks.
11. Calculate the centripetal acceleration of Moon towards the Earth.

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IV. Numerical Problems

1. A force of 50N act on the object of mass 20 kg. shown in the figure. Calculate the acceleration of the object in x and y directions.

Ans: a_x=2.165 ms^{-2}; a_y=1.25 ms^{-2}

2. A spider of mass 50 g is hanging on a string of a cob web as shown in the figure. What is the tension in the string?

Ans: T=0.49N

3. What is the reading shown in spring balance?

Ans: Zero, 9.8 N

4. The physics books are stacked on each other in the sequence: +1 volumes 1 and 2; +2 volumes 1 and 2 on a table.
   a) Identify the forces acting on each book and draw the free body diagram.
   b) Identify the forces exerted by each book on the other.

5. A bob attached to the string oscillates back and forth. Resolve the forces acting on the bob in to components. What is the acceleration experience by the bob at an angle θ.

   Ans: Tangential acceleration = g sinθ ; centripetal acceleration = $\frac{(T-mg\cos\theta)}{m}$.

6. Two masses m_1 and m_2 are connected with a string passing over a frictionless pulley fixed at the corner of the table as shown in the figure. The coefficient of static friction of mass m_1 with the table is μ_s. Calculate the minimum mass m_3 that may be placed on m1to prevent it from sliding. Check if m_1=15 kg, m_2=10 kg, m_3=25 kg and μ_s =0.2

Ans: $m_3 = \frac{m_2}{\mu_s} - m_1$, the combined masses m_1+m_3 will slide.

7. Calculate the acceleration of the bicycle of mass 25 kg as shown in Figures 1 and 2.

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Ans: a=4 ms^{-2}, zero

8. Apply Lami's theorem on sling shot and calculate the tension in each string ?

Ans: T= 28.868N.

9. A football player kicks a 0.8 kg ball and imparts it a velocity 12 ms^{-1}. The contact between the foot and ball is only for one- sixtieth of a second. Find the average kicking force.

Ans: 576N.

10. A stone of mass 2 kg is attached to a string of length 1 meter. The string can withstand maximum tension 200 N. What is the maximum speed that stone can have during the whirling motion?

Ans: v_{max}=10ms^{-1}

11. Imagine that the gravitational force between Earth and Moon is provided by an invisible string that exists between the Moon and Earth. What is the tension that exists in this invisible string due to Earth's centripetal force? (Mass of the Moon = 7.34×10^{22} kg, Distance between Moon and Earth = 3.84×10^{8} m)

Ans: $T \approx 2 \times 10^{20} N$.

12. Two bodies of masses 15 kg and 10 kg are connected with light string kept on a smooth surface. A horizontal force F=500 N is applied to a 15 kg as shown in the figure. Calculate the tension acting in the string

Ans: $T = 200N$.

13. People often say "For every action there is an equivalent opposite reaction". Here they meant 'action of a human'. Is it correct to apply Newton's third law to human actions? What is mean by 'action' in Newton third law? Give your arguments based on Newton's laws.

Ans: Newton's third law is applicable to only human's actions which involves physical force. Third law is not applicable to human's psychological actions or thoughts

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14. A car takes a turn with velocity 50 ms^{-1} on the circular road of radius of curvature 10 m. calculate the centrifugal force experienced by a person of mass 60kg inside the car?

Ans: 15,000 N

15. A long stick rests on the surface. A person standing 10 m away from the stick. With what minimum speed an object of mass 0.5 kg should he thrown so that it hits the stick. (Assume the coefficient of kinetic friction is 0.7).

Ans: 11.71 ms^{-1}

BOOKS FOR REFERENCE

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3. SomnathDatta, *Mechanics,* Pearson Publication
4. H.C.Verma, *Concepts of physics* volume 1 and Volume 2, Bharati Bhawan Publishers
5. Serway and Jewett, *Physics for scientist and Engineers with modern physics*, Brook/Coole publishers, Eighth edition
6. Halliday, Resnick & Walker, *Fundamentals of Physics*, Wiley Publishers, 10th edition

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ICT CORNER

Force and motion

Through this activity you will understand the Force and motion

STEPS:

- Open the browser and type the given URL to open the PhET simulation on force and motion. Click OK to open the java applet.
- Select the values of the applied force to observe the change.
- Observe the change of the ramp angle by changing the position of the object.
- You can also observe the variations in force and ramp angle by changing the weights.

Step1

Step2

Step3

Step4

PhET simulation's URL:

https://phet.colorado.edu/en/simulation/ramp-forces-and-motion

* Pictures are indicative only.

* If browser requires, allow **Flash Player** or **Java Script** to load the page.

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UNIT 4

WORK, ENERGY AND POWER

"Matter is Energy. Energy is Light. We are all Light Beings." – Albert Einstein

Learning Objectives

In this unit, student is exposed to

- definition of work
- work done by a constant and a variable force
- various types of energy
- law of conservation of energy
- vertical circular motion
- definition of power
- various types of collisions

8GPTA

4.1 INTRODUCTION

The term *work* is used in diverse contexts in daily life. It refers to both physical as well as mental work. In fact, any activity can generally be called as work. But in Physics, the term work is treated as a physical quantity with a precise definition. Work is said to be done by the force when the force applied on a body displaces it. To do work, *energy* is required. In simple words, *energy is defined as the ability to do work*. Hence, work and energy are equivalents and have same dimension. Energy, in Physics exists in different forms such as mechanical, electrical, thermal, nuclear and so on. Many machines consume one form of energy and deliver energy in a different form. In this chapter we deal mainly with mechanical energy and its two types namely kinetic energy and potential energy. The next quantity in this sequence of discussion is the rate of work done or the rate of energy delivered. *The rate of work done is called power*. A powerful strike in cricket refers to a hit on the ball at a fast rate. This chapter aims at developing a good understanding of these three physical quantities namely work, energy and power and their physical significance.

4.1.1 WORK

Let us consider a force ($\vec{F}$), acting on a body which moves it by a displacement in some direction ($d\vec{r}$) as shown in Figure 4.1

The expression for work done (W) by the force on the body is mathematically written as,

$$W = \vec{F} \cdot d\vec{r} \qquad (4.1)$$

Figure 4.1 Work done by a force

Here, the product $\vec{F} \cdot d\vec{r}$ is a *scalar product (or dot product)*. The scalar product of two vectors is a scalar (refer section 2.5.1). Thus, work done is a scalar quantity. It has only magnitude and no direction. In SI system, unit of work done is N m (or) joule (J). Its dimensional formula is $[ML^2T^{-2}]$.

The equation (4.1) is,

$$W = F\,dr\cos\theta \qquad (4.2)$$

which can be realised using Figure 4.2 (as $\vec{a} \cdot \vec{b} = ab\cos\theta$) where, θ is the angle between applied force and the displacement of the body.

The work done by the force depends on the force (F), displacement (dr) and the angle (θ) between them.

Figure 4.2 Calculating work done.

Work done is zero in the following cases.

(i) **When the force is zero (F = 0).** For example, a body moving on a horizontal smooth frictionless surface will continue to do so as no force (not even friction) is acting along the plane. (This is an ideal situation.)

(ii) **When the displacement is zero (dr = 0).** For example, when force is applied on a rigid wall it does not produce any displacement. Hence, the work done is zero as shown in Figure. 4.3(a).

Figure 4.3 Different cases of zero work done

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(iii) **When the force and displacement are perpendicular ($\theta = 90^o$) to each other.** when a body moves on a horizontal direction, the gravitational force (mg) does no work on the body, since it acts at right angles to the displacement as shown in Figure 4.3(b). In circular motion the centripetal force does not do work on the object moving on a circle as it is always perpendicular to the displacement as shown in Figure 4.3(c).

For a given force (F) and displacement (dr), the angle (θ) between them decides the value of work done as consolidated in Table 4.1.

There are many examples for the negative work done by a force. In a football game, the goalkeeper catches the ball coming towards him by applying a force such that the force is applied in a direction opposite to that of the motion of the ball till it comes to rest in his hands. During the time of applying the force, he does a negative work on the ball as shown in Figure 4.4. We will discuss many more situations of negative work further in this unit.

Figure 4.4 Negative work done

EXAMPLE 4.1

A box is pulled with a force of 25 N to produce a displacement of 15 m. If the angle between the force and displacement is 30°, find the work done by the force.

Solution

Force, F = 25 N
Displacement, dr = 15 m
Angle between F and dr, $\theta = 30^o$

Table 4.1 Angle (θ) and the nature of work

Angle (θ)	cosθ	Work
$\theta = 0^o$	1	Positive, Maximum
$0 < \theta < 90^o$ (acute)	$0 < \cos\theta < 1$	Positive
$\theta = 90^o$ (right angle)	0	Zero
$90^o < \theta < 180^o$	$-1 < \cos\theta < 0$	Negative
$\theta = 180^o$	-1	Negative, Maximum

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Work done, $W = F\,dr\cos\theta$

$$W = 25\times15\times\cos30^\circ = 25\times15\times\frac{\sqrt{3}}{2}$$

$$W = 324.76\ \text{J}$$

4.1.2 Work done by a constant force

When a constant force F acts on a body, the small work done (dW) by the force in producing a small displacement dr is given by the relation,

$$dW = (F\cos\theta)\ dr \qquad (4.3)$$

The total work done in producing a displacement from initial position $\mathbf{r}_i$ to final position $\mathbf{r}_f$ is,

$$W = \int_{r_i}^{r_f} dW \qquad (4.4)$$

$$W = \int_{r_i}^{r_f} (F\cos\theta)\ dr = (F\cos\theta)\int_{r_i}^{r_f} dr = (F\cos\theta)(r_f - r_i) \qquad (4.5)$$

The graphical representation of the work done by a constant force is shown in Figure 4.5. The area under the graph shows the work done by the constant force.

Figure 4.5 Work done by the constant force

EXAMPLE 4.2

An object of mass 2 kg falls from a height of 5 m to the ground. What is the work done by the gravitational force on the object? (Neglect air resistance; Take $g = 10\ \text{m s}^{-2}$)

Solution

In this case the force acting on the object is downward gravitational force $m\vec{g}$. This is a constant force.

Work done by gravitational force is

$$W = \int_{r_i}^{r_f} \vec{F}\cdot d\vec{r}$$

$$W = (F\cos\theta)\int_{r_i}^{r_f} dr = (mg\cos\theta)\,(r_f - r_i).$$

The object also moves downward which is in the direction of gravitational force ($\vec{F} = m\vec{g}$) as shown in figure. Hence, the angle between them is $\theta = 0^\circ$; $\cos 0^\circ = 1$ and the displacement, $(r_f - r_i) = 5\,\text{m}$

$$W = mg(r_f - r_i)$$

$$W = 2\times10\times5 = 100\ \text{J}$$

The work done by the gravitational force on the object is positive.

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EXAMPLE 4.3

An object of mass m=1 kg is sliding from top to bottom in the frictionless inclined plane of inclination angle $\theta = 30°$ and the length of inclined plane is 10 m as shown in the figure. Calculate the work done by gravitational force and normal force on the object. Assume acceleration due to gravity, g = 10 m s^{-2}

Solution

We calculated in the previous chapter that the acceleration experienced by the object in the inclined plane as $g\sin\theta$.

According to Newton's second law, the force acting on the mass along the inclined plane $F = mg\sin\theta$. Note that this force is constant throughout the motion of the mass.

The work done by the parallel component of gravitational force $(mg\ \sin\theta)$ is given by

$$W = \vec{F}.d\vec{r} = Fdr\cos\phi$$

where ϕ is the angle between the force (mg $\sin\theta$) and the direction of motion (dr). In this case, force (mg $\sin\theta$) and the displacement ($d\vec{r}$) are in the same direction. Hence $\phi = 0$ and $\cos\phi = 1$

$$W = F\ dr = (mg\ \sin\theta)\ (dr)$$

(dr = length of the inclined place)

$$W = 1 \times 10 \times \sin(30°) \times 10 = 100 \times \frac{1}{2} = 50\,J$$

The component mg $\cos\theta$ and the normal force N are perpendicular to the direction of motion of the object, so they do not perform any work.

EXAMPLE 4.4

If an object of mass 2 kg is thrown up from the ground reaches a height of 5 m and falls back to the Earth (neglect the air resistance). Calculate

(a) The work done by gravity when the object reaches 5 m height
(b) The work done by gravity when the object comes back to Earth
(c) Total work done by gravity both in upward and downward motion and mention the physical significance of the result.

Solution

When the object goes up, the displacement points in the upward direction whereas the gravitational force acting on the object points in downward direction. Therefore, the angle between gravitational force and displacement of the object is 180°.

(a) The work done by gravitational force in the upward motion.

Given that $dr = 5m$ and $F = mg$

$$W_{up} = Fdr\ \cos\theta = mgdr\ \cos 180°$$

$$W_{up} = 2 \times 10 \times 5 \times (-1) = -100 \text{ joule.}$$

$[\cos 180° = -1]$

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(b) When the object falls back, both the gravitational force and displacement of the object are in the same direction. This implies that the angle between gravitational force and displacement of the object is 0°.

$$W_{down} = Fdr\cos 0°$$
$$W_{down} = 2\times 10\times 5\times(1) = 100 \text{ joule}$$
[cos 0° = 1]

(c) The total work done by gravity in the entire trip (upward and downward motion)

$$W_{total} = W_{up} + W_{down}$$
$$= -100 \text{ joule} + 100 \text{ joule} = 0$$

It implies that the gravity does not transfer any energy to the object. When the object is thrown upwards, the energy is transferred to the object by the external agency, which means that the object gains some energy. As soon as it comes back and hits the Earth, the energy gained by the object is transferred to the surface of the Earth (i.e., dissipated to the Earth).

EXAMPLE 4.5

A weight lifter lifts a mass of 250 kg with a force 5000 N to the height of 5 m.

(a) What is the workdone by the weight lifter?
(b) What is the workdone by the gravity?
(c) What is the net workdone on the object?

Solution

(a) When the weight lifter lifts the mass, force and displacement are in the same direction, which means that the angle between them $\theta = 0^0$. Therefore, the work done by the weight lifter,

$$W_{\text{weight lifter}} = F_w h\ \cos\theta = F_w h\left(\cos 0^0\right)$$
$$= 5000\times 5\times(1) = 25{,}000 \text{ joule} = 25 \text{ kJ}$$

(b) When the weight lifter lifts the mass, the gravity acts downwards which means that the force and displacement are in opposite direction. Therefore, the angle between them $\theta = 180^0$

$$W_{gravity} = F_g h\ \cos\theta = mgh\left(\cos 180^0\right)$$
$$= 250\times 10\times 5\times(-1)$$
$$= -12{,}500 \text{ joule} = -12.5 \text{ kJ}$$

(c) The net workdone (or total work done) on the object

$$W_{net} = W_{\text{weight lifter}} + W_{gravity}$$
$$= 25 \text{ kJ} - 12.5 \text{ kJ} = +12.5 \text{ kJ}$$

4.1.3 Work done by a variable force

When the component of a variable force F acts on a body, the small work done (dW) by the force in producing a small displacement dr is given by the relation

$$dW = (F\cos\theta)dr$$
[F cos θ is the component of the variable force F]

where, F and θ are variables. The total work done for a displacement from initial position r_i to final position r_f is given by the relation,

$$W = \int_{r_i}^{r_f} dW = \int_{r_i}^{r_f} F\cos\theta\ dr \qquad (4.6)$$

A graphical representation of the work done by a variable force is shown in Figure 4.6. The area under the graph is the work done by the variable force.

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Figure 4.6 Work done by a variable force

EXAMPLE 4.6

A variable force $F = k\,x^2$ acts on a particle which is initially at rest. Calculate the work done by the force during the displacement of the particle from $x = 0$ m to $x = 4$ m. (Assume the constant $k = 1$ N m^{-2})

Solution

Work done,

$$W = \int_{x_i}^{x_f} F(x)\,dx = k\int_0^4 x^2 dx = \frac{64}{3}\ \text{N m}$$

4.2 ENERGY

Energy is defined as the capacity to do work. In other words, work done is the manifestation of energy. That is why work and energy have the same dimension (ML^2T^{-2})

Work ⇔ Energy

The important aspect of energy is that for an isolated system, the sum of all forms of energy i.e., the total energy remains the same in any process irrespective of whatever internal changes may take place. This means that the energy disappearing in one form reappears in another form. This is known as the law of conservation of energy. In this chapter we shall take up only the mechanical energy for discussion.

In a broader sense, mechanical energy is classified into two types

1. Kinetic energy
2. Potential energy

The energy possessed by a body due to its motion is called kinetic energy. The energy possessed by the body by virtue of its position is called potential energy.

The SI unit of energy is the same as that of work done i.e., N m (or) joule (J). The dimension of energy is also the same as that of work done. It is given by [ML^2T^{-2}]. The other units of energy and their SI equivalent values are given in Table 4.2.

Table 4.2 SI equivalent of other units of energy

Unit	Equivalent in joule
1 erg (CGS unit)	10^{-7} J
1 electron volt (eV)	1.6×10^{-19} J
1 calorie (cal)	4.186 J
1 kilowatt hour (kWh)	3.6×10^{6} J

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4.2.1 Kinetic energy

Kinetic energy is the energy possessed by a body by virtue of its motion. All moving objects have kinetic energy. A body that is in motion has the ability to do work. For example a hammer kept at rest on a nail does not push the nail into the wood. Whereas the same hammer when it strikes the nail, draws the nail into the wood as shown in Figure 4.7. Kinetic energy is measured by the amount of work that the body can perform before it comes to rest. The amount of work done by a moving body depends both on the mass of the body and the magnitude of its velocity. A body which is not in motion does not have kinetic energy.

4.2.2 Work–Kinetic Energy Theorem

Work and energy are equivalents. This is true in the case of kinetic energy also. To prove this, let us consider a body of mass m at rest on a frictionless horizontal surface.

The work (W) done by the constant force (F) for a displacement (s) in the same direction is,

$$W = Fs \qquad (4.7)$$

The constant force is given by the equation,

$$F = ma \qquad (4.8)$$

The third equation of motion (refer section 2.10.3) can be written as,

$$v^2 = u^2 + 2as$$

$$a = \frac{v^2 - u^2}{2s}$$

Substituting for a in equation (4.8),

$$F = m\left(\frac{v^2 - u^2}{2s}\right) \qquad (4.9)$$

Substituting equation (4.9) in (4.7),

$$W = m\left(\frac{v^2}{2s}s\right) - m\left(\frac{u^2}{2s}s\right)$$

$$W = \frac{1}{2}mv^2 - \frac{1}{2}mu^2 \qquad (4.10)$$

The expression for kinetic energy:

The term $\left(\frac{1}{2}mv^2\right)$ in the above equation is the kinetic energy of the body of mass (m) moving with velocity (v).

$$KE = \frac{1}{2}mv^2 \qquad (4.11)$$

Kinetic energy of the body is always positive. From equations (4.10) and (4.11)

$$\Delta KE = \frac{1}{2}mv^2 - \frac{1}{2}mu^2 \qquad (4.12)$$

Thus, $W = \Delta KE$

Figure 4.7 Demonstration of kinetic energy

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The expression on the right hand side (RHS) of equation (4.12) is the change in kinetic energy (ΔKE) of the body.

This implies that *the work done by the force on the body changes the kinetic energy of the body. This is called work-kinetic energy theorem.*

The work-kinetic energy theorem implies the following.

1. If the work done by the force on the body is positive then its kinetic energy increases.
2. If the work done by the force on the body is negative then its kinetic energy decreases.
3. If there is no work done by the force on the body then there is no change in its kinetic energy, which means that the body has moved at constant speed provided its mass remains constant.

4.2.3 Relation between Momentum and Kinetic Energy

Consider an object of mass m moving with a velocity $\vec{v}$. Then its linear momentum is $\vec{p} = m\vec{v}$ and its kinetic energy, $KE = \frac{1}{2}mv^2$.

$$KE = \frac{1}{2}mv^2 = \frac{1}{2}m(\vec{v}.\vec{v}) \quad (4.13)$$

Multiplying both the numerator and denominator of equation (4.13) by mass, m

$$KE = \frac{1}{2}\frac{m^2(\vec{v}.\vec{v})}{m}$$

$$= \frac{1}{2}\frac{(m\vec{v}).(m\vec{v})}{m} \quad [\vec{p} = m\vec{v}]$$

$$= \frac{1}{2}\frac{\vec{p}.\vec{p}}{m}$$

$$= \frac{p^2}{2m}$$

$$KE = \frac{p^2}{2m} \quad (4.14)$$

where $|\vec{p}|$ is the magnitude of the momentum. The magnitude of the linear momentum can be obtained by

$$|\vec{p}| = p = \sqrt{2m\,(KE)} \quad (4.15)$$

Note that if kinetic energy and mass are given, only the magnitude of the momentum can be calculated but not the direction of momentum. It is because the kinetic energy and mass are scalars.

EXAMPLE 4.7

Two objects of masses 2 kg and 4 kg are moving with the same momentum of 20 kg m s^{-1}.

(a) Will they have same kinetic energy?
(b) Will they have same speed?

Solution

(a) The kinetic energy of the mass is given by $KE = \frac{p^2}{2m}$

For the object of mass 2 kg, kinetic energy is $KE_1 = \frac{(20)^2}{2\times 2} = \frac{400}{4} = 100\,J$

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For the object of mass 4 kg, kinetic energy is $KE_2 = \frac{(20)^2}{2\times 4} = \frac{400}{8} = 50\,J$

Note that $KE_1 \neq KE_2$ i.e., even though both are having the same momentum, the kinetic energy of both masses is not the same. The kinetic energy of the heavier object has lesser kinetic energy than smaller mass. It is because the kinetic energy is inversely proportional to the mass ($KE \propto \frac{1}{m}$) for a given momentum.

(b) As the momentum, $p = mv$, the two objects will not have same speed.

4.2.4 Potential Energy

The potential energy of a body is associated with its position and configuration with respect to its surroundings. This is because the various forces acting on the body also depends on position and configuration.

"Potential energy of an object at a point P is defined as the amount of work done by an external force in moving the object at constant velocity from the point O (initial location) to the point P (final location). At initial point O potential energy can be taken as zero.

Mathematically, *potential energy* is defined as $U = \int \vec{F}_a . d\vec{r}$ (4.16)

where the limit of integration ranges from initial location point O to final location point P.

We have various types of potential energies. Each type is associated with a particular force. For example,

(i) The energy possessed by the body due to gravitational force gives rise to gravitational potential energy.

(ii) The energy due to spring force and other similar forces give rise to elastic potential energy.

(iii) The energy due to electrostatic force on charges gives rise to electrostatic potential energy.

We will learn more about conservative forces in the section 4.2.7. Now, we continue to discuss more about gravitational potential energy and elastic potential energy.

4.2.5 Potential energy near the surface of the Earth

The gravitational potential energy (U) at some height h is equal to the amount of work required to take the object from ground to that height h with constant velocity.

Let us consider a body of mass m being moved from ground to the height h against the gravitational force as shown in Figure 4.8.

Figure 4.8 Gravitational potential energy

The gravitational force $\vec{F}_g$ acting on the body is, $\vec{F}_g = -mg\,\hat{j}$ (as the force is in y direction, unit vector $\hat{j}$ is used). Here, negative sign implies that the force is acting

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vertically downwards. In order to move the body without acceleration (or with constant velocity), an external applied force $\vec{F}_a$ equal in magnitude but opposite to that of gravitational force $\vec{F}_g$ has to be applied on the body i.e., $\vec{F}_a = -\vec{F}_g$. This implies that $\vec{F}_a = +mg\,\hat{j}$. The positive sign implies that the applied force is in vertically upward direction. Hence, when the body is lifted up its velocity remains unchanged and thus its kinetic energy also remains constant.

The gravitational potential energy (U) at some height h is equal to the amount of work required to take the object from the ground to that height h.

$$U = \int \vec{F}_a . d\vec{r} = \int_0^h \left|\vec{F}_a\right| \left|d\vec{r}\right| \cos\theta \quad (4.17)$$

Since the displacement and the applied force are in the same upward direction, the angle between them, $\theta = 0°$. Hence, $\cos 0^0 = 1$ and $\left|\vec{F}_a\right| = mg$ and $\left|d\vec{r}\right| = dr$.

$$U = mg\int_0^h dr \quad (4.18)$$

$$U = mg\left[r\right]_0^h = mgh \quad (4.19)$$

Note that the potential energy stored in the object is defined through work done by the external force which is positive. Physically this implies that the agency which is applying the external force is transferring the energy to the object which is then stored as potential energy. If the object is allowed to fall from a height h then the stored potential energy is converted into kinetic energy.

Note

- **How can an object move with zero acceleration (constant velocity) when the external force is acting on the object?**

  It is possible when there is another force which acts exactly opposite to the external applied force. They both cancel each other and the resulting net force becomes zero, hence the object moves with zero acceleration.

- Why should the object be moved at constant velocity when we define potential energy?

  If the object does not move at constant velocity, then it will have different velocities at the initial and final locations. According to work-kinetic energy theorem, the external force will impart some extra kinetic energy. But we associate potential energy to the forces like gravitational force, spring force and coulomb force. So the external agency should not impart any kinetic energy when the object is taken from initial to final location.

EXAMPLE 4.8

An object of mass 2 kg is taken to a height 5 m from the ground $\left(g = 10\,m\,s^{-2}\right)$.

(a) Calculate the potential energy stored in the object.

(b) Where does this potential energy come from?

(c) What external force must act to bring the mass to that height?

(d) What is the net force that acts on the object while the object is taken to the height 'h'?

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Solution

(a) The potential energy $U = mgh$ = $2 \times 10 \times 5 = 100$ J

Here the positive sign implies that the energy is stored on the mass.

(b) This potential energy is transferred from external agency which applies the force on the mass.

(c) The external applied force $\vec{F}_a$ which takes the object to the height 5 m is $\vec{F}_a = -\vec{F}_g$

$$\vec{F}_a = -\left(-mg\hat{j}\right) = mg\hat{j}$$

where, $\hat{j}$ represents unit vector along vertical upward direction.

(d) From the definition of potential energy, the object must be moved at constant velocity. So the net force acting on the object is zero.

$$\vec{F}_g + \vec{F}_a = 0$$

4.2.6 Elastic Potential Energy

When a spring is elongated, it develops a restoring force. *The potential energy possessed by a spring due to a deforming force which stretches or compresses the spring is termed as elastic potential energy*. The work done by the applied force against the restoring force of the spring is stored as the elastic potential energy in the spring.

Consider a spring-mass system. Let us assume a mass, m lying on a smooth horizontal table as shown in Figure 4.9. Here, $x = 0$ is the equilibrium position. One end of the spring is attached to a rigid wall and the other end to the mass.

Figure 4.9 Potential energy of the spring (elastic potential energy)

As long as the spring remains in equilibrium position, its potential energy is zero. Now an external force $\vec{F}_a$ is applied so that it is stretched by a distance (x) in the direction of the force.

There is a restoring force called spring force $\vec{F}_s$ developed in the spring which tries to bring the mass back to its original position. This applied force and the spring force are equal in magnitude but opposite in direction i.e., $\vec{F}_a = -\vec{F}_s$. According Hooke's law, the restoring force developed in the spring is

$$\vec{F}_s = -k\vec{x} \qquad (4.20)$$

The negative sign in the above expression implies that the spring force is always opposite to that of displacement $\vec{x}$ and k

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is the force constant. Therefore applied force is $\vec{F}_a = +k\vec{x}$. The positive sign implies that the applied force is in the direction of displacement $\vec{x}$. The spring force is an example of variable force as it depends on the displacement $\vec{x}$. Let the spring be stretched to a small distance $d\vec{x}$. The work done by the applied force on the spring to stretch it by a displacement $\vec{x}$ is stored as elastic potential energy.

$$U = \int \vec{F}_a . d\vec{r} = \int_0^x |\vec{F}_a| \, |d\vec{r}| \cos\theta = \int_0^x F_a \, dx \cos\theta \qquad (4.21)$$

The applied force $\vec{F}_a$ and the displacement $d\vec{r}$ (i.e., here dx) are in the same direction. As, the initial position is taken as the equilibrium position or mean position, x=0 is the lower limit of integration.

$$U = \int_0^x kx\,dx \qquad (4.22)$$

$$U = k\left[\frac{x^2}{2}\right]_0^x \qquad (4.23)$$

$$U = \frac{1}{2}kx^2 \qquad (4.24)$$

If the initial position is not zero, and if the mass is changed from position x_i to x_f, then the elastic potential energy is

$$U = \frac{1}{2}k\left(x_f^2 - x_i^2\right) \qquad (4.25)$$

From equations (4.24) and (4.25), we observe that the potential energy of the stretched spring depends on the force constant k and elongation or compression x.

Note The potential energy stored in the spring does not depend on the mass that is attached to the spring.

Force-displacement graph for a spring

Since the restoring spring force and displacement are linearly related as $F = -kx$, and are opposite in direction, the graph between F and x is a straight line with dwelling only in the second and fourth quadrant as shown in Figure 4.10. The elastic potential energy can be easily calculated by drawing a F - x graph. The shaded area (triangle) is the work done by the spring force.

$$\text{Area} = \frac{1}{2}(\text{base})(\text{height}) = \frac{1}{2}\times(x)\times(kx). = \frac{1}{2}kx^2$$

Figure 4.10 Force–displacement graph for a spring

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Potential energy-displacement graph for a spring

A compressed or extended spring will transfer its stored potential energy into kinetic energy of the mass attached to the spring. The potential energy-displacement graph is shown in Figure 4.11.

Figure 4.11 Potential energy–displacement graph for a spring-mass system

In a frictionless environment, the energy gets transferred from kinetic to potential and potential to kinetic repeatedly such that the total energy of the system remains constant. At the mean position,

$$\Delta KE = \Delta U \qquad (4.26)$$

EXAMPLE 4.9

Let the two springs A and B be such that $k_A > k_B$. On which spring will more work has to be done if they are stretched by the same force?

Solution

$$F = k_A x_A = k_B x_B$$

$$x_A = \frac{F}{k_A}; \quad x_B = \frac{F}{k_B}$$

The work done on the springs are stored as potential energy in the springs.

$$U_A = \frac{1}{2} k_A x_A^2 \; ; \quad U_B = \frac{1}{2} k_B x_B^2$$

$$\frac{U_A}{U_B} = \frac{k_A x_A^2}{k_B x_B^2} = \frac{k_A \left(\frac{F}{k_A}\right)^2}{k_B \left(\frac{F}{k_B}\right)^2} = \frac{\frac{1}{k_A}}{\frac{1}{k_B}}$$

$$\frac{U_A}{U_B} = \frac{k_B}{k_A}$$

$k_A > k_B$ implies that $U_B > U_A$. Thus, more work is done on B than A.

EXAMPLE 4.10

A body of mass m is attached to the spring which is elongated to 25 cm by an applied force from its equilibrium position.

(a) Calculate the potential energy stored in the spring-mass system?

(b) What is the work done by the spring force in this elongation?

(c) Suppose the spring is compressed to the same 25 cm, calculate the potential energy stored and also the work done by the spring force during compression. (The spring constant, $k = 0.1\ N\ m^{-1}$).

Solution

The spring constant, $k = 0.1\ N\ m^{-1}$

The displacement, $x = 25\ cm = 0.25\ m$

(a) The potential energy stored in the spring is given by

$$U = \frac{1}{2} k x^2 = \frac{1}{2} \times 0.1 \times (0.25)^2 = 0.0031 J$$

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(b) The work done W_s by the spring force $\vec{F}_s$ is given by,

$$W_s = \int_0^x \vec{F}_s \cdot d\vec{r} = \int_0^x \left(-k\,x\hat{i}\right)\cdot\left(dx\,\hat{i}\right)$$

The spring force $\vec{F}_s$ acts in the negative x direction while elongation acts in the positive x direction.

$$W_s = \int_0^x (-kx)\,dx = -\frac{1}{2}kx^2$$

$$W_s = -\frac{1}{2}\times 0.1\times(0.25)^2 = -0.0031\,J$$

Note that the potential energy is defined through the work done by the external agency. The positive sign in the potential energy implies that the energy is transferred from the agency to the object. But the work done by the restoring force in this case is negative since restoring force is in the opposite direction to the displacement direction.

(c) During compression also the potential energy stored in the object is the same.

$$U = \frac{1}{2}kx^2 = 0.0031\,J.$$

Work done by the restoring spring force during compression is given by

$$W_s = \int_0^x \vec{F}_s.d\vec{r} = \int_0^x \left(k\,x\,\hat{i}\right).\left(-dx\,\hat{i}\right)$$

In the case of compression, the restoring spring force acts towards positive x-axis and displacement is along negative x direction.

$$W_s = \int_0^x (-kx)\,dx = -\frac{1}{2}kx^2 = -0.0031\,J$$

4.2.7 Conservative and non-conservative forces

Conservative force

A force is said to be a conservative force if the work done by or against the force in moving the body depends only on the initial and final positions of the body and not on the nature of the path followed between the initial and final positions.

Let us consider an object at point A on the Earth. It can be taken to another point B at a height h above the surface of the Earth by three paths as shown in Figure 4.12.

Whatever may be the path, the work done against the gravitational force is the same as long as the initial and final positions are the same. This is the reason why gravitational force is a conservative force. Conservative force is equal to the negative gradient of the potential energy. In one dimensional case,

Figure 4.12 Conservative force

$$F_x = -\frac{dU}{dx} \qquad (4.27)$$

Examples for conservative forces are elastic spring force, electrostatic force, magnetic force, gravitational force, etc.

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Table 4.3 Comparison of conservative and non-conservative forces

S.No	Conservative forces	Non-conservative forces
1.	Work done is independent of the path	Work done depends upon the path
2.	Work done in a round trip is zero	Work done in a round trip is not zero
3	Total energy remains constant	Energy is dissipated as heat energy
4	Work done is completely recoverable	Work done is not completely recoverable.
5	Force is the negative gradient of potential energy	No such relation exists.

Non-conservative force

A force is said to be non-conservative if the work done by or against the force in moving a body depends upon the path between the initial and final positions. This means that the value of work done is different in different paths.

1. Frictional forces are non-conservative forces as the work done against friction depends on the length of the path moved by the body.
2. The force due to air resistance, viscous force are also non-conservative forces as the work done by or against these forces depends upon the velocity of motion.

The properties of conservative and non-conservative forces are summarized in the Table 4.3.

EXAMPLE 4.11

Compute the work done by the gravitational force for the following cases

Solution

Force $\vec{F} = mg\left(-\hat{j}\right) = -mg\hat{j}$

Displacement vector $d\vec{r} = dx\,\hat{i} + dy\,\hat{j}$

(As the displacement is in two dimension; unit vectors $\hat{i}$ and $\hat{j}$ are used)

(a) Since the motion is only vertical, horizontal displacement component dx is zero. Hence, work done by the force along path 1 (of distance h).

$$W_{path\ 1} = \int_A^B \vec{F}.d\vec{r} = \int_A^B \left(-mg\hat{j}\right).\left(dy\,\hat{j}\right)$$

$$= -mg\int_0^h dy = -mgh$$

Total work done for path 2 is

$$W_{path\ 2} = \int_A^B \vec{F}.d\vec{r} = \int_A^C \vec{F}.d\vec{r} + \int_C^D \vec{F}.d\vec{r} + \int_D^B \vec{F}.d\vec{r}$$

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But

$$\int_A^C \vec{F}.d\vec{r} = \int_A^C \left(-mg\hat{j}\right).\left(dx\hat{i}\right) = 0$$

$$\int_C^D \vec{F}.d\vec{r} = \int_C^D \left(-mg\hat{j}\right).\left(dy\hat{j}\right)$$

$$= -mg\int_0^h dy = -mgh$$

$$\int_D^B \vec{F}.d\vec{r} = \int_D^B \left(-mg\hat{j}\right).\left(-dx\hat{i}\right) = 0$$

Therefore, the total work done by the force along the path 2 is

$$W_{\text{path 2}} = \int_A^B \vec{F}.d\vec{r} = -mgh$$

Note that the work done by the conservative force is independent of the path.

EXAMPLE 4.12

Consider an object of mass 2 kg moved by an external force 20 N in a surface having coefficient of kinetic friction 0.9 to a distance 10 m. What is the work done by the external force and kinetic friction ? Comment on the result. (Assume g = 10 ms^{-2})

Solution

m = 2 kg, d = 10 m, F_{ext} = 20 N, μ_k = 0.9.

When an object is in motion on the horizontal surface, it experiences two forces.

(a) External force, $F_{ext} = 20$ N

(b) Kinetic friction,

$$f_k = \mu_k mg = 0.9 \times (2) \times 10 = 18\,\text{N}.$$

The work done by the external force $W_{ext} = F\,d = 20 \times 10 = 200\,\text{J}$

The work done by the force of kinetic friction $W_k = f_k d = (-18) \times 10 = -180$ J. Here the negative sign implies that the force of kinetic friction is opposite to the direction of displacement.

The total work done on the object $W_{total} = W_{ext} + W_k = 200\text{ J} - 180\text{ J} = 20\text{ J}$.

Since the friction is a non-conservative force, out of 200 J given by the external force, the 180 J is lost and it can not be recovered.

4.2.8 Law of conservation of energy

When an object is thrown upwards its kinetic energy goes on decreasing and consequently its potential energy keeps increasing (neglecting air resistance). When it reaches the highest point its energy is completely potential. Similarly, when the object falls back from a height its kinetic energy increases whereas its potential energy decreases. When it touches the ground its energy is completely kinetic. At the intermediate points the energy is both kinetic and potential as shown in Figure 4.13. When the body reaches the ground the kinetic energy is completely dissipated into some other form of energy like sound, heat, light and deformation of the body etc.

In this example the energy transformation takes place at every point. The sum of kinetic energy and potential energy i.e., the total mechanical energy always remains constant, implying that the total energy is conserved. This is stated as the law of conservation of energy.

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Figure 4.13 Conservation of energy

The law of conservation of energy states that *energy can neither be created nor destroyed. It may be transformed from one form to another but the total energy of an isolated system remains constant.*

Figure 4.13 illustrates that, if an object starts from rest at height *h*, the total energy is purely potential energy (U=mgh) and the kinetic energy (KE) is zero at *h*. When the object falls at some distance *y*, the potential energy and the kinetic energy are not zero whereas, the total energy remains same as measured at height *h*. When the object is about to touch the ground, the potential energy is zero and total energy is purely kinetic.

EXAMPLE 4.13

An object of mass 1 kg is falling from the height h = 10 m. Calculate

(a) The total energy of an object at $h = 10$ m

(b) Potential energy of the object when it is at $h = 4$ m

(c) Kinetic energy of the object when it is at $h = 4$ m

(d) What will be the speed of the object when it hits the ground?

(Assume $g = 10$ m s^{-2})

Solution

(a) The gravitational force is a conservative force. So the total energy remains constant throughout the motion. At $h = 10$ m, the total energy E is entirely potential energy.

$$E = U = mgh = 1 \times 10 \times 10 = 100 \text{ J}$$

(b) The potential energy of the object at $h = 4$ m is

$$U = mgh = 1 \times 10 \times 4 = 40 \text{ J}$$

(c) Since the total energy is constant throughout the motion, the kinetic energy at $h = 4$ m must be $KE = E - U = 100 - 40 = 60 \text{ J}$

Alternatively, the kinetic energy could also be found from velocity of the object at 4 m. At the height 4 m, the object has fallen through a height of 6 m.

The velocity after falling 6 m is calculated from the equation of motion,

$$v = \sqrt{2gh} = \sqrt{2 \times 10 \times 6} = \sqrt{120} \text{ m s}^{-1};$$

$$v^2 = 120$$

The kinetic energy is $KE = \frac{1}{2}mv^2 = \frac{1}{2} \times 1 \times 120 = 60 \text{ J}$

(d) When the object is just about to hit the ground, the total energy is completely kinetic and the potential energy, $U = 0$.

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$$E = KE = \frac{1}{2}mv^2 = 100 \text{ J}$$

$$v = \sqrt{\frac{2}{m}KE} = \sqrt{\frac{2}{1} \times 100} = \sqrt{200} \approx 14.12 \text{ m s}^{-1}$$

EXAMPLE 4.14

A body of mass 100 kg is lifted to a height 10 m from the ground in two different ways as shown in the figure. What is the work done by the gravity in both the cases? Why is it easier to take the object through a ramp?

Path (1) straight up

Path (2) along the ramp

Solution

m = 100 kg, h = 10 m

Along path (1):

The minimum force F_1 required to move the object to the height of 10 m should be equal to the gravitational force, $F_1 = mg = 100 \times 10 = 1000$ N

The distance moved along path (1) is, $h = 10$ m

The work done on the object along path (1) is

$$W = F\,h = 1000 \times 10 = 10{,}000 \text{ J}$$

Along path (2):

In the case of the ramp, the minimum force F_2 that we apply on the object to take it up is not equal to mg, it is rather equal to $mg\sin\theta$. $(mg\sin\theta < mg)$.

Here, angle $\theta = 30^\circ$

Therefore, $F_2 = mg\sin\theta = 100 \times 10 \times \sin 30^\circ = 100 \times 10 \times 0.5 = 500$ N

Hence, $(mg\sin\theta < mg)$

The path covered along the ramp is, $l = \frac{h}{\sin 30^\circ} = \frac{10}{0.5} = 20\,m$

The work done on the object along path (2) is, $W = F_2 l = 500 \times 20 = 10{,}000$ J

Since the gravitational force is a conservative force, the work done by gravity on the object is independent of the path taken.

In both the paths the work done by the gravitational force is 10,000 J

Along path (1): more force needs to be applied against gravity to cover lesser distance .

Along path (2): lesser force needs to be applied against the gravity to cover more distance.

As the force needs to be applied along the ramp is less, it is easier to move the object along the ramp.

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EXAMPLE 4.15

An object of mass m is projected from the ground with initial speed v_0.

Find the speed at height h.

Solution

Since the gravitational force is conservative; the total energy is conserved throughout the motion.

	Initial	Final
Kinetic energy	$\frac{1}{2}mv_0^2$	$\frac{1}{2}mv^2$
Potential energy	0	mgh
Total energy	$\frac{1}{2}mv_0^2 + 0 = \frac{1}{2}mv_0^2$	$\frac{1}{2}mv^2 + mgh$

Final values of potential energy, kinetic energy and total energy are measured at the height *h*.

By law of conservation of energy, the initial and final total energies are the same.

$$\frac{1}{2}mv_0^2 = \frac{1}{2}mv^2 + mgh$$

$$v_0^2 = v^2 + 2gh$$

$$v = \sqrt{v_0^2 - 2gh}$$

Note that in section (2.11.2) similar result is obtained using kinematic equation based on calculus method. However, calculation through energy conservation method is much easier than calculus method.

EXAMPLE 4.16

An object of mass 2 kg attached to a spring is moved to a distance $x = 10$ m from its equilibrium position. The spring constant $k = 1$ N m^{-1} and assume that the surface is frictionless.

(a) When the mass crosses the equilibrium position, what is the speed of the mass?

(b) What is the force that acts on the object when the mass crosses the equilibrium position and extremum position $x = \pm 10$ m.

Solution

(a) Since the spring force is a conservative force, the total energy is constant. At $x = 10$ m, the total energy is purely potential.

$$E = U = \frac{1}{2}kx^2 = \frac{1}{2}\times(1)\times(10)^2 = 50 \text{ J}$$

When the mass crosses the equilibrium position $(x = 0)$, the potential energy

$$U = \frac{1}{2}\times 1\times(0) = 0 \text{ J}$$

The entire energy is purely kinetic energy at this position.

$$E = KE = \frac{1}{2}mv^2 = 50 \text{ J}$$

The speed

$$v = \sqrt{\frac{2KE}{m}} = \sqrt{\frac{2\times 50}{2}} = \sqrt{50}\, m\,s^{-1} \approx 7.07\, m\,s^{-1}$$

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(b) Since the restoring spring force is F = - kx, when the object crosses the equilibrium position, it experiences no force. Note that at equilibrium position, the object moves very fast. When the object is at x = +10 m (elongation), the force F = – k x

F = – (1) (10) = – 10 N. Here the negative sign implies that the force is towards equilibrium i.e., towards negative x-axis and when the object is at $x = -10m$ (compression), it experiences a forces F = – (1) (– 10) = +10 N. Here the positive sign implies that the force points towards positive x-axis.

The object comes to momentary rest at $x = \pm 10\,m$ even though it experiences a maximum force at both these points.

4.2.9 Motion in a vertical circle

Imagine that a body of mass (m) attached to one end of a massless and inextensible string executes circular motion in a vertical plane with the other end of the string fixed. The length of the string becomes the radius ($\vec{r}$) of the circular path (Figure 4.14).

Let us discuss the motion of the body by taking the free body diagram (FBD) at a position where the position vector ($\vec{r}$) makes an angle θ with the vertically downward direction and the instantaneous velocity is as shown in Figure 4.14.

There are two forces acting on the mass.

1. Gravitational force which acts downward
2. Tension along the string.

Applying Newton's second law on the mass,

In the tangential direction,

$$mg\sin\theta = m\,a_t$$
$$mg\sin\theta = -m\left(\frac{dv}{dt}\right) \qquad (4.28)$$

Figure 4.14 Motion in vertical circle

where, $a_t = -\dfrac{dv}{dt}$ is tangential retardation

In the radial direction,

$$T - mg\cos\theta = m\,a_r$$
$$T - mg\cos\theta = \frac{mv^2}{r} \qquad (4.29)$$

where, $a_r = \dfrac{v^2}{r}$ is the centripetal acceleration.

The circle can be divided into four sections A, B, C, D for better understanding of the motion. The four important facts to be understood from the two equations are as follows:

(i) The mass is having tangential acceleration (g sin θ) for all values of θ (except θ = 0°), it is clear that this vertical cirular motion is not a uniform circular motion.

(ii) From the equations (4.28) and (4.29) it is understood that as the magnitude of velocity is not a constant in the course of motion, the tension in the string is also not constant.

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(iii) The equation (4.29), $T = mg\cos\theta + \frac{mv^2}{r}$ highlights that in sections A and D of the circle, $\left(\text{for } -\frac{\pi}{2} < \theta < \frac{\pi}{2};\ \cos\theta \text{ is positive}\right)$, the term mg cosθ is always greater than zero. Hence the tension cannot vanish even when the velocity vanishes.

(iv) The equation (4.29), $\frac{mv^2}{r} = T - mg\cos\theta$; further highlights that in sections B and C of the circle, $\left(\text{for } \frac{\pi}{2} < \theta < \frac{3\pi}{2};\ \cos\theta \text{ is negative}\right)$, the second term is always greater than zero. Hence velocity cannot vanish, even when the tension vanishes.

These points are to be kept in mind while solving problems related to motion in vertical circle.

To start with let us consider only two positions, say the lowest point 1 and the highest point 2 as shown in Figure 4.15 for further analysis. Let the velocity of the body at the lowest point 1 be $\vec{v}_1$, at the highest point 2 be $\vec{v}_2$ and $\vec{v}$ at any other point. The direction of velocity is tangential to the circular path at all points. Let $\vec{T}_1$ be the tension in the string at the lowest point and $\vec{T}_2$ be the tension at the highest point and $\vec{T}$ be the tension at any other point. Tension at each point acts towards the centre. The tensions and velocities at these two points can be found by applying the law of conservation of energy.

For the lowest point (1)

When the body is at the lowest point 1, the gravitational force $m\vec{g}$ which acts on the body (vertically downwards) and another one is the tension $\vec{T}_1$ acting vertically upwards, i.e. towards the centre. From the equation (4.29), we get

$$T_1 - mg = \frac{mv_1^2}{r} \qquad (4.30)$$

$$T_1 = \frac{mv_1^2}{r} + mg \qquad (4.31)$$

Figure 4.15 Motion in vertical circle shown for lowest and highest points

For the highest point (2)

At the highest point 2, both the gravitational force $m\vec{g}$ on the body and the tension $\vec{T}_2$ act downwards, i.e. towards the centre again.

$$T_2 + mg = \frac{mv_2^2}{r} \qquad (4.32)$$

$$T_2 = \frac{mv_2^2}{r} - mg \qquad (4.33)$$

From equations (4.31) and (4.33), it is understood that $T_1 > T_2$. The difference in tension $T_1 - T_2$ is obtained by subtracting equation (4.33) from equation (4.31).

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$$T_1 - T_2 = \frac{mv_1^2}{r} + mg - \left(\frac{mv_2^2}{r} - mg\right)$$

$$= \frac{mv_1^2}{r} + mg - \frac{mv_2^2}{r} + mg$$

$$T_1 - T_2 = \frac{m}{r}\left[v_1^2 - v_2^2\right] + 2mg \qquad (4.34)$$

The term $\left[v_1^2 - v_2^2\right]$ can be found easily by applying law of conservation of energy at point 1 and also at point 2.

Note

The tension will not do any work on the mass as the tension and the direction of motion is always perpendicular.

The gravitational force is doing work on the mass, as it is a conservative force the total energy of the mass is conserved throughout the motion.

Total Energy at point 1(E_1) is same as the total energy at a point 2 (E_2)

$$E_1 = E_2 \qquad (4.35)$$

Potential Energy at point 1, $U_1 = 0$ (by taking reference as point 1)

Kinetic Energy at point 1, $KE_1 = \frac{1}{2}mv_1^2$

Total Energy at point 1, $E_1 = U_1 + KE_1 =$ $= 0 + \frac{1}{2}mv_1^2 = \frac{1}{2}mv_1^2$

Similarly, Potential Energy at point 2, $U_2 = mg\,(2r)$ (h is 2r from point 1)

Kinetic Energy at point 2, $KE_2 = \frac{1}{2}mv_2^2$

Total Energy at point 2, $E_2 = U_2 + KE_2 =$ $2\,mg\,r + \frac{1}{2}mv_2^2$

From the law of conservation of energy given in equation (4.35), we get

$$\frac{1}{2}mv_1^2 = 2mgr + \frac{1}{2}mv_2^2$$

After rearranging,

$$\frac{1}{2}m\left(v_1^2 - v_2^2\right) = 2mgr$$

$$v_1^2 - v_2^2 = 4gr \qquad (4.36)$$

Substituting equation (4.36) in equation (4.34) we get,

$$T_1 - T_2 = \frac{m}{r}\left[4gr\right] + 2mg$$

Therefore, the difference in tension is

$$T_1 - T_2 = 6\ mg \qquad (4.37)$$

Minimum speed at the highest point (2)

The body must have a minimum speed at point 2 otherwise, the string will slack before reaching point 2 and the body will not loop the circle. To find this minimum speed let us take the tension $T_2 = 0$ in equation (4.33).

$$0 = \frac{mv_2^2}{r} - mg$$

$$\frac{mv_2^2}{r} = mg$$

$$v_2^{\ 2} = rg$$

$$v_2 = \sqrt{gr} \qquad (4.38)$$

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The body must have a speed at point 2, $v_2 \geq \sqrt{gr}$ to stay in the circular path.

Minimum speed at the lowest point 1

To have this minimum speed $\left(v_2 = \sqrt{gr}\right)$ at point 2, the body must have minimum speed also at point 1.

By making use of equation (4.36) we can find the minimum speed at point 1.

$$v_1^2 - v_2^2 = 4gr$$

Substituting equation (4.38) in (4.36),

$$v_1^2 - gr = 4gr$$

$$v_1^2 = 5gr$$

$$v_1 = \sqrt{5gr} \qquad (4.39)$$

The body must have a speed at point 1, $v_1 \geq \sqrt{5gr}$ to stay in the circular path.

From equations (4.38) and (4.39), it is clear that the minimum speed at the lowest point 1 should be $\sqrt{5}$ times more than the minimum speed at the highest point 2, so that the body loops without leaving the circle.

EXAMPLE 4.17

Water in a bucket tied with rope is whirled around in a vertical circle of radius 0.5 m. Calculate the minimum velocity at the lowest point so that the water does not spill from it in the course of motion. ($g = 10\ ms^{-2}$)

Solution

Radius of circle r = 0.5 m

The required speed at the highest point $v_2 = \sqrt{gr} = \sqrt{10 \times 0.5} = \sqrt{5}\ ms^{-1}$. The speed at lowest point $v_1 = \sqrt{5gr} = \sqrt{5} \times \sqrt{gr} = \sqrt{5} \times \sqrt{5} = 5\ ms^{-1}$

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4.3 POWER

4.3.1 Definition of power

Power is a measure of how fast or slow a work is done. *Power is defined as the rate of work done or energy delivered.*

$$\text{Power } (P) = \frac{\text{work done}(W)}{\text{time taken}(t)}$$

$$P = \frac{W}{t}$$

Average power

The average power (P_{av}) is defined as the ratio of the total work done to the total time taken.

$$P_{av} = \frac{\text{total work done}}{\text{total time taken}}$$

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Instantaneous power

The instantaneous power (P_{inst}) is defined as the power delivered at an instant (as time interval approaches zero),

$$P_{inst} = \frac{dW}{dt}$$

4.3.2 Unit of power

Power is a scalar quantity. Its dimension is $[ML^2T^{-3}]$. The SI unit of power is watt (W), named after the inventor of the steam engine James Watt. *One watt is defined as* the power *when one joule of work is done in one second*, ($1\ W = 1\ J\ s^{-1}$).

The higher units are kilowatt(kW), megawatt(MW), and Gigawatt(GW).

$$1kW = 1000\ W = 10^3\ watt$$
$$1MW = 10^6\ watt$$
$$1GW = 10^9\ watt$$

For motors, engines and some automobiles an old unit of power still commercially in use which is called as the horse-power (hp). We have a conversion for horse-power (hp) into watt (W) which is,

$$1\ hp = 746\ W$$

All electrical goods come with a definite power rating in watt printed on them. A 100 watt bulb consumes 100 joule of electrical energy in one second. The energy measured in joule in terms of power in watt and time in second is written as, 1 J =1 W s. When electrical appliances are put in use for long hours, they consume a large amount of energy. Measuring the electrical energy in a small unit watt. second (W s) leads to handling large numerical values. Hence, electrical energy is measured in the unit called kilowatt hour (kWh).

$$1\ \text{electrical unit} = 1\ kWh = 1 \times (10^3\ W) \times (3600\ s)$$
$$1\ \text{electrical unit} = 3600\times10^3\ W\ s$$
$$1\ \text{electrical unit} = 3.6\times10^6\ J$$
$$1\ kWh = 3.6\times10^6\ J$$

Electricity bills are generated in units of kWh for electrical energy consumption. 1 unit of electrical energy is 1 kWh. (**Note**: kWh is unit of energy and not of power.)

EXAMPLE 4.18

Calculate the energy consumed in electrical units when a 75 W fan is used for 8 hours daily for one month (30 days).

Solution

Power, P = 75 W

Time of usage, t = 8 hour × 30 days = 240 hours

Electrical energy consumed is the product of power and time of usage.

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Electrical energy = power × time of usage = P × t

$$= 75 \text{ watt} \times 240 \text{ hour}$$
$$= 18000 \text{ watt hour}$$
$$= 18 \text{ kilowatt hour} = 18 \text{ kWh}$$
$$1 \text{ electrical unit} = 1 \text{ kWh}$$
$$\text{Electrical energy} = 18 \text{ unit}$$

DO YOU KNOW?

Incandescent lamps glow for 1000 hours. CFL lamps glow for 6000 hours. But LED lamps glow for 50000 hrs (almost 25 years at 5.5 hour per day).

4.3.3 Relation between power and velocity

The work done by a force $\vec{F}$ for a displacement $d\vec{r}$ is

$$W = \int \vec{F} \cdot d\vec{r} \qquad (4.40)$$

Left hand side of the equation (4.40) can be written as

$$W = \int dW = \int \frac{dW}{dt} dt$$

(multiplied and divided by dt) (4.41)

Since, velocity is $\vec{v} = \frac{d\vec{r}}{dt}$; $d\vec{r} = \vec{v}\, dt$. Right hand side of the equation (4.40) can be written as

$$\int \vec{F} \cdot d\vec{r} = \int \left(\vec{F} \cdot \frac{d\vec{r}}{dt} \right) dt = \int \left(\vec{F} \cdot \vec{v} \right) dt \quad \left[\vec{v} = \frac{d\vec{r}}{dt} \right] \qquad (4.42)$$

Substituting equation (4.41) and equation (4.42) in equation (4.40), we get

$$\int \frac{dW}{dt} dt = \int \left(\vec{F} \cdot \vec{v} \right) dt$$
$$\int \left(\frac{dW}{dt} - \vec{F} \cdot \vec{v} \right) dt = 0$$

This relation is true for any arbitrary value of dt. This implies that the term within the bracket must be equal to zero, i.e.,

$$\frac{dW}{dt} - \vec{F} \cdot \vec{v} = 0$$

Or

$$\frac{dW}{dt} = \vec{F} \cdot \vec{v} = P \qquad (4.43)$$

EXAMPLE 4.19

A vehicle of mass 1250 kg is driven with an acceleration 0.2 $m\,s^{-2}$ along a straight level road against an external resistive force 500 N. Calculate the power delivered by the vehicle's engine if the velocity of the vehicle is 30 $m\,s^{-1}$.

Solution

The vehicle's engine has to do work against resistive force and make vechile to move with an acceleration. Therefore, power delivered by the vehicle engine is

P = (resistive force + mass × acceleration) (velocity)

$$P = \vec{F}_{-tot} \cdot \vec{v} = (F_{resistive} + F)\vec{v}$$
$$P = \vec{F}_{tot} \cdot \vec{v} = (F_{resistive} + ma)\vec{v}$$
$$= (500 \text{ N} + (1250 \text{ kg}) \times (0.2\ ms^{-2}))\ (30 \text{ ms}^{-1}) = 22.5 \text{ kW}$$

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4.4 COLLISIONS

Collision is a common phenomenon that happens around us every now and then. For example, carom, billiards, marbles, etc.,. Collisions can happen between two bodies with or without physical contacts.

Linear momentum is conserved in all collision processes. When two bodies collide, the mutual impulsive forces acting between them during the collision time (Δt) produces a change in their respective momenta. That is, the first body exerts a force $\vec{F}_{21}$ on the second body. From Newton's third law, the second body exerts a force $\vec{F}_{12}$ on the first body. This causes a change in momentum $\Delta\vec{p}_1$ and $\Delta\vec{p}_2$ of the first body and second body respectively. Now, the relations could be written as,

$$\Delta\vec{p}_1 = \vec{F}_{12}\Delta t \qquad (4.44)$$

$$\Delta\vec{p}_2 = \vec{F}_{21}\Delta t \qquad (4.45)$$

Adding equation (4.44) and equation (4.45), we get

$$\Delta\vec{p}_1 + \Delta\vec{p}_2 = \vec{F}_{12}\Delta t + \vec{F}_{21}\Delta t = \left(\vec{F}_{12} + \vec{F}_{21}\right)\Delta t$$

According to Newton's third law, $\vec{F}_{12} = -\vec{F}_{21}$

$$\Delta\vec{p}_1 + \Delta\vec{p}_2 = 0$$

$$\Delta\left(\vec{p}_1 + \vec{p}_2\right) = 0$$

Dividing both sides by Δt and taking limit $\Delta t \to 0$, we get

$$\lim_{\Delta t \to 0} \frac{\Delta\left(\vec{p}_1 + \vec{p}_2\right)}{\Delta t} = \frac{d\left(\vec{p}_1 + \vec{p}_2\right)}{dt} = 0$$

The above expression implies that the total linear momentum is a conserved quantity. **Note:** The momentum is a vector quantity. Hence, vector addition has to be followed to find the total momentum of the individual bodies in collision.

4.4.1 Types of Collisions

In any collision process, the total linear momentum and total energy are always conserved whereas the total kinetic energy need not be conserved always. Some part of the initial kinetic energy is transformed to other forms of energy. This is because, the impact of collisions and deformation occurring due to collisions may in general, produce heat, sound, light etc. By taking these effects into account, we classify the types of collisions as follows:

(a) Elastic collision

(b) Inelastic collision

(a) Elastic collision

In a collision, the total initial kinetic energy of the bodies (before collision) is equal to the total final kinetic energy of the bodies (after collision) then, it is called as elastic collision. i.e.,

Total kinetic energy before collision = Total kinetic energy after collision

(b) Inelastic collision

In a collision, the total initial kinetic energy of the bodies (before collision) is not equal

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to the total final kinetic energy of the bodies (after collision) then, it is called as inelastic collision. i.e.,

Total kinetic energy before collision ≠ Total kinetic energy after collision

$$\begin{pmatrix} \textit{Total kinetic energy} \\ \textit{before collision} \end{pmatrix} - \begin{pmatrix} \textit{Total kinetic energy} \\ \textit{after collision} \end{pmatrix} = \begin{pmatrix} \textit{loss in energy} \\ \textit{during collision} \end{pmatrix} = \Delta Q$$

Even though kinetic energy is not conserved but the total energy is conserved. This is because the total energy contains the kinetic energy term and also a term ΔQ, which includes all the losses that take place during collision. Note that loss in kinetic energy during collision is transformed to another form of energy like sound, thermal, etc. Further, if the two colliding bodies stick together after collision such collisions are known as completely inelastic collision or perfectly inelastic collision. Such a collision is found very often. For example when a clay putty is thrown on a moving vehicle, the clay putty (or Bubblegum) sticks to the moving vehicle and they move together with the same velocity.

4.4.2 Elastic collisions in one dimension

Consider two elastic bodies of masses m_1 and m_2 moving in a straight line (along positive x direction) on a frictionless horizontal surface as shown in Figure 4.16.

Figure 4.16 Elastic collision in one dimension

Mass	Initial velocity	Final velocity
Mass m_1	u_1	v_1
Mass m_2	u_2	v_2

In order to have collision, we assume that the mass m_1 moves faster than mass m_2 i.e., $u_1 > u_2$. For elastic collision, the total linear momentum and kinetic energies of the two bodies before and after collision must remain the same.

Table 4.4 Comparison between elastic and inelastic collisions

S.No.	Elastic Collision	Inelastic Collision
1.	Total momentum is conserved	Total momentum is conserved
2.	Total kinetic energy is conserved	Total kinetic energy is not conserved
3.	Forces involved are conservative forces	Forces involved are non-conservative forces
4.	Mechanical energy is not dissipated.	Mechanical energy is dissipated into heat, light, sound etc.

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	Momentum of mass m_1	Momentum of mass m_2	Total linear momentum
Before collision	$p_{i1} = m_1 u_1$	$p_{i2} = m_2 u_2$	$p_i = p_{i1} + p_{i2}$ $p_i = m_1 u_1 + m_2 u_2$
After collision	$p_{f1} = m_1 v_1$	$p_{f2} = m_2 v_2$	$p_f = p_{f1} + p_{f2}$ $p_f = m_1 v_1 + m_2 v_2$

From the law of conservation of linear momentum,

Total momentum before collision (p_i) = Total momentum after collision (p_f)

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2 \quad (4.46)$$

$$\text{Or } m_1(u_1 - v_1) = m_2(v_2 - u_2) \quad (4.47)$$

Further,

	Kinetic energy of mass m_1	Kinetic energy of mass m_2	Total kinetic energy
Before collision	$KE_{i1} = \frac{1}{2} m_1 u_1^2$	$KE_{i2} = \frac{1}{2} m_2 u_2^2$	$KE_i = KE_{i1} + KE_{i2}$ $KE_i = \frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2$
After collision	$KE_{f1} = \frac{1}{2} m_1 v_1^2$	$KE_{f2} = \frac{1}{2} m_2 v_2^2$	$KE_f = KE_{f1} + KE_{f2}$ $KE_f = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$

For elastic collision,

Total kinetic energy before collision KE_i = Total kinetic energy after collision KE_f

$$\frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 \quad (4.48)$$

After simplifying and rearranging the terms,

$$m_1(u_1^2 - v_1^2) = m_2(v_2^2 - u_2^2)$$

Using the formula $a^2 - b^2 = (a+b)(a-b)$, we can rewrite the above equation as

$$m_1(u_1 + v_1)(u_1 - v_1) = m_2(v_2 + u_2)(v_2 - u_2) \quad (4.49)$$

Dividing equation (4.49) by (4.47) gives,

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$$\frac{m_1(u_1+v_1)(u_1-v_1)}{m_1(u_1-v_1)}=\frac{m_2(v_2+u_2)(v_2-u_2)}{m_2(v_2-u_2)}$$

$$u_1+v_1=v_2+u_2$$
$$u_1-u_2=v_2-v_1 \quad \text{Rearranging,} \quad (4.50)$$

Equation (4.50) can be rewritten as

$$u_1-u_2=-(v_1-v_2)$$

This means that for any elastic head on collision, the relative speed of the two elastic bodies after the collision has the same magnitude as before collision but in opposite direction. Further note that this result is independent of mass.

Rewriting the above equation for v_1 and v_2,

$$v_1=v_2+u_2-u_1 \quad (4.51)$$

Or

$$v_2=u_1+v_1-u_2 \quad (4.52)$$

To find the final velocities v_1 and v_2:
Substituting equation (4.52) in equation (4.47) gives the velocity of m_1 as

$$m_1(u_1-v_1)=m_2(u_1+v_1-u_2-u_2)$$
$$m_1(u_1-v_1)=m_2(u_1+v_1-2u_2)$$
$$m_1u_1-m_1v_1=m_2u_1+m_2v_1-2m_2u_2$$
$$m_1u_1-m_2u_1+2m_2u_2=m_1v_1+m_2v_1$$
$$(m_1-m_2)u_1+2m_2u_2=(m_1+m_2)v_1$$
$$\text{or } v_1=\left(\frac{m_1-m_2}{m_1+m_2}\right)u_1+\left(\frac{2m_2}{m_1+m_2}\right)u_2 \quad (4.53)$$

Similarly, by substituting (4.51) in equation (4.47) or substituting equation (4.53) in equation (4.52), we get the final velocity of m_2 as

$$v_2=\left(\frac{2m_1}{m_1+m_2}\right)u_1+\left(\frac{m_2-m_1}{m_1+m_2}\right)u_2 \quad (4.54)$$

Case 1: When bodies has the same mass i.e., $m_1=m_2$,

$$\text{equation (4.53)} \Rightarrow v_1=(0)\,u_1+\left(\frac{2m_2}{2m_2}\right)u_2$$
$$v_1=u_2 \quad (4.55)$$
$$\text{equation (4.54)} \Rightarrow v_2=\left(\frac{2m_1}{2m_1}\right)u_1+(0)u_2$$
$$v_2=u_1 \quad (4.56)$$

The equations (4.55) and (4.56) show that in one dimensional elastic collision, when two bodies of equal mass collide after the collision their velocities are exchanged.

Case 2: When bodies have the same mass i.e., $m_1=m_2$ and second body (usually called target) is at rest ($u_2=0$),

By substituting $m_1=m_2$ and $u_2=0$ in equations (4.53) and equations (4.54) we get,

$$\text{from equation (4.53)} \Rightarrow v_1=0 \quad (4.57)$$
$$\text{from equation (4.54)} \Rightarrow v_2=u_1 \quad (4.58)$$

Equations (4.57) and (4.58) show that when the first body comes to rest the second

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body moves with the initial velocity of the first body.

Case 3:

The first body is very much lighter than the second body

$\left(m_1 \ll m_2, \frac{m_1}{m_2} \ll 1\right)$ then the ratio $\frac{m_1}{m_2} \approx 0$

and also if the target is at rest $(u_2 = 0)$

Dividing numerator and denominator of equation (4.53) by m_2, we get

$$v_1 = \left(\frac{\frac{m_1}{m_2} - 1}{\frac{m_1}{m_2} + 1}\right) u_1 + \left(\frac{2}{\frac{m_1}{m_2} + 1}\right)(0)$$

$$v_1 = \left(\frac{0-1}{0+1}\right) u_1$$

$$v_1 = -u_1 \qquad (4.59)$$

Similarly,

Dividing numerator and denominator of equation (4.54) by m_2, we get

$$v_2 = \left(\frac{2\frac{m_1}{m_2}}{\frac{m_1}{m_2} + 1}\right) u_1 + \left(\frac{1 - \frac{m_1}{m_2}}{\frac{m_1}{m_2} + 1}\right)(0)$$

$$v_2 = (0) u_1 + \left(\frac{1 - \frac{m_1}{m_2}}{\frac{m_1}{m_2} + 1}\right)(0)$$

$$v_2 = 0 \qquad (4.60)$$

The equation (4.59) implies that the first body which is lighter returns back (rebounds) in the opposite direction with the same initial velocity as it has a negative sign. The equation (4.60) implies that the second body which is heavier in mass continues to remain at rest even after collision. For example, if a ball is thrown at a fixed wall, the ball will bounce back from the wall with the same velocity with which it was thrown but in opposite direction.

Case 4:

The second body is very much lighter than the first body

$\left(m_2 \ll m_1, \frac{m_2}{m_1} \ll 1\right)$ then the ratio $\frac{m_2}{m_1} \approx 0$

and also if the target is at rest $(u_2 = 0)$

Dividing numerator and denominator of equation (4.53) by m_1, we get

$$v_1 = \left(\frac{1 - \frac{m_2}{m_1}}{1 + \frac{m_2}{m_1}}\right) u_1 + \left(\frac{2\frac{m_2}{m_1}}{1 + \frac{m_2}{m_1}}\right)(0)$$

$$v_1 = \left(\frac{1-0}{1+0}\right) u_1 + \left(\frac{0}{1+0}\right)(0)$$

$$v_1 = u_1 \qquad (4.61)$$

Similarly,

Dividing numerator and denominator of equation (4.58) by m_1, we get

$$v_2 = \left(\frac{2}{1 + \frac{m_2}{m_1}}\right) u_1 + \left(\frac{\frac{m_2}{m_1} - 1}{1 + \frac{m_2}{m_1}}\right)(0)$$

$$v_2 = \left(\frac{2}{1+0}\right) u_1$$

$$v_2 = 2u_1 \qquad (4.62)$$

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The equation (4.61) implies that the first body which is heavier continues to move with the same initial velocity. The equation (4.62) suggests that the second body which is lighter will move with twice the initial velocity of the first body. It means that the lighter body is thrown away from the point of collision.

EXAMPLE 4.20

A lighter particle moving with a speed of 10 m s^{-1} collides with an object of double its mass moving in the same direction with half its speed. Assume that the collision is a one dimensional elastic collision. What will be the speed of both particles after the collision?

Solution

Let the mass of the first body be *m* which moves with an initial velocity, $u_1 = 10 \text{ m s}^{-1}$. Therefore, the mass of second body is *2m* and its initial velocity is $u_2 = \frac{1}{2} u_1 = \frac{1}{2}\left(10 \text{ m s}^{-1}\right)$,

Then, the final velocities of the bodies can be calculated from the equation (4.53) and equation (4.54)

$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \left(\frac{2m_2}{m_1 + m_2}\right)u_2$$

$$v_1 = \left(\frac{m - 2m}{m + 2m}\right)10 + \left(\frac{2 \times 2m}{m + 2m}\right)5$$

$$v_1 = -\left(\frac{1}{3}\right)10 + \left(\frac{4}{3}\right)5 = \frac{-10 + 20}{3} = \frac{10}{3}$$

$$v_1 = 3.33 \text{ ms}^{-1}$$

$$v_2 = \left(\frac{2m_1}{m_1 + m_2}\right)u_1 + \left(\frac{m_2 - m_1}{m_1 + m_2}\right)u_2$$

$$v_2 = \left(\frac{2m}{m + 2m}\right)10 + \left(\frac{2m - m}{m + 2m}\right)5$$

$$v_2 = \left(\frac{2}{3}\right)10 + \left(\frac{1}{3}\right)5 = \frac{20 + 5}{3} = \frac{25}{3}$$

$$v_2 = 8.33 \text{ ms}^{-1}$$

As the two speeds v_1 and v_2 are positive, they move in the same direction with the velocities, 3.33 m s^{-1} and 8.33 m s^{-1} respectively.

4.4.3 Perfect inelastic collision

In a perfectly inelastic or completely inelastic collision, the objects stick together permanently after collision such that they move with common velocity. Let the two bodies with masses m_1 and m_2 move with initial velocities u_1 and u_2 respectively before collision. After perfect inelastic collision both the objects move together with a common velocity v as shown in Figure (4.17).

Since, the linear momentum is conserved during collisions,

$$m_1 u_1 + m_2 u_2 = (m_1 + m_2)\, v$$

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Figure 4.17 Perfect inelastic collision in one dimension

	Velocity		Linear momentum	
	Initial	Final	Initial	Final
Mass m_1	u_1	v	m_1u_1	m_1v
Mass m_2	u_2	v	m_2u_2	m_2v
	Total		$m_1u_1 + m_2u_2$	$(m_1+m_2)\,v$

The common velocity can be computed by

$$v = \frac{m_1u_1 + m_2u_2}{(m_1 + m_2)} \qquad (4.63)$$

EXAMPLE 4.21

A bullet of mass 50 g is fired from below into a suspended object of mass 450 g. The object rises through a height of 1.8 m with bullet remaining inside the object. Find the speed of the bullet. Take g = 10 ms^{-2}.

Solution

m_1 = 50 g = 0.05 kg; m_2 = 450 g = 0.45 kg

The speed of the bullet is u_1. The second body is at rest $(u_2 = 0)$. Let the common velocity of the bullet and the object after the bullet is embedded into the object is v.

$$v = \frac{m_1u_1 + m_2u_2}{(m_1 + m_2)}$$

$$v = \frac{0.05u_1 + (0.45 \times 0)}{(0.05 + 0.45)} = \frac{0.05}{0.50}u_1$$

The combined velocity is the initial velocity for the vertical upward motion of the combined bullet and the object. From second equation of motion,

$$v = \sqrt{2gh}$$

$$v = \sqrt{2 \times 10 \times 1.8} = \sqrt{36}$$

$$v = 6\,ms^{-1}$$

Substituting this in the above equation, the value of u_1 is

$$6 = \frac{0.05}{0.50}u_1 \quad \text{or} \quad u_1 = \frac{0.50}{0.05} \times 6 = 10 \times 6$$

$$u_1 = 60\,ms^{-1}$$

4.4.4 Loss of kinetic energy in perfect inelastic collision

In perfectly inelastic collision, the loss in kinetic energy during collision is transformed to another form of energy like sound, thermal, heat, light etc. Let KE_i be the total kinetic energy before collision and KE_f be the total kinetic energy after collision.

Total kinetic energy before collision,

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$$KE_i = \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 \qquad (4.64)$$

Total kinetic energy after collision,

$$KE_f = \frac{1}{2}(m_1 + m_2)v^2 \qquad (4.65)$$

Then the loss of kinetic energy is
Loss of KE, $\Delta Q = KE_i - KE_f$

$$\Delta Q = \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 - \frac{1}{2}(m_1 + m_2)v^2 \qquad (4.66)$$

Substituting equation (4.63) in equation (4.66), and on simplifying (expand v by using the algebra $(a+b)^2 = a^2 + b^2 + 2ab$, we get

$$\text{Loss of KE, } \Delta Q = \frac{1}{2}\left(\frac{m_1m_2}{m_1 + m_2}\right)(u_1 - u_2)^2 \qquad (4.67)$$

4.4.5 Coefficient of restitution (e)

Suppose we drop a rubber ball and a plastic ball on the same floor. The rubber ball will bounce back higher than the plastic ball. This is because the loss of kinetic energy for an elastic ball is much lesser than the loss of kinetic energy for a plastic ball. The amount of kinetic energy after the collision of two bodies, in general, can be measured through a dimensionless number called the coefficient of restitution (COR).

It is defined as the ratio of velocity of separation (relative velocity) after collision to the velocity of approach (relative velocity) before collision, i.e.,

$$e = \frac{\text{velocity of separation (after collision)}}{\text{velocity of approach (before collision)}} = \frac{(v_2 - v_1)}{(u_1 - u_2)} \qquad (4.68)$$

In an elastic collision, we have obtained the velocity of separation is equal to the velocity of approach i.e.,

$$(u_1 - u_2) = (v_2 - v_1) \rightarrow e = \frac{(v_2 - v_1)}{(u_1 - u_2)} = 1$$

This implies that, coefficient of restitution for an elastic collision, $e = 1$. Physically, it means that there is no loss of kinetic energy after the collision. So, the body bounces back with the same kinetic energy which is usually called as perfect elastic.

In any real collision problems, there will be some losses in kinetic energy due to collision, which means e is not always equal to unity. If the ball is perfectly plastic, it will never bounce back and therefore their separation of velocity is zero after the collision. Hence, the value of coefficient of restitution, $e = 0$.

In general, the coefficient of restitution for a material lies between $0 < e < 1$.

EXAMPLE 4.22

Show that the ratio of velocities of equal masses in an inelastic collision when one of the masses is stationary is $\frac{v_1}{v_2} = \frac{1-e}{1+e}$.

Solution

$$e = \frac{\text{velocity of separation (after collision)}}{\text{velocity of approach (before collision)}}$$

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$$= \frac{(v_2 - v_1)}{(u_1 - u_2)} = \frac{(v_2 - v_1)}{(u_1 - 0)} = \frac{(v_2 - v_1)}{u_1}$$

$$\Rightarrow v_2 - v_1 = e\,u_1 \qquad (1)$$

From the law of conservation of linear momentum,

$$m\,u_1 = m\,v_1 + m\,v_2 \Rightarrow u_1 = v_1 + v_2 \quad (2)$$

Using the equation (2) for u_1 in (1), we get

$$v_2 - v_1 = e(v_1 + v_2)$$

On simplification, we get

$$\frac{v_1}{v_2} = \frac{1-e}{1+e}$$

SUMMARY

- When a force $\vec{F}$ acting on an object displaces it by $d\vec{r}$, then the work done(W) by the force is $W = \vec{F}.d\vec{r} = Fdr\cos\theta$.
- The work done by the variable force is defined by $\int_i^f \vec{F}.d\vec{r}$
- Work-kinetic energy theorem: The work done by a force on the object is equal to the change in its kinetic energy.
- The kinetic energy can also be defined in terms of momentum which is given by $K.E = \frac{p^2}{2m}$.
- The potential energy at a point P is defined as the amount of work required to move the object from some reference point O to the point P with constant velocity. It is given by $U = \int_O^P \vec{F}_{ext}.d\vec{r}$. The reference point can be taken as zero potential energy.
- The gravitational potential energy at a height h is given by $U = mgh$. When the elongation or compression is x, the spring potential energy is given by $U = \frac{1}{2}kx^2$. Here k is spring constant.
- The work done by a conservative force around the closed path is zero and for a non-conservative force it is not zero.
- The gravitational force, spring force and Coulomb force are all conservative but frictional force is non-conservative.
- In the conservative force field, the total energy of the object is conserved.
- In the vertical circular motion, the minimum speed required by the mass to complete the circle is $\sqrt{5gr}$. Where r, is the radius of the circle.
- Power is defined as the rate of work done or energy delivered. It is equal to $P = \frac{W}{t} = \vec{F}.\vec{v}$
- The total linear momentum of the system is always conserved for both the elastic and inelastic collisions.
- The kinetic energy of the system is conserved in elastic collisions.
- The coefficient of restitution = $\frac{velocity\ of\ separation(after\ collision)}{velocity\ of\ approach(before\ collision)}$

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CONCEPT MAP

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EVALUATION

I. Multiple Choice Questions

1. A uniform force of $(2\hat{i}+\hat{j})$ N acts on a particle of mass 1 kg. The particle displaces from position $(3\hat{j}+\hat{k})$ m to $(5\hat{i}+3\hat{j})$ m. The work done by the force on the particle is

(AIPMT model 2013)

(a) 9 J (b) 6 J

(c) 10 J (d) 12 J

2. A ball of mass 1 kg and another of mass 2 kg are dropped from a tall building whose height is 80 m. After, a fall of 40 m each towards Earth, their respective kinetic energies will be in the ratio of

(AIPMT model 2004)

(a) $\sqrt{2}:1$ (b) $1:\sqrt{2}$

(c) $2:1$ (d) $1:2$

3. A body of mass 1 kg is thrown upwards with a velocity 20 m s^{-1}. It momentarily comes to rest after attaining a height of 18 m. How much energy is lost due to air friction?.

(Take $g=10ms^{-2}$) (AIPMT 2009)

(a) 20 J (b) 30 J

(c) 40 J (d) 10 J

4. An engine pumps water continuously through a hose. Water leaves the hose with a velocity v and m is the mass per unit length of the water of the jet. What is the rate at which kinetic energy is imparted to water ?.

(AIPMT 2009)

(a) $\frac{1}{2}mv^3$ (b) mv^3

(c) $\frac{3}{2}mv^2$ (d) $\frac{5}{2}mv^2$

5. A body of mass 4 m is lying in xy-plane at rest. It suddenly explodes into three pieces. Two pieces each of mass m move perpendicular to each other with equal speed v. The total kinetic energy generated due to explosion is

(AIPMT 2014)

(a) mv^2 (b) $\frac{3}{2}mv^2$

(c) $2mv^2$ (d) $4mv^2$

6. The potential energy of a system increases, if work is done

(a) by the system against a conservative force

(b) by the system against a non-conservative force

(c) upon the system by a conservative force

(d) upon the system by a non-conservative force

7. What is the minimum velocity with which a body of mass m must enter a vertical loop of radius R so that it can complete the loop?.

(a) $\sqrt{2gR}$ (b) $\sqrt{3gR}$

(c) $\sqrt{5gR}$ (d) $\sqrt{gR}$

8. The work done by the conservative force for a closed path is

(a) always negative

(b) zero

(c) always positive

(d) not defined

D5PQK

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9. If the linear momentum of the object is increased by 0.1%, then the kinetic energy is increased by

(a) 0.1 % (b) 0.2%

(c) 0.4% (d) 0.01%

10. If the potential energy of the particle is $\alpha - \frac{\beta}{2}x^2$, then force experienced by the particle is

(a) $F = \frac{\beta}{2}x^2$ (b) $F = \beta x$

(c) $F = -\beta x$ (d) $F = -\frac{\beta}{2}x^2$

11. A wind-powered generator converts wind energy into electric energy. Assume that the generator converts a fixed fraction of the wind energy intercepted by its blades into electrical energy. For wind speed v, the electrical power output will be proportional to

(a) v (b) v^2

(c) v^3 (d) v^4

12. Two equal masses m_1 and m_2 are moving along the same straight line with velocities $5\,\text{ms}^{-1}$ and $-9\,\text{ms}^{-1}$ respectively. If the collision is elastic, then calculate the velocities after the collision of m_1 and m_2, respectively

(a) $-4\,\text{ms}^{-1}$ and $10\ \text{ms}^{-1}$

(b) 10ms^{-1} and $0\ \text{ms}^{-1}$

(c) $-9\,\text{ms}^{-1}$ and $5\ \text{ms}^{-1}$

(d) $5\ \text{ms}^{-1}$ and $1\ \text{ms}^{-1}$

13. A particle is placed at the origin and a force $F = kx$ is acting on it (where k is a positive constant). If $U(0) = 0$, the graph of $U(x)$ versus x will be (where U is the potential energy function)

(IIT 2004)

14. A particle which is constrained to move along x-axis, is subjected to a force in the same direction which varies with the distance x of the particle from the origin as $F(x) = -kx + ax^3$. Here, k and a are positive constants. For $x \geq 0$, the functional form of the potential energy $U(x)$ of the particle is

(IIT 2002)

15. A spring of force constant k is cut into two pieces such that one piece is double the length of the other. Then, the long piece will have a force constant of

(a) $\frac{2}{3}k$ (b) $\frac{3}{2}k$

(c) $3k$ (d) $6k$

Answers

1) c	**2)** d	**3)** a	**4)** a	**5)** b
6) a	**7)** c	**8)** b	**9)** b	**10)** b
11) c	**12)** c	**13)** c	**14)** d	**15)** b

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II. Short Answer Questions

1. Explain how the definition of work in physics is different from general perception.
2. Write the various types of potential energy. Explain the formulae.
3. Write the differences between conservative and Non-conservative forces. Give two examples each.
4. Explain the characteristics of elastic and inelastic collision.
5. Define the following
   a) Coefficient of restitution
   b) Power
   c) Law of conservation of energy
   d) loss of kinetic energy in inelastic collision.

III. Long Answer Questions

1. Explain with graphs the difference between work done by a constant force and by a variable force.
2. State and explain work energy principle. Mention any three examples for it.
3. Arrive at an expression for power and velocity. Give some examples for the same.
4. Arrive at an expression for elastic collision in one dimension and discuss various cases.
5. What is inelastic collision? In which way it is different from elastic collision. Mention few examples in day to day life for inelastic collision.

IV. Numerical Problems

1. Calculate the work done by a force of 30 N in lifting a load of 2kg to a height of 10m ($g = 10\ m\ s^{-2}$)

   Ans: 300J

2. A ball with a velocity of 5 ms^{-1} impinges at angle of 60° with the vertical on a smooth horizontal plane. If the coefficient of restitution is 0.5, find the velocity and direction after the impact.

   Ans: $v = 4.51\ m\ s^{-1}$

3. A bob of mass m is attached to one end of the rod of negligible mass and length r, the other end of which is pivoted freely at a fixed centre O as shown in the figure. What initial speed must be given to the object to reach the top of the circle? (Hint: Use law of conservation of energy). Is this speed less or greater than speed obtained in the section 4.2.9?

   Ans: $v = \sqrt{4gr}\ m\ s^{-1}$

4. Two different unknown masses A and B collide. A is initially at rest when B has a speed v. After collision B has a speed v/2 and moves at right angles to its original

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direction of motion. Find the direction in which A moves after collision.

Ans: $\theta = 26° 33'$

5. A bullet of mass 20 g strikes a pendulum of mass 5 kg. The centre of mass of pendulum rises a vertical distance of 10 cm. If the bullet gets embedded into the pendulum, calculate its initial speed.

Ans: $v = 351.4 ms^{-1}$

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ICT CORNER

Energy Skate

Through this activity you will understand about potential energy and kinetic energy

STEPS:

- Open the browser and type the given URL to open the PhET simulation on work power and energy. Click OK to open the activity.
- Select the height to observe the change in the kinetic energy, potential energy.
- Also observe the change by altering the mass.
- You can also create your own optional friction or playground to observe the change in potential energy and kinetic energy.

Step1

Step2

Step3

Step4

PhET simulation's URL:

https://phet.colorado.edu/en/simulation/energy-skate-park

*Pictures are indicative only.

*If browser requires, allow **Flash Player** or **Java Script** to load the page.

TDSWA

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UNIT 5

MOTION OF SYSTEM OF PARTICLES AND RIGID BODIES

"In nature, we have to deal not with material points but with material bodies ... – Max Planck

Learning Objectives

In this unit, the student is exposed to

- relevance of the centre of mass in various systems of particles
- torque and angular momentum in rotational motion
- types of equilibria with appropriate examples
- moment of inertia of different rigid bodies
- dynamics of rotation of rigid bodies
- distinguishing translational motion from rotational motion
- rolling motion, slipping and sliding motions.

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5.1 INTRODUCTION

Most of the objects that we come across in our day to day life consist of large number of particles. In the previous Units, we studied the motion of bodies without considering their size and shape. So far we have treated even the bulk bodies as only point objects. In this section, we will give importance to the size and shape of the bodies. These bodies are actually made up of a large number of particles. When such a body moves, we consider it as the motion of collection of particles as a whole. We define the concept of centre of mass to deal with such a system of particles.

The forces acting on these bulk bodies are classified into *internal* and *external forces*. Internal forces are the forces acting among the particles within a system that constitute the body. External forces are the forces acting on the particles of a system from outside. In this unit, we deal with such system of particles which make different rigid bodies. *A rigid body is the one which maintains its definite and fixed shape even when an external force acts on it*. This means that, the interatomic distances do not change in a rigid body when an external force is applied. However, in real life situation, we have bodies which are not ideally rigid, because the shape and size of the body change when forces act on them. For the rigid bodies we study here, we assume that such deformations are negligible. The deformations produced on non-rigid bodies are studied separately in Unit 7 under elasticity of solids.

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5.1.1 CENTRE OF MASS

When a rigid body moves, all particles that constitute the body need not take the same path. Depending on the type of motion, different particles of the body may take different paths. For example, when a wheel rolls on a surface, the path of the centre point of the wheel and the paths of other points of the wheel are different. In this Unit, we study about the translation, rotation and the combination of these motions of rigid bodies in detail.

5.1.2 Centre of Mass of a Rigid Body

When a bulk object (say a bat) is thrown at an angle in air as shown in Figure 5.1; do all the points of the body take a parabolic path? Actually, only one point takes the parabolic path and all the other points take different paths.

Figure 5.1 Centre of mass tracing the path of a parabola

The one point that takes the parabolic path is a very special point called *centre of mass* (CM) of the body. Its motion is like the motion of a single point that is thrown. *The centre of mass of a body is defined as a point where the entire mass of the body appears to be concentrated*. Therefore, this point can represent the entire body.

For bodies of regular shape and uniform mass distribution, the centre of mass is at the geometric centre of the body. As examples, for a circle and sphere, the centre of mass is at their centres; for square and rectangle, at the point their diagonals meet; for cube and cuboid, it is at the point where their body diagonals meet. For other bodies, the centre of mass has to be determined using some methods. The centre of mass could be well within the body and in some cases outside the body as well.

5.1.3 Centre of Mass for Distributed Point Masses

A point mass is a hypothetical point particle which has nonzero mass and no size or shape. To find the centre of mass for a collection of n point masses, say, $m_1, m_2, m_3 \ldots m_n$ we have to first choose an origin and an appropriate coordinate system as shown in Figure 5.2. Let, $x_1, x_2, x_3 \ldots x_n$ be the X-coordinates of the positions of these point masses in the X direction from the origin.

Figure 5.2 Centre of mass for distributed point masses

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The equation for the x coordinate of the centre of mass is,

$$x_{CM} = \frac{\sum m_i x_i}{\sum m_i}$$

where, $\sum m_i$ is the total mass M of all the particles, $\left(\sum m_i = M\right)$. Hence,

$$x_{CM} = \frac{\sum m_i x_i}{M} \qquad (5.1)$$

Similarly, we can also find y and z coordinates of the centre of mass for these distributed point masses as indicated in Figure (5.2).

$$y_{CM} = \frac{\sum m_i y_i}{M} \qquad (5.2)$$

$$z_{CM} = \frac{\sum m_i z_i}{M} \qquad (5.3)$$

Hence, the position of centre of mass of these point masses in a Cartesian coordinate system is (x_{CM}, y_{CM}, z_{CM}). In general, the position of centre of mass can be written in a vector form as,

$$\vec{r}_{CM} = \frac{\sum m_i \vec{r}_i}{M} \qquad (5.4)$$

where, $\vec{r}_{CM} = x_{CM}\hat{i} + y_{CM}\hat{j} + z_{CM}\hat{k}$ is the position vector of the centre of mass and $\vec{r}_i = x_i\hat{i} + y_i\hat{j} + z_i\hat{k}$ is the position vector of the distributed point mass; where, $\hat{i}$, $\hat{j}$ and $\hat{k}$ are the unit vectors along X, Y and Z-axes respectively.

5.1.4 Centre of Mass of Two Point Masses

With the equations for centre of mass, let us find the centre of mass of two point masses m_1 and m_2, which are at positions x_1 and x_2 respectively on the X-axis. For this case, we can express the position of centre of mass in the following three ways based on the choice of the coordinate system.

(i) When the masses are on positive X-axis:
The origin is taken arbitrarily so that the masses m_1 and m_2 are at positions x_1 and x_2 on the positive X-axis as shown in Figure 5.3(a). The centre of mass will also be on the positive X-axis at x_{CM} as given by the equation,

$$x_{CM} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

(ii) When the origin coincides with any one of the masses:
The calculation could be minimised if the origin of the coordinate system is made to coincide with any one of the masses as shown in Figure 5.3(b). When the origin coincides with the point mass m_1, its position x_1 is zero, (i.e. $x_1 = 0$). Then,

$$x_{CM} = \frac{m_1(0) + m_2 x_2}{m_1 + m_2}$$

The equation further simplifies as,

$$x_{CM} = \frac{m_2 x_2}{m_1 + m_2}$$

(iii) When the origin coincides with the centre of mass itself:
If the origin of the coordinate system is made to coincide with the centre of mass,

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then, $x_{CM} = 0$ and the mass m_1 is found to be on the negative X-axis as shown in Figure 5.3(c). Hence, its position x_1 is negative, (i.e. $-x_1$).

$$0 = \frac{m_1(-x_1) + m_2 x_2}{m_1 + m_2}$$

$$0 = m_1(-x_1) + m_2 x_2$$

$$m_1 x_1 = m_2 x_2$$

The equation given above is known as *principle of moments*. We will learn more about this in Section 5.3.3.

(a) When the masses are on positive X axis

(b) When the origin coincides with any one of the masses

(c) When the origin coincides with the center of mass itself

Figure 5.3 Centre of mass of two point masses determined by shifting the origin

EXAMPLE 5.1

Two point masses 3 kg and 5 kg are at 4 m and 8 m from the origin on X-axis. Locate the position of centre of mass of the two point masses (i) from the origin and (ii) from 3 kg mass.

Solution

Let us take, $m_1 = 3$ kg and $m_2 = 5$ kg

(i) To find centre of mass from the origin:

The point masses are at positions, $x_1 = 4$ m, $x_2 = 8$ m from the origin along X axis.

The centre of mass x_{CM} can be obtained using equation 5.4.

$$x_{CM} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

$$x_{CM} = \frac{(3 \times 4) + (5 \times 8)}{3 + 5}$$

$$x_{CM} = \frac{12 + 40}{8} = \frac{52}{8} = 6.5\,\text{m}$$

The centre of mass is located 6.5 m from the origin on X-axis.

(ii) To find the centre of mass from 3 kg mass:

The origin is shifted to 3 kg mass along X-axis. The position of 3 kg point mass is zero ($x_1 = 0$) and the position of 5 kg point mass is 4 m from the shifted origin ($x_2 = 4$ m).

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$$x_{CM} = \frac{(3\times 0)+(5\times 4)}{3+5}$$

$$x_{CM} = \frac{0+20}{8} = \frac{20}{8} = 2.5\,\text{m}$$

The centre of mass is located 2.5 m from 3 kg point mass, (and 1.5 m from the 5 kg point mass) on X-axis.

- This result shows that the centre of mass is located closer to larger mass.
- If the origin is shifted to the centre of mass, then the principle of moments holds good. $m_1x_1 = m_2x_2$; $3\times 2.5 = 5\times 1.5$; $7.5 = 7.5$

When we compare case (i) with case (ii), the $x_{CM} = 2.5\,\text{m}$ from 3 kg mass could also be obtained by subtracting 4 m (the position of 3 kg mass) from 6.5 m, where the centre of mass was located in case (i)

EXAMPLE 5.2

From a uniform disc of radius R, a small disc of radius $\frac{R}{2}$ is cut and removed as shown in the diagram. Find the centre of mass of the remaining portion of the disc.

Solution

Let us consider the mass of the uncut full disc be M. Its centre of mass would be at the geometric centre of the disc on which the origin coincides.

Let the mass of the small disc cut and removed be m and its centre of mass is at a position $\frac{R}{2}$ to the right of the origin as shown in the figure.

Hence, the remaining portion of the disc should have its centre of mass to the left of the origin; say, at a distance x. We can write from the principle of moments,

$$(M-m)x = (m)\frac{R}{2}$$

$$x = \left(\frac{m}{(M-m)}\right)\frac{R}{2}$$

If σ is the surface mass density (i.e. mass per unit surface area), $\sigma = \frac{M}{\pi R^2}$; then, the mass m of small disc is,

$$m = \text{surface mass density} \times \text{surface area}$$

$$m = \sigma \times \pi\left(\frac{R}{2}\right)^2$$

$$m = \left(\frac{M}{\pi R^2}\right)\pi\left(\frac{R}{2}\right)^2 = \frac{M}{\pi R^2}\pi\frac{R^2}{4} = \frac{M}{4}$$

substituting m in the expression for x

$$x = \frac{\frac{M}{4}}{\left(M-\frac{M}{4}\right)}\times\frac{R}{2} = \frac{\frac{M}{4}}{\left(\frac{3M}{4}\right)}\times\frac{R}{2}$$

$$x = \frac{R}{6}$$

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The centre of mass of the remaining portion is at a distance $\frac{R}{6}$ to the left from the centre of the disc.

- If, the small disc is removed concentrically from the large disc, what will be the position of the centre of mass of the remaining portion of disc?

EXAMPLE 5.3

The position vectors of two point masses 10 kg and 5 kg are $(-3\hat{i}+2\hat{j}+4\hat{k})$ m and $(3\hat{i}+6\hat{j}+5\hat{k})$ m respectively. Locate the position of centre of mass.

Solution

$$m_1 = 10\,kg$$

$$m_2 = 5\,kg$$

$$\vec{r}_1 = (-3\hat{i}+2\hat{j}+4\hat{k})\,m$$

$$\vec{r}_2 = (3\hat{i}+6\hat{j}+5\hat{k})\,m$$

$$\vec{r} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1+m_2}$$

$$\therefore \vec{r} = \frac{10(-3\hat{i}+2\hat{j}+4\hat{k})+5(3\hat{i}+6\hat{j}+5\hat{k})}{10+5}$$

$$= \frac{-30\hat{i}+20\hat{j}+40\hat{k}+15\hat{i}+30\hat{j}+25\hat{k}}{15}$$

$$= \frac{-15\hat{i}+50\hat{j}+65\hat{k}}{15}$$

$$\vec{r} = \left(-\hat{i}+\frac{10}{3}\hat{j}+\frac{13}{3}\hat{k}\right)\,m$$

The centre of mass is located at position $\vec{r}$.

5.1.5 Centre of mass for uniform distribution of mass

If the mass is uniformly distributed in a bulk object, then a small mass (Δm) of the body can be treated as a point mass and the summations can be done to obtain the expressions for the coordinates of centre of mass.

$$x_{CM} = \frac{\sum(\Delta m_i)x_i}{\sum \Delta m_i}$$

$$y_{CM} = \frac{\sum(\Delta m_i)y_i}{\sum \Delta m_i} \qquad (5.5)$$

$$z_{CM} = \frac{\sum(\Delta m_i)z_i}{\sum \Delta m_i}$$

On the other hand, if the small mass taken is infinitesimally* small (dm) then, the summations can be replaced by integrations as given below.

$$x_{cm} = \frac{\int x\,dm}{\int dm}$$

$$y_{cm} = \frac{\int y\,dm}{\int dm} \qquad (5.6)$$

$$z_{cm} = \frac{\int z\,dm}{\int dm}$$

EXAMPLE 5.4

Locate the centre of mass of a uniform rod of mass M and length ℓ.

Solution

Consider a uniform rod of mass M and length ℓ whose one end coincides with the origin as shown in Figure. The rod is kept along the x axis. To find the centre of mass

* Infinitesimal quantity is an extremely small quantity.

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of this rod, we choose an infinitesimally small mass dm of elemental length dx at a distance x from the origin.

λ is the linear mass density (i.e. mass per unit length) of the rod. $\lambda = \frac{M}{\ell}$

The mass of small element (dm) is, $dm = \frac{M}{\ell} dx$

Now, we can write the centre of mass equation for this mass distribution as,

$$x_{CM} = \frac{\int x dm}{\int dm}$$

$$x_{CM} = \frac{\int_0^{\ell} x\left(\frac{M}{\ell} dx\right)}{M} = \frac{1}{\ell}\int_0^{\ell} x dx$$

$$= \frac{1}{\ell}\left[\frac{x^2}{2}\right]_0^{l} = \frac{1}{\ell}\left(\frac{\ell^2}{2}\right)$$

$$x_{CM} = \frac{\ell}{2}$$

As the position $\frac{\ell}{2}$ is the geometric centre of the rod, it is concluded that the centre of mass of the uniform rod is located at its geometric centre itself.

5.1.6 Motion of Centre of Mass

When a rigid body moves, its centre of mass will also move along with the body. For kinematic quantities like velocity (v_{CM}) and acceleration (a_{CM}) of the centre of mass, we can differentiate the expression for position of centre of mass with respect to time once and twice respectively. For simplicity, let us take the motion along X direction only.

$$\vec{v}_{CM} = \frac{d\vec{x}_{CM}}{dt} = \frac{\sum m_i\left(\frac{d\vec{x}_i}{dt}\right)}{\sum m_i}$$

$$\vec{v}_{CM} = \frac{\sum m_i \vec{v}_i}{\sum m_i} \qquad (5.7)$$

$$\vec{a}_{CM} = \frac{d}{dt}\left(\frac{d\vec{x}_{CM}}{dt}\right) = \left(\frac{d\vec{v}_{CM}}{dt}\right) = \frac{\sum m_i\left(\frac{d\vec{v}_i}{dt}\right)}{\sum m_i}$$

$$\vec{a}_{CM} = \frac{\sum m_i \vec{a}_i}{\sum m_i} \qquad (5.8)$$

In the absence of external force, i.e. $\vec{F}_{ext} = 0$, the individual rigid bodies of a system can move or shift only due to the internal forces. This will not affect the position of the centre of mass. This means that the centre of mass will be in a state of rest or uniform motion. Hence, $\vec{v}_{CM}$ will be zero when centre of mass is at rest and constant when centre of mass has uniform motion $(\vec{v}_{CM} = 0$ or $\vec{v}_{CM} = \text{constant})$. There will be no acceleration of centre of mass, $(\vec{a}_{CM} = 0)$.

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From equation (5.7) and 5.8,

$$\vec{v}_{CM} = \frac{\sum m_i \vec{v}_i}{\sum m_i} = 0 \text{ (or) } \vec{v}_{CM} = \text{constant}$$

It implies

$$\vec{a}_{CM} = \frac{\sum m_i \vec{a}_i}{\sum m_i} = 0$$

Here, the individual particles may still move with their respective velocities and accelerations due to internal forces.

In the presence of external force, (i.e. $\vec{F}_{ext} \neq 0$), the centre of mass of the system will accelerate as given by the following equation.

$$\vec{F}_{ext} = \left(\sum m_i\right)\vec{a}_{CM}; \quad \vec{F}_{ext} = M\vec{a}_{CM}; \quad \vec{a}_{CM} = \frac{\vec{F}_{ext}}{M}$$

EXAMPLE 5.5

A man of mass 50 kg is standing at one end of a boat of mass 300 kg floating on still water. He walks towards the other end of the boat with a constant velocity of 2 m s^{-1} with respect to a stationary observer on land. What will be the velocity of the boat, (a) with respect to the stationary observer on land? (b) with respect to the man walking in the boat?

[Given: There is friction between the man and the boat and no friction between the boat and water.]

Solution

Mass of the man (m_1) is, m_1= 50 kg

Mass of the boat (m_2) is, m_2 = 300 kg

With respect to a stationary observer:

The man moves with a velocity, v_1 = 2 m s^{-1} and the boat moves with a velocity v_2 (which is to be found)

(i) To determine the velocity of the boat with respect to a stationary observer on land:

As there is no external force acting on the system, the man and boat move due to the friction, which is an internal force in the boat-man system. Hence, the velocity of the centre of mass is zero ($v_{CM} = 0$).

Using equation 5.7,

$$0 = \frac{\sum m_i v_i}{\sum m_i} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$$

$$0 = m_1 v_1 + m_2 v_2$$

$$-m_2 v_2 = m_1 v_1$$

$$v_2 = -\frac{m_1}{m_2} v_1$$

$$v_2 = -\frac{50}{300} \times 2 = -\frac{100}{300}$$

$$v_2 = -0.33 \text{ m s}^{-1}$$

The negative sign in the answer implies that the boat moves in a direction opposite to that of the walking man on the boat to a stationary observer on land.

(ii) To determine the velocity of the boat with respect to the walking man:

We can find the relative velocity as,

$$v_{21} = v_2 - v_1$$

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where, v_{21} is the relative velocity of the boat with respect to the walking man.

$$v_{21} = (-0.33) - (2)$$
$$v_{21} = -2.33\,ms^{-1}$$

The negative sign in the answer implies that the boat appears to move in the opposite direction to the man walking in the boat.

- The magnitude of the relative velocity of the boat with respect to the walking man is greater than the magnitude of the relative velocity of the boat with respect to the stationary observer.
- The negative signs in the two answers indicate the opposite direction of the boat with respect to the stationary observer and the walking man on the boat.

Centre of mass in explosions:

Many a times rigid bodies are broken in to fragments. If an explosion is caused by the internal forces in a body which is at rest or in motion, the state of the centre of mass is not affected. It continues to be in the same state of rest or motion. But, the kinematic quantities of the fragments get affected. If the explosion is caused by an external agency, then the kinematic quantities of the centre of mass as well as the fragments get affected.

EXAMPLE 5.6

A projectile of mass 5 kg, in its course of motion explodes on its own into two fragments. One fragment of mass 3 kg falls at three fourth of the range R of the projectile. Where will the other fragment fall?

Solution

It is an explosion of its own without any external influence. After the explosion, the centre of mass of the projectile will continue to complete the parabolic path even though the fragments are not following the same parabolic path. After the fragments have fallen on the ground, the centre of mass rests at a distance R (the range) from the point of projection as shown in the diagram.

If the origin is fixed to the final position of the centre of mass, the principle of moments holds good.

$$m_1x_1 = m_2x_2$$

where, $m_1 = 3$ kg, $m_2 = 2$ kg, $x_1 = \frac{1}{4}R$. The value of $x_2 = d$

$$3 \times \frac{1}{4}R = 2 \times d;$$
$$d = \frac{3}{8}R$$

The distance between the point of launching and the position of 2 kg mass is R+d.

$$R + d = R + \frac{3}{8}R = \frac{11}{8}R = 1.375R$$

The other fragment falls at a distance of 1.375R from the point of launching. (Here R is the range of the projectile.)

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5.2 TORQUE AND ANGULAR MOMENTUM

When a net force acts on a body, it produces linear motion in the direction of the applied force. If the body is fixed to a point or an axis, such a force rotates the body depending on the point of application of the force on the body. This ability of the force to produce rotational motion in a body is called torque or moment of force. Examples for such motion are plenty in day to day life. To mention a few; the opening and closing of a door about the hinges and turning of a nut using a wrench.

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The extent of the rotation depends on the magnitude of the force, its direction and the distance between the fixed point and the point of application. When torque produces rotational motion in a body, its angular momentum changes with respect to time. In this Section we will learn about the torque and its effect on rigid bodies.

5.2.1 Definition of Torque

Torque is defined as the moment of the external applied force about a point or axis of rotation. The expression for torque is,

$$\vec{\tau} = \vec{r} \times \vec{F} \qquad (5.9)$$

where, $\vec{r}$ is the position vector of the point where the force $\vec{F}$ is acting on the body as shown in Figure 5.4.

Figure 5.4 Torque on a rigid body

Here, the product of $\vec{r}$ and $\vec{F}$ is called the *vector product or cross product*. The vector product of two vectors results in another vector that is perpendicular to both the vectors (refer Section 2.5.2). Hence, torque $(\vec{\tau})$ is a vector quantity.

Torque has a magnitude $rF\sin\theta$ and direction perpendicular to $\vec{r}$ and $\vec{F}$. Its unit is N m.

$$\vec{\tau} = (rF\sin\theta)\hat{n} \qquad (5.10)$$

Here, θ is the angle between $\vec{r}$ and $\vec{F}$, and $\hat{n}$ is the unit vector in the direction of $\vec{\tau}$. Torque $(\vec{\tau})$ is sometimes called as a *pseudo vector* as it needs the other two vectors $\vec{r}$ and $\vec{F}$ for its existence.

The direction of torque is found using right hand rule. This rule says that if fingers of right hand are kept along the position vector with palm facing the direction of the force and when the fingers are curled the thumb points to the direction of the torque. This is shown in Figure 5.5.

The direction of torque helps us to find the type of rotation caused by the torque. For example, if the direction of torque is out

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Figure 5.5 Direction of torque using right hand rule

of the paper, then the rotation produced by the torque is anticlockwise. On the other hand, if the direction of the torque is into the paper, then the rotation is clockwise as shown in Figure 5.6.

In many cases, the direction and magnitude of the torque are found separately. For direction, we use the vector rule or right hand rule. For magnitude, we use scalar form as,

$$\tau = r\ F\ \sin\theta \qquad (5.11)$$

The expression for the magnitude of torque can be written in two different ways by associating $\sin\theta$ either with r or F in the following manner.

$$\tau = r\left(F\ \sin\theta\right) = r \times \left(F\perp\right) \qquad (5.12)$$

$$\tau = \left(r\ \sin\theta\right)F = \left(r\perp\right) \times F \qquad (5.13)$$

Here, $\left(F\sin\theta\right)$ is the component of $\vec{F}$ perpendicular to $\vec{r}$. Similarly, $\left(r\sin\theta\right)$ is the component of $\vec{r}$ perpendicular to $\vec{F}$. The two cases are shown in Figure 5.7.

Figure 5.6 Direction of torque and the type of rotation

Figure 5.7 Two ways of calculating the torque.

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Based on the angle θ between $\vec{r}$ and $\vec{F}$, the torque takes different values.

The torque is maximum when, $\vec{r}$ and $\vec{F}$ are perpendicular to each other. That is when $\theta = 90^\circ$ and $\sin 90^\circ = 1$, Hence, $\tau_{max} = rF$.

The torque is zero when $\vec{r}$ and $\vec{F}$ are parallel or antiparallel. If parallel, then $\theta = 0^\circ$ and $\sin 0^\circ = 0$. If antiparallel, then $\theta = 180^\circ$ and $\sin 180^\circ = 0$. Hence, $\tau = 0$.

The torque is zero if the force acts at the reference point. i.e. as $\vec{r} = 0$, $\tau = 0$. The different cases discussed are shown in Table 5.1.

Table 5.1 The Value of τ for different cases.

$\theta = 90^\circ;\ \tau_{max} = rF$	$\theta = 0^\circ; \tau = 0$
$\theta = 180^\circ; \tau = 0$	$r = 0; \tau = 0$

EXAMPLE 5.7

If the force applied is perpendicular to the handle of the spanner as shown in the diagram, find the (i) torque exerted by the force about the centre of the nut, (ii) direction of torque and (iii) type of rotation caused by the torque about the nut.

Solution

Arm length of the spanner, r = 15 cm $= 15\times10^{-2}$m

Force, F = 2.5 N

Angle between r and F, $\theta = 90^\circ$

(i) Torque, $\tau = rF\ \sin\theta$

$$\tau = 15\times10^{-2}\times2.5\times\sin(90^\circ)$$

[here, $\sin 90^\circ = 1$]

$$\tau = 37.5\times10^{-2}\ \text{N m}$$

(ii) As per the right hand rule, the direction of torque is out of the page.

(iii) The type of rotation caused by the torque is anticlockwise.

EXAMPLE 5.8

A force of $\left(4\hat{i} - 3\hat{j} + 5\hat{k}\right)$ N is applied at a point whose position vector is $\left(7\hat{i} + 4\hat{j} - 2\hat{k}\right)$ m. Find the torque of force about the origin.

Solution

$$\vec{r} = 7\hat{i} + 4\hat{j} - 2\hat{k}$$
$$\vec{F} = 4\hat{i} - 3\hat{j} + 5\hat{k}$$

Torque, $\vec{\tau} = \vec{r}\times\vec{F}$

$$\vec{\tau} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 7 & 4 & -2 \\ 4 & -3 & 5 \end{vmatrix}$$

$$\vec{\tau} = \hat{i}(20-6) - \hat{j}(35+8) + \hat{k}(-21-16)$$

$$\vec{\tau} = \left(14\hat{i} - 43\hat{j} - 37\hat{k}\right)\ \text{N m}$$

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EXAMPLE 5.9

A crane has an arm length of 20 m inclined at 30° with the vertical. It carries a container of mass of 2 ton suspended from the top end of the arm. Find the torque produced by the gravitational force on the container about the point where the arm is fixed to the crane. [Given: 1 ton = 1000 kg; neglect the weight of the arm. $g = 10\ m\ s^{-2}$]

Solution

Note In many problems, the angle θ between $\vec{r}$ and $\vec{F}$ will not be directly given. Thus, the students must get accustomed to identify and denote always the angle between the $\vec{r}$ and $\vec{F}$ as θ. The other angles in the arrangement may be denoted as α, β, ϕ etc.

The force F at the point of suspension is due to the weight of the hanging mass.

$$F = mg = 2 \times 1000 \times 10 = 20000\ N;$$
$$\text{The arm length, } r = 20\ m$$

We can solve this problem by three different methods.

Method – I

The angle (θ) between the arm length (r) and the force (F) is, θ = 150°

The torque (τ) about the fixed point of the arm is,

$$\tau = rF\ \sin\theta$$
$$\tau = 20 \times 20000 \times \sin(150^\circ)$$
$$= 400000 \times \sin(90^\circ + 60^\circ)$$
$$[\text{here, } \sin(90^\circ + \theta) = \cos\theta]$$
$$= 400000 \times \cos(60^\circ)$$
$$= 400000 \times \frac{1}{2} \quad \left[\cos 60^\circ = \frac{1}{2}\right]$$
$$= 200000\ N\,m$$
$$\tau = 2 \times 10^5\ N\,m$$

Method – II

Let us take the force and perpendicular distance from the point where the arm is fixed to the crane.

$$\tau = (r\perp)F$$
$$\tau = r\cos\phi\ mg$$
$$\tau = 20 \times \cos 60^\circ \times 20000$$
$$= 20 \times \frac{1}{2} \times 20000$$
$$= 200000\ Nm$$
$$\tau = 2 \times 10^5\ Nm$$

Method – III

Let us take the distance from the fixed point and perpendicular force.

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$$\tau = r(F\perp)$$

$$\tau = r\,mg\ \cos\phi$$

$$\tau = 20\times 20000\times\cos 60^{\circ}$$

$$= 20\times 20000\times\frac{1}{2}$$

$$= 200000\ \text{Nm}$$

$$\tau = 2\times 10^{5}\ \text{Nm}$$

All the three methods, give the same answer.

5.2.2 Torque about an Axis

In the earlier sections, we have dealt with the torque about a point. In this section we will deal with the torque about an axis. Let us consider a rigid body capable of rotating about an axis AB as shown in Figure 5.8. Let the force F act at a point P on the rigid body. *The force F may not be on the plane ABP*. We can take the origin O at any random point on the axis AB.

Figure 5.8 Torque about an axis

Tamil Nadu is known for creative and innovative traditional games played by children. One such very popular game is "silli" (சில்லி) or "sillukodu" (சில்லுக்கோடு). There is a rectangular area which is further partitioned as seen in the Figure. One has to hop through the rectangles. While doing so, children lean on one side, because of the reason that naturally the body takes this position to balance the gravitational force (mg) and normal force (N) acting on the body and to nullify the torque. Failing which, both these forces act along different lines leading to a net torque which makes one to fall.

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The torque of the force $\vec{F}$ about O is, $\vec{\tau} = \vec{r} \times \vec{F}$. The component of the torque $\vec{\tau}$ along the axis is the torque about the axis. To find it, we should first find the vector $\vec{\tau} = \vec{r} \times \vec{F}$ and then find the angle ϕ between $\vec{\tau}$ and the axis AB. (Remember here, the force $\vec{F}$ is not on the plane ABP). The torque about the axis AB is the parallel component of the torque along the axis AB, which is $|\vec{r} \times \vec{F}| \cos \phi$. The torque perpendicular to the axis AB is $|\vec{r} \times \vec{F}| \sin \phi$.

The torque about the axis will rotate the object about the axis and the torque perpendicular to the axis will turn or tilt the axis of rotation itself. When both components exist simultaneously on a rigid body, the body will have a *precession*. One can witness the precessional motion in a spinning top when it is about to come to rest as shown in Figure 5.9.

Figure 5.9 Precession of a spinning top

Study of precession is beyond the scope of the present course of study. Hence, it is assumed that there are constraints to cancel the effect of the perpendicular components of the torques, so that the fixed position of the axis is maintained. Therefore, perpendicular components of the torque need not be taken into account.

For the rest of the lesson, we consider rotation about only fixed axis. For this we shall,

1. *Consider forces that lie only on planes perpendicular to the axis (without intersecting in the axis).*
2. *Consider position vectors that are only perpendicular to the axis.*

Note

- Forces parallel to the axis will give torques perpendicular to the axis of rotation and need not be taken into account.
- Forces that intersect (pass through) the axis cannot produce torque as r = 0.
- Position vectors along the axis will result in torques perpendicular to the axis and need not be taken into account.

EXAMPLE 5.10

Two mutually perpendicular beams AB, CD, are joined at O to form a structure which is fixed to the ground firmly as shown in the Figure. A string is tied to the point D and its free end E is pulled with a force $\vec{F}$. Find the magnitude and direction of the torque produced by the force,

(i) about the points E, D, O and B,

(ii) about the axes DE, CD, AB and BG.

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Solution

(i) Torque about point E is zero. (as $\vec{F}$ passes through E).

Torque about point D is zero. (as $\vec{F}$ passes through D).

Torque about point O is $\left(\overrightarrow{OE}\right)\times\vec{F}$ which is perpendicular to axes AB and CD.

Torque about point B is $\left(\overrightarrow{BE}\right)\times\vec{F}$ which is perpendicular to axes AB and CD.

(ii) Torque about axis DE is zero (as $\vec{F}$ is parallel to DE).

Torque about axis CD is zero (as $\vec{F}$ intersects CD).

Torque about axis AB is zero (as $\vec{F}$ is parallel to AB).

Torque about axis BG is zero (as $\vec{F}$ intersects BG).

The torque of a force about an axis is independent of the choice of the origin as long as it is chosen on that axis itself. This can be shown as below.

Let O be the origin on the axis AB, which is the rotational axis of a rigid body. F is the force acting at the point P. Now, choose another point O' anywhere on the axis as shown in Figure 5.10.

Figure 5.10 Torque about an axis is independent of origin

Identify the direction of torque in country press shown in picture (in Tamil, 'Marasekku' மரச்செக்கு)

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The torque of F about O' is,

$$\overrightarrow{O'P}\times\vec{F}=\left(\overrightarrow{O'O}+\overrightarrow{OP}\right)\times\vec{F}$$
$$=\left(\overrightarrow{O'O}\times\vec{F}\right)+\left(\overrightarrow{OP}\times\vec{F}\right)$$

As $\overrightarrow{O'O}\times\vec{F}$ is perpendicular to $\overrightarrow{O'O}$, this term will not have a component along AB. Thus, the component of $\overrightarrow{O'P}\times\vec{F}$ is equal to that of $\overrightarrow{OP}\times\vec{F}$.

5.2.3 Torque and Angular Acceleration

Let us consider a rigid body rotating about a fixed axis. A point mass m in the body will execute a circular motion about a fixed axis as shown in Figure 5.11. A tangential force $\vec{F}$ acting on the point mass produces the necessary torque for this rotation. This force $\vec{F}$ is perpendicular to the position vector $\vec{r}$ of the point mass.

Figure 5.11 Torque and Angular acceleration

The torque produced by the force on the point mass m about the axis can be written as,

$$\tau=rF\sin 90^\circ=rF \qquad [\because \sin 90^\circ=1]$$
$$\tau=rma \qquad [\because (F=ma)]$$
$$\tau=rmr\alpha=mr^2\alpha \qquad [\because (a=r\alpha)]$$

$$\tau=\left(mr^2\right)\alpha \qquad (5.14)$$

Hence, the torque of the force acting on the point mass produces an angular acceleration (α) in the point mass about the axis of rotation.

In vector notation,

$$\vec{\tau}=\left(mr^2\right)\vec{\alpha} \qquad (5.15)$$

The directions of τ and α are along the axis of rotation. If the direction of τ is in the direction of α, it produces angular acceleration. On the other hand if, τ is opposite to α, angular deceleration or retardation is produced on the point mass.

The term mr^2 in equations 5.14 and 5.15 is called moment of inertia (I) of the point mass. A rigid body is made up of many such point masses. Hence, the moment of inertia of a rigid body is the sum of moments of inertia of all such individual point masses that constitute the body $\left(I=\sum m_i r_i^2\right)$. Hence, torque for the rigid body can be written as,

$$\vec{\tau}=\left(\sum m_i r_i^2\right)\vec{\alpha} \qquad (5.16)$$
$$\vec{\tau}=I\vec{\alpha} \qquad (5.17)$$

We will learn more about the moment of inertia and its significance for bodies with different shapes in section 5.4.

5.2.4 Angular Momentum

The angular momentum in rotational motion is equivalent to linear momentum in translational motion. The angular

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momentum of a point mass is defined as the moment of its linear momentum. In other words, the angular momentum L of a point mass having a linear momentum p at a position r with respect to a point or axis is mathematically written as,

$$\vec{L} = \vec{r} \times \vec{p} \qquad (5.18)$$

The magnitude of angular momentum could be written as,

$$L = r\,p\sin\theta \qquad (5.19)$$

where, θ is the angle between $\vec{r}$ and $\vec{p}$. $\vec{L}$ is perpendicular to the plane containing $\vec{r}$ and $\vec{p}$. As we have written in the case of torque, here also we can associate sin θ with either $\vec{r}$ or $\vec{p}$.

$$L = r\left(p\sin\theta\right) = r\left(p\perp\right) \qquad (5.20)$$

$$L = \left(r\sin\theta\right)p = \left(r\perp\right)p \qquad (5.21)$$

where, p⊥ is the component of linear momentum p perpendicular to r, and r⊥ is the component of position r perpendicular to p.

The angular momentum is zero $\left(L = 0\right)$, if the linear momentum is zero (p = 0) or if the particle is at the origin $\left(\vec{r} = 0\right)$ or if $\vec{r}$ and $\vec{p}$ are parallel or antiparallel to each other (θ = 0^0 or 180^0).

There is a misconception that the angular momentum is a quantity that is associated only with rotational motion. It is not true. The angular momentum is also associated with bodies in the linear motion. Let us understand the same with the following example.

EXAMPLE 5.11

A particle of mass (m) is moving with constant velocity (v). Show that its angular momentum about any point remains constant throughout the motion.

Solution

Let the particle of mass m move with constant velocity $\vec{v}$. As it is moving with constant velocity, its path is a straight line. Its momentum $\left(\vec{p} = m\vec{v}\right)$ is also directed along the same path. Let us fix an origin (O) at a perpendicular distance (d) from the path. At a particular instant, we can connect the particle which is at positon Q with a position vector $\left(\vec{r} = \overrightarrow{OQ}\right)$.

Take, the angle between the $\vec{r}$ and $\vec{p}$ as θ. The magnitude of angular momentum of that particle at that instant is,

$$L = OQ\,p\sin\theta = OQ\,mv\sin\theta = mv\left(OQ\sin\theta\right)$$

The term $\left(OQ\sin\theta\right)$ is the perpendicular distance (d) between the origin and line along which the mass is moving. Hence, the angular momentum of the particle about the origin is,

$$L = mvd$$

The above expression for angular momentum L, does not have the angle θ. As the momentum (p = mv) and the

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perpendicular distance (d) are constants, the angular momentum of the particle is also constant. Hence, the angular momentum is associated with bodies with linear motion also. If the straight path of the particle passes through the origin, then the angular momentum is zero, which is also a constant.

5.2.5 Angular Momentum and Angular Velocity

Let us consider a rigid body rotating about a fixed axis. A point mass m in the body will execute a circular motion about the fixed axis as shown in Figure 5.12.

Figure 5.12 Angular momentum and angular velocity

The point mass m is at a distance r from the axis of rotation. Its linear momentum at any instant is tangential to the circular path. Then the angular momentum $\vec{L}$ is perpendicular to $\vec{r}$ and $\vec{p}$. Hence, it is directed along the axis of rotation. The angle θ between $\vec{r}$ and $\vec{p}$ in this case is 90°. The magnitude of the angular momentum L could be written as,

$$L = r\,mv\sin 90^\circ = r\,mv$$

where, v is the linear velocity. The relation between linear velocity v and angular velocity ω in a circular motion is, $v = r\omega$. Hence,

$$L = rmr\omega$$

$$L = \left(mr^2\right)\omega \qquad (5.22)$$

The directions of L and ω are along the axis of rotation. The above expression can be written in the vector notation as,

$$\vec{L} = \left(mr^2\right)\vec{\omega} \qquad (5.23)$$

As discussed earlier, the term mr^2 in equations 5.22 and 5.23 is called moment of inertia (I) of the point mass. A rigid body is made up of many such point masses. Hence, the moment of inertia of a rigid body is the sum of moments of inertia of all such individual point masses that constitute the body $\left(I = \sum m_i r_i^2\right)$. Hence, the angular momentum of the rigid body can be written as,

$$\vec{L} = \left(\sum m_i r_i^2\right)\vec{\omega} \qquad (5.24)$$

$$\vec{L} = I\vec{\omega} \qquad (5.25)$$

The study about moment of inertia (I) is reserved for Section 5.4.

5.2.6 Torque and Angular Momentum

We have the expression for magnitude of angular momentum of a rigid body as, $L = I\omega$. The expression for magnitude of torque on a rigid body is, $\tau = I\alpha$

We can further write the expression for torque as,

$$\tau = I\frac{d\omega}{dt} \quad \because \left(\alpha = \frac{d\omega}{dt}\right) \qquad (5.26)$$

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Where, ω is angular velocity and α is angular acceleration. We can also write equation 5.26 as,

$$\tau = \frac{d(I\omega)}{dt}$$

$$\tau = \frac{dL}{dt} \qquad (5.27)$$

The above expression says that an external torque on a rigid body fixed to an axis produces rate of change of angular momentum in the body about that axis. This is the Newton's second law in rotational motion as it is in the form of $F = \frac{dp}{dt}$ which holds good for translational motion.

Conservation of angular momentum:
From the above expression we could conclude that in the absence of external torque, the angular momentum of the rigid body or system of particles is conserved.

$$\text{If } \tau = 0 \text{ then, } \frac{dL}{dt} = 0;\ L = \text{constant}$$

The above expression is known as law of conservation of angular momentum. We will learn about this law further in section 5.5.

5.3 EQUILIBRIUM OF RIGID BODIES

When a body is at rest without any motion on a table, we say that there is no force acting on the body. Actually it is wrong because, there is gravitational force acting on the body downward and also the normal force exerted by table on the body upward. These two forces cancel each other and thus there is no net force acting on the body. There is a lot of difference between the terms "no force" and "no net force" acting on a body. The same argument holds good for rotational conditions in terms of torque or moment of force.

A rigid body is said to be in mechanical equilibrium when both its linear momentum and angular momentum remain constant.

When the linear momentum remains constant, the net force acting on the body is zero.

$$\vec{F}_{net} = 0 \qquad (5.28)$$

In this condition, the body is said to be in translational equilibrium. This implies that the vector sum of different forces $\vec{F}_1, \vec{F}_2, \vec{F}_3 \ldots$ acting in different directions on the body is zero.

$$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \cdots + \vec{F}_n = 0 \qquad (5.29)$$

If the forces $\vec{F}_1, \vec{F}_2, \vec{F}_3 \ldots$ act in different directions on the body, we can resolve them into horizontal and vertical components and then take the resultant in the respective directions. In this case there will be horizontal as well as vertical equilibria possible.

Similarly, when the angular momentum remains constant, the net torque acting on the body is zero.

$$\vec{\tau}_{net} = 0 \qquad (5.30)$$

Under this condition, the body is said to be in rotational equilibrium. The vector sum of different torques $\vec{\tau}_1, \vec{\tau}_2, \vec{\tau}_3 \ldots$ producing different senses of rotation on the body is zero.

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$$\vec{\tau}_1 + \vec{\tau}_2 + \vec{\tau}_3 + \cdots + \vec{\tau}_n = 0 \qquad (5.31)$$

Thus, we can also conclude that a rigid body is in mechanical equilibrium when the net force and net torque acts on the body is zero.

$$\vec{F}_{net} = 0 \text{ and } \vec{\tau}_{net} = 0 \qquad (5.32)$$

As the forces and torques are vector quantities, the directions are to be taken with proper sign conventions.

5.3.1 Types of Equilibrium

Based on the above discussions, we come to a conclusion that different types of equilibrium are possible based on the different conditions. They are consolidated in Table 5.2.

Table 5.2 Different types of Equilibrium and their Conditions.

Type of equilibrium	Conditions
Translational equilibrium	▪ Linear momentum is constant. ▪ Net force is zero.
Rotational equilibrium	▪ Angular momentum is constant. ▪ Net torque is zero.
Static equilibrium	▪ Linear momentum and angular momentum are zero. ▪ Net force and net torque are zero.
Dynamic equilibrium	▪ Linear momentum and angular momentum are constant. ▪ Net force and net torque are zero.
Stable equilibrium	▪ Linear momentum and angular momentum are zero. ▪ The body tries to come back to equilibrium if slightly disturbed and released. ▪ The centre of mass of the body shifts slightly higher if disturbed from equilibrium. ▪ Potential energy of the body is minimum and it increases if disturbed.
Unstable equilibrium	▪ Linear momentum and angular momentum are zero. ▪ The body cannot come back to equilibrium if slightly disturbed and released. ▪ The centre of mass of the body shifts slightly lower if disturbed from equilibrium. ▪ Potential energy of the body is not minimum and it decreases if disturbed.
Neutral equilibrium	▪ Linear momentum and angular momentum are zero. ▪ The body remains at the same equilibrium if slightly disturbed and released. ▪ The centre of mass of the body does not shift higher or lower if disturbed from equilibrium. ▪ Potential energy remains same even if disturbed.

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EXAMPLE 5.12

Arun and Babu carry a wooden log of mass 28 kg and length 10 m which has almost uniform thickness. They hold it at 1 m and 2 m from the ends respectively. Who will bear more weight of the log? [g = 10 ms^{-2}]

Solution

Let us consider the log is in mechanical equilibrium. Hence, the net force and net torque on the log must be zero. The gravitational force acts at the centre of mass of the log downwards. It is cancelled by the normal reaction forces R_A and R_B applied upwards by Arun and Babu at points A and B respectively. These reaction forces are the weights borne by them.

The total weight, W = mg = 28 × 10 = 280 N, has to be borne by them together. The reaction forces are the weights borne by each of them separately. Let us show all the forces acting on the log by drawing a free body diagram of the log.

For translational equilibrium:

The net force acting on the log must be zero.

$$R_A + (-mg) + R_B = 0$$

Here, the forces R_A an R_B are taken positive as they act upward. The gravitational force acting downward is taken negative.

$$R_A + R_B = mg$$

For rotational equilibrium:

The net torque acting on the log must be zero. For ease of calculation, we can take the torque caused by all the forces about the point A on the log. The forces are perpendicular to the distances. Hence,

$$(0R_A) + (-4mg) + (7R_B) = 0.$$

Here, the reaction force R_A cannot produce any torque as the reaction forces pass through the point of reference A. The torque of force mg produces a clockwise turn about the point A which is taken negative and torque of force R_B causes anticlockwise turn about A which is taken positive.

$$7R_B = 4mg$$

$$R_B = \frac{4}{7}mg$$

$$R_B = \frac{4}{7} \times 28 \times 10 = 160 \text{ N}$$

By substituting for R_B we get,

$$R_A = mg - R_B$$

$$R_A = 28 \times 10 - 160 = 280 - 160 = 120 \text{ N}$$

As R_B is greater than R_A, it is concluded that Babu bears more weight than Arun. The one closer to centre of mass of the log bears more weight.

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5.3.2 Couple

Consider a thin uniform rod AB. Its centre of mass is at its midpoint C. Let two forces which are equal in magnitude and opposite in direction be applied at the two ends A and B of the rod perpendicular to it. The two forces are separated by a distance of 2r as shown in Figure 5.13.

Figure 5.13 Couple

As the two equal forces are opposite in direction, they cancel each other and the net force acting on the rod is zero. Now the rod is in translational equilibrium. But, the rod is not in rotational equilibrium. Let us see how it is not in rotational equilibrium. The moment of the force applied at the end A taken with respect to the centre point C, produces an anticlockwise rotation. Similarly, the moment of the force applied at the end B also produces an anticlockwise rotation. The moments of both the forces cause the same sense of rotation in the rod. Thus, the rod undergoes a rotational motion or turning even though the rod is in translational equilibrium.

A pair of forces which are equal in magnitude but opposite in direction and separated by a perpendicular distance so that their lines of action do not coincide that causes a turning effect is called a couple. We come across couple in many of our daily activities as shown in Figure 5.14.

Note

There are cases in which the two forces may not cancel each other. If the two forces are not equal or the direction of the forces is not exactly opposite, then the body will have both translational as well as rotational motion.

5.3.3 Principle of Moments

Consider a light rod of negligible mass which is pivoted at a point along its length. Let two parallel forces F_1 and F_2 act at the two ends at distances d_1 and d_2 from the point of pivot and the normal reaction force N at

Figure 5.14 Turning effect of Couple

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the point of pivot as shown in Figure 5.15. If the rod has to remain stationary in horizontal position, it should be in translational and rotational equilibrium. Then, both the net force and net torque must be zero.

Figure 5.15 Principle of Moments

For translational equilibrium,
net force has to be zero, $-F_1 + N - F_2 = 0$

$$N = F_1 + F_2$$

For rotational equilibrium,
net torque has to be zero, $d_1F_1 - d_2F_2 = 0$

$$d_1F_1 = d_2F_2 \qquad (5.33)$$

The above equation represents the *principle of moments*. This forms the principle for beam balance used for weighing goods with the condition $d_1 = d_2$; $F_1 = F_2$. We can rewrite the equation 5.33 as,

$$\frac{F_1}{F_2} = \frac{d_2}{d_1} \qquad (5.34)$$

If F_1 is the load and F_2 is our effort, we get advantage when, $d_1 < d_2$. This implies that $F_1 > F_2$. Hence, we could lift a large load with small effort. The ratio $\left(\frac{d_2}{d_1}\right)$ is called mechanical advantage of the simple lever. The pivoted point is called fulcrum.

$$\text{Mechanical Advantage}(MA) = \frac{d_2}{d_1} \qquad (5.35)$$

There are many simple machines that work on the above mentioned principle.

5.3.4 Centre of Gravity

Each rigid body is made up of several point masses. Such point masses experience gravitational force towards the centre of Earth. As the size of Earth is very large compared to any practical rigid body we come across in daily life, these forces appear to be acting parallelly downwards as shown in Figure 5.16.

Figure 5.16. Centre of gravity

The resultant of these parallel forces always acts through a point. This point is called centre of gravity of the body (with respect to Earth). The *centre of gravity of a body is the point at which the entire weight of the body acts irrespective of the position and orientation of the body*. The centre of gravity and centre of mass of a rigid body coincide when the gravitational field is uniform across the body. The concept of gravitational field is dealt in Unit 6.

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We can also determine the centre of gravity of a uniform lamina of even an irregular shape by pivoting it at various points by trial and error. The lamina remains horizontal when pivoted at the point where the net gravitational force acts, which is the centre of gravity as shown in Figure 5.17. When a body is supported at the centre of gravity, the sum of the torques acting on all the point masses of the rigid body becomes zero. Moreover the weight is compensated by the normal reaction force exerted by the pivot. The body is in static equilibrium and hence it remains horizontal.

Figure 5.17. Determination of centre of gravity of plane lamina by pivoting

There is also another way to determine the centre of gravity of an irregular lamina. If we suspend the lamina from different points like P, Q, R as shown in Figure 5.18, the vertical lines PP', QQ', RR' all pass through the centre of gravity. Here, reaction force acting at the point of suspension and the gravitational force acting at the centre of gravity cancel each other and the torques caused by them also cancel each other.

Figure 5.18 Determination of centre of gravity of plane lamina by suspending

5.3.5 Bending of Cyclist in Curves

Let us consider a cyclist negotiating a circular level road (not banked) of radius r with a speed v. The cycle and the cyclist are considered as one system with mass m. The centre gravity of the system is C and it goes in a circle of radius r with centre at O. Let us choose the line OC as X-axis and the vertical line through O as Z-axis as shown in Figure 5.19.

Figure 5.19 Bending of cyclist

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The system as a frame is rotating about Z-axis. The system is at rest in this rotating frame. To solve problems in rotating frame of reference, we have to apply a centrifugal force (pseudo force) on the system which will be $\frac{mv^2}{r}$. This force will act through the centre of gravity. The forces acting on the system are, (i) gravitational force (mg), (ii) normal force (N), (iii) frictional force (f) and (iv) centrifugal force $\left(\frac{mv^2}{r}\right)$. As the system is in equilibrium in the rotational frame of reference, the net external force and net external torque must be zero. Let us consider all torques about the point A in Figure 5.20.

Figure 5.20 Force diagrams for the cyclist in turns

For rotational equilibrium,

$$\vec{\tau}_{net} = 0$$

The torque due to the gravitational force about point A is $(mg\,AB)$ which causes a clockwise turn that is taken as negative. The torque due to the centrifugal force is $\left(\frac{mv^2}{r}BC\right)$ which causes an anticlockwise turn that is taken as positive.

$$-mg\,AB + \frac{mv^2}{r}BC = 0$$

$$mg\,AB = \frac{mv^2}{r}BC$$

From Δ ABC,

$AB = AC\sin\theta$ and $BC = AC\cos\theta$

$$mg\,AC\,\sin\theta = \frac{mv^2}{r}AC\cos\theta$$

$$\tan\theta = \frac{v^2}{rg}$$

$$\theta = \tan^{-1}\left(\frac{v^2}{rg}\right) \qquad (5.36)$$

While negotiating a circular level road of radius r at velocity v, a cyclist has to bend by an angle θ from vertical given by the above expression to stay in equilibrium (i.e. to avoid a fall).

EXAMPLE 5.13

A cyclist while negotiating a circular path with speed 20 m s^{-1} is found to bend an angle by 30° with vertical. What is the radius of the circular path? (given, $g = 10$ m s^{-2})

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Solution

Speed of the cyclist, $v = 20\ m\ s^{-1}$

Angle of bending with vertical, $\theta = 30^o$

Equation for angle of bending, $\tan\theta = \frac{v^2}{rg}$

Rewriting the above equation for radius

$$r = \frac{v^2}{\tan\theta\, g}$$

Substituting,

$$r = \frac{(20)^2}{(\tan 30^\circ)\times 10} = \frac{20\times 20}{(\tan 30^\circ)\times 10}$$

$$= \frac{400}{\left(\frac{1}{\sqrt{3}}\right)\times 10}$$

$$r = (\sqrt{3})\times 40 = 1.732\times 40$$

$$r = 69.28\,m$$

5.4 MOMENT OF INERTIA

In the expressions for torque and angular momentum for rigid bodies (which are considered as bulk objects), we have come across a term $\sum m_i r_i^2$. This quantity is called moment of inertia (I) of the bulk object. For point mass m_i at a distance r_i from the fixed axis, the moment of inertia is given as, $m_i r_i^2$.

Moment of inertia for point mass,

$$I = m_i r_i^2 \qquad (5.37)$$

Moment of inertia for bulk object,

$$I = \sum m_i r_i^2 \qquad (5.38)$$

In translational motion, mass is a measure of inertia; in the same way, for rotational motion, moment of inertia is a measure of rotational inertia. The unit of moment of inertia is, kg m^2. Its dimension is ML^2. In general, mass is an invariable quantity of matter (except for motion comparable to that of light). But, the moment of inertia of a body is not an invariable quantity. It depends not only on the mass of the body, but also on the way the mass is distributed around the axis of rotation.

To find the moment of inertia of a uniformly distributed mass; we have to consider an infinitesimally small mass (dm) as a point mass and take its position (r) with respect to an axis. The moment of inertia of this point mass can now be written as,

$$dI = (dm)r^2 \qquad (5.39)$$

We get the moment of inertia of the entire bulk object by integrating the above expression.

$$I = \int dI = \int (dm) r^2$$

$$I = \int r^2 dm \qquad (5.40)$$

We can use the above expression for determining the moment of inertia of some of the common bulk objects of interest like rod, ring, disc, sphere etc.

5.4.1 Moment of Inertia of a Uniform Rod

Let us consider a uniform rod of mass (M) and length (ℓ) as shown in Figure 5.21. Let us find an expression for moment of inertia of this rod about an axis that passes through the centre of mass and perpendicular to

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Figure 5.21 Moment of inertia of uniform rod

the rod. First an origin is to be fixed for the coordinate system so that it coincides with the centre of mass, which is also the geometric centre of the rod. The rod is now along the x axis. We take an infinitesimally small mass (dm) at a distance (x) from the origin. The moment of inertia (dI) of this mass (dm) about the axis is,

$$dI = (dm)x^2$$

As the mass is uniformly distributed, the mass per unit length (λ) of the rod is, $\lambda = \frac{M}{\ell}$

The (dm) mass of the infinitesimally small length as, $dm = \lambda\, dx = \frac{M}{\ell} dx$

The moment of inertia (I) of the entire rod can be found by integrating dI,

$$I = \int dI = \int (dm)x^2 = \int \left(\frac{M}{\ell} dx\right) x^2$$

$$I = \frac{M}{\ell}\int x^2 dx$$

As the mass is distributed on either side of the origin, the limits for integration are taken from $-\ell/2$ to $\ell/2$.

$$I = \frac{M}{\ell}\int_{-\ell/2}^{\ell/2} x^2 dx = \frac{M}{\ell}\left[\frac{x^3}{3}\right]_{-\ell/2}^{\ell/2}$$

$$I = \frac{M}{\ell}\left[\frac{\ell^3}{24} - \left(-\frac{\ell^3}{24}\right)\right] = \frac{M}{\ell}\left[\frac{\ell^3}{24} + \frac{\ell^3}{24}\right]$$

$$I = \frac{M}{\ell}\left[2\left(\frac{\ell^3}{24}\right)\right]$$

$$I = \frac{1}{12} M\ell^2 \qquad (5.41)$$

EXAMPLE 5.14

Find the moment of inertia of a uniform rod about an axis which is perpendicular to the rod and touches any one end of the rod.

Solution

The concepts to form the integrand to find the moment of inertia could be borrowed from the earlier derivation. Now, the origin is fixed to the left end of the rod and the limits are to be taken from 0 to ℓ.

$$I = \frac{M}{\ell}\int_0^{\ell} x^2 dx = \frac{M}{\ell}\left[\frac{x^3}{3}\right]_0^{\ell} = \frac{M}{\ell}\left[\frac{\ell^3}{3}\right]$$

$$I = \frac{1}{3} M\ell^2$$

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Note

The moment of inertia of the same uniform rod is different about different axes of reference. The reference axes could be even outside the object. We have two useful theorems to calculate the moments of inertia about different axes. We shall see these theorems in Section 5.4.5.

5.4.2 Moment of Inertia of a Uniform Ring

Let us consider a uniform ring of mass M and radius R. To find the moment of inertia of the ring about an axis passing through its centre and perpendicular to the plane, let us take an infinitesimally small mass (dm) of length (dx) of the ring. This (dm) is located at a distance R, which is the radius of the ring from the axis as shown in Figure 5.22.

Figure 5.22 Moment of inertia of a uniform ring

The moment of inertia (dI) of this small mass (dm) is,

$$dI = (dm)R^2$$

The length of the ring is its circumference $(2\pi R)$. As the mass is uniformly distributed, the mass per unit length (λ) is,

$$\lambda = \frac{\text{mass}}{\text{length}} = \frac{M}{2\pi R}$$

The mass (dm) of the infinitesimally small length is, $dm = \lambda\, dx = \frac{M}{2\pi R} dx$

Now, the moment of inertia (I) of the entire ring is,

$$I = \int dI = \int (dm)R^2 = \int \left(\frac{M}{2\pi R} dx\right) R^2$$

$$I = \frac{MR}{2\pi} \int dx$$

To cover the entire length of the ring, the limits of integration are taken from 0 to $2\pi R$.

$$I = \frac{MR}{2\pi} \int_0^{2\pi R} dx$$

$$I = \frac{MR}{2\pi} [x]_0^{2\pi R} = \frac{MR}{2\pi} [2\pi R - 0]$$

$$I = MR^2 \qquad (5.42)$$

5.4.3 Moment of Inertia of a Uniform Disc

Consider a disc of mass M and radius R. This disc is made up of many infinitesimally small rings as shown in Figure 5.23. Consider one such ring of mass (dm) and thickness (dr) and radius (r). The moment of inertia (dI) of this small ring is,

$$dI = (dm)r^2$$

As the mass is uniformly distributed, the mass per unit area (σ) is, $\sigma = \frac{\text{mass}}{\text{area}} = \frac{M}{\pi R^2}$

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Figure 5.23 Moment of inertia of a uniform disc

The mass of the infinitesimally small ring is,

$$dm = \sigma 2\pi r dr = \frac{M}{\pi R^2} 2\pi r dr$$

where, the term $(2\pi r\, dr)$ is the area of this elemental ring ($2\pi r$ is the length and dr is the thickness). $dm = \frac{2M}{R^2} r dr$

$$dI = \frac{2M}{R^2} r^3 dr$$

The moment of inertia (I) of the entire disc is,

$$I = \int dI$$

$$I = \int_0^R \frac{2M}{R^2} r^3 dr = \frac{2M}{R^2} \int_0^R r^3 dr$$

$$I = \frac{2M}{R^2}\left[\frac{r^4}{4}\right]_0^R = \frac{2M}{R^2}\left[\frac{R^4}{4} - 0\right]$$

$$I = \frac{1}{2} MR^2 \qquad (5.43)$$

5.4.4 Radius of Gyration

For bulk objects of regular shape with uniform mass distribution, the expression for moment of inertia about an axis involves their total mass and geometrical features like radius, length, breadth, which take care of the shape and the size of the objects. But, we need an expression for the moment of inertia which could take care of not only the mass, shape and size of objects, but also its orientation to the axis of rotation. Such an expression should be general so that it is applicable even for objects of irregular shape and non-uniform distribution of mass. The general expression for moment of inertia is given as,

$$I = MK^2 \qquad (5.44)$$

where, M is the total mass of the object and K is called the radius of gyration.

The radius of gyration of an object is the perpendicular distance from the axis of rotation to an equivalent point mass, which would have the same mass as well as the same moment of inertia of the object.

As the radius of gyration is distance, its unit is m. Its dimension is [L].

A rotating rigid body with respect to any axis, is considered to be made up of point masses m_1, m_2, m_3, . . .m_n at perpendicular distances (or positions) r_1, r_2, r_3 . . . r_n respectively as shown in Figure 5.24.

The moment of inertia of that object can be written as,

$$I = \sum_{i=1}^{n} m_i r_i^2 = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \cdots + m_n r_n^2$$

If we take all the n number of individual masses to be equal,

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Figure 5.24 Radius of gyration

$$m = m_1 = m_2 = m_3 = \ldots = m_n$$

then,

$$I = mr_1^2 + mr_2^2 + mr_3^2 + \cdots + mr_n^2$$

$$= m\left(r_1^2 + r_2^2 + r_3^2 + \cdots + r_n^2\right)$$

$$= nm\left(\frac{r_1^2 + r_2^2 + r_3^2 + \cdots + r_n^2}{n}\right)$$

$$I = MK^2$$

where, nm is the total mass M of the body and K is the radius of gyration.

$$K = \sqrt{\frac{r_1^2 + r_2^2 + r_3^2 + \cdots + r_n^2}{n}} \qquad (5.45)$$

The expression for radius of gyration indicates that it is the root mean square (rms) distance of the particles of the body from the axis of rotation.

In fact, the moment of inertia of any object could be expressed in the form, $I = MK^2$.

For example, let us take the moment of inertia of a uniform rod of mass M and length ℓ. Its moment of inertia with respect to a perpendicular axis passing through the centre of mass is, $I = \frac{1}{12}M\ell^2$

In terms of radius of gyration, $I = MK^2$

Hence, $MK^2 = \frac{1}{12}M\ell^2$

$$K^2 = \frac{1}{12}\ell^2$$

$$K = \frac{1}{\sqrt{12}}\ell \text{ or } K = \frac{1}{2\sqrt{3}}\ell \text{ or } K = (0.289)\ell$$

EXAMPLE 5.15

Find the radius of gyration of a disc of mass M and radius R rotating about an axis passing through the centre of mass and perpendicular to the plane of the disc.

Solution

The moment of inertia of a disc about an axis passing through the centre of mass and perpendicular to the disc is, $I = \frac{1}{2}MR^2$

In terms of radius of gyration, $I = MK^2$

Hence, $MK^2 = \frac{1}{2}MR^2$; $\quad K^2 = \frac{1}{2}R^2$

$$K = \frac{1}{\sqrt{2}}R \text{ or } K = \frac{1}{1.414}R \text{ or } K = (0.707)R$$

From the case of a rod and also a disc, we can conclude that the radius of gyration of the rigid body is always a geometrical feature like length, breadth, radius or their combinations with a positive numerical value multiplied to it.

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Obesity, torque and Moment of Inertia!

Obesity and associated ailments like back pain, joint pain etc. are due to the shift in centre of mass of the body. Due to this shift in centre of mass, unbalanced torque acting on the body leads to ailments. As the mass is spread away from centre of the body the moment of inertia is more and turning will also be difficult.

5.4.5 Theorems of Moment of Inertia

As the moment of inertia depends on the axis of rotation and also the orientation of the body about that axis, it is different for the same body with different axes of rotation. We have two important theorems to handle the case of shifting the axis of rotation.

(i) Parallel axis theorem:

Parallel axis theorem states that the moment of inertia of a body about any axis is equal to the sum of its moment of inertia about a parallel axis through its centre of mass and the product of the mass of the body and the square of the perpendicular distance between the two axes.

If I_C is the moment of inertia of the body of mass M about an axis passing through the centre of mass, then the moment of inertia I about a parallel axis at a distance d from it is given by the relation,

$$I = I_C + Md^2 \qquad (5.46)$$

Let us consider a rigid body as shown in Figure 5.25. Its moment of inertia about an axis AB passing through the centre of mass is I_C. DE is another axis parallel to AB at a perpendicular distance d from AB. The moment of inertia of the body about DE is I. We attempt to get an expression for I in terms of I_C. For this, let us consider a point mass m on the body at position x from its centre of mass.

Figure 5.25 Parallel axis theorem

The moment of inertia of the point mass about the axis DE is, $m(x+d)^2$.

The moment of inertia I of the whole body about DE is the summation of the above expression.

$$I = \sum m(x+d)^2$$

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This equation could further be written as,

$$I = \sum m\left(x^2 + d^2 + 2xd\right)$$

$$I = \sum\left(mx^2 + md^2 + 2dmx\right)$$

$$I = \sum mx^2 + \sum md^2 + 2d\sum mx$$

Here, $\sum mx^2$ is the moment of inertia of the body about the centre of mass. Hence, $I_C = \sum mx^2$

The term, $\sum mx = 0$ because, x can take positive and negative values with respect to the axis AB. The summation $\left(\sum mx\right)$ will be zero.

Thus, $I = I_C + \sum md^2 = I_C + \left(\sum m\right)d^2$

Here, $\sum m$ is the entire mass M of the object $\left(\sum m = M\right)$

$$I = I_C + Md^2$$

Hence, the parallel axis theorem is proved.

(ii) Perpendicular axis theorem:

This perpendicular axis theorem holds good only for plane laminar objects.

The theorem states that the moment of inertia of a plane laminar body about an axis perpendicular to its plane is equal to the sum of moments of inertia about two perpendicular axes lying in the plane of the body such that all the three axes are mutually perpendicular and have a common point.

Let the X and Y-axes lie in the plane and Z-axis perpendicular to the plane of the laminar object. If the moments of inertia of the body about X and Y-axes are I_X and I_Y respectively and I_Z is the moment of inertia about Z-axis, then the perpendicular axis theorem could be expressed as,

$$I_Z = I_X + I_Y \qquad (5.47)$$

To prove this theorem, let us consider a plane laminar object of negligible thickness on which lies the origin (O). The X and Y-axes lie on the plane and Z-axis is perpendicular to it as shown in Figure 5.26. The lamina is considered to be made up of a large number of particles of mass m. Let us choose one such particle at a point P which has coordinates (x, y) at a distance r from O.

Figure 5.26 Perpendicular axis theorem

The moment of inertia of the particle about Z-axis is, mr^2

The summation of the above expression gives the moment of inertia of the entire lamina about Z-axis as, $I_Z = \sum mr^2$

Here, $r^2 = x^2 + y^2$

Then, $I_Z = \sum m\left(x^2 + y^2\right)$

$$I_Z = \sum mx^2 + \sum my^2$$

In the above expression, the term $\sum mx^2$ is the moment of inertia of the body about the

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Y-axis and similarly the term $\sum my^2$ is the moment of inertia about X-axis. Thus,

$$I_X = \sum my^2 \quad \text{and} \quad I_Y = \sum mx^2$$

Substituting in the equation for I_z gives,

$$I_Z = I_X + I_Y$$

Thus, the perpendicular axis theorem is proved.

TJ7QM

EXAMPLE 5.16

Find the moment of inertia of a disc of mass 3 kg and radius 50 cm about the following axes.

(i) axis passing through the centre and perpendicular to the plane of the disc,

(ii) axis touching the edge and perpendicular to the plane of the disc and

(iii) axis passing through the centre and lying on the plane of the disc.

Solution

The mass, M = 3 kg, radius R = 50 cm = 50×10^{-2} m = 0.5 m

(i) The moment of inertia (I) about an axis passing through the centre and perpendicular to the plane of the disc is,

$$I = \frac{1}{2}MR^2$$

$$I = \frac{1}{2} \times 3 \times (0.5)^2 = 0.5 \times 3 \times 0.5 \times 0.5$$

$$I = 0.375\,\text{kg m}^2$$

(ii) The moment of inertia (I) about an axis touching the edge and perpendicular to the plane of the disc by parallel axis theorem is,

$$I = I_C + M d^2$$

where, $I_C = \frac{1}{2}MR^2$ and $d = R$

$$I = \frac{1}{2}MR^2 + MR^2 = \frac{3}{2}MR^2$$

$$I = \frac{3}{2} \times 3 \times (0.5)^2 = 1.5 \times 3 \times 0.5 \times 0.5$$

$$I = 1.125\,\text{kg m}^2$$

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(iii) The moment of inertia (I) about an axis passing through the centre and lying on the plane of the disc is,

$$I_Z = I_X + I_Y$$

where, $I_X = I_Y = I$ and $I_Z = \frac{1}{2}MR^2$

$$I_Z = 2I;\ I = \frac{1}{2}I_Z$$

$$I = \frac{1}{2} \times \frac{1}{2}MR^2 = \frac{1}{4}MR^2$$

$$I = \frac{1}{4} \times 3 \times (0.5)^2 = 0.25 \times 3 \times 0.5 \times 0.5$$

$$I = 0.1875\,\text{kg m}^2$$

- About which of the above axis it is easier to rotate the disc?
- It is easier to rotate the disc about an axis about which the moment of inertia is the least. Hence, it is case (iii).

EXAMPLE 5.17

Find the moment of inertia about the geometric centre of the given structure made up of one thin rod connecting two similar solid spheres as shown in Figure.

Solution

The structure is made up of three objects; one thin rod and two solid spheres.

The mass of the rod, M = 3 kg and the total length of the rod, ℓ = 80 cm = 0.8 m

The moment of inertia of the rod about its centre of mass is, $I_{rod} = \frac{1}{12}M\ell^2$

$$I_{rod} = \frac{1}{12} \times 3 \times (0.8)^2 = \frac{1}{4} \times 0.64$$

$$I_{rod} = 0.16\,\text{kg m}^2$$

The mass of the sphere, M = 5 kg and the radius of the sphere, R = 10 cm = 0.1 m

The moment of inertia of the sphere about its centre of mass is, $I_C = \frac{2}{5}MR^2$

The moment of inertia of the sphere about geometric centre of the structure is, $I_{sph} = I_C + Md^2$

Where, d = 40 cm + 10 cm = 50 cm = 0.5 m

$$I_{sph} = \frac{2}{5}MR^2 + Md^2$$

$$I_{sph} = \frac{2}{5} \times 5 \times (0.1)^2 + 5 \times (0.5)^2$$

$$I_{sph} = (2 \times 0.01) + (5 \times 0.25) = 0.02 + 1.25$$

$$I_{sph} = 1.27\,\text{kg m}^2$$

As there are one rod and two similar solid spheres we can write the total moment of inertia (I) of the given geometric structure as, $I = I_{rod} + (2 \times I_{sph})$

$$I = (0.16) + (2 \times 1.27) = 0.16 + 2.54$$

$$I = 2.7\,\text{kg m}^2$$

5.4.6 Moment of Inertia of Different Rigid Bodies

The moment of inertia of different objects about different axes is given in the Table 5.3.

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5.5 ROTATIONAL DYNAMICS

The relations among torque, angular acceleration, angular momentum, angular velocity and moment of inertia were seen in Section 5.2. In continuation to that, in this section, we will learn the relations among the other dynamical quantities like work, kinetic energy in rotational motion of rigid bodies. Finally a comparison between the translational and rotational quantities is made with a tabulation.

5.5.1 Effect of Torque on Rigid Bodies

A rigid body which has non zero external torque (τ) about the axis of rotation would have an angular acceleration (α) about that axis. The scalar relation between the torque and angular acceleration is,

$$\tau = I\alpha \qquad (5.48)$$

where, I is the moment of inertia of the rigid body. The torque in rotational motion is equivalent to the force in linear motion.

EXAMPLE 5.18

A disc of mass 500 g and radius 10 cm can freely rotate about a fixed axis as shown in figure. light and inextensible string is wound several turns around it and 100 g body is suspended at its free end. Find the acceleration of this mass. [Given: The string makes the disc to rotate and does not slip over it. $g = 10\ m\ s^{-2}$.]

Solution

Let the mass of the disc be m_1 and its radius R. The mass of the suspended body is m_2.

$$m_1 = 500\ g = 500\times10^{-3}\ kg = 0.5\ kg$$
$$m_2 = 100\ g = 100\times10^{-3}\ kg = 0.1\ kg$$
$$R = 10\ cm = 10\times10^{-2}\ m = 0.1\ m$$

As the light inextensible string is wound around the disc several times it makes the disc rotate without slipping over it. The translational acceleration of m_2 and tangential acceleration of m_1 will be the same. Let us draw the free body diagram (*FBD*) of m_1 and m_2 separately.

FBD of the disc:

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Table 5.3 Moment of Inertia of Different Rigid Bodies

No.	Object	About an axis	Diagram	Moment of Inertia (I) kg m^2	Radius of Gyration (K)	Ratio $\left(\frac{K^2}{R^2}\right)$
1.	**Thin Uniform Rod** Mass = M Length = ℓ	Passing through the centre and perpendicular to the length		$\frac{1}{12}M\ell^2$	$\frac{\ell}{\sqrt{12}}$	--
		Touching one end and perpendicular to the length		$\frac{1}{3}M\ell^2$	$\frac{\ell}{\sqrt{3}}$	
2.	**Thin Uniform Rectangular Sheet** Mass = M; Length = ℓ; Breadth = b	Passing through the centre and perpendicular to the plane of the sheet		$\frac{1}{12}M(\ell^2+b^2)$	$\sqrt{\frac{(\ell^2+b^2)}{12}}$	--
3.	Thin Uniform Ring Mass = M Radius = R	Passing through the centre and perpendicular to the plane		MR^2	R	1
		Touching the edge perpendicular to the plane (perpendicular tangent)		$2MR^2$	$\left(\sqrt{2}\right)R$	2
		Passing through the centre lying on the plane (along diameter)		$\frac{1}{2}MR^2$	$\left(\frac{1}{\sqrt{2}}\right)R$	$\frac{1}{2}$
		Touching the edge parallel to the plane (parallel tangent)		$\frac{3}{2}MR^2$	$\left(\sqrt{\frac{3}{2}}\right)R$	$\frac{3}{2}$
4.	**Thin Uniform Disc** Mass = M Radius = R	Passing through the centre and perpendicular to the plane		$\frac{1}{2}MR^2$	$\left(\frac{1}{\sqrt{2}}\right)R$	$\frac{1}{2}$
		Touching the edge perpendicular to the plane (perpendicular tangent to the plane)		$\frac{3}{2}MR^2$	$\left(\sqrt{\frac{3}{2}}\right)R$	$\frac{3}{2}$
		Passing through the centre lying on the plane (along diameter)		$\frac{1}{4}MR^2$	$\left(\frac{1}{2}\right)R$	$\frac{1}{4}$
		Touching the edge parallel to the plane (parallel tangent to the plane)		$\frac{5}{4}MR^2$	$\left(\sqrt{\frac{5}{4}}\right)R$	$\frac{5}{4}$

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5.	**Thin Uniform Hollow Cylinder** Mass = M Length = ℓ; Radius = R	Passing through the centre and along the axis of the cylinder		MR^2	R	1
		Passing perpendicular to the length and passing through the centre		$M\left(\frac{R^2}{2}+\frac{\ell^2}{12}\right)$	$\sqrt{\frac{R^2}{2}+\frac{\ell^2}{12}}$	- -
6.	Uniform Solid Cylinder Mass = M Length = ℓ; Radius = R	Passing through the centre and along the axis of the cylinder		$\frac{1}{2}MR^2$	$\left(\frac{1}{\sqrt{2}}\right)R$	$\frac{1}{2}$
		Passing perpendicular to the length and passing through the centre		$M\left(\frac{R^2}{4}+\frac{\ell^2}{12}\right)$	$\sqrt{\frac{R^2}{4}+\frac{\ell^2}{12}}$	- -
7.	Thin Hollow Sphere (Thin Spherical Shell) Mass = M Radius = R	Passing through the centre (along diameter)		$\frac{2}{3}MR^2$	$\left(\sqrt{\frac{2}{3}}\right)R$	$\frac{2}{3}$
		Touching the edge (tangent)		$\frac{5}{3}MR^2$	$\left(\sqrt{\frac{5}{3}}\right)R$	$\frac{5}{3}$
8.	Uniform Solid Sphere Mass = M Radius = R	Passing through the centre (along diameter)		$\frac{2}{5}MR^2$	$\left(\sqrt{\frac{2}{5}}\right)R$	$\frac{2}{5}$
		Touching the edge (tangent)		$\frac{7}{5}MR^2$	$\left(\sqrt{\frac{7}{5}}\right)R$	$\frac{7}{5}$

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Its gravitational force (m_1g) acts downward and normal force N exerted by the fixed support at the centre acts upward. The tension T acts downward at the edge. The gravitational force (m_1g) and the normal force (N) cancel each other. $m_1g = N$

The tension T produces a torque (R T), which produces a rotational motion in the disc with angular acceleration, $\left(\alpha = \frac{a}{R}\right)$. Here, a is the linear acceleration of a point at the edge of the disc. If the moment of inertia of the disc is I and its radius of gyration is K, then

$$RT = I\alpha; \quad RT = \left(m_1K^2\right)\frac{a}{R}$$

$$T = \left(m_1K^2\right)\frac{a}{R^2}$$

FBD of the body:

Its gravitational force (m_2g) acts downward and the tension T acts upward. As ($T < m_2g$), there is a resultant force (m_2a) acting on it downward.

$$m_2g - T = m_2a$$

Substituting for T from the equation for disc,

$$m_2g - \left(m_1K^2\right)\frac{a}{R^2} = m_2a$$

$$m_2g = \left(m_1K^2\right)\frac{a}{R^2} + m_2a$$

$$m_2g = \left[\left(m_1\frac{K^2}{R^2}\right) + m_2\right]a$$

$$a = \frac{m_2}{\left[\left(m_1\frac{K^2}{R^2}\right) + m_2\right]}g$$

The expression $\left(\frac{K^2}{R^2}\right)$ for a disc rotating about an axis passing through the centre and perpendicular to the plane is, $\frac{K^2}{R^2} = \frac{1}{2}$. (Ref Table 5.3) Now the expression for acceleration further simplifies as,

$$a = \frac{m_2}{\left[\left(\frac{m_1}{2}\right) + m_2\right]}g; \quad a = \frac{2m_2}{\left[m_1 + 2m_2\right]}g$$

substituting the values,

$$a = \frac{2 \times 0.1}{\left[0.5 + 0.2\right]} \times 10 = \frac{0.2}{0.7} \times 10$$

$$a = 2.857\,m\,s^{-2}$$

5.5.2 Conservation of Angular Momentum

When no external torque acts on the body, the net angular momentum of a rotating rigid body remains constant. This is known as law of conservation of angular momentum.

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$$\tau = \frac{dL}{dt}$$

If $\tau = 0$ then, $L = \text{constant}$ (5.49)

As the angular momentum is $L = I\omega$, the conservation of angular momentum could further be written for initial and final situations as,

$$I_i\omega_i = I_f\omega_f \text{ (or) } I\omega = \text{constant} \quad (5.50)$$

The above equations say that if I increases ω will decrease and vice-versa to keep the angular momentum constant.

There are several situations where the principle of conservation of angular momentum is applicable. One striking example is an ice dancer as shown in Figure 5.27. The dancer spins slowly when the hands are stretched out and spins faster when the hands are brought close to the body. Stretching of hands away from body increases moment of inertia, thus the angular velocity decreases resulting in slower spin. When the hands are brought close to the body, the moment of inertia decreases, and thus the angular velocity increases resulting in faster spin.

A diver while in air as in Figure 5.28 curls the body close to decrease the moment of inertia, which in turn helps to increase the number of somersaults in air.

Figure 5.28 Conservation of angular momentum for a diver

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Figure 5.27 Conservation of angular momentum for ice dancer

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EXAMPLE 5.19

A jester in a circus is standing with his arms extended on a turn table rotating with angular velocity ω. He brings his arms closer to his body so that his moment of inertia is reduced to one third of the original value. Find his new angular velocity. [Given: There is no external torque on the turn table in the given situation.]

Solution

Let the moment of inertia of the jester with his arms extended be I. As there is no external torque acting on the jester and the turn table, his total angular momentum is conserved. We can write the equation,

$$I_i\omega_i = I_f\omega_f$$

$$I_i\,\omega_i = \frac{1}{3}I_i\,\omega_f \qquad \left(\because I_f = \frac{1}{3}I_i\right)$$

$$\omega_f = 3\omega_i$$

The above result tells that the final angular velocity is three times that of initial angular velocity.

5.5.3 Work done by Torque

Let us consider a rigid body rotating about a fixed axis. Figure 5.29 shows a point P on the body rotating about an axis perpendicular to the plane of the page. A tangential force F is applied on the body.

It produces a small displacement ds on the body. The work done (dw) by the force is,

Figure 5.29 Work done by torque

$$dw = F\,ds$$

As the distance ds, the angle of rotation dθ and radius r are related by the expression,

$$ds = r\,d\theta$$

The expression for work done now becomes,

$$dw = F\,ds; \quad dw = F\,r\,d\theta$$

The term (Fr) is the torque τ produced by the force on the body.

$$dw = \tau\,d\theta \qquad (5.51)$$

This expression gives the work done by the external torque τ, which acts on the body rotating about a fixed axis through an angle dθ.

The corresponding expression for work done in translational motion is,

$$dw = F\,ds$$

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5.5.4 Kinetic Energy in Rotation

Let us consider a rigid body rotating with angular velocity ω about an axis as shown in Figure 5.30. Every particle of the body will have the same angular velocity ω and different tangential velocities v based on its positions from the axis of rotation.

Figure 5.30 Kinetic energy in rotation

Let us choose a particle of mass m_i situated at distance r_i from the axis of rotation. It has a tangential velocity v_i given by the relation, $v_i = r_i\,\omega$. The kinetic energy KE_i of the particle is,

$$KE_i = \frac{1}{2} m_i v_i^2$$

Writing the expression with the angular velocity,

$$KE_i = \frac{1}{2} m_i (r_i\omega)^2 = \frac{1}{2}\left(m_i r_i^2\right)\omega^2$$

For the kinetic energy of the whole body, which is made up of large number of such particles, the equation is written with summation as,

$$KE = \frac{1}{2}\left(\sum m_i r_i^2\right)\omega^2$$

where, the term $\sum m_i r_i^2$ is the moment of inertia I of the whole body. $I = \sum m_i r_i^2$

Hence, the expression for KE of the rigid body in rotational motion is,

$$KE = \frac{1}{2} I\omega^2 \qquad (5.52)$$

This is analogous to the expression for kinetic energy in translational motion.

$$KE = \frac{1}{2} Mv^2$$

Relation between rotational kinetic energy and angular momentum

Let a rigid body of moment of inertia I rotate with angular velocity ω.

The angular momentum of a rigid body is, $L = I\omega$

The rotational kinetic energy of the rigid body is, $KE = \frac{1}{2} I\omega^2$

By multiplying the numerator and denominator of the above equation with I, we get a relation between L and KE as,

$$KE = \frac{1}{2}\frac{I^2\omega^2}{I} = \frac{1}{2}\frac{(I\omega)^2}{I}$$

$$KE = \frac{L^2}{2I} \qquad (5.53)$$

EXAMPLE 5.20

Find the rotational kinetic energy of a ring of mass 9 kg and radius 3 m rotating with 240 rpm about an axis passing through its centre and perpendicular to its plane. (rpm is a unit of speed of rotation which means revolutions per minute)

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Solution

The rotational kinetic energy is, $KE = \frac{1}{2} I\omega^2$

The moment of inertia of the ring is, $I = MR^2$

$$I = 9 \times 3^2 = 9 \times 9 = 81\,\text{kg m}^2$$

The angular speed of the ring is,

$$\omega = 240\,\text{rpm} = \frac{240 \times 2\pi}{60}\,\text{rad s}^{-1}$$

$$KE = \frac{1}{2} \times 81 \times \left(\frac{240 \times 2\pi}{60}\right)^2 = \frac{1}{2} \times 81 \times (8\pi)^2$$

$$KE = \frac{1}{2} \times 81 \times 64 \times (\pi)^2 = 2592 \times (\pi)^2$$

$$KE \approx 25920\,\text{J} \quad \because (\pi)^2 \approx 10$$

$$KE = 25.920\,\text{kJ}$$

5.5.5 Power Delivered by Torque

Power delivered is the work done per unit time. If we differentiate the expression for work done with respect to time, we get the instantaneous power (P).

$$P = \frac{dw}{dt} = \tau \frac{d\theta}{dt} \quad \because (dw = \tau d\theta)$$

$$P = \tau\omega \qquad (5.54)$$

The analogous expression for instantaneous power delivered in translational motion is,

$$P = \vec{F} \cdot \vec{v}$$

5.5.6 Comparison of Translational and Rotational Quantities

Many quantities in rotational motion have expressions similar to that of translational motion. The rotational terms are compared with the translational equivalents in Table 5.4.

Table 5.4 Comparison of Translational and Rotational Quantities

S.No	Translational Motion	Rotational motion about a fixed axis
1	Displacement, x	Angular displacement, θ
2	Time, t	Time, t
3	Velocity, $v = \frac{dx}{dt}$	Angular velocity, $\omega = \frac{d\theta}{dt}$
4	Acceleration, $a = \frac{dv}{dt}$	Angular acceleration, $\alpha = \frac{d\omega}{dt}$
5	Mass, m	Moment of inertia, I
6	Force, F = ma	Torque, $\tau = I\,\alpha$
7	Linear momentum, p = mv	Angular momentum, $L = I\omega$
8	Impulse, $F\,\Delta t = \Delta p$	Angular impulse, $\tau\Delta t = \Delta L$
9	Work done, w = F s	Work done, $w = \tau\,\theta$
10	Kinetic energy, $KE = \frac{1}{2} m v^2$	Kinetic energy, $KE = \frac{1}{2} I\omega^2$
11	Power, P = F v	Power, $P = \tau\omega$

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5.6 ROLLING MOTION

The rolling motion is the most commonly observed motion in daily life. The motion of wheel is an example of rolling motion. Round objects like ring, disc, sphere etc. are most suitable for rolling .

Let us study the rolling of a disc on a horizontal surface. Consider a point P on the edge of the disc. While rolling, the point undergoes translational motion along with its centre of mass and rotational motion with respect to its centre of mass.

5.6.1 Combination of Translation and Rotation

We will now see how these translational and rotational motions are related in rolling. If the radius of the rolling object is R, in one full rotation, the centre of mass is displaced by $2\pi R$ (its circumference). One would agree that not only the centre of mass, but all the points on the disc are displaced by the same $2\pi R$ after one full rotation. The only difference is that the centre of mass takes a straight path; but, all the other points undergo a path which has a combination of the translational and rotational motion. Especially the point on the edge undergoes a path of a cycloid as shown in the Figure 5.31.

As the centre of mass takes only a straight line path, its velocity v_{CM} is only translational velocity v_{TRANS} ($v_{CM} = v_{TRANS}$). All the other points have two velocities. One is the translational velocity v_{TRANS}, (which is also the velocity of centre of mass) and the other is the rotational velocity v_{ROT} ($v_{ROT} = r\omega$). Here, r is the distance of the point from the centre of mass and ω is the angular velocity. The rotational velocity v_{ROT} is perpendicular to the instantaneous position vector from the centre of mass as shown in Figure 5.32(a). The resultant of these two velocities is v. This resultant velocity v is perpendicular to the position vector from the point of contact of the rolling object

Figure 5.31 Rolling is combination of translation and rotation

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(a) with respect to centre of mass

(b) with respect to point of contact

Figure 5.32 Resultant velocity at a point

with the surface on which it is rolling as shown in Figure 5.32(b).

We shall now give importance to the point of contact. In pure rolling, the point of the rolling object which comes in contact with the surface is at momentary rest. This is the case with every point that is on the edge of the rolling object. As the rolling proceeds, all the points on the edge, one by one come in contact with the surface; remain at momentary rest at the time of contact and then take the path of the cycloid as already mentioned.

Hence, we can consider the pure rolling in two different ways.

(i) The combination of translational motion and rotational motion about the centre of mass.

(or)

(ii) The momentary rotational motion about the point of contact.

As the point of contact is at momentary rest in pure rolling, its resultant velocity v is zero (v = 0). For example, in Figure 5.33, at the point of contact, v_{TRANS} is forward (to right) and v_{ROT} is backwards (to the left).

Figure 5.33 In pure rolling, the point of contact is at rest

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That implies that, v_{TRANS} and v_{ROT} are equal in magnitude and opposite in direction ($v = v_{TRANS} - v_{ROT} = 0$). Hence, we conclude that in pure rolling, for all the points on the edge, the magnitudes of v_{TRANS} and v_{ROT} are equal ($v_{TRANS} = v_{ROT}$). As $v_{TRANS} = v_{CM}$ and $v_{ROT} = R\omega$, in pure rolling we have,

$$v_{CM} = R\,\omega \qquad (5.55)$$

We should remember the special feature of the equation 5.55. In rotational motion, as per the relation $v = r\omega$, the centre point will not have any velocity as r is zero. But in rolling motion, it suggests that the centre point has a velocity v_{CM} given by equation 5.55.

For the topmost point, the two velocities v_{TRANS} and v_{ROT} are equal in magnitude and in the same direction (to the right). Thus, the resultant velocity v is the sum of these two velocities, $v = v_{TRANS} + v_{ROT}$. In other form, $v = 2\,v_{CM}$ as shown in Figure 5.34.

Figure 5.34 Velocity of different point in pure rolling

5.6.2 Slipping and Sliding

When the round object moves, it always tends to roll on any surface which has a coefficient of friction any value greater than zero ($\mu > 0$). The friction that enabling the rolling motion is called rolling friction. In pure rolling, there is no relative motion of the point of contact with the surface. When the rolling object speeds up or slows down,

Perfect wheels were made with high precision using conventional tools when sophisticated instruments were not avaliable.

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it must accelerate or decelerate respectively. If this suddenly happens it makes the rolling object to slip or slide.

Sliding

Sliding is the case when $v_{CM} > R\omega$ (or $v_{TRANS} > v_{ROT}$). The translation is more than the rotation. This kind of motion happens when sudden break is applied in a moving vehicles, or when the vehicle enters into a slippery road. In this case, the point of contact has more of v_{TRANS} than v_{ROT}. Hence, it has a resultant velocity v in the forward direction as shown in Figure 5.35. The kinetic frictional force (f_k) opposes the relative motion. Hence, it acts in the opposite direction of the relative velocity. This frictional force reduces the translational velocity and increases the rotational velocity till they become equal and the object sets on pure rolling. Sliding is also referred as forward slipping.

Figure 5.35 Sliding

Slipping

Slipping is the case when $v_{CM} < R\omega$ (or $v_{TRANS} < v_{ROT}$). The rotation is more than the translation. This kind of motion happens when we suddenly start the vehicle from rest or the vehicle is stuck in mud. In this case, the point of contact has more of v_{ROT} than v_{TRANS}. It has a resultant velocity v in the backward direction as shown in Figure 5.36. The kinetic frictional force (f_k) opposes the relative motion. Hence it acts in the opposite direction of the relative velocity. This frictional force reduces the rotational velocity and increases the translational velocity till they become equal and the object sets pure rolling. Slipping is sometimes empahasised as backward slipping.

Figure 5.36 Slipping

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EXAMPLE 5.21

A rolling wheel has velocity of its centre of mass as 5 m s^{-1}. If its radius is 1.5 m and angular velocity is 3 rad s^{-1}, then check whether it is in pure rolling or not.

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Solution

Translational velocity (v_{TRANS}) or velocity of centre of mass, $v_{CM} = 5\ m\ s^{-1}$

The radius is, R = 1.5 m and the angular velocity is, $\omega = 3\ rad\ s^{-1}$

Rotational velocity, $v_{ROT} = R\omega$

$$v_{ROT} = 1.5 \times 3$$

$$v_{ROT} = 4.5\ m\ s^{-1}$$

As $v_{CM} > R\omega$ (or) $v_{TRANS} > R\omega$, It is not in pure rolling, but sliding.

5.6.3 Kinetic Energy in Pure Rolling

In genreal pure rolling is the combination of translational and rotational motion, we can write the total kinetic energy (KE) as the sum of kinetic energy due to translational motion (KE_{TRANS}) and kinetic energy due to rotational motion (KE_{ROT}).

$$KE = KE_{TRANS} + KE_{ROT} \qquad (5.56)$$

If the mass of the rolling object is M, the velocity of centre of mass is v_{CM}, its moment of inertia about centre of mass is I_{CM} and angular velocity is ω, then

$$KE = \frac{1}{2}Mv_{CM}^2 + \frac{1}{2}I_{CM}\omega^2 \qquad (5.57)$$

With centre of mass as reference:

The moment of inertia (I_{CM}) of a rolling object about the centre of mass is, $I_{CM} = MK^2$ and $v_{CM} = R\omega$. Here, K is radius of gyration.

$$KE = \frac{1}{2}Mv_{CM}^2 + \frac{1}{2}\left(MK^2\right)\frac{v_{CM}^2}{R^2}$$

$$KE = \frac{1}{2}Mv_{CM}^2 + \frac{1}{2}Mv_{CM}^2\left(\frac{K^2}{R^2}\right) \qquad (5.58)$$

$$KE = \frac{1}{2}Mv_{CM}^2\left(1 + \frac{K^2}{R^2}\right) \qquad (5.59)$$

With point of contact as reference:

We can also arrive at the same expression by taking the momentary rotation happening with respect to the point of contact (another approach to rolling). If we take the point of contact as O, then,

$$KE = \frac{1}{2}I_o\omega^2$$

Here, I_o is the moment of inertia of the object about the point of contact. By parallel axis theorem, $I_o = I_{CM} + MR^2$. Further we can write, $I_o = MK^2 + MR^2$. With $v_{CM} = R\omega$ or $\omega = \frac{v_{CM}}{R}$

$$KE = \frac{1}{2}\left(MK^2 + MR^2\right)\frac{v_{CM}^2}{R^2}$$

$$KE = \frac{1}{2}Mv_{CM}^2\left(1 + \frac{K^2}{R^2}\right) \qquad (5.60)$$

As the two equations 5.59 and 5.60 are the same, it is once again confirmed that the

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pure rolling problems could be solved by considering the motion as any one of the following two cases.

(i) The combination of translational motion and rotational motion about the centre of mass. (or)

(ii) The momentary rotational motion about the point of contact.

EXAMPLE 5.22

A solid sphere is undergoing pure rolling. What is the ratio of its translational kinetic energy to rotational kinetic energy?

Solution

The expression for total kinetic energy in pure rolling is,

$$KE = KE_{TRANS} + KE_{ROT}$$

For any object the total kinetic energy as per equation 5.58 and 5.59 is,

$$KE = \frac{1}{2}Mv_{CM}^2 + \frac{1}{2}Mv_{CM}^2\left(\frac{K^2}{R^2}\right)$$

$$KE = \frac{1}{2}Mv_{CM}^2\left(1+\frac{K^2}{R^2}\right)$$

Then,

$$\frac{1}{2}Mv_{CM}^2\left(1+\frac{K^2}{R^2}\right) = \frac{1}{2}Mv_{CM}^2 + \frac{1}{2}Mv_{CM}^2\left(\frac{K^2}{R^2}\right)$$

The above equation suggests that in pure rolling the ratio of total kinetic energy, translational kinetic energy and rotational kinetic energy is given as,

$$KE : KE_{TRANS} : KE_{ROT} :: \left(1+\frac{K^2}{R^2}\right) : 1 : \left(\frac{K^2}{R^2}\right)$$

Now, $KE_{TRANS} : KE_{ROT} :: 1 : \left(\frac{K^2}{R^2}\right)$

For a solid sphere, $\frac{K^2}{R^2} = \frac{2}{5}$

Then, $KE_{TRANS} : KE_{ROT} :: 1 : \frac{2}{5}$ or

$$KE_{TRANS} : KE_{ROT} :: 5 : 2$$

5.6.4 Rolling on Inclined Plane

Let us assume a round object of mass m and radius R is rolling down an inclined plane without slipping as shown in Figure 5.37. There are two forces acting on the object along the inclined plane. One is the component of gravitational force (mg sinθ) and the other is the static frictional force (f). The other component of gravitation force (mg cosθ) is cancelled by the normal force (N) exerted by the plane. As the motion is happening along the incline, we shall write the equation for motion from the free body diagram (*FBD*) of the object.

Figure 5.37 Rolling on inclined plane

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For translational motion, mg sinθ is the supporting force and f is the opposing force,

$$mg\sin\theta - f = ma \qquad (5.61)$$

For rotational motion, let us take the torque with respect to the centre of the object. Then mg sinθ cannot cause torque as it passes through it but the frictional force f can set torque of Rf.

$$Rf = I\alpha$$

By using the relation, $a = r\,\alpha$, and moment of inertia $I = mK^2$, we get,

$$Rf = mK^2\frac{a}{R}; \quad f = ma\left(\frac{K^2}{R^2}\right)$$

Now equation (5.61) becomes,

$$mg\sin\theta - ma\left(\frac{K^2}{R^2}\right) = ma$$

$$mg\sin\theta = ma + ma\left(\frac{K^2}{R^2}\right)$$

$$a\left(1+\frac{K^2}{R^2}\right) = g\sin\theta$$

After rewriting it for acceleration, we get,

$$a = \frac{g\sin\theta}{\left(1+\frac{K^2}{R^2}\right)} \qquad (5.62)$$

We can also find the expression for final velocity of the rolling object by using third equation of motion for the inclined plane. $v^2 = u^2 + 2as$. If the body starts rolling from rest, $u = 0$. When h is the vertical height of the incline, the length of the incline s is, $s = \frac{h}{\sin\theta}$

$$v^2 = 2\frac{g\sin\theta}{\left(1+\frac{K^2}{R^2}\right)}\left(\frac{h}{\sin\theta}\right) = \frac{2gh}{\left(1+\frac{K^2}{R^2}\right)}$$

By taking square root,

$$v = \sqrt{\frac{2gh}{\left(1+\frac{K^2}{R^2}\right)}} \qquad (5.63)$$

The time taken for rolling down the incline could also be written from first equation of motion as, $v = u + at$. For the object which starts rolling from rest, $u = 0$. Then,

$$t = \frac{v}{a}$$

$$t = \left(\sqrt{\frac{2gh}{\left(1+\frac{K^2}{R^2}\right)}}\right)\left(\frac{\left(1+\frac{K^2}{R^2}\right)}{g\sin\theta}\right)$$

$$t = \sqrt{\frac{2h\left(1+\frac{K^2}{R^2}\right)}{g\sin^2\theta}} \qquad (5.64)$$

The equation suggests that for a given incline, the object with the least value of radius of gyration K will reach the bottom of the incline first.

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EXAMPLE 5.23

Four round objects namely a ring, a disc, a hollow sphere and a solid sphere with same radius R start to roll down an incline at the same time. Find out which object will reach the bottom first.

Solution

For all the four objects namely the ring, disc, hollow sphere and solid sphere, the radii of gyration K are R, $\sqrt{\frac{1}{2}}$R, $\sqrt{\frac{2}{3}}$R, $\sqrt{\frac{2}{5}}$R (ref Table (5.3)). With numerical values the radius of gyration K are 1R, 0.707R, 0.816R, 0.632R respectively. The expression for time taken for rolling has the radius of gyration K in the numerator as per equation 5.63

$$t = \sqrt{\frac{2h\left(1+\frac{K^2}{R^2}\right)}{g\sin^2\theta}}$$

The one with least value of radius of gyration K will take the shortest time to reach the bottom of the inclined plane. The order of objects reaching the bottom is first, solid sphere; second, disc; third, hollow sphere and last, ring.

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SUMMARY

- A rigid body is the one in which the distances between different particles remain constant.
- For regular shaped bodies with uniform mass distribution, centre of mass always lies at the geometrical centre.
- Net torque produces turning motion in rigid object.
- A rigid body is in translational equilibrium if the total external force on it is zero. It is in rotational equilibrium if the total external torque on it is zero.
- The centre of gravity of an extended body is that point where the total gravitational torque on the body is zero.
- If the external torque acting on the body is zero, the component of angular momentum along the axis of rotation is constant.
- There are rotational equivalents for all the translational quantities.
- Rolling motion is the combination of translational and rotational motions.
- Rolling can also be treated as the momentary rotation about the point of contact.
- In pure rolling, the total kinetic energy is the sum of kinetic energies of translational and rotational motions.
- In sliding the translational motion is more than rotational motion.
- In slipping the rotational motion is more than translational motion.

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CONCEPT MAP

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EVALUATION

I. Multiple Choice Questions

1. The centre of mass of a system of particles does not depend upon,
   (a) position of particles
   (b) relative distance between particles
   (c) masses of particles
   (d) force acting on particle
   [AIPMT 1997, AIEEE 2004]

2. A couple produces,
   (a) pure rotation
   (b) pure translation
   (c) rotation and translation
   (d) no motion
   [AIPMT 1997]

3. A particle is moving with a constant velocity along a line parallel to positive X-axis. The magnitude of its angular momentum with respect to the origin is,
   (a) zero
   (b) increasing with x
   (c) decreasing with x
   (d) remaining constant
   [IIT 2002]

4. A rope is wound around a hollow cylinder of mass 3 kg and radius 40 cm. What is the angular acceleration of the cylinder if the rope is pulled with a force 30 N?
   (a) 0.25 rad s^{-2} (b) 25 rad s^{-2}
   (c) 5 m s^{-2} (d) 25 m s^{-2}.
   [NEET 2017]

5. A closed cylindrical container is partially filled with water. As the container rotates in a horizontal plane about a perpendicular bisector, its moment of inertia,
   (a) increases (b) decreases
   (c) remains constant
   (d) depends on direction of rotation.
   [IIT 1998]

6. A rigid body rotates with an angular momentum L. If its kinetic energy is halved, the angular momentum becomes,
   (a) L (b) L/2
   (c) 2L (d) $L/\sqrt{2}$
   [AFMC 1998, AIPMT 2015]

7. A particle undergoes uniform circular motion. The angular momentum of the particle remain conserved about,
   (a) the centre point of the circle.
   (b) the point on the circumference of the circle.
   (c) any point inside the circle.
   (d) any point outside the circle.
   [IIT 2003]

8. When a mass is rotating in a plane about a fixed point, its angular momentum is directed along,
   (a) a line perpendicular to the plane of rotation
   (b) the line making an angle of 45° to the plane of rotation
   (c) the radius
   (d) tangent to the path
   [AIPMT 2012]

9. Two discs of same moment of inertia rotating about their regular axis passing through centre and perpendicular to

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the plane of disc with angular velocities ω_1 and ω_2. They are brought in to contact face to face coinciding the axis of rotation. The expression for loss of energy during this process is,

(a) $\frac{1}{4}I(\omega_1-\omega_2)^2$ (b) $I(\omega_1-\omega_2)^2$

(c) $\frac{1}{8}I(\omega_1-\omega_2)^2$ (d) $\frac{1}{2}I(\omega_1-\omega_2)^2$

[NEET 2017]

10. A disc of moment of inertia I_a is rotating in a horizontal plane about its symmetry axis with a constant angular speed ω. Another discinitially at rest of moment of inertia I_b is dropped coaxially on to the rotating disc. Then, both the discs rotate with same constant angular speed. The loss of kinetic energy due to friction in this process is,

(a) $\frac{1}{2}\frac{I_b^2}{(I_a+I_b)}\omega^2$

(b) $\frac{I_b^2}{(I_a+I_b)}\omega^2$

(c) $\frac{(I_b-I_a)^2}{(I_a+I_b)}\omega^2$

(d) $\frac{1}{2}\frac{I_bI_b}{(I_a+I_b)}\omega^2$ [AIPMT 2001]

11. The ratio of the acceleration for a solid sphere (mass m and radius R) rolling down an incline of angle θ without slipping and slipping down the incline without rolling is,

(a) 5:7 (b)2:3 (c) 2:5 (d) 7:5

[AIPMT 2014]

12. From a disc of radius R a mass M, a circular hole of diameter R, whose rim passes through the centre is cut. What is the moment of inertia of the remaining part of the disc about a perpendicular axis passing through it

(a) $15MR^2/32$ (b) $13MR^2/32$

(c) $11MR^2/32$ (d) $9MR^2/32$

[NEET 2016]

13. The speed of a solid sphere after rolling down from rest without sliding on an inclined plane of vertical height h is,

(a) $\sqrt{\frac{4}{3}gh}$ (b) $\sqrt{\frac{10}{7}gh}$

(c) $\sqrt{2gh}$ (d) $\sqrt{\frac{1}{2}gh}$

14. The speed of the centre of a wheel rolling on a horizontal surface is v_o. A point on the rim in level with the centre will be moving at a speed of speed of,

(a) zero (b) v_o

(c) $\sqrt{2}\,v_o$ (d) $2v_o$

[PMT 1992, PMT 2003, IIT 2004]

15. A round object of mass M and radius R rolls down without slipping along an inclined plane. The frictional force,

(a) dissipates kinetic energy as heat.

(b) decreases the rotational motion.

(c) decreases the rotational and transnational motion

(d) converts transnational energy into rotational energy

[PMT 2005]

Answers:

1) d	2) a	3) d	4) b
5) a	6) d	7) a	8) a
9) a	10) d	11) a	12) b
13) b	14) c	15) d	

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II. Short Answer Questions

1. Define centre of mass.
2. Find out the centre of mass for the given geometrical structures.
   a) Equilateral triangle
   b) Cylinder c) Square
3. Define torque and mention its unit.
4. What are the conditions in which force can not produce torque?
5. Give any two examples of torque in day-to-day life.
6. What is the relation between torque and angular momentum?
7. What is equilibrium?
8. How do you distinguish between stable and unstable equilibrium?
9. Define couple.
10. State principle of moments.
11. Define centre of gravity.
12. Mention any two physical significance of moment of inertia
13. What is radius of gyration?
14. State conservation of angular momentum.
15. What are the rotational equivalents for the physical quantities, (i) mass and (ii) force?
16. What is the condition for pure rolling?
17. What is the difference between sliding and slipping?

III. Long Answer Questions

1. Explain the types of equilibrium with suitable examples.
2. Explain the method to find the centre of gravity of a irregularly shaped lamina.
3. Explain why a cyclist bends while negotiating a curve road? Arrive at the expression for angle of bending for a given velocity.
4. Derive the expression for moment of inertia of a rod about its centre and perpendicular to the rod.
5. Derive the expression for moment of inertia of a uniform ring about an axis passing through the centre and perpendicular to the plane.
6. Derive the expression for moment of inertia of a uniform disc about an axis passing through the centre and perpendicular to the plane.
7. Discuss conservation of angular momentum with example.
8. State and prove parallel axis theorem.
9. State and prove perpendicular axis theorem.
10. Discuss rolling on inclined plane and arrive at the expression for the acceleration.

IV. Numerical Problems

1. A uniform disc of mass 100g has a diameter of 10 cm. Calculate the total energy of the disc when rolling along a horizontal table with a velocity of 20 cms^{-1}. (take the surface of table as reference)

   Ans: 0.005 J

2. A particle of mass 5 units is moving with a uniform speed of $v = 3\sqrt{2}$ units in the XOY plane along the line y = x + 4. Find the magnitude of angular momentum.

   Ans: 60 units

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3. A fly wheel rotates with a uniform angular acceleration. If its angular velocity increases from 20π rad/s to 40π rad/s in 10 seconds. Find the number of rotations in that period.

Ans: 150 rotations

4. A uniform rod of mass m and length ℓ makes a constant angle θ with an axis of rotation which passes through one end of the rod. Find the moment of inertia about this axis.

Ans: $\frac{1}{3}M\ell^2 \sin^2\theta$

5. Two particles P and Q of mass 1kg and 3 kg respectively start moving towards each other from rest under mutual attraction. What is the velocity of their centre of mass?

Ans: Zero

6. Find the moment of inertia of a hydrogen molecule about an axis passing through its centre of mass and perpendicular to the inter-atomic axis. Given: mass of hydrogen atom 1.7×10^{-27} kg and inter atomic distance is equal to 4×10^{-10}m.

Ans: 1.36×10^{-46} kg m^2

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ICT CORNER

Moment of inertia

Which is harder to rotate:
Circular ring or Circular disc?

STEPS:

- Open the browser and type the given URL to open the PhET simulation on Torque. Click the picture link or the download button. Once downloaded, click ok to open the java applet.
- Set platform mass 0.1 kg, Outer radius 4 m. (Keep inner radius = 0). Now it is a circular disc. Click the button 'go' to get the value of moment of inertia.
- Adjust the values of mass and radius and then observe how moment of inertia changes in the middle graph.
- Keep the inner and outer radius same (say R = r = 4 m.) and mass 0.1 kg. Now it becomes circular ring. Click the button 'go' to start the calculation.
- Observe the moment of inertia from the middle graph. Compare the moment of inertia of a circular disc and circular ring with same mass and radius.

Hint: If moment of inertia is relatively large, it is very difficult to accelerate in angular direction.

Step1

Step2

Step3

Step4

PhET simulation's URL:

https://phet.colorado.edu/en/simulation/torque

* Pictures are indicative only.

*If browser requires, allow Flash Player or Java Script to load the page.

PVTKN

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APPENDIX 1

SOLVED EXAMPLE UNIT-1

1. Find the dimensions of a and b in the formula $[P + \frac{a}{V^2}][V - b] = RT$ where P is pressure and V is the volume of the gas

Solution:

By the principle of homogeneity, a/V^2 is of the dimensions of pressure and b is of the dimensions of volume.

$$[a] = [\text{pressure}]\,[V^2] = [ML^{-1}T^{-2}]\,[L^6]$$
$$= [ML^5T^{-2}]$$
$$[b] = [V] = [L^3]$$

2. Show that $(P^{-5/6}\rho^{1/2}E^{1/3})$ is of the dimension of time. Here P is the pressure, ρ is the density and E is the energy of a bubble)

Solution:

Dimension of Pressure $= [ML^{-1}T^{-2}]$

Dimension of density $= [ML^{-3}]$

Dimension of Energy $= [ML^2T^{-2}]$

By substituting in the given equation,

$$= [ML^{-1}T^{-2}]^{-5/6}[ML^{-3}]^{1/2}[ML^2T^{-2}]^{1/3}$$
$$= M^{-5/6+1/2+1/3}L^{5/6-3/2+2/3}T^{5/3-2/3}$$
$$= M^0L^0T^1 = [T]$$

3. Find the dimensions of mass in terms of Energy, length and time

Solution:

Let the dimensions of Energy, Length and Time be $[E],[L],[T]$ respectively.

We know that Force = mass x acceleration

$$\text{Mass} = \frac{Force}{acceleration}$$
$$= \frac{Workdone\ (or)\ Energy}{acceleration \times displacement}$$
$$[m] = \frac{Energy}{[acceleration][displacement]}$$
$$= \frac{[E]}{[LT^{-2}][L]} = \frac{[E]}{L^2T^{-2}} = [EL^{-2}T^2]$$

4. A physical quantity Q is found to depend on quantities x,y,z obeying

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relation Q = $\frac{x^2y^3}{z^1}$. The percentage errors in x, y and z are 2%, 3% and 1% respectively. Find the percentage error in Q.

Solution:

$$\text{Let, } Q = \frac{x^2y^3}{z}.$$

$$\text{It is given, } \frac{\Delta x}{x} = 2\% \quad \frac{\Delta y}{y} = 3\% \quad \frac{\Delta z}{z} = 1\%$$

$$\frac{\Delta Q}{Q} = 2\left(\frac{\Delta x}{x}\right) + 3\left(\frac{\Delta y}{y}\right) + 1\left(\frac{\Delta z}{z}\right)$$

$$= 2(2\%) + 3\ (3\%) + 1(1\%)$$

$$\frac{\Delta Q}{Q} = 4\% + 9\% + 1\% = 14\%$$

5. The mass and volume of a body are found to be 4±.03 kg and 5±.01 m^3 respectively. Then find the maximum possible percentage error in density.

Solution:

$$\text{Mass } m = 4 \pm 0.03 \text{ kg } (m + \Delta m)$$

$$\text{Volume } V = 5 \pm .01 \text{ m}^3\ (V + \Delta V)$$

$$\text{Density} = ?$$

$$\text{Error in mass} = \frac{\Delta m}{m} = \frac{0.03}{4} \times 100$$

$$= 0.75\%$$

$$\text{Error in volume} = \frac{\Delta V}{V} = \frac{0.01}{5} \times 100$$

$$= 0.2\%$$

$$\text{Density} = \frac{mass}{volume}.$$

Error in density = error in mass + error in volume

$$= 0.75\% + 0.2\% = 0.95\%$$

6. Using a Vernier Callipers, the length of a cylinder in different measurements is found to be 2.36 cm, 2.27 cm, 2.26 cm, 2.28 cm, 2.31 cm, 2.28 cm and 2.29 cm. Find the mean value, absolute error, the relative error and the percentage error of the cylinder.

Solution:

The given readings are 2.36 cm, 2.27 cm, 2.26 cm, 2.28 cm, 2.31 cm, 2.28 cm and 2.29 cm

The Mean value $\overline{l}$ =

$$\frac{2.36 + 2.27 + 2.26 + 2.28 + 2.31 + 2.28 + 2.29}{7}$$

$$\frac{16.05}{7} = 2.29 \text{ cm}$$

Absolute errors in the measurement are,

$$|\Delta l_1| = |2.29 - 2.36| = 0.07$$
$$|\Delta l_2| = |2.29 - 2.27| = 0.02$$
$$|\Delta l_3| = |2.29 - 2.26| = 0.03$$
$$|\Delta l_4| = |2.29 - 2.28| = 0.01$$
$$|\Delta l_5| = |2.29 - 2.31| = 0.02$$
$$|\Delta l_6| = |2.29 - 2.28| = 0.01$$
$$|\Delta l_7| = |2.29 - 2.29| = 0.00$$

Mean Absolute error

$$\Delta l_{mean} = \frac{0.07 + 0.02 + 0.03 + 0.01 + 0.02 + 0.01 + 0.00}{7}$$

$$= \frac{.16}{7} = .02$$

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Relative error

$$= \frac{\Delta l mean}{\bar{l}} = \pm\frac{.02}{2.29} = \pm 8.7\times 10^{-3}$$

Percentage error $= \pm 8.7\times 10^{-3}\times 100 =$

$0.87\%\times 100 = \pm(8.7\times 10^{-1}) = 0.9\%$

7. The shadow of a pole standing on a level ground is found to be 45 m longer when the sun's altitude is 30° than when it was 60°. Determine the height of the pole. [Given $\sqrt{3} = 1.73$]

Solution:

Let the height of the pole be h.

Solution $\frac{x+45}{h} = \cot 30° \Rightarrow h = \frac{x+45}{\cot 30°}$

$\frac{x}{h} = \cot 60° \Rightarrow x = h\cot 60°$

Substituting the values of x in the above equation

$$h = \frac{h\cot 60° + 45}{\cot 30°}$$

$$h \cot 30° = h \cot 60° + 45$$

$$h(\cot 30° - \cot 60°) = 45$$

$$h = \frac{45}{\cot 30° - \cot 60°} = \frac{45}{\sqrt{3} - \frac{1}{\sqrt{3}}} = 38.97\,\text{m}$$

8. Calculate the number of times a human heart beats in the life of 100 years old man. Time of one heart beat = 0.8s.

Solution:

Life of the man = 100 years

100 years includes 76 normal years and 24 leap years

Total no of days $= 76\times 365 + 24\times 366$ = 36524 days

Number of seconds = 36524 × 24 × 3600 = 3.155 x 10^9 second

Number of heart beats =

$$\frac{\textit{Total no of Seconds}}{\textit{Time period of heart beat}}$$

$$= \frac{3.155\times 10^9}{0.8s} = 3.94\times 10^9$$

9. The parallax of a heavenly body measured from two points diametrically opposite on equator of earth is 2′. Calculate the distance of the heavenly body. [Given radius of the earth = 6400km] [1″ = 4.85 x 10^{-6} rad]

Solution:

Angle $\theta = 2^1 = 2$ x 60″ = 120″ = 120 × 4.85 × 10^{-6} rad

$\theta = 5.82\times 10^{-4}$ rad;

The distance of heavenly body

$$D = \frac{d}{\theta} = \frac{12800\times 10^3}{5.82\times 10^{-4}}$$

$$D = 2.19\times 10^{10}\,\text{m}$$

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10. Convert a velocity of 72 kmh^{-1} into m s^{-1} with the help of dimensional analysis.

Solution:

$n_1 = 72\ kmh^{-1}$ $\quad n_2 = ?\ m\ s^{-1}$

$L_1 = 1Km$ $\quad L_2 = 1m$

$T_1 = 1h$ $\quad T_2 = 1s$

$$n_2 = n_1\left[\frac{L_1}{L_2}\right]^a\left[\frac{T_1}{T_2}\right]^b$$

The dimensional formula for velocity is $[L\ T^{-1}]$

$$a = 1\ b = -1$$

$$n_2 = 72\left[\frac{1Km}{1m}\right]^1\left[\frac{1h}{1s}\right]^{-1}$$

$$n_2 = 72\left[\frac{1000m}{1m}\right]^1\left[\frac{3600s}{1s}\right]^{-1}$$

$$= 72 \times 1000 \times 1/3600 = 20m\ s^{-1}]$$

$$72\ kmh^{-1} = 20\ m\ s^{-1}$$

11. Check the correctness of the following equation using dimensional analysis. Make a comment on it.
$S = ut + 1/4\ at^2$ where s is the displacement, u is the initial velocity, t is the time and a is the acceleration produced.

Solution:

Dimension for distance s = [L]
Dimension for initial velocity

$$v = [LT^{-1}]$$

Dimension for time t = [T]
Dimension for acceleration

$$a = [LT^{-2}]$$

According to the principle of homogeneity,

Dimensions of LHS = Dimensions of RHS

Substituting the dimensions in the given formula

$S = ut + 1/4\ at^2$, $\frac{1}{4}$ is a number. It has no dimensions

$$[L] = [LT^{-1}]\ [T^1] + [LT^{-2}]\ [T^2]$$
$$[L] = [L] + [L]$$

As the dimensional formula of LHS is same as that of RHS, the equation is dimensionally correct.
Comment:

But actually it is a ***wrong*** equation. We know that the equation of motion is $s = ut + 1/2\ at^2$

So, a dimensionally correct equation ***need not*** be the true (or) actual equation

But a true equation is ***always*** dimensionally correct.

12. Round - off the following numbers as indicated.
a) 17.234 to 3 digits
b) 3.996×10^5 to 3 digits
c) 3.6925×10^{-3} to 2 digits
d) 124783 to 5 digits.

Solution:

a) 17.2 $\quad$ b) 4.00×10^5
c) 3.7×10^{-3} $\quad$ d) 124780

13. Solve the following with regard to significant figures.
a) $\sqrt{4.5 - 3.31}$
b) $5.9 \times 10^5 - 2.3 \times 10^4$

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c) 7.18 + 4.3
d) 6.5 + .0136

Solution:

a) Among the two, the least number of significant after decimal is one

$$\sqrt{4.5-3.31} = \sqrt{1.19} = 1.09$$

b) The number of minimum significant figures is 2

$$5.9\times10^5 - 2.3\times10^4$$
$$= 5.9\times10^5 - 0.23\times10^5$$
$$= 5.67\times10^5 = 5.7\times10^5$$

c) The lowest least number of significant digit after decimal is one

7.18 + 4.3 = 11.48 Rounding off we get 11.5

d) The lowest least number of significant digit after decimal is one

$$6.5 + .0136 = 6.5136 = 6.5$$

14. Arrive at Einstein's mass-energy relation by dimensional method ($E = mc^2$)

Solution:

Let us assume that the Energy E depends on mass m and velocity of light c.

$$E \alpha m^a c^b$$

$E = km^a c^b$ where K a constant

Dimensions of E = $[ML^2T^{-2}]$

Dimensions of m = [M]

Dimensions of c = $[LT^{-1}]$

Substituting the values in the above equation

$$[ML^2T^{-2}] = K\,[M]^a[LT^{-1}]^b$$

By equating the dimensions,

a = 1
b = 2
-b = -2
$E = k.mc^2$

The value of constant k = 1

$E = mc^2$ This is Einstein's mass energy relation

15. The velocity of a body is given by the equation $v = b/t + ct^2 + dt^3$. Find the dimensional formula for b.

Solution:

(b/t) should have the dimensions of velocity
b has the dimensions of (velocity x time)

$$[b] = [LT^{-1}][T] = [L] = [M^0L^1T^0]$$

16. The initial and final temperatures of a liquid in a container are observed to be 75.4 ±0.5°C and 56.8 ±0.2°C. Find the fall in the temperature of the liquid.

Solution:

$t_1 = (75.4 \pm 0.5)°C$
$t_2 = (56.8 \pm 0.2)°C$
Fall in temperature = (75.4 ±0.5°C) – (56.8 ±0.2°C)

$$t = (18.6 \pm 0.7)°C$$

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17. Two resistors of resistances R_1 = 150 ± 2 Ohm and R_2 = 220 ± 6 Ohm are connected in parallel combination. Calculate the equivalent resistance.

Hint: $\frac{1}{R'} = \frac{1}{R_1} + \frac{1}{R_2}$

Solution:

The equivalent resistance of a parallel combination

$$R' = \frac{R_1 R_2}{R_1 + R_2}$$

$$= \frac{150 \times 220}{150 + 220} = \frac{33000}{370} = 89.1\ Ohm$$

We know that, $\frac{1}{R'} = \frac{1}{R_1} + \frac{1}{R_2}$

$$\frac{\Delta R'}{(R')^2} = \frac{\Delta R_1}{R_1^2} + \frac{\Delta R_2}{R_2^2}$$

$$\Delta R' = (R')^2 \frac{\Delta R_1}{R_1^2} + (R')^2 \frac{\Delta R_2}{R_2^2}$$

$$= \left(\frac{R'}{R_1}\right)^2 \Delta R_1 + \left(\frac{R'}{R_2}\right)^2 \Delta R_2$$

Substituting the value,

$$\Delta R' = \left[\frac{89.1}{150}\right]^2 \times 2 + \left[\frac{89.1}{220}\right]^2 \times 6$$

$$= 0.070 + 0.098 = 0.168$$

$$R' = 89.1 \pm 0.168 \text{ Ohm.}$$

18. A capacitor of capacitance C = $3.0 \pm 0.1 \mu F$ is charged to a voltage of V = $18 \pm 0.4 Volt$. Calculate the charge Q [Use Q= CV]

Solution:

$(C + \Delta C) = (3.0 \pm 0.1)\ \mu f$

$(V + \Delta V) = (18 \pm 0.4)\ V$

$Q = CV$

$$Q = 3.0 \times 10^{-6} \times 18 = 54 \times 10^{-6} \text{ coulomb}$$

$$\text{Error in C} = \frac{\Delta C}{C} \times 100$$

$$= \frac{0.1}{3} \times 100 = 3.3\%$$

$$\text{Error in V} = \frac{\Delta V}{V} \times 100$$

$$= \frac{0.4}{18} \times 100 = 2.2\%$$

Error in Q = Error in C + Error in V

= 3.3% + 2.2% =5.5%

$\therefore$ Charge $Q = (54 \times 10^{-6} \pm 5.5\%)$ coulomb

SOLVED EXAMPLE UNIT-2

1. The position vector for a particle is represented be $\vec{r} = 3t^2\hat{i} + 5t\hat{j} + 6\hat{k}$, find the velocity and speed of the particle at t = 3 sec?

Solution:

$$\vec{v} = \frac{\overrightarrow{dr}}{dt} = 6t\hat{i} + 5\hat{j}.$$

The velocity at any time '*t*' is given by $\vec{v} = 6t\hat{i} + 5\hat{j}$.

The magnitude of velocity is speed. The speed at any time '*t*' is then given by

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$$\text{Speed} = \sqrt{(6t)^2 + 5^2} = \sqrt{36t^2 + 25}$$

Now the velocity at t = 3sec is given by

$$\vec{v} = 6(3)\hat{i} + 5\hat{j} = 18\hat{i} + 5\hat{j}.$$

and speed at t = 3 sec, is given by

$$\text{speed} = \sqrt{349}\ m s^{-1}$$

2. A gun is fired from a place which is at distance 1.2 km from a hill. The echo of the sound is heard back at the same place of firing after 8 second. Find the speed of sound.

Solution:

The echo will be heard when the sound reaches back at the place of firing. So, the total distance travelled by sound is 2 × 1.2 km = 2.4 km = 2400 m.

$$\text{speed} = \frac{2400\ \text{m}}{8s} = 300\ \text{m s}^{-1}$$

3. A train 100 m long is moving with a speed of 60 kmh^{-1}. In how many seconds will it cross a bridge of 1 km long?

Solution:

Total distance to be covered = 1 km + 100 m = 1100 m (including both bridge and time)

Then, Speed = 60 km h^{-1}

$$= 60 \times \frac{5}{18}\ \text{m s}^{-1} = \frac{150}{9}\ m s^{-1}$$

Then, Time taken to cover this distance $= \dfrac{1100}{150/9}$ s = 66 s

4. Draw the resultant direction of the two unit vectors $\hat{i}$ and $\hat{j}$. Use a 2-dimensional Cartesian co-ordinate system. Is $\hat{i} + \hat{j}$ a unit vector?

By using the triangular law of addition $\hat{i} + \hat{j}$ as shown in the following figure,

The definition of unit vector is $\hat{A}.\hat{A} = 1$

But here,

$$(\hat{i} + \hat{j}).(\hat{i} + \hat{j}) = \hat{i}.\hat{i} + \hat{i}.\hat{j} + \hat{j}.\hat{i} + \hat{j}.\hat{j}$$
$$= 1 + 0 + 0 + 1 = 2$$

So, $\hat{i} + \hat{j}$ is not a unit vector.

To make any vector to a unit vector, must divide the vector by its magnitude,

$$\hat{A} = \frac{\vec{A}}{|\vec{A}|}$$

The norm of the vector $\hat{i} + \hat{j} = \sqrt{2}$.

Hence, the unit vector is $\dfrac{\hat{i} + \hat{j}}{\sqrt{2}}$

5. A swimmer moves across the Cauvery river of 750 m wide. The velocity of the swimmer relative to water ($\vec{v}_{sw}$) is 1.5 m s^{-1} and directed perpendicular to the water current. The velocity of water relative to the bank ($\vec{v}_{wb}$) is 1 m s^{-1}. Calculate the
   (a) velocity of the swimmer with respect to the bank of the river ($\vec{v}_{sb}$).
   (b) time taken by the swimmer to cross the Cauvery river.

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Solution:

(a) We can draw the following picture from the given data in the problem.

The velocity of the swimmer relative to the bank $\vec{v}_{sb} = \vec{v}_{sw} + \vec{v}_{wb}$

Since the swimmer travels in the perpendicular direction against the water current

The magnitude is given by

$$\left|\vec{v}_{sb}\right| = \sqrt{v_{sw}^2 + v_{wb}^2} = \sqrt{1.5^2 + 1^2} = \sqrt{3.25}\, m\, s^{-1} \cong 1.802\, m\, s^{-1}$$

The direction of the swimmer relative to the bank is given by

$$tan\theta = \frac{v_{sw}}{v_{wb}} = \frac{1.5}{1} = 1.5$$

$$\theta = tan^{-1}(1.5) \approx 56°$$

(c) The time taken by the swimmer to cross the river is equal to the total distance covered by the swimmer with velocity 1.802 ms^{-1}.

The total distance covered by him,

$$d = \frac{width\ of\ the\ river}{\sin 56^\circ} = \frac{750}{0.829} = 904.7\, m$$

The time taken by the swimmer,

$$T = \frac{d}{v_{sb}} = \frac{904.7}{1.802} \approx 502\, s$$

6. A monkey hangs on a tree. A hunter aims a gun at the monkey and fires the bullet with velocity v_0 which makes angle θ with horizontal direction. At the instant gun fires, monkey leaves the branch and falls straight down to escape from the bullet as shown in the figure. Will bullet hit the monkey or will the monkey escape the bullet? (ignore air resistance)

$$y_m = y_d = v_{oy}t - \frac{1}{2}gt^2 = h - \frac{1}{2}gt^2$$

As soon as the monkey begins to fall, it will have downward vertical motion with acceleration due to gravity g.

Its equation of motion at any time t is given by

$$y_m = h - \frac{1}{2}gt^2 \qquad (1)$$

When the bullet comes out of the gun, it has both vertical and horizontal components of velocity given by

$$v_{0x} = v_0\, cos\theta; v_{0y} = v_0\, sin\theta \qquad (2)$$

Let us assume the horizontal distance between the monkey and hunter is 'd'.

At time t, the horizontal distance travelled by the bullet $x = v_0\ cos\theta\ t$.

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When the horizontal position of bullet, $x = d$, the time $d = v_{0x}T$. It implies that $T = d / v_{0x}$

At this time T, the vertical distance covered by the bullet is

$$y_b = v_{0y}T - \frac{1}{2}gT^2 = \frac{v_{0y}d}{v_{0x}} - \frac{1}{2}gT^2.$$

$$y_b = \frac{d\,v_0\ sin\theta}{v_0\ cos\theta} - \frac{1}{2}gT^2$$

$$= d\ tan\theta - \frac{1}{2}gT^2 \qquad (3)$$

But from the figure we can write, $tan\theta = \frac{h}{d}$.

$$h = d\ tan\theta\,.$$

By substituting this in the equation (3), we get,

$$y_b = h - \frac{1}{2}gT^2 \qquad (4)$$

At this same time T, the vertical position of the monkey can be calculated from the equation (1)

$$y_m = h - \frac{1}{2}gT^2 \qquad (5)$$

Note that at the time T, the y coordinate of both monkey and bullet is same. It implies that the bullet will hit the monkey.

7. A three storey building of height 100 m is located on Earth and a similar building is also located on Moon. If two persons jump from the top of these buildings on Earth and Moon simultaneously, when they reach the ground, what will be their speed? ($g = 10\ m\ s^{-2}$)

Solution:

For both persons, the Kinematic equations are the same, with u = 0, $a_e = g$ and $a_{moon} = \frac{g}{6}$. Then

$$a_e = g\ and\ a_m = \frac{g}{6}$$

For a person on earth, $V_{earth} = \sqrt{2gh}$ $= \sqrt{2g100} = \sqrt{2\times10\times100}$

$Hence, V_{earth} = \sqrt{2000}\ m\ s^{-1}$ gives the velocity at the ground, on earth.

Similarly, for a person on the moon,

$$V_{moon} = \sqrt{\frac{2gh}{6}} = \frac{\sqrt{2000}}{\sqrt{6}}\ m\ s^{-1}$$

The person on earth reaches ground with greater velocity than the person on the moon

8. The following graphs represent position – time graphs. Arrange the graphs in ascending order of increasing speed.

(a) (b)

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(c) (d)

The slope in the position – time graph will give the speed of the particle.

In the graph (a) slope is zero. Graph (c) has higher slope than graphs (b) and (d). So we can arrange the speeds in ascending order as

$$v_a < v_d < v_b < v_c$$

SOLVED EXAMPLE UNIT-3

1. A body of mass 100 kg is moving with an acceleration of 50 cm s^{-2}. Calculate the force experienced by it.

Solution:

Mass m = 100 kg

Acceleration a = 50 cm s^{-2} = 0.5 m s^{-2}

Using Newton's second law,

$$F = ma$$

$$F = 100 \text{ kg} \times 0.5 \text{ m s}^{-2} = 50 \text{ N}$$

2. Identify the free body diagram that represents the particle accelerating in positive *x* direction in the following.

The relative magnitude of forces should be indicated when the free body diagram for mass m is drawn.

Case (a):

The forces F_1 and F_2 have equal length but opposite direction. So net force along y-direction is zero. Since the force is zero, acceleration is also zero along Y-direction (Newton's second law). Similarly in the x direction, F_3 and F_4 have equal length and opposite in direction. So net force is zero in the x direction. So there is no acceleration in x direction.

Case (b):

The forces F_1 and F_2 are not equal in length and act opposite to each other. The figure (b) shows that there are unbalanced forces along the y-direction. So the particle has acceleration in the -y direction. The forces F_3 and F_4 are having equal length and act in opposite directions. So there is no net force along the x direction. So the particle has no acceleration in the x direction.

Case (c):

The forces F_1 and F_2 are equal in magnitude and act opposite to each

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other. The net force is zero in y direction. So in y-direction there is no acceleration. The forces F_3 and F_4 are not equal in magnitude and F_3 is greater than F_4. So there is a net acceleration in negative x direction

Case (d):
The forces F_1 and F_2 are equal in magnitude and act opposite to each other. The net force is zero in y direction. So there is no acceleration in y-direction. The forces F_3 and F_4 are not equal in magnitude. The force F_4 is greater than the force F_3. So there is a net acceleration in the positive x direction.

3. A gun weighing 25 kg fires a bullet weighing 30 g with the speed of 200 ms^{-1}. What is the speed of recoil of the gun.

Solution:

Mass of the gun M = 25 kg
Mass of the bullet m = 30 g = 30×10^{-3} kg
Speed of bullet v = 200 m s^{-1}
Speed of gun V = ?
The motion is in one dimension.
As per law of conservation of momentum,

$$MV + mv = 0$$

$$V = \frac{-mv}{M}$$

$$V = \frac{-30 \times 10^{-3} \times 200}{25} = -240 \times 10^{-3} m s^{-1}$$

The negative sign shows that the gun moves in the opposite direction of the bullet. Further the magnitude of the recoil speed is very small compared to the bullet's speed.

4. A wooden box is lying on an inclined plane. What is the coefficient of friction if the box starts sliding when the angle of inclination is 45°.

Solution:

Angle of inclination $\Theta = 45°$
$\therefore$ Coefficient of friction $\mu = \tan \Theta = \tan 45° = 1$

5. Two masses m_1 = 5 kg and m_2 = 4 kg tied to a string are hanging over a light frictionless pulley. What is the acceleration of each mass when left free to move? (g = 10 m s^{-2})

$$a = \frac{m_1 - m_2}{m_1 + m_2} \times g$$

$$= \frac{5-4}{5+4} \times 10 = \frac{1}{9} \times 10 = 1.1\ m s^{-2}$$

6. A block of mass m is pushed momentarily along a horizontal surface with an initial velocity u. If μ_k is the coefficient of kinetic friction between the object and surface, find the time at which the block comes to rest.

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Solution:

When the block slides, the force acting on the block is kinetic friction which is equal to $f_k = \mu_k mg$

From Newton's second law $ma = -\mu_k mg$

The negative sign implies that force acts on the opposite direction of motion.

The acceleration of the block while sliding $a = -\mu_k g$.

The negative sign implies that the acceleration is in opposite direction of the velocity.

Note that the acceleration depends only on g and the coefficient of kinetic friction μ_k

We can apply the following kinematic equation

$$v = u + at$$

The final velocity is zero.

$$0 = u - \mu_k gt$$

$$t = \frac{u}{\mu_k g}$$

7. Three blocks of masses 10 kg, 7 kg and 2 kg are placed in contact with each other on a frictionless table. A force of 50 N is applied on the heaviest mass. What is the acceleration of the system?

Solution:

We know that

$$a = \left[\frac{F}{m_1 + m_2 + m_3}\right] = \frac{50N}{10\,kg + 7kg + 2\,kg}$$

$$= \frac{50}{19} = 2.63\,ms^{-2}$$

8. The coefficient of friction between a block and plane is $\frac{1}{\sqrt{3}}$. If the inclination of the plane gradually increases, at what angle will the object begin to slide?

Since the coefficient of friction is $\frac{1}{\sqrt{3}}$

$$\text{Tan}\,\Theta = \frac{1}{\sqrt{3}} \Rightarrow \Theta = 30°$$

9. Find the maximum speed at which a car can turn round a curve of 36 m radius on a level road. Given the coefficient of friction between the tyre and the road is 0.53.

Radius of the curve r = 36 m

Coefficient of friction μ = 0.53

Acceleration due to gravity g = 10 ms^{-2}

$$v_{max} = \sqrt{\mu rg} = \sqrt{0.53 \times 36 \times 10}$$

$$= 13.81\,ms^{-1}$$

10. Calculate the centripetal acceleration of the Earth which orbits around the

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Sun. The Sun to Earth distance is appriximately 150 million km. (Assume the orbit of Earth to be circular)

The centripetal acceleration $a_c = \frac{v^2}{r}$

V - velocity of Earth around the orbit

r - radius of orbit or distance of Earth to Sun

Velocity of Earth is written in terms of angular velocity (ω) as

$$v = \omega r$$

By substituting in the centripetal acceleration formula, $a_c = \frac{\omega^2 r^2}{r} = \omega^2 r$

But $\omega = \frac{2\pi}{T}$ where T is time for the Earth to orbit around the sun, which is one year.

$$T = 365 \text{ days} = 365 \times 24 \times 60 \times 60 \text{ s} = 3.1 \times 10^7 \, s$$

$$\omega = 2.02 \times 10^{-7} \, rad \; per \; sec$$

$$a_c = \left(2.02 \times 10^{-7}\right)^2 \times \left(150 \times 10^9\right)$$

$$a_c = 6.12 \times 10^{-3} \, m s^{-2}$$

11. A block 1 of mass m_1, constrained to move along a plane inclined at an angle θ to the horizontal, is connected via a frictionless, massless and inextensible string that passes over a massless pulley, to a second block 2 of mass m_2. Assume the coefficient of static friction between the block and the inclined plane is μ_s and the coefficient of kinetic friction is μ_k

What is the relation between the masses of block 1 and block 2 such that the system just starts to slip?

Solution:

For all parts of this problem, it will be convenient to use different coordinate systems for the two different blocks. For block 1, take the positive x -direction to be up the incline, parallel to the plane, and the positive y -direction to be perpendicular to the plane, directed with a positive upward component. Take the positive direction of the position of block 2 to be downward.

The normal component N of the contact force between block 1 and the ramp will be

$$N = m_1 g \cos\theta \, . \qquad (1)$$

The net x -component of the force on block 1 is then

$$F_{1x} = T - f_{\text{friction}} - m_1 g \sin\theta \qquad (2)$$

where T is the tension in the string

For the just-slipping condition, the frictional force has magnitude

$$f_{\text{friction}} = \mu_s N = \mu_s m_1 g \cos\theta. \qquad (3)$$

The tension in the string is the gravitational force of the suspended mass,

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$$T = m_2 g. \quad (4)$$

For the just-slipping condition, the net force on block 1 must be zero. Equations (2), (3) and (4) gives

$0 = m_2 g - \mu_s m_1 g \cos\theta - m_1 g \sin\theta$

$m_2 = m_1(\mu_s \cos\theta + \sin\theta)$

12. Consider two objects of masses 5 kg and 20 kg which are initially at rest. A force 100 N is applied on the two objects for 5 second.

a) What is the momentum gained by each object after 5 s.

b) What is the speed gained by each object after 5 s.

Final momentum on each object $\Delta P = F\Delta t = 100 \text{X } 5 = 500 \text{ kgms}^{-1}$

Final speed on the object of mass 5 kg = 500 /5 =100 m s^{-1}

Final speed on the object of mass 20 kg =500/20 =25 m s^{-1}

Note that momentum on each object is the same after 5 seconds but speed is not the same after 5 seconds. The heavier mass acquires lesser speed than the one with lower mass.

13. An object of mass 5 kg is initially at rest on the surface. The surface has coefficient of kinetic friction $\mu_k = 0.6$. What initial velocity must be given to the object so that it travels 10 m before coming to rest?

When the object moves on the surface it will experience three forces.

a) Downward gravitational force (mg)

b) Upward normal force (N)

c) Frictional force opposite to the motion of the object.

Since there is no motion along the vertical direction, magnitude of normal force is equal to the magnitude of gravitational force.

$$N = mg$$

Applying Newton's second law along the x direction

$$m\vec{a} = -\mu_k mg\hat{i}$$

The acceleration is $\vec{a} = -\mu_k mg\hat{i}$

Note that the acceleration is along the x direction since the frictional force acts along the negative x direction.

Or $\quad a = -\mu_k g$

Note that the acceleration is uniform during the entire motion. We can use Newton's kinematic equation to find the final velocity.

Along the x direction $v^2 = u^2 + 2as$

Here v = final velocity and u = initial velocity to be given to travel a distance s.

In this problem s = 10 m

Since the particle comes to rest, the final velocity v = 0

$$0 = u^2 - 2\mu_k gs$$

$$u = \sqrt{2\mu_k gs}$$

$$u = \sqrt{2 \times 0.6 \times 9.8 \times 10} = 10.8 \ ms^{-1}$$

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14. In the section 3.7.3 (Banking of road) we have not included the friction exerted by the road on the car. Suppose the coefficient of static friction between the car tyre and the surface of the road is μ_s, calculate the minimum speed with which the car can take safe turn?

When the car takes turn in the banked road, the following three forces act on the car.

(1) The gravitational force mg acting downwards
(2) The normal force N acting perpendicular to the surface of the road
(3) The static frictional force f acting on the car along the surface.

The following figure shows the forces acting on the horizontal and vertical direction.

When the car takes turn with the speed v, the centripetal force is exerted by horizontal component of normal force and static frictional force. It is given by

$$N\sin\theta + f\cos\theta = \frac{mv^2}{r} \qquad (1)$$

In the vertical direction, there is no acceleration. It implies that the vertical component of normal force is balanced by downward gravitational force and downward vertical component of frictional force. This can be expressed as

$$N\cos\theta = mg + f\sin\theta$$

Or $$N\cos\theta - f\sin\theta = mg \qquad (2)$$

Diving the equation (1) by equation (2), we get

$$\frac{N\sin\theta + f\cos\theta}{N\cos\theta - f\sin\theta} = \frac{v^2}{rg} \qquad (3)$$

To calculate the maximum speed for the safe turn, we can use the maximum static friction is given by $f = \mu_s N$. By substituting this relation in equation (3), we get

$$\frac{N\sin\theta + \mu_s N\cos\theta}{N\cos\theta - \mu_s N\sin\theta} = \frac{v_{max}^{\ 2}}{rg}$$

By simplifying this equation,

$$\frac{N\cos\theta\left\{\left(\frac{N\sin\theta}{N\cos\theta}\right) + \mu_s\right\}}{N\cos\theta\left(1 - \mu_s\frac{N\sin\theta}{N\cos\theta}\right)} = \frac{v_{max}^{\ 2}}{rg}$$

$$\frac{(\tan\theta + \mu_s)}{1 - \mu_s\tan\theta} = \frac{v_{max}^{\ 2}}{rg}$$

The Maximum speed for safe turn is given by $v_{max} = \sqrt{rg\frac{(\tan\theta + \mu_s)}{(1 - \mu_s\tan\theta)}}$ (4)

Suppose we neglect the effect of friction ($\mu_s = 0$), then safe speed

$$v_{safe} = \sqrt{rg\tan\theta} \qquad (5)$$

Note that the maximum speed with which the car takes safe turn is increased by friction (equation (4)). Suppose the car turns with speed $v < v_{safe}$, then the static friction acts up in the slope to prevent from inward skidding.

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If the car turns with the speed little greater than, then the static friction acts down the slope to prevent outward skidding. But if the car turns with the speed much greater than v_{safe} then static friction cannot prevent from outward skidding.

SOLVED EXAMPLE UNIT-4

1. A force $\vec{F} = \hat{i} + 2\hat{j} + 3\hat{k}$ N acts on a particle and displaces it through a distance $\vec{S} = 4\hat{i} + 6\hat{j}$ m. Calculate the work done.

Solution:

Force $\vec{F} = \hat{i} + 2\hat{j} + 3\hat{k}$ N

Distance $\vec{S} = 4\hat{i} + 6\hat{j}$ m

Work done = $\vec{F} \cdot \vec{S} = (\hat{i} + 2\hat{j} + 3\hat{k}) \cdot (4\hat{i} + 6\hat{j})$

= 4+12+0 = 16 J

2. A particle moves along X- axis from x=0 to x=8 under the influence of a force given by F= $3x^2 - 4x + 5$. Find the work done in the process.

Solution:

Work done in moving a particle from x=0 to x=8 will be

$$W = \int_0^8 F dx = \int_0^8 (3x^2 - 4x + 5) dx$$

$$= \left[\frac{3x^3}{3} - \frac{4x^2}{2} + 5x \right]_0^8$$

$$W = \left[3\frac{(8)^3}{3} - 4\left(\frac{8^2}{2}\right) + 40 \right]$$

$$= [512 - 128 + 40] = 424 J$$

3. A body of mass 10kg at rest is subjected to a force of 16N. Find the kinetic energy at the end of 10 s.

Solution:

Mass m = 10 kg

Force F = 16 N

time t = 10 s

$$a = F/m = \frac{16}{10} = 1.6 m s^{-2}$$

we know that, $v = u + at$

$= 0 + 1.6 \times 10 = 16\ m\ s^{-1}$

Kinetic energy K.E = $\frac{1}{2} m v^2$

$= \frac{1}{2} \times 10 \times 16 \times\ 16$

$= 1280 J$

4. A body of mass 5kg is thrown up vertically with a kinetic energy of 1000 J. If acceleration due to gravity is 10 $m\ s^{-2}$, find the height at which the kinetic energy becomes half of the original value.

Solution:

Mass m = 5 kg

K.E E = 1000 J

g = 10 $m\ s^{-2}$

At a height 'h', mgh = $\frac{E}{2}$

$$5 \times 10 \times h = \frac{1000}{2}$$

$$h = \frac{500}{50} = 10 m$$

5. Two bodies of masses 60 kg and 30 kg moving in the same direction along straight line with velocity 40 cm s^{-1} and 30 cm s^{-1}

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respectively suffer one dimensional elastic collision. Find their velocities after collision.

Solution:

Mass m_1 = 60 kg

Mass m_2 = 30 kg

$$V_1 = 40\ cms^{-1}$$

$$V_2 = 30\ cms^{-1}$$

Solution:

$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \left(\frac{2m_2}{m_1 + m_2}\right)u_2$$

$$v_2 = \frac{(m_2 - m_1)}{m_1 + m_2}u_2 + \frac{2m_1}{m_1 + m_2}u_1$$

Substituting the values, we get,

$$v_1 = \frac{(60-30)}{90} \times 40 + \frac{2 \times 30}{90} \times 30$$

$$v_1 = \frac{1}{90}\ [1200 + 1800]$$

$$= \frac{3000}{90} = 33.3 cms^{-1}$$

Likewise,

$$v_2 = \frac{(30-60)}{90} \times 30 + \frac{2 \times 60}{90} \times 40$$

$$v_2 = \frac{1}{90}[-900 + 4800]$$

$$= \frac{3900}{90} = 43.3 cms^{-1}$$

6. A particle strikes a horizontal frictionless floor with a speed u at an angle θ with the vertical and rebounds with the speed v at an angle ϕ with the vertical. The coefficient of restitution between the particle and floor is e. What is the magnitude of v?

Solution:

Applying component of velocities,

The x - component of velocity is

$$u\sin\theta = v\sin\phi \qquad (1)$$

The magnitude of y – component of velocity is not same, therefore, using coefficient of restitution,

$$e = \frac{v\cos\varphi}{u\cos\theta} \qquad (2)$$

Squaring (1) and (2) and adding we get

$$v^2 \sin^2\varphi = u^2 \sin^2\theta$$

$$v^2 \cos^2\varphi = e^2 u^2 \cos^2\theta$$

adding

$$v^2 = u^2 \sin^2\theta + e^2 u^2 \cos^2\theta$$

$$\therefore v^2 = u^2\left[\sin^2\theta + e^2\cos^2\theta\right]$$

$$v = u\sqrt{\sin^2\theta + e^2\cos^2\theta}$$

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7. A particle of mass m is fixed to one end of a light spring of force constant k and un-stretched length l. It is rotated with an angular velocity ω in horizontal circle. What will be the length increase in the spring?

Solution:

Mass of particle = m

Force constant = k

Un-stretched length = l

Angular velocity = ω

Let 'x' be the increase in the length of the spring.

The new length = (l+x) = r

When the spring is rotated in a horizontal circle,

Spring force = centripetal force.

$$kx = m\omega^2(l+x)$$

$$x = \frac{m\omega^2 l}{k - m\omega^2}$$

8. A gun fires 8 bullets per second into a target X. If the mass of each bullet is 3 g and its speed 600 ms^{-1}, then calculate the power delivered by the bullets.

Solution:

Power = work done per second = total kinetic energy of 8 bullets per second

$P = 8 \times$ (kinetic energy of each bullet per second)

$$= 8 \times \frac{1}{2} \times \left(3 \times 10^{-3}\right) \times \left(600\right)^2$$

P = 4320 W

P = 4.320 kW

SOLVED EXAMPLE UNIT-5

1. Three particles of masses m_1 = 1 kg, m_2 = 2 kg and m_3 = 3 kg are placed at the corners of an equilateral triangle of side 1m as shown in Figure. Find the position of centre of mass.

Solution:

The centre of mass of an equilateral triangle lies at its geometrical centre G.

The positions of the mass m_1, m_2 and m_3 are at positions A, B and C as shown in the Figure.

From the given position of the masses, the coordinates of the masses m_1 and m_2 are easily marked as (0,0) and (1,0) respectively.

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To find the position of m_3 the Pythagoras theorem is applied. As the ΔDBC is a right angle triangle,

$$BC^2 = CD^2 + DB^2$$

$$CD^2 = BC^2 - DB^2$$

$$CD^2 = 1^2 - \left(\frac{1}{2}\right)^2 = 1 - \left(\frac{1}{4}\right) = \frac{3}{4}$$

$$CD = \frac{\sqrt{3}}{2}$$

The position of mass m_3 is

$$\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right) \text{ or } \left(0.5, 0.5\sqrt{3}\right)$$

X Coordinate of centre of mass,

$$x_{CM} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$$

$$x_{CM} = \frac{(1\times 0) + (2\times 1) + (3\times 0.5)}{1+2+3} = \frac{3.5}{6}$$

$$x_{CM} = \frac{7}{12} \text{ m}$$

Y Coordinate of centre of mass,

$$y_{CM} = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3}$$

$$y_{CM} = \frac{(1\times 0) + (2\times 0) + (3\times 0.5\times\sqrt{3})}{1+2+3} = \frac{1.5\sqrt{3}}{6}$$

$$y_{CM} = \frac{\sqrt{3}}{4} \text{ m.}$$

∴ The coordinates of centre of mass G $\left(x_{CM}, y_{CM}\right)$ is $\left(\frac{7}{12}, \frac{\sqrt{3}}{4}\right)$

2. An electron of mass $9\times10^{-31}\,kg$ revolves around a nucleus in a circular orbit of radius 0.53 Å. What is the angular momentum of the electron? (Velocity of electron is, $v = 2.2\times10^{6}\,m\,s^{-1}$)

Solution:

Mass of the electron, $m = 9\times10^{-31}\,kg$

Radius of the electron, r = 0.53 Å = $0.53\times10^{-10}\,m$

Velocity of the electron, $v = 2.2\times10^{6}\,m\,s^{-1}$

Angular momentum of electron is, $L = I\,\omega$

Electron is considered as a point mass. Hence, its moment of inertia is, $I = m\,r^2$

The relation, $\omega = \frac{v}{r}$ could be used.

Angular momentum, $L = mr^2\times\frac{v}{r}$

$= mvr$

$$= 9.1\times10^{-31}\times2.2\times10^{6}\times0.53\times10^{-10}$$

$$L = 1.06\times10^{-34}\,kg\,m^2\,s^{-1}$$

3. A solid sphere of mass 20 kg and radius 0.25 m rotates about an axis passing through the centre. What is the angular momentum if the angular velocity is 5 rad s^{-1}

Solution:

Mass of the sphere, m = 20 kg

Radius r = 0.25 m

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Angular velocity $\omega = 5 \text{ rad s}^{-1}$

Solution:

Angular momentum $L = I\omega = \frac{2}{5}mr^2\omega$

$$= \frac{2}{5} \times 20 \times (0.25)^2 \times 5 = 40 \times (0.0625) = 2.5$$

$$L = 2.5 \text{ kg m}^2\text{s}^{-1}$$

4. A solid cylinder when dropped from a height of 2 m acquires a velocity while reaching the ground. If the same cylinder is rolled down from the top of an inclined plane to reach the ground with same velocity, what must be the height of the inclined plane? Also compute the velocity.

Solution:

In the first case,

potential energy = kinetic energy

$$mgh = \frac{1}{2}mv^2$$

$$mg \times 2 = \frac{1}{2}mv^2 \qquad (1)$$

In second case,

potential energy = translational kinetic energy + rotational kinetic energy

$$mgh' = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$mgh' = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{mr^2}{2}\right)\left(\frac{v^2}{r^2}\right)$$

$$\therefore mgh' = \frac{3}{4}mv^2 \qquad (2)$$

Dividing (2) by (1),

$$\frac{mgh'}{mg \times 2} = \frac{\frac{3}{4}mv^2}{\frac{1}{2}mv^2} = \frac{3}{4} \times \frac{2}{1} = \frac{3}{2}$$

$$h' = 3\,m$$

From equation (1), $2mg = \frac{1}{2}mv^2$

$$v = \sqrt{4g} = 2\sqrt{g}$$

$$v = 2 \times \sqrt{9.81}$$

$$v = 6.3\,m\,s^{-1}$$

5. A small particle of mass m is projected with an initial velocity v at an angle θ with x axis in X-Y plane as shown in Figure. Find the angular momentum of the particle.

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Solution:

Let the particle of mass m cross a horizontal distance x in time t.

Angular momentum $\vec{L} = \int \vec{\tau}\, dt$

But $\vec{\tau} = \vec{r} \times \vec{F}$

$\vec{r} = x\hat{i} + y\hat{j}$ and $\vec{F} = -mg\hat{j}$

$$\therefore \vec{\tau} = \left(x\hat{i} + y\hat{j}\right) \times \left(-mg\hat{j}\right)$$

$$\vec{\tau} = -mgx\left(\hat{i} \times \hat{j}\right) = -mgx\hat{k}$$

$$\vec{L} = -mg\int (x\,dt)\,\hat{k} = -mgv\cos\theta\left(\int t\,dt\right)\hat{k}$$

Let initial time t = 0 and final time t = t_f

$$\therefore \vec{L} = -mgv\cos\theta\left(\int_0^{t_f} t\,dt\right)\hat{k} = -\frac{1}{2}mgv\cos\theta\, t_f^2\ \hat{k}$$

Negative sign indicates, $\vec{L}$ point inwards

6. From a complete ring of mass M and radius R, a sector angle θ is removed. What is the moment of inertia of the incomplete ring about axis passing through the centre of the ring and perpendicular to the plane of the ring?

Solution:

Let R be the radius of the ring and M be the total mass of the complete ring.

Let m be the mass of the section removed from the ring then, mass of the incomplete ring is M-m

Let us introduce a positive integer (n), such that, $n\theta = 360^\circ$, or $n = \dfrac{360^\circ}{\theta}$

$$mass\ of\ incomplete\ ring = M - m$$

$$m = \frac{M}{360} \times \theta$$

$$\therefore mass\ of\ incomplete\ ring = M - \frac{M}{360} \times \theta$$

$$mass\ of\ incomplete\ ring = M - \frac{M}{n} = M\frac{(n-1)}{n}$$

For example, *a) when* $\theta = 60^\circ$; $n = \dfrac{360^\circ}{60^\circ} = 6$

$$\therefore n - 1 = 5$$

mass of incomplete ring $= \dfrac{5}{6}M$

b) when $\theta = 30^\circ$, $n = \dfrac{360^\circ}{30^\circ} = 12$

$$n - 1 = 11$$

mass of incomplete ring $= \dfrac{11}{12}M$

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The moment of inertia of the incomplete ring is, $I = M\frac{(n-1)}{n}R^2$

7. A massless right angled triangle is suspended with its right angle corner. A mass of 100 kg is suspended from another corner B which subtends an angle 53°. Find the mass m that should be suspended from other corner C so that BC (hypotenuse) remains horizontal.

Solution:

From the principle of moments,

$$100 \times g \times x_1 = m \times g \times x_2$$

$$100 \times \cos 53^\circ = m \times \cos 37^\circ \qquad (1)$$

Where, x_1 and x_2 are the arm lengths.

The right angle triangle with angles 37°, 53° and 90° is a special triangle which has the respective sides in the ratio, 3:4:5 as shown in the diagram.

Substituting the values in equation (1),

$$100 \times \cos 53^\circ = m \times \cos 37^\circ$$

$$100 \times \frac{3}{5} = m \times \frac{4}{5}$$

$$m = 100 \times \frac{3}{4}$$

$$m = 75\,kg$$

8. If energy of 1000 J is spent in increasing the speed of a flywheel from 30 rpm to 720 rpm, find the moment of inertia of the wheel.

Solution:

$$\omega_1 = 30rpm = 2\pi \times \frac{30}{60} rads^{-1} = \pi\ rads$$

$$\omega_2 = 720rpm$$

$$= 2\ \pi \times \frac{720}{60} rads^{-1} = 24\pi\ rads$$

Change in kinetic energy,

$$\Delta KE = \frac{1}{2} I(\omega_2^{\ 2} - \omega_1^{\ 2})$$

$$I = \frac{2 \times \Delta KE}{(\omega_2^{\ 2} - \omega_1^{\ 2})} = \frac{2 \times 1000}{(24\pi)^2 - (\pi)^2}$$

$$I = \frac{2000}{25\pi \times 23\pi}$$

Remember:

$a^2 - b^2 = (a + b)\ (a - b)$

$I \approx 0.35\ kg\ m^2$ as $\pi^2 \approx 10$

9. Consider two cylinders with same radius and same mass. Let one of the cylinders be solid and another one be hollow. When subjected to same torque, which one among them gets more angular acceleration than the other?

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Solution:

Moment of inertia of a solid cylinder about its axis $I_s = \frac{1}{2}MR^2$

Moment of inertia of a hollow cylinder about its axis $I_h = MR^2$

$$I_s = \frac{1}{2}I_h \text{ or } I_h = 2I_s$$

$$torque\ \tau = I\alpha$$

$$\alpha = \frac{\tau}{I}$$

$$\alpha_s = \frac{\tau}{I_s} \text{ and } \alpha_h = \frac{\tau}{I_h}$$

$$\alpha_s I_s = \alpha_h I_h \Rightarrow \alpha_s = \alpha_h \frac{I_h}{I_s}$$

Since, $I_h > I_s \Rightarrow \frac{I_h}{I_s} > 1$

$$\therefore \alpha_s > \alpha_h$$

For the same torque, a solid cylinder gets more acceleration than a hollow cylinder.

Note: **The above two cylinders must be made up of materials of different density. (Say why?)**

10. **A thin horizontal circular disc is rotating about a vertical axis passing through its centre. An insect goes from A to point B along its diameter as shown in Figure. Discuss how the angular speed of the circular disc changes?**

Solution:

As the disc is freely rotating, with the insect on it, the angular momentum of the system is conserved.

$$L = I\omega = \text{constant}$$

When the insect moves towards the centre (from A to O), the moment of inertia (I) decreases. Thus, the angular velocity (ω) increases. When it moves away from centre (from O to B), the moment of inertia (I) increases. Thus, the angular velocity (ω) decreases.

11. (i) What is the shape of the graph between $\sqrt{E_{kr}}$ and L? (E_{kr} is the rotational kinetic energy and L is angular momentum)

(ii) What information can you get from the slope of the graph?

(iii) You are given the graph of $\sqrt{E_{kr}}$ and L for two bodies A and B. Which one has more moment of inertia?

Solution:

i) We know that, Rotational kinetic Energy

$$E_{kr} = \frac{1}{2}I\omega^2$$

$$= \frac{1}{2}I\omega \times \omega = \frac{1}{2}.L.\omega = \frac{1}{2}\frac{L^2}{I} \qquad \because L = I\omega \quad \omega = L/I$$

$$E_{kr} = \frac{L^2}{2I}$$

$$L^2 = 2IE_{kr}$$

$$L = \sqrt{2IE_{kr}} = \sqrt{2I}.\sqrt{E_{kr}}$$

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Graph of $\sqrt{E_{kr}}$ and L:

The shape of the graph is a straight line

ii) The slope of the graph gives the value of moment of inertia I.

iii) We know that the slope gives the value of moment of Inertia. The line A has higher slope and hence more moment of Inertia.

12. **Consider a thin uniform circular ring rolling down in an inclined plane without slipping. Compute the linear acceleration along the inclined plane if the angle of inclination is 45˚.**

Solution:

The linear acceleration along the inclined plane can be computed by

$$a = \frac{g\sin\theta}{1+\frac{K^2}{R^2}}$$

For a thin uniform circular ring, axis passing through its centre is $I = MR^2$

$$\therefore K^2 = R^2 \Rightarrow \frac{K^2}{R^2} = 1.$$

And the angle of inclination, θ = 45˚

$\Rightarrow (\sin 45^\circ = \frac{1}{\sqrt{2}})$

Hence,

$$a = \frac{\frac{g}{\sqrt{2}}}{1+1}$$

$$a = \frac{g}{2\sqrt{2}} ms^{-2}$$

COMPETITIVE EXAM CORNER

MLZYF

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APPENDIX 2

Table A1.1 Systematic developments in Physics over centuries

Observation of motion of Sun, Stars, Planets and Moon	Around 3000 BC (Early Greeks)
In the universe everything is changing. Nothing remains in the same state indefinitely. The idea of 'time' started with this understanding.	Around 500 BC (Heraclitus).
Everything is composed of entirely small indivisible elements called 'atoms'. Development of theory of 'atomism'. But that time it is a hypothesis. No experimental proof.	End of 500 BC (Democritus and Leucippus)
▪ Idea of natural laws behind every day phenomena ▪ Idea of motion with gravity ▪ Theory of four elements(Everything is made up of Earth, water, fire, air able to inter-transform into each other) ▪ Earth is centre of universe(Hypothesis) ▪ Force is required to move the object ▪ Heavier object falls faster towards Earth than lighter object	Around 3rd century BC (Aristotle)
▪ Earth is sphere ▪ Measurement of radius of the Earth almost accurately	Around 240 BC (Eratosthenes)

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Table A1.1 Systematic developments in physics over centuries (*Cont*)

▪ Sun is centre of the solar system (Hypothesis. Could not be proven by experimental evidence) ▪ Idea of Earth rotation around its own axis.	Around 2nd century BC (Aristarchus of Samos) Around 2nd century BC (Seleucia)
▪ Foundation of hydrostatics ▪ Idea of lever ▪ Mechanical work using pulleys ▪ Law of Buoyancy known as a Archimedes principle ▪ First accurate value of the number 'pi'	Around 3rd century BC (Archimedes)
▪ Focused on motion of planets and stars ▪ Prediction of solar eclipses ▪ Calculation of distance of Earth to Moon, Earth to Sun ▪ Astronomical observation were recorded	End of 2nd century BC (Hippachrus)
▪ Geo centric model (Not hypothesis. Explained a lot of naked eye observation) ▪ Explanation of Planets 'retrograde motion' ▪ "Almagest"- First book on astronomy	Around 100 AD(CE) (Ptolemy)
▪ Idea of Earth's rotation about its own axis ▪ Idea of zero ▪ Contribution to mathematics	5th Century AD (Aryabhatta-India)
▪ Understanding of early optics ▪ Boon on 'Treasury of astronomy'- Accurate astronomical table than Ptolemy's data	9th century AD (Ibn al-Hayatham- Arabia) 12th Century AD (Nasir al-Din- Persian astronomer)
▪ From 7th century to 14th century major development in science happened in muslim countries (Arabia, Persia, Iran etc.)	

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Table A1.1 Systematic developments in physics over centuries (*Cont*)

▪ Copernicun Revolution ▪ Heliocentric model (Not hypothesis. It provides simplest explanation than Ptolemy model for motion of stars and Planets) ▪ Accurate astronomical datas ▪ Laws of Planetary motion	15th Century AD AD 1543 (Copernicus) Tycho Brahe Kepler
▪ Law of inertia ▪ Telescope observation(Founder of modern observational astronomy) ▪ Calculation of time period of the Moons of the Jupiter ▪ Earth is not flat ▪ All object fall to Earth at the same rate (Disproved Aristotle's argument) ▪ Law of inertia(Force need not required to maintain the motion- disproved Aristotle's argument) ▪ Pendulum, inclined plane experiments ▪ Study of projectile motion ▪ Introduction of 'controlled experiments'	Gallieo Galilei (1564- 1642)
▪ Introduction of Cartesian coordinate system ▪ Idea of analytical geometry	Rane Descarte (1596-1650)
▪ 17th and 18thcentury Development	
▪ Laws of motion ▪ Quantitative idea of motion ▪ Law of gravitation ▪ Development of calculus independent of Leibnitz ▪ Founder of modern optics (reflection, dispersion, prism) ▪ Light consists of minute particles 'corpuscules' ▪ Derivation of Kepler laws ▪ Greatest book 'The principia mathematica'(1687) ▪ Wave theory of light	Isaac Newton (1642-1727) Christian Huygens; Father of modern optics

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Table A1.1 Systematic developments in physics over centuries (*Cont*)

▪ Work on magnetism	William Gilbert (Around 1600)
▪ Behavior of gases	Robert Boyle (1627- 1691)& Robert Hooke
▪ Fluid dynamics - Bernoulli's theorem (1734)	Daniel Bernoulli(1700- 1782)
▪ Early ideas of kinetic theory of gases	Daniel Bernoulli (1700 - 1782)
▪ Study of spring motion(Hooke's law)	Robert Hooke
▪ Derivation of frequency of vibration in strings(1714)	Taylor
▪ Reformulation of Newtonian mechanics using Energy approach (Lagrangian mechanics)	De Alembert, Lagrange
▪ Invention of steam engine(1781)-	James Watt
▪ Force between charges (Coulomb's law)	Coulomb
▪ 19th century development	
▪ Early ideas of Thermodynamics(1840s)	James Joule, Carnot
▪ Caloric theory	
▪ Laws of thermodynamics(1850s)	Kelvin, Clausius
▪ Behavior of gases, velocity and speed(1860s)	James Clark Maxwell
▪ Foundation of statistical mechanics and entropy formula (around 1870s)	Boltzmann
▪ Wave nature of light- Experiments	Young and Fresnel
▪ Behavior of electric charges	
▪ Magnetic effect of electric current(1820s)	Oersted
▪ Force between two parallel currents	Ampere
▪ Principle of Least action, Hamilton mechanics (1821)	William Hamilton
▪ Electric powered motor, electricity demonstration	Micheal Faraday
▪ Theory of electromagnetism(1873)- Bridge between electricity and magnetism	James Clerk Maxwell
▪ Maxwell equations –Paved way to modern technology	
▪ Alternating current	Tesla

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Table A1.1 Systematic developments in physics over centuries (*Cont*)

▪ 20th century developments	
▪ Study of black body radiation ▪ Discovery of an electron ▪ Rutherford atomic model ▪ Study on radioactivity	Max planck(around 1900) J.J.Thomson Rutherford(1910s) Marie Curie(1920s and 30s)
▪ Special theory of relativity, Photo electric effect, Existence of atom(1905), $E=mc^2$ ▪ Revolution of physics after Newton ▪ New Idea of space and time ▪ The General theory of relativity(Greatest theory of 20th century)- 1915 ▪ Study of specific heat capacities	Albert Einstein
▪ Study of atoms ▪ Bohr atom model (1912) ▪ Behavior of electron, proton – Schrodinger equation ▪ Uncertainty principle ▪ Formulation of Quantum Mechanics ▪ Formulation of quantum field theory ▪ Particle physics, standard model ▪ X-ray diffraction(1930s)- Paved way to understanding the materials ▪ Raman effect ▪ Study of stars and black holes ▪ Invention of transistor(1947) ▪ Classification of stars using temperature (Astro thermodynamics), Saha ionization formula ▪ Field of Cosmology (1920s) ▪ Expanding universe model(1922) ▪ Discovery of redshift	 Niels Bohr Schrodinger Heisenberg Paul Dirac Dirac, Feynman, Schwinger Gellman, Weinberg, Abdus salam C.V. Raman(India) Chandrasekhar(Indian origin) John Bardeen, Walter Brattain,William Schokley Megnad Saha(India) Eddington, Schwarchild Thomas Friedmann Edwin Hubble

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Table A1.1 Systematic developments in physics over centuries (*Cont*)

▪ Birth of materials science ▪ Nanotechnology, Condensed matter physics ▪ Gravitational waves, Dark energy, Dark matter, String theory	

APPENDIX A1.2

vi) Error involving the division of two quantities (using the method of differentiation)

Consider $Z = \frac{A^n}{B^m}$

Taking logarithms of both the sides,

$$\log Z = \log A^n - \log B^m$$
$$\log Z = n \log A - m \log B$$

Differentiating both sides, we get,

$$\frac{dZ}{Z} = n\frac{dA}{A} - m\frac{dB}{B}$$

In terms of fractional error, this may be written as,

$$\pm\frac{\Delta Z}{Z} = \pm n\frac{\Delta A}{A} \mp m\frac{\Delta B}{B}$$

Maximum possible relative error in Z is given by

$$\frac{\Delta Z}{Z} = n\frac{\Delta A}{A} + m\frac{\Delta B}{B}$$

vii) Error involving the product of three quantities (using calculus method)

let $u = x^m y^n Z^p$

Taking logarithms on both sides, we get

$$\log u = \log x^m + \log y^n + \log Z^p$$
$$\log u = m \log x + n \log y + p \log Z$$

Differentiating both sides, we get

$$\frac{du}{u} = m\frac{dx}{x} + n\frac{dy}{y} + p\frac{dz}{z}$$

In terms of fractional error, the equation maybe rewritten as,

$$\pm\frac{\Delta u}{u} = \pm m\frac{\Delta x}{x} \pm n\frac{\Delta y}{y} \pm p\frac{\Delta z}{z}$$

APPENDIX A2.1

Parallelogram Law of Vector addition:
If two non-zero vectors $\vec{A}$ and $\vec{B}$ are represented by the two adjacent sides of a parallelogram then the resultant is given by the diagonal of the parallelogram passing through the point of intersection of the two vectors.

Consider the figure 2.19 below. Here the two vectors $\vec{A}$ and $\vec{B}$ are connected by a common tail at an angle θ. The parallelogram OACB is next constructed.

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Then the diagonal OC is the resultant ($\vec{R}$) passing through the common tail O.

Figure 2.19 Magnitude of resultant vector by parallelogram method

We next find the magnitude and direction of this resultant vector.

(i) Magnitude:

First extend OA to the point N, so that we get ON. Then CN is drawn perpendicular to this ON, from C. Then ONC is a right angled triangle. We can write $R^2 = ON^2 + CN^2$

$$R^2 = (OA + AN)^2 + CN^2$$

$$\Rightarrow R^2 = A^2 + B^2 + 2AB\cos\theta$$

$$\therefore R = |\vec{R}| = |\vec{A} + \vec{B}| = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

Special cases:

When $\theta = 0°$, then R = A+B

When $\theta = 180°$, then R = A-B

When $\theta = 90°$, then $R = \sqrt{A^2 + B^2}$

(ii) Direction

Let β be the angle between the vectors $\vec{A}$ and $\vec{R}$. Then

$$\tan\beta = \frac{CN}{ON} = \frac{B\sin\theta}{A + B\cos\theta}$$

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APPENDIX 3

Some important formulae for Integral

(1) $\int dx = x$; $\frac{d}{dx}(x) = 1$

(2) $\int x^n dx = \left(\frac{x^{n+1}}{n+1}\right)$; $\frac{d}{dx}\left(\frac{x^{n+1}}{n+1}\right) = x^n$

(3) $\int cudx = c\int udx$ where c is a constant

(4) $\int (u \pm v \pm w)dx = \int udx \pm \int vdx \pm \int wdx$

(5) $\int \frac{1}{x}dx = \ln x + C$

(6) $\int e^x dx = e^x + C$

(7) $\int \cos\theta d\theta = \sin\theta + C$

(8) $\int \sin\theta d\theta = -\cos\theta + C$

Some important formulae in Differential calculus

1. $\frac{d}{dx}(c) = 0$, if c is a constant

2. If y = cu, where c is a constant and u is a function of x then
$$\frac{dy}{dx} = \frac{d}{dx}(cu) = c\frac{du}{dx}$$

3. If y = u ±v ±w where u,v and w are functions of x then
$$\frac{dy}{dx} = \frac{d}{dx}(u \pm v \pm w) = \frac{du}{ds} \pm \frac{dv}{dx} \pm \frac{dw}{dx}$$

4. If y = x^n, where n is the real number then
$$\frac{dy}{dx} = \frac{d}{dx}(x^n) = nx^{n-1}$$

5. If y = uv, where u and v are functions of x then
$$\frac{dy}{dx} = \frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$

6. If y is a function of x, then $dy = \frac{dy}{dx}.dx$

7. $\frac{d}{dx}(e^x) = e^x$

8. $\frac{d}{dx}(\ln x) = \frac{1}{x}$

9. $\frac{d}{d\theta}(\sin\theta) = \cos\theta$

10. $\frac{d}{d\theta}(\cos\theta) = -\sin\theta$

11. If y is a trigonometric function of θ and θ is the function of t, then
$$\frac{d}{dt}(\sin\theta) = \cos\theta\frac{d\theta}{dt}$$
$$\frac{d}{dt}(\cos\theta) = -\sin\theta\frac{d\theta}{dt}$$

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APPENDIX 4

(THE GREEK ALPHABET)

(கிரேக்க எழுத்துகள்)

The Greek Alphabet	Upper Case	Lower Case
Alpha	Α	α
Beta	Β	β
Gamma	Γ	γ
Delta	Δ	δ
Epsilon	Ε	ε
Zeta	Ζ	ζ
Eta	Η	η
Theta	Θ	θ
Iota	Ι	ι
Kappa	Κ	κ
Lambda	Λ	λ
Mu	Μ	μ
Nu	Ν	ν
Xi	€	ξ
Omicron	Ο	ο
Pi	Π	π
Rho	Ρ	ρ
Sigma	Σ	σ
Tau	Τ	τ
Upsilon	Υ	υ
Phi	Φ	ϕ
Chi	Χ	χ
Psi	Ψ	ψ
Omega	Ω	ω

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SOME IMPORTANT CONSTANTS IN PHYSICS

Name	Symbols	Value
Speed of light in vacuum	c	2.9979×10^{8} m s^{-1}
Gravitational constant	G	6.67×10^{-11} N m^{2} kg^{-2}
Acceleration due to gravity (sea level, at 45° latitude)	g	9.8 m s^{-2}
Planck constant	h	6.626×10^{-34} J s
Boltzmann constant	k	1.38×10^{-23} J K^{-1}
Avogadro number	N_A	6.023×10^{23} mol^{-1}
Universal gas constant	R	8.31 J mol^{-1} K^{-1}
Stefan – Boltmann constant	σ	5.67×10^{-8} W m^{-2} K^{-4}
Wien's constant	b	2.898×10^{-3} m K
Permeability of free space	μ_0	$4\pi \times 10^{-7}$ H m^{-1}
Standard atmospheric pressure	1 *atm*	1.013×10^{5} Pa

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GLOSSARY
கலைச்சொற்கள்

1.	Acceleration	-	முடுக்கம்
2.	Angular Momentum	-	கோண உந்தம்
3.	Astro physics	-	வானியற்பியல்
4.	Average Velocity	-	சராசரித் திசைவேகம்
5.	Angular displacement	-	கோண இடப்பெயர்ச்சி
6.	Angular velocity	-	கோணத் திசைவேகம்
7.	Angular acceleration	-	கோண முடுக்கம்
8.	Angle of friction	-	உராய்வுக் கோணம்
9.	Angle of repose	-	சறுக்குக் கோணம்
10.	Centripetal acceleration	-	மையநோக்கு முடுக்கம்
11.	Circular Motion	-	வட்ட இயக்கம்
12.	Concurrent forces	-	மைய விசைகள்
13.	Coplanar force	-	ஒரு தள விசைகள்
14.	Centripetal force	-	மையநோக்கு விசை
15.	Centrifugal force	-	மையவிலக்கு விசை
16.	Coefficient of friction	-	உராய்வுக் குணகம்
17.	Centre of mass	-	நிறை மையம்
18.	Couple	-	இரட்டை
19.	Centre of gravity	-	ஈர்ப்பு மையம்
20.	Circular Motion	-	வட்ட இயக்கம்
21.	Collision	-	மோதல்
22.	Conservative force	-	ஆற்றல் மாற்றா விசை
23.	Dimensional Analysis	-	பரிமாண பகுப்பாய்வு
24.	Displacement	-	இடப்பெயர்ச்சி
25.	Escape -Speed	-	விடுபடு வேகம்
26.	Elastic potential energy	-	மீட்சியழுத்த ஆற்றல்
27.	Elastic collision	-	மீட்சி மோதல்
28.	Free body diagram	-	தனித்த பொருளின் விசைப்படம்
29.	Gravitational Constant	-	ஈர்ப்பியல் மாறிலி
30.	Gravitational field	-	ஈர்ப்புப்புலம்
31.	Gravitational potential	-	ஈர்ப்பழுத்தம்
32.	Gravitational Potential energy	-	ஈர்ப்பழுத்த ஆற்றல்
33.	Gross Error	-	மொத்த பிழை
34.	Geo stationary satellite	-	புவி நிலைத் துணைக்கோள்/ செயற்கைக்கோள்
35.	Horizontal plane	-	கிடைத்தளம்
36.	Horse power	-	குதிரைத்திறன்

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37.	Instantaneous Velocity	-	உடனடி / கணத் திசைவேகம்
38.	Inclined plane	-	சாய்தளம்
39.	Inertial frame	-	நிலைமக்குறிப்பாயம்
40.	Impulse	-	கணத்தாக்கு
41.	Instantaneous power	-	உடனடித்திறன்
42.	Kinetic friction	-	இயக்க உராய்வு
43.	Linear motion	-	நேர்கோட்டு இயக்கம்
44.	Linear density	-	நீள் அடர்த்தி
45.	Linear momentum	-	நேர்கோட்டு உந்தம்
46.	Least Count error	-	மீச்சிற்றளவு பிழை
47.	Motion	-	இயக்கம்
48.	Moment of inertia	-	நிலைமத்திருப்புத் திறன்
49.	Non conservative force	-	ஆற்றல் மாற்றும் விசை
50.	Orbital speed	-	சுற்றியக்க வேகம்
51.	One dimensional motion	-	ஒரு பரிமாண இயக்கம்
52.	accuracy	-	துல்லியத்தன்மை
53.	Parallax Method	-	இடமாறு தோற்றமுறை
54.	Projectile	-	எறியம்
55.	Point mass	-	புள்ளி நிறை
56.	Precison	-	நுட்பத்தன்மை
57.	Planetary Motion	-	கோள்களின் இயக்கம்
58.	Polar satellite	-	துருவ மைய துணைக்கோள்/ செயற்கைக்கோள்
59.	Precession	-	அச்சுச் சுழற்சி
60.	Pulley	-	கப்பி
61.	Rolling	-	உருளுதல்
62.	Random error	-	சமவாய்ப்பு பிழை
63.	Rounding off	-	முழுமைபடுத்துதல்
64.	Rest	-	ஓய்வு
65.	Retardation	-	எதிர் முடுக்கம்
66.	Relative velocity	-	சார்புத்திசைவேகம்
67.	Range	-	கிடைத்தள தூரம்
68.	Rolling friction	-	உருள்தலில் உராய்வு
69.	Rigid body	-	திண்மப்பொருள்
70.	Radius of gyration	-	சுழற்சி ஆரம்
71.	Rotational Motion	-	சுழற்சி இயக்கம்
72.	Spring Constant	-	சுருள் மாறிலி
73.	Sliping	-	நழுவுதல்
74.	Sliding	-	சறுக்குதல்
75.	Systematic error	-	முறையான பிழை
76.	Static friction	-	நிலை உராய்வு
77.	Significant Number	-	முக்கிய எண்ணுரு
78.	Time of flight	-	பறக்கும் நேரம்
79.	Torque	-	திருப்பு விசை
80.	Translational Motion	-	இடம்பெயர்வு இயக்கம்
81.	Tension	-	இழுவிசை
82.	Velocity	-	திசைவேகம்

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LOGARITHM TABLE

											Mean Difference								
	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	0.000	0.004	0.009	0.013	0.017	0.021	0.025	0.029	0.033	0.037	4	8	11	17	21	25	29	33	37
11	0.041	0.045	0.049	0.053	0.057	0.061	0.064	0.068	0.072	0.076	4	8	11	15	19	23	26	30	34
12	0.079	0.083	0.086	0.090	0.093	0.097	0.100	0.104	0.107	0.111	3	7	10	14	17	21	24	28	31
13	0.114	0.117	0.121	0.124	0.127	0.130	0.134	0.137	0.140	0.143	3	6	10	13	16	19	23	26	29
14	0.146	0.149	0.152	0.155	0.158	0.161	0.164	0.167	0.170	0.173	3	6	9	12	15	18	21	24	27
15	0.176	0.179	0.182	0.185	0.188	0.190	0.193	0.196	0.199	0.201	3	6	8	11	14	17	20	22	25
16	0.204	0.207	0.210	0.212	0.215	0.217	0.220	0.223	0.225	0.228	3	5	8	11	13	16	18	21	24
17	0.230	0.233	0.236	0.238	0.241	0.243	0.246	0.248	0.250	0.253	2	5	7	10	12	15	17	20	22
18	0.255	0.258	0.260	0.262	0.265	0.267	0.270	0.272	0.274	0.276	2	5	7	9	12	14	16	19	21
19	0.279	0.281	0.283	0.286	0.288	0.290	0.292	0.294	0.297	0.299	2	4	7	9	11	13	16	18	20
20	0.301	0.303	0.305	0.307	0.310	0.312	0.314	0.316	0.318	0.320	2	4	6	8	11	13	15	17	19
21	0.322	0.324	0.326	0.328	0.330	0.332	0.334	0.336	0.338	0.340	2	4	6	8	10	12	14	16	18
22	0.342	0.344	0.346	0.348	0.350	0.352	0.354	0.356	0.358	0.360	2	4	6	8	10	12	14	15	17
23	0.362	0.364	0.365	0.367	0.369	0.371	0.373	0.375	0.377	0.378	2	4	6	7	9	11	13	15	17
24	0.380	0.382	0.384	0.386	0.387	0.389	0.391	0.393	0.394	0.396	2	4	5	7	9	11	12	14	16
25	0.398	0.400	0.401	0.403	0.405	0.407	0.408	0.410	0.412	0.413	2	3	5	7	9	10	12	14	15
26	0.415	0.417	0.418	0.420	0.422	0.423	0.425	0.427	0.428	0.430	2	3	5	7	8	10	11	13	15
27	0.431	0.433	0.435	0.436	0.438	0.439	0.441	0.442	0.444	0.446	2	3	5	6	8	9	11	13	14
28	0.447	0.449	0.450	0.452	0.453	0.455	0.456	0.458	0.459	0.461	2	3	5	6	8	9	11	12	14
29	0.462	0.464	0.465	0.467	0.468	0.470	0.471	0.473	0.474	0.476	1	3	4	6	7	9	10	12	13
30	0.477	0.479	0.480	0.481	0.483	0.484	0.486	0.487	0.489	0.490	1	3	4	6	7	9	10	11	13
31	0.491	0.493	0.494	0.496	0.497	0.498	0.500	0.501	0.502	0.504	1	3	4	6	7	8	10	11	12
32	0.505	0.507	0.508	0.509	0.511	0.512	0.513	0.515	0.516	0.517	1	3	4	5	7	8	9	11	12
33	0.519	0.520	0.521	0.522	0.524	0.525	0.526	0.528	0.529	0.530	1	3	4	5	6	8	9	10	12
34	0.531	0.533	0.534	0.535	0.537	0.538	0.539	0.540	0.542	0.543	1	3	4	5	6	8	9	10	11
35	0.544	0.545	0.547	0.548	0.549	0.550	0.551	0.553	0.554	0.555	1	2	4	5	6	7	9	10	11
36	0.556	0.558	0.559	0.560	0.561	0.562	0.563	0.565	0.566	0.567	1	2	3	5	6	7	8	10	11
37	0.568	0.569	0.571	0.572	0.573	0.574	0.575	0.576	0.577	0.579	1	2	3	5	6	7	8	9	10
38	0.580	0.581	0.582	0.583	0.584	0.585	0.587	0.588	0.589	0.590	1	2	3	5	6	7	8	9	10
39	0.591	0.592	0.593	0.594	0.595	0.597	0.598	0.599	0.600	0.601	1	2	3	4	5	7	8	9	10
40	0.602	0.603	0.604	0.605	0.606	0.607	0.609	0.610	0.611	0.612	1	2	3	4	5	6	8	9	10
41	0.613	0.614	0.615	0.616	0.617	0.618	0.619	0.620	0.621	0.622	1	2	3	4	5	6	7	8	9
42	0.623	0.624	0.625	0.626	0.627	0.628	0.629	0.630	0.631	0.632	1	2	3	4	5	6	7	8	9
43	0.633	0.634	0.635	0.636	0.637	0.638	0.639	0.640	0.641	0.642	1	2	3	4	5	6	7	8	9
44	0.643	0.644	0.645	0.646	0.647	0.648	0.649	0.650	0.651	0.652	1	2	3	4	5	6	7	8	9
45	0.653	0.654	0.655	0.656	0.657	0.658	0.659	0.660	0.661	0.662	1	2	3	4	5	6	7	8	9
46	0.663	0.664	0.665	0.666	0.667	0.667	0.668	0.669	0.670	0.671	1	2	3	4	5	6	7	7	8
47	0.672	0.673	0.674	0.675	0.676	0.677	0.678	0.679	0.679	0.680	1	2	3	4	5	5	6	7	8
48	0.681	0.682	0.683	0.684	0.685	0.686	0.687	0.688	0.688	0.689	1	2	3	4	4	5	6	7	8
49	0.690	0.691	0.692	0.693	0.694	0.695	0.695	0.696	0.697	0.698	1	2	3	4	4	5	6	7	8
50	0.699	0.700	0.701	0.702	0.702	0.703	0.704	0.705	0.706	0.707	1	2	3	4	4	5	6	7	8
51	0.708	0.708	0.709	0.710	0.711	0.712	0.713	0.713	0.714	0.715	1	2	3	4	4	5	6	7	8
52	0.716	0.717	0.718	0.719	0.719	0.720	0.721	0.722	0.723	0.723	1	2	2	4	4	5	6	7	7
53	0.724	0.725	0.726	0.727	0.728	0.728	0.729	0.730	0.731	0.732	1	2	2	4	4	5	6	6	7
54	0.732	0.733	0.734	0.735	0.736	0.736	0.737	0.738	0.739	0.740	1	2	2	4	4	5	6	6	7

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LOGARITHM TABLE

											Mean Difference								
55	0.740	0.741	0.742	0.743	0.744	0.744	0.745	0.746	0.747	0.747	1	2	2	4	4	5	5	6	7
56	0.748	0.749	0.750	0.751	0.751	0.752	0.753	0.754	0.754	0.755	1	2	2	4	4	5	5	6	7
57	0.756	0.757	0.757	0.758	0.759	0.760	0.760	0.761	0.762	0.763	1	2	2	4	4	5	5	6	7
58	0.763	0.764	0.765	0.766	0.766	0.767	0.768	0.769	0.769	0.770	1	1	2	4	4	4	5	6	7
59	0.771	0.772	0.772	0.773	0.774	0.775	0.775	0.776	0.777	0.777	1	1	2	3	4	4	5	6	7
60	0.778	0.779	0.780	0.780	0.781	0.782	0.782	0.783	0.784	0.785	1	1	2	3	4	4	5	6	6
61	0.785	0.786	0.787	0.787	0.788	0.789	0.790	0.790	0.791	0.792	1	1	2	3	4	4	5	6	6
62	0.792	0.793	0.794	0.794	0.795	0.796	0.797	0.797	0.798	0.799	1	1	2	3	3	4	5	6	6
63	0.799	0.800	0.801	0.801	0.802	0.803	0.803	0.804	0.805	0.806	1	1	2	3	3	4	5	5	6
64	0.806	0.807	0.808	0.808	0.809	0.810	0.810	0.811	0.812	0.812	1	1	2	3	3	4	5	5	6
65	0.813	0.814	0.814	0.815	0.816	0.816	0.817	0.818	0.818	0.819	1	1	2	3	3	4	5	5	6
66	0.820	0.820	0.821	0.822	0.822	0.823	0.823	0.824	0.825	0.825	1	1	2	3	3	4	5	5	6
67	0.826	0.827	0.827	0.828	0.829	0.829	0.830	0.831	0.831	0.832	1	1	2	3	3	4	5	5	6
68	0.833	0.833	0.834	0.834	0.835	0.836	0.836	0.837	0.838	0.838	1	1	2	3	3	4	4	5	6
69	0.839	0.839	0.840	0.841	0.841	0.842	0.843	0.843	0.844	0.844	1	1	2	2	3	4	4	5	6
70	0.845	0.846	0.846	0.847	0.848	0.848	0.849	0.849	0.850	0.851	1	1	2	2	3	4	4	5	6
71	0.851	0.852	0.852	0.853	0.854	0.854	0.855	0.856	0.856	0.857	1	1	2	2	3	4	4	5	5
72	0.857	0.858	0.859	0.859	0.860	0.860	0.861	0.862	0.862	0.863	1	1	2	2	3	4	4	5	5
73	0.863	0.864	0.865	0.865	0.866	0.866	0.867	0.867	0.868	0.869	1	1	2	2	3	4	4	5	5
74	0.869	0.870	0.870	0.871	0.872	0.872	0.873	0.873	0.874	0.874	1	1	2	2	3	4	4	5	5
75	0.875	0.876	0.876	0.877	0.877	0.878	0.879	0.879	0.880	0.880	1	1	2	2	3	3	4	5	5
76	0.881	0.881	0.882	0.883	0.883	0.884	0.884	0.885	0.885	0.886	1	1	2	2	3	3	4	5	5
77	0.886	0.887	0.888	0.888	0.889	0.889	0.890	0.890	0.891	0.892	1	1	2	2	3	3	4	4	5
78	0.892	0.893	0.893	0.894	0.894	0.895	0.895	0.896	0.897	0.897	1	1	2	2	3	3	4	4	5
79	0.898	0.898	0.899	0.899	0.900	0.900	0.901	0.901	0.902	0.903	1	1	2	2	3	3	4	4	5
80	0.903	0.904	0.904	0.905	0.905	0.906	0.906	0.907	0.907	0.908	1	1	2	2	3	3	4	4	5
81	0.908	0.909	0.910	0.910	0.911	0.911	0.912	0.912	0.913	0.913	1	1	2	2	3	3	4	4	5
82	0.914	0.914	0.915	0.915	0.916	0.916	0.917	0.918	0.918	0.919	1	1	2	2	3	3	4	4	5
83	0.919	0.920	0.920	0.921	0.921	0.922	0.922	0.923	0.923	0.924	1	1	2	2	3	3	4	4	5
84	0.924	0.925	0.925	0.926	0.926	0.927	0.927	0.928	0.928	0.929	1	1	2	2	3	3	4	4	5
85	0.929	0.930	0.930	0.931	0.931	0.932	0.932	0.933	0.933	0.934	1	1	2	2	3	3	4	4	5
86	0.934	0.935	0.936	0.936	0.937	0.937	0.938	0.938	0.939	0.939	1	1	2	2	3	3	4	4	5
87	0.940	0.940	0.941	0.941	0.942	0.942	0.943	0.943	0.943	0.944	0	1	1	2	2	3	3	4	5
88	0.944	0.945	0.945	0.946	0.946	0.947	0.947	0.948	0.948	0.949	0	1	1	2	2	3	3	4	4
89	0.949	0.950	0.950	0.951	0.951	0.952	0.952	0.953	0.953	0.954	0	1	1	2	2	3	3	4	4
90	0.954	0.955	0.955	0.956	0.956	0.957	0.957	0.958	0.958	0.959	0	1	1	2	2	3	3	4	4
91	0.959	0.960	0.960	0.960	0.961	0.961	0.962	0.962	0.963	0.963	0	1	1	2	2	3	3	4	4
92	0.964	0.964	0.965	0.965	0.966	0.966	0.967	0.967	0.968	0.968	0	1	1	2	2	3	3	4	4
93	0.968	0.969	0.969	0.970	0.970	0.971	0.971	0.972	0.972	0.973	0	1	1	2	2	3	3	4	4
94	0.973	0.974	0.974	0.975	0.975	0.975	0.976	0.976	0.977	0.977	0	1	1	2	2	3	3	4	4
95	0.978	0.978	0.979	0.979	0.980	0.980	0.980	0.981	0.981	0.982	0	1	1	2	2	3	3	4	4
96	0.982	0.983	0.983	0.984	0.984	0.985	0.985	0.985	0.986	0.986	0	1	1	2	2	3	3	4	4
97	0.987	0.987	0.988	0.988	0.989	0.989	0.989	0.990	0.990	0.991	0	1	1	2	2	3	3	4	4
98	0.991	0.992	0.992	0.993	0.993	0.993	0.994	0.994	0.995	0.995	0	1	1	2	2	3	3	4	4
99	0.996	0.996	0.997	0.997	0.997	0.998	0.998	0.999	0.999	1.000	0	1	1	2	2	3	3	3	4

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ANTILOG TABLE

											Mean Difference								
	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
0.00	1.000	1.002	1.005	1.007	1.009	1.012	1.014	1.016	1.019	1.021	0	0	1	1	1	1	2	2	2
0.01	1.023	1.026	1.028	1.030	1.033	1.035	1.038	1.040	1.042	1.045	0	0	1	1	1	1	2	2	2
0.02	1.047	1.050	1.052	1.054	1.057	1.059	1.062	1.064	1.067	1.069	0	0	1	1	1	1	2	2	2
0.03	1.072	1.074	1.076	1.079	1.081	1.084	1.086	1.089	1.091	1.094	0	0	1	1	1	1	2	2	2
0.04	1.096	1.099	1.102	1.104	1.107	1.109	1.112	1.114	1.117	1.119	0	1	1	1	1	2	2	2	2
0.05	1.122	1.125	1.127	1.130	1.132	1.135	1.138	1.140	1.143	1.146	0	1	1	1	1	2	2	2	2
0.06	1.148	1.151	1.153	1.156	1.159	1.161	1.164	1.167	1.169	1.172	0	1	1	1	1	2	2	2	2
0.07	1.175	1.178	1.180	1.183	1.186	1.189	1.191	1.194	1.197	1.199	0	1	1	1	1	2	2	2	2
0.08	1.202	1.205	1.208	1.211	1.213	1.216	1.219	1.222	1.225	1.227	0	1	1	1	1	2	2	2	3
0.09	1.230	1.233	1.236	1.239	1.242	1.245	1.247	1.250	1.253	1.256	0	1	1	1	1	2	2	2	3
0.10	1.259	1.262	1.265	1.268	1.271	1.274	1.276	1.279	1.282	1.285	0	1	1	1	1	2	2	2	3
0.11	1.288	1.291	1.294	1.297	1.300	1.303	1.306	1.309	1.312	1.315	0	1	1	1	2	2	2	2	3
0.12	1.318	1.321	1.324	1.327	1.330	1.334	1.337	1.340	1.343	1.346	0	1	1	1	2	2	2	2	3
0.13	1.349	1.352	1.355	1.358	1.361	1.365	1.368	1.371	1.374	1.377	0	1	1	1	2	2	2	3	3
0.14	1.380	1.384	1.387	1.390	1.393	1.396	1.400	1.403	1.406	1.409	0	1	1	1	2	2	2	3	3
0.15	1.413	1.416	1.419	1.422	1.426	1.429	1.432	1.435	1.439	1.442	0	1	1	1	2	2	2	3	3
0.16	1.445	1.449	1.452	1.455	1.459	1.462	1.466	1.469	1.472	1.476	0	1	1	1	2	2	2	3	3
0.17	1.479	1.483	1.486	1.489	1.493	1.496	1.500	1.503	1.507	1.510	0	1	1	1	2	2	2	3	3
0.18	1.514	1.517	1.521	1.524	1.528	1.531	1.535	1.538	1.542	1.545	0	1	1	1	2	2	2	3	3
0.19	1.549	1.552	1.556	1.560	1.563	1.567	1.570	1.574	1.578	1.581	0	1	1	1	2	2	3	3	3
0.20	1.585	1.589	1.592	1.596	1.600	1.603	1.607	1.611	1.614	1.618	0	1	1	1	2	2	3	3	3
0.21	1.622	1.626	1.629	1.633	1.637	1.641	1.644	1.648	1.652	1.656	0	1	1	2	2	2	3	3	3
0.22	1.660	1.663	1.667	1.671	1.675	1.679	1.683	1.687	1.690	1.694	0	1	1	2	2	2	3	3	3
0.23	1.698	1.702	1.706	1.710	1.714	1.718	1.722	1.726	1.730	1.734	0	1	1	2	2	2	3	3	4
0.24	1.738	1.742	1.746	1.750	1.754	1.758	1.762	1.766	1.770	1.774	0	1	1	2	2	2	3	3	4
0.25	1.778	1.782	1.786	1.791	1.795	1.799	1.803	1.807	1.811	1.816	0	1	1	2	2	2	3	3	4
0.26	1.820	1.824	1.828	1.832	1.837	1.841	1.845	1.849	1.854	1.858	0	1	1	2	2	3	3	3	4
0.27	1.862	1.866	1.871	1.875	1.879	1.884	1.888	1.892	1.897	1.901	0	1	1	2	2	3	3	3	4
0.28	1.905	1.910	1.914	1.919	1.923	1.928	1.932	1.936	1.941	1.945	0	1	1	2	2	3	3	4	4
0.29	1.950	1.954	1.959	1.963	1.968	1.972	1.977	1.982	1.986	1.991	0	1	1	2	2	3	3	4	4
0.30	1.995	2.000	2.004	2.009	2.014	2.018	2.023	2.028	2.032	2.037	0	1	1	2	2	3	3	4	4
0.31	2.042	2.046	2.051	2.056	2.061	2.065	2.070	2.075	2.080	2.084	0	1	1	2	2	3	3	4	4
0.32	2.089	2.094	2.099	2.104	2.109	2.113	2.118	2.123	2.128	2.133	0	1	1	2	2	3	3	4	4
0.33	2.138	2.143	2.148	2.153	2.158	2.163	2.168	2.173	2.178	2.183	0	1	1	2	2	3	3	4	4
0.34	2.188	2.193	2.198	2.203	2.208	2.213	2.218	2.223	2.228	2.234	1	1	2	2	3	3	4	4	5
0.35	2.239	2.244	2.249	2.254	2.259	2.265	2.270	2.275	2.280	2.286	1	1	2	2	3	3	4	4	5
0.36	2.291	2.296	2.301	2.307	2.312	2.317	2.323	2.328	2.333	2.339	1	1	2	2	3	3	4	4	5
0.37	2.344	2.350	2.355	2.360	2.366	2.371	2.377	2.382	2.388	2.393	1	1	2	2	3	3	4	4	5
0.38	2.399	2.404	2.410	2.415	2.421	2.427	2.432	2.438	2.443	2.449	1	1	2	2	3	3	4	4	5
0.39	2.455	2.460	2.466	2.472	2.477	2.483	2.489	2.495	2.500	2.506	1	1	2	2	3	3	4	5	5
0.40	2.512	2.518	2.523	2.529	2.535	2.541	2.547	2.553	2.559	2.564	1	1	2	2	3	4	4	5	5
0.41	2.570	2.576	2.582	2.588	2.594	2.600	2.606	2.612	2.618	2.624	1	1	2	2	3	4	4	5	5
0.42	2.630	2.636	2.642	2.649	2.655	2.661	2.667	2.673	2.679	2.685	1	1	2	2	3	4	4	5	6
0.43	2.692	2.698	2.704	2.710	2.716	2.723	2.729	2.735	2.742	2.748	1	1	2	3	3	4	4	5	6
0.44	2.754	2.761	2.767	2.773	2.780	2.786	2.793	2.799	2.805	2.812	1	1	2	3	3	4	4	5	6
0.45	2.818	2.825	2.831	2.838	2.844	2.851	2.858	2.864	2.871	2.877	1	1	2	3	3	4	5	5	6
0.46	2.884	2.891	2.897	2.904	2.911	2.917	2.924	2.931	2.938	2.944	1	1	2	3	3	4	5	5	6
0.47	2.951	2.958	2.965	2.972	2.979	2.985	2.992	2.999	3.006	3.013	1	1	2	3	3	4	5	5	6
0.48	3.020	3.027	3.034	3.041	3.048	3.055	3.062	3.069	3.076	3.083	1	1	2	3	4	4	5	6	6
0.49	3.090	3.097	3.105	3.112	3.119	3.126	3.133	3.141	3.148	3.155	1	1	2	3	4	4	5	6	6

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ANTILOG TABLE

											Mean Difference								
0.50	3.162	3.170	3.177	3.184	3.192	3.199	3.206	3.214	3.221	3.228	1	1	2	3	4	4	5	6	7
0.51	3.236	3.243	3.251	3.258	3.266	3.273	3.281	3.289	3.296	3.304	1	2	2	3	4	5	5	6	7
0.52	3.311	3.319	3.327	3.334	3.342	3.350	3.357	3.365	3.373	3.381	1	2	2	3	4	5	5	6	7
0.53	3.388	3.396	3.404	3.412	3.420	3.428	3.436	3.443	3.451	3.459	1	2	2	3	4	5	6	6	7
0.54	3.467	3.475	3.483	3.491	3.499	3.508	3.516	3.524	3.532	3.540	1	2	2	3	4	5	6	6	7
0.55	3.548	3.556	3.565	3.573	3.581	3.589	3.597	3.606	3.614	3.622	1	2	2	3	4	5	6	7	7
0.56	3.631	3.639	3.648	3.656	3.664	3.673	3.681	3.690	3.698	3.707	1	2	2	3	4	5	6	7	8
0.57	3.715	3.724	3.733	3.741	3.750	3.758	3.767	3.776	3.784	3.793	1	2	3	3	4	5	6	7	8
0.58	3.802	3.811	3.819	3.828	3.837	3.846	3.855	3.864	3.873	3.882	1	2	3	4	4	5	6	7	8
0.59	3.890	3.899	3.908	3.917	3.926	3.936	3.945	3.954	3.963	3.972	1	2	3	4	5	5	6	7	8
0.60	3.981	3.990	3.999	4.009	4.018	4.027	4.036	4.046	4.055	4.064	1	2	3	4	5	6	6	7	8
0.61	4.074	4.083	4.093	4.102	4.111	4.121	4.130	4.140	4.150	4.159	1	2	3	4	5	6	7	8	9
0.62	4.169	4.178	4.188	4.198	4.207	4.217	4.227	4.236	4.246	4.256	1	2	3	4	5	6	7	8	9
0.63	4.266	4.276	4.285	4.295	4.305	4.315	4.325	4.335	4.345	4.355	1	2	3	4	5	6	7	8	9
0.64	4.365	4.375	4.385	4.395	4.406	4.416	4.426	4.436	4.446	4.457	1	2	3	4	5	6	7	8	9
0.65	4.467	4.477	4.487	4.498	4.508	4.519	4.529	4.539	4.550	4.560	1	2	3	4	5	6	7	8	9
0.66	4.571	4.581	4.592	4.603	4.613	4.624	4.634	4.645	4.656	4.667	1	2	3	4	5	6	7	9	10
0.67	4.677	4.688	4.699	4.710	4.721	4.732	4.742	4.753	4.764	4.775	1	2	3	4	5	7	7	9	10
0.68	4.786	4.797	4.808	4.819	4.831	4.842	4.853	4.864	4.875	4.887	1	2	3	4	5	7	8	9	10
0.69	4.898	4.909	4.920	4.932	4.943	4.955	4.966	4.977	4.989	5.000	1	2	3	4	5	7	8	9	10
0.70	5.012	5.023	5.035	5.047	5.058	5.070	5.082	5.093	5.105	5.117	1	2	3	4	5	7	8	9	11
0.71	5.129	5.140	5.152	5.164	5.176	5.188	5.200	5.212	5.224	5.236	1	2	4	5	6	7	8	10	11
0.72	5.248	5.260	5.272	5.284	5.297	5.309	5.321	5.333	5.346	5.358	1	2	4	5	6	7	9	10	11
0.73	5.370	5.383	5.395	5.408	5.420	5.433	5.445	5.458	5.470	5.483	1	3	4	5	6	8	9	10	11
0.74	5.495	5.508	5.521	5.534	5.546	5.559	5.572	5.585	5.598	5.610	1	3	4	5	6	8	9	10	12
0.75	5.623	5.636	5.649	5.662	5.675	5.689	5.702	5.715	5.728	5.741	1	3	4	5	7	8	9	10	12
0.76	5.754	5.768	5.781	5.794	5.808	5.821	5.834	5.848	5.861	5.875	1	3	4	5	7	8	9	11	12
0.77	5.888	5.902	5.916	5.929	5.943	5.957	5.970	5.984	5.998	6.012	1	3	4	5	7	8	10	11	12
0.78	6.026	6.039	6.053	6.067	6.081	6.095	6.109	6.124	6.138	6.152	1	3	4	6	7	8	10	11	13
0.79	6.166	6.180	6.194	6.209	6.223	6.237	6.252	6.266	6.281	6.295	1	3	4	6	7	9	10	11	13
0.80	6.310	6.324	6.339	6.353	6.368	6.383	6.397	6.412	6.427	6.442	1	3	4	6	7	9	10	12	13
0.81	6.457	6.471	6.486	6.501	6.516	6.531	6.546	6.561	6.577	6.592	2	3	5	6	8	9	11	12	14
0.82	6.607	6.622	6.637	6.653	6.668	6.683	6.699	6.714	6.730	6.745	2	3	5	6	8	9	11	12	14
0.83	6.761	6.776	6.792	6.808	6.823	6.839	6.855	6.871	6.887	6.902	2	3	5	6	8	9	11	13	14
0.84	6.918	6.934	6.950	6.966	6.982	6.998	7.015	7.031	7.047	7.063	2	3	5	6	8	10	11	13	15
0.85	7.079	7.096	7.112	7.129	7.145	7.161	7.178	7.194	7.211	7.228	2	3	5	7	8	10	12	13	15
0.86	7.244	7.261	7.278	7.295	7.311	7.328	7.345	7.362	7.379	7.396	2	3	5	7	8	10	12	13	15
0.87	7.413	7.430	7.447	7.464	7.482	7.499	7.516	7.534	7.551	7.568	2	3	5	7	9	10	12	14	16
0.88	7.586	7.603	7.621	7.638	7.656	7.674	7.691	7.709	7.727	7.745	2	3	5	7	9	10	12	14	16
0.89	7.762	7.780	7.798	7.816	7.834	7.852	7.870	7.889	7.907	7.925	2	4	5	7	9	11	12	14	16
0.90	7.943	7.962	7.980	7.998	8.017	8.035	8.054	8.072	8.091	8.110	2	4	6	7	9	11	13	15	17
0.91	8.128	8.147	8.166	8.185	8.204	8.222	8.241	8.260	8.279	8.299	2	4	6	8	9	11	13	15	17
0.92	8.318	8.337	8.356	8.375	8.395	8.414	8.433	8.453	8.472	8.492	2	4	6	8	10	12	14	15	17
0.93	8.511	8.531	8.551	8.570	8.590	8.610	8.630	8.650	8.670	8.690	2	4	6	8	10	12	14	16	18
0.94	8.710	8.730	8.750	8.770	8.790	8.810	8.831	8.851	8.872	8.892	2	4	6	8	10	12	14	16	18
0.95	8.913	8.933	8.954	8.974	8.995	9.016	9.036	9.057	9.078	9.099	2	4	6	8	10	12	15	17	19
0.96	9.120	9.141	9.162	9.183	9.204	9.226	9.247	9.268	9.290	9.311	2	4	6	8	11	13	15	17	19
0.97	9.333	9.354	9.376	9.397	9.419	9.441	9.462	9.484	9.506	9.528	2	4	7	9	11	13	15	17	20
0.98	9.550	9.572	9.594	9.616	9.638	9.661	9.683	9.705	9.727	9.750	2	4	7	9	11	13	16	18	20
0.99	9.772	9.795	9.817	9.840	9.863	9.886	9.908	9.931	9.954	9.977	2	5	7	9	11	14	16	18	20

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x	0	1	2	3	4	5	6	7	8	9
0	0	0.0017453	0.0034907	0.0052360	0.0069813	0.0087265	0.010472	0.012217	0.013962	0.015707
1	0.017452	0.019197	0.020942	0.022687	0.024432	0.026177	0.027922	0.029666	0.031411	0.033155
2	0.034899	0.036644	0.038388	0.040132	0.041876	0.043619	0.045363	0.047106	0.04885	0.050593
3	0.052336	0.054079	0.055822	0.057564	0.059306	0.061049	0.062791	0.064532	0.066274	0.068015
4	0.069756	0.071497	0.073238	0.074979	0.076719	0.078459	0.080199	0.081939	0.083678	0.085417
5	0.087156	0.088894	0.090633	0.092371	0.094108	0.095846	0.097583	0.09932	0.101056	0.102793
6	0.104528	0.106264	0.107999	0.109734	0.111469	0.113203	0.114937	0.116671	0.118404	0.120137
7	0.121869	0.123601	0.125333	0.127065	0.128796	0.130526	0.132256	0.133986	0.135716	0.137445
8	0.139173	0.140901	0.142629	0.144356	0.146083	0.147809	0.149535	0.151261	0.152986	0.15471
9	0.156434	0.158158	0.159881	0.161604	0.163326	0.165048	0.166769	0.168489	0.170209	0.171929
x	0	1	2	3	4	5	6	7	8	9
10	0.173648	0.175367	0.177085	0.178802	0.180519	0.182236	0.183951	0.185667	0.187381	0.189095
11	0.190809	0.192522	0.194234	0.195946	0.197657	0.199368	0.201078	0.202787	0.204496	0.206204
12	0.207912	0.209619	0.211325	0.21303	0.214735	0.21644	0.218143	0.219846	0.221548	0.22325
13	0.224951	0.226651	0.228351	0.23005	0.231748	0.233445	0.235142	0.236838	0.238533	0.240228
14	0.241922	0.243615	0.245307	0.246999	0.24869	0.25038	0.252069	0.253758	0.255446	0.257133
15	0.258819	0.260505	0.262189	0.263873	0.265556	0.267238	0.26892	0.2706	0.27228	0.273959
16	0.275637	0.277315	0.278991	0.280667	0.282341	0.284015	0.285688	0.287361	0.289032	0.290702
17	0.292372	0.29404	0.295708	0.297375	0.299041	0.300706	0.30237	0.304033	0.305695	0.307357
18	0.309017	0.310676	0.312335	0.313992	0.315649	0.317305	0.318959	0.320613	0.322266	0.323917
19	0.325568	0.327218	0.328867	0.330514	0.332161	0.333807	0.335452	0.337095	0.338738	0.34038
x	0	1	2	3	4	5	6	7	8	9
20	0.34202	0.34366	0.345298	0.346936	0.348572	0.350207	0.351842	0.353475	0.355107	0.356738
21	0.358368	0.359997	0.361625	0.363251	0.364877	0.366501	0.368125	0.369747	0.371368	0.372988
22	0.374607	0.376224	0.377841	0.379456	0.38107	0.382683	0.384295	0.385906	0.387516	0.389124
23	0.390731	0.392337	0.393942	0.395546	0.397148	0.398749	0.400349	0.401948	0.403545	0.405142
24	0.406737	0.40833	0.409923	0.411514	0.413104	0.414693	0.416281	0.417867	0.419452	0.421036
25	0.422618	0.424199	0.425779	0.427358	0.428935	0.430511	0.432086	0.433659	0.435231	0.436802
26	0.438371	0.439939	0.441506	0.443071	0.444635	0.446198	0.447759	0.449319	0.450878	0.452435
27	0.45399	0.455545	0.457098	0.45865	0.4602	0.461749	0.463296	0.464842	0.466387	0.46793
28	0.469472	0.471012	0.472551	0.474088	0.475624	0.477159	0.478692	0.480223	0.481754	0.483282
29	0.48481	0.486335	0.48786	0.489382	0.490904	0.492424	0.493942	0.495459	0.496974	0.498488
x	0	1	2	3	4	5	6	7	8	9
30	0.5	0.501511	0.50302	0.504528	0.506034	0.507538	0.509041	0.510543	0.512043	0.513541
31	0.515038	0.516533	0.518027	0.519519	0.52101	0.522499	0.523986	0.525472	0.526956	0.528438
32	0.529919	0.531399	0.532876	0.534352	0.535827	0.5373	0.538771	0.54024	0.541708	0.543174
33	0.544639	0.546102	0.547563	0.549023	0.550481	0.551937	0.553392	0.554844	0.556296	0.557745
34	0.559193	0.560639	0.562083	0.563526	0.564967	0.566406	0.567844	0.56928	0.570714	0.572146
35	0.573576	0.575005	0.576432	0.577858	0.579281	0.580703	0.582123	0.583541	0.584958	0.586372
36	0.587785	0.589196	0.590606	0.592013	0.593419	0.594823	0.596225	0.597625	0.599024	0.60042
37	0.601815	0.603208	0.604599	0.605988	0.607376	0.608761	0.610145	0.611527	0.612907	0.614285
38	0.615661	0.617036	0.618408	0.619779	0.621148	0.622515	0.62388	0.625243	0.626604	0.627963
39	0.62932	0.630676	0.632029	0.633381	0.634731	0.636078	0.637424	0.638768	0.64011	0.64145
x	0	1	2	3	4	5	6	7	8	9
40	0.642788	0.644124	0.645458	0.64679	0.64812	0.649448	0.650774	0.652098	0.653421	0.654741
41	0.656059	0.657375	0.658689	0.660002	0.661312	0.66262	0.663926	0.66523	0.666532	0.667833
42	0.669131	0.670427	0.671721	0.673013	0.674302	0.67559	0.676876	0.67816	0.679441	0.680721
43	0.681998	0.683274	0.684547	0.685818	0.687088	0.688355	0.68962	0.690882	0.692143	0.693402
44	0.694658	0.695913	0.697165	0.698415	0.699663	0.700909	0.702153	0.703395	0.704634	0.705872
45	0.707107	0.70834	0.709571	0.710799	0.712026	0.71325	0.714473	0.715693	0.716911	0.718126

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46	0.71934	0.720551	0.72176	0.722967	0.724172	0.725374	0.726575	0.727773	0.728969	0.730162
47	0.731354	0.732543	0.73373	0.734915	0.736097	0.737277	0.738455	0.739631	0.740805	0.741976
48	0.743145	0.744312	0.745476	0.746638	0.747798	0.748956	0.750111	0.751264	0.752415	0.753563
49	0.75471	0.755853	0.756995	0.758134	0.759271	0.760406	0.761538	0.762668	0.763796	0.764921
x	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
50	0.766044	0.767165	0.768284	0.7694	0.770513	0.771625	0.772734	0.77384	0.774944	0.776046
51	0.777146	0.778243	0.779338	0.78043	0.78152	0.782608	0.783693	0.784776	0.785857	0.786935
52	0.788011	0.789084	0.790155	0.791224	0.79229	0.793353	0.794415	0.795473	0.79653	0.797584
53	0.798636	0.799685	0.800731	0.801776	0.802817	0.803857	0.804894	0.805928	0.80696	0.80799
54	0.809017	0.810042	0.811064	0.812084	0.813101	0.814116	0.815128	0.816138	0.817145	0.81815
55	0.819152	0.820152	0.821149	0.822144	0.823136	0.824126	0.825113	0.826098	0.827081	0.82806
56	0.829038	0.830012	0.830984	0.831954	0.832921	0.833886	0.834848	0.835807	0.836764	0.837719
57	0.838671	0.83962	0.840567	0.841511	0.842452	0.843391	0.844328	0.845262	0.846193	0.847122
58	0.848048	0.848972	0.849893	0.850811	0.851727	0.85264	0.853551	0.854459	0.855364	0.856267
59	0.857167	0.858065	0.85896	0.859852	0.860742	0.861629	0.862514	0.863396	0.864275	0.865151
x	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
60	0.866025	0.866897	0.867765	0.868632	0.869495	0.870356	0.871214	0.872069	0.872922	0.873772
61	0.87462	0.875465	0.876307	0.877146	0.877983	0.878817	0.879649	0.880477	0.881303	0.882127
62	0.882948	0.883766	0.884581	0.885394	0.886204	0.887011	0.887815	0.888617	0.889416	0.890213
63	0.891007	0.891798	0.892586	0.893371	0.894154	0.894934	0.895712	0.896486	0.897258	0.898028
64	0.898794	0.899558	0.900319	0.901077	0.901833	0.902585	0.903335	0.904083	0.904827	0.905569
65	0.906308	0.907044	0.907777	0.908508	0.909236	0.909961	0.910684	0.911403	0.91212	0.912834
66	0.913545	0.914254	0.91496	0.915663	0.916363	0.91706	0.917755	0.918446	0.919135	0.919821
67	0.920505	0.921185	0.921863	0.922538	0.92321	0.92388	0.924546	0.92521	0.925871	0.926529
68	0.927184	0.927836	0.928486	0.929133	0.929776	0.930418	0.931056	0.931691	0.932324	0.932954
69	0.93358	0.934204	0.934826	0.935444	0.93606	0.936672	0.937282	0.937889	0.938493	0.939094
x	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
70	0.939693	0.940288	0.940881	0.941471	0.942057	0.942641	0.943223	0.943801	0.944376	0.944949
71	0.945519	0.946085	0.946649	0.94721	0.947768	0.948324	0.948876	0.949425	0.949972	0.950516
72	0.951057	0.951594	0.952129	0.952661	0.953191	0.953717	0.95424	0.954761	0.955278	0.955793
73	0.956305	0.956814	0.957319	0.957822	0.958323	0.95882	0.959314	0.959805	0.960294	0.960779
74	0.961262	0.961741	0.962218	0.962692	0.963163	0.96363	0.964095	0.964557	0.965016	0.965473
75	0.965926	0.966376	0.966823	0.967268	0.967709	0.968148	0.968583	0.969016	0.969445	0.969872
76	0.970296	0.970716	0.971134	0.971549	0.971961	0.97237	0.972776	0.973179	0.973579	0.973976
77	0.97437	0.974761	0.975149	0.975535	0.975917	0.976296	0.976672	0.977046	0.977416	0.977783
78	0.978148	0.978509	0.978867	0.979223	0.979575	0.979925	0.980271	0.980615	0.980955	0.981293
79	0.981627	0.981959	0.982287	0.982613	0.982935	0.983255	0.983571	0.983885	0.984196	0.984503
x	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
80	0.984808	0.985109	0.985408	0.985703	0.985996	0.986286	0.986572	0.986856	0.987136	0.987414
81	0.987688	0.98796	0.988228	0.988494	0.988756	0.989016	0.989272	0.989526	0.989776	0.990024
82	0.990268	0.990509	0.990748	0.990983	0.991216	0.991445	0.991671	0.991894	0.992115	0.992332
83	0.992546	0.992757	0.992966	0.993171	0.993373	0.993572	0.993768	0.993961	0.994151	0.994338
84	0.994522	0.994703	0.994881	0.995056	0.995227	0.995396	0.995562	0.995725	0.995884	0.996041
85	0.996195	0.996345	0.996493	0.996637	0.996779	0.996917	0.997053	0.997185	0.997314	0.997441
86	0.997564	0.997684	0.997801	0.997916	0.998027	0.998135	0.99824	0.998342	0.998441	0.998537
87	0.99863	0.998719	0.998806	0.99889	0.998971	0.999048	0.999123	0.999194	0.999263	0.999328
88	0.999391	0.99945	0.999507	0.99956	0.99961	0.999657	0.999701	0.999743	0.999781	0.999816
89	0.999848	0.999877	0.999903	0.999925	0.999945	0.999962	0.999976	0.999986	0.999994	0.999998
90	1	0.999998	0.999994	0.999986	0.999976	0.999962	0.999945	0.999925	0.999903	0.999877

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NATURAL COSINES

x	0	1	2	3	4	5	6	7	8	9
0	1	0.999998	0.999994	0.999986	0.999976	0.999962	0.999945	0.999925	0.999903	0.999877
1	0.999848	0.999816	0.999781	0.999743	0.999701	0.999657	0.99961	0.99956	0.999507	0.99945
2	0.999391	0.999328	0.999263	0.999194	0.999123	0.999048	0.998971	0.99889	0.998806	0.998719
3	0.99863	0.998537	0.998441	0.998342	0.99824	0.998135	0.998027	0.997916	0.997801	0.997684
4	0.997564	0.997441	0.997314	0.997185	0.997053	0.996917	0.996779	0.996637	0.996493	0.996345
5	0.996195	0.996041	0.995884	0.995725	0.995562	0.995396	0.995227	0.995056	0.994881	0.994703
6	0.994522	0.994338	0.994151	0.993961	0.993768	0.993572	0.993373	0.993171	0.992966	0.992757
7	0.992546	0.992332	0.992115	0.991894	0.991671	0.991445	0.991216	0.990983	0.990748	0.990509
8	0.990268	0.990024	0.989776	0.989526	0.989272	0.989016	0.988756	0.988494	0.988228	0.98796
9	0.987688	0.987414	0.987136	0.986856	0.986572	0.986286	0.985996	0.985703	0.985408	0.985109
x	0	1	2	3	4	5	6	7	8	9
10	0.984808	0.984503	0.984196	0.983885	0.983571	0.983255	0.982935	0.982613	0.982287	0.981959
11	0.981627	0.981293	0.980955	0.980615	0.980271	0.979925	0.979575	0.979223	0.978867	0.978509
12	0.978148	0.977783	0.977416	0.977046	0.976672	0.976296	0.975917	0.975535	0.975149	0.974761
13	0.97437	0.973976	0.973579	0.973179	0.972776	0.97237	0.971961	0.971549	0.971134	0.970716
14	0.970296	0.969872	0.969445	0.969016	0.968583	0.968148	0.967709	0.967268	0.966823	0.966376
15	0.965926	0.965473	0.965016	0.964557	0.964095	0.96363	0.963163	0.962692	0.962218	0.961741
16	0.961262	0.960779	0.960294	0.959805	0.959314	0.95882	0.958323	0.957822	0.957319	0.956814
17	0.956305	0.955793	0.955278	0.954761	0.95424	0.953717	0.953191	0.952661	0.952129	0.951594
18	0.951057	0.950516	0.949972	0.949425	0.948876	0.948324	0.947768	0.94721	0.946649	0.946085
19	0.945519	0.944949	0.944376	0.943801	0.943223	0.942641	0.942057	0.941471	0.940881	0.940288
x	0	1	2	3	4	5	6	7	8	9
20	0.939693	0.939094	0.938493	0.937889	0.937282	0.936672	0.93606	0.935444	0.934826	0.934204
21	0.93358	0.932954	0.932324	0.931691	0.931056	0.930418	0.929776	0.929133	0.928486	0.927836
22	0.927184	0.926529	0.925871	0.92521	0.924546	0.92388	0.92321	0.922538	0.921863	0.921185
23	0.920505	0.919821	0.919135	0.918446	0.917755	0.91706	0.916363	0.915663	0.91496	0.914254
24	0.913545	0.912834	0.91212	0.911403	0.910684	0.909961	0.909236	0.908508	0.907777	0.907044
25	0.906308	0.905569	0.904827	0.904083	0.903335	0.902585	0.901833	0.901077	0.900319	0.899558
26	0.898794	0.898028	0.897258	0.896486	0.895712	0.894934	0.894154	0.893371	0.892586	0.891798
27	0.891007	0.890213	0.889416	0.888617	0.887815	0.887011	0.886204	0.885394	0.884581	0.883766
28	0.882948	0.882127	0.881303	0.880477	0.879649	0.878817	0.877983	0.877146	0.876307	0.875465
29	0.87462	0.873772	0.872922	0.872069	0.871214	0.870356	0.869495	0.868632	0.867765	0.866897
x	0	1	2	3	4	5	6	7	8	9
30	0.866025	0.865151	0.864275	0.863396	0.862514	0.861629	0.860742	0.859852	0.85896	0.858065
31	0.857167	0.856267	0.855364	0.854459	0.853551	0.85264	0.851727	0.850811	0.849893	0.848972
32	0.848048	0.847122	0.846193	0.845262	0.844328	0.843391	0.842452	0.841511	0.840567	0.83962
33	0.838671	0.837719	0.836764	0.835807	0.834848	0.833886	0.832921	0.831954	0.830984	0.830012
34	0.829038	0.82806	0.827081	0.826098	0.825113	0.824126	0.823136	0.822144	0.821149	0.820152
35	0.819152	0.81815	0.817145	0.816138	0.815128	0.814116	0.813101	0.812084	0.811064	0.810042
36	0.809017	0.80799	0.80696	0.805928	0.804894	0.803857	0.802817	0.801776	0.800731	0.799685
37	0.798636	0.797584	0.79653	0.795473	0.794415	0.793353	0.79229	0.791224	0.790155	0.789084
38	0.788011	0.786935	0.785857	0.784776	0.783693	0.782608	0.78152	0.78043	0.779338	0.778243
39	0.777146	0.776046	0.774944	0.77384	0.772734	0.771625	0.770513	0.7694	0.768284	0.767165
x	0	1	2	3	4	5	6	7	8	9
40	0.766044	0.764921	0.763796	0.762668	0.761538	0.760406	0.759271	0.758134	0.756995	0.755853
41	0.75471	0.753563	0.752415	0.751264	0.750111	0.748956	0.747798	0.746638	0.745476	0.744312
42	0.743145	0.741976	0.740805	0.739631	0.738455	0.737277	0.736097	0.734915	0.73373	0.732543
43	0.731354	0.730162	0.728969	0.727773	0.726575	0.725374	0.724172	0.722967	0.72176	0.720551
44	0.71934	0.718126	0.716911	0.715693	0.714473	0.71325	0.712026	0.710799	0.709571	0.70834
45	0.707107	0.705872	0.704634	0.703395	0.702153	0.700909	0.699663	0.698415	0.697165	0.695913

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NATURAL COSINES

46	0.694658	0.693402	0.692143	0.690882	0.68962	0.688355	0.687088	0.685818	0.684547	0.683274
47	0.681998	0.680721	0.679441	0.67816	0.676876	0.67559	0.674302	0.673013	0.671721	0.670427
48	0.669131	0.667833	0.666532	0.66523	0.663926	0.66262	0.661312	0.660002	0.658689	0.657375
49	0.656059	0.654741	0.653421	0.652098	0.650774	0.649448	0.64812	0.64679	0.645458	0.644124
x	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
50	0.642788	0.64145	0.64011	0.638768	0.637424	0.636078	0.634731	0.633381	0.632029	0.630676
51	0.62932	0.627963	0.626604	0.625243	0.62388	0.622515	0.621148	0.619779	0.618408	0.617036
52	0.615661	0.614285	0.612907	0.611527	0.610145	0.608761	0.607376	0.605988	0.604599	0.603208
53	0.601815	0.60042	0.599024	0.597625	0.596225	0.594823	0.593419	0.592013	0.590606	0.589196
54	0.587785	0.586372	0.584958	0.583541	0.582123	0.580703	0.579281	0.577858	0.576432	0.575005
55	0.573576	0.572146	0.570714	0.56928	0.567844	0.566406	0.564967	0.563526	0.562083	0.560639
56	0.559193	0.557745	0.556296	0.554844	0.553392	0.551937	0.550481	0.549023	0.547563	0.546102
57	0.544639	0.543174	0.541708	0.54024	0.538771	0.5373	0.535827	0.534352	0.532876	0.531399
58	0.529919	0.528438	0.526956	0.525472	0.523986	0.522499	0.52101	0.519519	0.518027	0.516533
59	0.515038	0.513541	0.512043	0.510543	0.509041	0.507538	0.506034	0.504528	0.50302	0.501511
x	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
60	0.5	0.498488	0.496974	0.495459	0.493942	0.492424	0.490904	0.489382	0.48786	0.486335
61	0.48481	0.483282	0.481754	0.480223	0.478692	0.477159	0.475624	0.474088	0.472551	0.471012
62	0.469472	0.46793	0.466387	0.464842	0.463296	0.461749	0.4602	0.45865	0.457098	0.455545
63	0.45399	0.452435	0.450878	0.449319	0.447759	0.446198	0.444635	0.443071	0.441506	0.439939
64	0.438371	0.436802	0.435231	0.433659	0.432086	0.430511	0.428935	0.427358	0.425779	0.424199
65	0.422618	0.421036	0.419452	0.417867	0.416281	0.414693	0.413104	0.411514	0.409923	0.40833
66	0.406737	0.405142	0.403545	0.401948	0.400349	0.398749	0.397148	0.395546	0.393942	0.392337
67	0.390731	0.389124	0.387516	0.385906	0.384295	0.382683	0.38107	0.379456	0.377841	0.376224
68	0.374607	0.372988	0.371368	0.369747	0.368125	0.366501	0.364877	0.363251	0.361625	0.359997
69	0.358368	0.356738	0.355107	0.353475	0.351842	0.350207	0.348572	0.346936	0.345298	0.34366
x	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
70	0.34202	0.34038	0.338738	0.337095	0.335452	0.333807	0.332161	0.330514	0.328867	0.327218
71	0.325568	0.323917	0.322266	0.320613	0.318959	0.317305	0.315649	0.313992	0.312335	0.310676
72	0.309017	0.307357	0.305695	0.304033	0.30237	0.300706	0.299041	0.297375	0.295708	0.29404
73	0.292372	0.290702	0.289032	0.287361	0.285688	0.284015	0.282341	0.280667	0.278991	0.277315
74	0.275637	0.273959	0.27228	0.2706	0.26892	0.267238	0.265556	0.263873	0.262189	0.260505
75	0.258819	0.257133	0.255446	0.253758	0.252069	0.25038	0.24869	0.246999	0.245307	0.243615
76	0.241922	0.240228	0.238533	0.236838	0.235142	0.233445	0.231748	0.23005	0.228351	0.226651
77	0.224951	0.22325	0.221548	0.219846	0.218143	0.21644	0.214735	0.21303	0.211325	0.209619
78	0.207912	0.206204	0.204496	0.202787	0.201078	0.199368	0.197657	0.195946	0.194234	0.192522
79	0.190809	0.189095	0.187381	0.185667	0.183951	0.182236	0.180519	0.178802	0.177085	0.175367
x	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
80	0.173648	0.171929	0.170209	0.168489	0.166769	0.165048	0.163326	0.161604	0.159881	0.158158
81	0.156434	0.15471	0.152986	0.151261	0.149535	0.147809	0.146083	0.144356	0.142629	0.140901
82	0.139173	0.137445	0.135716	0.133986	0.132256	0.130526	0.128796	0.127065	0.125333	0.123601
83	0.121869	0.120137	0.118404	0.116671	0.114937	0.113203	0.111469	0.109734	0.107999	0.106264
84	0.104528	0.102793	0.101056	0.09932	0.097583	0.095846	0.094108	0.092371	0.090633	0.088894
85	0.087156	0.085417	0.083678	0.081939	0.080199	0.078459	0.076719	0.074979	0.073238	0.071497
86	0.069756	0.068015	0.066274	0.064532	0.062791	0.061049	0.059306	0.057564	0.055822	0.054079
87	0.052336	0.050593	0.04885	0.047106	0.045363	0.043619	0.041876	0.040132	0.038388	0.036644
88	0.034899	0.033155	0.031411	0.029666	0.027922	0.026177	0.024432	0.022687	0.020942	0.019197
89	0.017452	0.015707	0.013962	0.012217	0.010472	0.0087265	0.0069813	0.0052360	0.0034907	0.0017453
90	0	0.0017453	0.0034907	0.0052360	0.0069813	0.0087265	0.010472	0.012217	0.013962	0.015707

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NATURAL TANGENTS

x	0	1	2	3	4	5	6	7	8	9
0	0	0.0017453	0.0034907	0.0052360	0.0069814	0.0087269	0.010472	0.012218	0.013964	0.015709
1	0.017455	0.019201	0.020947	0.022693	0.024439	0.026186	0.027933	0.029679	0.031426	0.033173
2	0.034921	0.036668	0.038416	0.040164	0.041912	0.043661	0.04541	0.047159	0.048908	0.050658
3	0.052408	0.054158	0.055909	0.05766	0.059411	0.061163	0.062915	0.064667	0.06642	0.068173
4	0.069927	0.071681	0.073435	0.07519	0.076946	0.078702	0.080458	0.082215	0.083972	0.08573
5	0.087489	0.089248	0.091007	0.092767	0.094528	0.096289	0.098051	0.099813	0.101576	0.10334
6	0.105104	0.106869	0.108635	0.110401	0.112168	0.113936	0.115704	0.117473	0.119243	0.121013
7	0.122785	0.124557	0.126329	0.128103	0.129877	0.131652	0.133428	0.135205	0.136983	0.138761
8	0.140541	0.142321	0.144102	0.145884	0.147667	0.149451	0.151236	0.153022	0.154808	0.156596
9	0.158384	0.160174	0.161965	0.163756	0.165549	0.167343	0.169137	0.170933	0.17273	0.174528
x	0	1	2	3	4	5	6	7	8	9
10	0.176327	0.178127	0.179928	0.181731	0.183534	0.185339	0.187145	0.188952	0.19076	0.19257
11	0.19438	0.196192	0.198005	0.19982	0.201635	0.203452	0.205271	0.20709	0.208911	0.210733
12	0.212557	0.214381	0.216208	0.218035	0.219864	0.221695	0.223526	0.22536	0.227194	0.229031
13	0.230868	0.232707	0.234548	0.23639	0.238234	0.240079	0.241925	0.243774	0.245624	0.247475
14	0.249328	0.251183	0.253039	0.254897	0.256756	0.258618	0.26048	0.262345	0.264211	0.266079
15	0.267949	0.269821	0.271694	0.273569	0.275446	0.277325	0.279205	0.281087	0.282971	0.284857
16	0.286745	0.288635	0.290527	0.29242	0.294316	0.296213	0.298113	0.300014	0.301918	0.303823
17	0.305731	0.30764	0.309552	0.311465	0.313381	0.315299	0.317219	0.319141	0.321065	0.322991
18	0.32492	0.32685	0.328783	0.330718	0.332656	0.334595	0.336537	0.338481	0.340428	0.342377
19	0.344328	0.346281	0.348237	0.350195	0.352156	0.354119	0.356084	0.358052	0.360022	0.361995
x	0	1	2	3	4	5	6	7	8	9
20	0.36397	0.365948	0.367928	0.369911	0.371897	0.373885	0.375875	0.377869	0.379864	0.381863
21	0.383864	0.385868	0.387874	0.389884	0.391896	0.39391	0.395928	0.397948	0.399971	0.401997
22	0.404026	0.406058	0.408092	0.41013	0.41217	0.414214	0.41626	0.418309	0.420361	0.422417
23	0.424475	0.426536	0.428601	0.430668	0.432739	0.434812	0.436889	0.438969	0.441053	0.443139
24	0.445229	0.447322	0.449418	0.451517	0.45362	0.455726	0.457836	0.459949	0.462065	0.464185
25	0.466308	0.468434	0.470564	0.472698	0.474835	0.476976	0.47912	0.481267	0.483419	0.485574
26	0.487733	0.489895	0.492061	0.494231	0.496404	0.498582	0.500763	0.502948	0.505136	0.507329
27	0.509525	0.511726	0.51393	0.516138	0.518351	0.520567	0.522787	0.525012	0.52724	0.529473
28	0.531709	0.53395	0.536195	0.538445	0.540698	0.542956	0.545218	0.547484	0.549755	0.55203
29	0.554309	0.556593	0.558881	0.561174	0.563471	0.565773	0.568079	0.57039	0.572705	0.575026
x	0	1	2	3	4	5	6	7	8	9
30	0.57735	0.57968	0.582014	0.584353	0.586697	0.589045	0.591398	0.593757	0.59612	0.598488
31	0.600861	0.603239	0.605622	0.60801	0.610403	0.612801	0.615204	0.617613	0.620026	0.622445
32	0.624869	0.627299	0.629734	0.632174	0.634619	0.63707	0.639527	0.641989	0.644456	0.646929
33	0.649408	0.651892	0.654382	0.656877	0.659379	0.661886	0.664398	0.666917	0.669442	0.671972
34	0.674509	0.677051	0.679599	0.682154	0.684714	0.687281	0.689854	0.692433	0.695018	0.69761
35	0.700208	0.702812	0.705422	0.708039	0.710663	0.713293	0.71593	0.718573	0.721223	0.723879
36	0.726543	0.729213	0.731889	0.734573	0.737264	0.739961	0.742666	0.745377	0.748096	0.750821
37	0.753554	0.756294	0.759041	0.761796	0.764558	0.767327	0.770104	0.772888	0.77568	0.778479
38	0.781286	0.7841	0.786922	0.789752	0.79259	0.795436	0.79829	0.801151	0.804021	0.806898
39	0.809784	0.812678	0.81558	0.818491	0.821409	0.824336	0.827272	0.830216	0.833169	0.83613
x	0	1	2	3	4	5	6	7	8	9
40	0.8391	0.842078	0.845066	0.848062	0.851067	0.854081	0.857104	0.860136	0.863177	0.866227
41	0.869287	0.872356	0.875434	0.878521	0.881619	0.884725	0.887842	0.890967	0.894103	0.897249
42	0.900404	0.903569	0.906745	0.90993	0.913125	0.916331	0.919547	0.922773	0.92601	0.929257
43	0.932515	0.935783	0.939063	0.942352	0.945653	0.948965	0.952287	0.955621	0.958966	0.962322
44	0.965689	0.969067	0.972458	0.975859	0.979272	0.982697	0.986134	0.989582	0.993043	0.996515
45	1	1.0035	1.00701	1.01053	1.01406	1.01761	1.02117	1.02474	1.02832	1.03192

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46	1.03553	1.03915	1.04279	1.04644	1.0501	1.05378	1.05747	1.06117	1.06489	1.06862
47	1.07237	1.07613	1.0799	1.08369	1.08749	1.09131	1.09514	1.09899	1.10285	1.10672
48	1.11061	1.11452	1.11844	1.12238	1.12633	1.13029	1.13428	1.13828	1.14229	1.14632
49	1.15037	1.15443	1.15851	1.16261	1.16672	1.17085	1.175	1.17916	1.18334	1.18754
x	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
50	1.19175	1.19599	1.20024	1.20451	1.20879	1.2131	1.21742	1.22176	1.22612	1.2305
51	1.2349	1.23931	1.24375	1.2482	1.25268	1.25717	1.26169	1.26622	1.27077	1.27535
52	1.27994	1.28456	1.28919	1.29385	1.29853	1.30323	1.30795	1.31269	1.31745	1.32224
53	1.32704	1.33187	1.33673	1.3416	1.3465	1.35142	1.35637	1.36134	1.36633	1.37134
54	1.37638	1.38145	1.38653	1.39165	1.39679	1.40195	1.40714	1.41235	1.41759	1.42286
55	1.42815	1.43347	1.43881	1.44418	1.44958	1.45501	1.46046	1.46595	1.47146	1.47699
56	1.48256	1.48816	1.49378	1.49944	1.50512	1.51084	1.51658	1.52235	1.52816	1.534
57	1.53986	1.54576	1.5517	1.55766	1.56366	1.56969	1.57575	1.58184	1.58797	1.59414
58	1.60033	1.60657	1.61283	1.61914	1.62548	1.63185	1.63826	1.64471	1.6512	1.65772
59	1.66428	1.67088	1.67752	1.68419	1.69091	1.69766	1.70446	1.71129	1.71817	1.72509
x	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
60	1.73205	1.73905	1.7461	1.75319	1.76032	1.76749	1.77471	1.78198	1.78929	1.79665
61	1.80405	1.8115	1.81899	1.82654	1.83413	1.84177	1.84946	1.8572	1.86499	1.87283
62	1.88073	1.88867	1.89667	1.90472	1.91282	1.92098	1.9292	1.93746	1.94579	1.95417
63	1.96261	1.97111	1.97966	1.98828	1.99695	2.00569	2.01449	2.02335	2.03227	2.04125
64	2.0503	2.05942	2.0686	2.07785	2.08716	2.09654	2.106	2.11552	2.12511	2.13477
65	2.14451	2.15432	2.1642	2.17416	2.18419	2.1943	2.20449	2.21475	2.2251	2.23553
66	2.24604	2.25663	2.2673	2.27806	2.28891	2.29984	2.31086	2.32197	2.33317	2.34447
67	2.35585	2.36733	2.37891	2.39058	2.40235	2.41421	2.42618	2.43825	2.45043	2.4627
68	2.47509	2.48758	2.50018	2.51289	2.52571	2.53865	2.5517	2.56487	2.57815	2.59156
69	2.60509	2.61874	2.63252	2.64642	2.66046	2.67462	2.68892	2.70335	2.71792	2.73263
x	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
70	2.74748	2.76247	2.77761	2.79289	2.80833	2.82391	2.83965	2.85555	2.87161	2.88783
71	2.90421	2.92076	2.93748	2.95437	2.97144	2.98868	3.00611	3.02372	3.04152	3.0595
72	3.07768	3.09606	3.11464	3.13341	3.1524	3.17159	3.191	3.21063	3.23048	3.25055
73	3.27085	3.29139	3.31216	3.33317	3.35443	3.37594	3.39771	3.41973	3.44202	3.46458
74	3.48741	3.51053	3.53393	3.55761	3.5816	3.60588	3.63048	3.65538	3.68061	3.70616
75	3.73205	3.75828	3.78485	3.81177	3.83906	3.86671	3.89474	3.92316	3.95196	3.98117
76	4.01078	4.04081	4.07127	4.10216	4.1335	4.1653	4.19756	4.2303	4.26352	4.29724
77	4.33148	4.36623	4.40152	4.43735	4.47374	4.51071	4.54826	4.58641	4.62518	4.66458
78	4.70463	4.74534	4.78673	4.82882	4.87162	4.91516	4.95945	5.00451	5.05037	5.09704
79	5.14455	5.19293	5.24218	5.29235	5.34345	5.39552	5.44857	5.50264	5.55777	5.61397
x	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
80	5.67128	5.72974	5.78938	5.85024	5.91236	5.97576	6.04051	6.10664	6.17419	6.24321
81	6.31375	6.38587	6.45961	6.53503	6.61219	6.69116	6.77199	6.85475	6.93952	7.02637
82	7.11537	7.20661	7.30018	7.39616	7.49465	7.59575	7.69957	7.80622	7.91582	8.02848
83	8.14435	8.26355	8.38625	8.51259	8.64275	8.77689	8.9152	9.05789	9.20516	9.35724
84	9.51436	9.6768	9.84482	10.0187	10.1988	10.3854	10.5789	10.7797	10.9882	11.2048
85	11.4301	11.6645	11.9087	12.1632	12.4288	12.7062	12.9962	13.2996	13.6174	13.9507
86	14.3007	14.6685	15.0557	15.4638	15.8945	16.3499	16.8319	17.3432	17.8863	18.4645
87	19.0811	19.7403	20.4465	21.2049	22.0217	22.9038	23.8593	24.8978	26.0307	27.2715
88	28.6363	30.1446	31.8205	33.6935	35.8006	38.1885	40.9174	44.0661	47.7395	52.0807
89	57.29	63.6567	71.6151	81.847	95.4895	114.589	143.237	190.984	286.478	572.957
90	∞									

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This book has been printed on 80 G.S.M.
Elegant Maplitho paper.
Printed by offset at:

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First Edition - 2018

Revised Edition - 2019, 2020, 2022

(Published Under New Syllabus)

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Content Creation

State Council of Educational Research and Training

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Printing & Publishing

Tamil NaduTextbook and Educational Services Corporation

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	• Additional facts related to the topics covered to facilitate curiosity driven learning
Example problems	• To ensure understanding, problems/illustrations are given at every stage before advancing to next level
	• Visual representation of concepts with illustrations • Videos, animations, and tutorials
ICT	• To harness the digital skills to class room learning and experimenting
Summary	• Recap of salient points of the lesson
Concept Map	• Schematic outline of salient learning of the unit
Evaluation	• Evaluate students' understanding and get them acquainted with the application of physical concepts to numerical and conceptual questions
Books for Reference	• List of relevant books for further reading
Solved examples	• Solutions to exercise problems are accessible here. In addition, a few solved examples are given to facilitate students to apply the concepts learnt.
Competitive Exam corner	• Model Questions - To motivate students aspiring to take up competitive examinations such as NEET, JEE, Physics Olympiad, JIPMER etc
Practical	• List of practical and the description of each is appended for easy access.
Glossary	• Scientific terms frequently used with their Tamil equivalents

Back wrapper: Solvay Conference 1927, Belgium

Photograph of the attendees of the most famous fifth solvay international conference held in october 1972. 17 of the 29 scientists found in this photograph are Nobel laureates (shown in bold).

Front row (L to R) : **I. Langmuir**, **M. Planck**, **Marie Curie**, **H.A. Lorentz**, **A. Einstein**, P. Langevin, Ch.E. Guye, **C.T.R. Wilson**, **O.W. Richardson**

Middle row (L to R) : **P. Debye**, M. Knudsen, **W.L. Bragg**, H.A. Kramers, **P.A.M. Dirac**, **A.H. Compton**, **L.de Broglie**, **M. Born**, **N. Bohr**

Back row (L to R) : A. Piccard, E. Henriot, P. Ehrenfest, Ed. Herzen, Th.De Donder, **E. Schrödinger**, E.Verschaffelt, **W. Pauli**, **W. Heisenberg**, R.H. Fowler, L. Brillouin

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- Physics Olympiad Exam
- NEET- National Eligibility and Entrance Test
- NEST- National Entrance Screening Test
- AIEEE- All India Engineering Entrance Exam
- AIIMS- All India Institute of Medical Science (Entrance Examination)
- JIPMER- Jawaharlal institute of Postgraduate Medical Education and Research (Entrance Examination)
- KVPY- Kishore Vaigyanik Protsahan Yojana
- JAM- Joint Admission Test
- TIFR GS - Tata Institute of Fundamental Research Graduate School Admissions Examination
- JEST- Joint Entrance Screening Test
- NET- National Eligibility Test (CSIR and UGC)
- GATE-Graduate Aptitude Test in Engineering
- ICAR -AIEEA-Indian Council of Agricultural Research All India Entrance Examination

After completing +2

- B.Sc (Physics)
- Integrated M.Sc (Physics) (Central Universities)
- Integrated M.Sc (in Central Research Institutes through NEST and KVPY with stipend)
- B.Sc./B.S./B.Stat./B.Math./M.S. in Mathematics, Chemistry and Biology. (KVPY)
- B.E/B.Tech/ B.Arch (JEE, AIEEE in IITs and NITs) MBBS/ B.D.S/B.Pharm (NEET, JIPMER, AIIMS)
- B.Sc. (Agriculture) (ICAR -AIEEA)
- Dual Degree Program BS & MS (JEE, JEST in IITs and IISERs)
- B.Sc (Hospitality administration)
- B.Sc (Optoectronics)
- B.Sc (Optometry)
- B.Tech (Optics and Optoelectronics)

After completing undergraduate course in Physics (B.Sc Physics)

- M.Sc. (Physics) (In Central and State Universities and Colleges)
- M.Sc. Physics (JAM in IISc ,IITs and NITs)
- M.Sc. (In State and Central Universities)
- Medical Physics
- Materials Science
- Energy
- Earth Sciences
- Space science
- Oceanography
- Remote sensing
- Electronics
- Photonics
- Optoelectronics
- Acoustics
- Applied electronics
- Astronomy and Astrophysics
- Nanoscience and Nanotechnology
- Biostatistics
- Bio informatics
- Vacuum sciences

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Opportunities after B.Sc. Physics

Jobs in Government Sector

- Indian Forest Services
- Scientist Job in ISRO, DRDO, CSIR labs
- Union Public Service Commission
- Staff Selection Commission
- Indian Defence Services etc.
- Public Sector Bank
- State PCS
- Grade III & Compiler Post
- Tax Assistant
- Statistical Investigator
- Combined Higher Secondary
- Combined Graduate Level Exam etc.

Financial assistance to pursue higher education

Scholarships for graduate and post graduate courses

- International Olympiad: for getting stipend for Higher Education in Science and Mathematics
- DST – INSPIRE Scholarships (for UG and PG)
- DST – INSPIRE Fellowships (for Ph.D)
- UGC National Fellowship (for Ph.D)
- Indira Gandhi Fellowship for Single Girl Child (for UG and PG)
- Moulana Azad Fellowship for Minorities (for Ph.D)
- In addition various fellowships for SC/ST/PWD, OBC etc. are available.
- Visit website of University Grants Commission (UGC) and Department of Science and Technology (DST)

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Institutes in india to pursue research in physics

After completing Post Graduate course in Physics (M.Sc Physics)

Topics of Research

- Quantum Physics and Quantum Optics
- Astrophysics, Astronomy
- String theory, Quantum Gravity
- Mathematical Physics, Statistical Mechanics
- Quantum Field Theory
- Particle Physics and Quantum Thermodynamics
- Quantum Information Theory
- Condensed Matter Physics, Materials Science
- Electro Magnetic Theory
- Black Holes, Cosmology
- Crystal Growth, Crystallography
- Spectroscopy, Atomic, Molecular and Optical Physics
- Nano Science and Nanotechnology
- Energy and Environment Studies
- Biophysics, Medical Physics
- Cryptography, Spintronics
- Optics and Photonics
- Meteorology and Atmospheric Science

Research Institutions in various areas of science

Name of the Institution	Website
Indian Institute of Science (IISc) Bangalore	www.iisc.ac.in
Raman Research Institute (RRI) Bangalore	www.rri.res.in
Institute of Mathematical Sciences (IMSc) Chennai	www.imsc.res.in
Indian Association for Cultivation of Science (IACS) Calcutta	www.iacs.res.in
Chennai Mathematical Institute (CMI) Chennai	www.cmi.ac.in
Tata Institute of Fundamental Research (TIFR) Mumbai	www.tifr.res.in
Bhaba Atomic Research Centre (BARC) Mumbai	www.barc.gov.in
SN Bose centre Basic Natural science Calcutta	www.bose.res.in
Indian Institute of Space Science and Technology (IIST) Trivandrum	www.iist.ac.in
Physics Research Laboratory (PRL) Ahmedabad	www.prl.res.in
Indian Institute of Astrophysics (IIA) Bangalore	www.iiap.res.in
Institute of Physics (IOP) Bhubaneswar	www.iopb.res.in
Institute for Plasma Research (IPR) Gujarat	www.ipr.res.in
Inter University Centre for Astronomy and Astrophysics (IUCAA) Pune	www.iucaa.in
Indira Gandhi Centre for Atomic Research (IGCAR), Kalpakkam	www.igcar.gov.in
Hyderabad Central University, Hyderabad	www.uohyd.ac.in
Delhi University, Delhi	www.du.ac.in
Mumbai University, Mumbai	www.mu.ac.in
SavithiribaiPhule Pune university, Pune	www.unipune.ac.in
National Institute of Science Education and Research (NISER), Bhubaneshwar	www.niser.ac.in
IISER Educational Institutions	www.iiseradmission.in
Indian Institute of Technology in various places (IIT's)	www.iitm.ac.in
National Institute of Technology (NITs)	www.nitt.edu
Jawaharlal Nehru University (JNU)	www.jnu.ac.in
Central Universities	www.ugc.ac.in
State Universities	https://www.ugc.ac.in
CSIR – Academy (National laboratories, Delhi, Hyderabad, Trivandrum, Chennai, Calcutta etc)	

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UNIT 6

GRAVITATION

"The most remarkable discovery in all of astronomy is that the stars are made up of atoms of same kind as those in the Earth" – Richard Feynman

Learning Objectives

In this unit, the student is exposed to

- Kepler's laws for planetary motion
- Newton's law of gravitation
- connection between Kepler's laws and law of gravitation
- calculation of gravitational field and potential
- calculation of variation of acceleration due to gravity
- calculation of escape speed and energy of satellites
- concept of weightlessness
- advantage of heliocentric system over geocentric system
- measurement of the radius of Earth using Eratosthenes method
- recent developments in gravitation and astrophysics

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6.1 INTRODUCTION

We are amazed looking at the glittering sky; we wonder how the Sun rises in the East and sets in the West, why there are comets or why stars twinkle. The sky has been an object of curiosity for human beings from time immemorial. We have always wondered about the motion of stars, the Moon, and the planets. From Aristotle to Stephen Hawking, great minds have tried to understand the movement of celestial objects in space and what causes their motion.

The 'Theory of Gravitation' was developed by Newton in the late 17th century to explain the motion of celestial objects and terrestrial objects and answer most of the queries raised. In spite of the study of gravitation and its effect on celestial objects, spanning last three centuries, "gravitation" is still one of the active areas of research in physics today. In 2017, the Nobel Prize in Physics was given for the detection of 'Gravitational waves' which was theoretically predicted by Albert Einstein in the year 1915. Understanding planetary motion, the formation of stars and galaxies, and recently massive objects like black holes and their life cycle have remained the focus of study for the past few centuries in physics.

Geocentric Model of Solar System

In the second century, Claudius Ptolemy, a famous Greco-Roman astronomer,

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developed a theory to explain the motion of celestial objects like the Sun, the Moon, Mars, Jupiter etc. This theory was called the geocentric model. According to the geocentric model, the Earth is at the center of the universe and all celestial objects including the Sun, the Moon, and other planets orbit the Earth. Ptolemy's model closely matched with the observations of the sky with our naked eye. But later, astronomers found that even though Ptolemy's model successfully explained the motion of the Sun and the Moon up to a certain level, the motion of Mars and Jupiter could not be explained effectively.

Heliocentric Model of Nicholas Copernicus

In the 15th century, a Polish astronomer, Nicholas Copernicus (1473-1543) proposed a new model called the 'Heliocentric model' in which the Sun was considered to be at the center of the solar system and all planets including the Earth orbited the Sun in circular orbits. This model successfully explained the motion of all celestial objects.

Around the same time, Galileo, a famous Italian physicist discovered that all objects close to Earth were accelerated towards the Earth at the same rate. Meanwhile, a noble man called Tycho Brahe (1546-1601) spent his entire lifetime in recording the observations of the stellar and planetary positions with his naked eye. The data that he compiled were analyzed later by his assistant Johannes Kepler (1571–1630) and eventually the analysis led to the deduction of the laws of the planetary motion. These laws are termed as 'Kepler's laws of planetary motion'.

6.1.1 Kepler's Laws of Planetary Motion

Kepler's laws are stated as follows:

1. **Law of orbits:**
   Each planet moves around the Sun in an elliptical orbit with the Sun at one of the foci.

Figure 6.1 An ellipse traced out by a planet around the Sun.

The closest point of approach of the planet to the Sun 'P' is called perihelion and the farthest point 'A' is called aphelion (Figure 6.1). The semi-major axis is 'a' and semi-minor axis is 'b'. In fact, both Copernicus and Ptolemy considered planetary orbits to be circular, but Kepler discovered that the actual orbits of the planets are elliptical.

Figure 6.2 Motion of a planet around the Sun depicting 'law of area'.

2. **Law of area:**

The radial vector (line joining the Sun to a planet) sweeps equal areas in equal intervals of time.

In Figure 6.2, the white shaded portion is the area ΔA swept in a small interval of time Δt, by a planet around the Sun. Since the Sun is not at the center of the ellipse, *the planets travel faster when they are nearer to the Sun and slower when they are farther from it, to cover equal area in equal intervals of time.* Kepler discovered the law of area by carefully noting the variation in the speed of planets.

3. **Law of period:**

The square of the time period of revolution of a planet around the Sun in its elliptical orbit is directly proportional to the cube of the semi-major axis of the ellipse. It can be written as:

$$T^2 \propto a^3 \qquad (6.1)$$

$$\frac{T^2}{a^3} = constant \qquad (6.2)$$

where, T is the time period of revolution for a planet and *a* is the semi-major axis. Physically this law implies that as the distance of the planet from the Sun increases, the time period also increases but not at the same rate.

In Table 6.1, the time period of revolution of planets around the Sun along with their semi-major axes are given. From column four, we can realize that $\frac{T^2}{a^3}$ is nearly a constant endorsing Kepler's third law.

Table 6.1 The time period of revolution of the planets revolving around the Sun and their semi-major axes.

Planet	a ($10^{10}\,m$)	T (years)	$\frac{T^2}{a^3}$
Mercury	5.79	0.24	2.95
Venus	10.8	0.615	3.00
Earth	15.0	1	2.96
Mars	22.8	1.88	2.98
Jupiter	77.8	11.9	3.01
Saturn	143	29.5	2.98
Uranus	287	84	2.98
Neptune	450	165	2.99

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Points to Contemplate

DATA			PROBLEM
Planet	**a**	**T**	What is the law connecting *a* and T?
A	1	3	
B	2	6	
C	4	18	

Comment on the relation between *a* and T for these imaginary planets

6.1.2 Universal Law of Gravitation

Even though Kepler's laws were able to explain the planetary motion, they failed to explain the forces responsible for it. It was Isaac Newton who analyzed Kepler's laws, Galileo's observations and deduced the law of gravitation.

Newton's law of gravitation states that a particle of mass M_1 attracts any other particle of mass M_2 in the universe with an attractive force. The strength of this force of attraction was found to be directly proportional to the product of their masses and is inversely proportional to the square of the distance between them. In mathematical form, it can be written as:

$$\vec{F} = -\frac{GM_1M_2}{r^2}\hat{r} \qquad (6.3)$$

where $\hat{r}$ is the unit vector from M_1 towards M_2 as shown in Figure 6.3, and G is the Gravitational constant that has the value of $6.67\times10^{-11}\,N\,m^2kg^{-2}$, and r is the distance between the two masses M_1 and M_2. In Figure 6.3, the vector $\vec{F}$ denotes the gravitational force experienced by M_2 due to M_1. Here the negative sign indicates that the gravitational force is always attractive in nature and the direction of the force is along the line joining the two masses.

Figure 6.3 Attraction of two masses towards each other.

In cartesian coordinates, the square of the distance is expressed as $r^2 = (x^2 + y^2 + z^2)$. This is dealt in unit 2.

EXAMPLE 6.1

Consider two point masses m_1 and m_2 which are separated by a distance of 10 meter as shown in the following figure. Calculate the force of attraction between them and draw the directions of forces on each of them. Take m_1= 1 kg and m_2 = 2 kg

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Solution

The force of attraction is given by

$$\vec{F} = -\frac{Gm_1m_2}{r^2}\hat{r}$$

From the figure, r =10 m.

First, we can calculate the magnitude of the force

$$F = \frac{Gm_1m_2}{r^2} = \frac{6.67\times10^{-11}\times1\times2}{100}$$
$$= 13.34\times10^{-13}\,\text{N}$$

It is to be noted that this force is very small. This is the reason we do not feel the gravitational force of attraction between each other. The small value of G plays a very crucial role in deciding the strength of the force.

The force of attraction ($\vec{F}_{21}$) experienced by the mass m_2 due to m_1 is in the negative 'y' direction ie., $\hat{r} = -\hat{j}$. According to Newton's third law, the mass m_2 also exerts equal and opposite force on m_1. So the force of attraction ($\vec{F}_{12}$) experienced by m_1 due to m_2 is in the direction of positive 'y' axis ie., $\hat{r} = \hat{j}$.

$$\vec{F}_{21} = -13.34\times10^{-13}\,\hat{j}$$

$$\vec{F}_{12} = 13.34\times10^{-13}\,\hat{j}$$

The direction of the force is shown in the figure,

Gravitational force of attraction between m_1 and m_2

$\vec{F}_{12} = -\vec{F}_{21}$ which confirms Newton's third law.

Important features of gravitational force:

- As the distance between two masses increases, the strength of the force tends to decrease because of inverse dependence on r^2. Physically it implies that the planet Uranus experiences less gravitational force from the Sun than the Earth since Uranus is at larger distance from the Sun compared to the Earth.

Figure 6.4 Variation of gravitational force with distance

- The gravitational forces between two particles always constitute an action-reaction pair. It implies that the gravitational force exerted by the Sun on the Earth is always towards the Sun. The reaction-force is exerted by the Earth on the Sun. The direction of this reaction force is towards Earth.

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- The torque experienced by the Earth due to the gravitational force of the Sun is given by

$$\vec{\tau} = \vec{r} \times \vec{F} = \vec{r} \times \left(-\frac{GM_S M_E}{r^2} \hat{r} \right) = 0$$

Since $\vec{r} = r\,\hat{r}$, $(\hat{r} \times \hat{r}) = 0$

So $\vec{\tau} = \frac{d\vec{L}}{dt} = 0$. It implies that angular momentum $\vec{L}$ is a constant vector. The angular momentum of the Earth about the Sun is constant throughout the motion. It is true for all the planets. In fact, this constancy of angular momentum leads to the Kepler's second law.

- The expression $\vec{F} = -\frac{GM_1M_2}{r^2}\hat{r}$ has one inherent assumption that both M_1 and M_2 are treated as point masses. When it is said that Earth orbits around the Sun due to Sun's gravitational force, we assumed Earth and Sun to be point masses. This assumption is a good approximation because the distance between the two bodies is very much larger than their diameters. For some irregular and extended objects separated by a small distance, we cannot directly use the equation (6.3). Instead, we have to invoke separate mathematical treatment which will be brought forth in higher classes.
- However, this assumption about point masses holds even for small distance for one special case. To calculate force of attraction between a hollow sphere of mass M with uniform density and point mass m kept outside the hollow sphere, we can replace the hollow sphere of mass M as equivalent to a point mass M located at the center of the hollow sphere. The force of attraction between the hollow sphere of mass M and point mass m can be calculated by treating the hollow sphere also as another point mass. Essentially the entire mass of the hollow sphere appears to be concentrated at the center of the hollow sphere. It is shown in the Figure 6.5(a).
- There is also another interesting result. Consider a hollow sphere of mass M. If we place another object of mass 'm' inside this hollow sphere as in Figure 6.5(b), the force experienced by this mass 'm' will be zero. This calculation will be dealt with in higher classes.

Figure 6.5 A mass placed in a hollow sphere.

- The triumph of the law of gravitation is that it concludes that the mango that is falling down and the Moon orbiting the Earth are due to the same gravitational force.

Newton's inverse square Law:

Newton considered the orbits of the planets as circular. For circular orbit of radius r, the centripetal acceleration towards the center is

$$a = -\frac{v^2}{r} \qquad (6.4)$$

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Figure 6.6 Point mass orbiting in a circular orbit.

Here v is the velocity and r, the distance of the planet from the center of the orbit (Figure 6.6).

The velocity in terms of known quantities r and T, is

$$v = \frac{2\pi r}{T} \tag{6.5}$$

Here T is the time period of revolution of the planet. Substituting this value of v in equation (6.4) we get,

$$a = -\frac{\left(\frac{2\pi r}{T}\right)^2}{r} = -\frac{4\pi^2 r}{T^2} \tag{6.6}$$

Substituting the value of 'a' from (6.6) in Newton's second law, $F = ma$, where 'm' is the mass of the planet.

$$F = -\frac{4\pi^2 mr}{T^2} \tag{6.7}$$

From Kepler's third law,

$$\frac{r^3}{T^2} = k\left(constant\right) \tag{6.8}$$

$$\frac{r}{T^2} = \frac{k}{r^2} \tag{6.9}$$

By substituting equation 6.9 in the force expression, we can arrive at the law of gravitation.

$$F = -\frac{4\pi^2 mk}{r^2} \tag{6.10}$$

Here negative sign implies that the force is attractive and it acts towards the center. In equation (6.10), mass of the planet 'm' comes explicitly. But Newton strongly felt that according to his third law, if Earth is attracted by the Sun, then the Sun must also be attracted by the Earth with the same magnitude of force. So he felt that the Sun's mass (M) should also occur explicitly in the expression for force (6.10). From this insight, he equated the constant $4\pi^2 k$ *to* GM which turned out to be the law of gravitation.

$$F = -\frac{GMm}{r^2}$$

Again the negative sign in the above equation implies that the gravitational force is attractive.

In the above discussion we assumed that the orbit of the planet to be circular which is not true as the orbit of the planet around the Sun is elliptical. But this circular orbit assumption is justifiable because planet's orbit is very close to being circular and there is only a very small deviation from the circular shape.

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Points to Contemplate

If Kepler's third law was "r^3T^2 = constant" instead of "$\frac{r^3}{T^2}$ = constant" what would be the new law of gravitation? Would it still be an inverse square law? How would the gravitational force change with distance? In this new law of gravitation, will Neptune experience greater gravitational force or lesser gravitational force when compared to the Earth?

EXAMPLE 6.2

Moon and an apple are accelerated by the same gravitational force due to Earth. Compare the acceleration of the two.

The gravitational force experienced by the apple due to Earth

$$F = -\frac{GM_E M_A}{R^2}$$

Here M_A– Mass of the apple, M_E– Mass of the Earth and R – Radius of the Earth.

Equating the above equation with Newton's second law,

$$M_A a_A = -\frac{GM_E M_A}{R^2}.$$

Simplifying the above equation we get,

$$a_A = -\frac{GM_E}{R^2}$$

Here a_A is the acceleration of apple that is equal to 'g'.

Similarly the force experienced by Moon due to Earth is given by

$$F = -\frac{GM_E M_m}{R_m^2}.$$

Here R_m- distance of the Moon from the Earth, M_m – Mass of the Moon

The acceleration experienced by the Moon is given by

$$a_m = -\frac{GM_E}{R_m^2}.$$

The ratio between the apple's acceleration to Moon's acceleration is given by

$$\frac{a_A}{a_m} = \frac{R_m^2}{R^2}.$$

From the Hipparchrus measurement, the distance to the Moon is 60 times that of Earth radius. R_m = 60R.

$$a_A / a_m = \frac{(60R)^2}{R^2} = 3600.$$

The apple's acceleration is 3600 times the acceleration of the Moon.

The same result was obtained by Newton using his gravitational formula. The apple's acceleration is measured easily and it is 9.8 m s^{-2}. Moon orbits the Earth once in 27.3 days and by using the centripetal acceleration formula, (Refer unit 3).

$$\frac{a_A}{a_m} = \frac{9.8}{0.00272} = 3600$$

which is exactly what he got through his law of gravitation.

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Note The above calculation depends on knowing the distance between the Earth and the Moon and the radius of the Earth. The radius of the Earth was measured by Greek librarian Eratosthenes and distance between the Earth and the Moon was measured by Greek astronomer Hipparchrus 2400 years ago. It is very interesting to note that in order to measure these distances he used only high school geometry and trigonometry. These details are discussed in the astronomy section (6.5).

6.1.3 Gravitational Constant

In the law of gravitation, the value of gravitational constant G plays a very important role. The value of G explains why the gravitational force between the Earth and the Sun is so great while the same force between two small objects (for example between two human beings) is negligible.

The force experienced by a mass 'm' which is on the surface of the Earth (Figure 6.7) is given by

$$F = -\frac{GM_E m}{R_E^{\,2}} \qquad (6.11)$$

M_E-mass of the Earth, m - mass of the object, R_E- radius of the Earth.

Equating Newton's second law, $F = -mg$, to equation (6.11) we get,

$$-mg = -\frac{GM_E m}{R_E^{\,2}}$$

$$g = \frac{GM_E}{R_E^{\,2}} \qquad (6.12)$$

Figure 6.7 Force experienced by a mass on the (i) surface of the Earth (ii) at a distance from the centre of the Earth

Now the force experienced by some other object of mass M at a distance r from the center of the Earth is given by,

$$F = -\frac{GM_E M}{r^2}$$

Using the value of g in equation (6.12), the force F will be,

$$F = -gM\frac{R_E^{\,2}}{r^2} \qquad (6.13)$$

From this it is clear that the force can be calculated simply by knowing the value of g. It is to be noted that in the above calculation G is not required.

Do you know? In the year 1798, Henry Cavendish experimentally determined the value of gravitational constant 'G' by using a torsion balance. He calculated the value of 'G' to be equal to $6.75 \times 10^{-11}\,\mathrm{N\,m^2kg^{-2}}$. Using modern techniques a more accurate value of G could be measured. The currently accepted value of G is $6.67259 \times 10^{-11}\,\mathrm{N\,m^2kg^{-2}}$.

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6.2 GRAVITATIONAL FIELD AND GRAVITATIONAL POTENTIAL

6.2.1 Gravitational field

Force is basically due to the interaction between two particles. Depending upon the type of interaction we can have two kinds of forces: Contact forces and Non-contact forces (Figure 6.8).

Figure 6.8 Depiction of contact and non-contact forces

Contact forces are the forces applied where one object is in physical contact with the other. The movement of the object is caused by the physical force exerted through the contact between the object and the agent which exerts force.

Consider the case of Earth orbiting around the Sun. Though the Sun and the Earth are not physically in contact with each other, there exists an interaction between them. This is because of the fact that the Earth experiences the gravitational force of the Sun. This gravitational force is a non-contact force.

It sounds mysterious that the Sun attracts the Earth despite being very far from it and without touching it. For contact forces like push or pull, we can calculate the strength of the force since we can feel or see. But how do we calculate the strength of non-contact force at different distances? To understand and calculate the strength of non-contact forces, the concept of 'field' is introduced.

The gravitational force on a particle of mass 'm_2' due to a particle of mass 'm_1' is

$$\vec{F}_{21} = -\frac{Gm_1m_2}{r^2}\hat{r} \qquad (6.14)$$

where $\hat{r}$ is a unit vector that points from m_1 to m_2 along the line joining the masses m_1 and m_2.

The gravitational field intensity $\vec{E}_1$ (here after called as gravitational field) at a point which is at a distance r from m_1 is defined as the gravitational force experienced by unit mass placed at that point. It is given by the ratio $\frac{\vec{F}_{21}}{m_2}$ (where m_2 is the mass of the object on which $\vec{F}_{21}$ acts)

Using $\vec{E}_1 = \frac{\vec{F}_{21}}{m_2}$ in equation (6.14) we get,

$$\vec{E}_1 = -\frac{Gm_1}{r^2}\hat{r} \qquad (6.15)$$

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$\vec{E}_1$ is a vector quantity that points towards the mass m_1 and is independent of mass m_2, Here m_2 is taken to be of unit magnitude . The unit $\hat{r}$ is along the line between m_1 and the point in question. The field $\vec{E}_1$ is due to the mass m_1.

In general, the gravitational field intensity due to a mass M at a distance r is given by

$$\vec{E} = -\frac{GM}{r^2}\hat{r} \qquad (6.16)$$

Now in the region of this gravitational field, a mass 'm' is placed at a point P (Figure 6.9). Mass 'm' interacts with the field $\vec{E}$ and experiences an attractive force due to M as shown in Figure 6.9. The gravitational force experienced by 'm' due to 'M' is given by

Figure 6.9 Gravitational Field intensity measured with an object of unit mass

$$\vec{F}_m = m\vec{E} \qquad (6.17)$$

Now we can equate this with Newton's second law $\vec{F} = m\vec{a}$

$$m\vec{a} = m\vec{E} \qquad (6.18)$$

$$\vec{a} = \vec{E} \qquad (6.19)$$

In other words, equation (6.18) implies that the gravitational field at a point is equivalent to the acceleration experienced by a particle at that point. However, it is to be noted that $\vec{a}$ and $\vec{E}$ are separate physical quantities that have the same magnitude and direction. The gravitational field $\vec{E}$ is the property of the source and acceleration $\vec{a}$ is the effect experienced by the test mass (unit mass) which is placed in the gravitational field $\vec{E}$. The non-contact interaction between two masses can now be explained using the concept of "Gravitational field".

Points to be noted:

i) The strength of the gravitational field decreases as we move away from the mass M as depicted in the Figure 6.10. The magnitude of $\vec{E}$ decreases as the distance r increases.

Figure 6.10 Strength of the Gravitational field lines decreases with distance

Figure 6.10 shows that the strength of the gravitational field at points P, Q, and R is given by $|\vec{E}_P| < |\vec{E}_Q| < |\vec{E}_R|$. It can be understood by comparing the length of the vectors at points P, Q, and R.

ii) The "field" concept was introduced as a mathematical tool to calculate gravitational interaction. Later it was found that field is a real physical quantity and it carries energy and momentum in

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space. The concept of field is inevitable in understanding the behavior of charges.

iii) The unit of gravitational field is Newton per kilogram (N/kg) or m s^{-2}.

6.2.2 Superposition principle for Gravitational field

Consider 'n' particles of masses $m_1, m_2, \ldots m_n$, distributed in space at positions $\vec{r}_1, \vec{r}_2, \vec{r}_3 \ldots$ etc, with respect to point P. The total gravitational field at a point P due to all the masses is given by the vector sum of the gravitational field due to the individual masses (Figure 6.11). This principle is known as superposition of gravitational fields.

$$\vec{E}_{total} = \vec{E}_1 + \vec{E}_2 + \ldots \vec{E}_n$$

$$= -\frac{Gm_1}{r_1^2}\hat{r}_1 - \frac{Gm_2}{r_2^2}\hat{r}_2 - \ldots - \frac{Gm_n}{r_n^2}\hat{r}_n$$

$$= -\sum_{i=1}^{n} \frac{Gm_i}{r_i^2}\hat{r}_i . \qquad (6.20)$$

Figure 6.11 Superposition of two gravitational field intensities giving resultant field.

Instead of discrete masses, if we have continuous distribution of a total mass M, then the gravitational field at a point P is calculated using the method of integration.

EXAMPLE 6.3

(a) Two particles of masses m_1 *and* m_2 are placed along the x and y axes respectively at a distance 'a' from the origin. Calculate the gravitational field at a point P shown in figure below.

Solution

Gravitational field due to m_1 at a point P is given by,

$$\vec{E}_1 = -\frac{Gm_1}{a^2}\hat{j}$$

Gravitational field due to m_2 at the point p is given by,

$$\vec{E}_2 = -\frac{Gm_2}{a^2}\hat{i}$$

$$\vec{E}_{total} = -\frac{Gm_1}{a^2}\hat{j} - \frac{Gm_2}{a^2}\hat{i}$$

$$= -\frac{G}{a^2}\left(m_1\hat{j} + m_2\hat{i}\right)$$

The direction of the total gravitational field is determined by the relative value of m_1 and m_2.

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When $m_1 = m_2 = m$

$$\vec{E}_{total} = -\frac{Gm}{a^2}(\hat{i}+\hat{j})$$

($\hat{i}+\hat{j} = \hat{j}+\hat{i}$ as vectors obeys commutation law).

$\vec{E}_{total}$ points towards the origin of the co-ordinate system and the magnitude of $\vec{E}_{total}$ is $\sqrt{2}\frac{Gm}{a^2}$.

EXAMPLE 6.4

Qualitatively indicate the gravitational field of Sun on Mercury, Earth, and Jupiter shown in figure.

Since the gravitational field decreases as distance increases, Jupiter experiences a weak gravitational field due to the Sun. Since Mercury is the nearest to the Sun, it experiences the strongest gravitational field.

Solar System

6.2.3 Gravitational Potential Energy

The concept of potential energy and its physical meaning were dealt in unit 4. The gravitational force is a conservative force and hence we can define a gravitational potential energy associated with this conservative force field.

Two masses m_1 and m_2 are initially separated by a distance r'. Assuming m_1 to be fixed in its position, work must be done on m_2 to move the distance from r' to r as shown in Figure 6.12(a)

Figure 6.12 Two distant masses changing the linear distance

To move the mass m_2 through an infinitesimal displacement $d\vec{r}$ from $\vec{r}$ to $\vec{r}+d\vec{r}$ (shown in the Figure 6.12(b)), work has to be done externally. This infinitesimal work is given by

$$dW = \vec{F}_{ext}.d\vec{r} \qquad (6.21)$$

The work is done against the gravitational force, therefore,

$$\left|\vec{F}_{ext}\right| = \left|\vec{F}_G\right| = \frac{Gm_1m_2}{r^2} \qquad (6.22)$$

Substituting Equation (6.22) in 6.21, we get

$$dW = \frac{Gm_1m_2}{r^2}\hat{r}.d\vec{r} \qquad (6.23)$$

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Also we know,

$$d\vec{r} = dr\,\hat{r} \qquad (6.24)$$

$$\Rightarrow dW = \frac{Gm_1m_2}{r^2}\hat{r}.(dr\,\hat{r}) \qquad (6.25)$$

$$\hat{r}.\hat{r} = 1(\textit{since both are unit vectors})$$

$$\therefore \quad dW = \frac{Gm_1m_2}{r^2}dr \qquad (6.26)$$

Thus the total work done for displacing the particle from r' to r is

$$W = \int_{r'}^{r} dW = \int_{r'}^{r} \frac{Gm_1m_2}{r^2}dr \qquad (6.27)$$

$$W = -\left(\frac{Gm_1m_2}{r}\right)_{r'}^{r}$$

$$W = -\frac{Gm_1m_2}{r} + \frac{Gm_1m_2}{r'} \qquad (6.28)$$

$$W = U(r) - U(r')$$

where $U(r) = \dfrac{-Gm_1m_2}{r}$

This work done W gives the gravitational potential energy difference of the system of masses m_1 and m_2 when the separation between them are r and r' respectively.

Case 1: If $r < r'$

Since gravitational force is attractive, m_2 is attracted by m_1.Then m_2 can move from r' to r without any external work (Figure 6.13(a)). Here work is done by the system spending its internal energy and hence the work done is said to be negative.

Figure 6.13 Cases for calculation of work done by gravity

Case 2: If $r > r'$

Work has to be done against gravity to move the object from r' to r (Figure 6.13(b)). Therefore work is done on the body by external force and hence work done is positive.

It is to be noted that only potential energy difference has physical significance. Now gravitational potential energy can be discussed by choosing one point as the reference point.

Let us choose $r' = \infty$. Then the second term in the equation (6.28) becomes zero.

$$W = -\frac{Gm_1m_2}{r} + 0 \qquad (6.29)$$

Now we can define gravitational potential energy of a system of two masses m_1 and m_2 separated by a distance r as the amount of work done to take the mass m_2 from a distance r to infinity assuming m_1 to be fixed in its position and is written as $U(r) = -\dfrac{Gm_1m_2}{r}$. It is to be noted that the gravitational potential energy of the system consisting of two masses m_1 and m_2 separated by a distance r, is the gravitational potential energy difference of the system when the masses are

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separated by an infinite distance and by distance r. $U(r) = U(r) - U(\infty)$. Here we choose $U(\infty) = 0$ as the reference point. The gravitational potential energy $U(r)$ is always negative because when two masses come together slowly from infinity, work is done by the system.

The unit of gravitational potential energy $U(r)$ is Joule and it is a scalar quantity. The gravitational potential energy depends upon the two masses and the distance between them.

6.2.4 Gravitational potential energy near the surface of the Earth

It is already discussed in chapter 4 that when an object of mass m is raised to a height h, the potential energy stored in the object is mgh (Figure 6.14). This can be derived using the general expression for gravitational potential energy.

Figure 6.14 Mass placed at a distance r from the center of the Earth

Consider the Earth and mass system, with *r*, the distance between the mass m and the Earth's centre. Then the gravitational potential energy,

$$U = -\frac{GM_e m}{r} \qquad (6.30)$$

Here r = R_e+h, where R_e is the radius of the Earth. h is the height above the Earth's surface

$$U = -G\frac{M_e m}{(R_e + h)} \qquad (6.31)$$

If h << R_e, equation (6.31) can be modified as

$$U = -G\frac{M_e m}{R_e(1 + h/R_e)}$$

$$U = -G\frac{M_e m}{R_e}(1 + h/R_e)^{-1} \qquad (6.32)$$

By using Binomial expansion and neglecting the higher order terms, we get

$$U = -G\frac{M_e m}{R_e}\left(1 - \frac{h}{R_e}\right). \qquad (6.33)$$

We know that, for a mass m on the Earth's surface,

$$G\frac{M_e m}{R_e} = mgR_e \qquad (6.34)$$

Substituting equation (6.34) in (6.33) we get,

$$U = -mgR_e + mgh \qquad (6.35)$$

It is clear that the first term in the above expression is independent of the height h. For example, if the object is taken from

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height h_1 to h_2, then the potential energy at h_1 is

$$U(h_1) = -mgR_e + mgh_1 \qquad (6.36)$$

and the potential energy at h_2 is

$$U(h_2) = -mgR_e + mgh_2 \qquad (6.37)$$

The potential energy difference between h_1 and h_2 is

$$U(h_2) - U(h_1) = mg(h_2 - h_1). \qquad (6.38)$$

The term mgR_e in equations (6.36) and (6.37) plays no role in the result. Hence in the equation (6.35) the first term can be omitted or taken to zero. Thus it can be stated that The gravitational potential energy stored in the particle of mass m at a height h from the surface of the Earth is $U = mgh$. On the surface of the Earth, $U = 0$, since h is zero.

It is to be noted that mgh is the work done on the particle when we take the mass m from the surface of the Earth to a height h. This work done is stored as a gravitational potential energy in the mass m. Even though mgh is gravitational potential energy of the system (Earth and mass m), we can take mgh as the gravitational potential energy of the mass m since Earth is stationary when the mass moves to height h.

6.2.5 Gravitational potential V(r)

It is explained in the previous sections that the gravitational field $\vec{E}$ depends only on the source mass which creates the field. It is a vector quantity. We can also define a scalar quantity called "gravitational potential" which depends only on the source mass.

The gravitational potential at a distance r due to a mass is defined as the amount of work required to take unit mass from the distance r to infinity and it is denoted as V(r). In other words, the gravitational potential at distance r is equivalent to gravitational potential energy per unit mass at the same distance r. It is a scalar quantity and its unit is J kg^{-1}

We can determine gravitational potential from gravitational potential energy. Consider two masses m_1 and m_2 separated by a distance r which has gravitational potential energy $U(r)$ (Figure 6.15). The gravitational potential due to mass m_1 at a point P which is at a distance r from m_1 is obtained by making m_2 equal to unity (m_2 = 1kg). Thus the gravitational potential $V(r)$ due to mass m_1 at a distance r is

$$V(r) = -\frac{Gm_1}{r} \qquad (6.39)$$

Figure 6.15 Point mass placed at a distance

Gravitational field and gravitational force are vector quantities whereas the gravitational potential and gravitational potential energy are scalar quantities. The motion of particles can be easily analyzed using scalar quantities than vector quantities. Consider the example of a falling apple:

Figure 6.16 shows an apple which falls on Earth due to Earth's gravitational force. This can be explained using the concept of gravitational potential $V(r)$ as follows.

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Figure 6.16 Apple falling freely under gravity

The gravitational potential $V(r)$ at a point of height h from the surface of the Earth is given by,

$$V(r=R+h)=-\frac{GM_e}{(R+h)} \qquad (6.40)$$

The gravitational potential $V(r)$ on the surface of Earth is given by,

$$V(r=R)=-\frac{GM_e}{R} \qquad (6.41)$$

Thus we see that

$$V(r=R)<V(r=R+h). \qquad (6.42)$$

It is already discussed in the previous section that the gravitational potential energy near the surface of the Earth at height h is mgh. The gravitational potential at this point is simply $V(h)=U(h)/m = gh$. In fact, the gravitational potential on the surface of the Earth is zero since h is zero. So the apple falls from a region of a higher gravitational potential to a region of lower gravitational potential. In general, the mass will move from a region of higher gravitational potential to a region of lower gravitational potential.

EXAMPLE 6.5

Water falls from the top of a hill to the ground. Why?

This is because the top of the hill is a point of higher gravitational potential than the surface of the Earth i.e. $V_{hill} > V_{ground}$

Water falling from hill top

The motion of particles can be analyzed more easily using scalars like $U(r)$ or $V(r)$ than vector quantities like $\vec{F}$ or $\vec{E}$. In modern theories of physics, the concept of potential plays a vital role.

EXAMPLE 6.6

Consider four masses m_1, m_2, m_3, and m_4 arranged on the circumference of a circle as shown in figure below

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Calculate

(a) The gravitational potential energy of the system of 4 masses shown in figure.

(b) The gravitational potential at the point O due to all the 4 masses.

Solution

The gravitational potential energy $U(r)$ can be calculated by finding the sum of gravitational potential energy of each pair of particles.

$$U = -\frac{Gm_1m_2}{r_{12}} - \frac{Gm_1m_3}{r_{13}} - \frac{Gm_1m_4}{r_{14}} - \frac{Gm_2m_3}{r_{23}} - \frac{Gm_2m_4}{r_{24}} - \frac{Gm_3m_4}{r_{34}}$$

Here r_{12}, r_{13} ... are distance between pair of particles

$$r_{14}^{\ 2} = R^2 + R^2 = 2R^2$$

$$r_{14} = \sqrt{2}R = r_{12} = r_{23} = r_{34}$$

$$r_{13} = r_{24} = 2R$$

$$U = -\frac{Gm_1m_2}{\sqrt{2}R} - \frac{Gm_1m_3}{2R} - \frac{Gm_1m_4}{\sqrt{2}R} - \frac{Gm_2m_3}{\sqrt{2}R} - \frac{Gm_2m_4}{2R} - \frac{Gm_3m_4}{\sqrt{2}R}$$

$$U = -\frac{G}{R}\left[\frac{m_1m_2}{\sqrt{2}} + \frac{m_1m_3}{2} + \frac{m_1m_4}{\sqrt{2}} + \frac{m_2m_3}{\sqrt{2}} + \frac{m_2m_4}{2} + \frac{m_3m_4}{\sqrt{2}}\right]$$

If all the masses are equal, then $m_1 = m_2 = m_3 = m_4 = M$

$$U = -\frac{GM^2}{R}\left[\frac{1}{\sqrt{2}} + \frac{1}{2} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} + \frac{1}{2} + \frac{1}{\sqrt{2}}\right]$$

$$U = -\frac{GM^2}{R}\left[1 + \frac{4}{\sqrt{2}}\right]$$

$$U = -\frac{GM^2}{R}\left[1 + 2\sqrt{2}\right]$$

The gravitational potential V(r) at a point O is equal to the sum of the gravitational potentials due to individual mass. Since potential is a scalar, the net potential at point O is the algebraic sum of potentials due to each mass.

$$V_O(r) = -\frac{Gm_1}{R} - \frac{Gm_2}{R} - \frac{Gm_3}{R} - \frac{Gm_4}{R}$$

If $m_1 = m_2 = m_3 = m_4 = M$

$$V_O(r) = -\frac{4GM}{R}$$

6.3 ACCELERATION DUE TO GRAVITY OF THE EARTH

When objects fall on the Earth, the acceleration of the object is towards the Earth. From Newton's second law, an object is accelerated only under the action of a force. In the case of Earth, this force is the gravitational pull of Earth. This force produces a constant acceleration near the Earth's surface in all bodies, irrespective of their masses. The gravitational force exerted by Earth on the mass m near the surface of the Earth is given by

$$\vec{F} = -\frac{GmM_e}{R_e^{\ 2}}\hat{r}$$

Now equating Gravitational force to Newton's second law,

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$$m\vec{a} = -\frac{GmM_e}{R_e^2}\hat{r}$$

hence, acceleration is,

$$\vec{a} = -\frac{GM_e}{R_e^2}\hat{r} \qquad (6.43)$$

The acceleration experienced by the object near the surface of the Earth due to its gravity is called acceleration due to gravity. It is denoted by the symbol g. The magnitude of acceleration due to gravity is

$$|g| = \frac{GM_e}{R_e^2}. \qquad (6.44)$$

It is to be noted that the acceleration experienced by any object is independent of its mass. The value of g depends only on the mass and radius of the Earth. Infact, Galileo arrived at the same conclusion 400 years ago that *all objects fall towards the Earth with the same acceleration* through various quantitative experiments. The acceleration due to gravity g is found to be 9.8 m s^{-2} on the surface of the Earth near the equator.

6.3.1 Variation of g with altitude, depth and latitude

Consider an object of mass m at a height h from the surface of the Earth. Acceleration experienced by the object due to Earth is

$$g' = \frac{GM}{(R_e + h)^2} \qquad (6.45).$$

Figure 6.17(a) Mass at a height h from the center of the Earth

$$g' = \frac{GM}{R_e^2\left(1+\frac{h}{R_e}\right)^2}$$

$$g' = \frac{GM}{R_e^2}\left(1+\frac{h}{R_e}\right)^{-2}$$

If $h \ll R_e$

We can use Binomial expansion. Taking the terms upto first order

$$g' = \frac{GM}{R_e^2}\left(1-2\frac{h}{R_e}\right)$$

$$g' = g\left(1-2\frac{h}{R_e}\right) \qquad (6.46)$$

We find that $g' < g$. This means that as altitude h increases the acceleration due to gravity g decreases.

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EXAMPLE 6.7

1. Calculate the value of g in the following two cases:
   (a) If a mango of mass ½ kg falls from a tree from a height of 15 meters, what is the acceleration due to gravity when it begins to fall?

Solution

$$g' = g\left(1 - 2\frac{h}{R_e}\right)$$

$$g' = 9.8\left(1 - \frac{2\times 15}{6400\times 10^3}\right)$$

$$g' = 9.8\left(1 - 0.469\times 10^{-5}\right)$$

But $1 - 0.00000469 \cong 1$

Therefore $g' = g$

   (b) Consider a satellite orbiting the Earth in a circular orbit of radius 1600 km above the surface of the Earth. What is the acceleration experienced by the satellite due to Earth's gravitational force?

Solution

$$g' = g\left(1 - 2\frac{h}{R_e}\right)$$

$$g' = g\left(1 - \frac{2\times 1600\times 10^3}{6400\times 10^3}\right)$$

$$g' = g\left(1 - \frac{2}{4}\right)$$

$$g' = g\left(1 - \frac{1}{2}\right) = g/2$$

The above two examples show that the acceleration due to gravity is a constant near the surface of the Earth.

Note

Can we substitute h = R_e in the equation 6.46? No. To get equation 6.46 we assumed that h << R_e. However h = R_e can be substituted in equation 6.45.

Variation of g with depth:

Consider a particle of mass m which is in a deep mine on the Earth. (Example: coal mines in Neyveli). Assume the depth of the mine as d. To calculate g' at a depth d, consider the following points.

Figure 6.17(b) Particle in a mine

The part of the Earth which is above the radius $(R_e - d)$ do not contribute to the acceleration. The result is proved earlier and is given as

$$g' = \frac{GM'}{(R_e - d)^2} \qquad (6.47)$$

Here M' is the mass of the Earth of radius $(R_e - d)$

Assuming the density of Earth ρ to be constant,

$$\rho = \frac{M}{V} \qquad (6.48)$$

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where M is the mass of the Earth and V its volume, Thus,

$$\rho = \frac{M'}{V'}$$

$$\frac{M'}{V'} = \frac{M}{V} \text{ and } M' = \frac{M}{V}V'$$

$$M' = \left(\frac{M}{\frac{4}{3}\pi R_e^{\,3}}\right)\left(\frac{4}{3}\pi(R_e - d)^3\right)$$

$$M' = \frac{M}{R_e^{\,3}}(R_e - d)^3 \qquad (6.49)$$

Substituting equation (6.49) in equation (6.47)

$$g' = G\frac{M}{R_e^{\,3}}(R_e - d)^3 . \frac{1}{(R_e - d)^2}$$

$$g' = GM\frac{R_e\left(1 - \frac{d}{R_e}\right)}{R_e^{\,3}}$$

$$g' = GM\frac{\left(1 - \frac{d}{R_e}\right)}{R_e^{\,2}}$$

Thus

$$g' = g\left(1 - \frac{d}{R_e}\right) \qquad (6.50)$$

Here also $g' < g$. As depth increases, g' decreases. It is very interesting to know that acceleration due to gravity is maximum on the surface of the Earth but decreases when we go either upward or downward.

Variation of g with latitude:

Whenever we analyze the motion of objects in rotating frames [explained in chapter 3] we must take into account the centrifugal force. Even though we treat the Earth as an inertial frame, it is not exactly correct because the Earth spins about its own axis. So when an object is on the surface of the Earth, it experiences a centrifugal force that depends on the latitude of the object on Earth. If the Earth were not spinning, the force on the object would have been mg. However, the object experiences an additional centrifugal force due to spinning of the Earth.

Figure 6.18 Variation of g with latitude

This centrifugal force is given by $m\omega^2 R'$.

$$R' = R\cos\lambda \qquad (6.51)$$

where λ is the latitude. The component of centrifugal acceleration experienced by the object in the direction opposite to g is

$$a_{PQ} = \omega^2 R'\cos\lambda = \omega^2 R\cos^2\lambda$$

$$\text{since } R' = R\cos\lambda$$

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Therefore,

$$g' = g - \omega^2 R \cos^2 \lambda \qquad (6.52)$$

From the expression (6.52), we can infer that at equator, $\lambda = 0$; $g' = g - \omega^2 R$. The acceleration due to gravity is minimum. At poles $\lambda = 90°$; $g' = g$, it is maximum. At the equator, g' is minimum.

EXAMPLE 6.8

Find out the value of g' in your school laboratory?

Solution

Calculate the latitude of the city or village where the school is located. The information is available in Google search. For example, the latitude of Chennai is approximately 13 degree.

$$g' = g - \omega^2 R \cos^2 \lambda$$

Here $\omega^2 R = (2\text{x}3.14/86400)^2 \text{x} (6400\text{x}10^3) = 3.4\text{x}10^{-2}$ m s^{-2}.

It is to be noted that the value of λ should be in radian and not in degree. 13 degree is equivalent to 0.2268 rad.

$$g' = 9.8 - \left(3.4 \times 10^{-2}\right) \times \left(\cos 0.2268\right)^2$$

$$g' = 9.7677 \text{ m s}^{-2}$$

Points to Contemplate

Suppose you move towards east-west along the same latitude. Will the value of g' change?

6.4 ESCAPE SPEED AND ORBITAL SPEED

Hydrogen and helium are the most abundant elements in the universe but Earth's atmosphere consists mainly of nitrogen and oxygen. The following discussion brings forth the reason why hydrogen and helium are not found in abundance on the Earth's atmosphere. When an object is thrown up with some initial speed it will reach a certain height after which it will fall back to Earth. If the same object is thrown again with a higher speed, it reaches a greater height than the previous one and falls back to Earth. This leads to the question of what should be the speed of an object thrown vertically up such that it escapes the Earth's gravity and would never come back. This speed is called "Escape speed" and it is defined as "the minimum speed required for a body to escape from the earth's gravitational pull".

Consider an object of mass M on the surface of the Earth. When it is thrown up with an initial speed v_i, the initial total energy of the object is

$$E_i = \frac{1}{2} M v_i^2 - \frac{GMM_E}{R_E} \qquad (6.53)$$

where, M_E is the mass of the Earth and R_E- the radius of the Earth. The term $-\frac{GMM_E}{R_E}$ is the potential energy of the mass M.

When the object reaches a height far away from Earth and hence treated as approaching infinity, the gravitational potential energy becomes zero $[U(\infty) = 0]$ and the kinetic energy becomes zero as well. Therefore the final total energy of the

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object becomes zero. This is for minimum energy and for minimum speed to escape. Otherwise Kinetic energy can be nonzero.

$$E_f = 0$$

According to the law of energy conservation,

$$E_i = E_f \qquad (6.54)$$

Substituting (6.53) in (6.54) we get,

$$\frac{1}{2}Mv_i^2 - \frac{GMM_E}{R_E} = 0$$

$$\frac{1}{2}Mv_i^2 = \frac{GMM_E}{R_E} \qquad (6.55)$$

Consider the escape speed, the minimum speed required by an object to escape Earth's gravitational field, hence replace v_i with v_e. i.e,

$$\frac{1}{2}Mv_e^2 = \frac{GMM_E}{R_E}$$

$$v_e^2 = \frac{GMM_E}{R_E}.\frac{2}{M}$$

$$v_e^2 = \frac{2GM_E}{R_E}$$

Using $g = \frac{GM_E}{R_E^2}$,

$$v_e^2 = 2gR_E$$

$$v_e = \sqrt{2gR_E} \qquad (6.56)$$

From equation (6.56) the escape speed depends on two factors: acceleration due to gravity and radius of the Earth. It is completely independent of the mass of the object. By substituting the values of g (9.8 $m\,s^{-2}$) and $R_e = 6400\,km$, the escape speed of the Earth is $v_e = 11.2\,km\,s^{-1}$. The escape speed is independent of the direction in which the object is thrown. Irrespective of whether the object is thrown vertically up, radially outwards or tangentially it requires the same initial speed to escape Earth's gravity. It is shown in Figure 6.19

Figure 6.19 Escape speed independent of angle

Lighter molecules such as hydrogen and helium have enough speed to escape from the Earth, unlike the heavier ones such as nitrogen and oxygen. (The average speed of hydrogen and helium atoms compaired with the escape speed of the Earth,is presented in the kinetic theory of gases, unit 9).

6.4.1 Satellites, orbital speed and time period

We are living in a modern world with sophisticated technological gadgets and are able to communicate to any place on Earth. This advancement was made possible because of our understanding of solar system. Communication mainly depends on the satellites that orbit the Earth (Figure 6.20). Satellites revolve around the Earth just like the planets revolve around the Sun. Kepler's laws are applicable to man-made satellites also.

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For a satellite of mass M to move in a circular orbit, centripetal force must be acting on the satellite. This centripetal force is provided by the Earth's gravitational force.

Figure 6.20 Satellite revolving around the Earth.

$$\frac{Mv^2}{(R_E+h)} = \frac{GMM_E}{(R_E+h)^2} \qquad (6.57)$$

$$v^2 = \frac{GM_E}{(R_E+h)}$$

$$v^2 = \sqrt{\frac{GM_E}{(R_E+h)}} \qquad (6.58)$$

As h increases, the speed of the satellite decreases.

Time period of the satellite:

The distance covered by the satellite during one rotation in its orbit is equal to $2\pi(R_E+h)$ and time taken for it is the time period, T. Then

$$Speed\ v = \frac{Distance\ travelled}{Time\ taken} = \frac{2\pi(R_E+h)}{T}$$

From equation (6.58)

$$\sqrt{\frac{GM_E}{(R_E+h)}} = \frac{2\pi(R_E+h)}{T} \qquad (6.59)$$

$$T = \frac{2\pi}{\sqrt{GM_E}}(R_E+h)^{3/2} \qquad (6.60)$$

Squaring both sides of the equation (6.60), we get

$$T^2 = \frac{4\pi^2}{GM_E}(R_E+h)^3$$

$$\frac{4\pi^2}{GM_E} = \text{constant say } c$$

$$T^2 = c(R_E+h)^3 \qquad (6.61)$$

Equation (6.61) implies that a satellite orbiting the Earth has the same relation between time and distance as that of Kepler's law of planetary motion. For a satellite orbiting near the surface of the Earth, h is negligible compared to the radius of the Earth R_E. Then,

$$T^2 = \frac{4\pi^2}{GM_E}R_E^3$$

$$T^2 = \frac{4\pi^2}{GM_E/R_E^2}R_E$$

$$T^2 = \frac{4\pi^2}{g}R_E$$

$$since\ GM_E/R_E^2 = g$$

$$T = 2\pi\sqrt{\frac{R_E}{g}} \qquad (6.62)$$

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By substituting the values of $R_E = 6.4 \times 10^6$ m and g = 9.8 m s^{-2}, the orbital time period is obtained as T ≅ 85 minutes.

EXAMPLE 6.9

Moon is the natural satellite of Earth and it takes 27 days to go once around its orbit. Calculate the distance of the Moon from the surface of the Earth assuming the orbit of the Moon as circular.

Solution

We can use Kepler's third law,

$$T^2 = c\left(R_E + h\right)^3$$

$$T^{2/3} = c^{1/3}\left(R_E + h\right)$$

$$\left(\frac{T^2}{c}\right)^{1/3} = \left(R_E + h\right)$$

$$\left(\frac{T^2 GM_E}{4\pi^2}\right)^{\frac{1}{3}} = \left(R_E + h\right);$$

$$c = \frac{4\pi^2}{GM_E}$$

$$h = \left(\frac{T^2 GM_E}{4\pi^2}\right)^{1/3} - R_E$$

Here h is the distance of the Moon from the surface of the Earth. Here,

R_E – radius of the Earth = 6.4×10^6 m

M_E – mass of the Earth = 6.02×10^{24} kg

G – Universal gravitational constant = $6.67 \times 10^{-11} \frac{\text{Nm}^2}{\text{kg}^2}$

By substituting these values, the distance to the Moon from the surface of the Earth is calculated to be 3.77×10^5 km.

6.4.2 Energy of an Orbiting Satellite

The total energy of a satellite orbiting the Earth at a distance h from the surface of Earth is calculated as follows; The total energy of the satellite is the sum of its kinetic energy and the gravitational potential energy. The potential energy of the satellite is,

$$U = -\frac{GM_s M_E}{\left(R_E + h\right)} \qquad (6.63)$$

Here M_s - mass of the satellite, M_E - mass of the Earth, R_E - radius of the Earth.

The Kinetic energy of the satellite is

$$K.E = \frac{1}{2} M_s v^2 \qquad (6.64)$$

Here v is the orbital speed of the satellite and is equal to

$$v = \sqrt{\frac{GM_E}{\left(R_E + h\right)}} \qquad (6.65)$$

Substituting the value of v in (6.64), the kinetic energy of the satellite becomes,

$$K.E = \frac{1}{2}\frac{GM_E M_s}{\left(R_E + h\right)}$$

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Therefore the total energy of the satellite is

$$E = \frac{1}{2}\frac{GM_E M_s}{(R_E + h)} - \frac{GM_s M_E}{(R_E + h)}$$

$$E = -\frac{GM_s M_E}{2(R_E + h)} \qquad (6.66)$$

The negative sign in the total energy implies that the satellite is bound to the Earth and it cannot escape from the Earth.

As h approaches ∞, the total energy tends to zero. Its physical meaning is that the satellite is completely free from the influence of Earth's gravity and is not bound to Earth at large distances.

EXAMPLE 6.10

Calculate the energy of the (i) Moon orbiting the Earth and (ii) Earth orbiting the Sun.

Solution

Assuming the orbit of the Moon to be circular, the energy of Moon is given by,

$$E_m = -\frac{GM_E M_m}{2R_m}$$

where M_E is the mass of Earth 6.02×10^{24} kg; M_m is the mass of Moon 7.35×10^{22} kg; and R_m is the distance between the Moon and the center of the Earth $3.84 \times 10^5\, km$

$$G = 6.67 \times 10^{-11} \text{N m}^2 \text{kg}^{-2}.$$

$$E_m = -\frac{6.67 \times 10^{-11} \times 6.02 \times 10^{24} \times 7.35 \times 10^{22}}{2 \times 3.84 \times 10^5 \times 10^3}$$

$$E_m = -38.42 \times 10^{-19} \times 10^{46}$$

$$E_m = -38.42 \times 10^{27} \text{ Joule}$$

The negative energy implies that the Moon is bound to the Earth.

Same method can be used to prove that the energy of the Earth is also negative.

6.4.3 Geo-stationary and polar satellite

The satellites orbiting the Earth have different time periods corresponding to different orbital radii. Can we calculate the orbital radius of a satellite if its time period is 24 hours?

Kepler's third law is used to find the radius of the orbit.

$$T^2 = \frac{4\pi^2}{GM_E}(R_E + h)^3$$

$$(R_E + h)^3 = \frac{GM_E T^2}{4\pi^2}$$

$$R_E + h = \left(\frac{GM_E T^2}{4\pi^2}\right)^{1/3}$$

Substituting for the time period (24 hrs = 86400 seconds), mass, and radius of the Earth, h turns out to be 36,000 km. Such satellites are called "geo-stationary satellites", since they appear to be stationary when seen from Earth.

India uses the INSAT group of satellites that are basically geo-stationary satellites for the purpose of telecommunication. Another type of satellite which is placed at a distance

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Figure 6.21 Polar orbit and geostationary satellite

of 500 to 800 km from the surface of the Earth orbits the Earth from north to south direction. This type of satellite that orbits Earth from North Pole to South Pole is called a polar satellite. The time period of a polar satellite is nearly 100 minutes and the satellite completes many revolutions in a day. A Polar satellite covers a small strip of area from pole to pole during one revolution. In the next revolution it covers a different strip of area since the Earth would have moved by a small angle. In this way polar satellites cover the entire surface area of the Earth.

Figure 6.22 Strip of communication region, covered by a polar satellite.

6.4.4 Weightlessness

Weight of an object

Objects on Earth experience the gravitational force of Earth. The gravitational force acting on an object of mass m is mg. This force always acts downwards towards the center of the Earth. When we stand on the floor, there are two forces acting on us. One is the gravitational force, acting downwards and the other is the normal force exerted by the floor upwards on us to keep us at rest. The weight of an object $\vec{W}$ is defined as the downward force whose magnitude W is equal to that of upward force that must be applied to the object to hold it at rest or at constant velocity relative to the earth. The direction of weight is in the direction of gravitational force. So the magnitude of

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weight of an object is denoted as, W=N=mg. Note that even though magnitude of weight is equal to mg, it is not same as gravitational force acting on the object.

Apparent weight in elevators

Everyone who used an elevator would have felt a jerk when the elevator takes off or stops. Why does it happen? Understanding the concept of weight is crucial for explaining this effect. Let us consider a man inside an elevator in the following scenarios.

When a man is standing in the elevator, there are two forces acting on him.

1. Gravitational force which acts downward. If we take the vertical direction as positive y direction, the gravitational force acting on the man is $\vec{F}_G = -mg\hat{j}$
2. The normal force exerted by floor on the man which acts vertically upward, $\vec{N} = N\hat{j}$

Case (i) When the elevator is at rest

The acceleration of the man is zero. Therefore the net force acting on the man is zero. With respect to inertial frame (ground), applying Newton's second law on the man,

$$\vec{F}_G + \vec{N} = 0$$
$$-mg\hat{j} + N\hat{j} = 0$$

By comparing the components, we can write

$$N - mg = 0 \text{ (or) } N = mg \qquad (6.67)$$

Since weight, W =N, the apparent weight of the man is equal to his actual weight.

Case (ii) When the elevator is moving uniformly in the upward or downward direction

In uniform motion (constant velocity), the net force acting on the man is still zero. Hence, in this case also the apparent weight of the man is equal to his actual weight. It is shown in Figure 6.23(a)

Case (iii) When the elevator is accelerating upwards

If an elevator is moving with upward acceleration $(\vec{a} = a\hat{j})$ with respect to inertial frame (ground), applying Newton's second law on the man,

$$\vec{F}_G + \vec{N} = m\vec{a}$$

Writing the above equation in terms of unit vector in the vertical direction,

$$-mg\hat{j} + N\hat{j} = ma\hat{j}$$

By comparing the components,

$$N = m\,(g + a) \qquad (6.68)$$

Therefore, apparent weight of the man is greater than his actual weight. It is shown in Figure 6.23(b)

Case (iv) When the elevator is accelerating downwards

If the elevator is moving with downward acceleration $(\vec{a} = -a\hat{j})$, by applying Newton's second law on the man, we can write

$$\vec{F}_G + \vec{N} = m\vec{a}$$

Writing the above equation in terms of unit vector in the vertical direction,

$$-mg\hat{j} + N\hat{j} = -ma\hat{j}$$

By comparing the components,

$$N = m\,(g - a) \qquad (6.69)$$

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Therefore, apparent weight W = N = m(g-a) of the man is lesser than his actual weight. It is shown in Figure 6.23(c)

Weightlessness of freely falling bodies

Freely falling objects experience only gravitational force. As they fall freely, they are not in contact with any surface (by neglecting air friction). The normal force acting on the object is zero. The downward acceleration is equal to the acceleration due to the gravity of the Earth. i.e (a = g). From equation (6.69) we get.

$$a = g \quad \therefore N = m\,(g - g) = 0.$$

This is called the state of weightlessness. When the lift falls (when the lift wire cuts) with downward acceleration a = g, the person inside the elevator is in the state of weightlessness or free fall. It is shown in Figure 6.23(d)

When the apple was falling from the tree it was weightless.As soon as it hit Newton's head, it gained weight! and Newton gained physics!

Weightlessness in satellites:

There is a wrong notion that the astronauts in satellites experience no gravitational force because they are far away from the Earth. Actually the Earth satellites that orbit very close to Earth experience only gravitational force. The astronauts inside the satellite also experience the same gravitational force. Because of this, they cannot exert any force on the floor of the satellite. Thus, the floor of the satellite also cannot exert any normal force on the astronaut. Therefore, the astronauts inside a satellite are in the state of weightlessness. Not only the astronauts, but all the objects in the satellite will be in the state of weightlessness which is similar to that of a free fall. It is shown in the Figure 6.24.

Figure 6.23 Apparent weight in the lift

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Figure 6.24 The well known scientist Stephen Hawking in the state of weightlessness. https://www.youtube.com/watch?v=OCsuHvv_D0s

6.5 ELEMENTARY IDEAS OF ASTRONOMY

Astronomy is one of the oldest sciences in the history of mankind. In the olden days, astronomy was an inseparable part of physical science. It contributed a lot to the development of physics in the 16th century. In fact Kepler's laws and Newton's theory of gravitation were formulated and verified using astronomical observations and data accumulated over the centuries by famous astronomers like Hippachrus, Aristachrus, Ptolemy, Copernicus and Tycho Brahe. Without Tycho Brahe's astronomical observations, Kepler's laws would not have emerged. Without Kepler's laws, Newton's theory of gravitation would not have been formulated.

It was mentioned in the beginning of this chapter that Ptolemy's geocentric model was replaced by Copernicus' heliocentric model. It is important to analyze and explain the shortcoming of the geocentric model over heliocentric model.

6.5.1 Heliocentric system over geocentric system

When the motion of the planets are observed in the night sky by naked eyes over a period of a few months, it can be seen that the planets move eastwards and reverse their motion for a while and return to eastward motion again. This is called "retrograde motion" of planets.

Figure 6.25 shows the retrograde motion of the planet Mars. Careful observation for a period of a year clearly shows that Mars initially moves eastwards (February to June), then reverses its path and moves backwards (July, August, September). It changes its direction of motion once again and continues its forward motion (October onwards). In olden days, astronomers recorded the retrograde motion of all

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Figure 6.25 Retrograde motion of planets

visible planets and tried to explain the motion. According to Aristotle, the other planets and the Sun move around the Earth in the circular orbits. If it was really a circular orbit it was not known how the planet could reverse its motion for a brief interval. To explain this retrograde motion, Ptolemy introduced the concept of "epicycle" in his geocentric model. According to this theory, while the planet orbited the Earth, it also underwent another circular motion termed as "epicycle". A combination of epicycle and circular motion around the Earth gave rise to retrograde motion of the planets with respect to Earth (Figure 6.26). Essentially Ptolemy retained the Earth centric idea of Aristotle and added the epicycle motion to it.

Figure 6.26 "Epicycle" motion of planetary objects around Earth, depicted with respect to months of observation.

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But Ptolemy's model became more and more complex as every planet was found to undergo retrograde motion. In the 15th century, the Polish astronomer Copernicus proposed the heliocentric model to explain this problem in a simpler manner. According to this model, the Sun is at the center of the solar system and all planets orbited the Sun. The retrograde motion of planets with respect to Earth is because of the relative motion of the planet with respect to Earth. The retrograde motion from the heliocentric point of view is shown in Figure 6.27.

Figure 6.27 shows that the Earth orbits around the Sun faster than Mars. Because of the relative motion between Mars and Earth, Mars appears to move backwards from July to October. In the same way the retrograde motion of all other planets was explained successfully by the Copernicus model. It was because of its simplicity, the heliocentric model slowly replaced the geocentric model. Historically, if any natural phenomenon has one or more explanations, the simplest one is usually accepted. Though this was not the only reason to disqualify the geocentric model, a detailed discussion on correctness of the Copernicus model over to Ptolemy's model can be found in astronomy books.

Activity

Students are encouraged to observe the motion of the planet Mars by naked eye and identify its retrograde motion. As mentioned above, to observe the retrograde motion six to seven months are required. So students may start their observation of Mars from the month of June and continue till April next year. Mars is the little bright planet with reddish color. The position of the planet Mars in the sky can be easily taken from 'Google'.

Figure 6.27 'Retrograde motion' in heliocentric model

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6.5.2 Kepler's Third Law and The Astronomical Distance

When Kepler derived his three laws, he strongly relied on Tycho Brahe's astronomical observation. In his third law, he formulated the relation between the distance of a planet from the Sun to the time period of revolution of the planet. Astronomers cleverly used geometry and trigonometry to calculate the distance of a planet from the Sun in terms of the distance between Earth and Sun. Here we can see how the distance of Mercury and Venus from the Sun were measured. The Venus and Mercury, being inner planets with respect to Earth, the maximum angular distance they can subtend at a point on Earth with respect to the Sun is 46 degree for Venus and 22.5 degree for Mercury. It is shown in the Figure 6.28

Figure 6.29 shows that when Venus is at maximum elongation (i.e., 46 degree) with respect to Earth, Venus makes 90 degree to Sun. This allows us to find the distance between Venus and Sun. The distance between Earth and Sun is taken as one Astronomical unit (1 AU).

Figure 6.28 Angle of elevation for Venus and Mercury from horizon

The trigonometric relation satisfied by this right angled triangle is shown in Figure 6.29.

$$\sin\theta = \frac{r}{R}$$

where R = 1 AU.

$$r = R\sin\theta = (1AU)(\sin 46°)$$

Figure 6.29 Angle of elevation for Mercury from horizon

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Here $\sin 46° = 0.72$. Hence Venus is at a distance of 0.72 AU from Sun. Similarly, the distance between Mercury (θ is 22.5 degree) and Sun is calculated as 0.38 AU. To find the distance of exterior planets like Mars and Jupiter, a slightly different method is used. The distances of planets from the Sun is given in the table below.

Table 6.2 a^3/T^2 for different planets

Planet	semi major axis of the orbit(a)	Period T (days)	a^3/T^2
Mercury	0.389 AU	87.77	7.64
Venus	0.724 AU	224.70	7.52
Earth	1.000 AU	365.25	7.50
Mars	1.524 AU	686.98	7.50
Jupiter	5.200 AU	4332.62	7.49
Saturn	9.510 AU	10,759.20	7.40

It is to be noted that to verify the Kepler's law we need only high school level geometry and trigonometry.

Activity

Venus can be observed with the naked eye. We can see Venus during sunrise or sunset. Students are encouraged to observe the motion of Venus and verify that the maximum elevation is at 46 degree and calculate the distance of Venus from the Sun. As pointed out already Google or Stellarium will be helpful in locating the position of Venus in the sky.

6.5.3 Measurement of radius of the Earth

Around 225 B.C a Greek librarian "Eratosthenes" who lived at Alexandria measured the radius of the Earth with a small error when compared with results using modern measurements. The technique he used involves lower school geometry and

Figure 6.30 Measuring radius of The Earth

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brilliant insight. He observed that during noon time of summer solstice the Sun's rays cast no shadow in the city Syne which was located 500 miles away from Alexandria. At the same day and same time he found that in Alexandria the Sun's rays made 7.2 degree with local vertical as shown in the Figure 6.30. He realized that this difference of 7.2 degree was due to the curvature of the Earth.

The angle 7.2 degree is equivalent to $\frac{1}{8}$ radian. So $\theta = \frac{1}{8}$ rad;

If S is the length of the arc between the cities of Syne and Alexandria, and if R is radius of Earth, then

$$S = R\theta = 500 \text{ miles},$$

so radius of the Earth

$$R = \frac{500}{\theta} \text{ miles}$$

$$R = \frac{500}{\left(\frac{1}{8}\right)} \text{ miles}$$

R = 4000 miles

1 mile is equal to 1.609 km. So, he measured the radius of the Earth to be equal to R = 6436 km, which is amazingly close to the correct value of 6378 km.

The distance of the Moon from Earth was measured by a famous Greek astronomer Hipparchus in the 3rd century BC.

Activity

To measure the radius of the Earth, choose two different places (schools) that are separated by at least 500 km. It is important to note that these two places have to be along the same longitude of the Earth (For example Hosur and Kanyakumari lie along the same longitude of 77.82° E). Take poles of known length (h) and fix them vertically in the ground (it may be in the school playgrounds) at both the places. At exactly noon in both the places the length of the shadow (L) cast by each pole has to be noted down. Draw the picture like in Figure 6.30. By using the equation $\tan\theta = \frac{L}{h}$, the angle in radian can be found at each place. The difference in angle (θ') is due to the curvature of the Earth. Now the distance between the two schools can be obtained from 'Google maps'. Divide the distance with the angle (θ' in radians) which will give the radius of the Earth.

6.5.4 Interesting Astronomical Facts

1. **Lunar eclipse and measurement of shadow of Earth**

   On January 31, 2018 there was a total lunar eclipse which was observed from various places including Tamil Nadu. It is possible to measure the radius of shadow of the Earth at the point where the Moon crosses. Figure 6.31 illustrates this.

   When the Moon is inside the umbra shadow, it appears red in color. As soon as the Moon exits from the umbra

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Figure 6.31 Total lunar eclipse

shadow, it appears in crescent shape. Figure 6.32 is the photograph taken by digital camera during Moon's exit from the umbra shadow.

Figure 6.32 Image of the Moon when it exits from umbra shadow

By finding the apparent radii of the Earth's umbra shadow and the Moon, the ratio of the these radii can be calculated. This is shown in Figures 6.33 and 6.34.

Figure 6.33 Schematic diagram of umbra disk radius

The apparent radius of Earth's umbra shadow = R_s = 13.2 cm

The apparent radius of the Moon = R_m = 5.15 cm

The ratio $\frac{R_s}{R_m} \approx 2.56$

The radius of the Earth's umbra shadow is $R_s = 2.56 \times R_m$

The radius of Moon $R_m = 1737\,\text{km}$

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Figure 6.34 Calculation of umbra radius

The radius of the Earth's umbra shadow is $R_s = 2.56 \times 1737\,\text{km} \cong 4446\,\text{km}$.

The correct radius is 4610 km.

The percentage of error in the calculation

$$= \frac{4610 - 4446}{4610} \times 100 = 3.5\%.$$

The error will reduce if the pictures taken using a high quality telescope are used. It is to be noted that this calculation is done using very simple mathematics.

Early astronomers proved that Earth is spherical in shape by looking at the shape of the shadow cast by Earth on the Moon during lunar eclipse.

2. **Why there are no lunar eclipse and solar eclipse every month?**
If the orbits of the Moon and Earth lie on the same plane, during full Moon of every month, we can observe lunar eclipse. If this is so during new Moon we can observe solar eclipse. But Moon's orbit is tilted 5° with respect to Earth's orbit. Due to this 5° tilt, only during certain periods of the year, the Sun, Earth and Moon align in straight line leading to either lunar eclipse or solar eclipse depending on the alignment. This is shown in Figure 6.35

3. **Why do we have seasons on Earth?**
The common misconception is that 'Earth revolves around the Sun, so when the Earth is very far away, it is winter and when the Earth is nearer, it is summer'. Actually, the seasons in the Earth arise due to the rotation of Earth around the Sun with 23.5° tilt. This is shown in Figure 6.36

Due to this 23.5° tilt, when the northern part of Earth is farther to the Sun, the southern part is nearer to the Sun. So when it is summer in the northern hemisphere, the southern hemisphere experience winter.

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Figure 6.35 Orbital tilt of the Moon

Figure 6.36 Seasons on Earth

Note Pole star is a star located exactly above the Earth's axis of rotation, hence it appears to be stationary. The Star Polaris is our pole star.

4. **Star's apparent motion and spinning of the Earth**

The Earth's spinning motion can be proved by observing star's position over a night. Due to Earth's spinning motion, the stars in sky appear to move in circular motion about the pole star as shown in Figure 6.37

Point to ponder

Using Sun rays and shadows, How will you prove that the Earth's tilt is 23.5° ?

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Rotation of the earth about PQ explains the apparent rotation of stars (in opposite direction) about PQ.

If we observe the stars, it slowly changes its position and appears to move in a circle over the night.

Figure 6.37 Star's apparent circular motion due to Earth's rotation.

6.5.5 Recent developments of astronomy and gravitation

Till the 19th century astronomy mainly depended upon either observation with the naked eye or telescopic observation. After the discovery of the electromagnetic spectrum at the end of the 19th century, our understanding of the universe increased enormously. Because of this development in the late 19th century it was found that Newton's law of gravitation could not explain certain phenomena and showed some discrepancies. Albert Einstein formulated his 'General theory of relativity' which was one of the most successful theories of 20th century in the field of gravitation.

In the twentieth century both astronomy and gravitation merged together and have grown in manifold. The birth and death of stars were more clearly understood. Many Indian physicists made very important contributions to the field of astrophysics and gravitation. Subramanian Chandrasekar formulated the theory of black holes and explained the life of stars. These studies brought him the Nobel prize in the year 1983. Another very notable Indian astrophysicist Meghnad Saha discovered the ionization formula which was useful in classifying stars. This formula is now known as "Saha ionization formula". In the field of gravitation Amal Kumar Raychaudhuri solved an equation now known as "Raychaudhuri equation" which was a very important contribution. Another notable Indian Astrophysicist Jayant V Narlikar made pioneering contribution in the field of astrophysics and has written interesting books on astronomy and astrophysics. IUCAA (Inter University Center for Astronomy and Astrophysics) is one of the important Indian research institutes where active research in astrophysics and gravitation are conducted. The institute was founded by Prof. J.V. Narlikar. Students are encouraged to read more about the recent developments in these fields.

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SUMMARY

- The motion of planets can be explained using Kepler's laws.
- **Kepler's first law**: All the planets in the solar system orbit the Sun in elliptical orbits with the Sun at one of the foci.
- **Kepler's second law:** The radial vector line joining the Sun to a planet sweeps equal areas in equal intervals of time.
- **Kepler's third law:** The ratio of the square of the time period of planet to the cubic power of semi major axis is constant for all the planets in the solar system.
- **Newton's law of gravitation** states that the gravitational force between two masses is directly proportional to product of masses and inversely proportional to square of the distance between the masses. In vector form it is given by $\vec{F} = -\frac{Gm_1m_2}{r^2}\hat{r}$
- Gravitational force is a central force.
- Kepler's laws can be derived from Newton's law of gravitation.
- The gravitational field due to a mass m at a point which is at a distance r from mass m is given by $\vec{E} = -\frac{Gm}{r^2}\hat{r}$. It is a vector quantity.
- The gravitational potential energy of two masses is given by $U = -\frac{Gm_1m_2}{r}$. It is a scalar quantity.
- The gravitational potential at a point which is at a distance r from mass m is given by $V = -\frac{Gm}{r}$. It is a scalar quantity.
- The acceleration due to Earth's gravity decreases as altitude increases and as depth increases.
- Due to rotation of the Earth, the acceleration due to gravity is maximum at poles and minimum at Earth's equator.
- The (escape) speed of any object required to escape from the Earth's gravitational field is $v_e = \sqrt{2gR_e}$. It is independent of mass of the object.
- The energy of the satellite is negative. It implies that the satellite is bound to Earth's gravitational force.
- Copernicus model explained that retrograde motion is due to relative motion between planets. This explanation is simpler than Ptolemy's epicycle explanation which is complicated
- Copernicus and Kepler measured the distance between a planet and the Sun using simple geometry and trigonometry.
- 2400 years ago, Eratosthenes measured the radius of the Earth using simple geometry and trigonometry.

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CONCEPT MAP

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EVALUATION

I. Multiple Choice Questions

1. The linear momentum and position vector of the planet is perpendicular to each other at

(a) perihelion and aphelion

(b) at all points

(c) only at perihelion

(d) no point

2. If the masses of the Earth and Sun suddenly double, the gravitational force between them will

(a) remain the same

(b) increase 2 times

(c) increase 4 times

(d) decrease 2 times

3. A planet moving along an elliptical orbit is closest to the Sun at distance r_1 and farthest away at a distance of r_2. If v_1 and v_2 are linear speeds at these points respectively. Then the ratio $\frac{v_1}{v_2}$ is

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(a) $\frac{r_2}{r_1}$ (b) $\left(\frac{r_2}{r_1}\right)^2$

(c) $\frac{r_1}{r_2}$ (d) $\left(\frac{r_1}{r_2}\right)^2$

4. The time period of a satellite orbiting Earth in a cirular orbit is independent of .

(a) Radius of the orbit

(b) The mass of the satellite

(c) Both the mass and radius of the orbit

(d) Neither the mass nor the radius of its orbit

5. If the distance between the Earth and Sun were to be doubled from its present value, the number of days in a year would be

(a) 64.5 (b) 1032

(c) 182.5 (d) 730

6. According to Kepler's second law, the radial vector to a planet from the Sun sweeps out equal areas in equal intervals of time. This law is a consequence of

(a) conservation of linear momentum

(b) conservation of angular momentum

(c) conservation of energy

(d) conservation of kinetic energy

7. The gravitational potential energy of the Moon with respect to Earth is

(a) always positive

(b) always negative

(c) can be positive or negative

(d) always zero

8. The kinetic energies of a planet in an elliptical orbit about the Sun, at positions A, B and C are K_A, K_B and K_C respectively. AC is the major axis and SB is perpendicular to AC at the position of the Sun S as shown in the figure. Then

(NEET 2018)

(a) $K_A > K_B > K_C$

(b) $K_B < K_A < K_C$

(c) $K_A < K_B < K_C$

(d) $K_B > K_A > K_C$

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9. The work done by the Sun's gravitational force on the Earth is
   (a) always zero
   (b) always positive
   (c) can be positive or negative
   (d) always negative

10. If the mass and radius of the Earth are both doubled, then the acceleration due to gravity g'
   (a) remains same (b) $\frac{g}{2}$
   (c) 2 g (d) 4 g

11. The magnitude of the Sun's gravitational field as experienced by Earth is
   (a) same over the year
   (b) decreases in the month of January and increases in the month of July
   (c) decreases in the month of July and increases in the month of January
   (d) increases during day time and decreases during night time.

12. If a person moves from Chennai to Trichy, his weight
   (a) increases
   (b) decreases
   (c) remains same
   (d) increases and then decreases

13. An object of mass 10 kg is hanging on a spring scale which is attached to the roof of a lift. If the lift is in free fall, the reading in the spring scale is
   (a) 98 N (b) zero
   (c) 49 N (d) 9.8 N

14. If the acceleration due to gravity becomes 4 times its original value, then escape speed
   (a) remains same
   (b) 2 times of original value
   (c) becomes halved
   (d) 4 times of original value

15. The kinetic energy of the satellite orbiting around the Earth is
   (a) equal to potential energy
   (b) less than potential energy
   (c) greater than kinetic energy
   (d) zero

Answers

1) a	**2)** c	**3)** a	**4)** b	**5)** b
6) b	**7)** b	**8)** a	**9)** c	**10)** b
11) c	**12)** b	**13)** b	**14)** b	**15)** b

II. Short Answer Questions

1. State Kepler's three laws.
2. State Newton's Universal law of gravitation.
3. Will the angular momentum of a planet be conserved? Justify your answer.
4. Define the gravitational field. Give its unit.
5. What is meant by superposition of gravitational field?
6. Define gravitational potential energy.
7. Is potential energy the property of a single object? Justify.
8. Define gravitational potential.
9. What is the difference between gravitational potential and gravitational potential energy?
10. What is meant by escape speed in the case of the Earth?
11. Why is the energy of a satellite (or any other planet) negative?
12. What are geostationary and polar satellites?
13. Define weight

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14. Why is there no lunar eclipse and solar eclipse every month?
15. How will you prove that Earth itself is spinning?

III. Long Answer Questions

1. Discuss the important features of the law of gravitation.
2. Explain how Newton arrived at his law of gravitation from Kepler's third law.
3. Explain how Newton verified his law of gravitation.
4. Derive the expression for gravitational potential energy.
5. Prove that at points near the surface of the Earth, the gravitational potential energy of the object is $U = mgh$
6. Explain in detail the idea of weightlessness using lift as an example.
7. Derive an expression for escape speed.
8. Explain the variation of g with lattitude.
9. Explain the variation of g with altitude.
10. Explain the variation of g with depth from the Earth's surface.
11. Derive the time period of satellite orbiting the Earth.
12. Derive an expression for energy of satellite.
13. Explain in detail the geostationary and polar satellites.
14. Explain how geocentric theory is replaced by heliocentric theory using the idea of retrograde motion of planets.
15. Explain in detail the Eratosthenes method of finding the radius of Earth.
16. Describe the measurement of Earth's shadow (umbra) radius during total lunar eclipse

IV. Exercises

1. An unknown planet orbits the Sun with distance twice the semi major axis distance of the Earth's orbit. If the Earth's time period is T_1, what is the time period of this unknown planet?

   Ans: $T_2 = 2\sqrt{2}T_1$

2. Assume that you are in another solar system and provided with the set of data given below consisting of the planets' semi major axes and time periods. Can you infer the relation connecting semi major axis and time period?

Planet (imaginary)	Time period(T) (in year)	Semi major axis (a) (in AU)
Kurinji	2	8
Mullai	3	18
Marutham	4	32
Neithal	5	50
Paalai	6	72

   Ans: $a \propto 2T^2$

3. If the masses and mutual distance between the two objects are doubled, what is the change in the gravitational force between them?

   Ans: No change

4. Two bodies of masses m and 4m are placed at a distance r. Calculate the gravitational potential at a point on the line joining them where the gravitional field is zero.

   Ans: $V = -\dfrac{9Gm}{r}$

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5. If the ratio of the orbital distance of two planets $\frac{d_1}{d_2}$ =2, what is the ratio of gravitational field experienced by these two planets?

Ans: $E_2 = 4\,E_1$

6. The Moon Io orbits Jupiter once in 1.769 days. The orbital radius of the Moon Io is 421700 km. Calculate the mass of Jupiter?

Ans: 1.898×10^{27} kg

7. If the angular momentum of a planet is given by $\vec{L} = 5t^2\hat{i} - 6t\hat{j} + 3\hat{k}$ What is the torque experienced by the planet? Will the torque be in the same direction as that of the angular momentum?

Ans: $\vec{\tau} = 10t\hat{i} - 6\hat{j}$

8. Four particles, each of mass M and equidistant from each other, move along a circle of radius R under the action of their mutual gravitational attraction. Calculate the speed of each particle

Ans: $\frac{1}{2}\sqrt{\frac{GM}{R}\left(1+2\sqrt{2}\right)}$

9. Suppose unknowingly you wrote the universal gravitational constant value as $G = 6.67 \times 10^{11}$ instead of the correct value $G = 6.67 \times 10^{-11}$, what is the acceleration due to gravity g' for this incorrect G? According to this new acceleration due to gravity, what will be your weight W'?

Ans: g' $= 10^{22}$ g, W' $= 10^{22}$W

10. Calculate the gravitational field at point O due to three masses m_1,m_2 and m_3 whose positions are given by the following figure. If the masses m_1 and m_2 are equal what is the change in gravitational field at the point O?

Ans: $\vec{E} = \frac{G}{a^2}\left[(m_1 - m_2)i + m_3 j\right]$

if $m_1 = m_2$, $\vec{E} = \frac{G}{a^2}\left[m_3\,\hat{j}\right]$

11. What is the gravitational potential energy of the Earth and Sun? The Earth to Sun distance is around 150 million km. The mass of the Earth is 5.9×10^{24} kg and mass of the Sun is 1.9×10^{30} kg.

Ans: U= -49.84×10^{32} Joule

12. Earth revolves around the Sun at 30 km s^{-1}. Calculate the kinetic energy of the Earth. In the previous example you calculated the potential energy of the Earth. What is the total energy of the Earth in that case? Is the total energy positive? Give reasons.

Ans: K.E = 26.5×10^{32} J

E = -23.29×10^{32} J

(–) ve implies that Earth is bounded with Sun

13. An object is thrown from Earth in such a way that it reaches a point at infinity with non-zero kinetic energy

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$\left[K.E(r=\infty)=\frac{1}{2}Mv_{\infty}^{2}\right]$, with what velocity should the object be thrown from Earth?

Ans: $v_e=\sqrt{v_\infty^2+2gR_E}$

14. Suppose we go 200 km above and below the surface of the Earth, what are the g values at these two points? In which case, is the value of g small?

Ans: $g_{down} = 0.96\ g$

$g_{up} = 0.94\ g$

15. Calculate the change in g value in your district of Tamil nadu. (Hint: Get the latitude of your district of Tamil nadu from the Google). What is the difference in g values at Chennai and Kanyakumari?

Ans: $g_{chennai} = 9.7677\ m\ s^{-2}$

$g_{Kanyakumari} = 9.7667\ m\ s^{-2}$

$\Delta g = 0.001\ m\ s^{-2}$

V. Conceptual Questions

1. In the following, what are the quantities which that are conserved?

a) Linear momentum of planet

b) Angular momentum of planet

c) Total energy of planet

d) Potential energy of a planet

2. The work done by Sun on Earth in one year will be

a) Zero b) Non zero

c) positive d) negative

3. The work done by Sun on Earth at any finite interval of time is

a) positive, negative or zero

b) Strictly positive

c) Strictly negative

d) It is always zero

4. If a comet suddenly hits the Moon and imparts energy which is more than the total energy of the Moon, what will happen?

5. If the Earth's pull on the Moon suddenly disappears, what will happen to the Moon?

6. If the Earth has no tilt, what happens to the seasons of the Earth?

7. A student was asked a question 'why are there summer and winter for us? He replied as 'since Earth is orbiting in an elliptical orbit, when the Earth is very far away from the Sun(aphelion) there will be winter, when the Earth is nearer to the Sun(perihelion) there will be winter'. Is this answer correct? If not, what is the correct explanation for the occurrence of summer and winter?

8. The following photographs are taken from the recent lunar eclipse which occurred on January 31, 2018. Is it possible to prove that Earth is a sphere from these photographs?

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BOOKS FOR REFERENCE

1. Mechanics by Charles Kittel, Walter Knight, Malvin Ruderman, Carl Helmholtz and Moyer
2. Newtonian Mechanics by A.P. French
3. Introduction to Mechanics by Daniel Kepler and Robert Kolenkow
4. Mechanics by Somnath Datta
5. Concepts of Physics volume 1 and Volume 2 by H.C. Verma
6. Physics for Scientist and Engineers with Modern physics by Serway and Jewett
7. Physics for Scientist and Engineers by Paul Tipler and Gene Mosca
8. Physics for the Inquiring Mind by Eric Rogers
9. Fundamental laws of Mechanics by Irodov.
10. Question and Problems in School Physics by Tarasov and Tarasova

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ICT CORNER

Gravitation

Through this activity you will be able to learn about the gravitational force and orbital paths.

STEPS:

- Click the URL or scan the QR code to launch the activity page. Click on the box labelled "Model" to start the activity.
- In the activity window, a diagram of sun and earth is given. Click the play icon to see the motion of earth.
- We can change the objects by selecting objects from the table given in the right side window.
- The path of the gravity, velocity and the object in motion can be viewed. Check on the relevant boxes given in the table.

Step1

Step2

Step3

Step4

URL:

https://phet.colorado.edu/sims/html/gravity-and-orbits/latest/gravity-and-orbits_en.html

* Pictures are indicative only.

* If browser requires, allow **Flash Player** or **Java Script** to load the page.

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UNIT 7

PROPERTIES OF MATTER

Many of the greatest advances that have been made from the beginning of the world to the present time have been made in the earnest desire to turn the knowledge of the properties of matter to some purpose useful to mankind– Lord Kelvin

Learning Objectives

In this unit, the student is exposed to

- inter atomic or intermolecular forces in matter
- stress, strain and elastic modulus
- surface tension
- viscosity
- properties of fluids and their applications

7.1 INTRODUCTION

One of the oldest dams in the world is Kallanai (கல்லணை) located at Trichy. Kallanai was built across river Kaveri for the purpose of irrigation. During heavy floods, the velocity of water is generally very high in river Kaveri. The stability and utility of Kallanai dam reveal the intuitive scientific understanding of Tamils who designed this dam as early as 2nd century AD. The other example known for insightful constructions of the past is the pyramids in Egypt. The flyovers and overbridges are common worldwide today. Heavy vehicles ply over and hence, the bridges are always under stress. Without effective design using suitable materials, the bridges and flyovers will not

Kallanai (கல்லணை)

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be stable. The growth of human civilization is due to the understanding of various forms of matter (solid, liquid, and gas).

The study of properties of matter is very essential in selecting a material for a specific application. For example, in technology, the materials used for space applications should be of lightweight but should be strong. Materials used for artificial human organ replacements should be biocompatible. Artificial body fluids are used as tissue substitute for radiotherapy analysis in medicine. Fluids used as lubricants or fuel should possess certain properties. Such salient macroscopic properties are decided by the microscopic phenomena within matter. This unit deals with the properties of solids and fluids and the laws governing the behaviour of matter.

7.2 MICROSCOPIC UNDERSTANDING OF VARIOUS STATES OF MATTER

Even though various forms of matter such as solid food, liquids like water, and the air that we breathe are familiar in the day – to – day lifestyle for the past several thousand years, the microscopic understanding of solids, liquids, and gases was established only in 20th century. In the universe, everything is made up of atoms. If so, why the same materials exist in three states? For example, water exists in three forms as solid ice, liquid water, and gaseous steam. Interestingly ice, water, and steam are made up of same water molecules; two hydrogen atoms and one oxygen atom form a water molecule. Physics helps us to explore this beauty of nature at the microscopic level. The distance between the atoms or molecules determines whether it exists in solid, liquid or gaseous state.

Solids

In solids, atoms or molecules are tightly fixed. In the solid formation, atoms get bound together through various types of bonding. Due to the interaction between the atoms, they position themselves at a particular interatomic distance. This position of atoms in this bound condition is called their mean positions.

Liquids

When the solid is not given any external energy such as heat, it will remain as a solid due to the bonding between atoms. When heated, atoms of the solid receive thermal energy and vibrate about their mean positions. When the solid is heated above its melting point, the heat energy will break the bonding between atoms and eventually the atoms receive enough energy and wander around. Here also the intermolecular (or interatomic) forces are important, but the molecules will have enough energy to move around, which makes the structure mobile.

Gases

When a liquid is heated at constant pressure to its boiling point or when the pressure is reduced at a constant temperature it will convert to a gas. This process of a liquid changing to a gas is called evaporation. The gas molecules have either very weak bonds or no bonds at all. This enables them to move freely and quickly. Hence, the gas will conform to the shape of its container and also will expand to fill the container. The transition from solid to liquid to gaseous states with the variation in external energy is schematically shown in Figure 7.1.

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Figure 7.1 Schematic representations of the transition from solid to liquid to gaseous states with a change in external energy

In addition to the three physical states of matter (solid, liquid, and gas), in extreme environments, matter can exist in other states such as plasma, Bose-Einstein condensates. Additional states, such as quark-gluon plasmas are also believed to be possible. A major part of the atomic matter of the universe is hot plasma in the form of rarefied interstellar medium and dense stars.

In the study of Newtonian mechanics (Volume 1), we assumed the objects to be either as point masses or perfect rigid bodies (collection of point masses). Both these are idealized models. In rigid bodies, changes in the shape of the bodies are so small that they are neglected. In real materials, when a force is applied on the objects, there could be some deformations due to the applied force. It is very important to know how materials behave when a deforming force is applied.

7.2.1 Elastic behaviour of materials

In a solid, interatomic forces bind two or more atoms together and the atoms occupy the positions of stable equilibrium. When a deforming force is applied on a body, its atoms are pulled apart or pushed closer. When the deforming force is removed, interatomic forces of attraction or repulsion restore the atoms to their equilibrium positions. **If a body regains its original shape and size after the removal of deforming force, it is said to be elastic and the property is called elasticity**. The force which changes the size or shape of a body is called a deforming force.

Examples: Rubber, metals, steel ropes.

Figure 7.2 Elasticity

Plasticity:

If a body does not regain its original shape and size after removal of the deforming force, it is said to be a plastic body and the property is called plasticity.

Example: Glass

7.2.2 Stress and strain

(a) Stress:

When a force is applied, the size or shape or both may change due to the change in relative positions of atoms or molecules. This deformation may not be noticeable to

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our naked eyes but it exists in the material itself. When a body is subjected to such a deforming force, internal force is developed in it, called as restoring force. The force per unit area is called as stress.

$$\text{Stress, } \sigma = \frac{\text{Force}}{\text{Area}} = \frac{F}{A} \qquad (7.1)$$

The SI unit of stress is N m^{-2} or pascal (Pa) and its dimension is [ML^{-1}T^{-2}]. Stress is a tensor.

(i) Longitudinal stress and shearing stress:

Let us consider a body as shown in Figure 7.3. When many forces act on the system (body), the center of mass (defined in unit 5) remains at rest. However, the body gets deformed due to these forces and so the internal forces appear. Let ΔA be the cross sectional area of the body. The parts of the body on two sides of ΔA exert internal forces $\vec{F}$ and $-\vec{F}$ on each other which is due to deformation. The force can be resolved in two components; F_n normal to the surface ΔA (perpendicular to the surface) and F_t tangential to the surface ΔA (tangent to the surface). The normal stress or longitudinal stress (σ_n) over the area is defined as

$$\sigma_n = \frac{F_n}{\Delta A}$$

Figure 7.3 Longitudinal stress

Similarly, the tangential stress or shearing stress σ_t over the area is defined as

$$\sigma_t = \frac{F_t}{\Delta A}$$

Longitudinal stress can be classified into two types, tensile stress and compressive stress.

Tensile stress

Internal forces on the two sides of ΔA may pull each other, i.e., it is stretched by equal and opposite forces. Then, the longitudinal stress is called tensile stress.

Figure 7.4 Tensile stress

Compressive stress

When forces acting on the two sides of ΔA push each other, ΔA is pushed by equal and opposite forces at the two ends. In this case, ΔA is said to be under compression. Then, the longitudinal stress is called compressive stress.

Figure 7.5 Compressive stress

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(ii) Volume stress

This happens when a body is acted by forces everywhere on the surface such that the force at any point is normal to the surface and the magnitude of the force on a small surface area is proportional to the area. For instance, when a solid is immersed in a fluid, the pressure at the location of the solid is P, the force on any area ΔA is

$$F = P\,\Delta A$$

Where, F is perpendicular to the area. Thus, force per unit area is called volume stress.

$$\sigma_v = \frac{F}{A}$$

which is the same as the pressure.

(b) Strain:

Strain measures how much an object is stretched or deformed when a force is applied. Strain deals with the fractional change in the size of the object, in other words, strain measures the degree of deformation. As an example, in one dimension, consider a rod of length L when it stretches to a new length ΔL then

$$\text{Strain, } \varepsilon = \frac{\text{Change in size}}{\text{Original size}} = \frac{\Delta L}{L} \qquad (7.2)$$

ε is a dimensionless quantity and has no unit. Strain is classified into three types.

(1) Longitudinal strain

When a rod of length L is pulled by equal and opposite forces, the longitudinal strain is defined as

$$\varepsilon_l = \frac{\text{Increase in length of the rod}}{\text{Original or natural length of the rod}} = \frac{\Delta L}{L} \qquad (7.3)$$

Longitudinal strain can be classified into two types

(i) Tensile strain: If the length is increased from its natural length then it is known as tensile strain.

(ii) Compressive strain: If the length is decreased from its natural length then it is known as compressive strain.

(2) Shearing strain

Consider a cuboid as shown in Figure 7.6. Let us assume that the body remains in translational and rotational equilibrium. Let us apply the tangential force F along AD such that the cuboid deforms as shown in Figure 7.6. Hence, shearing strain or shear is (ε_s)

Figure 7.6 Shearing strain

$$\varepsilon_s = \frac{AA'}{BA} = \frac{x}{h} = \tan\theta \qquad (7.4)$$

For small angle, $\tan\theta \approx \theta$

Therefore, shearing strain or shear,

$$\varepsilon_s = \frac{x}{h} = \theta = \text{Angle of shear}$$

(3) Volume strain

If the body is subjected to a volume stress, the volume will change. Let V be the original volume of the body before stress and $V + \Delta V$ be the change in volume due to stress. The volume strain which measures the fractional change in volume is

$$\text{Volume strain, } \varepsilon_v = \frac{\Delta V}{V} \qquad (7.5)$$

Elastic Limit

The maximum stress within which the body regains its original size and shape after the

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removal of deforming force is called the elastic limit.

If the deforming force exceeds the elastic limit, the body acquires a permanent deformation. For example, rubber band loses its elasticity if pulled apart too much. It changes its size and becomes misfit to be used again.

7.2.3 Hooke's law and its experimental verification

Hooke's law states that for a small deformation within the elastic limit, the strain produced in a body is directly proportional to the stress that produces it.

It can be verified in a simple way by stretching a thin straight wire (stretches like spring) of length L and uniform cross-sectional area A suspended from a fixed point O. A pan and a pointer are attached at the free end of the wire as shown in Figure 7.7 (a).

The extension produced on the wire is measured using a vernier scale arrangement. The experiment shows that for a given load, the corresponding stretching force is F and the elongation produced on the wire is ΔL. It is directly proportional to the original length L and inversely proportional to the area of cross section A. A graph is plotted using F on the X- axis and ΔL on the Y- axis. This graph is a straight line passing through the origin as shown in Figure 7.7 (b).

Figure 7.7 (a) Experimental verification of Hooke's law

Figure 7.7 (b) Variation of ΔL with F

Therefore,

$$\Delta L = (\text{slope})F$$

Multiplying and dividing by volume,

$$V = A\,L,$$

$$F\,(\text{slope}) = \frac{AL}{AL}\Delta L$$

Rearranging, we get

$$\frac{F}{A} = \left(\frac{L}{A(slope)}\right)\frac{\Delta L}{L}$$

Therefore, $$\frac{F}{A} \propto \left(\frac{\Delta L}{L}\right)$$

Comparing with equation (7.1) and equation (7.2), we get equation (7.5) as

$$\sigma \propto \varepsilon$$

i.e., the stress is proportional to the strain in the elastic limit.

Stress – Strain profile curve:

The stress versus strain profile is a plot in which stress and strain are noted for each load and a graph is drawn taking strain along the X-axis and stress along the Y-axis. The elastic characteristics of the materials can be analyzed from the stress-strain profile.

Figure 7.8 Stress-Strain profile

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(a) Portion OA:

In this region, stress is very small such that stress is proportional to strain, which means Hooke's law is valid. The point A is called *limit of proportionality* because above this point Hooke's law is not valid. The slope of the line OA gives the Young's modulus of the wire.

(b) Portion AB:

This region is reached if the stress is increased by a very small amount. In this region, stress is not proportional to the strain. But once the stretching force is removed, the wire will regain its original length. This behaviour ends at point B and hence, the point B is known as *yield point* (elastic limit). The elastic behaviour of the material (here wire) in stress-strain curve is OAB.

(c) Portion BC:

If the wire is stretched beyond the point B (elastic limit), stress increases and the wire will not regain its original length after the removal of stretching force.

(d) Portion CD:

With further increase in stress (beyond the point C), the strain increases rapidly and reaches the point D. Beyond D, the strain increases even when the load is removed and breaks (ruptures) at the point E. Therefore, the maximum stress (here D) beyond which the wire breaks is called *breaking stress* or *tensile strength*. The corresponding point D is known as *fracture point*. The region BCDE represents the plastic behaviour of the material of the wire.

7.2.4 Moduli of elasticity

From Hooke's law,

$$\text{stress} \propto \text{strain}$$

$\frac{\text{stress}}{\text{strain}}$ = a constant, known as modulus of elasticity. Its SI unit is Nm^{-2} or pascal and its dimensional formula is $ML^{-1}T^{-2}$.

In this section, we shall define the elastic modulus of a given material. There are three types of elastic modulus.

(a) Young's modulus

(b) Rigidity modulus (or Shear modulus)

(c) Bulk modulus

Young's modulus:

When a wire is stretched or compressed, then the ratio between tensile stress (or compressive stress) and tensile strain (or compressive strain) is defined as Young's modulus.

Young modulus of a material

$$= \frac{\text{Tensile stress or compressive stress}}{\text{Tensile strain or compressive strain}}$$

$$Y = \frac{\sigma_t}{\varepsilon_t} \quad \text{or} \quad Y = \frac{\sigma_c}{\varepsilon_c} \qquad (7.6)$$

EXAMPLE 7.1

Within the elastic limit, the stretching strain produced in wires A, B, and C due to stress is shown in the figure. Assume the load applied are the same and discuss the elastic property of the material.

Write down the elastic modulus in ascending order.

Solution

Here, the elastic modulus is Young modulus and due to stretching, stress is tensile stress and strain is tensile strain.

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Within the elastic limit, stress is proportional to strain (obey Hooke's law). Therefore, it shows a straight line behaviour. So, Young modulus can be computed by taking slope of these straight lines. Hence, calculating the slope for the straight line, we get

Slope of A > Slope of B > Slope of C

Which implies,

Young modulus of C < Young modulus of B < Young modulus of A

Notice that larger the slope, lesser the strain (fractional change in length). So, the material is much stiffer. Hence, the elasticity of wire A is greater than wire B which is greater than C. From this example, we have understood that Young's modulus measures the resistance of solid to a change in its length.

EXAMPLE 7.2

A wire 10 m long has a cross-sectional area 1.25×10^{-4} m^2. It is subjected to a load of 5 kg. If Young's modulus of the material is 4×10^{10} N m^{-2}, calculate the elongation produced in the wire. Take g = 10 ms^{-2}.

Solution

We know that $\frac{F}{A} = Y \times \frac{\Delta L}{L}$

$$\Delta L = \left(\frac{F}{A}\right)\left(\frac{L}{Y}\right)$$

$$= \left(\frac{50}{1.25\times10^{-4}}\right)\left(\frac{10}{4\times10^{10}}\right) = 10^{-4}\text{ m}$$

Bulk modulus:

The bulk modulus is defined as the ratio of the volume stress to the volume strain.

Bulk modulus, K =

$$\frac{\text{Normal (Perpendicular) stress or Pressure}}{\text{Volume strain}}$$

The normal stress or pressure is

$$\sigma_n = \frac{F_n}{\Delta A} = \Delta p$$

The volume strain is

$$\varepsilon_v = \frac{\Delta V}{V}$$

Therefore, Bulk modulus is

$$K = -\frac{\sigma_n}{\varepsilon_v} = -\frac{\Delta P}{\frac{\Delta V}{V}} \qquad (7.7)$$

The negative sign in the equation (7.7) means that when pressure is applied on the body, its volume decreases. Further, the equation (7.7) implies that a material can be easily compressed if it has a small value of bulk modulus. In other words, bulk modulus measures the resistance of solids to change in their volume.

For an example, we know that gases can be easily compressed than solids. This means that gases have small value of bulk modulus compared to solids.

Compressibility

The reciprocal of the bulk modulus is called compressibility. It is defined as the fractional change in volume per unit increase in pressure.

From equation (7.7), we can say that the compressibility

$$C = \frac{1}{K} = -\frac{\varepsilon_v}{\sigma_n} = -\frac{\frac{\Delta V}{V}}{\Delta P} \qquad (7.8)$$

Since gases have small value of bulk modulus than solids, their values of compressibility is very high.

After pumping the air in the cycle tyre, usually we press the cycle tyre to check whether it has

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enough air. What is checked here is essentially the compressibility of air. The tyre should be less compressible for its easy rolling

In fact the rear tyre is less compressible than front tyre for a smooth ride.

EXAMPLE 7.3

A metallic cube of side 100 cm is subjected to a uniform force acting normal to the whole surface of the cube. The pressure is 10^6 pascal. If the volume changes by 1.5×10^{-5} m^3, calculate the bulk modulus of the material.

Solution

By definition, $$K = \frac{\frac{F}{A}}{\frac{\Delta V}{V}} = P\frac{V}{\Delta V}$$

$$K = \frac{10^6 \times 1}{1.5 \times 10^{-5}} = 6.67 \times 10^{10}\ \text{N m}^{-2}$$

Rigidity modulus or shear modulus:
The rigidity modulus is defined as the ratio of the shearing stress to the shearing strain.

$$\eta_R = \frac{\text{Shearing stress}}{\text{Angle of shear or shearing strain}}$$

The shearing stress is

$$\sigma_s = \frac{\text{Tangential force}}{\text{Area over which it is applied}} = \frac{F_t}{\Delta A}$$

The angle of shear or shearing strain

$$\varepsilon_s = \frac{x}{h} = \theta$$

Therefore, Rigidity modulus is

$$\eta_R = \frac{\sigma_s}{\varepsilon_s} = \frac{\frac{F_t}{\Delta A}}{\frac{x}{h}} = \frac{F_t}{\Delta A \theta} \qquad (7.9)$$

Further, the equation (7.9) implies that a material can be easily twisted if it has small value of rigidity modulus. For example, consider a wire twisted through an angle θ, a restoring torque (τ) developed is

$$\tau \propto \theta$$

This means that for a larger torque, wire will twist by a larger amount (angle of shear θ is large). Since the rigidity modulus is inversely proportional to angle of shear, the modulus of rigidity is small.

For the best understanding, the elastic coefficients of some of the important materials are listed in Table 7.1.

Table 7.1 Elastic coefficient of some materials

Material	Young's modulus (Y) (10^{10}N m^{-2})	Bulk modulus (K) (10^{10} N m^{-2})	Shear modulus (η_R) (10^{10} N m^{-2})
Steel	20.0	15.8	8.0
Aluminium	7.0	7.0	2.5
Copper	12.0	12.0	4.0
Iron	19.0	10	6.4
Glass	7.0	3.6	2.6

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EXAMPLE 7.4

A metal cube of side 0.20 m is subjected to a shearing force of 4000 N. The top surface is displaced through 0.50 cm with respect to the bottom. Calculate the shear modulus of elasticity of the metal.

Solution

Here, L = 0.20 m, F = 4000 N, x = 0.50 cm = 0.005 m and Area A = L^2 = 0.04 m^2

Therefore,

$$\eta_R = \frac{F}{A} \times \frac{L}{x} = \frac{4000}{0.04} \times \frac{0.20}{0.005} = 4 \times 10^6 \, \text{N m}^{-2}$$

7.2.5 Poisson's ratio

Figure 7.9 Lateral strain versus longitudinal strain

Suppose we stretch a wire, its length increases (elongation) but its diameter decreases (contraction). Similarly, when we stretch a rubber band (elongation), it becomes noticeably thinner (contraction). That is, deformation of the material in one direction produces deformation in another direction. To quantify this, French Physicist S.D. Poisson proposed a ratio, known as Poisson's ratio.

It is defined as the ratio of relative contraction (lateral strain) to relative expansion (longitudinal strain). It is denoted by the symbol μ.

$$\text{Poisson's ratio, } \mu = \frac{\text{Lateral strain}}{\text{Longitudinal strain}} \quad (7.10)$$

Consider a wire of length L with diameter D. Due to applied force, wire stretches and let the increase in length be l and decrease in diameter be d. Then

$$\mu = -\frac{\frac{d}{D}}{\frac{l}{L}} = -\frac{L}{l} \times \frac{d}{D}$$

Negative sign indicates the elongation is along longitudinal and the contraction along lateral dimension. Further, notice that it is the ratio between quantities of the same dimension. So, Poisson's ratio has no unit and no dimension (dimensionless number). The Poisson's ratio values of some of the materials are listed in Table 7.2.

Table 7.2 Poisson's ratio of some of the materials

Material	Poisson's ratio
Rubber	0.4999
Gold	0.42 -0.44
Copper	0.33
Stainless steel	0.30-0.31
Steel	0.27-0.30
Cast iron	0.21-0.26
Concrete	0.1-0.2
Glass	0.18-0.3
Foam	0.10-0.50
Cork	0.0

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7.2.6 Elastic energy

When a body is stretched, work is done against the restoring force (internal force). This work done is stored in the body in the form of elastic energy.

Consider a wire whose un-stretch length is L and area of cross section is A. Let a force produce an extension l and further assume that the elastic limit of the wire has not been exceeded and there is no loss in energy. Then, the work done by the force F is equal to the energy gained by the wire.

The work done in stretching the wire by dl,
dW = F dl

The total work done in stretching the wire from 0 to l is

$$W = \int_0^l F\,dl \qquad (7.11)$$

From Young's modulus of elasticity,

$$Y = \frac{F}{A} \times \frac{L}{l} \Rightarrow F = \frac{YAl}{L} \qquad (7.12)$$

Substituting equation (7.12) in equation (7.11), we get

$$W = \int_0^l \frac{YAl}{L} dl$$

Since l is the dummy variable in the integration, we can change l to l' (not in limits). Therefore

$$W = \int_0^l \frac{YAl'}{L} dl' = \frac{YA}{L}\left(\frac{l'^2}{2}\right)\Bigg|_0^l = \frac{YA}{L}\frac{l^2}{2} = \frac{1}{2}\left(\frac{YAl}{L}\right)l = \frac{1}{2}Fl$$

$$W = \frac{1}{2}Fl = \text{Elastic potential energy}$$

The energy per unit volume is called energy density which is is given by,

$$u = \frac{\text{Elastic potential energy}}{\text{Volume}} = \frac{\frac{1}{2}Fl}{AL}$$

$$= \frac{1}{2}\frac{F}{A}\frac{l}{L} = \frac{1}{2} \times \text{Stress} \times \text{Strain} \qquad (7.13)$$

EXAMPLE 7.5

A wire of length 2 m with the area of cross-section 10^{-6} m^2 is used to suspend a load of 980 N. Calculate i) the stress developed in the wire ii) the strain and iii) the energy stored. Given: Y=12 × 10^{10}N m^{-2}.

Solution

(i) stress $= \frac{F}{A} = \frac{980}{10^{-6}} = 98 \times 10^7\, N\,m^{-2}$

(ii) strain $= \frac{\text{stress}}{Y} = \frac{98 \times 10^7}{12 \times 10^{10}} = 8.17 \times 10^{-3}$ (no unit)

(iii) Since volume = 2 × $10^{-6} m^3$,

$$\text{Energy} = \frac{1}{2}(\text{stress} \times \text{strain}) \times \text{volume}$$

$$= \frac{1}{2}(98 \times 10^7) \times (8.17 \times 10^{-3}) \times 2 \times 10^{-6} = 8\text{ J}$$

7.2.7 Applications of elasticity

The mechanical properties of materials play a very vital role in everyday life. The elastic behaviour is one such property which especially decides the structural design of the columns and beams of a building.

As far as the structural engineering is concerned, the amount of stress that the design could withstand is a primary safety factor. A bridge has to be designed in such a way that it should have the capacity to withstand the load of the flowing traffic, the force of winds, and even its own weight. The elastic behaviour or in other words, the bending of beams is a major concern over the stability of the buildings or bridges.

For an example, to reduce the bending of a beam for a given load, one should use the material with a higher value of Young's modulus of elasticity. It is obvious from Table 7.1 that the Young's modulus of steel is greater than aluminium or copper. Iron comes next to steel. This is the reason why steel is mostly preferred in the design of heavy duty machines and iron rods in the construction of buildings.

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DO YOU KNOW?

Which one is more elastic? Rubber or steel? Steel is more elastic than rubber. If an equal stress is applied to both steel and rubber, the steel produces less strain. So the Young's modulus is higher for steel than rubber. The object which has higher Young's modulus is more elastic.

7.3 FLUIDS

7.3.1 Introduction

Fluids are found everywhere in the world. Earth's surface has about two-thirds of water and one-third of land. Fluids are different from solids. Unlike solid, fluid has no defined shape of its own. As far as fluid is concerned, liquid has fixed volume whereas gas fills the entire volume of the container.

Pressure of a fluid:

Fluid is a substance which begins to flow when an external force is applied on it. It offers a very small resistance to the applied force. If the force acts on a smaller area, then the impact will be more and vice versa. This particular idea decides yet another quantity called '*pressure*'.

Assume that an object is submerged in a fluid (say water) at rest. In this case, the fluid will exert a force on the surface of the object. This force is always normal to the surface of the object. If F is the magnitude of the normal force acting on the surface area A, then the pressure is defined as the '*force acting per unit area*'.

$$P = \frac{F}{A} \qquad (7.14)$$

Pressure is a scalar quantity. Its S.I. unit and dimensions are $N\,m^{-2}$ or pascal (Pa) and $[ML^{-1}T^{-2}]$ respectively.

Another common unit of pressure is atmosphere which is abbreviated as 'atm'. It is defined as the pressure exerted by the atmosphere at sea level.

i.e., 1 atm = 1.013×10^5 Pa or $N\,m^{-2}$.

Apart from pressure, there are two more parameters such as density and relative density (or specific gravity) which are used to describe the nature of fluids.

Density of a fluid:

The density of a fluid is defined as its mass per unit volume. For a fluid of mass m occupying volume V, the density $\rho = \frac{m}{V}$. The dimensions and S.I unit of ρ are $[ML^{-3}]$ and $kg\,m^{-3}$, respectively. It is a positive scalar quantity.

Mostly, a liquid is largely incompressible and hence its density is nearly constant at ambient pressure (i.e. at 1 atm. pressure). In the case of gases, there are variations in densities with reference to pressure.

Relative density or specific gravity:

The relative density of a substance is defined as the ratio of the density of a substance to the density of water at 4 °C. It is a dimensionless positive scalar quantity.

For example, the density of mercury is $13.6 \times 10^3 kg\,m^{-3}$. Its relative density is equal to $\frac{13.6\times10^3\,kg\,m^{-3}}{1.0\times10^3\,kg\,m^{-3}} = 13.6$.

[The density of water at 4 °C is 1000 $kg\,m^{-3}$]

EXAMPLE 7.6

A solid sphere has a radius of 1.5 cm and a mass of 0.038 kg. Calculate the specific gravity or relative density of the sphere.

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Solution

Radius of the sphere R = 1.5 cm

mass m = 0.038 kg

Volume of the sphere $V = \frac{4}{3}\pi R^3$

$= \frac{4}{3} \times (3.14) \times (1.5 \times 10^{-2})^3 \;\; = 1.413 \times 10^{-5} m^3$

Therefore, density

$$\rho = \frac{m}{V} = \frac{0.038\,kg}{1.413 \times 10^{-5}\,m^3} = 2690\ kg\,m^{-3}$$

Hence, the specific gravity of the sphere

$$= \frac{2690}{1000} = 2.69$$

7.3.2 Pressure due to fluid column at rest

A mountaineer climbing the mountain is able to experience a decrease in pressure with altitude. A person jumping into the swimming pool always realizes an increase in pressure with depth below the water surface. In both the cases, the pressure encountered by the mountaineer and diver is usually due to the hydrostatic pressure, because they are due to fluids that are static.

In order to understand the increase in pressure with depth below the water surface, consider a water sample of cross sectional area A in the form of a cylinder. Let h_1 and h_2 be the depths from the air-water interface to level 1 and level 2 of the cylinder respectively as shown in Figure 7.10(a). Let F_1 be the force acting downwards on level 1 and F_2 be the force acting upwards on level 2 such that $F_1 = P_1 A$ and $F_2 = P_2 A$. Let us assume the mass of the sample to be m and under equilibrium condition, the total upward force (F_2) is balanced by the total downward force (F_1 + mg). In other words, the gravitational force will act downward which is being exactly balanced by the difference between the force $F_2 - F_1$.

$$F_2 - F_1 = mg = F_G \qquad (7.15)$$

Let ρ be the density of the water, then the mass of water available in the sample element is

$$m = \rho V = \rho A (h_2 - h_1)$$
$$V = A (h_2 - h_1)$$

Figure 7.10 (a) A sample of water with base area A in a static fluid with its forces in equilibrium

Hence gravitational force,

$$F_G = \rho A (h_2 - h_1) g$$

On substituting the value of F_G in equation (7.15)

$$F_2 = F_1 + m g$$
$$P_2 A = P_1 A + \rho A (h_2 - h_1) g$$

Cancelling out A on both sides,

$$P_2 = P_1 + \rho (h_2 - h_1) g \qquad (7.16)$$

Figure 7.10 (b) The pressure (P) at a depth (h) below the water surface

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If we choose the level 1 at the surface of the liquid (i.e., air-water interface) and the level 2 at a depth 'h' below the surface (as shown in Figure 7.10(b)), then the value of h_1 becomes zero ($h_1 = 0$) and in turn P_1 assumes the value of atmospheric pressure (say P_a). In addition, the pressure (P_2) at a depth becomes P. Substituting these values in equation (7.16), we get

$$P = P_a + \rho g h \qquad (7.17)$$

Therefore, the pressure at a depth h is greater than the pressure on the surface of the liquid.

If the atmospheric pressure is neglected or ignored, then the pressure at a depth *h* is

$$P = \rho g h \qquad (7.18)$$

For a given liquid, ρ is fixed and g is also constant, then the pressure due to the fluid column is directly proportional to vertical distance or height of the fluid column. This implies that the height of the fluid column is more important to decide the pressure and not the cross sectional or base area or even the shape of the container.

Hydrostatic Paradox

When we talk about liquid at rest, the liquid pressure is the same at all points at the same horizontal level (or same depth). This statement can be demonstrated by an experiment called 'hydrostatic paradox'. Let us consider three vessels of different shapes A, B, and C as shown in Figure 7.11. These vessels are connected at the bottom by a horizontal pipe. When they are filled with a liquid (say water), it occupies the same level even though the vessels hold different amounts of water. It is true because the liquid at the bottom of each section of the vessel experiences the same pressure.

Figure 7.11 Illustration of hydrostatic parodox

The atmospheric pressure at a place is the gravitational force exerted by air above that place per unit surface area. It changes with height and weather conditions (i.e. density of air). In fact, the atmospheric pressure decreases with increasing elevation.

The decrease of atmospheric pressure with altitude has an unwelcome consequence in daily life. For example, it takes longer time to cook at higher altitudes. Nose bleeding is another common experience at higher altitude because of larger difference in atmospheric pressure and blood pressure. Its value on the surface of the Earth at sea level is 1atm.

1 atm pressure = 1.013×10^5 pa

Activity

Take a metal container with an opening. Connect a vacuum pump to the opening. Evacuate the air from inside the container. Why the shape of the metal container gets crumbled?

Inference:

Due to the force of atmospheric pressure acting on its outer surface, the shape of the container crumbles.

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ACTIVITY

Take a glass tumbler. Fill it with water to the brim. Slide a cardboard on its rim so that no air remains in between the cardboard and the tumbler. Invert the tumbler gently. The water does not fall down.

Inference:

This is due to the fact that the weight of water in the tumbler is balanced by the upward thrust caused due to the atmospheric pressure acting on the lower surface of the cardboard that is exposed to air.

7.3.3 Pascal's law and its applications

The French scientist Blaise Pascal observed that the pressure in a fluid at rest is the same at all points if they are at the same height. Pascal's law states that *if the pressure in a liquid is changed at a particular point, the change is transmitted to the entire liquid without being diminished in magnitude.*

Application of Pascal's law

Hydraulic lift

A practical application of Pascal's law is the hydraulic lift which is used to lift a heavy load with a small force. It is a force multiplier. It consists of two cylinders A and B connected to each other by a horizontal pipe, filled with a liquid (Figure 7.12). They are fitted with frictionless pistons of cross sectional areas A_1 and A_2 ($A_2 > A_1$).

Figure 7.12 Hydraulic lift

Suppose a downward force F is applied on the smaller piston, the pressure of the liquid under this piston increases to $P\left(\text{where, } P = \frac{F_1}{A_1}\right)$. According to Pascal's law, this increased pressure P is transmitted undiminished in all directions. Therefore, a pressure is exerted on piston B. Upward force on piston B is

$$F_2 = P \times A_2 = \frac{F_1}{A_1} \times A_2$$

$$F_2 = \frac{A_2}{A_1} \times F_1 \qquad (7.19)$$

Therefore by changing the force on the smaller piston A, the force on the piston B can be increased by the factor $\frac{A_2}{A_1}$ and this factor is called the mechanical advantage of the lift.

EXAMPLE 7.7

Two pistons of a hydraulic lift have diameters of 60 cm and 5 cm. What is the force exerted by the larger piston when 50 N is placed on the smaller piston?

Solution

Since, the diameter of the pistons are given, we can calculate the radius of the piston

$$r = \frac{D}{2}$$

Area of smaller piston, $A_1 = \pi\left(\frac{5}{2}\right)^2 = \pi(2.5)^2$

Area of larger piston, $A_2 = \pi\left(\frac{60}{2}\right)^2 = \pi(30)^2$

$$F_2 = \frac{A_2}{A_1} \times F_1 = (50N) \times \left(\frac{30}{2.5}\right)^2 = 7200\,N$$

This means that with the force of 50 N, the force of 7200 N can be lifted.

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7.3.4 Buoyancy

When a body is partially or fully immersed in a fluid, it displaces a certain amount of fluid. The displaced fluid exerts an upward force on the body. The upward force exerted by a fluid that opposes the weight of an immersed object in a fluid is called *upthrust* or *buoyant force* and the phenomenon is called *buoyancy*.

Archimedes principle:

Figure 7.13 Archimedes principle

It states that when a body is partially or wholly immersed in a fluid, it experiences an upward thrust equal to the weight of the fluid displaced by it and its upthrust acts through the centre of gravity of the liquid displaced.

upthrust or buoyant force = weight of liquid displaced.

Law of floatation

It is well-known that boats, ships, and some wooden objects move on the upper part of the water. We say that they float. Floatation can be defined as the tendency of an object to rise up to the upper levels of the fluid or to stay on the surface of the fluid.

The law of floatation states that a body will float in a liquid if the weight of the liquid displaced by the immersed part of the body equals the weight of the body. For example, a wooden object weighs 300 kg (about 3000 N) floats in water displaces 300 kg (about 3000 N) of water.

Note

If an object floats, the volume of fluid displaced is equal to the volume of the object submerged and the percentage of the volume of the object submerged is equal to the relative density of an object with respect to the density of the fluid in which it floats.

For example, if an ice cube of density 0.9 g cm^{-3} floats in the fresh water of density 1.0 g cm^{-3} then the percentage volume of an object submerged in fresh water is. $\frac{0.9\,\text{g cm}^{-3}}{1.0\,\text{g cm}^{-3}} \times 100\% = 90\%$.

Conversely, if the same ice cube floats in sea water of density 1.3 g cm^{-3}, then the percentage volume of the object submerged in seawater would be $\frac{0.9\,\text{g cm}^{-3}}{1.3\,\text{g cm}^{-3}} \times 100\% = 69.23\%$ only.

EXAMPLE 7.8

A cube of wood floating in water supports a 300 g mass at the centre of its top face. When the mass is removed, the cube rises by 3 cm. Determine the volume of the cube.

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Solution

Let each side of the cube be l. The volume occupied by 3 cm depth of cube,

$$V = (3\text{cm}) \times l^2 = 3l^2\text{cm}$$

According to the principle of floatation, we have

$$V\rho g = mg \Rightarrow V\rho = m$$

$$\rho \text{ is density of water} = 1000\ kg\ m^{-3}$$

$$\Rightarrow (3l^2 \times 10^{-2}\text{m}) \times (1000\ \text{kgm}^{-3}) = 300 \times 10^{-3}\text{kg}$$

$$l^2 = \frac{300\times10^{-3}}{3\times10^{-2}\times1000} m^2 \Rightarrow l^2 = 100\times10^{-4} m^2$$

$$l = 10 \times 10^{-2}\text{m} = 10\text{ cm}$$

Therefore, volume of cube $V = l^3 = 1000\ \text{cm}^3$

Submarines can sink or rise in the depth of water by controlling its buoyancy. To achieve this, the submarines have ballast tanks that can be filled with water or air alternatively. When the ballast tanks are filled with air, the overall density of the submarine becomes lesser than the surrounding water and it surfaces (positive buoyancy). If the tanks are flooded with water replacing air, the overall density becomes greater than the surrounding water and submarine begins to sink (negative buoyancy). To keep the submarine at any depth, tanks are filled with air and water (neutral buoyancy).

Examples of floating bodies:

i) A person can swim in sea water more easily than in river water.

ii) Ice floats on water.

iii) The ship is made of steel but its interior is made hollow by giving it a concave shape.

7.4 VISCOSITY

7.4.1 Introduction

In section 7.3, the behaviour of fluids at rest is discussed. Successive discussions will bring out the influence of fluid motion on different properties. A fluid in motion is a complex phenomenon as it possesses potential, kinetic, and gravitational energy besides causing friction viscous forces to come into play. Therefore, it is necessary to consider the case of an ideal liquid to simplify the task. An ideal liquid is incompressible (i.e., bulk modulus is infinity) and in which no shearing forces can be maintained (i.e., the coefficient of viscosity is zero).

Most of the fluids offer resistance towards motion. A frictional force acts at the contact surface when a fluid moves relative to a solid or when two fluids move relative to each other. This resistance to fluid motion is similar to the friction produced when a solid moves on a surface. The internal friction existing between the layers of a moving fluid is viscosity. So, viscosity is defined as the property of a fluid to oppose the relative motion between its layers.

Activity

Consider three steel balls of the same size, dropped simultaneously in three tall jars each filled with air, water, and oil respectively. It moves easily in air, but not so easily in water. Moving in oil would be even more difficult. There is a relative motion produced between the different layers of the liquid by the falling ball, which causes a viscous force. This frictional force depends

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on the density of the liquid. This property of a moving fluid to oppose the relative motion between its layers is called viscosity.

Cause of viscosity:

Consider a liquid flowing over a horizontal surface with two neighboring layers. The upper layer tends to accelerate the lower layer and in turn, the lower layer tends to retard the upper layer. As a result, a backward tangential force is set-up. This tends to destroy the relative motion. This accounts for the viscous behaviour of fluids.

Figure 7.14 Viscosity

Coefficient of viscosity:

Consider a liquid flowing steadily over a horizontal fixed layer (Figure 7.15). The velocities of the layers increase uniformly as we move away from the fixed layer. Consider any two parallel layers A and B. Let υ and $\upsilon + d\upsilon$ be the velocities of the neighboring layers at distances x and $x + dx$ respectively from the fixed layer.

Figure 7.15 Flow of liquid over the horizontal layers

The force of viscosity F acting tangentially between two layers is given by Newton's First law. This force is proportional to (i) area A of the liquid and (ii) the velocity gradient $\frac{d\upsilon}{dx}$

$$F \propto A \text{ and } F \propto \frac{d\upsilon}{dx}$$

$$\Rightarrow F = -\eta A \frac{d\upsilon}{dx} \qquad (7.20)$$

Where the constant of proportionality η is called the coefficient of viscosity of the liquid and the negative sign implies that the force is frictional and it opposes the relative motion. The dimensional formula for coefficient of viscosity is $\left[ML^{-1}T^{-1}\right]$.

Note Viscosity is similar to friction. The kinetic energy of the substance is dissipated as heat energy.

EXAMPLE 7.9

A metal plate of area 2.5×10^{-4} m^2 is placed on a 0.25×10^{-3} m thick layer of castor oil. If a force of 2.5 N is needed to move the plate with a velocity 3×10^{-2} m s^{-1}, calculate the coefficient of viscosity of castor oil.

Given: A=2.5×10^{-4} m^2, dx = 0.25×10^{-3}m, F=2.5 N and $d\upsilon = 3\times10^{-2}$ m s^{-1}

Solution

$$F = -\eta A \frac{d\upsilon}{dx}$$

In magnitude, $\eta = \frac{F}{A}\frac{dx}{d\upsilon}$

$$= \frac{(2.5N)}{(2.5\times10^{-4}\,\text{m}^2)}\frac{(0.25\times10^{-3}\,\text{m})}{(3\times10^{-2}\,\text{m s}^{-1})}$$

$= 0.083\times10^3$ N m^{-2}s

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7.4.2 Streamlined flow

The flow of fluids occurs in different ways. It can be a steady or streamlined flow, unsteady or turbulent flow, compressible or incompressible flow or even viscous or non-viscous flow. For example, consider a calm flow of water through a river. Careful observation reveals that the velocity of water at different locations of the river is quite different. It is almost faster at the center and slowest near the banks. However, the velocity of the particle at any point is constant. For better understanding, assume that the velocity of the particle is about 4 meter per second at the center of the river. Hence it will be of the same value for all other particles crossing through this point. In a similar way, if the velocity of the particle flowing near the bank of the river is 0.5 meter per second, then the succeeding particles flowing through it will have the same value.

When a liquid flows such that each particle of the liquid passing through a point moves along the same path with the same velocity as its predecessor then the flow of liquid is said to be a *streamlined flow*. It is also referred to as steady or laminar flow. The actual path taken by the particle of the moving fluid is called a streamline, which is a curve, the tangent to which at any point gives the direction of the flow of the fluid at that point as shown in Figure 7.16. It is named so because the flow looks like the flow of a stream or river under ideal conditions.

If we assume a bundle of streamlines having the same velocity over any cross section perpendicular to the direction of flow then such bundle is called a '*tube of flow*'. Thus, it is important to note that any particle in a tube of flow always remains in the tube throughout its motion and cannot mix with liquid in another tube. Always the axis of the tube of flow gives the streamline. The streamlines always represent the trajectories of the fluid particles. **The flow of fluid is streamlined up to a certain velocity called critical velocity.** This means a steady flow can be achieved at low flow speeds below the critical speed.

Figure 7.16 Flow is steady velocity at any point in the liquid remains constant

7.4.3 Turbulent flow

When the speed of the moving fluid exceeds the critical speed v_c, the motion becomes turbulent. In this case, the velocity changes both in magnitude and direction from particle to particle and hence the individual particles do not move in a streamlined path. Hence, the path taken by the particles in turbulent flow becomes erratic and whirlpool-like circles called eddy current or eddies (Figure 7.17 (a) and (b)). The flow of water just behind a boat or a ship and the air flow behind a moving bus are a few examples of turbulent flow.

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Figure 7.17 (a) turbulent flow around a sphere (when $v = v_c$) (b) turbulent flow around a sphere (when $v > v_c$)

The distinction between the two types of motion can be easily demonstrated by injecting a jet of ink axially in a wide tube through which water flows. When the velocity of the fluid is small, the ink will move in a straight line path. Conversely, when the velocity is increased beyond a certain value, the ink will spread out showing the disorderliness and hence the motion becomes turbulent. The zig-zag motion results in the formation of eddy currents and as a consequence, much energy is dissipated.

7.4.4 Reynold's number

We have learnt that the flow of a fluid becomes steady or laminar when the velocity of flow is less than the critical velocity v_c otherwise, the flow becomes turbulent. Osborne Reynolds (1842-1912) formulated an equation to find out the nature of the flow of fluid, whether it is streamlined or turbulent.

$$R_c = \frac{\rho v D}{\eta} \qquad (7.21)$$

It is a dimensionless number called '*Reynold's number*'. It is denoted by the symbol R_c or K. In the equation, ρ denotes the density of the fluid, v the velocity of the flowing fluid, D is the diameter of the pipe in which the fluid flow, and η is the coefficient of viscosity of the fluid. The value of R_c remains the same in any system of units.

Table 7.3 To understand the flow of liquid, Reynold has estimated the value of R_c as follows

S. No.	Reynold's number	Flow
1	$R_c < 1000$	Streamline
2	$1000 < R_c < 2000$	Unsteady
3	$R_c > 2000$	Turbulent

Hence, Reynold's number R_c is a critical variable which decides whether the flow of a fluid through a cylindrical pipe is streamlined or turbulent.

In fact, the critical value of R_c at which the turbulence sets is found to be the same for geometrically similar flows. For example, when two liquids (say oil and water) of different densities and viscosities flow in pipes of same shapes and sizes, the turbulence sets in at almost the same value of R_c. The above fact leads to the *Law of similarity* which states that when there are two geometrically similar flows, both are essentially equal to each other, as long as they embrace the same Reynold's number. The *Law of similarity* plays a very important role in technological applications. The

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shape of ships, submarines, racing cars, and airplanes are designed in such a way that their speed can be maximized.

7.4.5 Terminal velocity

To understand terminal velocity, consider a small metallic sphere falling freely from rest through a large column of a viscous fluid. The forces acting on the sphere are (i) gravitational force of the sphere acting vertically downwards, (ii) upthrust U due to buoyancy and (iii) viscous drag acting upwards (viscous force always acts in a direction opposite to the motion of the sphere).

Initially, the sphere is accelerated in the downward direction so that the upward force is less than the downward force. As the velocity of the sphere increases, the velocity of the viscous force also increases. A stage is reached when the net downward force balances the upward force and hence the resultant force on the sphere becomes zero. It now moves down with a constant velocity.

The maximum constant velocity acquired by a body while falling freely through a viscous medium is called the terminal velocity $\boldsymbol{v_t}$. In the Figure 7.18, a graph is drawn with velocity along y- axis and time along x- axis. It is evident from the graph that the sphere is accelerated initially

Figure 7.18 Velocity verses time graph

and in course of time it becomes constant, and attains terminal velocity (v_t).

Expression for terminal velocity:

Consider a sphere of radius r which falls freely through a highly viscous liquid of coefficient of viscosity η. Let the density of the material of the sphere be ρ and the density of the fluid be σ.

Figure 7.19 Forces acting on the sphere when it falls in a viscous liquid

Gravitational force acting on the sphere,

$F_G = mg = \frac{4}{3}\pi r^3 \rho g$ (downward force)

Up thrust, $U = \frac{4}{3}\pi r^3 \sigma g$ (upward force)

Viscous force, $F = 6\pi\eta r v_t$

At terminal velocity v_t,

The net downward force = upward force

$$F_G = U + F$$

$$F_G - U = F \Rightarrow \frac{4}{3}\pi r^3 \rho g - \frac{4}{3}\pi r^3 \sigma g = 6\pi\eta r v_t$$

$$v_t = \frac{2}{9} \times \frac{r^2(\rho - \sigma)}{\eta} g \Rightarrow v_t \propto r^2 \qquad (7.22)$$

Here, it should be noted that the terminal speed of the sphere is directly proportional to the square of its radius.

If σ is greater than ρ, then the term (ρ - σ) becomes negative leading to a negative terminal velocity. That is why air bubbles

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rise up through water or any fluid. This is also the reason for the clouds in the sky to move in the upward direction.

Point to ponder

i) The terminal speed of a sphere is directly proportional to the square of the radius of the sphere. Hence, larger raindrops fall with greater speed as compared to the smaller raindrops.
ii) If the density of the material of the sphere is less than the density of the medium, then the sphere shall attain terminal velocity in the upward direction. That is why gas bubbles rise up in soda water.

7.4.6 Stoke's law and its applications

When a body falls through a viscous medium, it drags the layer of the fluid immediately in contact with it. This produces a relative motion between the different layers of the liquid. Stoke performed many experiments on the motion of small spherical bodies in different fluids and concluded that the viscous force F acting on a spherical body of radius r depends directly on

i) radius (r) of the sphere
ii) velocity (υ) of the sphere and
iii) coefficient of viscosity η of the liquid

Therefore F $\propto \eta^x r^y \upsilon^z \Rightarrow F = k\eta^x r^y \upsilon^z$, where k is a dimensionless constant.

Using dimensions, the above equation can be written as

$$\left[MLT^{-2}\right] = k\left[ML^{-1}T^{-1}\right]^x \times \left[L\right]^y \times \left[LT^{-1}\right]^z$$

On solving, we get x=1, y=1, and z=1

Therefore, F=kη $r\upsilon$

Experimentally, Stoke found that the value of k = 6π

$$F = 6\pi\eta\, r\upsilon \qquad (7.23)$$

This relation is known as Stoke's law

Practical applications of Stoke's law

Since the raindrops are smaller in size and their terminal velocities are small, remain suspended in air in the form of clouds. As they grow up in size, their terminal velocities increase and they start falling in the form of rain.

This law explains the following:

a) Floatation of clouds
b) Larger raindrops hurt us more than the smaller ones
c) A man coming down with the help of a parachute acquires constant terminal velocity.

7.4.7 Poiseuille's equation

Poiseuille analyzed the steady flow of liquid through a capillary tube. He derived an expression for the volume of the liquid flowing per second through the capillary tube.

As per the theory, the following conditions must be retained while deriving the equation.

- The flow of liquid through the tube is streamlined.
- The tube is horizontal so that gravity does not influence the flow
- The layer in contact with the wall of the tube is at rest
- The pressure is uniform over any cross section of the tube

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We can derive Poiseuille's equation using dimensional analysis. Consider a liquid flowing steadily through a horizontal capillary tube. Let $\upsilon = \left(\frac{V}{t}\right)$ be the volume of the liquid flowing out per second through a capillary tube. It depends on (1) coefficient of viscosity (η) of the liquid, (2) radius of the tube (r), and (3) the pressure gradient $\left(\frac{P}{l}\right)$. Then,

$$\upsilon \propto \eta^a r^b \left(\frac{P}{l}\right)^c$$

$$\upsilon = k\eta^a r^b \left(\frac{P}{l}\right)^c \qquad (7.24)$$

where, k is a dimensionless constant. Therefore,

$$[\upsilon] = \frac{\text{volume}}{\text{time}} = \left[L^3T^{-1}\right], \left[\frac{dP}{dx}\right] = \frac{\text{Pressure}}{\text{distance}}$$

$$\left[ML^{-2}T^{-2}\right], [\eta] = \left[ML^{-1}T^{-1}\right] and [r] = [L]$$

Substituting in equation (7.24)

$$\left[L^3T^{-1}\right] = \left[ML^{-1}T^{-1}\right]^a [L]^b \left[ML^{-2}T^{-2}\right]^c$$

$$M^0L^3T^{-1} = M^{a+c}L^{-a+b-2c}T^{-a-2c}$$

So, equating the powers of M, L, and T on both sides, we get

$$a + c = 0, -a + b - 2c = 3, \text{ and } -a - 2c = -1$$

We have three unknowns a, b, and c. We have three equations, on solving, we get

$$a = -1, b = 4, \text{ and } c = 1$$

Therefore, equation (7.24) becomes,

$$\upsilon = k\eta^{-1}r^4\left(\frac{P}{l}\right)^1$$

Experimentally, the value of k is shown to be $\frac{\pi}{8}$, we have

$$\upsilon = \frac{\pi r^4 P}{8\eta l} \qquad (7.25)$$

The above equation is known as *Poiseuille's equation* for the flow of liquid through a narrow tube or a capillary tube. This relation holds good for the fluids whose velocities are lesser than the critical velocity (v_c).

7.4.8 Applications of viscosity

The importance of viscosity can be understood from the following examples.

(1) The oil used as a lubricant for heavy machinery parts should have a high viscous coefficient. To select a suitable lubricant, we should know its viscosity and how it varies with temperature [Note: As temperature increases, the viscosity of the liquid decreases]. Also, it helps to choose oils with low viscosity used in car engines (light machinery).

(2) The highly viscous liquid is used to damp the motion of some instruments and is used as brake oil in hydraulic brakes.

(3) Blood circulation through arteries and veins depends upon the viscosity of fluids.

(4) Millikan conducted the oil drop experiment to determine the charge of an electron. He used the knowledge of viscosity to determine the charge.

7.5 SURFACE TENSION

7.5.1 Intermolecular forces

Some liquids do not mix together due to their physical properties such as density, surface tension force, etc. For example, water and kerosene do not mix together. Mercury does not wet the glass but water sticks to it. Water

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rises up to the leaves through the stem. They are mostly related to the free surfaces of liquids. Liquids have no definite shape but have a definite volume. Hence they acquire a free surface when poured into a container. Therefore, the surfaces have some additional energy, called as surface energy. The phenomenon behind the above fact is called surface tension. Laplace and Gauss developed the theory of surface and motion of a liquid under various situations.

The molecules of a liquid are not rigidly fixed like in a solid. They are free to move about. The force between the like molecules which holds the liquid together is called '*cohesive force*'. When the liquid is in contact with a solid, the molecules of these solid and liquid will experience an attractive force which is called '*adhesive force*'.

These molecular forces are effective only when the distance between the molecules is very small about 10^{-9} m (i.e., 10 Å). The distance through which the influence of these molecular forces can be felt in all directions constitute a range and is called *sphere of influence*. The forces outside this range are rather negligible.

Consider three different molecules A, B, and C in a given liquid as shown in Figure 7.20. Let a molecule 'A' be considered well inside the liquid within the sphere of influence. Since this molecule interacts with all other molecules in all directions, the net force experienced by A is zero. Now consider a molecule 'B' in which three-fourth lies below the liquid surface and one–fourth on the air. Since B has more molecules towards its lower side than the upper side, it experiences a net force in the downward direction. In a similar way, if another molecule 'C' is chosen on the liquid surface (i.e, upper half in air and lower half in liquid), it experiences a maximum downward force due to the availability of more number of liquid molecules on the lower part. Hence it is obvious that all molecules of the liquid that falls within the molecular range inside the liquid interact with the molecule and hence experience a downward force.

Figure 7.20 Molecules at different levels of liquid

When any molecule is brought towards the surface from the interior of the liquid, work is done against the cohesive force among the molecules of the surface. This work is stored as potential energy in molecules. So the molecules on the surface will have greater potential energy than that of molecules in the interior of the liquid. But for a system to be under stable equilibrium, its potential energy (or surface energy) must be a minimum. Therefore, in order to maintain stable equilibrium, a liquid always tends to have a minimum number of molecules. In other words, the liquid tends to occupy a minimum surface area. This behaviour of the liquid gives rise to surface tension.

Examples for surface tension.

Water bugs and water striders walk on the surface of water (Figure 7.21). The water molecules are pulled inwards and the surface of water acts like a springy or stretched membrane. This balance the weight of water bugs and enables them to walk on the

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surface of water. We call this phenomenon as surface tension.

Figure 7.21 Water striders can walk on water because of the surface tension of water

The hairs of the painting brush cling together when taken out of water. This is because the water films formed on them tends to contract to a minimum area (Figure 7.22).

Figure 7.22 Painting brush hairs cling together due to surface tension

Activity

Needle floats on water surface

Take a greased needle of steel on a piece of blotting paper and place it gently over the water surface. Blotting paper soaks water and soon sinks down but the needle keeps floating.

Figure 7.23 Floating needle

The floating needle causes a little depression; the forces F_s due to the surface tension of the curved surface are inclined as shown in Figure 7.23. The vertical components of these two forces support the weight of the needle. Now add liquid soap to the water and stir it. We find that the needle sinks.

Activity

Take a plastic sheet and cut out a piece in the shape of a boat (Figure 7.24). A tapering and smooth front with a notch at the back is suggested. Put a piece of camphor into the notch of the boat. Gently release the boat on the surface of the water and we find that the boat is propelled forward when the camphor dissolves. The surface tension is lowered, as the camphor dissolves and produces a difference in surface tension in the water nearby the notch. This causes the water to flow away from the back of the boat, which moves the boat forward.

Figure 7.24 Camphor boat

7.5.2 Factors affecting the surface tension of a liquid

Surface tension for a given liquid varies in following situations

(1) ***The presence of any contamination or impurities*** considerably affects the force of surface tension depending upon the degree of contamination.

(2) ***The presence of dissolved substances*** can also affect the value of surface

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tension. For example, a highly soluble substance like sodium chloride (*NaCl*) when dissolved in water (H_20) increases the surface tension of water. But the sparingly soluble substance like phenol or soap solution when mixed in water decreases the surface tension of water.

(3) ***Electrification*** affects the surface tension. When a liquid is electrified, surface tension decreases. Since external force acts on the liquid surface due to electrification, area of the liquid surface increases which acts against the contraction phenomenon of the surface tension. Hence, it decreases.

(4) ***Temperature*** plays a very crucial role in altering the surface tension of a liquid. Obviously, the surface tension decreases linearly with the rise of temperature. For a small range of temperature, the surface tension at T_t at t °C is $T_t = T_0\ (1-\alpha\, t)$

Where, T_0 is the surface tension at temperature 0°C and α is the temperature coefficient of surface tension. It is to be noted that at the critical temperature, the surface tension is zero as the interface between liquid and vapour disappear. For example, the critical temperature of water is 374°C. Therefore, the surface tension of water is zero at that temperature. van der Wall suggested the important relation between the surface tension and the critical temperature as

$$T_t = T_0\left(1-\frac{t}{t_c}\right)^{\frac{3}{2}}$$

Generalizing the above relation, we get

$$T_t = T_0\left(1-\frac{t}{t_c}\right)^{n}$$

which gives more accurate value. Here, n varies for different liquids and t and t_c denote the temperature and critical temperature in absolute scale (Kelvin scale) respectively.

7.5.3 Surface energy (S.E.) and surface tension (S.T.)

Surface Energy

Consider a sample of liquid in a container. A molecule inside the liquid is being pulled in all direction by other molecules that surround it. However, near the surface, a molecule is pulled down only by the molecules below it and there is a net downward force. As a result, the entire surface of the liquid is being pulled inward. The liquid surface thus tends to have the least surface area. To increase the surface area, some molecules are brought from the interior to the surface. For this reason, work has to be done against the forces of attraction. The amount of work done is stored as potential energy. Thus, the molecules lying on the surface possess greater potential energy than other molecules. This excess energy per unit area of the free surface of the liquid is called '*surface energy*'. In other words, the work done in increasing the surface area per unit area of the liquid against the surface tension force is called the surface energy of the liquid.

$$\text{Surface energy} = \frac{\text{work done in increasing the surface area}}{\text{increase in surface area}}$$

$$= \frac{W}{\Delta A} \qquad (7.26)$$

It is expressed in J m^{-2} or N m^{-1}.

Surface tension

The surface tension of a liquid is defined as the force per unit length of the liquid or the energy per unit area of the surface of a liquid

$$T = \frac{F}{l} \qquad (7.27)$$

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The SI unit and dimensions of T are $N\,m^{-1}$ and $M\,T^{-2}$, respectively.

Relation between surface tension and surface energy:

Consider a rectangular frame of wire ABCD in a soap solution (Figure 7.25). Let AB be the movable wire. Suppose the frame is dipped in soap solution, soap film is formed which pulls the wire AB inward due to surface tension. Let F be the force due to surface tension, then

$$F = (2T)l$$

here, 2 is introduced because it has two free surfaces. Suppose AB is moved by a small distance Δx to new a position $A'B'$. Since the area increases, some work has to be done against the inward force due to surface tension.

Work done = Force × distance = $(2T\,l)\,(\Delta x)$

Increase in area of the film

$$\Delta A = (2l)\,(\Delta x) = 2l\,\Delta x$$

Therefore,

$$\text{surface energy} = \frac{\text{work done}}{\text{increase in surface area}}$$

$$= \frac{2Tl\,\Delta x}{2l\,\Delta x} = T \qquad (7.28)$$

Hence, the surface energy per unit area of a surface is numerically equal to the surface tension.

Figure 7.25 A horizontal soap film on a rectangular frame of wire ABCD

Note It should be remembered that a liquid drop has only one free surface. Therefore, the surface area of a spherical drop of radius r is equal to $4\pi r^2$, whereas, a bubble has two free surfaces and hence the surface area of a spherical bubble is equal to $2 \times 4\pi r^2$.

EXAMPLE 7.10

Let $2.4\times10^{-4}\,J$ of work is done to increase the area of a film of soap bubble from 50 cm² to 100 cm². Calculate the value of surface tension of soap solution.

Solution:

A soap bubble has two free surfaces, therefore increase in surface area $\Delta A = A_2 - A_1 = 2(100-50)\times10^{-4}\text{m}^2 = 100\times10^{-4}\text{m}^2$.

Since, work done $W = T\times\Delta A \Rightarrow T = \dfrac{W}{\Delta A}$

$$= \frac{2.4\times10^{-4}\,\text{J}}{100\times10^{-4}\,\text{m}^2} = 2.4\times10^{-2}\,\text{N m}^{-1}$$

7.5.4 Angle of contact

When the free surface of a liquid comes in contact with a solid, then the surface of the liquid becomes curved at the point of contact. Whenever the liquid surface becomes a curve, then the angle between the two medium (solid-liquid interface) comes in the picture. For an example, when a glass plate is dipped in water with sides vertical as shown in figure, we can observe that the water is drawn up to the plate. In the same manner, instead of water the glass plate is dipped in mercury, the surface is curved but now the curve is depressed as shown in Figure 7.29

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The angle between the tangent to the liquid surface at the point of contact and the solid surface inside the liquid is known as the *angle of contact between the solid and the liquid*. It is denoted by θ.

Its value is different at interfaces of different pairs of solids and liquids. In fact, it is the factor which decides whether a liquid will spread on the surface of a chosen solid or it will form droplets on it.

Let us consider three interfaces such as liquid-air, solid-air and solid-liquid with reference to the point of contact 'O' and the interfacial surface tension forces T_{sa}, T_{sl} and T_{la} on the respective interfaces as shown in Figure 7.26.

Figure 7.26 Angle of contact of a liquid

Since the liquid is stable under equilibrium, the surface tension forces between the three interfaces must also be in equilibrium. Therefore,

$$T_{sa} = T_{la}\cos\theta + T_{sl} \Rightarrow \cos\theta = \frac{T_{sa} - T_{sl}}{T_{la}} \quad (7.29)$$

From the above equation, there are three different possibilities which can be discussed as follows.

(i) If $T_{sa} > T_{sl}$ and $T_{sa} - T_{sl} > 0$ (water-plastic interface), then the angle of contact θ is acute angle (θ less than 90°) as cosθ is positive.

(ii) If $T_{sa} < T_{sl}$ and $T_{sa} - T_{sl} < 0$ (water-leaf interface), then the angle of contact is obtuse angle (θ less than 180°) as cosθ is negative.

(iii) If $T_{sa} > T_{la} + T_{sl}$, then there will be no equilibrium and liquid will spread over the solid.

Therefore, the concept of *angle of contact* between the solid-liquid interface leads to some practical applications in real life. For example, soaps and detergents are wetting agents. When they are added to an aqueous solution, they will try to minimize the angle of contact and in turn penetrate well in the cloths and remove the dirt. On the other hand, water proofing paints are coated on the outer side of the building so that it will enhance the angle of contact between the water and the painted surface during the rainfall.

7.5.5 Excess of pressure inside a liquid drop, a soap bubble, and an air bubble

As it is discussed earlier, the free surface of a liquid becomes curved when it has contact with a solid. Depending upon the nature of liquid-air or liquid-gas interface, the magnitude of interfacial surface tension varies. In other words, as a consequence of surface tension, the above such interfaces have energy and for a given volume, the surface will have a minimum energy with least area. Due to this reason, the liquid drop becomes spherical (for a smaller radius).

When the free surface of the liquid is curved, there is a difference in pressure between the inner and outer the side of the surface (Figure 7.27).

i) When the liquid surface is plane, the forces due to surface tension (*T, T*) act tangentially to the liquid surface in

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Figure7.27 Excess of pressure across a liquid surface

opposite directions. Hence, the resultant force on the molecule is zero. Therefore, in the case of a plane liquid surface, the pressure on the liquid side is equal to the pressure on the vapour side.

ii) When the liquid surface is curved, every molecule on the liquid surface experiences forces (F_T, F_T) due to surface tension along the tangent to the surface. Resolving these forces into rectangular components, we find that horizontal components cancel out each other while vertical components get added up. Therefore, the resultant force normal to the surface acts on the curved surface of the liquid. Similarly, for a convex surface, the resultant force is directed inwards towards the centre of curvature, whereas the resultant force is directed outwards from the centre of curvature for a concave surface. Thus, for a curved liquid surface in equilibrium, the pressure on its concave side is greater than the pressure on its convex side.

Excess of pressure inside a bubble and a liquid drop:

The small bubbles and liquid drops are spherical because of the forces of surface tension. The fact that a bubble or a liquid drop does not collapse due to the combined effect indicates that the pressure inside a bubble or a drop is greater than that outside it.

1) Excess of pressure inside air bubble in a liquid.

Consider an air bubble of radius R inside a liquid having surface tension T as shown in Figure 7.28 (a). Let P_1 and P_2 be the pressures outside and inside the air bubble, respectively. Now, the excess pressure inside the air bubble is $\Delta P = P_2 - P_1$.

Figure 7.28. (a) Air bubble

In order to find the excess pressure inside the air bubble, let us consider the forces acting on the air bubble. For the hemispherical portion of the bubble, considering the forces acting on it, we get,

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i) The force due to surface tension acting towards right around the rim of length $2\pi R$ is $F_T = 2\pi RT$

ii) The force due to outside pressure P_1 is to the right acting across a cross sectional area of πR^2 is $F_{P_1} = P_1\pi R^2$

iii) The force due to pressure P_2 inside the bubble, acting to the left is $F_{P_2} = P_2\pi R^2$.

As the air bubble is in equilibrium under the action of these forces, $F_{P_2} = F_T + F_{P_1}$

$$P_2\pi R^2 = 2\pi RT + P_1\pi R^2$$

$$\Rightarrow (P_2 - P_1)\pi R^2 = 2\pi RT$$

Excess pressure is $\Delta P = P_2 - P_1 = \dfrac{2T}{R}$ (7.30)

2) Excess pressure inside a soap bubble

Consider a soap bubble of radius R and the surface tension of the soap bubble be T as shown in Figure 7.28 (b). A soap bubble has two liquid surfaces in contact with air, one inside the bubble and other outside the bubble. Therefore, the force on the soap bubble due to surface tension is $2\times 2\pi RT$. The various forces acting on the soap bubble are,

i) Force due to surface tension $F_T = 4\pi RT$ towards right

ii) Force due to outside pressure, $F_{P_1} = P_1\pi R^2$ towards right

iii) Force due to inside pressure, $F_{P_2} = P_2\pi R^2$ towards left

As the bubble is in equilibrium, $F_{P_2} = F_T + F_{P_1}$

$$P_2\pi R^2 = 4\pi RT + P_1\pi R^2$$

$$\Rightarrow (P_2 - P_1)\pi R^2 = 4\pi RT$$

Excess pressure is $\Delta P = P_2 - P_1 = \dfrac{4T}{R}$ (7.31)

Figure7.28 (b) Soap bubble

3) Excess pressure inside the liquid drop

Consider a liquid drop of radius R and the surface tension of the liquid is T as shown in Figure 7.28 (c).

Figure7.28 (c) Liquid drop

The various forces acting on the liquid drop are,

i) Force due to surface tension $F_T = 2\pi RT$ towards right

ii) Force due to outside pressure, $F_{P_1} = P_1\pi R^2$ towards right

iii) Force due to inside pressure, $F_{P_2} = P_2\pi R^2$ towards left

As the drop is in equilibrium, $F_{P_2} = F_T + F_{P_1}$

$$P_2\pi R^2 = 2\pi RT + P_1\pi R^2$$

$$\Rightarrow (P_2 - P_1)\pi R^2 = 2\pi RT$$

Excess pressure is $\Delta P = P_2 - P_1 = \dfrac{2T}{R}$ (7.32)

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The smaller the radius of a liquid drop, the greater is the excess of pressure inside the drop. It is due to this excess of pressure inside, the tiny fog droplets are rigid enough to behave like solids.

When an ice-skater skate over the surface of the ice, some ice melts due to the pressure exerted by the sharp metal edges of the skates, the tiny droplets of water act as rigid ball- bearings and help the skaters to run along smoothly.

EXAMPLE 7.11

If excess pressure is balanced by a column of oil (with specific gravity 0.8) 4 *mm* high, where R = 2.0 *cm*, find the surface tension of the soap bubble.

Solution

The excess of pressure inside the soap bubble is $\Delta P = P_2 - P_1 = \frac{4T}{R}$

But $\Delta P = P_2 - P_1 = \rho gh \Rightarrow \rho gh = \frac{4T}{R}$

$\Rightarrow$ Surface tension,

$$T = \frac{\rho ghR}{4} = \frac{(800)(9.8)(4\times10^{-3})(2\times10^{-2})}{4} =$$

$$T = 15.68\times10^{-2}\,N\,m^{-1}$$

7.5.6 Capillarity

The word 'capilla' means hair in Latin. If the tubes were hair thin, then the rise would be very large. It means that the tube having a very small diameter is called a 'capillary tube'. When a glass capillary tube open at both ends is dipped vertically in water, the water in the tube will rise above the level of water in the vessel. In case of mercury, the liquid is depressed in the tube below the level of mercury in the vessel (shown in Figure 7.29). In a liquid whose angle of contact with solid is less than 90°, suffers capillary rise. On the other hand, in a liquid whose angle of contact is greater than 90°, suffers capillary fall (Table 7.4). The rise or fall of a liquid in a narrow tube is called capillarity or capillary action. Depending on the diameter of the capillary tube, liquid rises or falls to different heights.

Figure 7.29 Capillary rise or fall

Table 7.4 Capillary rise and fall

Contact angle	Strength of		Degree of wetting	Meniscus	Rise or fall of liquid in the capillary tube
	Cohesive force	Adhesive force			
θ=0 (A)	Weak	Strong	Perfect Wetting	Plane	Neither rises nor is depressed
θ<90 (B)	Weak	Strong	High	Concave	Rise of liquid
θ>90 (C)	Strong	Weak	Low	Convex	Fall of liquid

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Figure 7.30 (a) water on silver surface (b) glass plate on water (c) glass on mercury

Practical applications of capillarity

- Due to capillary action, oil rises in the cotton within an earthen lamp. Likewise, sap rises from the roots of a plant to its leaves and branches.
- Absorption of ink by a blotting paper
- Capillary action is also essential for the tear fluid from the eye to drain constantly.
- Cotton dresses are preferred in summer because cotton dresses have fine pores which act as capillaries for sweat.

7.5.7 Surface Tension by capillary rise method

The pressure difference across a curved liquid-air interface is the basic factor behind the rising up of water in a narrow tube (influence of gravity is ignored). The capillary rise is more dominant in the case of very fine tubes. But this phenomenon is the outcome of the force of surface tension. In order to arrive a relation between the capillary rise (h) and surface tension (T), consider a capillary tube which is held vertically in a beaker containing water; the water rises in the capillary tube to a height h due to surface tension (Figure 7.31).

The surface tension force F_T, acts along the tangent at the point of contact downwards and its reaction force upwards. Surface tension T, is resolved into two components i) Horizontal component $T\sin\theta$ and ii) Vertical component $T\cos\theta$ acting upwards, all along the whole circumference of the meniscus.

$$\text{Total upward force} = (T\cos\theta)\,(2\pi r) = 2\pi r T\cos\theta$$

where θ is the angle of contact, r is the radius of the tube. Let ρ be the density of water and h be the height to which the liquid rises inside the tube. Then,

$$\begin{pmatrix}\text{the volume of}\\ \text{liquid column in}\\ \text{the tube, V}\end{pmatrix} = \begin{pmatrix}\text{volume of the}\\ \text{liquid column of radius r}\\ \text{height h}\end{pmatrix} + \begin{pmatrix}\text{volume of liquid of radius r}\\ \text{and height r - Volume of the}\\ \text{hemisphere of radius r}\end{pmatrix}$$

Figure 7.31 Capillary rise by surface tension

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$$V = \pi r^2 h + \left(\pi r^2 \times r - \frac{2}{3}\pi r^3\right) \Rightarrow V = \pi r^2 h + \frac{1}{3}\pi r^3$$

The upward force supports the weight of the liquid column above the free surface, therefore,

$$2\pi r T\cos\theta = \pi r^2\left(h + \frac{1}{3}r\right)\rho g \Rightarrow T = \frac{r\left(h + \frac{1}{3}r\right)\rho g}{2\cos\theta}$$

If the capillary is a very fine tube of radius (i.e., radius is very small) then $\frac{r}{3}$ can be neglected when it is compared to the height h. Therefore,

$$T = \frac{r\rho g h}{2\cos\theta} \qquad (7.33)$$

Liquid rises through a height h

$$h = \frac{2T\cos\theta}{r\rho g} \Rightarrow h \propto \frac{1}{r} \qquad (7.34)$$

This implies that the capillary rise (h) is inversely proportional to the radius (r) of the tube. i.e, the smaller the radius of the tube greater will be the capillarity.

EXAMPLE 7.12

Water rises in a capillary tube to a height of 2.0cm. How much will the water rise through another capillary tube whose radius is one-third of the first tube?

Solution

From equation (7.34), we have

$$h \propto \frac{1}{r} \Rightarrow hr = \text{constant}$$

Consider two capillary tubes with radius r_1 and r_2 which on placing in a liquid, capillary rises to height h_1 and h_2, respectively. Then,

$$h_1 r_1 = h_2 r_2 = \text{constant}$$

$$\Rightarrow h_2 = \frac{h_1 r_1}{r_2} = \frac{\left(2\times10^{-2}m\right)\times r}{\frac{r}{3}} \Rightarrow h_2 = 6\times10^{-2}\text{m}$$

EXAMPLE 7.13

Mercury has an angle of contact equal to 140° with soda lime glass. A narrow tube of radius 2 mm, made of this glass is dipped in a trough containing mercury. By what amount does the mercury dip down in the tube relative to the liquid surface outside? Surface tension of mercury T=0.456 N m^{-1}; Density of mercury ρ = 13.6 × 10^3 kg m^{-3}

Solution

Capillary descent, cos140 = cos(90+50) −sin50 = −0.7660

$$h = \frac{2T\cos\theta}{r\rho g} = \frac{2\times(0.456\,N\,m^{-1})(\cos140^0)}{(2\times10^{-3}m)(13.6\times10^3)(9.8\,m\,s^{-2})}$$

$$= \frac{2\times0.456\times(-0.7660)}{2\times13.6\times9.8}$$

$$= \frac{-0.6986}{266.56} = -2.62\times10^{-3}m$$

where, negative sign indicates that there is fall of mercury (mercury is depressed) in glass tube.

7.5.8 Applications of surface tension

- Mosquitoes lay their eggs on the surface of water. To reduce the surface tension of water, a small amount of oil is poured. This breaks the elastic film of water surface and eggs are killed by drowning.
- Chemical engineers must finely adjust the surface tension of the liquid, so it forms droplets of designed size and so it adheres to the surface without smearing. This is used in desktop printing, to paint automobiles and decorative items.
- Specks of dirt get removed when detergents are added to hot water while washing clothes because surface tension is reduced.
- A fabric can be made waterproof, by adding suitable waterproof material (wax) to the fabric. This increases the angle of contact.

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7.6 BERNOULLI'S THEOREM

7.6.1 Equation of continuity

In order to discuss the mass flow rate through a pipe, it is necessary to assume that the flow of fluid is steady, the flow of the fluid is said to be steady if at any given point, the velocity of each passing fluid particle remains constant with respect to time. Under this condition, the path taken by the fluid particle is a streamline.

Consider a pipe AB of varying cross sectional area a_1 and a_2 such that $a_1 > a_2$. A non-viscous and incompressible liquid flows steadily through the pipe, with velocities v_1 and v_2 in area a_1 and a_2, respectively as shown in Figure 7.32.

Fig 7.32 A streamlined flow of fluid through a pipe of varying cross sectional area

Let m_1 be the mass of fluid flowing through section A in time Δt, $m_1 = (a_1 v_1 \Delta t)\,\rho$

Let m_2 be the mass of fluid flowing through section B in time Δt, $m_2 = (a_2 v_2 \Delta t)\,\rho$

For an incompressible liquid, mass is conserved $m_1 = m_2$

$$a_1 v_1\, \Delta t\, \rho = a_2 v_2 \Delta t\, \rho$$

$$a_1 v_1 = a_2 v_2 \Rightarrow a\, v = \text{constant} \qquad (7.35)$$

which is called the equation of continuity and it is a statement of conservation of mass in the flow of fluids.

In general, $a\ v$ = constant, which means that the volume flux or flow rate remains constant throughout the pipe. In other words, the smaller the cross section, greater will be the velocity of the fluid.

EXAMPLE 7.14

In a normal adult, the average speed of the blood through the aorta (radius $r = 0.8$ cm) is 0.33 ms^{-1}. From the aorta, the blood goes into major arteries, which are 30 in number, each of radius 0.4 cm. Calculate the speed of the blood through the arteries.

Solution:

$a_1 v_1 = 30\, a_2\, v_2 \Rightarrow \pi r_1^2 v_1 = 30\, \pi r_2^2 v_2$

$$v_2 = \frac{1}{30}\left(\frac{r_1}{r_2}\right)^2 v_1$$

$$\Rightarrow v_2 = \frac{1}{30} \times \left(\frac{0.8\times10^{-2}\,\text{m}}{0.4\times10^{-2}\,\text{m}}\right)^2 \times (0.33\,\text{ms}^{-1})$$

$v_2 = 0.044\ \text{m s}^{-1}$

7.6.2 Pressure, kinetic and potential energy of liquids

A liquid in a steady flow can possess three kinds of energy. They are (1) Kinetic energy, (2) Potential energy, and (3) Pressure energy, respectively.

i) **Kinetic energy:** The kinetic energy of a liquid of mass m moving with a velocity v is given by

$$KE = \frac{1}{2} mv^2$$

The kinetic energy per unit mass =

$$\frac{KE}{m} = \frac{\frac{1}{2}mv^2}{m} = \frac{1}{2}v^2$$

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Similarly, the kinetic energy per unit volume

$$= \frac{KE}{\text{volume}} = \frac{\frac{1}{2}mv^2}{V} = \frac{1}{2}\left(\frac{m}{V}\right)v^2 = \frac{1}{2}\rho v^2$$

ii) **Potential energy:** The potential energy of a liquid of mass m at a height h above the ground level is given by

$$PE = mgh$$

The potential energy per unit mass

$$= \frac{PE}{m} = \frac{mgh}{m} = gh$$

Similarly, the potential energy per unit volume $= \frac{PE}{\text{volume}} = \frac{mgh}{V} = \left(\frac{m}{V}\right)gh = \rho gh$

iii) **Pressure energy:** The energy acquired by a fluid by applying pressure on the fluid. We know that

$$\text{Pressure} = \frac{\text{Force}}{\text{Area}} \Rightarrow \text{Force} = \text{Pressure} \times \text{Area}$$

$$F \times d = (P\,A) \times d = P\,(A \times d)$$

$$\Rightarrow F \times d = \text{W} = \text{P V} = \text{pressure energy}$$

Therefore, pressure energy, $E_p = PV$

The pressure energy per unit mass =

$$\frac{E_P}{m} = \frac{\text{PV}}{m} = \frac{\text{P}}{\frac{\text{m}}{\text{V}}} = \frac{\text{P}}{\rho}$$

Similarly, the pressure energy per unit volume $= \frac{E_P}{\text{volume}} = \frac{PV}{V} = P$

7.6.3 Bernoulli's theorem and its applications

In 1738, the Swiss scientist Daniel Bernoulli developed a relationship for the flow of fluid through a pipe of varying cross section. He proposed a theorem for the streamline flow of a liquid based on the law of conservation of energy.

Bernoulli's theorem

According to Bernoulli's theorem, the sum of pressure energy, kinetic energy, and potential energy per unit mass of an incompressible, non-viscous fluid in a streamlined flow remains a constant. Mathematically,

$$\frac{P}{\rho} + \frac{1}{2}v^2 + \text{gh} = \text{constant} \qquad (7.36)$$

This is known as Bernoulli's equation.

Proof:

Figure 7.33 Flow of liquid through a pipe AB

Let us consider a flow of liquid through a pipe AB as shown in Figure 7.33. Let V be the volume of the liquid when it enters A in a time t which is equal to the volume of the liquid leaving B in the same time. Let a_A, v_A and P_A be the area of cross section of the tube, velocity of the liquid and pressure exerted by the liquid at A respectively.

Let the force exerted by the liquid at A is

$$F_A = P_A a_A$$

Distance travelled by the liquid in time t is

$$d = v_A\, t$$

Therefore, the work done is

$$W = F_A d = P_A a_A v_A\, t$$

But $a_A v_A t = a_A d = V$, volume of the liquid entering at A.

Thus, the work done is the pressure energy (at A), $W = F_A d = P_A V$

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Pressure energy per unit volume at A

$$\frac{\text{Pressure energy}}{\text{volume}} = \frac{P_A V}{V} = P_A$$

Pressure energy per unit mass at A

$$\frac{\text{Pressure energy}}{\text{mass}} = \frac{P_A V}{m} = \frac{P_A}{\frac{m}{V}} = \frac{P_A}{\rho}$$

Since m is the mass of the liquid entering at A in a given time, therefore, pressure energy of the liquid at A is

$$E_{PA} = P_A V = P_A V \times \left(\frac{m}{m}\right) = m\frac{P_A}{\rho}$$

Potential energy of the liquid at A,

$$PE_A = mg\,h_A,$$

Due to the flow of liquid, the kinetic energy of the liquid at A,

$$KE_A = \frac{1}{2} m\ v_A^2$$

Therefore, the total energy due to the flow of liquid at A, $E_A = E_{PA} + KE_A + PE_A$

$$E_A = m\frac{P_A}{\rho} + \frac{1}{2} mv_A^2 + mg\,h_A$$

Similarly, let a_B, v_B, and P_B be the area of cross section of the tube, velocity of the liquid, and pressure exerted by the liquid at *B*. Calculating the total energy at E_B, we get

$$E_B = m\frac{P_B}{\rho} + \frac{1}{2} mv_B^2 + mg\,h_B$$

From the law of conservation of energy,

$$E_A = E_B$$

$$m\frac{P_A}{\rho} + \frac{1}{2} mv_A^2 + mgh_A = m\frac{P_B}{\rho} + \frac{1}{2} m\,v_B^2 + mgh_B$$

$$\frac{P_A}{\rho} + \frac{1}{2} v_A^2 + gh_A = \frac{P_B}{\rho} + \frac{1}{2} v_B^2 + gh_B = \text{constant}$$

Thus, the above equation can be written as

$$\frac{P}{\rho g} + \frac{1}{2}\frac{v^2}{g} + h = \text{constant}$$

The above equation is the consequence of the conservation of energy which is true until there is no loss of energy due to friction. But in practice, some energy is lost due to friction. This arises due to the fact that in a fluid flow, the layers flowing with different velocities exert frictional forces on each other. This loss of energy is generally converted into heat energy. Therefore, Bernoulli's relation is strictly valid for fluids with zero viscosity or non-viscous liquids. Notice that when the liquid flows through a horizontal pipe, then $h = 0 \Rightarrow \frac{P}{\rho g} + \frac{1}{2}\frac{v^2}{g}$ = constant.

Applications of Bernoulli's Theorem

(a) Blowing off roofs during wind storm

In olden days, the roofs of the huts or houses were designed with a slope as shown in Figure.7.34. One important scientific reason is that as per the Bernoulli's principle, it will be safeguarded except roof during storm or cyclone.

Figure 7.34 Roofs of the huts or houses

During cyclonic condition, the roof is blown off without damaging the other parts of the house. In accordance with the Bernoulli's principle, the high wind blowing over the roof creates a low-pressure P_1. The pressure under the roof P_2 is greater. Therefore, this pressure difference (P_2–P_1) creates an up thrust and the roof is blown off.

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(b) Aerofoil lift

The wings of an airplane (aerofoil) are so designed that its upper surface is more curved than the lower surface and the front edge is broader than the real edge. As the aircraft moves, the air moves faster above the aerofoil than at the bottom as shown in Figure 7.35.

According to Bernoulli's Principle, the pressure of air below is greater than above, which creates an upthrust called the dynamic lift to the aircraft.

Figure 7.35 Aerofoil lift

(c) Bunsen burner

In this, the gas comes out of the nozzle with high velocity, hence the pressure in the stem decreases. So outside air reaches into the burner through an air vent and the mixture of air and gas gives a blue flame as shown in Figure 7.36.

Figure 7.36 **Bunsen burner**

(d) Venturimeter

This device is used to measure the rate of flow (or say flow speed) of the incompressible fluid flowing through a pipe. It works on the principle of Bernoulli's theorem. It consists of two wider tubes A and A' (with cross sectional area A) connected by a narrow tube B (with cross sectional area *a*). A manometer in the form of U-tube is also attached between the wide and narrow tubes as shown in Figure7.37. The manometer contains a liquid of density 'ρ_m'.

Figure 7.37 A schematic diagram of venturimeter

Let P_1 be the pressure of the fluid at the wider region of the tube A. Let us assume that the fluid of density 'ρ' flows from the pipe with speed 'v_1' and into the narrow region, its speed increases to 'v_2'. According to the Bernoulli's equation, this increase in speed is accompanied by a decrease in the fluid pressure P_2 at the narrow region of the tube B. Therefore, the pressure difference between the tubes A and B is noted by measuring the height difference ($\Delta P = P_1 - P_2$) between the surfaces of the manometer liquid.

From the equation of continuity, we can say that $Av_1 = a\,v_2$ which means that

$$v_2 = \frac{A}{a}\,v_1.$$

Using Bernoulli's equation,

$$P_1 + \rho\,\frac{v_1^2}{2} = P_2 + \rho\,\frac{v_2^2}{2} = P_2 + \rho\,\frac{1}{2}\left(\frac{A}{a}v_1\right)^2$$

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From the above equation, the pressure difference

$$\Delta P = P_1 - P_2 = \rho \frac{v_1^2}{2} \frac{(A^2 - a^2)}{a^2}$$

Thus, the speed of flow of fluid at the wide end of the tube A

$$v_1^2 = \frac{2(\Delta P)a^2}{\rho(A^2 - a^2)} \Rightarrow v_1 = \sqrt{\frac{2(\Delta P)a^2}{\rho(A^2 - a^2)}}$$

The volume of the liquid flowing out per second is

$$V = Av_1 = A\sqrt{\frac{2(\Delta P)a^2}{\rho(A^2 - a^2)}} = aA\sqrt{\frac{2(\Delta P)}{\rho(A^2 - a^2)}}$$

(e) Other applications

This Bernoulli's concept is mainly used in the design of carburetor of automobiles, filter pumps, atomizers, and sprayers. For example, the carburetor has a very fine channel called nozzle through which the air is allowed to flow in larger speed. In this case, the pressure is lowered at the narrow neck and in turn, the required fuel or petrol is sucked into the chamber so as to provide the correct mixture of air and fuel necessary for ignition process.

Activity

A bottle is filled with thermocol balls. One end of a flexible tube is kept inside the bottle immersed inside the balls. The free end is rotated and we find the balls sprayed all around. This explains the working of an atomizer or sprayer.

Do you know?

A spider web is much stronger than what we think. A single strand of spider silk can stop flying insects which are tens and thousands times its mass. The young's modulus of the spider web is approximately $4.5 \times 10^9\ N\,m^{-2}$. Compare this value with Young's modulus of wood.

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SUMMARY

- The force between the atoms of an element is called inter-atomic force whereas the force between the molecules of a compound is called inter-molecular force.
- *Hooke's law:* within the elastic limit, the stress is directly proportional to strain.
- The force per unit area is known as *stress*. If F is the force applied and A is the area of cross section of the body then the magnitude of *stress* is equal to F/A. Tensile or compressional stress can be expressed using a single term called *longitudinal stress*.
- The ratio of change in length to the original length of a cylinder is $\Delta L/L$, which is known as *longitudinal strain*
- Within the elastic limit, the ratio of longitudinal stress to the longitudinal strain is called the Young' s modulus of the material of the wire.
- Within the elastic limit, the ratio of volume stress to the volume strain is called the *bulk modulus*.
- Within the elastic limit, the ratio of shear stress to the shear strain is called the *rigidity modulus*.
- Poisson's ratio = lateral strain/longitudinal strain
- The elastic potential energy stored in the wire per unit volume is $U = \frac{1}{2} \times \text{stress} \times \text{strain} = \frac{1}{2} \times Y \times (\text{strain})^2$, where Y denotes Young's modulus of the material.
- If F is the magnitude of the normal force acting on the surface area A, then the pressure is defined as the '*force acting per unit area*'.
- The total pressure at a depth h below the liquid surface is $P = P_a + \rho gh$, where P_a is the atmospheric pressure which is equal to 1.013×10^5 Pa.
- Pascal's law states that the pressure in a fluid at rest is the same at all points if they are at the same height.
- The law of floatation states that a body will float in a liquid if the weight of the liquid displaced by the immersed part of the body is equal to or greater than the weight of the body.
- The coefficient of viscosity of a liquid is the viscous force acting tangentially per unit area of a liquid layer having a unit velocity gradient in a direction perpendicular to the direction of flow of the liquid.
- When a liquid flows such that each particle of the liquid passing a point moves along the same path and has the same velocity as its predecessor then the flow of liquid is said to be streamlined flow.
- During the flow of fluid, when the critical velocity is exceeded by the moving fluid, the motion becomes *turbulent*.
- Reynold's number has a significance as it decides which decides whether the flow of fluid through a cylindrical pipe is streamlined or turbulent.

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SUMMARY (cont.)

- Stokes formula $F = 6\pi\eta av$, where F is the viscous force acting on a sphere of radius a and v is the terminal velocity of the sphere.
- The surface tension of a liquid is defined as the force of tension acting on a unit length of an imaginary line drawn on the free surface of the liquid, the direction of the force being perpendicular to the line so drawn and acting parallel to the surface.
- The angle between tangents drawn at the point of contact to the liquid surface and solid surface inside the liquid is called the *angle of contact* for a pair of solid and liquid.
- The flow of a fluid is said to be steady if, at any given point, the velocity of each passing fluid particle remains constant with respect to time.
- The equation $a_1 v_1 = a_2 v_2$ is called the equation of continuity for a flow of fluid through a tube and it is due to the conservation of mass in the flow of fluids. It states that the sum of pressure energy, kinetic energy, and potential energy per unit mass of an incompressible, non-viscous fluid in a streamlined flow remains constant. i.e., $P/\rho + v^2/2 + gh = \text{constant}$.

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CONCEPT MAP

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EVALUATION

I. Multiple Choice Questions

1. Consider two wires X and Y. The radius of wire X is 3 times the radius of Y. If they are stretched by the same load then the stress on Y is

(a) equal to that on X

(b) thrice that on X

(c) nine times that on X

(d) Half that on X

2. If a wire is stretched to double of its original length, then the strain in the wire is

(a) 1 (b) 2

(c) 3 (d) 4

3. The load – elongation graph of three wires of the same material are shown in figure. Which of the following wire is the thickest?

(a) wire 1

(b) wire 2

(c) wire 3

(d) all of them have same thickness

4. For a given material, the rigidity modulus is $\left(\frac{1}{3}\right)^{rd}$ of Young's modulus. Its Poisson's ratio is

(a) 0 (b) 0.25

(c) 0.3 (d) 0.5

5. A small sphere of radius 2cm falls from rest in a viscous liquid. Heat is produced due to viscous force. The rate of production of heat when the sphere attains its terminal velocity is proportional to

(NEET model 2018)

(a) 2^2 (b) 2^3

(c) 2^4 (d) 2^5

6. Two wires are made of the same material and have the same volume. The area of cross sections of the first and the second wires are A and 2A respectively. If the length of the first wire is increased by Δl on applying a force F, how much force is needed to stretch the second wire by the same amount?

(NEET model 2018)

(a) 2 F (b) 4 F

(c) 8 F (d) 16 F

7. With an increase in temperature, the viscosity of liquid and gas, respectively will

(a) increase and increase

(b) increase and decrease

(c) decrease and increase

(d) decrease and decrease

8. The Young's modulus for a perfect rigid body is

(a) 0 (b) 1

(c) 0.5 (d) infinity

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9. Which of the following is not a scalar?

(a) viscosity

(b) surface tension

(c) pressure

(d) stress

10. If the temperature of the wire is increased, then the Young's modulus will

(a) remain the same

(b) decrease

(c) increase rapidly

(d) increase by very a small amount

11. Copper of fixed volume V is drawn into a wire of length l. When this wire is subjected to a constant force F, the extension produced in the wire is Δl. If Y represents the Young's modulus, then which of the following graphs is a straight line?

(NEET 2014 model)

(a) Δl versus V

(b) Δl versus Y

(c) Δl versus F

(d) Δl versus $\frac{1}{l}$

12. A certain number of spherical drops of a liquid of radius R coalesce to form a single drop of radius R and volume V. If T is the surface tension of the liquid, then

(a) energy = $4\,V\,T\left(\frac{1}{r}-\frac{1}{R}\right)$ is released

(b) energy = $3\,V\,T\left(\frac{1}{r}+\frac{1}{R}\right)$ is absorbed

(c) energy = $3\,V\,T\left(\frac{1}{r}-\frac{1}{R}\right)$ is released

(d) energy is neither released nor absorbed

13. The following four wires are made of the same material. Which of these will have the largest extension when the same tension is applied?

(a) length = 200 cm, diameter = 0.5 mm

(b) length= 200 cm, diameter = 1 mm

(c) length = 200 cm, diameter = 2 mm

(d) length= 200 cm, diameter = 3 m

14. The wettability of a surface by a liquid depends primarily on

(a) viscosity

(b) surface tension

(c) density

(d) angle of contact between the surface and the liquid

15. In a horizontal pipe of non-uniform cross section, water flows with a velocity of 1 m s^{-1} at a point where the diameter of the pipe is 20 cm. The velocity of water (1.5 m s^{-1}) at a point where the diameter of the pipe is (in cm)

(a) 8 (b) 16

(c) 24 (d) 32

Answers:

1) c	2) a	3) a	4) d
5) d	6) b	7) c	8) d
9) d	10) b	11) c	12) c
13) a	14) d	15) b	

II. Short Answer Questions

1. Define stress and strain.

2. State Hooke's law of elasticity.

3. Define Poisson's ratio.

4. Explain elasticity using intermolecular forces.

5. Which one of these is more elastic, steel or rubber? Why?

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6. A spring balance shows wrong readings after using for a long time. Why?
7. What is the effect of temperature on elasticity?
8. Write down the expression for the elastic potential energy of a stretched wire.
9. State Pascal's law in fluids.
10. State Archimedes principle.
11. What do you mean by upthrust or buoyancy?
12. State the law of floatation.
13. Define coefficient of viscosity of a liquid.
14. Distinguish between streamlined flow and turbulent flow.
15. What is Reynold's number? Give its significance.
16. Define terminal velocity.
17. Write down the expression for the Stoke's force and explain the symbols involved in it.
18. State Bernoulli's theorem.
19. What are the energies possessed by a liquid? Write down their equations.
20. Two streamlines cannot cross each other. Why?
21. Define surface tension of a liquid. Mention its S.I unit and dimension.
22. How is surface tension related to surface energy?
23. Define angle of contact for a given pair of solid and liquid.
24. Distinguish between cohesive and adhesive forces.
25. What are the factors affecting the surface tension of a liquid?
26. What happens to the pressure inside a soap bubble when air is blown into it?
27. What do you mean by capillarity or capillary action?
28. A drop of oil placed on the surface of water spreads out. But a drop of water place on oil contracts to a spherical shape. Why?
29. State the principle and usage of Venturimeter.

III. Long Answer Questions

1. State Hooke's law and verify it with the help of an experiment.
2. Explain the different types of modulus of elasticity.
3. Derive an expression for the elastic energy stored per unit volume of a wire.
4. Derive an equation for the total pressure at a depth 'h' below the liquid surface.
5. State and prove Pascal's law in fluids.
6. State and prove Archimedes principle.
7. Derive the expression for the terminal velocity of a sphere moving in a high viscous fluid using stokes force.
8. Derive Poiseuille's formula for the volume of a liquid flowing per second through a pipe under streamlined flow.
9. Obtain an expression for the excess of pressure inside a i) liquid drop ii) liquid bubble iii) air bubble.
10. What is capillarity? Obtain an expression for the surface tension of a liquid by capillary rise method.
11. Obtain an equation of continuity for a flow of fluid on the basis of conservation of mass.

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12. State and prove Bernoulli's theorem for a flow of incompressible, non-viscous, and streamlined flow of fluid.

13. Describe the construction and working of venturimeter and obtain an equation for the volume of liquid flowing per second through a wider entry of the tube.

IV. Exercises

1. A capillary of diameter d*mm* is dipped in water such that the water rises to a height of 30*mm*. If the radius of the capillary is made $\left(\frac{2}{3}\right)$ of its previous value, then compute the height up to which water will rise in the new capillary?

   (Answer: 45 mm)

2. A cylinder of length 1.5 m and diameter 4 cm is fixed at one end. A tangential force of 4×10^5 N is applied at the other end. If the rigidity modulus of the cylinder is 6×10^{10} N m^{-2} then, calculate the twist produced in the cylinder. (Answer: 45.60)

3. A spherical soap bubble A of radius 2 cm is formed inside another bubble B of radius 4 cm. Show that the radius of a single soap bubble which maintains the same pressure difference as inside the smaller and outside the larger soap bubble is lesser than radius of both soap bubbles A and B.

4. A block of Ag of mass x kg hanging from a string is immersed in a liquid of relative density 0.72. If the relative density of Ag is 10 and tension in the string is 37.12 N then compute the mass of Ag block. (Answer: $x = 4$ kg)

5. The reading of pressure meter attached with a closed pipe is 5×10^5 N m^{-2}. On opening the valve of the pipe, the reading of the pressure meter is 4.5×10^5 Nm^{-2}. Calculate the speed of the water flowing in the pipe.

   (Answer: 10 ms^{-1})

BOOKS FOR REFERENCE

1. Serway and Jewett, Physics for scientist and Engineers with modern physics, Brook/Coole publishers, Eighth edition
2. Paul Tipler and Gene Mosca, Physics for scientist and engineers with modern physics, Sixth edition, W.H.Freeman and Company
3. H.C.Verma, Concepts of physics volume 1 and Volume 2, Bharati Bhawan Publishers

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ICT CORNER

Properties of Matter

Through this activity you will be able to learn about the Viscosity.

STEPS:

- Use the URL or scan the QR code to open 'Viscosity' activity page.
- In the activity window , 'Select Viscosity' by dragging the ball in the meter in the side.
- Select the 'Start' button , sphere falls from top to bottom of the beaker. We can see the changes in Distance and Time.
- Spheres can be reset to the top of each beaker by clicking the 'Reset' button.

Step1

Step2

Step3

Step4

URL:

http://www.geo.cornell.edu/hawaii/220/PRI/viscosity.html

* Pictures are indicative only.

* If browser requires, allow **Flash Player** or **Java Script** to load the page.

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UNIT 8

HEAT AND THERMODYNAMICS

Classical thermodynamics.... is the only physical theory of universal content which I am convinced... will never be overthrown. – Albert Einstein

Learning Objectives

In this unit, a student is exposed to

- meaning of heat, work and temperature
- ideal gas laws
- concept of specific heat capacity
- thermal expansion of solids, liquids and gases
- various states of matter
- Newton's law of cooling
- Stefan's law and Wien's law
- meaning of thermodynamic equilibrium
- meaning of internal energy
- zeroth and first laws of thermodynamics
- various thermodynamic processes
- work done in various thermodynamic processes
- second law of thermodynamics
- working of carnot engine and refrigerator

8.1 HEAT AND TEMPERATURE

8.1.1 Introduction

Temperature and heat play very important role in everyday life. All species can function properly only if its body is maintained at a particular temperature. In fact life on Earth is possible because the Sun maintains its temperature. Understanding the meaning of temperature and heat are very crucial to understand the nature. Thermodynamics is a branch of physics which explains the phenomena of temperature, heat etc. The concepts presented in this chapter will help us to understand the terms 'hot' and 'cold' and also differentiate heat from temperature. In thermodynamics, heat and temperature are two different but closely related parameters.

8.1.2 Meaning of heat

When an object at higher temperature is placed in contact with another object at lower temperature, there will be a spontaneous flow of energy from the object

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at higher temperature to the one at lower temperature. This energy is called heat. This process of energy transfer from higher temperature object to lower temperature object is called heating. Due to flow of heat sometimes the temperature of the body will increase or sometimes it may not increase.

Note

There is a misconception that heat is a quantity of energy. People often talk 'this water has more heat or less heat'. These words are meaningless. Heat is not a quantity. Heat is an energy in transit which flows from higher temperature object to lower temperature object. Once the heating process is stopped we cannot use the word heat. When we use the word 'heat', it is the energy in transit but not energy stored in the body.

EXAMPLE 8.1

a. 'A lake has more rain'.

b. 'A hot cup of coffee has more heat'.

What is wrong in these two statements?

Solution

a. When it rains, lake receives water from the cloud. Once the rain stops, the lake will have more water than before raining. Here 'raining' is a process which brings water from the cloud. Rain is not a quantity rather it is water in transit. So the statement 'lake has more rain' is wrong, instead the 'lake has more water' will be appropriate.

b. When heated, a cup of coffee receives heat from the stove. Once the coffee is taken from the stove, the cup of coffee has more internal energy than before. 'Heat' is the energy in transit and which flows from an object at higher temperature to an object at lower temperature. Heat is not a quantity. So the statement 'A hot cup of coffee has more heat' is wrong, instead 'coffee is hot' will be appropriate.

8.1.3 Meaning of work

When you rub your hands against each other the temperature of the hands increases. You have done some work on your hands by rubbing. The temperature of the hands increases due to this work. Now if you place your hands on the chin, the temperature of the chin increases. This is because the hands are at higher temperature than the chin. In the above example, the temperature of hands is increased due to work and temperature of the chin is increased due to heat transfer from the hands to the chin. It is shown in the Figure 8.1

By doing work on the system, the temperature in the system will increase and sometimes may not. Like heat, work is also

Figure 8.1 Difference between work and heat

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not a quantity and through the work energy is transferred to the system . So we cannot use the word 'the object contains more work' or 'less work'.

Either the system can transfer energy to the surrounding by doing work on surrounding or the surrounding may transfer energy to the system by doing work on the system. For the transfer of energy from one body to another body through the process of work, they need not be at different temperatures.

8.1.4 Meaning of temperature

Temperature is the degree of hotness or coolness of a body. Hotter the body higher is its temperature. The temperature will determine the direction of heat flow when two bodies are in thermal contact.

The SI unit of temperature is ***kelvin*** **(K)**.

In our day to day applications, ***Celsius*** (˚C) and ***Fahrenheit*** (°F) scales are used.

Temperature is measured with a thermometer.

The conversion of temperature from one scale to other scale is given in Table 8.1

Table 8.1 Temperature conversion

Scale	To Kelvin	From Kelvin
Celsius	K=°C + 273.15	°C = K – 273.15
Fahrenheit	K=(°F + 459.67)÷1.8	°F=(K × 1.8)-459.67

Scale	To Fahrenheit	From Fahrenheit
Celsius	°F=(1.8 × °C) + 32	°C = (°F – 32)÷1.8

Scale	To Celsius	From Celsius
Fahrenheit	°C=(°F – 32)÷1.8	°F = (1.8 × °C) + 32

8.2 THERMAL PROPERTIES OF MATTER

8.2.1 Boyle's law, Charles' law and ideal gas law

For a given gas at low pressure (density) kept in a container of volume V, experiments revealed the following information.

- When the gas is kept at constant temperature, the pressure of the gas is inversely proportional to the volume. $P \propto \frac{1}{V}$. It was discovered by Robert Boyle (1627-1691) and is known as Boyle's law.
- When the gas is kept at constant pressure, the volume of the gas is directly proportional to absolute temperature. $V \propto T$. It was discovered by Jacques Charles (1743-1823) and is known as Charles' law.
- By combining these two equations we have

$PV = CT$. Here C is a positive constant.

We can infer that C is proportional to the number of particles in the gas container by considering the following argument. If we take two containers of same type of gas with same volume V, same pressure P and same temperature T, then the gas in each container obeys the above equation. $PV = CT$. If the two containers of gas is considered as a single system, then the pressure and temperature of this combined system will be same but volume will be twice and number of particles will also be double as shown in Figure 8.2

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Figure 8.2 Ideal gas law

For this combined system, V becomes 2V, so C should also double to match with the ideal gas equation $\frac{P(2V)}{T} = 2C$. It implies that C must depend on the number of particles in the gas and also should have the dimension of $\left[\frac{PV}{T}\right] = \text{JK}^{-1}$. So we can write the constant C as k times the number of particles N.

Here k is the Boltzmann constant (1.381×10^{-23} JK^{-1}) and it is found to be a universal constant.

So the ideal gas law can be stated as follows

$$PV = NkT \qquad (8.1)$$

The equation (8.1) can also be expressed in terms of mole.

Mole is the practical unit to express the amount of gas. One mole of any substance is the amount of that substance which contains Avogadro number (N_A) of particles (such as atoms or molecules). The Avogadro's number N_A is defined as the number of carbon atoms contained in exactly 12 g of ^{12}C.

Suppose if a gas contains μ mole of particles then the total number of particles can be written as

$$N = \mu N_A \qquad (8.2)$$

where N_A is Avogadro number (6.023×10^{23}mol^{-1})

Substituting for N from equation (8.2), the equation (8.1) becomes

$PV = \mu N_A kT$. Here $N_A k = R$ called universal gas constant and its value is 8.314 J /mol. K.

So the ideal gas law can be written for μ mole of gas as

$$PV = \mu RT \qquad (8.3)$$

This is called the equation of state for an ideal gas. It relates the pressure, volume and temperature of thermodynamic system at equilibrium.

EXAMPLE 8.2

A student comes to school by a bicycle whose tire is filled with air at a pressure 240 kPa at 27°C. She travels 8 km to reach the school and the temperature of the bicycle tire increases to 39°C. What is the change in pressure in the tire when the student reaches school?

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Solution

We can take air molecules in the tire as an ideal gas. The number of molecules and the volume of tire remain constant. So the air molecules at 27°C satisfies the ideal gas equation $P_1V_1 = NkT_1$ and at 39°C it satisfies $P_2V_2 = NkT_2$

But we know

$$V_1 = V_2 = V$$

$$\frac{P_1V}{P_2V} = \frac{NkT_1}{NkT_2}$$

$$\frac{P_1}{P_2} = \frac{T_1}{T_2}$$

$$P_2 = \frac{T_2}{T_1}P_1$$

$$P_2 = \frac{312K}{300K} \times 240 \times 10^3\,Pa = 249.6\,kPa$$

EXAMPLE 8.3

When a person breaths, his lungs can hold up to 5.5 Litre of air at body temperature 37°C and atmospheric pressure (1 atm =101 kPa). This Air contains 21% oxygen. Calculate the number of oxygen molecules in the lungs.

Solution

We can treat the air inside the lungs as an ideal gas. To find the number of molecules, we can use the ideal gas law.

$$PV = NkT$$

Here volume is given in the Litre. 1 Litre is volume occupied by a cube of side 10 cm.
1Litre = 10cm × 10cm × 10cm = 10^{-3} m^3

$$N = \frac{PV}{kT} = \frac{1.01\times10^5\,\text{Pa}\times5.5\times10^{-3}\,\text{m}^3}{1.38\times10^{-23}\,\text{JK}^{-1}\times310\text{K}}$$

$$= 1.29\times10^{23}\,\text{Molecules}$$

Only 21% of N are oxygen. The total number of oxygen molecules

$$= 1.29 \times 10^{23} \times \frac{21}{100}$$

Number of oxygen molecules
$= 2.7 \times 10^{22}$ molecules

EXAMPLE 8.4

Calculate the volume of one mole of any gas at STP and at room temperature (300K) with the same pressure 1 atm.

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Solution:

Here STP means standard temperature (T=273K or 0°C) and Pressure (P=1 atm or 101.3 kPa)

We can use ideal gas equation $V = \frac{\mu RT}{P}$.

Here μ = 1 mol and R =8.314 J/mol.K.

By substituting the values

$$V = \frac{(1\text{ mol})(8.314\frac{\text{J}}{\text{mol}}\text{K})(273\text{K})}{1.013\times10^{5}\text{Nm}^{-2}}$$

$=22.4 \times 10^{-3}\text{ m}^3$

We know that 1 Litre (L) = $=10^{-3}\text{m}^3$. So we can conclude that 1 mole of any ideal gas has volume 22.4 L.

By multiplying 22.4L by $\frac{300K}{273K}$ we get the volume of one mole of gas at room temperature. It is 24.6 L.

EXAMPLE 8.5

Estimate the mass of air in your class room at NTP. Here NTP implies normal temperature (room temperature) and 1 atmospheric pressure.

Solution

The average size of a class is 6m length, 5 m breadth and 4 m height. The volume of the room V = 6 × 5 × 4 = 120m³. We can determine the number of mole. At room temperature 300K, the volume of a gas occupied by any gas is equal to 24.6L.

The number of mole $\mu = \frac{120\text{m}^3}{24.6\times10^{-3}\text{m}^3}$ ≈4878 mol.

Air is the mixture of about 20% oxygen, 79% nitrogen and remaining one percent are argon, hydrogen, helium, and xenon. The molar mass of air is 29 gmol^{-1}.

So the total mass of air in the room $m = 4878 \times 29 \times 10^{-3} = 141.4$kg.

8.2.2 Heat capacity and specific heat capacity

Take equal amount of water and oil at temperature 27°C and heat both of them till they reach the temperature 50°C. Note down the time taken by the water and oil to reach the temperature 50°C. Obviously these times are not same. We can see that water takes more time to reach 50°C than oil. It implies that water requires more heat energy to raise its temperature than oil. Now take twice the amount of water at 27°C and heat it up to 50°C , note the time taken for this rise in temperature. The time taken by the water is now twice compared to the previous case.

We can define 'heat capacity' as the amount of heat energy required to raise the temperature of the given body from T to T + ΔT .

$$\text{Heat capacity S} = \frac{\Delta Q}{\Delta T}$$

Specific heat capacity of a substance is defined as the amount of heat energy required to raise the temperature of 1kg of a substance by 1 Kelvin or 1°C

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$$\Delta Q = m\, s \Delta T$$

Therefore,

$$s = \frac{1}{m}\left(\frac{\Delta Q}{\Delta T}\right)$$

Where s is known as *specific heat capacity* of a substance and its value depends only on the nature of the substance not amount of substance

ΔQ = Amount of heat energy

ΔT = Change in temperature

m = Mass of the substance

The SI unit for specific heat capacity is J kg^{-1} K^{-1}. Heat capacity and specific heat capacity are always positive quantities.

Table 8.2 Specific heat capacity of some common substances at 1 atm (20°C)

Material	Specific heat capacity ($Jkg^{-1}K^{-1}$)
Air	1005
Lead	130
Copper	390
Iron (steel)	450
Glass	840
Aluminium	900
Human body	3470
Water	4186

From the table 8.2 it is clear that water has the highest value of specific heat capacity. For this reason it is used as a coolant in power stations and reactors.

Note

The term heat capacity or specific heat capacity does not mean that object contains a certain amount of heat. Heat is energy transfer from the object at higher temperature to the object at lower temperature. The correct usage is 'internal energy capacity'. But for historical reason the term 'heat capacity' or 'specific heat capacity' are retained.

DO YOU KNOW?

When two objects of same mass are *heated* at equal rates, the object with *smaller specific heat capacity* will have a *faster temperature increase.*

When two objects of same mass are *left to cool down*, the temperature of the object with *smaller specific heat capacity* will *drop faster.*

When we study properties of gases, it is more practical to use molar specific heat capacity. Molar specific heat capacity is defined as heat energy required to increase the temperature of one mole of substance by 1K or 1°C. It can be written as follows

$$C = \frac{1}{\mu}\left(\frac{\Delta Q}{\Delta T}\right)$$

Here C is known as *molar specific heat capacity* of a substance and μ is number of moles in the substance.

The SI unit for molar specific heat capacity is J mol^{-1} K^{-1}. It is also a positive quantity.

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8.2.3 Thermal expansion of solids, liquids and gases

Thermal expansion is the tendency of matter to change in shape, area, and volume due to a change in temperature.

All three states of matter (solid, liquid and gas) expand when heated. When a solid is heated, its atoms vibrate with higher amplitude about their fixed points. The relative change in the size of solids is small. Railway tracks are given small gaps so that in the summer, the tracks expand and do not buckle. Railroad tracks and bridges have expansion joints to allow them to expand and contract freely with temperature changes. It is shown in Figure 8.3

Liquids, have less intermolecular forces than solids and hence they expand more than solids. This is the principle behind the mercury thermometers.

In the case of **gas** molecules, the intermolecular forces are almost negligible and hence they expand much more than solids. For example in hot air balloons when gas particles get heated, they expand and take up more space.

The increase in dimension of a body due to the increase in its temperature is called thermal expansion.

The expansion in length is called **linear expansion**. Similarly the expansion in area is termed as **area expansion** and the

Figure 8.3 Expansion joints for safety

Figure 8.4 Thermal expansions

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expansion in volume is termed as **volume expansion**. It is shown in Figure 8.4

Linear Expansion

In solids, for a small change in temperature ΔT, the fractional change in length $\left(\frac{\Delta L}{L_0}\right)$ is directly proportional to ΔT.

$$\frac{\Delta L}{L_0} = \alpha_L \Delta T$$

Therefore, $\alpha_L = \frac{\Delta L}{L_0 \Delta T}$

Where, α_L = coefficient of linear expansion.

ΔL = Change in length

L_0 = Original length

ΔT = Change in temperature.

- When the lid of a glass bottle is tight, keep the lid near the hot water which makes it easier to open. It is because the lid has higher thermal expansion than glass.
- When the hot boiled egg is dropped in cold water, the egg shell can be removed easily. It is because of the different thermal expansions of the shell and egg.

EXAMPLE 8.6

Eiffel tower is made up of iron and its height is roughly 300 m. During winter season (January) in France the temperature is 2°C and in hot summer its average temperature 25°C. Calculate the change in height of Eiffel tower between summer and winter. The linear thermal expansion coefficient for iron $\alpha = 10 \times 10^{-6}$ per °C

Solution

$$\frac{\Delta L}{L_0} = \alpha_L \Delta T$$

$$\Delta L = \alpha_L L_0 \Delta T$$

$\Delta L = 10 \times 10^{-6} \times 300 \times 23 = 0.069$ m=69 mm

Area Expansion

For a small change in temperature ΔT the fractional change in area $\left(\frac{\Delta A}{A_0}\right)$ of a substance is directly proportional to ΔT and it can be written as

$$\frac{\Delta A}{A_0} = \alpha_A \Delta T$$

Therefore, $\alpha_A = \frac{\Delta A}{A_0 \Delta T}$

Where, α_A = coefficient of area expansion.

ΔA = Change in area

A_0 = Original area

ΔT = Change in temperature

Volume Expansion

For a small change in temperature ΔT the fractional change in volume $\left(\frac{\Delta V}{V_0}\right)$ of a substance is directly proportional to ΔT.

$$\frac{\Delta V}{V_0} = \alpha_V \Delta T$$

Therefore, $\alpha_V = \frac{\Delta V}{V_0 \Delta T}$

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Where, α_V = coefficient of volume expansion.

ΔV = Change in volume

V_0 = Original volume

ΔT = Change in temperature

Unit of coefficient of linear, area and volumetric expansion of solids is $^\circ C^{-1}$ or K^{-1}

Note

For a given specimen,

$$\frac{\Delta L}{L_0} = \alpha_L \Delta T \text{ (Linear expansion)}$$

$$\frac{\Delta A}{A_0} \approx 2\,\alpha_L \Delta T \text{ (Area expansion} \approx 2 \times \text{Linear expansion)}$$

$$\frac{\Delta V}{V_0} \approx 3\,\alpha_L \Delta T \text{ (Volume expansion} \approx 3 \times \text{Linear expansion)}$$

8.2.4 Anomalous expansion of water

Liquids expand on heating and contract on cooling at moderate temperatures. But water exhibits an anomalous behavior. It contracts on heating between 0˚C and 4˚C. The volume of the given amount of water decreases as it is cooled from room temperature, until it reach 4˚C . Below 4˚C the volume increases and so the density decreases. This means that the water has a maximum density at 4˚C . This behavior of water is called anomalous expansion of water. It is shown in the Figure 8.5

In cold countries during the winter season, the surface of the lakes will be at lower temperature than the bottom as shown in the Figure 8.6. Since the solid water (ice) has lower density than its liquid form, below 4°C, the frozen water will be on the top surface above the liquid water (ice floats). This is due to the anomalous expansion of water. As the water in lakes and ponds freeze only at the top the species living in the lakes will be safe at the bottom.

Figure 8.5 Anomalous Expansion of water

Figure 8.6 Anomalous expansion of water in lakes

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8.2.5 Change of state

All matter exists normally in three states as solids, liquids or gases. Matter can be changed from one state to another either by heating or cooling.

Examples:

1. Melting (solid to liquid)
2. Evaporation (liquid to gas)
3. Sublimation (solid to gas)
4. Freezing / Solidification (liquid to solid)
5. Condensation (gas to liquid)

Figure 8.7 Change of states of matter

Latent heat capacity:

While boiling a pot of water, the temperature of the water increases until it reaches 100 °C which is the boiling point of water, and then the temperature remains constant until all the water changes from liquid to gas. During this process heat is continuously added to the water. But the temperature of water does not increase above its boiling point. This is the concept of latent heat capacity.

Latent heat capacity of a substance is defined as the amount of heat energy required to change the state of a unit mass of the material.

$$Q = m \times L$$

Therefore, $L = \dfrac{Q}{m}$

Where L = Latent heat capacity of the substance

Q = Amount of heat

m = mass of the substance

The SI unit for Latent heat capacity is J kg^{-1}.

Figure 8.8 Temperature versus heat for water

Note When heat is added or removed during a change of state, the temperature remains constant.

- The latent heat for a solid - liquid state change is called the latent heat of fusion (L_f)
- The latent heat for a liquid - gas state change is called the latent heat of vaporization (L_v)
- The latent heat for a solid - gas state change is called the latent heat of sublimation (L_s)

Triple point

The triple point of a substance is the temperature and pressure at which the three phases (gas, liquid and solid) of that substance coexist in thermodynamic equilibrium.

The triple point of water is at 273.1 K and a partial vapour pressure of 611.657 Pascal.

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8.2.6 Calorimetry

Calorimetry means the measurement of the amount of heat released or absorbed by thermodynamic system during the heating process. When a body at higher temperature is brought in contact with another body at lower temperature, the heat lost by the hot body is equal to the heat gained by the cold body. No heat is allowed to escape to the surroundings. It can be mathematically expressed as

$$Q_{gain} = -Q_{lost}$$

$$Q_{gain} + Q_{lost} = 0$$

Heat gained or lost is measured with a calorimeter. Usually the calorimeter is an insulated container of water as shown in Figure 8.9.

Figure 8.9 Calorimeter

A sample is heated at high temperature (T_1) and immersed into water at room temperature (T_2) in the calorimeter. After some time both sample and water reach a final equilibrium temperature T_f. Since the calorimeter is insulated, heat given by the hot sample is equal to heat gained by the water. It is shown in the Figure 8.10

Figure 8.10 Calorimeter with sample of block

$$Q_{gain} = -Q_{lost}$$

Note the sign convention. The heat lost is denoted by negative sign and heat gained is denoted as positive.

From the definition of specific heat capacity

$$Q_{gain} = m_2 s_2 (T_f - T_2)$$

$$Q_{lost} = m_1 s_1 (T_f - T_1)$$

Here s_1 and s_2 specific heat capacity of hot sample and water respectively.

So we can write

$$m_2 s_2 (T_f - T_2) = - m_1 s_1 (T_f - T_1)$$

$$m_2 s_2 T_f - m_2 s_2 T_2 = - m_1 s_1 T_f + m_1 s_1 T_1$$

$$m_2 s_2 T_f + m_1 s_1 T_f = m_2 s_2 T_2 + m_1 s_1 T_1$$

The final temperature

$$T_f = \frac{m_1 s_1 T_1 + m_2 s_2 T_2}{m_1 s_1 + m_2 s_2}$$

EXAMPLE 8.7

If 5 L of water at 50°C is mixed with 4L of water at 30°C, what will be the final temperature of water? Take the specific heat capacity of water as 4184 J kg^{-1} K^{-1}.

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Solution

We can use the equation

$$T_f = \frac{m_1 s_1 T_1 + m_2 s_2 T_2}{m_1 s_1 + m_2 s_2}$$

$m_1 = 5L = 5$kg and $m_2 = 4L = 4$kg, $s_1 = s_2$ and T_1=50°C =323K and T_2 = 30°C=303 K.

So

$$T_f = \frac{m_1 T_1 + m_2 T_2}{m_1 + m_2} = \frac{5 \times 323 + 4 \times 303}{5 + 4} = 314.11 \text{ K}$$

T_f = 314.11 K-273K ≈ 41°C.

Suppose if we mix equal amount of water ($m_1 = m_2$) with 50°C and 30°C, then the final temperature is average of two temperatures.

$$T_f = \frac{T_1 + T_2}{2} = \frac{323 + 303}{2} = 313\text{K} = 40°\text{C}$$

Suppose if both the water are at 30°C then the final temperature will also 30°C. It implies that they are at equilibrium and no heat exchange takes place between each other.

Note It is important to note that the final equilibrium temperature of mixing of gas or liquid depends on mass of the substances, their specific heat capacities and their temperatures. Only if we mix the same substances at equal amount, the final temperature will be an average of the individual temperatures.

8.2.7 Heat transfer

As we have seen already heat is a energy in transit which is transferred from one body to another body due to temperature difference. There are three modes of heat transfer: Conduction, Convection and Radiation.

Conduction

Conduction is the process of direct transfer of heat through matter due to temperature difference. When two objects are in direct contact with one another, heat will be transferred from the hotter object to the colder one. The objects which allow heat to travel easily through them are called conductors.

Thermal conductivity

Thermal conductivity is the ability to conduct heat.

The quantity of heat transferred through a unit length of a material in a direction normal to unit surface area due to a unit temperature difference under steady state conditions is known as thermal conductivity of a material.

Figure 8.11 Steady state heat flow by conduction.

In steady state, the rate of flow of heat Q is proportional to the temperature difference ΔT and the area of cross section A and is inversely proportional to the length L. So the rate of flow of heat is written as

$$\frac{Q}{t} = \frac{KA\Delta T}{L}$$

Where, K is known as the **coefficient of thermal conductivity.**

(Not to be confused with Kelvin represented by upper case K)

The SI unit of thermal conductivity is

$$\text{J s}^{-1}\text{ m}^{-1}\text{ K}^{-1} \text{ or W m}^{-1}\text{ K}^{-1}.$$

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Table 8.3: Thermal conductivities (in W m^{-1} K^{-1}) of some materials at 1 *atm*

Material	Thermal Conductivity	Material	Thermal Conductivity
Diamond	2300	Water	0.56
Silver	420	Human tissue	0.2
Copper	380	Wood	0.17
Aluminum	200	Helium	0.152
Steel	40	Cork	0.042
Glass	0.84	Air	0.023
Brick	0.84		
Ice	2		

Note

Steady state:

The state at which temperature attains constant value everywhere and there is no further transfer of heat anywhere is called steady state.

Thermal conductivity depends on the nature of the material. For example silver and aluminum have high thermal conductivities. So they are used to make cooking vessels.

Convection

Convection is the process in which heat transfer is by actual movement of molecules in fluids such as liquids and gases. In convection, molecules move freely from one place to another. It happens naturally or forcefully.

DO YOU KNOW?

During the day, sun rays warm up the land more quickly than sea water. It is because land has less specific heat capacity than water. As a result the air above the land becomes less dense and rises. At the same time the cooler air above the sea flows to land and it is called 'sea breeze'. During the night time the land gets cooled faster than sea due to the same reason (specific heat). The air molecules above sea are warmer than air molecules above the land. So air molecules above the sea are replaced by cooler air molecules from the land. It is called 'land breeze'.

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Boiling water in a cooking pot is an example of convection. Water at the bottom of the pot receives more heat. Due to heating, the water expands and the density of water decreases at the bottom. Due to this decrease in density, molecules rise to the top. At the same time the molecules at the top receive less heat and become denser and come to the bottom of the pot. This process goes on continuously. The back and forth movement of molecules is called convection current.

To keep the room warm, we use room heater. The air molecules near the heater will heat up and expand. As they expand, the density of air molecules will decrease and rise up while the higher density cold air will come down. This circulation of air molecules is called convection current.

Radiation:

When we keep our hands near the hot stove we feel the heat even though our hands are not touching the hot stove. Here heat transferred from the hot stove to our hands is in the form of radiation. We receive energy from the sun in the form of radiations. These radiations travel through vacuum and reach the Earth. It is the peculiar character of radiation which requires no medium to transfer energy from one object to another. The conduction or convection requires medium to transfer the heat.

Radiation is a form of energy transfer from one body to another by electromagnetic waves.

Example:
1. Solar energy from the Sun.
2. Radiation from room heater.

Note

The parameter temperature is generally thought to be associated with matter (solid, liquid and gas). But radiation is also considered as a thermodynamic system which has well defined temperature and pressure. The visible radiation coming from the Sun is at the temperature of 5700 K and the Earth re emits the radiation in the infrared range into space which is at a temperature of around 300K.

8.2.8 Newton's law of cooling

Newton's law of cooling states that the rate of loss of heat of a object is directly proportional to the difference in the temperature between that object and its surroundings .

$$\frac{dQ}{dt} \propto -(T - T_s) \qquad (8.4)$$

The negative sign indicates that the quantity of heat lost by object goes on decreasing with time. Where,

T = Temperature of the object

T_s = Temperature of the surrounding

Figure 8.12 Cooling of hot water with time

From the graph in Figure 8.12 it is clear that the rate of cooling is high initially and decreases with falling temperature.

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Let us consider an object of mass m and specific heat capacity s at temperature T. Let T_s be the temperature of the surroundings. If the temperature falls by a small amount dT in time dt, then the amount of heat lost is,

$$dQ = msdT \quad (8.5)$$

Dividing both sides of equation (8.5) by dt

$$\frac{dQ}{dt} = \frac{msdT}{dt} \quad (8.6)$$

From Newton's law of cooling

$$\frac{dQ}{dt} \propto -(T - T_s)$$

$$\frac{dQ}{dt} = -a(T - T_s) \quad (8.7)$$

Where a is some positive constant.
From equation (8.6) and (8.7)

$$-a(T - T_s) = ms\frac{dT}{dt}$$

$$\frac{dT}{T - T_s} = -\frac{a}{ms}dt \quad (8.8)$$

Integrating equation (8.8) on both sides,

$$\int \frac{dT}{T - T_s} = -\int \frac{a}{ms}dt$$

$$\ln(T - T_s) = -\frac{a}{ms}t + b_1$$

Where b_1 is the constant of integration. taking exponential both sides, we get

$$T = T_s + b_2 e^{-\frac{a}{ms}t} \quad (8.9)$$

here $b_2 = e^{b_1}$ = constant

EXAMPLE 8.8

A hot water cools from 92°C to 84°C in 3 minutes when the room temperature is 27°C. How long will it take for it to cool from 65°C to 60°C?

The hot water cools 8°C in 3 minutes. The average temperature of 92°C and 84°C is 88°C. This average temperature is 61°C above room temperature. Using equation (8.8)

$$\frac{dT}{T - T_s} = -\frac{a}{ms}dt \text{ or } \frac{dT}{dt} = -\frac{a}{ms}(T - T_s)$$

$$\frac{8^oC}{3\,\text{min}} = -\frac{a}{ms}(61^oC)$$

Similarly the average temperature of 65°C and 60°C is 62.5°C. The average temperature is 35.5°C above the room temperature. Then we can write

$$\frac{5^oC}{dt} = -\frac{a}{ms}(35.5^oC)$$

By diving both the equation, we get

$$\frac{8^oC}{3\,\text{min}} \Big/ \frac{5^oC}{dt} = -\frac{a}{ms}(61^oC) \Big/ -\frac{a}{ms}(35.5^oC)$$

$$\frac{8 \times dt}{3 \times 5} = \frac{61}{35.5}$$

$$dt = \frac{61 \times 15}{35.5 \times 8} = \frac{915}{284} = 3.22\,\text{min}$$

8.3 LAWS OF HEAT TRANSFER

8.3.1 Prevost theory of heat exchange

Every object emits heat radiations at all finite temperatures (except 0 K) as well as it absorbs radiations from the surroundings. For example, if you touch someone, they might feel your skin as either hot or cold.

A body at high temperature radiates more heat to the surroundings than it receives from it. Similarly, a body at a lower temperature receives more heat from the surroundings than it loses to it.

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Prevost applied the idea of 'thermal equilibrium' to radiation. He suggested that all bodies radiate energy but hot bodies radiate more heat than the cooler bodies. At one point of time the rate of exchange of heat from both the bodies will become the same. Now the bodies are said to be in 'thermal equilibrium'.

Only at absolute zero temperature a body will stop emitting. Therefore *Prevost theory states that all bodies emit thermal radiation at all temperatures above absolute zero irrespective of the nature of the surroundings.*

8.3.2 Stefan Boltzmann law

Stefan Boltzmann law states that, *the total amount of heat radiated per second per unit area of a black body is directly proportional to the fourth power of its absolute temperature.*

$$E \propto T^4 \text{ or } E = \sigma T^4 \qquad (8.10)$$

Where, σ is known as Stefan's constant. Its value is 5.67×10^{-8} W $m^{-2} k^{-4}$

Note

If a body is not a perfect black body, then

$$E = e\,\sigma\, T^4 \qquad (8.11)$$

Where 'e' is emissivity of surface.

Emissivity is defined as the ratio of the energy radiated from a material's surface to that radiated from a perfectly black body at the same temperature and wavelength.

8.3.3 Wien's displacement law

In the universe every object emits radiation. The wavelengths of these radiations depend on the object's absolute temperature. These radiations have different wavelengths and all the emitted wavelengths will not have equal intensity.

Wien's law states that, *the wavelength of maximum intensity of emission of a black body radiation is inversely proportional to the absolute temperature of the black body.*

$$\lambda_m \propto \frac{1}{T} \; (or) \; \lambda_m = \frac{b}{T} \qquad (8.12)$$

Where, b is known as Wien's constant. Its value is 2.898×10^{-3} m K

It implies that if temperature of the body increases, maximal intensity wavelength (λ_m) shifts towards lower wavelength (higher frequency) of electromagnetic spectrum. It is shown in Figure 8.13

Graphical representation

Figure 8.13 Wien's displacement law

From the graph it is clear that the peak of the wavelengths is inversely proportional to temperature. The curve is known as 'black body radiation curve'.

Wien's law and Vision:

Why our eye is sensitive to only visible wavelength (in the range 400 nm to 700nm)?

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The Sun is approximately taken as a black body. Since any object above 0 K will emit radiation, Sun also emits radiation. Its surface temperature is about 5700 K. By substituting this value in the equation (8.12),

$$\lambda_m = \frac{b}{T} = \frac{2.898\times10^{-3}}{5700} \approx 508\,nm$$

It is the wavelength at which maximum intensity is 508*nm*. Since the Sun's temperature is around 5700K, the spectrum of radiations emitted by Sun lie between 400 nm to 700 nm which is the visible part of the spectrum. It is shown in Figure 8.14

Figure 8.14 Wien's law and Human's vision

The humans evolved under the Sun by receiving its radiations. The human eye is sensitive only in the visible not in infrared or X-ray ranges in the spectrum.

Suppose if humans had evolved in a planet near the star Sirius (9940K), then they would have had the ability to see the Ultraviolet rays!

EXAMPLE 8.9

The power radiated by a black body A is E_A and the maximum energy radiated was at the wavelength λ_A. The power radiated by another black body B is $E_B = N\,E_A$ and the radiated energy was at the maximum wavelength, $\frac{1}{2}\lambda_A$. What is the value of N?

According to Wien's displacement law

$\lambda_{max}\,T$ = constant for both object A and B

$$\lambda_A T_A = \lambda_B T_B. \text{ Here } \lambda_B = \frac{1}{2}\lambda_A$$

$$\frac{T_B}{T_A} = \frac{\lambda_A}{\lambda_B} = \frac{\lambda_A}{\left(\frac{1}{2}\right)\lambda_A} = 2$$

$$T_B = 2T_A$$

From Stefan-Boltzmann law

$$\frac{E_B}{E_A} = \left(\frac{T_B}{T_A}\right)^4 = (2)^4 = 16 = N$$

Object B has emitted at lower wavelength compared to A. So the object B would have emitted more energetic radiation than A.

8.4 THERMODYNAMICS:

8.4.1 Introduction

In the previous sections we have studied about the heat, temperature and thermal properties of matter. Thermodynamics is

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a branch of physics which describes the laws governing the process of conversion of work into heat and conversion of heat into work. The laws of thermodynamics are formulated over three centuries of experimental works of Boyle, Charles, Bernoulli, Joule, Clausius, Kelvin, Carnot and Helmholtz. In our day to day life, the functioning of everything around us and even our body is governed by the laws of thermodynamics. Therefore thermodynamics is one of the essential branches of physics.

Thermodynamic system:

A thermodynamic system is a finite part of the universe. It is a collection of large number of particles (atoms and molecules) specified by certain parameters called pressure (P), Volume (V) and Temperature (T). The remaining part of the universe is called surrounding. Both are separated by a boundary. It is shown in Figure 8.15

Figure 8.15 Thermodynamic system

Examples:

A thermodynamic system can be liquid, solid, gas and radiation.

Thermodynamic system	Surrounding
Bucket of water	Open atmosphere
Air molecules in the room	Outside air
Human body	Open atmosphere
Fish in the sea	Sea of water

We can classify thermodynamics system into three types: It is given in Figure 8.16

Figure 8.16 Different types of thermodynamic systems

8.4.2 Thermal equilibrium

When a hot cup of coffee is kept in the room, heat flows from coffee to the surrounding air. After some time the coffee reaches the same temperature as the surrounding air and there will be no net heat flow from coffee to air or air to coffee. It implies that the coffee and surrounding air are in thermal equilibrium with each other.

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Two systems are said to be in thermal equilibrium with each other if they are at the same temperature, which will not change with time.

Mechanical equilibrium:

Consider a gas container with piston as shown in Figure 8.17. When some mass is placed on the piston, it will move downward due to downward gravitational force and after certain humps and jumps the piston will come to rest at a new position. When the downward gravitational force given by the piston is balanced by the upward force exerted by the gas, the system is said to be in mechanical equilibrium. A system is said to be in mechanical equilibrium if no unbalanced force acts on the thermo dynamic system or on the surrounding by thermodynamic system.

Figure 8.17 Mechanical equilibrium

Chemical equilibrium:

If there is no net chemical reaction between two thermodynamic systems in contact with each other then it is said to be in chemical equilibrium.

Thermodynamic equilibrium:

If two systems are set to be in thermodynamic equilibrium, then the systems are at thermal, mechanical and chemical equilibrium with each other. In a state of thermodynamic equilibrium the macroscopic variables such as pressure, volume and temperature will have fixed values and do not change with time.

8.4.3 Thermodynamic state variables

In mechanics velocity, momentum and acceleration are used to explain the state of any moving object (which you would have realized in Volume 1). In thermodynamics, the state of a thermodynamic system is represented by a set of variables called thermodynamic variables.

Examples: Pressure, temperature, volume and internal energy etc.

The values of these variables completely describe the equilibrium state of a thermodynamic system. Heat and work are not state variables rather they are process variables.

There are two types of thermodynamic variables: Extensive and Intensive

Extensive variable depends on the size or mass of the system.

Example: Volume, total mass, entropy, internal energy, heat capacity etc.

Intensive variables do not depend on the size or mass of the system.

Example: Temperature, pressure, specific heat capacity, density etc.

Equation of state:

The equation which connects the state variables in a specific manner is called equation of state. A thermodynamic equilibrium is completely specified by these state variables by the equation of state. If the system is not in thermodynamic equilibrium then these equations cannot specify the state of the system.

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An ideal gas obeys the equation PV = NkT at thermodynamic equilibrium. Since all four macroscopic variables (P,V,T and N) are connected by this equation, we cannot change one variable alone. For example, if we push the piston of a gas container, the volume of the gas will decrease but pressure will increase or if heat is supplied to the gas, its temperature will increase, pressure and volume of the gas may also increase.

There is another example of equation of state called van der Waals equation. Real gases obey this equation at thermodynamic equilibrium. The air molecules in the room truly obey van der Waals equation of state. But at room temperature with low density we can approximate it into an ideal gas.

8.5 ZEROTH LAW OF THERMODYNAMICS

The zeroth law of thermodynamics states that if two systems, *A* and *B*, are in thermal equilibrium with a third system, *C*, then *A* and *B* are in thermal equilibrium with each other.

Consider three systems A, B and C which are initially at different temperatures. Assume that A and B are not in thermal contact with each other as shown in Figure 8.18 (a) but each of them is in thermal contact with a third system C. After a lapse of time, system A will be in thermal equilibrium with C and B also will be in thermal equilibrium with C. In this condition, if the systems A and B are kept in thermal contact as shown in the Figure 8.18 (b), there is no flow of heat between the systems A and B. It implies that the system A and B are also in thermal equilibrium with each other. Once the three systems are at thermal equilibrium, there will be no heat flow between them as they are at the same temperature. This can be mathematically expressed as if, $T_A = T_C$ and $T_B = T_C$, it implies that $T_A = T_B$, where T_A, T_B and T_C are the temperatures of the systems A, B, and C respectively.

Temperature is the property which determines whether the system is in

Figure 8.18 (a) Two systems A and B in thermal contact with object C separately (b) If systems A and B are in thermal contact, they are also in thermal equilibrium with each other.

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thermal equilibrium with other systems or not. Zeroth law enables us to determine the temperature. For example, when a thermometer is kept in contact with a human body, it reaches thermal equilibrium with the body. At this condition, the temperature of the thermometer will be same as the human body. This principle is used in finding the body temperature.

ACTIVITY

We often associate the temperature as a measure of how hot or cold an object is while touching it. Can we use our sensory organs to determine the temperature of an object?

When you stand bare feet with one foot on the carpet and the other on the tiled floor, your foot on tiled floor feels cooler than the foot on the carpet even though both the tiled floor and carpet are at the same room temperature. It is because the tiled floor transfers the heat energy to your skin at higher rate than the carpet. So the skin is not measuring the actual temperature of the object; instead it measures the rate of heat energy transfer. But if we place a thermometer on the tiled floor or carpet it will show the same temperature.

8.6 INTERNAL ENERGY (U)

The internal energy of a thermodynamic system is the sum of kinetic and potential energies of all the molecules of the system with respect to the center of mass of the system.

The energy due to molecular motion including translational, rotational and vibrational motion is called internal kinetic energy (E_K)

The energy due to molecular interaction is called internal potential energy (E_p). Example: Bond energy.

$$U = E_K + E_P$$

- Since ideal gas molecules are assumed to have no interaction with each other the internal energy consists of only kinetic energy part (E_K) which depends on the temperature, number of particles and is independent of volume. However this is not true for real gases like Van der Waals gases.
- Internal energy is a state variable. It depends only on the initial and final states of the thermodynamic system. For example, if the temperature of water is raised from 30°C to 40°C either by heating or by stirring, the final internal energy depends only on the final temperature 40°C and not the way it is arrived at.

Note It is very important to note that the internal energy of a thermodynamic system is associated with only the kinetic energy of the individual molecule due to its random motion and the potential energy of molecules which depends on their chemical nature. The bulk kinetic energy of the entire system or gravitational potential energy of the system should not be mistaken as a part of internal energy. For example

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Note

(a) Consider two gas containers at the same temperature having the same internal energy, one is kept at rest on the ground and the other is kept in a moving train. Even though the gas container in the train is moving with the speed of the train, the internal energy of the gas in it will not increase.

(b) Consider two gas containers at the same temperature having the same internal energy, one is kept on the ground and the other is kept at some height h. Even though the container at height h is having higher gravitational potential energy, this has no influence on internal energy of the gas molecules.

EXAMPLE 8.10

When you mix a tumbler of hot water with one bucket of normal water, what will be the direction of heat flow? Justify.

The water in the tumbler is at a higher temperature than the bucket of normal water. But the bucket of normal water has larger internal energy than the hot water in the tumbler. This is because the internal energy is an extensive variable and it depends on the size or mass of the system.

Even though the bucket of normal water has larger internal energy than the tumbler of hot water, heat will flow from water in the tumbler to the water in the bucket. This is because heat flows from a body at higher temperature to the one at lower temperature and is independent of internal energy of the system.

Once the heat is transferred to an object it becomes internal energy of the object. The right way to say is 'object has certain amount of internal energy'. Heat is one of the ways to increase the internal energy of a system as shown in the Figure.

Note

It is to be noted that heat does not always increases the internal energy. Later we shall see that in ideal gases during isothermal process the internal energy will not increase even though heat flows in to the system.

8.6.1 Joule's mechanical equivalent of heat

The temperature of an object can be increased by heating it or by doing some work on it. In the eighteenth century, James Prescott Joule showed that mechanical energy can be converted into internal energy and vice versa. In his experiment, two masses were attached with a rope and a paddle wheel as shown in Figure 8.19. When these masses fall through a distance h due to gravity, both the masses lose potential energy equal to 2mgh. When the masses fall, the paddle wheel turns. Due to the turning of wheel inside water, frictional force comes in between the water and the paddle wheel. This causes

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Figure.8.19 Joule's experiment for determining the mechanical equivalent of heat energy.

a rise in temperature of the water. This implies that gravitational potential energy is converted to internal energy of water. The temperature of water increases due to the work done by the masses. In fact, Joule was able to show that the mechanical work has the same effect as giving heat. He found that to raise 1 g of an object by 1°C , 4.186 J of energy is required. In earlier days the heat was measured in calorie.

$$1 \text{ cal} = 4.186 \text{ J}$$

This is called Joule's mechanical equivalent of heat.

Note Before James Prescott Joule, people thought that heat was a kind of fluid called caloric fluid which flows from an object at high temperature to that in low temperature. According to caloric fluid idea, the hot object contains more caloric fluid and the cold object contains less caloric fluid since heat was treated as a quantity. Now we understand that heat is not a quantity but it is an energy in transit. So the word 'mechanical equivalent of heat' is wrong terminology. Because mechanical energy is a quantity and any object can contain more or less mechanical energy, but this is not so with heat as it is not a quantity. However this terminology is retained for historical reasons. A correct name would be 'Joule's mechanical equivalence of internal energy'. Joule Essentially converted mechanical energy to internal energy. In his experiment potential energy is converted to rotational kinetic energy of paddle wheel and this rotational kinetic energy is converted to internal energy of water.

EXAMPLE 8.11

A student had a breakfast of 200 food calories. He thinks of burning this energy by drawing water from the well and watering the trees in his school. Depth of the well is about 25 m. The pot can hold 25L of water and each tree requires one pot of water. How many trees can he water? (Neglect the mass of the pot and the energy spent by walking. Take g =10 m s^{-2})

Solution

To draw 25 L of water from the well, the student has to do work against gravity by burning his energy.

Mass of the water = 25 L = 25 kg (1L=1kg)

The work required to draw 25 kg of water = gravitational potential energy gained by water.

$W = mgh = 25\times10\times25 = 6250\ J$

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The total energy gained from the food = 200 food cal =200 kcal.

$= 200\times10^3 \times 4.186\ J = 8.37 \times10^5\ J$

If we assume that by using this energy the student can drawn 'n' pots of water from the well, the total energy spent by him = $8.37 \times 10^5\ J = nmgh$

$$n = \frac{8.37\times10^5 J}{6250 J} \approx 134.$$

This n is also equal to the number of trees that he can water.

Is it possible to draw 134 pots of water from the well just by having breakfast? No. Actually the human body does not convert entire food energy into work. It is only approximately 20% efficient. It implies that only 20% of 200 food calories is used to draw water from the well. So 20 % of the 134 is only 26 pots of water. It is quite meaningful. So he can water only 26 trees.

The remaining energy is used for blood circulation and other functions of the body. It is to be noted that some energy is always 'wasted'. Why is it that the body cannot have 100% efficiency? You will find the answer in section 8.9

8.6.2 First law of thermodynamics

The first law of thermodynamics is a statement of the law of conservation of energy. In Newtonian mechanics conservation of energy involves kinetic and potential energies of bulk objects. But the first law of thermodynamics includes heat also. This law states that '*Change in internal energy (ΔU) of the system is equal to heat supplied to the system (Q) minus the work done by the system (W) on the surroundings*'. Mathematically it is written as

$$\Delta U = Q - W \qquad (8.13)$$

The internal energy of a thermodynamic system can be changed either by heating or by work as shown below.

Heat flows into the system	Internal energy increases
Heat flows out of the system	Internal energy decreases
Work is done on the system	Internal energy increases
Work is done by the system	Internal energy decreases

Based on the above table the sign convention is introduced to use first law of thermodynamics appropriately. It is shown in the following table and the Figure 8.20.

System gains heat	Q is positive
System loses heat	Q is negative
Work done on the system	W is negative
Work done by the system	W is positive

Figure 8.20 The Sign convention for heat and work

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Even though we often explain first law of thermodynamics using gases, this law is universal and applies to liquids and solids also.

Note Some book presents the first law of thermodynamics as $\Delta U = Q + W$. Here the work done by the system is taken as negative and work done on the system is positive. However both the conventions are correct and we can follow any one of the convention.

EXAMPLE 8.12

A person does 30 kJ work on 2 kg of water by stirring using a paddle wheel. While stirring, around 5 kcal of heat is released from water through its container to the surface and surroundings by thermal conduction and radiation. What is the change in internal energy of the system?

Solution

Work done on the system (by the person while stirring), W = -30 kJ = −30,000J

Heat flowing out of the system, $Q = -5 \text{ kcal} = -5 \times 4184\ J = -20920\ J$

Using First law of thermodynamics

$$\Delta U = Q - W$$

$$\Delta U = -20{,}920\ J - (-30{,}000)\ J$$

$$\Delta U = -20{,}920\ J + 30{,}000\ J = 9080\ J$$

Here, the heat lost is less than the work done on the system, so the change in internal energy is positive.

EXAMPLE 8.13

Jogging every day is good for health. Assume that when you jog a work of 500 kJ is done and 230 kJ of heat is given off. What is the change in internal energy of your body?

Solution

Work done by the system (body),

$$W = +500 \text{ kJ}$$

Heat released from the system (body),

$$Q = -230 \text{ kJ}$$

The change in internal energy of a body

$= \Delta U = -230\ kJ - 500\ kJ = -730\ kJ$

8.6.3 Quasi-static process

Consider a system of an ideal gas kept in a cylinder of volume V at pressure P and temperature T. When the piston attached to the cylinder moves outward the volume of the gas will change. As a result the temperature and pressure will also change because all three variables P,T and V are related by the equation of state PV = NkT. If a block of some mass is kept on the piston, it will suddenly push the piston downward. The pressure near the piston will be larger than other parts of the system. It implies that the gas is in non-equilibrium state. We cannot determine pressure, temperature or internal energy of the system until it reaches another equilibrium state. But if the piston is pushed very slowly such that at every stage it is still in equilibrium with surroundings, we can use the equation of state to calculate the internal energy, pressure or temperature. This kind of process is called quasi-static process.

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A quasi-static process is an infinitely slow process in which the system changes its variables (P,V,T) so slowly such that it remains in thermal, mechanical and chemical equilibrium with its surroundings throughout. By this infinite slow variation, the system is always almost close to equilibrium state.

EXAMPLE 8.14

Give an example of a quasi-static process.

Consider a container of gas with volume V, pressure P and temperature T. If we add sand particles one by one slowly on the top of the piston, the piston will move inward very slowly. This can be taken as almost a quasi-static process. It is shown in the figure

Sand particles added slowly- quasi-static process

8.6.4 Work done in volume changes

Consider a gas contained in the cylinder fitted with a movable piston. Suppose the gas is expanded quasi-statically by pushing the piston by a small distance dx as shown in Figure 8.21. Since the expansion occurs quasi-statically the pressure, temperature and internal energy will have unique values at every instant.

The small work done by the gas on the piston

$$dW = Fdx \qquad (8.14)$$

The force exerted by the gas on the piston F = PA. Here A is area of the piston and P is pressure exerted by the gas on the piston.

Figure 8.21 Work done by the gas

Equation (8.14) can be rewritten as

$$dW = PA\,dx \qquad (8.15)$$

But Adx = dV= change in volume during this expansion process.

So the small work done by the gas during the expansion is given by

$$dW = PdV \qquad (8.16)$$

Note here that dV is positive since the volume is increased. So dW is positive.

In general the work done by the gas by increasing the volume from V_i to V_f is given by

$$W = \int_{V_i}^{V_f} PdV \qquad (8.17)$$

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Suppose if the work is done on the system, then $V_i > V_f$. Then, W is negative.

Note here the pressure P is inside the integral in equation (8.17). It implies that while the system is doing work, the pressure need not be constant. To evaluate the integration we need to first express the pressure as a function of volume and temperature using the equation of state.

8.6.5 PV diagram

PV diagram is a graph between pressure P and volume V of the system. The P-V diagram is used to calculate the amount of work done by the gas during expansion or on the gas during compression. In Unit 2, we have seen that the area under the curve will give integration of the function from lower limit to upper limit. The area under the PV diagram will give the work done during expansion or compression which is shown in Figure 8.22

Figure 8.22 Work done by the gas during expansion

The shape of PV diagram depends on the nature of the thermodynamic process.

EXAMPLE 8.15

A gas expands from volume $1m^3$ to $2m^3$ at constant atmospheric pressure.

(a) Calculate the work done by the gas.

(b) Represent the work done in PV diagram.

Solution

(a) The pressure P = 1 atm = 101 kPa, $V_f = 2\ m^3$ and $V_i = 1m^3$

From equation (8.17)

$$W = \int_{V_i}^{V_f} PdV = P\int_{V_i}^{V_f} dV$$

Since P is constant. It is taken out of the integral.

$$W = P(V_f - V_i) = 101\times10^3 \times (2-1) = 101\ kJ$$

(b) Since the pressure is kept constant, PV diagram is straight line as shown in the figure. The area is equal to work done by the gas.

Note the arrow mark in the curve. Suppose the work is done on the system, then volume will decreases and the arrow will point in the opposite direction.

8.7 SPECIFIC HEAT CAPACITY OF A GAS

Specific heat capacity of a given system plays a very important role in determining

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the structure and molecular nature of the system. Unlike solids and liquids, gases have two specific heats: specific heat capacity at constant pressure (s_p) and specific heat capacity at constant volume (s_v).

8.7.1 Specific heat capacity

Specific heat capacity at constant pressure (s_p):

The amount of heat energy required to raise the temperature of one kg of a substance by 1 K or 1°C by keeping the pressure constant is called specific heat capacity of at constant pressure. When the heat energy is supplied to the gas, it expands to keep the pressure constant as shown in Figure 8.23

Figure 8.23 Specific heat capacity at constant pressure

In this process a part of the heat energy is used for doing work (expansion) and the remaining part is used to increase the internal energy of the gas.

Specific heat capacity at constant volume (s_v):

The amount of heat energy required to raise the temperature of one kg of a substance by 1 K or 1°C by keeping the volume constant is called specific heat capacity at constant volume.

If the volume is kept constant, then the supplied heat is used to increase only the internal energy. No work is done by the gas as shown in Figure 8.24.

Figure 8.24 Specific heat capacity at constant volume

It implies that to increase the temperature of the gas at constant volume requires less heat than increasing the temperature of the gas at constant pressure. In other words s_p is always greater than s_v.

Molar Specific heat capacities

Sometimes it is useful to calculate the molar heat capacities C_p and C_v. The amount of heat required to raise the temperature of one mole of a substance by 1K or 1°C at constant volume is called molar specific heat capacity at constant volume (C_v). If pressure is kept constant, it is called molar specific heat capacity at constant pressure (C_p).

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If Q is the heat supplied to mole of a gas at constant volume and if the temperature changes by an amount ΔT, we have

$$Q = \mu C_v \Delta T. \quad (8.18)$$

By applying the first law of thermodynamics for this constant volume process (W=0, since dV=0), we have

$$Q = \Delta U - 0 \quad (8.19)$$

By comparing the equations (8.18) and (8.19),

$$\Delta U = \mu C_v \Delta T \text{ or } C_v = \frac{1}{\mu}\frac{\Delta U}{\Delta T}$$

If the limit ΔT goes to zero, we can write

$$C_v = \frac{1}{\mu}\frac{dU}{dT} \quad (8.20)$$

Since the temperature and internal energy are state variables, the above relation holds true for any process.

8.7.2 Meyer's relation

Consider μ mole of an ideal gas in a container with volume V, pressure P and temperature T.

When the gas is heated at constant volume the temperature increases by dT. As no work is done by the gas, the heat that flows into the system will increase only the internal energy. Let the change in internal energy be dU.

If C_v is the molar specific heat capacity at constant volume, from equation (8.20)

$$dU = \mu C_v dT \quad (8.21)$$

Suppose the gas is heated at constant pressure so that the temperature increases by dT. If 'Q' is the heat supplied in this process and 'dV' the change in volume of the gas.

$$Q = \mu C_p dT \quad (8.22)$$

If W is the workdone by the gas in this process, then

$$W = PdV \quad (8.23)$$

But from the first law of thermodynamics,

$$Q = dU + W \quad (8.24)$$

Substituting equations (8.21), (8.22) and (8.23) in (8.24), we get,

$$\mu C_p dT = \mu C_v dT + PdV \quad (8.25)$$

For mole of ideal gas, the equation of state is given by

$$PV = \mu RT \Rightarrow PdV + VdP = \mu RdT \quad (8.26)$$

Since the pressure is constant, dP=0

$$\therefore C_p dT = C_v dT + RdT$$

$$\therefore C_p = C_v + R \quad \text{(or)} \quad C_p - C_v = R \quad (8.27)$$

This relation is called Meyer's relation

It implies that the molar specific heat capacity of an ideal gas at constant pressure is greater than molar specific heat capacity at constant volume.

The relation shows that specific heat at constant pressure (s_p) is always greater that specific heat at constant volume (s_v).

8.8 THERMODYNAMIC PROCESSES

8.8.1 Isothermal process

It is a process in which the temperature remains constant but the pressure and volume of a thermodynamic system will change. The ideal gas equation is

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$$PV = \mu RT$$

Here, T is constant for this process

So the equation of state for isothermal process is given by

$$PV = constant \qquad (8.28)$$

This implies that if the gas goes from one equilibrium state (P_1, V_1) to another equilibrium state (P_2, V_2) the following relation holds for this process

$$P_1 V_1 = P_2 V_2 \qquad (8.29)$$

Since PV = constant, P is inversely proportional to V $(P \propto \frac{1}{V})$. This implies that PV graph is a hyperbola. The pressure-volume graph for constant temperature is also called isotherm.

Figure 8.25 shows the PV diagram for quasi-static isothermal expansion and quasi-static isothermal compression.

We know that for an ideal gas the internal energy is a function of temperature only. For an isothermal process since temperature is constant, the internal energy is also constant. This implies that dU or $\Delta U = 0$.

For an isothermal process, the first law of thermodynamics can be written as follows,

$$Q = W \qquad (8.30)$$

From equation (8.30), we infer that the heat supplied to a gas is used to do only external work. It is a common misconception that when there is flow of heat to the system, the temperature will increase. For isothermal process this is not true. The isothermal compression takes place when the piston of the cylinder is pushed. This will increase the internal energy which will flow out of the system through thermal contact. This is shown in Figure 8.26.

Figure 8.26 Isothermal expansion and isothermal compression

Figure 8.25 (a) Quasi-static isothermal expansion (b) Quasi-static isothermal compression

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Examples:

(i) When water is heated, at the boiling point, even when heat flows to water, the temperature will not increase unless the water completely evaporates. Similarly, at the freezing point, when the ice melts to water, the temperature of ice will not increase even when heat is supplied to ice.

(ii) All biological processes occur at constant body temperature (37°C).

Work done in an isothermal process:

Consider an ideal gas which is allowed to expand quasi-statically at constant temperature from initial state (P_i, V_i) to the final state (P_f, V_f). We can calculate the work done by the gas during this process. From equation (8.17) the work done by the gas,

$$W = \int_{V_i}^{V_f} P dV \qquad (8.31)$$

As the process occurs quasi-statically, at every stage the gas is at equilibrium with the surroundings. Since it is in equilibrium at every stage the ideal gas law is valid. Writing pressure in terms of volume and temperature,

$$P = \frac{\mu RT}{V} \qquad (8.32)$$

Substituting equation (8.32) in (8.31) we get

$$W = \int_{V_i}^{V_f} \frac{\mu RT}{V} \, dV$$

$$W = \mu RT \int_{V_i}^{V_f} \frac{dV}{V} \qquad (8.33)$$

In equation (8.33), we take μRT out of the integral, since it is constant throughout the isothermal process.

By performing the integration in equation (8.33), we get

$$W = \mu RT \, ln \left(\frac{V_f}{V_i}\right) \qquad (8.34)$$

Since we have an isothermal expansion, $\frac{V_f}{V_i} > 1$, so $ln\left(\frac{V_f}{V_i}\right) > 0$. As a result the work done by the gas during an isothermal expansion is positive.

The above result in equation (8.34) is true for isothermal compression also. But in an isothermalcompression $\frac{V_f}{V_i} < 1$.So $ln\left(\frac{V_f}{V_i}\right) < 0$.

As a result the work done on the gas in an isothermal compression is negative.

In the PV diagram the work done during the isothermal expansion is equal to the area under the graph as shown in Figure 8.27

Figure 8.27 Work done in an isothermal process.

Similarly for an isothermal compression, the area under the PV graph is equal to the

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work done on the gas which turns out to be the area with a negative sign.

Note

To calculate the work done in an isothermal process, we assume that the process is quasi-static. If it is not quasi-static we cannot substitute $P = \frac{\mu RT}{V}$ in equation (8.31). It is because the ideal gas law is not valid for non equilibrium states. But equation (8.34) is valid even when the isothermal process is not quasi-static. This is because the state variables like pressure and volume depend on initial and final state alone of an ideal gas and do not depend on the way the final state is reached. The assumption of 'quasi-static' requires to do the integration.

EXAMPLE 8.16

A 0.5 mole of gas at temperature 300 K expands isothermally from an initial volume of 2 L to 6 L

(a) What is the work done by the gas?

(b) Estimate the heat added to the gas?

(c) What is the final pressure of the gas?

(The value of gas constant, R = 8.31 J mol^{-1} K^{-1})

Solution

(a) We know that work done by the gas in an isothermal expansion

Since μ = 0.5

$$W = 0.5mol \times \frac{8.31J}{mol.K} \times 300\ K \ln\left(\frac{6L}{2L}\right)$$

$$W = 1.369\ kJ$$

Note that W is positive since the work is done by the gas.

(b) From the First law of thermodynamics, in an isothermal process the heat supplied is spent to do work.

Therefore, $Q = W = 1.369\ kJ$. Thus Q is also positive which implies that heat flows in to the system.

(c) For an isothermal process

$$P_i V_i = P_f V_f = \mu RT$$

$$P_f = \frac{\mu RT}{V_f} = 0.5\text{mol} \times \frac{8.31J}{mol.\text{K}} \times \frac{300K}{6\times10^{-3}m^3}$$

$$=207.75\ k\,Pa$$

EXAMPLE 8.17

The following PV curve shows two isothermal processes for two different temperatures and. Identify the higher temperature of these two.

Solution

To determine the curve corresponding to higher temperature, draw a horizontal line parallel to x axis as shown in the figure. This is the constant pressure line. The volumes V_1 and V_2 belong to same pressure as the vertical lines from V_1 and V_2 meet the constant pressure line.

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At constant pressure, higher the volume of the gas, higher will be the temperature. From the figure, as $V_1 > V_2$ we conclude $T_1 > T_2$. In general the isothermal curve closer to the origin, has lower temperature.

8.8.2 Adiabatic process

This is a process in which no heat flows into or out of the system (Q=0). But the gas can expand by spending its internal energy or gas can be compressed through some external work. So the pressure, volume and temperature of the system may change in an adiabatic process.

For an adiabatic process, the first law becomes $\Delta U = -W$.

This implies that the work is done by the gas at the expense of internal energy or work is done on the system which increases its internal energy.

The adiabatic process can be achieved by the following methods

(i) Thermally insulating the system from surroundings so that no heat flows into or out of the system; for example, when thermally insulated cylinder of gas is compressed (adiabatic compression) or expanded (adiabatic expansion) as shown in the Figure 8.28

(ii) If the process occurs so quickly that there is no time to exchange heat with surroundings even though there is no thermal insulation. A few examples are shown in Figure 8.29.

Examples:

Figure 8.29 (a) When the tyre bursts the air expands so quickly that there is no time to exchange heat with the surroundings.

Figure 8.29 (b): When the gas is compressed or expanded so fast, the gas cannot exchange heat with surrounding even though there is no thermal insulation.

Figure 8.28 Adiabatic compression and expansion

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Figure 8.29 (c): When the warm air rises from the surface of the Earth, it adiabatically expands. As a result the water vapor cools and condenses into water droplets forming a cloud.

The equation of state for an adiabatic process is given by

$$PV^{\gamma} = constant \qquad (8.35)$$

Here γ is called adiabatic exponent ($\gamma = C_p/C_v$) which depends on the nature of the gas.

The equation (8.35) implies that if the gas goes from an equilibrium state (P_i, V_i) to another equilibrium state (P_f, V_f) adiabatically then it satisfies the relation

$$P_i V_i^{\gamma} = P_f V_f^{\gamma} \qquad (8.36)$$

The PV diagram of an adiabatic expansion and adiabatic compression process are shown in Figure 8.30. The PV diagram for an adiabatic process is also called *adiabat*

Note that the PV diagram for isothermal (Figure 8.25) and adiabatic (Figure 8.30) processes look similar. But actually the adiabatic curve is steeper than isothermal curve.

We can also rewrite the equation (8.35) in terms of T and V. From ideal gas equation, the pressure $P = \frac{\mu RT}{V}$. Substituting this equation in the equation (8.35), we have

$$\frac{\mu RT}{V} V^{\gamma} = \text{constant (or)} \ \frac{T}{V} V^{\gamma} = \frac{constant}{\mu R}$$

Note here that is another constant. So it can be written as

$$TV^{\gamma-1} = constant. \qquad (8.37)$$

The equation (8.37) implies that if the gas goes from an initial equilibrium state (T_i, V_i) to final equilibrium state (T_f, V_f) adiabatically then it satisfies the relation

$$T_i V_i^{\gamma-1} = T_f V_f^{\gamma-1} \qquad (8.38)$$

The equation of state for adiabatic process can also be written in terms of T and P as

$$T^{\gamma} P^{1-\gamma} = constant. \qquad (8.39)$$

Figure 8.30 PV diagram for adiabatic expansion and adiabatic compression

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(The proof of equation (8.39) left as an exercise).

EXAMPLE 8.18

We often have the experience of pumping air into bicycle tyre using hand pump. Consider the air inside the pump as a thermodynamic system having volume V at atmospheric pressure and room temperature, 27°C. Assume that the nozzle of the tyre is blocked and you push the pump to a volume 1/4 of V. Calculate the final temperature of air in the pump? (For air , since the nozzle is blocked air will not flow into tyre and it can be treated as an adiabatic compression). Take γ for air = 1.4

Solution

Here, the process is adiabatic compression. The volume is given and temperature is to be found. we can use the equation (8.38)

$$T_i V_i^{\gamma-1} = T_f V_f^{\gamma-1}.$$

$$T_i = 300\ K \quad (273+27°C = 300\ K)$$

$$V_i = V\ \&\ V_f = \frac{V}{4}$$

$$T_f = T_i\left(\frac{V_i}{V_f}\right)^{\gamma-1} = 300\ K \times 4^{1.4-1} = 300K \times 1.741$$

T2 ≈ 522 K or 249^{0}C

This temperature is higher than the boiling point of water. So it is very dangerous to touch the nozzle of blocked pump when you pump air.

DO YOU KNOW?

When the piston is compressed so quickly that there is no time to exchange heat to the surrounding, the temperature of the gas increases rapidly. This is shown in the figure. This principle is used in the diesel engine. The air-gasoline mixer is compressed so quickly (adiabatic compression) that the temperature increases enormously, which is enough to produce a spark.

Work done in an adiabatic process:

Consider μ moles of an ideal gas enclosed in a cylinder having perfectly non conducting walls and base. A frictionless and insulating piston of cross sectional area A is fitted in the cylinder as shown in Figure 8.31.

Figure 8.31 Work done in an adiabatic process

Let W be the work done when the system goes from the initial state (P_i, V_i, T_i) to the final state (P_f, V_f, T_f) adiabatically.

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$$W = \int_{V_i}^{V_f} PdV \qquad (8.40)$$

By assuming that the adiabatic process occurs quasi-statically, at every stage the ideal gas law is valid. Under this condition, the adiabatic equation of state is PV^{γ} = constant (or)

P = $\frac{\text{constant}}{V^{\gamma}}$ can be substituted in the equation (8.40), we get

$$\therefore W_{adia} = \int_{v_i}^{V_f} \frac{\text{constant}}{V^{\gamma}} dV$$

$$= \text{constant} \int_{V_i}^{Vf} V^{-\gamma} dV$$

$$= \text{constant} \left[\frac{V^{-\gamma+1}}{-\gamma+1} \right]_{V_i}^{V_f}$$

$$= \frac{\text{constant}}{1-\gamma} \left[\frac{1}{V_f^{\gamma-1}} - \frac{1}{V_i^{\gamma-1}} \right]$$

$$= \frac{1}{1-\gamma} \left[\frac{\text{constant}}{V_f^{\gamma-1}} - \frac{\text{constant}}{V_i^{\gamma-1}} \right]$$

But, $P_i V_i^{\gamma} = P_f V_f^{\gamma} = constant.$

$$\therefore W_{adia} = \frac{1}{1-\gamma} \left[\frac{P_f V_f^{\gamma}}{V_f^{\gamma-1}} - \frac{P_i V_i^{\gamma}}{V_i^{\gamma-1}} \right]$$

$$W_{adia} = \frac{1}{1-\gamma} [P_f V_f - P_i V_i] \qquad (8.41)$$

From ideal gas law,

$$P_f V_f = \mu R T_f \text{ and } P_i V_i = \mu R T_i$$

Substituting in equation (8.41), we get

$$\therefore W_{adia} = \frac{\mu R}{\gamma - 1} [T_i - T_f] \qquad (8.42)$$

In adiabatic expansion, work is done by the gas. i.e., W_{adia} is positive. As $T_i > T_f$, the gas cools during adiabatic expansion.

In adiabatic compression, work is done on the gas. i.e., W_{adia} is negative. As $T_i < T_f$, the temperature of the gas increases during adiabatic compression.

Note

Even though we have derived equations (8.41) and (8.42) by assuming that the adiabatic process is quasi-static, both the equations are valid even if the process is not quasi-static. This is because P and V are state variables and are independent of how the state is arrived.

In the adiabatic PV diagram shown in the Figure 8.32, the area under the adiabatic curve from initial state to final state will give the total work done in adiabatic process.

Figure 8.32 PV diagram -Work done in the adiabatic process

To differentiate between isothermal and adiabatic curves in (Figure 8.32) the adiabatic curve is drawn along with isothermal curve for T_f and T_i. Note that adiabatic curve is steeper than isothermal curve. This is because $\gamma > 1$ always.

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8.8.3 Isobaric process

This is a thermodynamic process that occurs at constant pressure. Even though pressure is constant in this process, temperature, volume and internal energy are not constant. From the ideal gas equation, we have

$$V = \left(\frac{\mu R}{P}\right) T \qquad (8.43)$$

Here $\frac{\mu R}{P} = constant$

In an isobaric process the temperature is directly proportional to volume.

$$V \propto T \quad \text{(Isobaric process)} \qquad (8.44)$$

This implies that for a isobaric process, the V-T graph is a straight line passing through the origin.

If a gas goes from a state (V_i, T_i) to (V_f, T_f) at constant pressure, then the system satisfies the following equation

$$\frac{T_f}{V_f} = \frac{T_i}{V_i} \qquad (8.45)$$

Examples for Isobaric process:

(i) When the gas is heated and pushes the piston so that it exerts a force equivalent to atmospheric pressure plus the force due to gravity then this process is isobaric. This is shown in Figure 8.33

Figure 8.33 Isobaric process

(ii) Most of the cooking processes in our kitchen are isobaric processes. When the food is cooked in an open vessel, the pressure above the food is always at atmospheric pressure. This is shown in Figure 8.34

Figure 8.34 Isobaric process

The PV diagram for an isobaric process is a horizontal line parallel to volume axis as shown in Figure 8.35.

Figure 8.35 (a) represents isobaric process where volume decreases

Figure 8.35 (b) represents isobaric process where volume increases

Figure 8.35 PV diagram for an isobaric process

The work done in an isobaric process:

Work done by the gas

$$W = \int_{V_i}^{V_f} P dV \qquad (8.46)$$

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In an isobaric process, the pressure is constant, so P comes out of the integral,

$$W = P\int_{V_i}^{V_f} dV \qquad (8.47)$$

$$W = P[V_f - V_i] = P\Delta V \qquad (8.48)$$

Where ΔV denotes change in the volume. If ΔV is negative, W is also negative. This implies that the work is done on the gas. If ΔV is positive, W is also positive, implying that work is done by the gas.

The equation (8.48) can also be rewritten using the ideal gas equation.

From ideal gas equation

$$PV = \mu RT \text{ and } V = \frac{\mu RT}{P}$$

Substituting this in equation (8.48) we get

$$W = \mu RT_f\left(1 - \frac{T_i}{T_f}\right) \qquad (8.49)$$

In the PV diagram, area under the isobaric curve is equal to the work done in isobaric process. The shaded area in the following Figure8.36 is equal to the work done by the gas.

Figure 8.36 Work done in an isobaric process

The first law of thermodynamics for isobaric process is given by

$$\Delta U = Q - P\Delta V \qquad (8.50)$$

EXAMPLE 8.19

The following graph shows a V-T graph for isobaric processes at two different pressures. Identify which one occurs at higher pressure.

Solution

From the ideal gas equation, $V = \left(\frac{\mu R}{P}\right)T$

V-T graph is a straight line passing the origin.

The slope = $\frac{\mu R}{P}$

The slope of V-T graph is inversely proportional to the pressure. If the slope is greater, lower is the pressure.

Here P_1 has larger slope than P_2. So $P_2 > P_1$.

Suppose the graph is drawn between T and V (Temperature along the x-axis and Volume along the y-axis) then will we still have $P_2 > P_1$?

EXAMPLE 8.20

One mole of an ideal gas initially kept in a cylinder at pressure 1 MPa and temperature 27°C is made to expand until its volume is doubled.

(a) How much work is done if the expansion is (i) adiabatic (ii) isobaric (iii) isothermal?

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(b) Identify the processes in which work done is least and is maximum.

(c) Show each process on a PV diagram.

(d) Name the processes in which the heat transfer is maximum and minimum.

(Take $\gamma = \frac{5}{3}$ and R=8.3 J mol^{-1} K^{-1})

Solution

(a) (i) In an adiabatic process the work done by the system is

$$W_{adia} = \frac{\mu R}{\gamma - 1}[T_i - T_f]$$

To find the final temperature T_f, we can use adiabatic equation of state.

$$T_f V_f^{\gamma-1} = T_i V_i^{\gamma-1}$$

$$T_f = T_i\left(\frac{V_i}{V_f}\right)^{\gamma-1} = 300\times\left(\frac{1}{2}\right)^{\frac{2}{3}}$$

$$= 0.63 \times 300K = 189.8\text{K}$$

$$W = 1 \times 8.3 \times \frac{3}{2}(300 - 189.8) = 1.37\text{kJ}$$

(ii) In an isobaric process the work done by the system

$$W = P\Delta V = P(V_f - V_i)$$

and $V_f = 2V_i$ so $W = 2PV_i$

To find V_i, we can use the ideal gas law for initial state. $P_i V_i = RT_i$

$$V_i = \frac{RT_i}{P_i} = 8.3 \times \frac{300}{1} \times 10^{-6} = 24.9\times10^{-4}\text{m}^3$$

The work done during isobaric process, $W = 2 \times 10^6 \times 24.9 \times 10^{-4} = 4.9\ kJ$

(iii) In an isothermal process the work done by the system,

$$W = \mu RT\ ln\left(\frac{V_f}{V_i}\right)$$

In an isothermal process the initial room temperature is constant.

$$W = 1 \times 8.3 \times 300 \times ln(2) = 1.7kJ$$

(b) Comparing all three processes, we see that the work done in the isobaric process is the greatest, and work done in the adiabatic process is the least.

(c) The PV diagram is shown in the Figure.

The area under the curve AB = Work done during the isobaric process

The area under the curve AC = Work done during the isothermal process

The area under the curve AD= Work done during the adiabatic process

From the PV diagram the area under the curve AB is more, implying that the work done in isobaric process is highest and work done in adiabatic process is least.

(d) In an adiabatic process no heat enters into the system or leaves from the system. In an isobaric process the work done is more so heat supplied should be more compared to an isothermal process.

8.8.4 Isochoric process

This is a thermodynamic process in which the volume of the system is kept constant. But pressure, temperature and internal energy continue to be variables.

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The pressure - volume graph for an isochoric process is a vertical line parallel to pressure axis as shown in Figure 8.37.

Figure 8.37 Isochoric process with (a) increased pressure and (b) decreased pressure

The equation of state for an isochoric process is given by

$$P = \left(\frac{\mu R}{V}\right) T \qquad (8.51)$$

where $\left(\frac{\mu R}{V}\right)$ = constant

We can infer that the pressure is directly proportional to temperature. This implies that the P-T graph for an isochoric process is a straight line passing through origin.

If a gas goes from state (P_i, T_i) to (P_f, T_f) at constant volume, then the system satisfies the following equation

$$\frac{P_i}{T_i} = \frac{P_f}{T_f} \qquad (8.52)$$

For an isochoric processes, ΔV=0 and W=0. Then the first law becomes

$$\Delta U = Q \qquad (8.53)$$

Implying that the heat supplied is used to increase only the internal energy. As a result the temperature increases and pressure also increases. This is shown in Figure 8.38

Figure 8.38 Isochoric process

Suppose a system loses heat to the surroundings through conducting walls by keeping the volume constant, then its internal energy decreases. As a result the temperature decreases; the pressure also decreases.

Examples:

1. When food is cooked by closing with a lid as shown in figure.

Table 8.4: Summary of various thermodynamic processes

S. No.	Process		Heat Q	Temperature & internal energy	Pressure
1	Isothermal	Expansion	Q > 0	Constant	decreases
		Compression	Q < 0	Constant	Increases
2	Isobaric	Expansion	Q > 0	increases	Constant
		Compression	Q < 0	decreases	Constant
3	Isochoric		Q > 0	Increases	increases
			Q < 0	Decreases	decreases
4	Adiabatic	Expansion	Q = 0	Decreases	Decreases
		Compression	Q = 0	Increases	increases

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	Volume	Equation of state	Work done (ideal gas)	Indicator diagram (PV diagram)
	increases	PV = Constant	$W=\mu RT \ln\left(\frac{V_f}{V_i}\right) > 0$	
	Decreases		$W=\mu RT \ln\left(\frac{V_f}{V_i}\right) < 0$	
	increases	$\frac{V}{T}$ = Constant	$W=P\,[V_f-V_i]=P\Delta V > 0$	
	decreases		$W=P\,[V_f-V_i]=P\Delta V < 0$	
	Constant	$\frac{P}{T}$ = Constant	Zero	
	increases	PV^{γ} = Constant	$W=\frac{\mu R}{\gamma-1}(T_i-T_f) > 0$	
	Decreases		$W=\frac{\mu R}{\gamma-1}(T_i-T_f) < 0$	

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When food is being cooked in this closed position, after a certain time you can observe the lid is being pushed upwards by the water steam. This is because when the lid is closed, the volume is kept constant. As the heat continuously supplied, the pressure increases and water steam tries to push the lid upwards.

2. In automobiles the petrol engine undergoes four processes. First the piston is adiabatically compressed to some volume as shown in the Figure (a). In the second process (Figure (b)), the volume of the air-fuel mixture is kept constant and heat is being added. As a result the temperature and pressure are increased. This is an isochoric process. For a third stroke (Figure (c)) there will be an adiabatic expansion, and fourth stroke again isochoric process by keeping the piston immoveable (Figure (d)).

The summary of various thermodynamic processes is give in the Table 8.4.

EXAMPLE 8.21

500 g of water is heated from 30°C to 60°C. Ignoring the slight expansion of water, calculate the change in internal energy of the water? (specific heat of water 4184 J/kg.K)

Solution

When the water is heated from 30°C to 60°C,there is only a slight change in its volume. So we can treat this process as isochoric. In an isochoric process the work done by the system is zero. The given heat supplied is used to increase only the internal energy.

$$\Delta U = Q = ms_v \Delta T$$

The mass of water = 500 g =0.5 kg

The change in temperature = 30K

The heat Q = 0.5×4184×30 = 62.76 *kJ*

8.8.5 Cyclic processes

This is a thermodynamic process in which the thermodynamic system returns to its initial state after undergoing a series of changes. Since the system comes back to the initial state, the change in the internal energy is zero. In cyclic process, heat can flow in to system and heat flow out of the system. From the first law of thermodynamics, the net heat transferred to the system is equal to work done by the gas.

$$Q_{net} = Q_{in} - Q_{out} = W \quad \text{(for a cyclic process)} \tag{8.54}$$

8.8.6 PV diagram for a cyclic process

In the PV diagram the cyclic process is represented by a closed curve.

Let the gas undergo a cyclic process in which it returns to the initial stage after an expansion and compression as shown in Figure 8.39

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Figure 8.39 PV diagram for cyclic process

Let W_1 be the work done by the gas during expansion from volume V_1 to volume V_2. It is equal to area under the graph CBA as shown in Figure 8.40 (a) .

Figure 8.40 (a) W for path CBA

Figure 8.40 (b) W for path ADC

Let W_2 be the work done on the gas during compression from volume V_2 to volume V_1. It is equal to the area under the graph ADC as shown in Figure 8.40 (b)

The total work done in this cyclic process = W_1 - W_2 = Green shaded area inside the loop, as shown in Figure 8.41.

Figure 8.41 Net work done in a cyclic process

Thus the net work done during the cyclic process shown above is not zero. In general the net work done can be positive or negative. If the net work done is positive, then work done by the system is greater than the work done on the system. If the net work done is negative then the work done by the system is less than the work done on the system.

Note Further, in a cyclic process the net work done is positive if the process goes clockwise and net work done is negative if the process goes anti-clockwise. In Figure 8.41the process goes clockwise.

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EXAMPLE 8.22

The PV diagrams for a thermodynamical system is given in the figure below. Calculate the total work done in each of the cyclic processes shown.

Solution

In the case (a) the closed curve is anticlockwise. So the net work done is negative, implying that the work done on the system is greater than the work done by the system. The area under the curve BC will give work done on the gas (isobaric compression) and area under the curve DA (work done by the system) will give the total work done by the system.

Area under the curve BC = Area of rectangle BC12 = $1 \times 4 = -4J$

Area under the curve DA = $1 \times 2 = +2J$

Net work done in cyclic process
$= -4 + 2 = -2$ J

In the case (b) the closed curve is clockwise. So the net work done is positive, implying that the work done on the system is less than the work done by the system. Area under the curve BC will give work done on the gas (isobaric compression) and area under the curve AB will give the total work done by the system.

Area under the curve AB = rectangle area+ triangle area = $(1\times2) + \frac{1}{2} \times 1\times2 = +3J$

Area under the curve BC = rectangle area $= 1 \times 2 = -2J$

Network done in the cyclic process = 1 J, which is positive.

In the case (c) the closed curve is anticlockwise. So the net work done is negative, implying that the work done on the system is greater than work done by the system. The area under the curve AB will give the work done on the gas (isobaric compression) and area under the curve CA (work done by the system) will give the total work done by the system.

The area under the curve AB =Rectangle of area $= 4 \times 1 = -4J$

The area under the curve CA = Rectangle area + triangle area = $(1\times2) + \frac{1}{2} \times 1\times2 = +3J$

The total work in the cyclic process = -1 J. It is negative

8.8.7 Limitations of first law of thermodynamics

The first law of thermodynamics explains well the inter convertibility of heat and work. But it does not indicate the direction of change.

For example,

a. When a hot object is in contact with a cold object, heat always flows from the hot object to cold object but not in the reverse direction. According to first law, it is possible for the energy to flow from hot object to cold object or from cold object to hot object. But in nature the direction of heat flow is always from higher temperature to lower temperature.

b. When brakes are applied, a car stops due to friction and the work done against friction is converted into heat. But this

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heat is not reconverted to the kinetic energy of the car.

So the first law is not sufficient to explain many of natural phenomena.

8.8.8 Reversible process

A thermodynamic process can be considered reversible only if it possible to retrace the path in the opposite direction in such a way that the system and surroundings pass through the same states as in the initial,direct process.

Example: A quasi–static isothermal expansion of gas, slow compression and expansion of a spring.

Conditions for reversible process:

1. The process should proceed at an extremely slow rate.
2. The system should remain in mechanical, thermal and chemical equilibrium state at all the times with the surroundings, during the process.
3. No dissipative forces such as friction, viscosity, electrical resistance should be present.

Note All reversible processes are quasi-static but all quasi-static processes need not be reversible. For example when we push the piston very slowly, if there is friction between cylinder wall and piston some amount of energy is lost to surroundings, which cannot be retrieved back.

Irreversible process:

All natural processes are irreversible. Irreversible process cannot be plotted in a PV diagram, because these processes cannot have unique values of pressure, temperature at every stage of the process.

The first law of thermodynamics is the statement about conservation of energy in a thermodynamic process. For example, if a hotter object is placed on a colder object, heat flows from hotter to colder object. Why does heat not flow from the colder object to hotter object? Even if energy flows from colder object to hotter object, the first law of thermodynamics is not violated. For example, if 5 J of heat flows form hotter to colder or from colder to hotter objects the total internal energy of this combined system remains the same. But 5 J of heat never flows from the colder object to hotter object. In nature all such process occur only in one direction but not in the reverse direction, even if the energy is conserved in both the processes. Thus the first law of thermodynamics has no explanation for this irreversibility. When the scientists of the eighteenth century tried to explain this irreversibility, they discovered a new law of nature. This is called the second law of thermodynamics. According to second law of thermodynamics

"Heat always flows from hotter object to colder object spontaneously". This is known as the Clausius form of second law of thermodynamics.

EXAMPLE 8.23

Give some examples of irreversible processes.

All naturally occuring processes are irreversible. Here we give some interesting examples.

(a) When we open a gas bottle, the gas molecules slowly spread into the entire room. These gas molecules can never get back in to the bottle.

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(b) Suppose one drop of an ink is dropped in water, the ink droplet slowly spreads in the water. It is impossible to get the ink droplet back.

(c) When an object falls from some height, as soon as it hits the earth it comes to rest. All the kinetic energy of the object is converted to kinetic energy of molecules of the earth surface, molecules of the object and small amount goes as sound energy. The spreaded kinetic energy to the molecules never collected back and object never goes up by itself.

Note that according to first law of thermodynamics all the above processes are possible in both directions. But second law of thermodynamics forbids The processes to occur in the reverse direction. The second law of thermodynamics is one of the very important laws of nature. It controls the way the natural processes occur.

8.9 HEAT ENGINE

In the modern technological world, the role of automobile engines plays a vital role in for transportation. In motor bikes and cars there are engines which take in petrol or diesel as input, and do work by rotating wheels. Most of these automobile engines have efficiency not greater than 40%. The second law of thermodynamics puts a fundamental restriction on efficiency of engines. Therefore understanding heat engines is very important.

Reservoir:

It is defined as a thermodynamic system which has very large heat capacity. By taking in heat from reservoir or giving heat to reservoir, the reservoir's temperature does not change.

Example: Pouring a tumbler of hot water in to lake will not increase the temperature of the lake. Here the lake can be treated as a reservoir.

When a hot cup of coffee attains equilibrium with the open atmosphere, the temperature of the atmosphere will not appreciably change. The atmosphere can be taken as a reservoir.

We can define heat engine as follows.

Heat engine is a device which takes heat as input and converts this heat in to work by undergoing a cyclic process.

A heat engine has three parts:

(a) Hot reservoir
(b) Working substance
(c) Cold reservoir

A Schematic diagram for heat engine is given below in the figure 8.42.

Figure 8.42 Heat Engine

1. Hot reservoir (or) Source: It supplies heat to the engine. It is always maintained at a high temperature T_H
2. Working substance: It is a substance like gas or water, which converts the heat supplied into work.

   A simple example of a heat engine is a steam engine. In olden days steam engines were used to drive trains. The working substance in these is water which absorbs heat from the burning of coal. The heat converts the water into steam. This steam is does work by rotating the wheels of the train, thus making the train move.
3. Cold reservoir (or) Sink: The heat engine ejects some amount of heat (Q_L) in to cold reservoir after it doing work. It is always maintained at a low temperature T_L.

   For example, in the automobile engine, the cold reservoir is the surroundings at room temperature. The automobile ejects heat to these surroundings through a silencer.

The heat engine works in a cyclic process. After a cyclic process it returns to the same state. Since the heat engine returns to the same state after it ejects heat, the change in the internal energy of the heat engine is zero.

The efficiency of the heat engine is defined as the ratio of the work done (out put) to the heat absorbed (input) in one cylic process.

Let the working substance absorb heat Q_H units from the source and reject Q_L units to the sink after doing work W units, as shown in the Figure 8.43.

Figure 8.43 Heat engine

We can write

Input heat = Work done + ejected heat

$$Q_H = W + Q_L$$

$$W = Q_H - Q_L$$

Then the efficiency of heat engine

$$\eta = \frac{output}{input} = \frac{W}{Q_H} = \frac{Q_H - Q_L}{Q_H}$$

$$\eta = \frac{output}{input} = \frac{W}{Q_H} = 1 - \frac{Q_L}{Q_H} \qquad (8.55)$$

Note here that Q_H, Q_L and W all are taken as positive, a sign convention followed in this expression.

Since $Q_L < Q_H$, the efficiency (η) always less than 1. This implies that heat absorbed is not completely converted into work. The second law of thermodynamics placed fundamental restrictions on converting heat completely into work.

We can state the heat engine statement of second law of thermodynamics. This is also called Kelvin-Planck's statement.

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Kelvin-Planck statement:

It is impossible to construct a heat engine that operates in a cycle, whose sole effect is to convert the heat completely into work. This implies that no heat engine in the universe can have 100% efficiency.

Note

According to first law of thermodynamics, in an isothermal process the given heat is completely converted into work ($Q = W$). Is it a violation of the second law of thermodynamics? No. For non-cyclic process like an isothermal expansion, the heat can be completely converted into work. But Second law of thermodynamics implies that 'In a cyclic process only a portion of the heat absorbed is converted into work'. All heat engines operate in a cyclic process.

EXAMPLE 8.24

During a cyclic process, a heat engine absorbs 500 J of heat from a hot reservoir, does work and ejects an amount of heat 300 J into the surroundings (cold reservoir). Calculate the efficiency of the heat engine?

Solution

The efficiency of heat engine is given by

$$\eta = 1 - \frac{Q_L}{Q_H}$$

$$\eta = 1 - \frac{300}{500} = 1 - \frac{3}{5}$$

$$\eta = 1 - 0.6 = 0.4$$

The heat engine has 40% efficiency, implying that this heat engine converts only 40% of the input heat into work.

8.9.1 Carnot's ideal heat engine

In the previous section we have seen that the heat engine cannot have 100% efficiency. What is the maximum possible efficiency can a heat engine have?. In the year 1824 a young French engineer Sadi Carnot proved that a certain reversible engine operated in cycle between hot and cold reservoir can have maximum efficiency. This engine is called Carnot engine.

A reversible heat engine operating in a cycle between two temperatures in a particular way is called a Carnot Engine.

The carnot engine has four parts which are given below.

i. Source: It is the source of heat maintained at constant high temperature T_H. Any amount of heat can be extracted from it, without changing its temperature.

ii. Sink: It is a cold body maintained at a constant low temperature T_L. It can absorb any amount of heat.

iii. Insulating stand: It is made of perfectly non-conducting material. Heat is not conducted through this stand.

iv. Working substance: It is an ideal gas enclosed in a cylinder with perfectly non-conducting walls and perfectly conducting bottom. A non-conducting and frictionless piston is fitted in it.

The four parts are shown in the following Figure 8.44

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Figure 8.44 Carnot engine

Carnot's cycle:

The working substance is subjected to four successive reversible processes forming what is called Carnot's cycle.

Let the initial pressure, volume of the working substance be P_1, V_1.

Step A to B: Quasi-static isothermal expansion from (P_1, V_1, T_H) to (P_2, V_2, T_H):

The cylinder is placed on the source. The heat (Q_H) flows from source to the working substance (ideal gas) through the bottom of the cylinder. Since the process is isothermal, the internal energy of the working substance will not change. The input heat increases the volume of the gas. The piston is allowed to move out very slowly(quasi-statically). It is shown in the figure 8.47(a).

W_1 is the work done by the gas in expanding from volume V_1 to volume V_2 with a decrease of pressure from P_1 to P_2. This is represented by the P-V diagram along the path AB as shown in the Figure 8.45.

Figure 8.45 PV diagram for Carnot cycle

Then the work done by the gas (working substance) is given by

$$\therefore Q_H = W_{A\to B} = \int_{V_1}^{V_2} PdV$$

Since the process occurs quasi-statically, the gas is in equilibrium with the source till it reaches the final state. The work done in the isothermal expansion is given by the equation (8.34)

$$W_{A\to B} = \mu R T_H \ln\left(\frac{V_2}{V_1}\right) = \text{Area under the curve AB} \qquad (8.56)$$

This is shown in Figure 8.46 (a)

Figure 8.46 Work done in Carnot cycle

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Figure 8.47 Carnot cycle

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Step B to C: Quasi-static adiabatic expansion from (P_2, V_2, T_H) to (P_3, V_3, T_L)

The cylinder is placed on the insulating stand and the piston is allowed to move out. As the gas expands adiabatically from volume V_2 to volume V_3 the pressure falls from P_2 to P_3. The temperature falls to T_L. This adiabatic expansion is represented by curve *BC* in the P-V diagram. This adiabatic process also occurs quasi-statically and implying that this process is reversible and the ideal gas is in equilibrium throughout the process. It is shown in the figure 8.47(b). From the equation (8.42)

The work done by the gas in an adiabatic expansion is given by,

$$W_{B\to C} = \int_{V_2}^{V_3} PdV = \frac{\mu R}{\gamma - 1}\left[T_H - T_L\right] =$$

Area under the curve BC (8.57)

This is shown in Figure 8.46 (b)

Step C → D: Quasi-static isothermal compression from (P_3, V_3, T_L) to (P_4, V_4, T_L): It is shown in the figure 8.47(c)

The cylinder is placed on the sink and the gas is isothermally compressed until the pressure and volume become P_4 and V_4 respectively. This is represented by the curve *CD* in the PV diagram as shown in Figure 8.45. Let $W_{C\to D}$ be the work done on the gas. According to first law of thermodynamics

$$Q_L = W_{C\to D} = \int_{V_3}^{V_4} PdV = \mu R T_L \ln\left(V_4/V_3\right) = -\mu R T_L \ln\left(\frac{V_3}{V_4}\right)$$

= −Area under the curve CD (8.58)

This is shown in Figure 8.46 (c)

Here V_3 is greater than V_4. So the work done is negative, implying work is done on the gas.

Step D→A: Quasi-static adiabatic compression from (P_4, V_4, T_L) to (P_1, V_1, T_H): It is shown in the figure 8.47(d)

The cylinder is placed on the insulating stand again and the gas is compressed adiabatically till it attains the initial pressure P_1, volume V_1 and temperature T_H. This is shown by the curve *DA* in the P-V diagram.

$$\therefore W_{D\to A} = \int_{V_4}^{V_1} PdV = -\frac{\mu R}{\gamma - 1}\left[T_H - T_L\right] =$$

−Area under the curve DA (8.59)

In the adiabatic compression also work is done on the gas so it is negative, as is shown in Figure 8.46 (d)

Let 'W' be the net work done by the working substance in one cycle

$$= W_{A\to B} + W_{B\to C} + W_{C\to D} + W_{D\to A}$$

since $W_{D\to A} = -W_{B\to C}$

$$= W_{A\to B} + W_{C\to D}$$

The net work done by the Carnot engine in one cycle $W = |W|_{A\to B} - |W|_{C\to D}$ (8.60)

Equation (8.60) shows that the net work done by the working substance in one cycle is equal to the area (enclosed by ABCD) of the P-V diagram (Figure 8.48)

Figure 8.48 Net work done in Carnot cycle

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It is very important to note that after one cycle the working substance returns to the initial temperature T_H. This implies that the change in internal energy of the working substance after one cycle is zero.

8.9.2 Efficiency of a Carnot engine

Efficiency is defined as the ratio of work done by the working substance in one cycle to the amount of heat extracted from the source.

$$\eta = \frac{\textit{work done}}{\textit{Heat extracted}} = \frac{W}{Q_H} \qquad (8.61)$$

From the first law of thermodynamics, $W = Q_H - Q_L$

$$\therefore \eta = \frac{Q_H - Q_L}{Q_H} = 1 - \frac{Q_L}{Q_H} \qquad (8.62)$$

Applying isothermal conditions, we get,

$$Q_H = \mu R T_H \ln\left(V_2/V_1\right)$$
$$Q_L = \mu R T_L \ln\left(V_3/V_4\right) \qquad (8.63)$$

Here we omit the negative sign. Since we are interested in only the amount of heat (Q_L) ejected into the sink, we have

$$\therefore \frac{Q_L}{Q_H} = \frac{T_L \ln\left(V_3/V_4\right)}{T_H \ln\left(V_2/V_1\right)} \qquad (8.64)$$

By applying adiabatic conditions, we get,

$$T_H V_2^{\gamma-1} = T_L V_3^{\gamma-1}$$
$$T_H V_1^{\gamma-1} = T_L V_4^{\gamma-1}$$

By dividing the above two equations, we get

$$\left(\frac{V_2}{V_1}\right)^{\gamma-1} = \left(\frac{V_3}{V_4}\right)^{\gamma-1}$$

Which implies that $\frac{V_2}{V_1} = \frac{V_3}{V_4}$ (8.65)

Substituting equation (8.65) in (8.64), we get

$$\frac{Q_L}{Q_H} = \frac{T_L}{T_H} \qquad (8.66)$$

$$\therefore \text{The efficiency } \eta = 1 - \frac{T_L}{T_H} \qquad (8.67)$$

Note : T_L and T_H should be expressed in Kelvin scale.

Important results:

1. η is always less than 1 because T_L is less than T_H. This implies the efficiency cannot be 100%. It can be 1 or 100% only when $T_L = 0K$ (absolute zero of temperature) which is impossible to attain practically.
2. The efficiency of the Carnot's engine is independent of the working substance. It depends only on the temperatures of the source and the sink. The greater the difference between the two temperatures, higher the efficiency.
3. When $T_H = T_L$ the efficiency $\eta = 0$. No engine can work having source and sink at the same temperature.
4. The entire process is reversible in the Carnot engine cycle. So Carnot engine is itself a reversible engine and has maximum efficiency. But all practical heat engines like diesel engine, petrol engine and steam engine have cycles which are not perfectly reversible. So their efficiency is always less than the Carnot efficiency. This can be stated in the form of the Carnot theorem. It is stated as follows '*Between two constant temperatures reservoirs, only Carnot engine can have maximum efficiency. All real heat engines will have efficiency less than the Carnot engine*'

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EXAMPLE 8.25

(a) A steam engine boiler is maintained at 250°C and water is converted into steam. This steam is used to do work and heat is ejected to the surrounding air at temperature 300K. Calculate the maximum efficiency it can have?

Solution

The steam engine is not a Carnot engine, because all the process involved in the steam engine are not perfectly reversible. But we can calculate the maximum possible efficiency of the steam engine by considering it as a Carnot engine.

$$\eta = 1 - \frac{T_L}{T_H} = 1 - \frac{300\,K}{523\,K} = 0.43$$

The steam engine can have maximum possible 43% of efficiency, implying this steam engine can convert 43% of input heat into useful work and remaining 57% is ejected as heat. In practice the efficiency is even less than 43%.

EXAMPLE 8.26

There are two Carnot engines A and B operating in two different temperature regions. For Engine A the temperatures of the two reservoirs are 150°C and 100°C. For engine B the temperatures of the reservoirs are 350°C and 300°C. Which engine has lesser efficiency?

Solution

The efficiency for engine A = $1 - \frac{373}{423}$ = 0.11. Engine A has 11% efficiency

The efficiency for engine B = $1 - \frac{573}{623}$ = 0.08

Engine B has only 8% efficiency.

Even though the differences between the temperature of hot and cold reservoirs in both engines is same, the efficiency is not same. The efficiency depends on the ratio of the two temperature and not on the difference in the temperature. The engine which operates in lower temperature has highest efficiency.

Diesel engines used in cars and petrol engines used in our motor bikes are all real heat engines. The efficiency of diesel engines has maximum up to 44% and the efficiency of petrol engines are maximum up to 30%. Since these engines are not ideal heat engines (Carnot engine), their efficiency is limited by the second law of thermodynamics. Now a days typical bikes give a mileage of 50 km per Liter of petrol. This implies only 30% of 1 Liter of petrol is converted into mechanical work and the remaining 70% goes out as wasted heat and ejected into the surrounding atmosphere!

8.9.3 Entropy and second law of thermodynamics

We have seen in the equation (8.66) that the quantity $\frac{Q_H}{T_H}$ is equal to $\frac{Q_L}{T_L}$. The quantity $\frac{Q}{T}$ is called entropy. It is a very important thermodynamic property of a system. It is also a state variable. $\frac{Q_H}{T_H}$ is the entropy received by the Carnot engine from hot reservoir and $\frac{Q_L}{T_L}$ is entropy given out by the Carnot engine to the cold reservoir. For reversible engines (Carnot Engine) both entropies should be same, so that the change

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in entropy of the Carnot engine in one cycle is zero. This is proved in equation (8.66). But for all practical engines like diesel and petrol engines which are not reversible engines, they satisfy the relation $\frac{Q_L}{T_L} > \frac{Q_H}{T_H}$. In fact we can reformulate the second law of thermodynamics as follows

"For all the processes that occur in nature (irreversible process), the entropy always increases. For reversible process entropy will not change". Entropy determines the direction in which natural process should occur.

We now come back to the question: Why does heat always flows from a state of higher temperature to one of lower temperature and not in the opposite direction? Because entropy increases when heat flows from hot object to cold object. If heat were to flow from a cold to a hot object, entropy will decrease leading to violation of second law thermodynamics.

Entropy is also called 'measure of disorder'. All natural process occur such that the disorder should always increases.

Consider a bottle with a gas inside. When the gas molecules are inside the bottle it has less disorder. Once it spreads into the entire room it leads to more disorder. In other words when the gas is inside the bottle the entropy is less and once the gas spreads into entire room, the entropy increases. From the second law of thermodynamics, entropy always increases. If the air molecules go back in to the bottle, the entropy should decrease, which is not allowed by the second law of thermodynamics. The same explanation applies to a drop of ink diffusing into water. Once the drop of ink spreads, its entropy is increased. The diffused ink can never become a drop again. So the natural processes occur in such a way that entropy should increase for all irreversible process.

8.10 REFRIGERATOR

A refrigerator is a Carnot's engine working in the reverse order. It is shown in the figure 8.49.

Working Principle:

Fig 8.49 (a) Schematic diagram of a refrigerator (b) Actual refrigerator

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The working substance (gas) absorbs a quantity of heat Q_L from the cold body (sink) at a lower temperature T_L. A certain amount of work W is done on the working substance by the compressor and a quantity of heat Q_H is rejected to the hot body (source) ie, the atmosphere at T_H. When you stand beneath of refrigerator, you can feel warmth air.

From the first law of thermodynamics , we have

$$Q_L + W = Q_H \qquad (8.68)$$

As a result the cold reservoir (refrigerator) further cools down and the surroundings (kitchen or atmosphere) gets hotter.

Coefficient of performance (COP) (β):

COP is a measure of the efficiency of a refrigerator. It is defined as the ratio of heat extracted from the cold body (sink) to the external work done by the compressor W.

$$\text{COP} = \beta = \frac{Q_L}{W} \qquad (8.69)$$

From the equation (8.68)

$$\beta = \frac{Q_L}{Q_H - Q_L}$$

$$\beta = \frac{1}{\frac{Q_H}{Q_L} - 1} \qquad (8.70)$$

But we know that $\frac{Q_H}{Q_L} = \frac{T_H}{T_L}$

Substituting this equation into equation (8.70) we get

$$\beta = \frac{1}{\frac{T_H}{T_L} - 1} = \frac{T_L}{T_H - T_L}$$

Inferences:

1. The greater the COP, the better is the condition of the refrigerator. A typical refrigerator has COP around 5 to 6.
2. Lesser the difference in the temperatures of the cooling chamber and the atmosphere, higher is the COP of a refrigerator.
3. In the refrigerator the heat is taken from cold object to hot object by doing external work. Without external work heat cannot flow from cold object to hot object. It is not a violation of second law of thermodynamics, because the heat is ejected to surrounding air and total entropy of (refrigerator + surrounding) is always increased.

EXAMPLE 8.27

A refrigerator has COP of 3. How much work must be supplied to the refrigerator in order to remove 200 J of heat from its interion?

$$\text{COP} = \beta = \frac{Q_L}{W}$$

$$W = \frac{Q_L}{COP} = \frac{200}{3} = 66.67\,J$$

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Green House Effect

The presence of atmosphere in the earth plays very important role in human lives. Top of the atmosphere is at -19°C and bottom of the atmosphere is at +14°C. The increase in 33°C from top to bottom is due to some gases present in the atmosphere. These gases are called Greenhouse gases and this effect is called Greenhouse effect.

The greenhouse gases are mainly CO_2, water vapour, Ne, He, NO_2, CH_4, Xe, Kr, ozone and NH_3. Except CO_2 and water vapor, all others are present only in very small amount in the atmosphere. The radiation from the Sun is mainly in the visible region of the spectrum. The earth absorbs these radiations and reradiate in the infrared region. Carbon dioxide and water Vapour are good absorbers of infrared radiation since they have more vibrational degree of freedom compared to nitrogen and oxygen (you will learn in unit 9) which keeps earth warmer as shown in Figure.

The amount of CO_2 present in the atmosphere is increased from 20% to 40% due to human activities since 1900s. The major emission of CO_2 comes from burning of fossil fuels. The increase in automobile usage worldwide causes this damage. Due to this increase in the CO_2 content in the atmosphere, the average temperature of the earth increases by 1°C. This effect is called global warming. It has serious influence and alarming effect on ice glaciers on Arctic and Antarctic regions. In addition, the CO_2 content is also increasing in ocean which is very dangerous to species in the oceans.

In addition to CO_2, another very important greenhouse gas is Chloro flouro carbon(CFC) which is used as coolant in refrigerators worldwide. In the human made greenhouse gases CO_2 is 55%, CFCs are 24%. Nitrogen oxide is 6% and methane is 15%. CFCs also has made huge damage to ozone layer.

Lot of efforts are taken internationally to reduce the emission of CO_2 and CFCs in various countries. Nowadays a lot of research is going to replace non fossil fuels to replace the fossil-fuels in automobile industry. The major emission of CO_2 comes from developed countries like USA and European countries. Various treaties are formed between countries to reduce the emission of CO_2 to considerable level before 2020s. But still global warming is not taken seriously in various countries.

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In hot summer, we use earthern pots to drink cold water. The pot reduces the temperature of water inside it. Does the earthern pot act as a refrigerator? No. cyclic process is the basic necessity for heat engine or refrigerator. In earthern pot, the cooling process is not due to any cyclic process. The cooling occurs due to evaporation of water molecules which oozes out through pores of the pot. Once the water molecules evaporate, they never come back to the pot. Even though the heat flows from cold water to open atmosphere, it is not a violation of second law of thermodynamics. The water inside the pot is an open thermodynamic system, so the entropy of water + surrounding always increases.

SUMMARY

- Heat is energy in transit which flows from hot object to cold object. However it is not a quantity.
- Work is a process to transfer energy from one object to another object.
- Temperature is a measure of hotness of the object. It determines the direction of the flow of heat.
- The ideal gas law is $PV = NkT$ or $PV = \mu RT$. The Ideal gas law holds for only at thermodynamic equilibrium. For non-equilibrium process, it is not valid.
- Heat capacity is the amount of heat energy required to increase the object's temperature by 1°C or 1K. It is denoted by S.
- Specific heat capacity is the amount of heat energy required to increase the 1 kg of object's temperature by 1°C or 1K. It is denoted by s.
- Molar specific heat capacity is the amount of heat energy requires to increase the 1 mole of substance's temperature by 1°C or 1K. It is denoted by C.
- Thermal expansion is a tendency of an object to change its shape, area, and volume due to change in temperature.
- Water has an anomalous behavior of expansion.
- Latent heat capacity is the amount of heat energy required to change the phase of the substance.
- Calorimetry is the measurement of the amount of heat energy released or absorbed by a thermodynamic system during the heating process.
- Heat transfers in three different modes: conduction, convection and radiation
- Stefan-Boltmann law: $E = \sigma T^4$ and Wien's law: $\lambda_{max} T = b$
- Thermodynamic equilibrium: thermal, mechanical and chemical equilibrium
- Thermodynamic variables : Pressure, temperature, volume, internal energy and entropy

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- Zeroth law of thermodynamics: If two objects are separately in thermal equilibrium with the third object, then these two are in thermal equilibrium. Temperature is a property which is the same for both the systems.
- Internal energy is the sum of kinetic and potential energies of molecules in the thermodynamic system.
- Joule converted mechanical energy to internal energy of the thermodynamic system
- First law of thermodynamics is a statement of conservation of energy. It included heat energy of the thermodynamic system.
- A quasi-static process is an infinitely slow process in which the system is always at equilibrium with the surrounding.
- When the volume of the system changes, the work done $W = \int P\, dV$
- The area under the PV diagram gives the work done by the system or work done on the system.
- Specific heat capacity at constant volume is always less than specific heat capacity at constant pressure.
- Isothermal process: T = constant, Isobaric process: P=constant, Isochoric process: V= constant, Adiabatic process Q = 0
- Work done in the isobaric process is most and work done in the adiabatic process is least
- In a cyclic process, change in internal energy is zero.
- The total work done in the cyclic process is given by a closed area in PV diagram
- A reversible process is an ideal process.
- All natural processes are irreversible.
- Heat engine takes input from the hot reservoir, performs work and rejects some amount of heat energy into sink.
- Carnot engine is a reversible engine. It has the highest efficiency. No real heat engine can have the efficiency of that of a Carnot engine.
- A refrigerator is reverse of a Carnot engine. COP (coefficient of performance) of the practical refrigerator is always less than ideal refrigerator.

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CONCEPT MAP

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EVALUATION

I. Multiple choice questions:

1. In hot summer after a bath, the body's

a) internal energy decreases

b) internal energy increases

c) heat decreases

d) no change in internal energy and heat

2. The graph between volume and temperature in Charles' law is

a) an ellipse b) a circle

c) a straight line d) a parabola

3. When a cycle tyre suddenly bursts, the air inside the tyre expands. This process is

a) isothermal b) adiabatic

c) isobaric d) isochoric

4. An ideal gas passes from one equilibrium state (P_1, V_1, T_1, N) to another equilibrium state ($2P_1$, $3V_1$, T_2, N). Then

a) $T_1 = T_2$ b) $T_1 = \frac{T_2}{6}$

c) $T_1 = 6T_2$ d) $T_1 = 3T_2$

5. When a uniform rod is heated, which of the following quantity of the rod will increase

a) mass

b) weight

c) center of mass

d) moment of inertia

6. When food is cooked in a vessel by keeping the lid closed, after some time the steam pushes the lid outward. By considering the steam as a thermodynamic system, then in the cooking process

a) $Q > 0$, $W > 0$,

b) $Q < 0$, $W > 0$,

c) $Q > 0$, $W < 0$,

d) $Q < 0$, $W < 0$,

7. When you exercise in the morning, by considering your body as thermodynamic system, which of the following is true?

a) $\Delta U > 0$, $W > 0$,

b) $\Delta U < 0$, $W > 0$,

c) $\Delta U < 0$, $W < 0$,

d) $\Delta U = 0$, $W > 0$,

8. A hot cup of coffee is kept on the table. After some time it attains a thermal equilibrium with the surroundings. By considering the air molecules in the room as a thermodynamic system, which of the following is true

a) $\Delta U > 0$, $Q = 0$

b) $\Delta U > 0$, $W < 0$

c) $\Delta U > 0$, $Q > 0$

d) $\Delta U = 0$, $Q > 0$

9. An ideal gas is taken from (P_i,V_i) to (P_f,V_f) in three different ways. Identify the process in which the work done on the gas the most.

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a) Process A

b) Process B

c) Process C

d) Equal work is done in Process A,B &C

10. The V-T diagram of an ideal gas which goes through a reversible cycle $A \rightarrow B \rightarrow C \rightarrow D$ is shown below. (Processes $D \rightarrow A$ and $B \rightarrow C$ are adiabatic)

The corresponding PV diagram for the process is (all figures are schematic)

a)

b)

c)

d)

11. A distant star emits radiation with maximum intensity at 350 nm. The temperature of the star is

a) 8280 K b) 5000K

c) 7260 K d) 9044 K

12. Identify the state variables given here?

a) Q, T, W b) P, T, U

c) Q, W d) P, T, Q

13. In an isochoric process, we have

a) W = 0 b) Q = 0

c) $\Delta U = 0$ d) $\Delta T = 0$

14. The efficiency of a heat engine working between the freezing point and boiling point of water is

(NEET 2018)

a) 6.25% b) 20%

c) 26.8% d) 12.5%

15. An ideal refrigerator has a freezer at temperature −12°C. The coefficient of performance of the engine is 5. The temperature of the air (to which the heat ejected) is

a) 50°C b) 45.2°C

c) 40.2°C d) 37.5°C

Answers:

1) a	2) c	3) b	4) b
5) d	6) a	7) b	8) c
9) b	10) a	11) a	12) b
13) a	14) c	15) c	

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II. Short answer questions:

1. 'An object contains more heat'- is it a right statement? If not why?
2. Obtain an ideal gas law from Boyle's and Charles' law.
3. Define one mole.
4. Define specific heat capacity and give its unit.
5. Define molar specific heat capacity.
6. What is a thermal expansion?
7. Give the expressions for linear, area and volume thermal expansions.
8. Define latent heat capacity. Give its unit.
9. State Stefan-Boltzmann law.
10. What is Wien's law?
11. Define thermal conductivity. Give its unit.
12. What is a black body?
13. What is a thermodynamic system? Give examples.
14. What are the different types of thermodynamic systems?
15. What is meant by 'thermal equilibrium'?
16. What is mean by state variable? Give example.
17. What are intensive and extensive variables? Give examples.
18. What is an equation of state? Give an example.
19. State Zeroth law of thermodynamics.
20. Define the internal energy of the system.
21. Are internal energy and heat energy the same? Explain.
22. Define one calorie.
23. Did joule converted mechanical energy to heat energy? Explain.
24. State the first law of thermodynamics.
25. Can we measure the temperature of the object by touching it?
26. Give the sign convention for Q and W.
27. Define the quasi-static process.
28. Give the expression for work done by the gas.
29. What is PV diagram?
30. Explain why the specific heat capacity at constant pressure is greater than the specific heat capacity at constant volume.
31. Give the equation of state for an isothermal process.
32. Give an expression for work done in an isothermal process.
33. Express the change in internal energy in terms of molar specific heat capacity.
34. Apply first law for (a) an isothermal (b) adiabatic (c) isobaric processes.
35. Give the equation of state for an adiabatic process.
36. Give an equation state for an isochoric process.
37. If the piston of a container is pushed fast inward. Will the ideal gas equation be valid in the intermediate stage? If not, why?
38. Draw the PV diagram for
    a. Isothermal process
    b. Adiabatic process
    c. Isobaric process
    d. Isochoric process
39. What is a cyclic process?

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40. What is meant by a reversible and irreversible processes?
41. State Clausius form of the second law of thermodynamics
42. State Kelvin-Planck statement of second law of thermodynamics.
43. Define heat engine.
44. What are processes involves in a Carnot engine?
45. Can the given heat energy be completely converted to work in a cyclic process? If not, when can the heat can completely converted to work?
46. State the second law of thermodynamics in terms of entropy.
47. Why does heat flow from a hot object to a cold object?
48. Define the coefficient of performance.

III. Long answer Questions:

1. Explain the meaning of heat and work with suitable examples.
2. Discuss the ideal gas laws.
3. Explain in detail the thermal expansion.
4. Describe the anomalous expansion of water. How is it helpful in our lives?
5. Explain Calorimetry and derive an expression for final temperature when two thermodynamic systems are mixed.
6. Discuss various modes of heat transfer.
7. Explain in detail Newton's law of cooling.
8. Explain Wien's law and why our eyes are sensitive only to visible rays?
9. Discuss the
   a. thermal equilibrium
   b. mechanical equilibrium
   c. Chemical equilibrium
   d. thermodynamic equilibrium.
10. Explain Joule's Experiment of the mechanical equivalent of heat.
11. Derive the expression for the work done in a volume change in a thermodynamic system.
12. Derive Mayer's relation for an ideal gas.
13. Explain in detail the isothermal process.
14. Derive the work done in an isothermal process
15. Explain in detail an adiabatic process.
16. Derive the work done in an adiabatic process
17. Explain the isobaric process and derive the work done in this process
18. Explain in detail the isochoric process.
19. What are the limitations of the first law of thermodynamics?
20. Explain the heat engine and obtain its efficiency.
21. Explain in detail Carnot heat engine.
22. Derive the expression for Carnot engine efficiency.
23. Explain the second law of thermodynamics in terms of entropy.
24. Explain in detail the working of a refrigerator.

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IV. Exercises

1. Calculate the number of moles of air is in the inflated balloon at room temperature as shown in the figure.

The radius of the balloon is 10 cm, and pressure inside the balloon is 180 kPa.

Answer: $\mu \cong 0.3$ *mol*

2. In the planet Mars, the average temperature is around -53°C and atmospheric pressure is 0.9 kPa. Calculate the number of moles of the molecules in unit volume in the planet Mars? Is this greater than that in earth?

Answer: $\mu_{Mars} = 0.49$ *mol*

$\mu_{Earth} \cong 40$ *mol*

3. An insulated container of gas has two chambers separated by an insulating partition. One of the chambers has volume V_1 and contains ideal gas at pressure P_1 and temperature T_1. The other chamber has volume V_2 and contains ideal gas at pressure P_2 and temperature T_2. If the partition is removed without doing any work on the gases, calculate the final equilibrium temperature of the container.

Answer: $T = \dfrac{T_1T_2(P_1V_1 + P_2V_2)}{P_1V_1T_2 + P_2V_2T_1}$

4. The temperature of a uniform rod of length L having a coefficient of linear expansion α_L is changed by ΔT. Calculate the new moment of inertia of the uniform rod about axis passing through its center and perpendicular to an axis of the rod.

Answer: $I' = I\,(1 + \alpha_L\,\Delta T)^2$

5. Draw the TP diagram (P-x axis, T-y axis), VT(T-x axis, V-y axis) diagram for

   a. Isochoric process

   b. Isothermal process

   c. isobaric process

6. A man starts bicycling in the morning at a temperature around 25°C, he checked the pressure of tire which is equal to be 500 kPa. Afternoon he found that the absolute pressure in the tyre is increased to 520 kPa. By assuming the expansion of tyre is negligible, what is the temperature of tyre at afternoon?

Answer: T= 36.9°C

7. Normal human body of the temperature is 98.6°F. During high fever if the temperature increases to 104°F, what is the change in peak wavelength that emitted by our body? (Assume human body is a black body)

Answer: (a) $\lambda_{max} \approx 9348$ nm at 98.6°F

(b) $\lambda_{max} \approx 9258$ nm at 104°F

8. In an adiabatic expansion of the air, the volume is increased by 4%, what is percentage change in pressure?

(For air $\gamma = 1.4$)

Answer: 5.6%

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9. In a petrol engine, (internal combustion engine) air at atmospheric pressure and temperature of 20°C is adiabatically compressed in the cylinder by the piston to 1/8 of its original volume. Calculate the temperature of the compressed air.

(For air $\gamma = 1.4$)

Answer: T ≅ 400°C

10. Consider the following cyclic process consist of isotherm, isochoric and isobar which is given in the figure.

Draw the same cyclic process qualitatively in the V-T diagram where T is taken along x direction and V is taken along y-direction. Analyze the nature of heat exchange in each process.

Answer:

Process 1 to 2 = increase in volume. So heat must be added.

Process 2 to 3 = Volume remains constant. Increase in temperature. The given heat is used to increase the internal energy.

Process 3 to 1 : Pressure remains constant. Volume and Temperature are reduced. Heat flows out of the system. It is an isobaric compression where the work is done on the system.

11. An ideal gas is taken in a cyclic process as shown in the figure. Calculate

(a) work done by the gas.

(b) work done on the gas

(c) Net work done in the process

Answer: (a) $W = +1.5$kJ

(b) $W = -1.2$kJ

(c) $W = +300$J.

12. For a given ideal gas $6 \times 10^5 J$ heat energy is supplied and the volume of gas is increased from 4 m³ to 6 m³ at atmospheric pressure. Calculate (a) the work done by the gas (b) change in internal energy of the gas (c) graph this process in PV and TV diagram.

Answer: (a) $W = +202.6\ kJ$

(b) $dU = 397.4\ kJ$

(c)

13. Suppose a person wants to increase the efficiency of the reversible heat engine that is operating between 100°C and 300°C. He had two ways to increase the efficiency. (a) By decreasing the cold reservoir temperature from 100°C to 50°C and keeping the hot reservoir temperature constant (b) by increasing

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the temperature of the hot reservoir from 300°C to 350°C by keeping the cold reservoir temperature constant. Which is the suitable method?

Answer: Initial efficiency = 34.9%

Efficiency in method (a) =43.6%

Efficiency in method (b) =40.1%

Method (a) is more efficient.

14. A Carnot engine whose efficiency is 45% takes heat from a source maintained at a temperature of 327°C. To have an engine of efficiency 60% what must be the intake temperature for the same exhaust (sink) temperature?

Answer: 552°C

15. An ideal refrigerator keeps its content at 0°C while the room temperature is 27°C. Calculate its coefficient of performance.

Answer: β=10.11

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ICT CORNER

Heat and Thermodynamics

Through this activity you will be able to learn the PV diagrams for various thermodynamic process.

STEPS:

- Use the URL or scan the QR code to open interactive simulation on 'Pressure and Volume Diagram ".
- At selected temperature, change the "Volume" given below the graph and click play button.
- Now select a different temperature, change the 'Volume' again and find the change in the pressure both in the left image and graph.
- Repeat the same with different values and try drawing the graph accordingly. This also helps to understand isothermal process.

Step1

Step2

Step3

Step4

URL:

http://physics.bu.edu/~duffy/HTML5/PV_diagram.html

* Pictures are indicative only.

* If browser requires, allow **Flash Player** or **Java Script** to load the page.

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UNIT 9

KINETIC THEORY OF GASES

"With thermodynamics one can calculate almost everything crudely; with kinetic theory, one can calculate fewer things, but more accurately." - Eugene Wigner

Learning Objectives

In this unit, the student is exposed to

- necessity of kinetic theory of gases
- the microscopic origin of pressure and temperature
- correlate the internal energy of the gas and translational kinetic energy of gas molecules
- meaning of degrees of freedom
- calculate the total degrees of freedom for mono atomic, diatomic and triatomic molecules
- law of equipartition of energy
- calculation of the ratio of C_p and C_V
- mean free path and its dependence with pressure, temperature and number density
- Brownian motion and its microscopic origin

9.1 KINETIC THEORY

9.1.1 Introduction

Thermodynamics is basically a macroscopic science. We discussed macroscopic parameters like pressure, temperature and volume of thermodynamical systems in unit 8. In this unit we discuss the microscopic origin of pressure and temperature by considering a thermodynamic system as collection of particles or molecules. Kinetic theory relates pressure and temperature to molecular motion of sample of a gas and it is a bridge between Newtonian mechanics and thermodynamics. The present chapter introduces the kinetic nature of gas molecules.

9.1.2 Postulates of kinetic theory of gases

Kinetic theory is based on certain assumptions which makes the mathematical treatment simple. None of these assumptions are strictly true yet the model based on these assumptions can be applied to all gases.

1. All the molecules of a gas are identical, elastic spheres.
2. The molecules of different gases are different.
3. The number of molecules in a gas is very large and the average separation between them is larger than size of the gas molecules.
4. The molecules of a gas are in a state of continuous random motion.
5. The molecules collide with one another and also with the walls of the container.

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6. These collisions are perfectly elastic so that there is no loss of kinetic energy during collisions.
7. Between two successive collisions, a molecule moves with uniform velocity.
8. The molecules do not exert any force of attraction or repulsion on each other except during collision. The molecules do not possess any potential energy and the energy is wholly kinetic.
9. The collisions are instantaneous. The time spent by a molecule in each collision is very small compared to the time elapsed between two consecutive collisions.
10. These molecules obey Newton's laws of motion even though they move randomly.

9.2 PRESSURE EXERTED BY A GAS

9.2.1 Expression for pressure exerted by a gas

Consider a monoatomic gas of N molecules each having a mass m inside a cubical container of side l as shown in the Figure 9.1 (a).

Figure 9.1 (a) Container of gas molecules

Figure 9.1 (b) Collision of a molecule with the wall

The molecules of the gas are in random motion. They collide with each other and also with the walls of the container. As the collisions are elastic in nature, there is no loss of kinetic energy, but a change in momentum occurs.

The molecules of the gas exert pressure on the walls of the container due to collision on it. During each collision, the molecules impart certain momentum to the wall. Due to transfer of momentum, the walls experience a continuous force. The force experienced per unit area of the walls of the container determines the pressure exerted by the gas. It is essential to determine the total momentum transferred by the molecules in a short interval of time.

A molecule of mass m moving with a velocity $\vec{v}$ having components (v_x, v_y, v_z) hits the right side wall. Since we have assumed that the collision is elastic, the particle rebounds with same speed and its x-component is reversed. This is shown in the Figure 9.1 (b). The components of velocity of the molecule after collision are ($-v_x$, v_y, v_z).

The x-component of momentum of the molecule before collision = mv_x

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The x-component of momentum of the molecule after collision = $-mv_x$

The change in momentum of the molecule in x direction

=Final momentum – initial momentum = $-mv_x - mv_x = -2mv_x$

According to law of conservation of linear momentum, the change in momentum of the wall = $2mv_x$

Note In x direction, the total momentum of the system before collision is equal to momentum of the molecule (mv_x) since the momentum of the wall is zero. According to the law of conservation of momentum the total momentum of system after the collision must be equal to total momentum of system before collision. The momentum of the molecule (in x direction) after the collision is $-mv_x$ and the momentum of the wall after the collision is $2mv_x$. So total momentum of the system after the collision is $(2mv_x - mv_x) = mv_x$ which is same as the total momentum of the system before collision.

The number of molecules hitting the right side wall in a small interval of time Δt is calculated as follows.

The molecules within the distance of $v_x \Delta t$ from the right side wall and moving towards the right will hit the wall in the time interval Δt. This is shown in the Figure 9.2. The number of molecules that will hit the right side wall in a time interval Δt is equal to the product of volume ($Av_x \Delta t$) and number density of the molecules (n). Here A is area of the wall and n is number of molecules per unit volume $\left(\frac{N}{V}\right)$. We have assumed that the number density is the same throughout the cube.

Figure 9.2 Number of molecules hitting the wall

Not all the n molecules will move to the right, therefore on an average only half of the n molecules move to the right and the other half moves towards left side.

The number of molecules that hit the right side wall in a time interval Δt

$$= \frac{n}{2} A v_x \Delta t \qquad (9.1)$$

In the same interval of time Δt, the total momentum transferred by the molecules

$$\Delta p = \frac{n}{2} A v_x \Delta t \times 2mv_x = Av_x^2 mn\Delta t \qquad (9.2)$$

From Newton's second law, the change in momentum in a small interval of time gives rise to force.

The force exerted by the molecules on the wall (in magnitude)

$$F = \frac{\Delta p}{\Delta t} = nmAv_x^2 \qquad (9.3)$$

Pressure, P = force divided by the area of the wall

$$P = \frac{F}{A} = nmv_x^2 \qquad (9.4)$$

Since all the molecules are moving completely in random manner, they do not have same speed. So we can replace the term v_x^2 by the average $\overline{v_x^2}$ in equation (9.4)

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$$P = nm\overline{v_x^2} \quad (9.5)$$

Since the gas is assumed to move in random direction, it has no preferred direction of motion (the effect of gravity on the molecules is neglected). It implies that the molecule has same average speed in all the three direction. So, $\overline{v_x^2} = \overline{v_y^2} = \overline{v_z^2}$. The mean square speed is written as

$$\overline{v^2} = \overline{v_x^2} + \overline{v_y^2} + \overline{v_z^2} = 3\overline{v_x^2}$$

$$\overline{v_x^2} = \frac{1}{3}\overline{v^2}$$

Using this in equation (9.5), we get

$$P = \frac{1}{3}nm\overline{v^2} \text{ or } P = \frac{1}{3}\frac{N}{V}m\overline{v^2} \quad (9.6)$$

$$\text{as } \left[n = \frac{N}{V}\right]$$

The following inference can be made from the above equation. The pressure exerted by the molecules depends on

(i) **Number density** $n = \frac{N}{V}$. It implies that if the number density increases then pressure will increase. For example when we pump air inside the cycle tyre or car tyre essentially the number density increases and as a result the pressure increases.

(ii) **Mass of the molecule** Since the pressure arises due to momentum transfer to the wall, larger mass will have larger momentum for a fixed speed. As a result the pressure will increase.

(iii) **Mean square speed** For a fixed mass if we increase the speed, the average speed will also increase. As a result the pressure will increase.

For simplicity the cubical container is taken into consideration. The above result is true for any shape of the container as the area A does not appear in the final expression (9.6). Hence the pressure exerted by gas molecules on the wall is independent of area of the wall (A).

9.2.2 Kinetic interpretation of temperature

To understand the microscopic origin of temperature in the same way,

Rewrite the equation (9.6)

$$P = \frac{1}{3}\frac{N}{V}m\overline{v^2}$$

$$PV = \frac{1}{3}Nm\overline{v^2} \quad (9.7)$$

Comparing the equation (9.7) with ideal gas equation $PV=NkT$,

$$NkT = \frac{1}{3}Nm\overline{v^2}$$

$$kT = \frac{1}{3}m\overline{v^2} \quad (9.8)$$

Multiply the above equation by 3/2 on both sides,

$$\frac{3}{2}kT = \frac{1}{2}m\overline{v^2} \quad (9.9)$$

R.H.S of the equation (9.9) is called average kinetic energy of a single molecule ($\overline{KE}$).

The average kinetic energy per molecule

$$\overline{KE} = \in = \frac{3}{2}kT \quad (9.10)$$

Equation (9.9) implies that the temperature of a gas is a measure of the average translational kinetic energy per molecule of the gas.

Note Compare this with the definition of temperature studied in lower classes: Temperature is the degree of hotness or coldness!

Equation 9.10 is a very important result from kinetic theory of gas. We can infer the following from this equation.

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(i) The average kinetic energy of the molecule is directly proportional to absolute temperature of the gas. The equation (9.9) gives the connection between the macroscopic world (temperature) to microscopic world (motion of molecules).

(ii) The average kinetic energy of each molecule depends only on temperature of the gas not on mass of the molecule. In other words, if the temperature of an ideal gas is measured using thermometer, the average kinetic energy of each molecule can be calculated without seeing the molecule through naked eye.

By multiplying the total number of gas molecules with average kinetic energy of each molecule, the internal energy of the gas is obtained.

Internal energy of ideal gas $U = N\left(\frac{1}{2}m\overline{v^2}\right)$

By using equation (9.9)

$$U = \frac{3}{2}NkT \qquad (9.11)$$

From equation (9.11), we understand that the internal energy of an ideal gas depends only on absolute temperature and is independent of pressure and volume.

EXAMPLE 9.1

A football at 27°C has 0.5 mole of air molecules. Calculate the internal energy of air in the ball.

Solution

The internal energy of ideal gas = $\frac{3}{2}\ NkT$.

The number of air molecules is given in terms of number of moles so, rewrite the expression as follows

$$U = \frac{3}{2}\mu RT$$

Since $Nk = \mu R$. Here μ is number of moles.

Gas constant $R = 8.31\ \frac{J}{mol\,k}$

Temperature T =273+27=300K

$U = \frac{3}{2} \times 0.5 \times 8.31 \times 300 = 1869.75J$

This is approximately equivalent to the kinetic energy of a man of 57 kg running with a speed of 8 m s^{-1}.

9.2.3 Relation between pressure and mean kinetic energy

From earlier section, the internal energy of the gas is given by

$$U = \frac{3}{2}NkT$$

The above equation can also be written as

$$U = \frac{3}{2}\ PV$$

since $PV = NkT$

$$P = \frac{2}{3}\frac{U}{V} = \frac{2}{3}u \qquad (9.12)$$

From the equation (9.12), we can state that the pressure of the gas is equal to two thirds of internal energy per unit volume or internal energy density ($u = \frac{U}{V}$).

Writing pressure in terms of mean kinetic energy density using equation (9.6)

$$P = \frac{1}{3}nm\overline{v^2} = \frac{1}{3}\rho\overline{v^2} \qquad (9.13)$$

where $\rho = nm$ = *mass density* (Note n is number density)

Multiply and divide R.H.S of equation (9.13) by 2, we get

$$P = \frac{2}{3}\left(\frac{\rho}{2}\overline{v^2}\right) \qquad (9.14)$$

$$P = \frac{2}{3}\overline{KE}$$

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From the equation (9.14), pressure is equal to 2/3 of mean kinetic energy per unit volume.

9.2.4 Some elementary deductions from kinetic theory of gases

Boyle's law:

From equation (9.12), we know that $PV = \frac{2}{3}U$

But the internal energy of an ideal gas is equal to N times the average kinetic energy ($\in$) of each molecule.

$$U = N\in$$

For a fixed temperature, the average translational kinetic energy $\in$ will remain constant. It implies that

$$PV = \frac{2}{3}N\in \qquad \text{Thus} \quad \text{PV = constant}$$

Therefore, *pressure of a given gas is inversely proportional to its volume provided the temperature remains constant. This is Boyle's law.*

Charles' law:

From the equation (9.12), we get $PV = \frac{2}{3}U$

For a fixed pressure, the volume of the gas is proportional to internal energy of the gas or average kinetic energy of the gas and the average kinetic energy is directly proportional to absolute temperature. It implies that

$$V \propto T \text{ or } \frac{V}{T} = \text{constant}$$

This is Charles' law.

Avogadro's law:

This law states that at constant temperature and pressure, equal volumes of all gases contain the same number of molecules. For two different gases at the same temperature and pressure, according to kinetic theory of gases,

From equation (9.6)

$$P = \frac{1}{3}\frac{N_1}{V}m_1\overline{v_1^2} = \frac{1}{3}\frac{N_2}{V}m_2\overline{v_2^2} \qquad (9.15)$$

where $\overline{v_1^2}$ and $\overline{v_2^2}$ are the mean square speed for two gases and N_1 and N_2 are the number of gas molecules in two different gases.

At the same temperature, average kinetic energy per molecule is the same for two gases.

$$\frac{1}{2}m_1\overline{v_1^2} = \frac{1}{2}m_2\overline{v_2^2} \qquad (9.16)$$

Dividing the equation (9.15) by (9.16) we get $N_1 = N_2$

This is Avogadro's law. It is sometimes referred to as Avogadro's hypothesis or Avogadro's Principle.

9.2.5 Root mean square speed (v_{rms})

Root mean square speed (v_{rms}) is defined as the square root of the mean of the square of speeds of all molecules. It is denoted by $v_{\text{rms}} = \sqrt{\overline{v^2}}$

Equation (9.8) can be re-written as,

$$\text{mean square speed } \overline{v^2} = \frac{3kT}{m} \qquad (9.17)$$

root mean square speed,

$$v_{\text{rms}} = \sqrt{\frac{3kT}{m}} = 1.73\sqrt{\frac{kT}{m}} \qquad (9.18)$$

From the equation (9.18) we infer the following

(i) rms speed is directly proportional to square root of the temperature and inversely proportional to square root of mass of the molecule. At a given temperature the molecules of lighter mass move faster on an average than the molecules with heavier masses.

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Example: Lighter molecules like hydrogen and helium have high 'v_{rms}' than heavier molecules such as oxygen and nitrogen at the same temperature.

(ii) Increasing the temperature will increase the r.m.s speed of molecules

We can also write the v_{rms} in terms of gas constant R. Equation (9.18) can be rewritten as follows

$v_{rms} = \sqrt{\frac{3N_A kT}{N_A m}}$ Where N_A is Avogadro number.

Since $N_A k = R$ and $N_A m = M$ (molar mass)

The root mean square speed or r.m.s speed

$$v_{rms} = \sqrt{\frac{3RT}{M}} \quad (9.19)$$

The equation (9.6) can also be written in terms of rms speed $P = \frac{1}{3} nmv^2_{rms}$

since $v^2_{rms} = \overline{v^2}$

Note Root mean square speed is not the same as average speed. Average speed is 0.92 times of r.m.s speed.

Impact of v_{rms} in nature:

1. **Moon has no atmosphere.**

   The escape speed of gases on the surface of Moon is much less than the root mean square speeds of gases due to low gravity. Due to this all the gases escape from the surface of the Moon.

2. **No hydrogen in Earth's atmosphere**.

   As the root mean square speed of hydrogen is much greater than that of nitrogen, it easily escapes from the earth's atmosphere.

   In fact, the presence of nonreactive nitrogen instead of highly combustible hydrogen deters many disastrous consequences.

EXAMPLE 9.2

A room contains oxygen and hydrogen molecules in the ratio 3:1. The temperature of the room is 27°C. The molar mass of 0_2 is 32 g mol^{-1} and of H_2 is 2 g mol^{-1}. The value of gas constant R is 8.32 J mol^{-1} K^{-1}

Calculate

(a) rms speed of oxygen and hydrogen molecule

(b) Average kinetic energy per oxygen molecule and per hydrogen molecule

(c) Ratio of average kinetic energy of oxygen molecules and hydrogen molecules

Solution

(a) Absolute Temperature
T=27°C =27+273=300 K.

Gas constant R=8.32 J mol^{-1} k^{-1}

For Oxygen molecule: Molar mass M=32 g =32 x 10^{-3} kg mol^{-1}

rms speed $v_{rms} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3\times 8.32\times 300}{32\times 10^{-3}}}$

$= 483.73\ m\ s^{-1} \approx 484\ m\ s^{-1}$

For Hydrogen molecule:

Molar mass M = 2 × 10^{-3} kg mol^{-1}

rms speed $v_{rms} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3\times 8.32\times 300}{2\times 10^{-3}}}$

$= 1934\, m\, s^{-1} = 1.93\ k\, m\ \ s^{-1}$

Note that the rms speed is inversely proportional to $\sqrt{M}$ and the molar mass of oxygen is 16 times higher than molar mass of hydrogen. It implies that the rms speed of hydrogen is 4 times greater than rms speed of oxygen at the same temperature. $\frac{1934}{484} \approx 4$.

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(b) The average kinetic energy per molecule is $\frac{3}{2}kT$. It depends only on absolute temperature of the gas and is independent of the nature of molecules. Since both the gas molecules are at the same temperature, they have the same average kinetic energy per molecule. k is Boltzmaan constant.

$$\frac{3}{2}kT = \frac{3}{2}\times 1.38\times 10^{-23}\times 300 = 6.21\times 10^{-21}\,J$$

(c) Average kinetic energy of total oxygen molecules = $\frac{3}{2}N_O kT$ where N_o- number of oxygen molecules in the room

Average kinetic energy of total hydrogen molecules = $\frac{3}{2}N_H kT$ where N_H- number of hydrogen molecules in the room.

It is given that the number of oxygen molecules is 3 times more than number of hydrogen molecules in the room. So the ratio of average kinetic energy of oxygen molecules with average kinetic energy of hydrogen molecules is 3:1.

9.2.6 Mean (or) average speed ($\bar{v}$)

It is defined as the mean (or) average of all the speeds of molecules

If v_1, v_2, v_3.....v_N are the individual speeds of molecules then

$$\bar{v} = \frac{v_1 + v_2 + v_3 ..... + v_n}{N} = \sqrt{\frac{8RT}{\pi M}} = \sqrt{\frac{8kT}{\pi m}} \quad (9.20)$$

Here M- Molar Mass and m – mass of the molecule.

$$\bar{v} = 1.60\sqrt{\frac{kT}{m}} \quad (9.21)$$

9.2.7. Most probable speed (V_{mp})

It is defined as the speed acquired by most of the molecules of the gas.

$$v_{mp} = \sqrt{\frac{2RT}{M}} = \sqrt{\frac{2kT}{m}} \quad (9.22)$$

$$v_{mp} = 1.41\sqrt{\frac{kT}{m}} \quad (9.23)$$

The derivation of equations (9.20), (9.22) is beyond the scope of the book

Comparison of v_{rms}, $\bar{v}$ and v_{mp}

Among the speeds v_{rms} is the largest and v_{mp} is the least

$$v_{rms} > \bar{v} > v_{mp}$$

Ratio-wise,

$$v_{rms} : \bar{v} : v_{mp} = \sqrt{3} : \sqrt{8/\pi} : \sqrt{2} = 1.732 : 1.6 : 1.414$$

EXAMPLE 9.3

Ten particles are moving at the speed of 2, 3, 4, 5, 5, 5, 6, 6, 7 and 9 m s^{-1}. Calculate rms speed, average speed and most probable speed.

Solution

The average speed

$$\bar{v} = \frac{2+3+4+5+5+5+6+6+7+9}{10} = 5.2\,m\,s^{-1}$$

To find the rms speed, first calculate the mean square speed $\overline{v^2}$

$$\overline{v^2} = \frac{2^2+3^2+4^2+5^2+5^2+5^2+6^2+6^2+7^2+9^2}{10}$$

$$= 30.6\,m^2\,s^{-2}$$

The rms speed

$$v_{rms} = \sqrt{\overline{v^2}} = \sqrt{30.6} = 5.53\,m\,s^{-1}$$

The most probable speed is 5 m s^{-1} because three of the particles have that speed.

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EXAMPLE 9.4

Calculate the rms speed, average speed and the most probable speed of 1 mole of hydrogen molecules at 300 K. Neglect the mass of electron.

Solution

The hydrogen atom has one proton and one electron. The mass of electron is negligible compared to the mass of proton.

Mass of one proton = $1.67 \times 10^{-27} kg$.

One hydrogen molecule = 2 hydrogen atoms = $2 \times 1.67 \times 10^{-27} kg$.

The average speed

$$\bar{v} = \sqrt{\frac{8kT}{\pi m}} = 1.60\sqrt{\frac{kT}{m}} =$$

$$= 1.60\sqrt{\frac{(1.38\times10^{-23})\times(300)}{2(1.67\times10^{-27})}} = 1.78\times10^{3}\, m\, s^{-1}$$

(Boltzmann Constant k = $1.38 \times 10^{-23}\ J\ K^{-1}$)

The rms speed $\quad v_{rms} = \sqrt{\frac{3kT}{m}} = 1.73\sqrt{\frac{kT}{m}}$

$$= 1.73\sqrt{\frac{(1.38\times10^{-23})\times(300)}{2(1.67\times10^{-27})}} = 1.93\times10^{3}\, m\, s^{-1}$$

Most probable speed $\quad v_{mp} = \sqrt{\frac{2kT}{m}} = 1.41\sqrt{\frac{kT}{m}}$

$$= 1.41\sqrt{\frac{(1.38\times10^{-23})\times(300)}{2(1.67\times10^{-27})}} = 1.57\times10^{3}\, m\, s^{-1}$$

Note that $\quad v_{rms} > \bar{v} > v_{mp}$

9.2.8 Maxwell-Boltzmann speed distribution function

In a classroom, the air molecules are moving in random directions. The speed of each molecule is not the same even though macroscopic parameters like temperature and pressure are fixed. Each molecule collides with every other molecule and they exchange their speed. In the previous section we calculated the rms speed of each molecule and not the speed of each molecule which is rather difficult. In this scenario we can find the number of gas molecules that move with the speed of 5 m s^{-1} to 10 m s^{-1} or 10 m $s^{-1}$to 15 m s^{-1} etc. In general our interest is to find how many gas molecules have the range of speed from v to $v + dv$. This is given by Maxwell's speed distribution function.

$$N_v = 4\pi N\left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}} v^2 e^{-\frac{mv^2}{2kT}} \qquad (9.24)$$

The above expression is graphically shown as follows

Figure 9.3 Maxwell's molecular speed distribution

From the Figure 9.3, it is clear that, for a given temperature the number of molecules having lower speed increases parabolically (v^2) but decreases exponentially ($e^{-\frac{mv^2}{2kT}}$) after reaching most probable speed. The rms speed, average speed and most probable speed are indicated in the Figure 9.3. It can be seen that the rms speed is greatest among the three.

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To know the number of molecules in the range of speed between 50 m s^{-1} and 60 m s^{-1}, we need to integrate $\int_{50}^{60} 4\pi N\left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}} v^2 e^{-\frac{mv^2}{2kT}} dv =$ $= N(50\ to\ 60\ ms^{-1})$. In general the number of molecules within the range of speed v and v+dv is given by

$$\int_{v}^{v+dv} 4\pi N\left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}} v^2 e^{-\frac{mv^2}{2kT}} dv = N(v\ to\ v+dv)\ .$$

The exact integration is beyond the scope of the book. But we can infer the behavior of gas molecules from the graph.

(i) The area under the graph will give the total number of gas molecules in the system

(ii) Figure 9.4 shows the speed distribution graph for two different temperatures. As temperature increases, the peak of the curve is shifted to the right. It implies that the average speed of each molecule will increase. But the area under each graph is same since it represents the total number of gas molecules.

Note Interestingly once the gas molecule attains equilibrium, the number of molecules in the given range of speeds are fixed. For example if a molecule initially moving with speed 12 m s^{-1}, collides with some other molecule and changes its speed to 9 m s^{-1}, then the other molecule initially moving with different speed reaches the speed 12 m s^{-1} due to another collision. So in general once the gas molecules attain equilibrium, the number of molecules that lie in the range of v to v+dv is always fixed.

9.3 DEGREES OF FREEDOM

9.3.1 Definition

The minimum number of independent coordinates needed to specify the position and configuration of a thermo-dynamical system in space is called the degree of freedom of the system.

Figure 9.4 Maxwell distribution graph for two different temperatures

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Example:

1. A free particle moving along x-axis needs only one coordinate to specify it completely. So its degree of freedom is one.
2. Similarly a particle moving over a plane has two degrees of freedom.
3. A particle moving in space has three degrees of freedom.

Suppose if we have N number of gas molecules in the container, then the total number of degrees of freedom is f = 3N.

But, if the system has q number of constraints (restrictions in motion) then the degrees of freedom decreases and it is equal to $f = 3N\text{-}q$ where N is the number of particles.

9.3.2 Monoatomic molecule

A monoatomic molecule by virtue of its nature has only three translational degrees of freedom.

Therefore $f = 3$

Example: Helium, Neon, Argon

9.3.3 Diatomic molecule

There are two cases.

1. At Normal temperature

A molecule of a diatomic gas consists of two atoms bound to each other by a force of attraction. Physically the molecule can be regarded as a system of two point masses fixed at the ends of a massless elastic spring.

The center of mass lies in the center of the diatomic molecule. So, the motion of the center of mass requires three translational degrees of freedom (figure 9.5 a). In addition, the diatomic molecule can rotate about three mutually perpendicular axes (figure 9.5 b). But the moment of inertia about its own axis of rotation is negligible (about y axis in the figure 9.5). Therefore, it has only two rotational degrees of freedom (one rotation is about Z axis and another rotation is about X axis). Therefore totally there are five degrees of freedom.

Figure 9.5 Degree of freedom of diatomic molecule

$$f = 5$$

2. At High Temperature

At a very high temperature such as 5000 K, the diatomic molecules possess additional

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two degrees of freedom due to vibrational motion[one due to kinetic energy of vibration and the other is due to potential energy] (Figure 9.5c). So totally there are seven degrees of freedom.

$$f = 7$$

Examples: Hydrogen, Nitrogen, Oxygen.

9.3.4 Triatomic molecules

There are two cases.

Linear triatomic molecule

In this type, two atoms lie on either side of the central atom as shown in the Figure 9.6

Figure 9.6 A linear triatomic molecule.

Linear triatomic molecule has three translational degrees of freedom. It has two rotational degrees of freedom because it is similar to diatomic molecule except there is an additional atom at the center. At normal temperature, linear triatomic molecule will have five degrees of freedom. At high temperature it has two additional vibrational degrees of freedom. So a linear triatomic molecule has seven degrees of freedom.

Example: Carbon dioxide.

Non-linear triatomic molecule

In this case, the three atoms lie at the vertices of a triangle as shown in the Figure 9.7

Figure 9.7 A non-linear triatomic molecule

It has three translational degrees of freedom and three rotational degrees of freedom about three mutually orthogonal axes. The total degrees of freedom, f = 6

Example: Water, Sulphurdioxide.

9.4 LAW OF EQUIPARTITION OF ENERGY

We have seen in Section 9.2.1 that the average kinetic energy of a molecule moving in x direction is $\frac{1}{2}m\overline{v_x^2} = \frac{1}{2}kT$.

Similarly, when the motion is in y direction, $\frac{1}{2}m\overline{v_y^2} = \frac{1}{2}kT$ and for the motion along z direction, $\frac{1}{2}m\overline{v_z^2} = \frac{1}{2}kT$.

According to kinetic theory, the average kinetic energy of system of molecules in thermal equilibrium at temperature T is uniformly distributed to all degrees of freedom (x or y or

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z directions of motion) so that each degree of freedom will get $\frac{1}{2}kT$ *of energy. This is called law of equipartition of energy.*

Average kinetic energy of a monatomic molecule (with f=3) = $3\times\frac{1}{2}kT=\frac{3}{2}kT$

Average kinetic energy of diatomic molecule at low temperature (with f = 5) = $5\times\frac{1}{2}kT=\frac{5}{2}kT$

Average kinetic energy of a diatomic molecule at high temperature (with f =7) $=7\times\frac{1}{2}kT=\frac{7}{2}kT$

Average kinetic energy of linear triatomic molecule (with f = 7) = $7\times\frac{1}{2}kT=\frac{7}{2}kT$

Average kinetic energy of non linear triatomic molecule (with f = 6) = $6\times\frac{1}{2}kT=3kT$

9.4.1 Application of law of equipartition energy in specific heat of a gas

Meyer's relation $C_P - C_V = R$ connects the two specific heats for one mole of an ideal gas.

Equipartition law of energy is used to calculate the value of $C_P - C_V$ and the ratio between them $\gamma = \frac{C_P}{C_V}$. Here γ is called adiabatic exponent.

i) Monatomic molecule

Average kinetic energy of a molecule

$$=\left[\frac{3}{2}kT\right]$$

Total energy of a mole of gas

$$=\frac{3}{2}kT\times N_A=\frac{3}{2}RT$$

For one mole, the molar specific heat at constant volume $C_V=\frac{dU}{dT}=\frac{d}{dT}\left[\frac{3}{2}RT\right]$

$$C_V=\left[\frac{3}{2}R\right]$$

$$C_P=C_V+R=\frac{3}{2}R+R=\frac{5}{2}R$$

The ratio of specific heats,

$$\gamma=\frac{C_P}{C_V}=\frac{\frac{5}{2}R}{\frac{3}{2}R}=\frac{5}{3}=1.67$$

ii) Diatomic molecule

Average kinetic energy of a diatomic molecule at low temperature = $\frac{5}{2}kT$

Total energy of one mole of gas

$$=\frac{5}{2}kT\times N_A=\frac{5}{2}RT$$

(Here, the total energy is purely kinetic)

For one mole Specific heat at constant volume

$$C_V=\frac{dU}{dT}=\left[\frac{5}{2}RT\right]=\frac{5}{2}R$$

But $C_P=C_V+R=\frac{5}{2}R+R=\frac{7}{2}R$

$$\therefore\gamma=\frac{C_P}{C_V}=\frac{\frac{7}{2}R}{\frac{5}{2}R}=7/5=1.40$$

Energy of a diatomic molecule at high temperature is equal to $\frac{7}{2}RT$

$$C_V=\frac{dU}{dT}=\left[\frac{7}{2}RT\right]=\frac{7}{2}R$$

$$\therefore C_P=C_V+R=7/2\,R+R$$

$$C_P=9/2\,R$$

Note that the C_V and C_P are higher for diatomic molecules than the mono atomic molecules. It implies that to increase the temperature of

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diatomic gas molecules by 1°C it require more heat energy than monoatomic molecules.

$$\therefore \gamma = \frac{C_P}{C_V} = \frac{9/2\, R}{7/2\, R} = 9/7 = 1.28$$

iii) Triatomic molecule

a) Linear molecule

Energy of one mole= $\frac{7}{2}kT \times N_A = \frac{7}{2}RT$

$$C_V = \frac{dU}{dT} = \frac{d}{dT}\left[\frac{7}{2}RT\right]$$

$$C_V = \frac{7}{2}R$$

$$C_P = C_V + R = \frac{7}{2}R + R = \frac{9R}{2}$$

$$\therefore \gamma = \frac{C_P}{C_V} = \frac{\frac{9}{2}R}{\frac{7}{2}R} = \frac{9}{7} = 1.28$$

b) Non-linear molecule

Energy of a mole = $\frac{6}{2}kT \times N_A = \frac{6}{2}RT = 3RT$

$$C_V = \frac{dU}{dT} = 3R$$

$$C_P = C_V + R = 3R + R = 4R$$

$$\therefore \gamma = \frac{C_P}{C_V} = \frac{4R}{3R} = \frac{4}{3} = 1.33$$

Note that according to kinetic theory model of gases the specific heat capacity at constant volume and constant pressure are independent of temperature. But in reality it is not sure. The specific heat capacity varies with the temperature.

EXAMPLE 9.5

Find the adiabatic exponent γ for mixture of μ_1 moles of monoatomic gas and μ_2 moles of a diatomic gas at normal temperature (27°C) .

Solution

The specific heat of one mole of a monoatomic gas $C_V = \frac{3}{2}R$

For μ_1 mole, $C_V = \frac{3}{2}\mu_1 R \quad C_P = \frac{5}{2}\mu_1 R$

The specific heat of one mole of a diatomic gas

$$C_V = \frac{5}{2}R$$

For μ_2 mole, $C_V = \frac{5}{2}\mu_2 R \quad C_P = \frac{7}{2}\mu_2 R$

The specific heat of the mixture at constant volume $C_V = \frac{3}{2}\mu_1 R + \frac{5}{2}\mu_2 R$

The specific heat of the mixture at constant pressure $C_P = \frac{5}{2}\mu_1 R + \frac{7}{2}\mu_2 R$

The adiabatic exponent $\gamma = \frac{C_P}{C_V} = \frac{5\mu_1 + 7\mu_2}{3\mu_1 + 5\mu_2}$

9.5 MEAN FREE PATH

Usually the average speed of gas molecules is several hundred meters per second even at room temperature (27°C). Odour from an open perfume bottle takes some time to reach us even if we are closer to the room. The time delay is because the odour of the molecules cannot travel straight to us as it undergoes a lot of collisions with the nearby air molecules and moves in a zigzag path. This *average distance travelled by the molecule between two successive collisions is called mean free path (λ). We can calculate the mean free path based on kinetic theory.*

Expression for mean free path

We know from postulates of kinetic theory that the molecules of a gas are in random motion and they collide with each other. Between two successive collisions, a molecule

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moves along a straight path with uniform velocity. This path is called mean free path.

Consider a system of molecules each with diameter d. Let n be the number of molecules per unit volume.

Assume that only one molecule is in motion and all others are at rest as shown in the Figure 9.8

Figure 9.8 Mean free path

If a molecule moves with average speed v in a time t, the distance travelled is vt. In this time t, consider the molecule to move in an imaginary cylinder of volume $\pi d^2 vt$. It collides with any molecule whose center is within this cylinder. Therefore, the number of collisions is equal to the number of molecules in the volume of the imaginary cylinder. It is equal to $\pi d^2 vtn$. The total path length divided by the number of collisions in time t is the mean free path.

Mean free path, $\lambda = \dfrac{\textit{distance travelled}}{\textit{Number of collisions}}$

$$\lambda = \frac{vt}{n\pi d^2 vt} = \frac{1}{n\pi d^2} \qquad (9.25)$$

Though we have assumed that only one molecule is moving at a time and other molecules are at rest, in actual practice all the molecules are in random motion. So the average relative speed of one molecule with respect to other molecules has to be taken into account. After some detailed calculations (you will learn in higher classes) the correct expression for mean free path

$$\therefore \lambda = \frac{1}{\sqrt{2}n\pi d^2} \qquad (9.26)$$

The equation (9.26) implies that the mean free path is inversely proportional to number density. When the number density increases the molecular collisions increases so it decreases the distance travelled by the molecule before collisions.

Case1: Rearranging the equation (9.26) using 'm' (mass of the molecule)

$$\therefore \lambda = \frac{m}{\sqrt{2}\pi d^2 mn}$$

But mn=mass per unit volume = ρ (density of the gas)

$$\therefore \lambda = \frac{m}{\sqrt{2}\,\pi d^2 \rho} \qquad (9.27)$$

Also we know that PV = NkT

$$P = \frac{N}{V}kT = nkT$$

$$\therefore n = \frac{P}{kT}$$

Substituting n= $\dfrac{P}{kT}$ in equation (9.26), we get

$$\lambda = \frac{kT}{\sqrt{2}\pi d^2 P} \qquad (9.28)$$

The equation (9.28) implies the following

1. Mean free path increases with increasing temperature. As the temperature increases, the average speed of each molecule will increase. It is the reason why the smell of hot sizzling food reaches several meter away than smell of cold food.
2. Mean free path increases with decreasing pressure of the gas and diameter of the gas molecules.

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EXAMPLE 9.6

An oxygen molecule is travelling in air at 300 K and 1 atm, and the diameter of oxygen molecule is $1.2 \times 10^{-10} m$. Calculate the mean free path of oxygen molecule.

Solution

From (9.26) $$\lambda = \frac{1}{\sqrt{2}\pi n d^2}$$

We have to find the number density n

By using ideal gas law

$$n = \frac{N}{V} = \frac{P}{kT} = \frac{101.3 \times 10^3}{1.381 \times 10^{-23} \times 300}$$

$$= 2.449 \times 10^{25}\ molecues/m^3$$

$$\lambda = \frac{1}{\sqrt{2} \times \pi \times 2.449 \times 10^{25} \times (1.2 \times 10^{-10})^2}$$

$$= \frac{1}{15.65 \times 10^5}$$

$$\lambda = 0.63 \times 10^{-6} m$$

9.6 BROWNIAN MOTION

In 1827, Robert Brown, a botanist reported that grains of pollen suspended in a liquid moves randomly from one place to other. The random (Zig - Zag path) motion of pollen suspended in a liquid is called Brownian motion. In fact we can observe the dust particle in water moving in random directions. This discovery puzzled scientists for long time. There were a lot of explanations for pollen or dust to move in random directions but none of these explanations were found adequate. After a systematic study, Wiener and Gouy proposed that Brownian motion is due to the bombardment of suspended particles by molecules of the surrounding fluid. But during 19th century people did not accept that every matter is made up of small atoms or molecules. In the year 1905, Einstein gave systematic theory of Brownian motion based on kinetic theory and he deduced the average size of molecules.

According to kinetic theory, any particle suspended in a liquid or gas is continuously bombarded from all the directions so that the mean free path is almost negligible. This leads to the motion of the particles in a random and zig–zag manner as shown in Figure 9.9. But when we put our hand in water it causes no random motion because the mass of our hand is so large that the momentum transferred by the molecular collision is not enough to move our hand.

Figure 9.9 Particles in Brownian motion

Factors affecting Brownian Motion

1. Brownian motion increases with increasing temperature.
2. Brownian motion decreases with bigger particle size, high viscosity and density of the liquid (or) gas.

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Note The experimental verification on Einstein's theoretical explanation of Brownian motion was done by Jean Perrin in the year 1908. The Einstein's explanation on Brownian motion and Perrin experiment was of great importance in physics because it provided direct evidence of reality of atoms and molecules.

SUMMARY

- Kinetic theory explains the microscopic origin of macroscopic parameters like temperature, pressure.
- The pressure exerted on the walls of gas container is due to the momentum imparted by the gas molecules on the walls.
- The pressure $P = \frac{1}{3} nm\overline{v^2}$. The pressure is directly proportional to the number density, mass of molecule and mean square speed.
- The temperature of a gas is a measure of the average translational kinetic energy per molecule of the gas. The average kinetic energy per molecule is directly proportional to absolute temperature of gas and independent of nature of molecules.
- The pressure is also equal to 2/3 of internal energy per unit volume.
- The rms speed of gas molecules = $v_{rms} = \sqrt{\frac{3kT}{m}} = 1.73\sqrt{\frac{kT}{m}}$
- The average speed of gas molecules $\bar{v} = \sqrt{\frac{8kT}{\pi m}} = 1.60\sqrt{\frac{kT}{m}}$
- The most probable speed of gas molecules $v_{mp} = \sqrt{\frac{2kT}{m}} = 1.41\sqrt{\frac{kT}{m}}$
- Among the speeds v_{rms} is the largest and v_{mp} is the least

$$v_{rms} > \bar{v} > v_{mp}$$

- The number of gas molecules in the range of speed v to v+dv is given by Maxwell-Boltzmann distribution

$$N_v dv = 4\pi N\left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}} v^2 e^{-\frac{mv^2}{2kT}} dv$$

- The minimum number of independent coordinates needed to specify the position and configuration of a thermodynamical system in space is called the degrees of freedom of the system. If a sample of gas has N molecules, then the total degrees of freedom f = 3N. If there are q number of constraints then total degrees of freedom f = 3N-q.
- For a monoatomic molecule, f = 3

  For a diatomic molecule (at normal temperature), f = 5

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For a diatomic molecule (at high temperature), f = 7

For a triatomic molecule (linear type), f = 7

For a triatomic molecule (non-linear type), f = 6

- The average kinetic energy of sample of gas is equally distributed to all the degrees of freedom. It is called law of equipartition of energy. Each degree of freedom will get $\frac{1}{2}kT$ energy.
- The ratio of molar specific heat at constant pressure and constant volume of a gas $\left[\gamma = \frac{C_p}{C_v}\right]$

For

Monoatomic molecule: 1.67

Diatomic molecule (Normal temperature) : 1.40

Diatomic molecule (High temperature): 1.28

Triatomic molecule (Linear type): 1.28.

Triatomic molecule (Non-linear type): 1.33

- The mean free path $\lambda = \frac{kT}{\sqrt{2}\pi d^2 P}$. The mean free path is directly proportional to temperature and inversely proportional to size of the molecule and pressure of the molecule
- The Brownian motion explained by Albert Einstein is based on kinetic theory. It proves the reality of atoms and molecules.

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CONCEPT MAP

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EVALUATION

I. Multiple choice questions

1. A particle of mass m is moving with speed u in a direction which makes 60° with respect to x axis. It undergoes elastic collision with the wall. What is the change in momentum in x and y direction?

(a) $\Delta p_x = -mu, \Delta p_y = 0$

(b) $\Delta p_x = -2mu, \Delta p_y = 0$

(c) $\Delta p_x = 0, \Delta p_y = mu$

(d) $\Delta p_x = mu, \Delta p_y = 0$

2. A sample of ideal gas is at equilibrium. Which of the following quantity is zero?

(a) rms speed

(b) average speed

(c) average velocity

(d) most probable speed

3. An ideal gas is maintained at constant pressure. If the temperature of an ideal gas increases from 100K to 10000K then the rms speed of the gas molecules

(a) increases by 5 times

(b) increases by 10 times

(c) remains same

(d) increases by 7 times

4. Two identically sized rooms A and B are connected by an open door. If the room A is air conditioned such that its temperature is 4°C lesser than room B, which room has more air in it?

(a) Room A

(b) Room B

(c) Both room has same air

(d) Cannot be determined

5. The average translational kinetic energy of gas molecules depends on

(a) number of moles and T

(b) only on T

(c) P and T

(d) P only

6. If the internal energy of an ideal gas U and volume V are doubled then the pressure

(a) doubles

(b) remains same

(c) halves

(d) quadruples

7. The ratio $\gamma = \frac{C_p}{C_v}$ for a gas mixture consisting of 8 g of helium and 16 g of oxygen is (Physics Olympiad -2005)

(a) 23/15

(b) 15/23

(c) 27/17

(d) 17/27

8. A container has one mole of monoatomic ideal gas. Each molecule has f degrees of freedom. What is the ratio of $\gamma = \frac{C_p}{C_v}$

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(a) f (b) $\frac{f}{2}$

(c) $\frac{f}{f+2}$ (d) $\frac{f+2}{f}$

9. If the temperature and pressure of a gas is doubled the mean free path of the gas molecules

(a) remains same

(b) doubled

(c) tripled

(d) quadrapoled

10. Which of the following shows the correct relationship between the pressure and density of an ideal gas at constant temperature?

11. A sample of gas consists of μ_1 moles of monoatomic molecules, μ_2 moles of diatomic molecules and μ_3 moles of linear triatomic molecules. The gas is kept at high temperature. What is the total number of degrees of freedom?

(a) $[3\mu_1 + 7(\mu_2 + \mu_3)]\, N_A$

(b) $[3\mu_1 + 7\,\mu_2 + 6\mu_3]\, N_A$

(c) $[7\mu_1 + 3(\mu_2 + \mu_3)]\, N_A$

(d) $[3\mu_1 + 6(\mu_2 + \mu_3)]\, N_A$

12. If s_P and s_V denote the specific heats of nitrogen gas per unit mass at constant pressure and constant volume respectively, then (JEE 2007)

(a) $s_P - s_V = 28R$ (b) $s_P - s_V = R/28$

(c) $s_P - s_V = R/14$ (d) $s_P - s_V = R$

13. Which of the following gases will have least rms speed at a given temperature?

(a) Hydrogen

(b) Nitrogen

(c) Oxygen

(d) Carbon dioxide

14. For a given gas molecule at a fixed temperature, the area under the Maxwell-Boltzmann distribution curve is equal to

(a) $\frac{PV}{kT}$ (b) $\frac{kT}{PV}$

(c) $\frac{P}{NkT}$ (d) PV

15. The following graph represents the pressure versus number density for ideal gas at two different temperatures T_1 and T_2. The graph implies

(a) $T_1 = T_2$

(b) $T_1 > T_2$

(c) $T_1 < T_2$

d) Cannot be determined

Answers:

1) a	2) c	3) b	4) a
5) a	6) b	7) c	8) d
9) a	10) d	11) a	12) b
13) d	14) a	15) b	

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II. Short answer questions

1. What is the microscopic origin of pressure?
2. What is the microscopic origin of temperature?
3. Why moon has no atmosphere?
4. Write the expression for rms speed, average speed and most probable speed of a gas molecule.
5. What is the relation between the average kinetic energy and pressure?
6. Define the term degrees of freedom.
7. State the law of equipartition of energy.
8. Define mean free path and write down its expression.
9. Deduce Charles' law based on kinetic theory.
10. Deduce Boyle's law based on kinetic theory.
11. Deduce Avogadro's law based on kinetic theory.
12. List the factors affecting the mean free path.
13. What is the reason for Brownian motion?

III. Long answer questions

1. Write down the postulates of kinetic theory of gases.
2. Derive the expression of pressure exerted by the gas on the walls of the container.
3. Explain in detail the kinetic interpretation of temperature.
4. Describe the total degrees of freedom for monoatomic molecule, diatomic molecule and triatomic molecule.
5. Derive the ratio of two specific heat capacities of monoatomic, diatomic and triatomic molecules
6. Explain in detail the Maxwell Boltzmann distribution function.
7. Derive the expression for mean free path of the gas.
8. Describe the Brownian motion.

IV Numerical Problems

1. A fresh air is composed of nitrogen N_2(78%) and oxygen O_2(21%). Find the rms speed of N_2 and O_2 at 20°C.

   Ans: For N_2, $v_{rms} = 511\ m\ s^{-1}$

   For O_2, $v_{rms} = 478\ m\ s^{-1}$
2. If the rms speed of methane gas in the Jupiter's atmosphere is 471.8 m s^{-1}, show that the surface temperature of Jupiter is sub-zero.

   Ans: -130°C
3. Calculate the temperature at which the rms velocity of a gas triples its value at S.T.P. (standard temperature $T_1 = 273$ K)

   Ans: $T_2 = 2457$ K
4. A gas is at temperature 80°C and pressure $5 \times 10^{-10} N\,m^{-2}$. What is the number of molecules per m^3 if Boltzmann's constant is $1.38 \times 10^{-23}\ J\,K^{-1}$

   Ans: 1.02×10^{11}
5. If 10^{20} oxygen molecules per second strike 4 cm^2 of wall at an angle of 30° with the normal when moving at a speed of 2×10^3 m s^{-1}, find the pressure exerted on the wall. (mass of one oxygen atom = 2.67×10^{-26} kg)

   Ans: 46.2 N m^{-2}
6. During an adiabatic process, the pressure of a mixture of monatomic and diatomic gases is found to be

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proportional to the cube of the temperature. Find the value of $\gamma = (C_p/C_V)$

Ans: 3/2

7. Calculate the mean free path of air molecules at STP. The diameter of N_2 and O_2 is about $3 \times 10^{-10}\, m$

Ans: $\lambda \approx 9.3 \times 10^{-8}\, m$

8. A gas made of a mixture of 2 moles of oxygen and 4 moles of argon at temperature T. Calculate the energy of the gas in terms of RT. Neglect the vibrational modes.

Ans: 11RT

9. Estimate the total number of air molecules in a room of capacity of 25 m^3 at a temperature of 27°C with 1atm pressure.

Ans: 6.1×10^{26} molecules

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3. H.C.Verma, Concepts of physics -Volume 2, Bharati Bhawan Publishers
4. Douglas C. Giancoli, Physics for scientist & Engineers, Pearson Publications, Fourth Edition
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ICT CORNER

Kinetic Theory of Gases

Through this activity you will be able to learn about the Brownian motion of the particles.

STEPS:

- Use the URL or scan the QR code to open 'interactive' simulation on "Brownian motion".
- Observe the movement of particles (Big balls) suspended in the gas at the initial stage. Observe the molecules in the gas by dragging the first slider. 'Drag to see what's actually going on'
- Find the variants such as "Energy", "Size Ratio" and "Mass Ratio" below the first slider. These variants can be lowered or increased.
- By dragging to the appropriate values of the variants, Brownian motion of the particles shall be observed.

Step1

Step2

Step3

Step4

URL:

http://labs.minutelabs.io/Brownian-Motion/

* Pictures are indicative only.

* If browser requires, allow **Flash Player** or **Java Script** to load the page.

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UNIT 10 OSCILLATIONS

Life is a constant oscillation between the sharp horns of a dilemma – H.L. Mencken

Learning Objectives

In this unit, the student is exposed to

- oscillatory motion – periodic motion and non-periodic motion
- simple harmonic motion
- angular harmonic motion
- linear harmonic oscillator – both horizontal and vertical
- combination of springs – series and parallel
- simple pendulum
- expression of energy – potential energy, kinetic energy and total energy
- graphical representation of simple harmonic motion
- types of oscillation – free, damped, maintained and forced oscillations
- concept of resonance

10.1 INTRODUCTION

Figure 10.1. Thanjavur dancing doll

Have you seen the Thanjavur Dancing Doll (In Tamil, it is called 'Thanjavur thalayatti bommai')?. It is a world famous Indian cultural doll (Figure 10.1). What does this doll do when disturbed? It will dance such that the head and body move continuously in a to and fro motion, until the movement gradually stops. Similarly, when we walk on the road, our hands and legs will move front and back. Again similarly, when a mother swings a cradle to make her child sleep, the cradle is made to move in to and fro motion. All these motions are different from the motion that we have discussed so far. These motions are shown in Figure 10.2. Generally, they are known as oscillatory motion or vibratory motion. A similar motion occurs even at atomic levels.

When the temperature is raised, the atoms in a solid vibrate about their mean position or equilibrium position. The study of vibrational motion is very important in engineering applications, such as, designing the structure of building, mechanical equipments, etc.

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Figure 10.2. Oscillatory or vibratory motions

10.1.1 Periodic and non-periodic motion

Motion in physics can be classified as repetitive (periodic motion) and non-repetitive (non-periodic motion).

1. **Periodic motion**
   Any motion which repeats itself in a fixed time interval is known as periodic motion.
   Examples : Hands in pendulum clock, swing of a cradle, the revolution of the Earth around the Sun, waxing and waning of Moon, etc.
2. **Non-Periodic motion**
   Any motion which does not repeat itself after a regular interval of time is known as non-periodic motion.
   Example : Occurance of Earth quake, eruption of volcano, etc.

EXAMPLE 10.1

Classify the following motions as periodic and non-periodic motions?.

a. Motion of Halley's comet.
b. Motion of clouds.
c. Moon revolving around the Earth.

Solution

a. Periodic motion
b. Non-periodic motion
c. Periodic motion

EXAMPLE 10.2

Which of the following functions of time represent periodic and non-periodic motion?.

a. $\sin \omega t + \cos \omega t$
b. $\ln \omega t$

Solution

a. Periodic
b. Non-periodic

Question to ponder

Discuss "what will happen if the motion of the Earth around the Sun is not a periodic motion".

10.1.2 Oscillatory motion

When an object or a particle moves back and forth repeatedly for some duration of time its motion is said to be oscillatory (or vibratory). Examples; our heart beat, swinging motion of the wings of an insect, grandfather's clock (pendulum clock), etc. Note that all oscillatory motion are periodic whereas all periodic motions need not be oscillation in nature. see Figure 10.3

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Figure 10.3 Oscillatory or vibratory motions

10.2 SIMPLE HARMONIC MOTION (SHM)

Figure 10.4 Simple Harmonic Motion

Note

A simple harmonic motion is a special type of oscillatory motion. But all oscillatory motions need not be simple harmonic .

Simple harmonic motion is a special type of oscillatory motion in which the acceleration or force on the particle is directly proportional to its displacement from a fixed point and is always directed towards that fixed point. In one dimensional case, let x be the displacement of the particle and a_x be the acceleration of the particle, then

$$a_x \propto x \qquad (10.1)$$

$$a_x = -\,b\,x \qquad (10.2)$$

where b is a constant which measures acceleration per unit displacement and dimensionally it is equal to T^{-2}. By multiplying by mass of the particle on both sides of equation (10.2) and from Newton's second law, the force is

$$F_x = -\,k\,x \qquad (10.3)$$

where k is a force constant which is defined as force per unit length. The negative sign indicates that displacement and force (or acceleration) are in opposite directions. This means that when the displacement of the particle is taken towards right of equilibrium position (x takes positive value), the force (or acceleration) will point towards equilibrium (towards left) and similarly, when the

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displacement of the particle is taken towards left of equilibrium position (x takes negative value), the force (or acceleration) will point towards equilibrium (towards right). This type of force is known as ***restoring force*** *because it always directs the particle executing simple harmonic motion to restore to its original (equilibrium or mean) position*. This force (restoring force) is central and attractive whose center of attraction is the equilibrium position.

In order to represent in two or three dimensions, we can write using vector notation

$$\vec{F} = -k\vec{r} \qquad (10.4)$$

where $\vec{r}$ is the displacement of the particle from the chosen origin. Note that the force and displacement have a linear relationship. This means that the exponent of force $\vec{F}$ and the exponent of displacement $\vec{r}$ are unity. The sketch between cause (magnitude of force $|\vec{F}|$) and effect (magnitude of displacement $|\vec{r}|$) is a straight line passing through second and fourth quadrant as shown in Figure 10.5. By measuring slope $\frac{1}{k}$, one can find the numerical value of force constant k.

Figure 10.5 Force verses displacement graph

10.2.1 The projection of uniform circular motion on a diameter of SHM

Consider a particle of mass m moving with uniform speed v along the circumference of a circle whose radius is r in anti-clockwise direction (as shown in Figure 10.6). Let us assume that the origin of the coordinate system coincides with the center O of the circle. If ω is the angular velocity of the particle and θ the angular displacement of the particle at any instant of time t, then $\theta = \omega t$. By projecting the uniform circular motion on its diameter gives a simple harmonic motion. This means that we can associate

Figure 10.6 Projection of moving particle on a circle on a diameter

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a map (or a relationship) between uniform circular (or revolution) motion to vibratory motion. Conversely, any vibratory motion or revolution can be mapped to uniform circular motion. In other words, these two motions are similar in nature.

Let us first project the position of a particle moving on a circle, on to its vertical diameter or on to a line parallel to vertical diameter as shown in Figure 10.7. Similarly, we can do it for horizontal axis or a line parallel to horizontal axis.

The following figures explain the position of particle at different time :

Figure 10.7 The location of a particle at each instant as projected on a vertical axis

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As a specific example, consider a spring mass system (or oscillation of pendulum) as shown in Figure 10.8. When the spring moves up and down (or pendulum moves to and fro), the motion of the mass or bob is mapped to points on the circular motion.

Figure 10.8 Motion of spring mass (or simple pendulum) related to uniform circular motion

Thus, if a particle undergoes uniform circular motion then the projection of the particle on the diameter of the circle (or on a line parallel to the diameter) traces straightline motion which is simple harmonic in nature. The circle is known as reference circle of the simple harmonic motion. *The simple harmonic motion can also be defined as the motion of the projection of a particle on any diameter of a circle of reference.*

Activity

a. Sketch the projection of ***spiral in*** motion as a wave form.
b. Sketch the projection of ***spiral out*** motion as a wave form.

10.2.2 Displacement, velocity, acceleration and its graphical representation - SHM

Figure 10.9 Displacement, velocity and acceleration of a particle at some instant of time

The distance travelled by the vibrating particle at any instant of time t from its mean position is known as displacement.

Let P be the position of the particle on a circle of radius A at some instant of time t as shown in Figure 10.9. Then its displacement y at that instant of time t can be derived as follows

In ΔOPN

$$\sin\theta = \frac{ON}{OP} \Rightarrow ON = OP\sin\theta \qquad (10.5)$$

But $\theta = \omega t$, ON = y and OP = A

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$$y = A \sin \omega t \qquad (10.6)$$

The displacement y takes maximum value (which is equal to A) when $\sin \omega t = 1$. This *maximum displacement from the mean position is known as amplitude (A) of the vibrating particle.* For simple harmonic motion, the amplitude is constant. But, in general, for any motion other than simple harmonic, the amplitude need not be constant, it may vary with time.

Velocity

The rate of change of displacement is velocity. Taking derivative of equation (10.6) with respect to time, we get

$$v = \frac{dy}{dt} = \frac{d}{dt}(A \sin \omega t)$$

For circular motion (of constant radius), amplitude A is a constant and further, for uniform circular motion, angular velocity ω is a constant. Therefore,

$$v = \frac{dy}{dt} = A\,\omega \cos \omega t \qquad (10.7)$$

Using trigonometry identity,

$\sin^2 \omega t + \cos^2 \omega t = 1 \Rightarrow \cos \omega t = \sqrt{1-\sin^2 \omega t}$

we get

$$v = A\omega\sqrt{1-\sin^2 \omega t}$$

From equation (10.6),

$$\sin \omega t = \frac{y}{A}$$

$$v = A\omega\sqrt{1-\left(\frac{y}{A}\right)^2}$$

$$v = \omega\sqrt{A^2 - y^2} \qquad (10.8)$$

From equation (10.8), when the displacement $y = 0$, the velocity $v = \omega A$ (maximum) and for the maximum displacement $y = A$, the velocity $v = 0$ (minimum). As displacement increases from zero to maximum, the velocity decreases from maximum to zero. This is repeated. Since velocity is a vector quantity, equation (10.7) can also be deduced by resolving in to components.

Acceleration

The rate of change of velocity is acceleration. Taking derivative of equation 10.7 with respect to time,

$$a = \frac{dv}{dt} = \frac{d}{dt}\left(A\omega \cos \omega t\right)$$

$$a = -\omega^2 A \sin \omega t = -\omega^2 y \qquad (10.9)$$

$$\therefore \quad a = \frac{d^2 y}{dt^2} = -\omega^2 y \qquad (10.10)$$

From the Table 10.1 and figure 10.10, we observe that at the mean position

Table 10.1 Displacement, velocity and acceleration at different instant of time.

Time	0	$\frac{T}{4}$	$\frac{2T}{4}$	$\frac{3T}{4}$	$\frac{4T}{4} = T$
ωt	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
Displacement $y = A \sin \omega t$	0	A	0	$-A$	0
Velocity $v = A\,\omega \cos \omega t$	$A\,\omega$	0	$-A\,\omega$	0	$A\,\omega$
Acceleration $a = -A\,\omega^2 \sin \omega t$	0	$-A\,\omega^2$	0	$A\,\omega^2$	0

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Figure 10.10 Variation of displacement, velocity and acceleration at different instant of time

(y = 0), velocity of the particle is maximum but the acceleration of the particle is zero. At the extreme position ($y = \pm A$), the velocity of the particle is zero but the acceleration is maximum $\mp A\omega^2$ acting in the opposite direction.

EXAMPLE 10.3

Which of the following represent simple harmonic motion?

(i) $x = A \sin \omega t + B \cos \omega t$

(ii) $x = A \sin \omega t + B \cos 2\omega t$

(iii) $x = A\, e^{i\omega t}$

(iv) $x = A \ln \omega t$

Solution

(i) $x = A \sin \omega t + B \cos \omega t$

$$\frac{dx}{dt} = A\,\omega \cos \omega t - B\,\omega \sin \omega t$$

$$\frac{d^2x}{dt^2} = -\,\omega^2\,(A \sin \omega t + B \cos \omega t)$$

$$\frac{d^2x}{dt^2} = -\,\omega^2 x$$

This differential equation is similar to the differential equation of SHM (equation 10.10).

Therefore, $x = A \sin \omega t + B \cos \omega t$ represents SHM.

(ii) $x = A \sin \omega t + B \cos 2\omega t$

$$\frac{dx}{dt} = A\,\omega \cos \omega t - B\,(2\omega) \sin 2\omega t$$

$$\frac{d^2x}{dt^2} = -\,\omega^2\,(A \sin \omega t + 4B \cos 2\omega t)$$

$$\frac{d^2x}{dt^2} \neq -\,\omega^2 x$$

This differential equation is not like the differential equation of a SHM (equation 10.10). Therefore, $x = A \sin \omega t + B \cos 2\omega t$ does not represent SHM.

(iii) $x = A\, e^{i\omega t}$

$$\frac{dx}{dt} = A\, i\omega e^{i\omega t}$$

$$\frac{d^2x}{dt^2} = -A\,\omega^2\, e^{i\omega t} = -\omega^2 x \qquad (\because\ i^2 = -1)$$

This differential equation is like the differential equation of SHM (equation 10.10). Therefore, $x = A\, e^{i\omega t}$ represents SHM.

(iv) $x = A \ln \omega t$

$$\frac{dx}{dt} = \left(\frac{A}{\omega t}\right)\omega = \frac{A}{t}$$

$$\frac{d^2x}{dt^2} = -\frac{A}{t^2} \Rightarrow \frac{d^2x}{dt^2} \neq -\omega^2 x$$

This differential equation is not like the differential equation of a SHM (equation 10.10). Therefore, $x = A \ln \omega t$ does not represent SHM.

EXAMPLE 10.4

Consider a particle undergoing simple harmonic motion. The velocity of the particle at position x_1 is v_1 and velocity of the particle at position x_2 is v_2. Show that the ratio of time period and amplitude is

$$\frac{T}{A} = 2\pi\sqrt{\frac{x_2^2 - x_1^2}{v_1^2 x_2^2 - v_2^2 x_1^2}}$$

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Solution

Using equation (10.8)

$v = \omega\sqrt{A^2 - x^2} \Rightarrow v^2 = \omega^2 (A^2 - x^2)$

Therefore, at position x_1,

$$v_1^2 = \omega^2(A^2 - x_1^2) \quad (1)$$

Similarly, at position x_2,

$$v_2^2 = \omega^2(A^2 - x_2^2) \quad (2)$$

Subtrating (2) from (1), we get

$$v_1^2 - v_2^2 = \omega^2(A^2 - x_1^2) - \omega^2(A^2 - x_2^2)$$

$$= \omega^2(x_2^2 - x_1^2)$$

$$\omega = \sqrt{\frac{v_1^2 - v_2^2}{x_2^2 - x_1^2}} \Rightarrow T = 2\pi\sqrt{\frac{x_2^2 - x_1^2}{v_1^2 - v_2^2}} \quad (3)$$

Dividing (1) and (2), we get

$$\frac{v_1^2}{v_2^2} = \frac{\omega^2\left(A^2 - x_1^2\right)}{\omega^2\left(A^2 - x_2^2\right)} \Rightarrow A = \sqrt{\frac{v_1^2 x_2^2 - v_2^2 x_1^2}{v_1^2 - v_2^2}} \quad (4)$$

Dividing equation (3) and equation (4), we have

$$\frac{T}{A} = 2\pi\sqrt{\frac{x_2^2 - x_1^2}{v_1^2 x_2^2 - v_2^2 x_1^2}}$$

10.2.3 Time period, frequency, phase, phase difference and epoch in SHM.

(i) **Time period**

The time period is defined as the time taken by a particle to complete one oscillation. It is usually denoted by T. For one complete revolution, the time taken is t = T, therefore

$$\omega T = 2\pi \Rightarrow T = \frac{2\pi}{\omega} \quad (10.11)$$

Then, the displacement of a particle executing simple harmonic motion can be written either as sine function or cosine function.

$$y(t) = A \sin \frac{2\pi}{T} t \text{ or } y(t) = A \cos \frac{2\pi}{T} t$$

where T represents the time period. Suppose the time t is replaced by $t + T$, then the function

$$y(t + T) = A \sin \frac{2\pi}{T} (t + T)$$

$$= A \sin\left(\frac{2\pi}{T} t + 2\pi\right)$$

$$= A \sin \frac{2\pi}{T} t = y(t)$$

$$y(t + T) = y(t)$$

Thus, the function repeats after one time period. This $y(t)$ is an example of periodic function.

(ii) **Frequency and angular frequency**

The number of oscillations produced by the particle per second is called frequency. It is denoted by f. SI unit for frequency is s^{-1} or hertz (In symbol, Hz).

Mathematically, frequency is related to time period by

$$f = \frac{1}{T} \quad (10.12)$$

The number of cycles (or revolutions) per second is called angular frequency. It is usually denoted by the Greek small letter 'omega', ω.

Comparing equation (10.11) and equation (10.12), angular frequency and frequency are related by

$$\omega = 2\pi f \quad (10.13)$$

SI unit for angular frequency is rad s^{-1}. (read it as radian per second)

(iii) **Phase**

The phase of a vibrating particle at any instant completely specifies the state of the particle. It expresses the position and direction of motion of the particle at that instant with respect to its mean position (Figure 10.11).

$$y = A \sin(\omega t + \varphi_0) \quad (10.14)$$

where $\omega t + \varphi_0 = \varphi$ *is called the phase of the vibrating particle. At time* $t = 0\,s$ *(initial time), the phase* $\varphi = \varphi_0$ *is called epoch (initial phase) where* φ_0 *is called the angle of epoch.*

Phase difference: Consider two particles executing simple harmonic motions. Their

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equations are $y_1 = A \sin(\omega t + \varphi_1)$ and $y_2 = A \sin(\omega t + \varphi_2)$, then the phase difference $\Delta\varphi = (\omega t + \varphi_2) - (\omega t + \varphi_1) = \varphi_2 - \varphi_1$.

Figure 10.11 The phase of vibrating particle at two instant of time.

EXAMPLE 10.5

A nurse measured the average heart beats of a patient and reported to the doctor in terms of time period as 0.8 *s*. Express the heart beat of the patient in terms of number of beats measured per minute.

Solution

Let the number of heart beats measured be *f*. Since the time period is inversely proportional to the heart beat, then

$$f = \frac{1}{T} = \frac{1}{0.8} = 1.25\,s^{-1}$$

One minute is 60 second,

$$\left(1 \text{ second} = \frac{1}{60} \text{ minute} \Rightarrow 1\ s^{-1} = 60\ min^{-1}\right)$$

$f = 1.25\ s^{-1} \Rightarrow f = 1.25 \times 60\ min^{-1} = 75$ beats per minute

EXAMPLE 10.6

Calculate the amplitude, angular frequency, frequency, time period and initial phase for the simple harmonic oscillation given below

a. $y = 0.3 \sin(40\pi t + 1.1)$

b. $y = 2 \cos(\pi t)$

c. $y = 3 \sin(2\pi t - 1.5)$

Solution

Simple harmonic oscillation equation is $y = A \sin(\omega t + \varphi_0)$ or $y = A \cos(\omega t + \varphi_0)$

a. For the wave, $y = 0.3 \sin(40\pi t + 1.1)$

Amplitude is $A = 0.3$ unit

Angular frequency $\omega = 40\pi$ rad s^{-1}

Frequency $f = \frac{\omega}{2\pi} = \frac{40\pi}{2\pi} = 20\,Hz$

Time period $T = \frac{1}{f} = \frac{1}{20} = 0.05\,s$

Initial phase is $\varphi_0 = 1.1$ rad

b. For the wave, $y = 2 \cos(\pi t)$

Amplitude is $A = 2$ unit

Angular frequency $\omega = \pi$ rad s^{-1}

Frequency $f = \frac{\omega}{2\pi} = \frac{\pi}{2\pi} = 0.5\,Hz$

Time period $T = \frac{1}{f} = \frac{1}{0.5} = 2\,s$

Initial phase is $\varphi_0 = 0$ rad

c. For the wave, $y = 3 \sin(2\pi t + 1.5)$

Amplitude is $A = 3$ unit

Angular frequency $\omega = 2\pi$ rad s^{-1}

Frequency $f = \frac{\omega}{2\pi} = \frac{2\pi}{2\pi} = 1\,Hz$

Time period $T = \frac{1}{f} = \frac{1}{1} = 1\,s$

Initial phase is $\varphi_0 = 1.5$ rad

EXAMPLE 10.7

Show that for a simple harmonic motion, the phase difference between

a. displacement and velocity is $\frac{\pi}{2}$ radian or 90°.

b. velocity and acceleration is $\frac{\pi}{2}$ radian or 90°.

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c. displacement and acceleration is π radian or 180°.

Solution

a. The displacement of the particle executing simple harmonic motion $y = A \sin\omega t$

Velocity of the particle is

$$v = A\,\omega\cos\omega t = A\,\omega\sin\left(\omega t + \frac{\pi}{2}\right)$$

The phase difference between displacement and velocity is $\frac{\pi}{2}$.

b. The velocity of the particle is

$$v = A\,\omega\cos\omega t$$

Acceleration of the particle is

$$a = -A\omega^2\sin\omega t = A\omega^2\cos\left(\omega t + \frac{\pi}{2}\right)$$

The phase difference between velocity and acceleration is $\frac{\pi}{2}$.

c. The displacement of the particle is $y = A \sin\omega t$

Acceleration of the particle is

$$a = -A\,\omega^2\sin\omega t = A\,\omega^2\sin(\omega t + \pi)$$

The phase difference between displacement and acceleration is π radian.

10.3 ANGULAR SIMPLE HARMONIC MOTION

10.3.1 Time period and frequency of angular SHM

When a body is allowed to rotate freely about a given axis then the oscillation is known as the angular oscillation. The point

Figure 10.12 A body (disc) allowed to rotate freely about an axis

at which the resultant torque acting on the body is taken to be zero is called mean position. If the body is displaced from the mean position, then the resultant torque acts such that it is proportional to the angular displacement and this torque has a tendency to bring the body towards the mean position. (Note: Torque is explained in unit 5)

Let $\vec{\theta}$ be the angular displacement of the body and the resultant torque $\vec{\tau}$ acting on the body is

$$\vec{\tau} \propto \vec{\theta} \qquad (10.15)$$

$$\vec{\tau} = -\kappa\vec{\theta} \qquad (10.16)$$

κ is the restoring torsion constant, which is torque per unit angular displacement. If I is the moment of inertia of the body and $\vec{\alpha}$ is the angular acceleration then

$$\vec{\tau} = I\vec{\alpha} = -\kappa\vec{\theta}$$

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But $\vec{\alpha}=\frac{d^2\vec{\theta}}{dt^2}$ and therefore,

$$\frac{d^2\vec{\theta}}{dt^2}=-\frac{\kappa}{I}\vec{\theta} \qquad (10.17)$$

This differential equation resembles simple harmonic differential equation.

So, comparing equation (10.17) with simple harmonic motion given in equation (10.10), we have

$$\omega=\sqrt{\frac{\kappa}{I}}\, rad\, s^{-1} \qquad (10.18)$$

The frequency of the angular harmonic motion (from equation 10.13) is

$$f=\frac{1}{2\pi}\sqrt{\frac{\kappa}{I}}\, Hz \qquad (10.19)$$

The time period (from equation 10.12) is

$$T=2\pi\sqrt{\frac{I}{\kappa}} \text{ second} \qquad (10.20)$$

10.3.2 Comparison of Simple Harmonic Motion and Angular Simple Harmonic Motion

In linear simple harmonic motion, the displacement of the particle is measured in terms of linear displacement $\vec{r}$ The restoring force is $\vec{F}=-k\vec{r}$, where k is a spring constant or force constant which is force per unit displacement. In this case, the inertia factor is mass of the body executing simple harmonic motion.

In angular simple harmonic motion, the displacement of the particle is measured in terms of angular displacement $\vec{\theta}$. Here, the spring factor stands for torque constant i.e., the moment of the couple to produce unit angular displacement or the restoring torque per unit angular displacement. In this case, the inertia factor stands for moment of inertia of the body executing angular simple harmonic oscillation.

Table 10.2 Comparision of simple harmonic motion and angular harmonic motion

S.No	Simple Harmonic Motion	Angular Harmonic Motion
1.	The displacement of the particle is measured in terms of linear displacement $\vec{r}$.	The displacement of the particle is measured in terms of angular displacement $\vec{\theta}$ (also known as angle of twist).
2.	Acceleration of the particle is $\vec{a}=-\omega^2\vec{r}$	Angular acceleration of the particle is $\vec{\alpha}=-\omega^2\vec{\theta}$.
3.	Force, $\vec{F}=m\vec{a}$, where m is called mass of the particle.	Torque, $\vec{\tau}=I\vec{\alpha}$, where I is called moment of inertia of a body.
4.	The restoring force $\vec{F}=-k\vec{r}$, where k is restoring force constant.	The restoring torque $\vec{\tau}=-\kappa\vec{\theta}$, where the symbol κ (Greek alphabet is pronounced as '*kappa*') is called restoring torsion constant. It depends on the property of a particular torsion fiber.
5.	Angular frequency, $\omega=\sqrt{\frac{k}{m}}$ rad s^{-1}	Angular frequency, $\omega=\sqrt{\frac{\kappa}{I}}$ rad s^{-1}

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10.4 LINEAR SIMPLE HARMONIC OSCILLATOR (LHO)

10.4.1 Horizontal oscillations of a spring-mass system

Figure 10.13 Horizontal oscillation of a spring-mass system

Consider a system containing a block of mass m attached to a massless spring with stiffness constant or force constant or spring constant k placed on a smooth horizontal surface (frictionless surface) as shown in Figure 10.13. Let x_0 be the equilibrium position or mean position of mass m when it is left undisturbed. Suppose the mass is displaced through a small displacement x towards right from its equilibrium position and then released, it will oscillate back and forth about its mean position x_0. Let F be the restoring force (due to stretching of the spring) which is proportional to the amount of displacement of block. For one dimensional motion, mathematically, we have

$$F \propto x$$
$$F = -k\,x$$

where negative sign implies that the restoring force will always act opposite to the direction of the displacement. This equation is called Hooke's law (refer to unit 7). Notice that, the restoring force is linear with the displacement (i.e., the exponent of force and displacement are unity). This is not always true; in case if we apply a very large stretching force, then the amplitude of oscillations becomes very large (which means, force is proportional to displacement containing higher powers of x) and therefore, the oscillation of the system is not linear and hence, it is called non-linear oscillation. We restrict ourselves only to linear oscillations throughout our discussions, which means Hooke's law is valid (force and displacement have a linear relationship).

From Newton's second law, we can write the equation for the particle executing simple harmonic motion

$$m\frac{d^2x}{dt^2} = -k\,x$$

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x \qquad (10.21)$$

Comparing the equation (10.21) with simple harmonic motion equation (10.10), we get

$$\omega^2 = \frac{k}{m}$$

which means the angular frequency or natural frequency of the oscillator is

$$\omega = \sqrt{\frac{k}{m}}\ \text{rad}\ s^{-1} \qquad (10.22)$$

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The frequency of the oscillation is

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \text{ Hertz} \qquad (10.23)$$

and the time period of the oscillation is

$$T = \frac{1}{f} = 2\pi\sqrt{\frac{m}{k}} \text{ seconds} \qquad (10.24)$$

Notice that ***in simple harmonic motion, the time period of oscillation is independent of amplitude***. This is valid only if the amplitude of oscillation is small.

The solution of the differential equation of a SHM may be written as

$$x(t) = A\sin(\omega t + \varphi) \qquad (10.25)$$

Or

$$x(t) = A\cos(\omega t + \varphi) \qquad (10.26)$$

where A, ω and ϕ are constants. General solution for differential equation 10.21 is $x(t) = A\sin(\omega t + \varphi) + B\cos(\omega t + \varphi)$ where A and B are contants.

Note

(a) Since, mass is inertial property and spring constant is an elastic property,

Time period is $T = 2\pi\sqrt{\frac{m}{k}}$

$$T = 2\pi\sqrt{\frac{\text{Inertial property}}{\text{Elastic property}}} = 2\pi\sqrt{\left|\frac{\text{displacement}}{\text{acceleration}}\right|}$$

(b) $\frac{\text{Displacement}}{\text{acceleration}} = \frac{x}{\frac{d^2x}{dt^2}} = -\frac{m}{k}$, whose modulus value or magnitude is $\frac{m}{k}$

hence, time period $T = 2\pi\sqrt{\frac{m}{k}}$

10.4.2 Vertical oscillations of a spring

Figure 10.14 Springs

Figure 10.15 A massless spring with stiffness constant k

Let us consider a massless spring with stiffness constant or force constant k attached to a ceiling as shown in Figure 10.15. Let the length of the spring before loading mass m be L. If the block of mass m is attached to the other end of spring, then the spring elongates by a length l. Let F_1 be the restoring force due to stretching of spring. Due to mass m, the gravitational force acts vertically downward. We can draw free-body diagram for this system as shown in Figure 10.15. When the system is under equilibrium,

$$F_1 + mg = 0 \qquad (10.27)$$

But the spring elongates by small displacement l, therefore,

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$$F_1 \propto l \Rightarrow F_1 = -k\,l \qquad (10.28)$$

Substituting equation (10.28) in equation (10.27), we get

$$-k\,l + mg = 0$$
$$mg = kl$$
or
$$\frac{m}{k} = \frac{l}{g} \qquad (10.29)$$

Suppose we apply a very small external force on the mass such that the mass further displaces downward by a displacement y, then it will oscillate up and down. Now, the restoring force due to this stretching of spring (total extension of spring is $y + l$) is

$$F_2 \propto (y + l)$$
$$F_2 = -k\,(y + l) = -ky - kl \qquad (10.30)$$

Since, the mass moves up and down with acceleration $\frac{d^2y}{dt^2}$, by drawing the free body diagram for this case, we get

$$-ky - kl + mg = m\frac{d^2y}{dt^2} \qquad (10.31)$$

The net force acting on the mass due to this stretching is

$$F = F_2 + mg$$
$$F = -ky - kl + mg \qquad (10.32)$$

The gravitational force opposes the restoring force. Substituting equation (10.29) in equation (10.32), we get

$$F = -ky - kl + kl = -ky$$

Applying Newton's law, we get

$$m\frac{d^2y}{dt^2} = -k\,y$$
$$\frac{d^2y}{dt^2} = -\frac{k}{m}y \qquad (10.33)$$

The above equation is in the form of simple harmonic differential equation. Therefore, we get the time period as

$$T = 2\pi\sqrt{\frac{m}{k}} \text{ second} \qquad (10.34)$$

Note The time period obtained for horizontal oscillations of spring and for vertical oscillations of spring are found to be equal.

The time period can be rewritten using equation (10.29)

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{l}{g}} \text{ second} \qquad (10.35)$$

The acceleration due to gravity g can be computed from the formula

$$g = 4\pi^2\left(\frac{l}{T^2}\right) \text{ m s}^{-2} \qquad (10.36)$$

EXAMPLE 10.8

A spring balance has a scale which ranges from 0 to 25 kg and the length of the scale is 0.25m. It is taken to an unknown planet X where the acceleration due to gravity is 11.5 m s^{-2}. Suppose a body of mass M kg is suspended in this spring and made to oscillate with a period of 0.50 s. Compute the gravitational force acting on the body.

Solution

Let us first calculate the stiffness constant of the spring balance by using equation (10.29),

$$k = \frac{mg}{l} = \frac{25 \times 11.5}{0.25} = 1150\ N\,m^{-1}$$

The time period of oscillations is given by $T = 2\pi\sqrt{\frac{M}{k}}$, where M is the mass of the body.

Since, M is unknown, rearranging, we get

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$$M = \frac{kT^2}{4\pi^2} = \frac{(1150)(0.5)^2}{4\pi^2} = 7.3\,kg$$

The gravitational force acting on the body is $W = Mg = 7.3 \times 11.5 = 83.95\ N \approx 84\ N$

10.4.3 Combinations of springs

Figure 10.16 Combination of spring as a shock-absorber in the motor cycle

Spring constant or force constant, also called as stiffness constant, is a measure of the stiffness of the spring.

Larger the value of the spring constant, stiffer is the spring. This implies that we need to apply more force to compress or elongate the spring. Similarly, smaller the value of spring constant, the spring can be stretched (elongated) or compressed with lesser force.

Springs can be connected in two ways. Either the springs can be connected end to end, also known as series connection, or alternatively, connected in parallel. In the following subsection, we compute the effective spring constant when

a. Springs are connected in series
b. Springs are connected in parallel

a. **Springs connected in series**

When two or more springs are connected in series, we can replace (by removing) all the springs in series with an equivalent spring (effective spring) whose net effect is the same as if all the springs are in series connection. Given the value of individual spring constants k_1, k_2, k_3,... (known quantity), we can establish a mathematical relationship to find out an effective (or equivalent) spring constant k_s (unknown quantity). For simplicity, let us consider only two springs whose spring constant are k_1 and k_2 and which can be attached to a mass m as shown in Figure 10.17. The results thus obtained can be generalized for any number of springs in series.

Figure 10.17 Springs are connected in series

Let F be the applied force towards right as shown in Figure 10.18. Since the spring constants for different spring are different and the connection points between them is not rigidly fixed, the strings can stretch in different lengths. Let x_1 and x_2 be the elongation of springs from their equilibrium position (un-stretched position) due to the applied force F. Then, the net displacement of the mass point is

$$x = x_1 + x_2 \qquad (10.37)$$

From Hooke's law, the net force

Figure 10.18 Effective spring constant in series connection

$$F = -k_s(x_1 + x_2) \Rightarrow x_1 + x_2 = -\frac{F}{k_s} \quad (10.38)$$

For springs in series connection

$$-k_1x_1 = -k_2x_2 = F$$

$$\Rightarrow x_1 = -\frac{F}{k_1} \text{ and } x_2 = -\frac{F}{k_2} \quad (10.39)$$

Therefore, substituting equation (10.39) in equation (10.38), the *effective spring constant* can be calculated as

$$-\frac{F}{k_1} - \frac{F}{k_2} = -\frac{F}{k_s}$$

$$\frac{1}{k_s} = \frac{1}{k_1} + \frac{1}{k_2}$$

Or

$$k_s = \frac{k_1k_2}{k_1 + k_2} \text{ Nm}^{-1} \quad (10.40)$$

Suppose we have "*n*" springs connected in series, the effective spring constant in series is

$$\frac{1}{k_s} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} + \ldots + \frac{1}{k_n} = \sum_{i=1}^{n} \frac{1}{k_i} \quad (10.41)$$

If all spring constants are identical i.e., $k_1 = k_2 = \ldots = k_n = k$ then

$$\frac{1}{k_s} = \frac{n}{k} \Rightarrow k_s = \frac{k}{n} \quad (10.42)$$

This means that the effective spring constant reduces by the factor "*n*". Hence, for springs in series connection, the effective spring constant is lesser than the individual spring constants.

From equation (10.39), we have,

$$k_1x_1 = k_2x_2$$

Then the ratio of compressed distance or elongated distance x_1 and x_2 is

$$\frac{x_2}{x_1} = \frac{k_1}{k_2} \quad (10.43)$$

The elastic potential energy stored in first and second springs are $U_1 = \frac{1}{2}k_1x_1^2$ and $U_2 = \frac{1}{2}k_2x_2^2$ respectively. Then, their ratio is

$$\frac{U_1}{U_2} = \frac{\frac{1}{2}k_1x_1^2}{\frac{1}{2}k_2x_2^2} = \frac{k_1}{k_2}\left(\frac{x_1}{x_2}\right)^2 = \frac{k_2}{k_1} \quad (10.44)$$

Note

The reciprocal of stiffness constant is called flexibility constant or compliance, denoted by C. It is measured in m N^{-1}

If n springs are connected in series :

$$\text{net compliance } C_s = \sum_{i=1}^{n} C_i$$

If n springs are connected in parallel :

$$\frac{1}{C_p} = \sum_{i=1}^{n} \frac{1}{C_i}$$

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EXAMPLE 10.9

Consider two springs whose force constants are 1 N m^{-1} and 2 N m^{-1} which are connected in series. Calculate the effective spring constant (k_s) and comment on k_s.

Solution

$$k_1 = 1 \text{ N m}^{-1},\ k_2 = 2 \text{ N m}^{-1}$$

$$k_s = \frac{k_1 k_2}{k_1 + k_2} \text{ N m}^{-1}$$

$$k_s = \frac{1 \times 2}{1+2} = \frac{2}{3} \text{ N m}^{-1}$$

$$k_s < k_1 \text{ and } k_s < k_2$$

Therefore, the effective spring constant is lesser than both k_1 and k_2.

b. **Springs connected in parallel**

When two or more springs are connected in parallel, we can replace (by removing) all these springs with an equivalent spring (effective spring) whose net effect is same as if all the springs are in parallel connection. Given the values of individual spring constants to be k_1, k_2, k_3, ... (known quantities), we can establish a mathematical relationship to find out an effective (or equivalent) spring constant k_p (unknown quantity). For simplicity, let us consider only two springs of spring constants k_1 and k_2 attached to a mass m as shown in Figure 10.19. The results can be generalized to any number of springs in parallel.

Figure 10.19 Springs connected in parallel

Let the force F be applied towards right as shown in Figure 10.20. In this case, both the springs elongate or compress by the same amount of displacement. Therefore, net force for the displacement of mass m is

$$F = -k_p x \qquad (10.45)$$

where k_p is called ***effective spring constant***. Let the first spring be elongated by a displacement x due to force F_1 and second spring be elongated by the same displacement x due to force F_2, then the net force

$$F = -k_1 x - k_2 x \qquad (10.46)$$

Equating equations (10.46) and (10.45), we get

$$k_p = k_1 + k_2 \qquad (10.47)$$

Generalizing, for n springs connected in parallel,

$$k_p = \sum_{i=1}^{n} k_i \qquad (10.48)$$

Figure 10.20 Effective spring constant in parallel connection

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If all spring constants are identical i.e., $k_1 = k_2 = ... = k_n = k$ then

$$k_p = n\,k \qquad (10.49)$$

This implies that the effective spring constant increases by a factor n. Hence, for the springs in parallel connection, the effective spring constant is greater than individual spring constant.

Note

The spring constant is inversely proportional to the length of the spring

$$k \propto \frac{1}{\textit{length of the spring}}$$

If the spring is cut into two pieces, one piece with length l_1 and other with length l_2, such that $l_1 = nl_2$, then spring constant of first length is $k_1 = \frac{k(n+1)}{n}$ and spring constant of second length is $k_2 = (n+1)\,k$, where k is the original spring constant before cutting into pieces.

EXAMPLE 10.10

Consider two springs with force constants 1 N m^{-1} and 2 N m^{-1} connected in parallel. Calculate the effective spring constant (k_p) and comment on k_p.

Solution

$$k_1 = 1 \text{ N m}^{-1},\ k_2 = 2 \text{ N m}^{-1}$$

$$k_p = k_1 + k_2 \text{ N m}^{-1}$$

$$k_p = 1 + 2 = 3 \text{ N m}^{-1}$$

$$k_p > k_1 \text{ and } k_p > k_2$$

Therefore, the effective spring constant is greater than both k_1 and k_2.

EXAMPLE 10.11

Calculate the equivalent spring constant for the following systems and also compute if all the spring constants are equal:

Solution

a. Since k_1 and k_2 are parallel, $k_u = k_1 + k_2$
Similarly, k_3 and k_4 are parallel, therefore, $k_d = k_3 + k_4$
But k_u and k_d are in series,

therefore, $k_{eq} = \frac{k_u k_d}{k_u + k_d}$

If all the spring constants are equal then, $k_1 = k_2 = k_3 = k_4 = k$
Which means, $k_u = 2k$ and $k_d = 2k$

Hence, $k_{eq} = \frac{4k^2}{4k} = k$

b. Since k_1 and k_2 are parallel, $k_A = k_1 + k_2$
Similarly, k_4 and k_5 are parallel, therefore, $k_B = k_4 + k_5$
But k_A, k_3, k_B, and k_6 are in series,

therefore, $\frac{1}{k_{eq}} = \frac{1}{k_A} + \frac{1}{k_3} + \frac{1}{k_B} + \frac{1}{k_6}$

If all the spring constants are equal then, $k_1 = k_2 = k_3 = k_4 = k_5 = k_6 = k$
which means, $k_A = 2k$ and $k_B = 2k$

$$\frac{1}{k_{eq}} = \frac{1}{2k} + \frac{1}{k} + \frac{1}{2k} + \frac{1}{k} = \frac{3}{k}$$

$$k_{eq} = \frac{k}{3}$$

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EXAMPLE 10.12

A mass m moves with a speed v on a horizontal smooth surface and collides with a nearly massless spring whose spring constant is k. If the mass stops after collision, compute the maximum compression of the spring.

Solution

When the mass collides with the spring, from the law of conservation of energy "the loss in kinetic energy of mass is gain in elastic potential energy by spring".

Let x be the distance of compression of spring, then the law of conservation of energy

$$\frac{1}{2}m\,v^2 = \frac{1}{2}k\,x^2 \Rightarrow x = v\sqrt{\frac{m}{k}}$$

10.4.4 Oscillations of a simple pendulum in SHM and laws of simple pendulum

Simple pendulum

Figure 10.21 Simple pendulum

A pendulum is a mechanical system which exhibits periodic motion. It has a bob with mass m suspended by a long string (assumed to be massless and inextensible string) and the other end is fixed on a stand as shown in Figure 10.21 (a). At equilibrium, the pendulum does not oscillate and hangs vertically downward. Such a position is known as mean position or equilibrium position. When a pendulum is displaced through a small displacement from its equilibrium position and released, the bob of the pendulum executes to and fro motion. Let l be the length of the pendulum which is taken as the distance between the point of suspension and the centre of gravity of the bob. Two forces act on the bob of the pendulum at any displaced position, as shown in the Figure 10.21 (d),

(i) The gravitational force acting on the body ($\vec{F}=m\vec{g}$) which acts vertically downwards.

(ii) The tension in the string $\vec{T}$ which acts along the string to the point of suspension.

Resolving the gravitational force into its components:

a. **Normal component:** The component along the string but in opposition to the direction of tension, $F_{as} = mg\cos\theta$.

b. **Tangential component:** The component perpendicular to the string i.e., along tangential direction of arc of swing, $F_{ps} = mg\sin\theta$.

Therefore, The normal component of the force is, along the string,

$$T - F_{as} = m\frac{v^2}{l}$$

Here v is speed of bob

$$T - mg\cos\theta = m\frac{v^2}{l} \qquad (10.50)$$

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Note From Newton's 2nd law, $\vec{F} = m\vec{a}$ Here, the net force on the L.H.S is T-F_{as} In R.H.S, m$\vec{a}$ is equivalent to the centripetal force = $\frac{mv^2}{l}$ which makes the bob oscillate.

From the Figure 10.21, we can observe that the tangential component W_{ps} of the gravitational force always points towards the equilibrium position i.e., the direction in which it always points opposite to the direction of displacement of the bob from the mean position. Hence, in this case, the tangential force is nothing but the restoring force. Applying Newton's second law along tangential direction, we have

$$m\frac{d^2s}{dt^2} + F_{ps} = 0 \Rightarrow m\frac{d^2s}{dt^2} = -F_{ps}$$

$$m\frac{d^2s}{dt^2} = -mg\sin\theta \qquad (10.51)$$

where, s is the position of bob which is measured along the arc. Expressing arc length in terms of angular displacement i.e.,

$$s = l\theta \qquad (10.52)$$

then its acceleration,

$$\frac{d^2s}{dt^2} = l\frac{d^2\theta}{dt^2} \qquad (10.53)$$

Substituting equation (10.53) in equation (10.51), we get

$$l\frac{d^2\theta}{dt^2} = -g\sin\theta$$

$$\frac{d^2\theta}{dt^2} = -\frac{g}{l}\sin\theta \qquad (10.54)$$

Because of the presence of sin θ in the above differential equation, it is a non-linear differential equation (Here, homogeneous second order). Assume "the small oscillation approximation", sin $\theta \approx \theta$, the above differential equation becomes linear differential equation.

$$\frac{d^2\theta}{dt^2} = -\frac{g}{l}\theta \qquad (10.55)$$

This is the well known oscillatory differential equation. Therefore, the angular frequency of this oscillator (natural frequency of this system) is

$$\omega^2 = \frac{g}{l} \qquad (10.56)$$

$$\Rightarrow \quad \omega = \sqrt{\frac{g}{l}} \text{ rad } s^{-1} \qquad (10.57)$$

The frequency of oscillations is

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{l}} \text{ Hz} \qquad (10.58)$$

and time period of oscillations is

$$T = 2\pi\sqrt{\frac{l}{g}} \text{ second} \qquad (10.59)$$

Laws of simple pendulum

The time period of a simple pendulum

a. Depends on the following laws

(i) Law of length

For a given value of acceleration due to gravity, the time period of a simple pendulum is directly proportional to the square root of length of the pendulum.

$$T \propto \sqrt{l} \qquad (10.60)$$

(ii) Law of acceleration

For a fixed length, the time period of a simple pendulum is inversely proportional to square root of acceleration due to gravity.

$$T \propto \frac{1}{\sqrt{g}} \qquad (10.61)$$

b. Independent of the following factors

(i) Mass of the bob

The time period of oscillation is independent of mass of the simple

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pendulum. This is similar to free fall. Therefore, in a pendulum of fixed length, it does not matter whether an elephant swings or an ant swings. Both of them will swing with the same time period.

(ii) Amplitude of the oscillations

For a pendulum with small angle approximation (angular displacement is very small), the time period is independent of amplitude of the oscillation.

EXAMPLE 10.13

In simple pendulum experiment, we have used small angle approximation . Discuss the small angle approximation.

θ (in degrees)	θ(in radian)	sin θ
0	0	0
5	0.087	0.087
10	0.174	0.174
15	0.262	0.256
20	0.349	0.342
25	0.436	0.422
30	0.524	0.500
35	0.611	0.574
40	0.698	0.643
45	0.785	0.707

For θ in radian, $\sin\theta \approx \theta$ for very small angles

This means that "for θ as large as 10 degrees, sin θ is nearly the same as θ when θ is expressed in radians". As θ increases in value sinθ gradually becomes different from θ

Pendulum length due to effect of temperature

Suppose the suspended wire is affected due to change in temperature. The rise in temperature affects length by

$$l = l_o(1 + \alpha\,\Delta t)$$

where l_o is the original length of the wire and l is final length of the wire when the temperature is raised. Let Δt is the change in temperature and α is the co-efficient of linear expansion.

Then, $$T = 2\pi\sqrt{\frac{l}{g}} = 2\pi\sqrt{\frac{l_0(1+\alpha\Delta t)}{g}}$$

$$= 2\pi\sqrt{\frac{l_0}{g}}\sqrt{(1+\alpha\Delta t)}$$

$$T = T_0\,(1+\alpha\,\Delta t)^{\frac{1}{2}} \approx T_0\,(1+\frac{1}{2}\alpha\,\Delta t)$$

$$\Rightarrow \quad \frac{T}{T_0} - 1 = \frac{T - T_0}{T_0} = \frac{\Delta T}{T_0} = \frac{1}{2}\alpha\,\Delta t$$

where ΔT is the change in time period due to the effect of temperature and T_0 is the time period of the simple pendulum with original length l_0.

EXAMPLE 10.14

If the length of the simple pendulum is increased by 44% from its original length, calculate the percentage increase in time period of the pendulum.

Solution

Since

$$T \propto \sqrt{l}$$

Therefore,

$$T = \text{constant}\ \sqrt{l}$$

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$$\frac{T_f}{T_i}=\sqrt{\frac{l+\frac{44}{100}l}{l}}=\sqrt{1.44}=1.2$$

Therefore, $T_f = 1.2\ T_i = T_i + 20\%\ T_i$

Oscillation of liquid in a U-tube:

Figure 10.22 U-shaped glass tube

Consider a U-shaped glass tube which consists of two open arms with uniform cross-sectional area A. Let us pour a non-viscous uniform incompressible liquid of density ρ in the U-shaped tube to a *h*eight *h* as shown in the Figure 10.22. If the liquid and tube are not disturbed then the liquid surface will be in equilibrium position *O*. It means the pressure as measured at any point on the liquid is the same and also at the surface on the arm (edge of the tube on either side), which balances with the atmospheric pressure. Due to this the level of liquid in each arm will be the same. By blowing air one can provide sufficient force in one arm, and the liquid gets disturbed from equilibrium position *O*, which means, the pressure at blown arm is higher than the other arm. This creates difference in pressure which will cause the liquid to oscillate for a very short duration of time about the mean or equilibrium position and finally comes to rest.

Time period of the oscillation is

$$T = 2\pi\sqrt{\frac{l}{2g}}\ \text{second} \qquad (10.62)$$

Where *l* is the total length of liquid column in U- tube.

10.5 ENERGY IN SIMPLE HARMONIC MOTION

a. Expression for Potential Energy

For the simple harmonic motion, the force and the displacement are related by Hooke's law

$$\vec{F} = -k\vec{r}$$

Since force is a vector quantity, in three dimensions it has three components. Further, the force in the above equation is a conservative force field; such a force can be derived from a scalar function which has only one component. In one dimensional case

$$F = -\,k\,x \qquad (10.63)$$

As we have discussed in unit 4 of volume I, the work done by the conservative force field is independent of path. The potential energy U can be calculated from the following expression.

$$F=-\frac{dU}{dx} \qquad (10.64)$$

Comparing (10.63) and (10.64), we get

$$-\frac{dU}{dx} = -\,kx$$

$$dU = k\,x\,dx$$

Dummy variable

The integrating variable x' (read x' as "x prime")is a dummy variable

$$\int_0^y t\,dt=\int_0^y x\,dx=\int_0^y p\,dp=\frac{y^2}{2}$$

Notice that the integrating variables like *t*, *x* and *p* are dummy variables because, in this integration, whether we put *t* or *x* or *p* as variable for integration, we get the same answer.

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This work done by the force F during a small displacement dx stores as potential energy

$$U(x)=\int_0^x k\,x'\,dx'=\frac{1}{2}k\left(x'\right)^2\Big|_0^x=\frac{1}{2}kx^2 \quad (10.65)$$

From equation (10.22), we can substitute the value of force constant $k = m\,\omega^2$ in equation (10.65),

$$U(x)=\frac{1}{2}m\omega^2x^2 \quad (10.66)$$

where ω is the natural frequency of the oscillating system. For the particle executing simple harmonic motion from equation (10.6), we get

$$x = A \sin \omega t$$

$$U(\mathrm{t})=\frac{1}{2}m\omega^2A^2\sin^2\omega t \quad (10.67)$$

This variation of U is shown below.

Figure 10.23 Variation of potential energy with time t

Question to think over

"If the potential energy is minimum then its second derivative is positive, why?"

b. Expression for Kinetic Energy

Kinetic energy

$$KE=\frac{1}{2}mv_x^2=\frac{1}{2}m\left(\frac{dx}{dt}\right)^2 \quad (10.68)$$

Since the particle is executing simple harmonic motion, from equation (10.6)

$$x = A \sin \omega t$$

Therefore, velocity is

$$v_x=\frac{dx}{dt}=A\omega\cos\omega t \quad (10.69)$$

$$=A\omega\sqrt{1-\left(\frac{x}{A}\right)^2}$$

$$v_x=\omega\sqrt{A^2-x^2} \quad (10.70)$$

Hence,

$$KE=\frac{1}{2}mv_x^2=\frac{1}{2}m\omega^2\left(A^2-x^2\right) \quad (10.71)$$

$$KE=\frac{1}{2}m\omega^2A^2\cos^2\omega t \quad (10.72)$$

This variation with time is shown below.

Figure 10.24 Variation of kinetic energy with time t.

c. Expression for Total Energy

Total energy is the sum of kinetic energy and potential energy

$$E = KE + U \quad (10.73)$$

$$E=\frac{1}{2}m\omega^2(\mathrm{A}^2-x^2)+\frac{1}{2}m\omega^2x^2$$

Hence, cancelling x^2 term,

$$E=\frac{1}{2}m\omega^2A^2=\text{ constant} \quad (10.74)$$

Alternatively, from equation (10.67) and equation (10.72), we get the total energy as

$$E=\frac{1}{2}m\omega^2A^2\sin^2\omega t+\frac{1}{2}m\omega^2A^2\cos^2\omega t$$

$$=\frac{1}{2}m\omega^2A^2(\sin^2\omega\mathrm{t}+\cos^2\omega t)$$

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From trigonometry identity,
$(\sin^2 \omega t + \cos^2 \omega t) = 1$

$$E = \frac{1}{2} m\omega^2 A^2 = \text{constant}$$

which gives the law of conservation of total energy. This is depicted in Figure 10.26

Figure 10.25 Both kinetic energy and potential energy vary but total energy is constant

Thus the amplitude of simple harmonic oscillator, can be expressed in terms of total energy.

$$A = \sqrt{\frac{2E}{m\omega^2}} = \sqrt{\frac{2E}{k}} \qquad (10.75)$$

EXAMPLE 10.15

Write down the kinetic energy and total energy expressions in terms of linear momentum, For one-dimensional case.

Solution

Kinetic energy is $KE = \frac{1}{2} m v_x^2$

Multiply numerator and denominator by m

$$KE = \frac{1}{2m} m^2 v_x^2 = \frac{1}{2m} (m v_x)^2 = \frac{1}{2m} p_x^2$$

where, p_x is the linear momentum of the particle executing simple harmonic motion.

Total energy can be written as sum of kinetic energy and potential energy, therefore, from equation (10.73) and also from equation (10.75), we get

$$E = KE + U(x) = \frac{1}{2m} p_x^2 + \frac{1}{2} m\omega^2 x^2 = \text{constant}$$

Figure 10.26 Conservation of energy – spring mass system and simple pendulum system

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Note

Conservation of energy

Both the kinetic energy and potential energy are periodic functions, and repeat their values after a time period $\frac{T}{2}$. But total energy is constant for all the values of x or t. The kinetic energy and the potential energy for a simple harmonic motion are always positive. Note that kinetic energy cannot take negative value because it is proportional to the square of velocity. The measurement of any physical quantity must be a real number. Therefore, if kinetic energy is negative then the numerical value of velocity becomes an imaginary number, which is physically not acceptable. At equilibrium, it is purely kinetic energy and at extreme positions it is purely potential energy.

EXAMPLE 10.16

Compute the position of an oscillating particle when its kinetic energy and potential energy are equal.

Solution

Since the kinetic energy and potential energy of the oscillating particle are equal,

$$\frac{1}{2}m\omega^2(A^2 - x^2) = \frac{1}{2}m\omega^2 x^2$$

$$A^2 - x^2 = x^2$$

$$2x^2 = A^2$$

$$\Rightarrow x = \pm\frac{A}{\sqrt{2}}$$

10.6 TYPES OF OSCILLATIONS:

10.6.1 Free oscillations

When the oscillator is allowed to oscillate by displacing its position from equilibrium position, it oscillates with a frequency which is equal to the natural frequency of the oscillator. Such an oscillation or vibration is known as free oscillation or free vibration. In this case, the amplitude, frequency and the energy of the vibrating object remains constant.

Examples:

(i) Vibration of a tuning fork.
(ii) Vibration in a stretched string.
(iii) Oscillation of a simple pendulum.
(iv) Oscillationsof a spring-mass system.

10.6.2 Damped oscillations

During the oscillation of a simple pendulum (in previous case), we have assumed that the amplitude of the oscillation is constant and also the total energy of the oscillator is constant. But in reality, in a medium, due to the presence of friction and air drag, the amplitude of oscillation decreases as time progresses. It implies that the oscillation is not sustained and the energy of the SHM decreases gradually indicating the loss of energy. The energy lost is absorbed by the surrounding medium. This type of

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oscillatory motion is known as damped oscillation. In other words, if an oscillator moves in a resistive medium, its amplitude goes on decreasing and the energy of the oscillator is used to do work against the resistive medium. The motion of the oscillator is said to be damped and in this case, the resistive force (or damping force) is proportional to the velocity of the oscillator.

Figure 10.27 Damped harmonic oscillator – amplitude decreases as time increases.

Examples

(i) The oscillations of a pendulum (including air friction) or pendulum oscillating inside an oil filled container.

(ii) Electromagnetic oscillations in a tank circuit.

(iii) Oscillations in a dead beat and ballistic galvanometers.

10.6.3 Maintained oscillations

While playing in swing, the oscillations will stop after a few cycles, this is due to damping. To avoid damping we have to supply a push to sustain oscillations. By supplying energy from an external source, the amplitude of the oscillation can be made constant. Such vibrations are known as maintained vibrations.

Example:

The vibration of a tuning fork getting energy from a battery or from external power supply.

10.6.4 Forced oscillations

Any oscillator driven by an external periodic agency to overcome the damping is known as forced oscillator or driven oscillator.

In this type of vibration, the body executing vibration initially vibrates with its natural frequency and due to the presence of external periodic force, the body later vibrates with the frequency of the applied periodic force. Such vibrations are known as forced vibrations.

Example:

Sound boards of stringed instruments.

10.6.5 Resonance

It is a special case of forced vibrations where the frequency of external periodic force (or driving force) matches with the natural frequency of the vibrating body (driven). As a result the oscillating body begins to vibrate such that its amplitude increases at each step and ultimately it has a large amplitude. Such a phenomenon is known as resonance and the corresponding vibrations are known as resonance vibrations.

Example

The breaking of glass due to sound

Note The concept of resonance is used in Tuning of station (or channel) in a radio (or Television) circuits.

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DO YOU KNOW?

Soldiers are not allowed to march on a bridge.

This is to avoid resonant vibration of the bridge.

While crossing a bridge, if the period of stepping on the ground by marching soldiers equals the natural frequency of the bridge, it may result in resonance vibrations. This may be so large that the bridge may collapse.

Extra:

Pendulum in a lift:

(i) Lift moving upwards with acceleration a:

Effective acceleration due to gravity is $g_{eff} = g + a$

Then time period is $T = 2\pi\sqrt{\frac{l}{g_{eff}}} = 2\pi\sqrt{\frac{l}{(g+a)}}$

Since the time period is inversely related to acceleration due to gravity, time period will decrease when lift moves upward.

(ii) Lift moving downwards with acceleration a:

Effective acceleration due to gravity is $g_{eff} = g - a$

Then time period is $T = 2\pi\sqrt{\frac{l}{g_{eff}}} = 2\pi\sqrt{\frac{l}{(g-a)}}$

Since the time period is inversely related to acceleration due to gravity, time period will increase when lift moves downward.

(iii) Lift falls with acceleration $a > g$:

The effective acceleration is $g_{eff} = a - g$

Then time period is $T = 2\pi\sqrt{\frac{l}{g_{eff}}} = 2\pi\sqrt{\frac{l}{(a-g)}}$

in this case, the pendulum will turn upside down and will oscillate about higest point.

(iv) Lift falls with acceleration $a = g$:

The effective acceleration is $g_{eff} = g - g = 0$

Then time period is $T \rightarrow \infty$ which means pendulum does not oscillate and its motion is arrested.

(v) If the simple pendulum is kept in a car which moves horizontally with acceleration a:

The effective acceleration is $g_{eff} = \sqrt{g^2 + a^2}$

Time period is $T = 2\pi\sqrt{\frac{l}{g_{eff}}} = 2\pi\sqrt{\frac{l}{\sqrt{g^2+a^2}}}$

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SUMMARY

- When an object or a particle moves back and forth repeatedly about a reference point for some duration of time it is said to have Oscillatory (or vibratory) motion.
- For a SHM, the acceleration or force on the particle is directly proportional to its displacement from a fixed point and always directed towards that fixed point. The force is
$$F_x = -kx$$
where k is a constant whose dimension is force per unit length, called as force constant.
- In Simple harmonic motion, the displacement, $y = A \sin \omega t$.
- In Simple harmonic motion, the velocity, $v = A\,\omega \cos \omega t = \omega\sqrt{A^2 - y^2}$.
- In Simple harmonic motion, the acceleration, $a = \frac{d^2y}{dt^2} = -\omega^2 y$.
- The time period is defined as the time taken by a particle to complete one oscillation. It is usually denoted by T. Time period $T = \frac{2\pi}{\omega}$.
- The number of oscillations produced by the particle per second is called frequency. It is denoted by f. SI unit for frequency is S^{-1} or hertz (In symbol, Hz). Mathematically, frequency is related to time period by $f = \frac{1}{T}$.
- The frequency of the angular harmonic motion is $f = \frac{1}{2\pi}\sqrt{\frac{\kappa}{I}}$ Hz
- For n springs connected in series, the effective spring constant in series is
$$\frac{1}{k_s} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} + \ldots + \frac{1}{k_n} = \sum_{i=0}^{n} \frac{1}{k_i}$$
- For n springs connected in parallel, the effective spring constant is
$$k_p = \sum_{i=1}^{n} k_i$$
- The time period for U-tube oscillation is $T = 2\pi\sqrt{\frac{l}{2g}}$ second.
- For a conservative system in one dimension, the force field can be derived from a scalar potential energy: $F = -\frac{dU}{dx}$.
- In a simple harmonic motion, potential energy is $U(x) = \frac{1}{2}m\omega^2 x^2$.
- In a simple harmonic motion, kinetic energy is $KE = \frac{1}{2}mv_x^2 = \frac{1}{2}m\omega^2(A^2 - x^2)$.
- Total energy for a simple harmonic motion is $E = \frac{1}{2}m\omega^2 A^2 =$ constant.
- Types of oscillations – Free oscillations, Damped oscillations, Maintained oscillations and Forced oscillations.
- Resonance is a special case of forced oscillations.

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CONCEPT MAP

Oscillation

Simple Harmonic Motion (SHM)

$F = -kx \quad x = +A \sin\omega t$

$$\frac{d^2x}{dt^2} + \omega^2 x = 0$$

$a = -\omega^2 A \sin\omega t$

$a = -\omega^2 y$

$V = A\omega \cos\omega t$

$$v = \omega\sqrt{A^2 - y^2}$$

$$F = -\frac{dU}{dx} \qquad U = \frac{1}{2}kx^2$$

$$TE = \frac{1}{2}kA^2 \qquad KE = \frac{1}{2}mv^2$$

$$T = \frac{2\pi}{\omega}$$

Angular SHM

$$\omega = \sqrt{\frac{\kappa}{I}}$$

Linear SHM

$$\omega = \sqrt{\frac{k}{m}}$$

Simple Pendulum

Differential equation:

$$\frac{d^2\theta}{dt^2} = -\frac{g}{l}\theta$$

Time Period:

$$T = 2\pi\sqrt{\frac{l}{g}}$$

U - Tube

Differential equation:

$$\frac{d^2y}{dt^2} = -\frac{2g}{l}y$$

Time Period:

$$T = 2\pi\sqrt{\frac{l}{2g}}$$

Spring mass

Combination of Springs

(1) Series:

$$k_s = \frac{k_1 k_2}{k_1 + k_2}$$

(2) Parallel:

$$k_p = k_1 + k_2$$

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EVALUATION

I. Multiple Choice Questions

1. In a simple harmonic oscillation, the acceleration against displacement for one complete oscillation will be

(model NSEP 2000-01)

a) an ellipse b) a circle

c) a parabola d) a straight line

2. A particle executing SHM crosses points A and B with the same velocity. Having taken 3 s in passing from A to B, it returns to B after another 3 s. The time period is

a) 15 s b) 6 s

c) 12 s d) 9 s

3. The length of a second's pendulum on the surface of the Earth is 0.9 m. The length of the same pendulum on surface of planet X such that the acceleration of the planet X is n times greater than the Earth is

a) $0.9n$ b) $\frac{0.9}{n}m$

c) $0.9n^2m$ d) $\frac{0.9}{n^2}$

4. A simple pendulum is suspended from the roof of a school bus which moves in a horizontal direction with an acceleration a, then the time period is

a) $T \propto \frac{1}{g^2+a^2}$ b) $T \propto \sqrt{\frac{1}{\sqrt{g^2+a^2}}}$

c) $T \propto \sqrt{g^2+a^2}$ d) $T \propto (g^2+a^2)$

5. Two bodies A and B whose masses are in the ratio 1:2 are suspended from two separate massless springs of force constants k_A and k_B respectively. If the two bodies oscillate vertically such that their maximum velocities are in the ratio 1:2, the ratio of the amplitude A to that of B is

a) $\sqrt{\frac{k_B}{2k_A}}$ b) $\sqrt{\frac{k_B}{8k_A}}$

c) $\sqrt{\frac{2k_B}{k_A}}$ d) $\sqrt{\frac{8k_B}{k_A}}$

6. A spring is connected to a mass m suspended from it and its time period for vertical oscillation is T. The spring is now cut into two equal halves and the same mass is suspended from one of the halves. The period of vertical oscillation is

a) $T' = \sqrt{2}\,T$ b) $T' = \frac{T}{\sqrt{2}}$

c) $T' = \sqrt{2T}$ d) $T' = \sqrt{\frac{T}{2}}$

7. The displacement of a simple harmonic motion is given by $y(t) = A\ sin\ (\omega t + \varphi)$ where A is amplitude of the oscillation, ω is the angular frequency and φ is the phase. Let the amplitude of the oscillation be 8 cm and the time period of the oscillation is 24 s. If the displacement at initial time (t = 0 s) is 4 cm, then the displacement at t = 6 s is

(a) 8 cm

(b) 4 cm

(c) $4\sqrt{3}$ cm

(d) $8\sqrt{3}$ cm

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8. A simple pendulum has a time period T_1. When its point of suspension is moved vertically upwards according as $y = k\,t^2$, where y is vertical distance covered and $k = 1\ ms^{-2}$, its time period becomes T_2. Then, $\frac{T_1^2}{T_2^2}$ is (g = 10 m s^{-2}) (IIT 2005)

a) $\frac{5}{6}$ b) $\frac{11}{10}$

c) $\frac{6}{5}$ d) $\frac{5}{4}$

9. An ideal spring of spring constant k, is suspended from the ceiling of a room and a block of mass M is fastened to its lower end. If the block is released when the spring is un-stretched, then the maximum extension in the spring is (IIT 2002)

a) $4\frac{Mg}{k}$ b) $\frac{Mg}{k}$

c) $2\frac{Mg}{k}$ d) $\frac{Mg}{2k}$

10. A pendulum is hung in a very high building oscillates to and fro motion freely like a simple harmonic oscillator. If the acceleration of the bob is 16 ms^{-2} at a distance of 4 m from the mean position, then the time period is (NEET 2018 model)

a) 2 s b) 1 s

c) 2πs (d) πs

11. A hollow sphere is filled with water. It is hung by a long thread. As the water flows out of a hole at the bottom, the period of oscillation will

a) first increase and then decrease

b) first decrease and then increase

c) increase continuously

d) decrease continuously

12. The damping force on an oscillator is directly proportional to the velocity. The units of the constant of proportionality are (AIPMT 2012)

a) kg m s^{-1} b) kg m s^{-2}

c) kg s^{-1} (d) kg s

13. Let the total energy of a particle executing simple harmonic motion with angular frequency is 1 rad s^{-1} is 0.256 J. If the displacement of the particle at time $t = \frac{\pi}{2}\,s$ is $8\sqrt{2}$ cm then the amplitude of motion is

a) 8 cm b) 16 cm

c) 32 cm d) 64 cm

14. A particle executes simple harmonic motion and displacement y at time t_0, $2t_0$ and $3t_0$ are A, B and C, respectively. Then the value of $\left(\frac{A+C}{2B}\right)$ is

(a) cos ωt_0 (b) cos $2\omega t_0$

(c) cos $3\omega t_0$ (d) 1

15. A mass of 3 kg is attached at the end of a spring moves with simple harmonic motion on a horizontal frictionless table with time period 2π and with amplitude of 2m, then the maximum fore exerted on the spring is

(a) 1.5 N (b) 3 N

(c) 6 N (d) 12 N

Answers:

1) d	2) c	3) a	4) b
5) b	6) b	7) d	8) c
9) c	10) d	11) a	12) c
13) b	14) a	15) c	

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II. Short Answers Questions

1. What is meant by periodic and non-periodic motion?. Give any two examples, for each motion.
2. What is meant by force constant of a spring?.
3. Define time period of simple harmonic motion.
4. Define frequency of simple harmonic motion.
5. What is an epoch?.
6. Write short notes on two springs connected in series.
7. Write short notes on two springs connected in parallel.
8. Write down the time period of simple pendulum.
9. State the laws of simple pendulum?.
10. Write down the equation of time period for linear harmonic oscillator.
11. What is meant by free oscillation?.
12. Explain damped oscillation. Give an example.
13. Define forced oscillation. Give an example.
14. What is meant by maintained oscillation?. Give an example.
15. Explain resonance. Give an example.

III. Long Answers Questions

1. What is meant by simple harmonic oscillation?. Give examples and explain why every simple harmonic motion is a periodic motion whereas the converse need not be true.
2. Describe Simple Harmonic Motion as a projection of uniform circular motion.
3. What is meant by angular harmonic oscillation?. Compute the time period of angular harmonic oscillation.
4. Write down the difference between simple harmonic motion and angular simple harmonic motion.
5. Discuss the simple pendulum in detail.
6. Explain the horizontal oscillations of a spring.
7. Describe the vertical oscillations of a spring.
8. Write short notes on the oscillations of liquid column in U-tube.
9. Discuss in detail the energy in simple harmonic motion.
10. Explain in detail the four different types of oscillations.

IV. Exercises

1. Given an one dimensional system with total energy $E = \frac{p^2 x}{2m} + V(x) = \text{constant}$, where p_x is the x component of the linear momentum and $V(x)$ is the potential energy of the system. Show that total time derivative of energy gives us force $F_x = -\frac{d}{dx} V(x)$. Verify Hooke's law by choosing potential energy $V(x) = \frac{1}{2} kx^2$.
2. Consider a simple pendulum of length $l = 0.9\ m$ which is properly placed on a trolley rolling down on a inclined plane which is at $\theta = 45°$ with the horizontal. Assuming that the inclined plane is frictionless, calculate the time period of oscillation of the simple pendulum.

   Answer: 0.86 s
3. A piece of wood of mass m is floating erect in a liquid whose density is ρ. If it is slightly pressed down and released,

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then executes simple harmonic motion. Show that its time period of oscillation is $T = 2\pi\sqrt{\frac{m}{Ag\rho}}$

4. Consider two simple harmonic motion along x and y-axis having same frequencies but different amplitudes as $x = A\sin(\omega t + \varphi)$ (along x axis) and $y = B\sin\omega t$ (along y axis). Then show that

$$\frac{x^2}{A^2} + \frac{y^2}{B^2} - \frac{2xy}{AB}\cos\varphi = \sin^2\varphi$$

and also discuss the special cases when

a. $\varphi = 0$ b. $\varphi = \pi$ c. $\varphi = \frac{\pi}{2}$

d. $\varphi = \frac{\pi}{2}$ and $A = B$ (e) $\varphi = \frac{\pi}{4}$

Note: *when a particle is subjected to two simple harmonic motion at right angle to each other the particle may move along different paths. Such paths are called* ***Lissajous figures****.*

Answer :

a. $y = \frac{B}{A}x$, equation is a straight line passing through origin with positive slope.

b. $y = -\frac{B}{A}x$ equation is a straight line passing through origin with negative slope.

c. $\frac{x^2}{A^2} + \frac{y^2}{B^2} = 1$, equation is an ellipse whose center is origin.

d. $x^2 + y^2 = A^2$, equation is a circle whose center is origin .

e. $\frac{x^2}{A^2} + \frac{y^2}{B^2} - \frac{2xy}{AB}\frac{1}{\sqrt{2}} = \frac{1}{2}$, equation is an ellipse (oblique ellipse which means tilted ellipse)

5. Show that for a particle executing simple harmonic motion

a. the average value of kinetic energy is equal to the average value of potential energy.

b. average potential energy = average kinetic energy = $\frac{1}{2}$(total energy)

Hint : *average kinetic energy* = <*kinetic energy*> = $\frac{1}{T}\int_0^T (Kinetic\,energy)\,dt$

and

average Potential energy = <*Potential energy*> = $\frac{1}{T}\int_0^T (Potential\ energy)\,dt$

6. Compute the time period for the following system if the block of mass m is slightly displaced vertically down from its equilibrium position and then released. Assume that the pulley is light and smooth, strings and springs are light.

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Hint and answer:

Case(a)

Pulley is fixed rigidly here. When the mass displace by y and the spring will also stretch by y. Therefore, $F = T = ky$

$$T = 2\pi\sqrt{\frac{m}{k}}$$

Case(b)

Mass displace by y, pulley also displaces by y. $T = 4ky$.

$$T = 2\pi\sqrt{\frac{m}{4k}}$$

BOOKS FOR REFERENCE

1. Vibrations and Waves – A. P. French, CBS publisher and Distributors Pvt. Ltd.
2. Concepts of Physics – H. C. Verma, Volume 1 and Volume 2, Bharati Bhawan Publisher.
3. Fundamentals of Physics – Halliday, Resnick and Walker, Wiley Publishers, 10th edition.
4. Physics for Scientist and Engineers with Modern Physics – Serway and Jewett, Brook/ Coole Publishers, Eighth Edition.

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ICT CORNER

Oscillations

Through this activity you will be able to learn about the resonance.

STEPS:

- Use the URL or scan the QR code to open 'PhET' simulation on 'Resonance'. Click the play button.
- In the activity window a diagram of resonator is given. Click the play icon and move the slider on 'sim speed' given below to see the resonance.
- Move the slider to change 'Number of Resonators', 'Mass' and 'Spring constant' on the right side window and see the 'frequency'.
- Select the 'On', 'Off' button on 'Gravity' to see the different resonance.

Step1

Step2

Step3

Step4

URL:

https://phet.colorado.edu/en/simulation/legacy/resonance

* Pictures are indicative only.

* If browser requires, allow **Flash Player** or **Java Script** to load the page.

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UNIT 11 WAVES

We are slowed down sound and light waves, a walking bundle of frequencies tuned into the cosmos. We are souls dressed up in sacred biochemical garments and our bodies are the instruments through which our souls play their music – Albert Einstein

Learning Objectives

In this unit, the student is exposed to

- waves and their types (transverse and longitudinal)
- basic terms like wavelength, frequency, time period and amplitude of a wave
- velocity of transverse waves and longitudinal waves
- velocity of sound waves
- reflection of sound waves from plane and curved surfaces and its applications
- progressive waves and their graphical representation
- superposition principle, interference of waves, beats and standing waves
- characteristics of stationary waves, sonometer
- fundamental frequency, harmonics and overtones
- intensity and loudness
- vibration of air column – closed organ pipe, open organ pipe and resonance air column
- Doppler effect and its applications

11.1 INTRODUCTION

In the previous chapter, we have discussed the oscillation of a particle. Consider a medium which consists of a collection of particles. If the disturbance is created at one end, it propagates and reaches the other end. That is, the disturbance produced at the first mass point is transmitted to the next neighbouring mass point, and so on. Notice that here, only the disturbance is transmitted, not the mass points. Similarly, the speech we deliver is due to the vibration of our vocal chord inside the throat. This leads to the vibration of the surrounding air molecules and hence, the effect of speech (information) is transmitted from one point in space to another point in space without the medium carrying the particles. Thus, *the disturbance which carries energy and momentum from one point in space to another point in space without the transfer of the medium is known as a* wave.

Figure 11.1 Standing waves in a violin

Figure 11.2 Waves formed in (a) ocean, (b) standing waves in plucking rubber band and (c) ripples formed on water surface

Standing near a beach, one can observe waves in the ocean reaching the seashore with a similar wave pattern; hence they are called ocean waves. A rubber band when plucked vibrates like a wave which is an example of a standing wave. These are shown in Figure 11.2. Other examples of waves are light waves (electromagnetic waves), through which we see and enjoy the beauty of nature and sound waves using which we hear and enjoy pleasant melodious songs. Day to day applications of waves are numerous, such as mobile phone communication, laser surgery, etc.

11.1.1 Ripples and wave formation on the water surface

Figure 11.3 Ripples formed on the surface of water

Suppose we drop a stone in a trough of still water, we can see a disturbance produced at the place where the stone strikes the water surface as shown in Figure 11.3. We find that this disturbance spreads out (diverges out) in the form of concentric circles of ever increasing radii (ripples) and strike the boundary of the trough. This is because some of the kinetic energy of the stone is transmitted to the water molecules on the surface. Actually the particles of the water (medium) themselves do not move outward with the disturbance. This can be observed by keeping a paper strip on the water surface. The strip moves up and down when the disturbance (wave) passes on the water surface. This shows that the water molecules only undergo vibratory motion about their mean positions.

11.1.2 Formation of waves on stretched string

Let us take a long string and tie one end of the string to the wall as shown in Figure 11.4 (a). If we give a quick jerk, a bump (like pulse) is produced in the string as shown in Figure 11.4 (b). Such a disturbance is sudden and it lasts for a short duration, hence it is known as a wave pulse. If jerks are given continuously then the waves produced are standing waves. Similar waves are produced by a plucked string in a guitar.

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Figure 11.4: Wave pulse created during jerk produced on one end of the string

11.1.3 Formation of waves in a tuning fork

When we strike a tuning fork on a rubber pad, the prongs of the tuning fork vibrate about their mean positions. The prong vibrating about a mean position means moving outward and inward, as indicated in the Figure 11.5. When a prong moves outward, it pushes the layer of air in its neighbourhood which means there is more accumulation of air molecules in this region. Hence, the density and also the pressure increase. These regions are known as compressed regions or compressions. This compressed air layer moves forward and compresses the next neighbouring layer in a similar manner. Thus a wave of compression advances or passes through air. When the prong moves inwards, the particles of the medium are moved to the right. In this region both density and pressure are low. It is known as a rarefaction or elongation.

Figure 11.5 Waves due to strike of a tuning fork on a rubber pad

11.1.4 Characteristics of wave motion

- For the propagation of the waves, the medium must possess both inertia and elasticity, which decide the velocity of the wave in that medium.
- In a given medium, the velocity of a wave is a constant whereas the constituent particles in that medium move with different velocities at different positions. Velocity is maximum at their mean position and zero at extreme positions.
- Waves undergo reflections, refraction, interference, diffraction and polarization.

Point to ponder

The medium possesses both inertia and elasticity for propagation of waves.

Light is an electromagnetic wave. what is the medium for its transmission?

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11.1.5 Mechanical wave motion and its types

Wave motion can be classified into two types

a. **Mechanical wave** – Waves which require a medium for propagation are known as mechanical waves.

   Examples: sound waves, ripples formed on the surface of water, etc.

b. **Non mechanical wave** – Waves which do not require any medium for propagation are known as non-mechanical waves.

   Example: light waves, Infra red rays etc.

Further, waves can also be classified into two types

a. Transverse waves

b. Longitudinal waves

11.1.6 Transverse wave motion

Figure 11.6 Transverse wave

In transverse wave motion, the constituents of the medium oscillate or vibrate about their mean positions in a direction perpendicular to the direction of propagation (direction of energy transfer) of waves as shown in Figure 11.6.

Example: light (electromagnetic waves)

11.1.7 Longitudinal wave motion

In longitudinal wave motion, the constituents of the medium oscillate or vibrate about their mean positions in a direction parallel to the direction of propagation (direction of energy transfer) of waves as shown in Figure 11.7.

Example: Sound waves travelling in air.

Discuss with your Teacher

- Tsunami (pronounced soo-nah-mee in Japanese) means Harbour waves.
- Tsunami is a series of huge and giant waves which come with great speed and huge force. What happened on 26th December2004 in southern part of India? - Discuss
- Gravitational waves and LIGO (Laser Interferometer Gravitational wave Observatory) experiment.
- Nobel Prize winners in Physics 2017 are Prof. Rainer Weiss, Prof. Barry C. Barish and Prof. Kip S. Thorne for decisive contributions to the LIGO detector and observation of gravitational forces.

Figure 11.7 Longitudinal waves

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Table 11.1: Comparison of transverse and longitudinal waves

S.No.	Transverse waves	Longitudinal waves
1.	The direction of vibration of particles of the medium is perpendicular to the direction of propagation of waves.	The direction of vibration of particles of the medium is parallel to the direction of propagation of waves.
2.	The disturbances are in the form of crests and troughs.	The disturbances are in the form of compressions and rarefactions.
3.	Transverse waves are possible in elastic medium.	Longitudinal waves are possible in all types of media (solid, liquid and gas).

NOTE:

1. Absence of medium is also known as vacuum. Only electromagnetic waves can travel through vacuum.
2. Rayleigh waves are considered to be mixture of transverse and longitudinal.

11.2 TERMS AND DEFINITIONS USED IN WAVE MOTION

Figure 11.8 Two different sinusoidal waves

Suppose we have two waves as shown in Figure 11.8. Are these two waves identical? No. Though, the two waves are both sinusoidal, there are many difference between them. Therefore, we have to define some basic terminologies to distinguish one wave from another.

Consider a wave produced in a stretched string as shown in Figure 11.9.

Figure 11.9 Crest and Trough of a wave

If we are interested in counting the number of waves created, let us put a reference level (mean position) as shown in Figure 11.9. Here the mean position is the horizontal line shown. The highest point in the shaded portion is called *crest*. With respect to the reference level, the lowest point on the un-shaded portion is called *trough*. This wave contains repetition of a section O to B and hence we define the length of the smallest section without repetition as one *wavelength* as shown in Figure 11.10. In Figure 11.10 the length OB or length BD is one wavelengh. A Greek letter lambda λ is used to denote one wavelength.

Figure 11.10 Defining wavelength

For transverse waves (as shown in Figure 11.11), the distance between two neighbouring crests or troughs is known as the *wavelength*.

For longitudinal waves, (as shown in Figure 11.12) the distance between two neighbouring compressions or rarefactions is known as the wavelength. The SI unit of wavelength is *meter*.

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Figure 11.11 Wavelength for transverse waves

Figure 11.12 Wavelength for longitudinal waves

EXAMPLE 11.1

Which of the following has longer wavelength?

Answer is (c)

In order to understand frequency and time period, let us consider waves (made of three wavelengths) as shown in Figure 11.13 (a). At time $t = 0$ *s*, the wave reaches the point A from left. After time $t = 1$ *s* (shown in figure 11.13(b)), the number of waves which have crossed the point A is two. Therefore, the frequency is defined as the number of waves crossing a point per second. It is measured in *hertz* whose symbol is *Hz*. In this example,

$$f = 2 \text{ Hz} \tag{11.1}$$

Figure 11.13 A wave consisting of three wavelengths passing a point A at time (a) $t = 0$ s and (b) after time $t = 1$ s

If two waves take one second (time) to cross the point A then the time taken by one wave to cross the point A is half a second. This defines the time period T as

$$T = \frac{1}{2} = 0.5 \text{ s} \tag{11.2}$$

From equation (11.1) and equation (11.2), *frequency and time period are inversely related* i.e.,

$$T = \frac{1}{f} \tag{11.3}$$

Time period is defined as *the time taken by one wave to cross a point*.

EXAMPLE 11.2

Three waves are shown in the figure below.

Write down

(a) the frequency in ascending order

(b) the wavelength in ascending order

Solution

(a) $f_c < f_a < f_b$

(b) $\lambda_b < \lambda_a < \lambda_c$

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From the example 11.2, we observe that the frequency is inversely related to the wavelength, $f \propto \frac{1}{\lambda}$.

Then, $f\lambda$ is equal to what?

$$[\text{(i.e) } f\lambda = ?]$$

A simple dimensional argument will help us to determine this unknown physical quantity.

Dimension of wavelength is, $[\lambda] = L$

Frequency $f = \frac{1}{\text{Time period}}$, which implies that the dimension of frequency is,

$$[f] = \frac{1}{[T]} = T^{-1}$$

$$\Rightarrow [\lambda f] = [\lambda][f] = LT^{-1} = [\text{velocity}]$$

Therefore,

$$\text{Velocity,} \quad \lambda f = v \qquad (11.4)$$

where v is known as the *wave velocity* or *phase velocity*. This is the velocity with which the wave propagates. *Wave velocity* is the distance travelled by a wave in one second.

Note:

1. The number of cycles (or revolutions) per unit time is called *angular frequency*.

   Angular frequency, $\omega = \frac{2\pi}{T} = 2\pi f$ (unit is radians/second)

2. The number of cycles per unit distance or number of waves per unit distance is called *wave number*.

   wave number, $k = \frac{2\pi}{\lambda}$ (unit is radians/meter)

   The velocity v, angular frequency ω and wave number k are related as:

   velocity, $v = \lambda f = \frac{\lambda}{2\pi}(2\pi f) = \frac{(2\pi f)}{2\pi/\lambda} = \frac{\omega}{k}$

EXAMPLE 11.3

The average range of frequencies at which human beings can hear sound waves varies from 20 Hz to 20 kHz. Calculate the wavelength of the sound wave in these limits. (Assume the speed of sound to be 340 m s^{-1}.

Solution

$$\lambda_1 = \frac{v}{f_1} = \frac{340}{20} = 17\text{m}$$

$$\lambda_2 = \frac{v}{f_2} = \frac{340}{20\times10^3} = 0.017\,\text{m}$$

Therefore, the audible wavelength region is from 0.017 m to 17 m when the velocity of sound in that region is 340 m s^{-1}.

EXAMPLE 11.4

A man saw a toy duck on a wave in an ocean. He noticed that the duck moved up and down 15 times per minute. He roughly measured the wavelength of the ocean wave as 1.2 m. Calculate the time taken by the toy duck for going one time up and down and also the velocity of the ocean wave.

Solution

Given that the number of times the toy duck moves up and down is 15 times per minute. This information gives us frequency (the number of times the toy duck moves up and down)

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$$f = \frac{15 \text{ times toy duck moves up and down}}{\text{one minute}}$$

But one minute is 60 second, therefore, expressing time in terms of second

$$f = \frac{15}{60} = \frac{1}{4} = 0.25\,\text{Hz}$$

The time taken by the toy duck for going one time up and down is time period which is inverse of frequency

$$T = \frac{1}{f} = \frac{1}{0.25} = 4\text{ s}$$

The velocity of ocean wave is

$v = \lambda f = 1.2 \times 0.25 = 0.3\text{ m s}^{-1}$.

Amplitude of a wave:

Figure 11.14 Waves of different amplitude

The waves shown in the Figure 11.14 have same wavelength, same frequency and same time period and also move with same velocity. The only difference between two waves is the height of either crest or trough. This means, the height of the crest or trough also signifies a wave character. So we define a quantity called an amplitude of the wave, as the maximum displacement of the medium with respect to a reference axis (for example in this case x-axis). Here, it is denoted by A.

EXAMPLE 11.5

Consider a string whose one end is attached to a wall. Then compute the following in both situations given in figure (assume waves crosses the distance in one second)

(a) Wavelength, (b) Frequency and (c) Velocity

Solution

	First case	Second case
(a) Wavelength	$\lambda = 6$ m	$\lambda = 2$ m
(b) Frequency	$f = 2$ Hz	$f = 6$ Hz
(c) Velocity	$v = 6 \times 2$ $= 12\text{ m s}^{-1}$	$v = 2 \times 6$ $= 12\text{ m s}^{-1}$

This means that the speed of the wave along a string is a constant. Higher the frequency, shorter the wavelength and vice versa, and their product is velocity which remains the same.

11.3 VELOCITY OF WAVES IN DIFFERENT MEDIA

Suppose a hammer is struck on long rails at a distance and when a person keeps his ear near the rails at the other end he/she will hear two sounds, at different instants. The sound that is heard through the rails (solid medium)

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is faster than the sound we hear through the air (gaseous medium). This implies the velocity of sound is different in different media.

In this section, we shall derive the velocity of waves in two different cases:

1. The velocity of a transverse waves along a stretched string.
2. The velocity of a longitudinal waves in an elastic medium.

11.3.1 Velocity of transverse waves in a stretched string

Let us compute the velocity of transverse travelling waves on a string. When a jerk is given at one end (left end) of the rope, the wave pulses move towards right end with a velocity v with respect to an observer who is at rest frame.

Consider an elemental segment in the string as shown in the Figure 11.15. Let A and B be two points on the string at an instant of time. Let dl and dm be the length and mass of the elemental string, respectively. By definition, linear mass density, μ is

$$\mu = \frac{dm}{dl} \tag{11.5}$$

$$dm = \mu\, dl \tag{11.6}$$

The elemental string AB has a curvature which looks like an arc of a circle with centre at O, radius R and the arc subtending an angle θ at the origin O as shown in Figure 11.15(b). The angle θ can be written in terms of arc length and radius as $\theta = \frac{dl}{R}$. The centripetal acceleration supplied by the tension in the string is

$$a_{cp} = \frac{v^2}{R} \tag{11.7}$$

Then, centripetal force is

$$F_{cp} = \frac{(dm)v^2}{R} \tag{11.8}$$

From eqn 11.6,

$$\frac{(dm)v^2}{R} = \frac{\mu v^2\, dl}{R} \tag{11.9}$$

The tension T acts along the tangent of the elemental segment of the string at A and B. Since the arc length is very small, variation

Figure 11.15 Elemental segment in a stretched string is zoomed and the pulse seen from an observer frame who moves with velocity v.

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in the tension force can be ignored. We can resolve T into horizontal component $T\cos\left(\frac{\theta}{2}\right)$ and vertical component $T\sin\left(\frac{\theta}{2}\right)$. The horizontal components at A and B are equal in magnitude but opposite in direction; therefore, they cancel each other. Since the elemental arc length AB is taken to be very small, the vertical components at A and B appears to acts vertical towards the centre of the arc and hence, they add up. The net radial force F_r is

$$F_r = 2T\sin\left(\frac{\theta}{2}\right) \quad (11.10)$$

Since the amplitude of the wave is very small when it is compared with the length of the string, $\sin\left(\frac{\theta}{2}\right) \approx \frac{\theta}{2}$. Hence,

$$F_r = 2T \times \frac{\theta}{2} = T\theta \quad (11.11)$$

But $\theta = \frac{dl}{R}$, we get

$$F_r = T\frac{dl}{R} \quad (11.12)$$

Applying Newton's second law to the elemental string in the radial direction, under equilibrium, the radial component of the force is equal to the centripetal force. Hence equating equation (11.9) and equation (11.12), we have

$$T\frac{dl}{R} = \mu v^2 \frac{dl}{R}$$

$$v = \sqrt{\frac{T}{\mu}} \quad (11.13)$$

Observations:

- The velocity of the string is
  a. directly proportional to the square root of the tension force
  b. inversely proportional to the square root of linear mass density
  c. independent of shape of the waves.

EXAMPLE 11.6

Calculate the velocity of the travelling pulse as shown in the figure below. The linear mass density of pulse is 0.25 kg m^{-1}. Further, compute the time taken by the travelling pulse to cover a distance of 30 cm on the string.

Solution

The tension in the string is $T = mg = 1.2 \times 9.8 = 11.76$ N

The mass per unit length is $\mu = 0.25$ kg m^{-1}

Therefore, velocity of the wave pulse is

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{11.76}{0.25}} = 6.858\ m\,s^{-1} = 6.8\ m\,s^{-1}$$

The time taken by the pulse to cover the distance of 30 cm is

$$t = \frac{d}{v} = \frac{30\times10^{-2}}{6.8} = 0.044\,s = 44\ ms$$ where, ms = milli second.

11.3.2 Velocity of longitudinal waves in an elastic medium

Consider an elastic medium (here we assume air) having a fixed mass contained in a long tube (cylinder) whose cross sectional area is A and maintained under a pressure P. One can generate longitudinal waves in the fluid either by displacing the fluid using a piston or by keeping a vibrating tuning fork at one end of the tube. Let us assume that the direction of propagation of waves coincides with the axis of the cylinder. Let ρ be the density of

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Figure 11.16 Longitudinal waves in the fluid by displacing the fluid using a piston

the fluid which is initially at rest. At $t = 0$, the piston at left end of the tube is set in motion toward the right with a speed u.

Let u be the velocity of the piston and v be the velocity of the elastic wave. In time interval Δt, the distance moved by the piston $\Delta d = u\ \Delta t$. Now, the distance moved by the elastic disturbance is $\Delta x = v\Delta t$. Let Δm be the mass of the air that has attained a velocity v in a time Δt. Therefore,

$$\Delta m = \rho\ A\ \Delta x = \rho\ A\ (v\ \Delta t)$$

Then, the momentum imparted due to motion of piston with velocity u is

$$\Delta p = [\rho\ A\ (v\ \Delta t)]u$$

But the change in momentum is impulse.

The net impulse is

$$I = (\Delta P\ A)\Delta t$$

Or $$(\Delta P\ A)\Delta t = [\rho\ A\ (v\ \Delta t)]u$$

$$\Delta P = \rho\ v\ u \qquad (11.14)$$

When the sound wave passes through air, the small volume element (ΔV) of the air undergoes regular compressions and rarefactions. So, the change in pressure can also be written as

$$\Delta P = K\frac{\Delta V}{V}$$

where, V is original volume and K is known as bulk modulus of the elastic medium.

But $V = A\ \Delta x = A\ v\ \Delta t$ and

$\Delta V = A\ \Delta d = A\ u\ \Delta t$

Therefore,

$$\Delta P = K\frac{A u\,\Delta t}{A v\,\Delta t} = K\frac{u}{v} \qquad (11.15)$$

Comparing equation (11.14) and equation (11.15), we get

$$\rho v u = K\frac{u}{v} \text{ or } v^2 = \frac{K}{\rho}$$

$$\Rightarrow v = \sqrt{\frac{K}{\rho}} \qquad (11.16)$$

In general, the velocity of a longitudinal wave in elastic medium is $v = \sqrt{\frac{E}{\rho}}$, where E is the modulus of elasticity of the medium.

Cases: For a solid :

(i) one dimensional rod (1D)

$$v = \sqrt{\frac{Y}{\rho}} \qquad (11.17)$$

where Y is the Young's modulus of the material of the rod and ρ is the density of the rod. The 1D rod will have only Young's modulus.

(ii) Three dimensional rod (3D) The speed of longitudinal wave in a solid is

$$v = \sqrt{\frac{K + \frac{4}{3}\eta}{\rho}} \qquad (11.18)$$

where η is the modulus of rigidity, K is the bulk modulus and ρ is the density of the rod.

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Cases: For liquids:

$$v = \sqrt{\frac{K}{\rho}} \qquad (11.19)$$

where, K (or) B is the bulk modulus and ρ is the density of the rod.

EXAMPLE 11.7

Calculate the speed of sound in a steel rod whose Young's modulus $Y = 2 \times 10^{11}$ N m^{-2} and $\rho = 7800$ kg m^{-3}.

Solution

$$v = \sqrt{\frac{Y}{\rho}} = \sqrt{\frac{2\times10^{11}}{7800}} = \sqrt{0.2564\times10^{8}}$$

$$= 0.506\times10^{4} ms^{-1} = 5\times10^{3} ms^{-1}$$

Therefore, longitudinal waves travel faster in a solid than in a liquid or a gas. Now you may understand why a shepherd checks before crossing railway track by keeping his ears on the rails to safegaurd his cattle.

EXAMPLE 11.8

An increase in pressure of 100 kPa causes a certain volume of water to decrease by 0.005% of its original volume.

(a) Calculate the bulk modulus of water?

(b) Compute the speed of sound (compressional waves) in water?

Solution

(a) Bulk modulus

$$B = V\left|\frac{\Delta P}{\Delta V}\right| = \frac{100\times10^{3}}{0.005\times10^{-2}} =$$

$$= \frac{100\times10^{3}}{5\times10^{-5}} = 2000\, MPa, \text{ where}$$

MPa is mega pascal

(b) Speed of sound in water is

$$v = \sqrt{\frac{K}{\rho}} = \sqrt{\frac{2000\times10^{6}}{1000}} = 1414 ms^{-1}$$

Note The velocities of both transverse waves and longitudinal waves depend on elastic property (like string tension T or bulk modulus K) and inertial property (like density or mass per unit length) i.e., $v = \sqrt{\frac{\text{Elastic property}}{\text{Inertial property}}}$

Table 11.2: Speed of sound in various media

S.No.	Medium	Speed in m s^{-1}
	Solids	
1.	Rubber	1600
2.	Gold	3240
3.	Brass	4700
4.	Copper	5010
5.	Iron	5950
6.	Aluminum	6420
	Liquids at 25°C	
1.	Kerosene	1324
2.	Mercury	1450
3.	Water	1493
4.	Sea Water	1533
	Gas (at 0°C)	
1.	Oxygen	317
2.	Air	331
3.	Helium	972
4.	Hydrogen	1286
	Gas (at 20°C)	
1.	Air	343

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11.4 PROPAGATION OF SOUND WAVES

We know that sound waves are longitudinal waves, and when they propagate compressions and rarefactions are formed. In the following section, we compute the speed of sound in air by Newton's method and also discuss the Laplace correction and the factors affecting sound in air.

11.4.1 Newton's formula for speed of sound waves in air

Sir Isaac Newton assumed that when sound propagates in air, the formation of compression and rarefaction takes place in a very slow manner so that the process is isothermal in nature. That is, the heat produced during compression (pressure increases, volume decreases), and heat lost during rarefaction (pressure decreases, volume increases) occur over a period of time such that the temperature of the medium remains constant. Therefore, by treating the air molecules to form an ideal gas, the changes in pressure and volume obey Boyle's law, Mathematically

$$PV = \text{Constant} \qquad (11.20)$$

Differentiating equation (11.20), we get

$$PdV + VdP = 0$$

$$\text{or,} \quad P = -V\frac{dP}{dV} = K_I \qquad (11.21)$$

where, K_I is an isothermal bulk modulus of air. Substituting equation (11.21) in equation (11.16), the speed of sound in air is

$$v_T = \sqrt{\frac{K_I}{\rho}} = \sqrt{\frac{P}{\rho}} \qquad (11.22)$$

Since P is the pressure of air whose value at NTP (Normal Temperature and Pressure) is 76 cm of mercury, we have $P = h\rho g$

$$P = (0.76 \times 13.6 \times 10^3 \times 9.8)\ \text{N m}^{-2}$$

$\rho = 1.293\ kg\ m^{-3}$. Here ρ is density of air

Then the speed of sound in air at Normal Temperature and Pressure (NTP) is

$$v_T = \sqrt{\frac{(0.76 \times 13.6 \times 10^3 \times 9.8)}{1.293}}$$

$= 279.80\ \text{m s}^{-1} \approx 280\ \text{ms}^{-1}$ (theoretical value)

But the speed of sound in air at 0°C is experimentally observed as $332\ \text{m s}^{-1}$ which is close upto 16% more than theoretical value (Percentage error is $\frac{(332-280)}{332} \times 100\% = 15.6\%$). This error is not small

11.4.2 Laplace's correction

In 1816, Laplace satisfactorily corrected this discrepancy by assuming that when the sound propagates through a medium, the particles oscillate very rapidly such that the compression and rarefaction occur very fast. Hence the exchange of heat produced due to compression and cooling effect due to rarefaction do not take place, because, air (medium) is a bad conductor of heat. Since, temperature is no longer considered as a constant here, sound propagation is an adiabatic process. By adiabatic considerations, the gas obeys Poisson's law (not Boyle's law as Newton assumed), which is

$$PV^{\gamma} = \text{constant} \qquad (11.23)$$

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where, $\gamma = \frac{C_P}{C_v}$, which is the ratio between specific heat at constant pressure and specific heat at constant volume.

Differentiating equation (11.23) on both the sides, we get

$$V^{\gamma}\, dP + P\,(\gamma V^{\gamma-1}\, dV) = 0$$

$$\text{or,} \quad \gamma P = -V\frac{dp}{dV} = K_A \qquad (11.24)$$

where, K_A is the adiabatic bulk modulus of air. Now, substituting equation (11.24) in equation (11.16), the speed of sound in air is

$$v_A = \sqrt{\frac{K_A}{\rho}} = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\gamma}\, v_T \qquad (11.25)$$

Since air contains mainly, nitrogen, oxygen, hydrogen etc, (diatomic gas), we take γ = 1.4. Hence, speed of sound in air is $v_A = (\sqrt{1.4})(280 \text{ m s}^{-1}) = 331.30 \text{ m s}^{-1}$, which is very much closer to experimental data.

11.4.3 Factors affecting speed of sound in gases

Let us consider an ideal gas whose equation of state is

$$PV = \mu R T \qquad (11.26)$$

where, P is pressure, V is volume, T is temperature, μ is number of mole and R is universal gas constant. For a given mass of a molecule, equation (11.26) can be written as

$$\frac{PV}{T} = \text{Constant} \qquad (11.27)$$

For a fixed mass m, density of the gas inversely varies with volume. i.e.,

$$\rho \propto \frac{1}{V}, \quad V = \frac{m}{\rho} \qquad (11.28)$$

Substituting equation (11.28) in equation (11.27), we get

$$\frac{P}{\rho} = cT \qquad (11.29)$$

where c is constant.

The speed of sound in air given in equation (11.25) can be written as

$$v = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\gamma c T} \qquad (11.30)$$

From the above relation we observe the following

(a) Effect of pressure :

For a fixed temperature, when the pressure varies, correspondingly density also varies such that the ratio $\left(\frac{P}{\rho}\right)$ becomes constant. This means that the speed of sound is independent of pressure for a fixed temperature. If the temperature remains same at the top and the bottom of a mountain then the speed of sound will remain same at these two points. But, in practice, the temperatures are not same at top and bottom of a mountain; hence, the speed of sound is different at different points.

(b) Effect of temperature :

Since $v \propto \sqrt{T}$,

the speed of sound varies directly to the square root of temperature in kelvin.

Let v_0 be the speed of sound at temperature at 0° C or 273 K and v be the speed of sound at any arbitrary temperature T (in kelvin), then

$$\frac{v}{v_0} = \sqrt{\frac{T}{273}} = \sqrt{\frac{273+t}{273}}$$

$$v = v_0\sqrt{1+\frac{t}{273}} \cong v_0\left(1+\frac{t}{546}\right)$$

(using binomial expansion)

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Since $v_0 = 331 m s^{-1}$ at 0°C, v at any temperature in t°C is

$$v = (331 + 0.61t)\ m s^{-1}$$

Thus the speed of sound in air increases by 0.61 $m s^{-1}$ per degree celcius rise in temperature. Note that when the temperature is increased, the molecules will vibrate faster due to gain in thermal energy and hence, speed of sound increases.

(c) Effect of density :

Let us consider two gases with different densities having same temperature and pressure. Then the speed of sound in the two gases are

$$v_1 = \sqrt{\frac{\gamma_1 P}{\rho_1}} \qquad (11.31)$$

and

$$v_2 = \sqrt{\frac{\gamma_2 P}{\rho_2}} \qquad (11.32)$$

Taking ratio of equation (11.31) and equation (11.32), we get

$$\frac{v_1}{v_2} = \frac{\sqrt{\frac{\gamma_1 P}{\rho_1}}}{\sqrt{\frac{\gamma_2 P}{\rho_2}}} = \sqrt{\frac{\gamma_1 \rho_2}{\gamma_2 \rho_1}}$$

For gases having same value of γ,

$$\frac{v_1}{v_2} = \sqrt{\frac{\rho_2}{\rho_1}} \qquad (11.33)$$

Thus the velocity of sound in a gas is inversely proportional to the square root of the density of the gas.

(d) Effect of moisture (humidity):

We know that density of moist air is 0.625 of that of dry air, which means the presence of moisture in air (increase in humidity) decreases its density. Therefore, speed of sound increases with rise in humidity. From equation (11.30)

$$v = \sqrt{\frac{\gamma p}{\rho}}$$

Let ρ_1, v_1 and ρ_2, v_2 be the density and speeds of sound in dry air and moist air, respectively. Then

$$\frac{v_1}{v_2} = \frac{\sqrt{\frac{\gamma_1 P}{\rho_1}}}{\sqrt{\frac{\gamma_2 P}{\rho_2}}} = \sqrt{\frac{\rho_2}{\rho_1}} \quad \text{if } \gamma_1 = \gamma_2$$

Since P is the total atmospheric pressure, According to Dalton's law of partial pressure, it can be shown that

$$\frac{\rho_2}{\rho_1} = \frac{P}{p_1 + 0.625 p_2}$$

where p_1 and p_2 are the partial pressures of dry air and water vapour respectively. Then

$$v_1 = v_2 \sqrt{\frac{P}{p_1 + 0.625 p_2}} \qquad (11.34)$$

(e) Effect of wind:

The speed of sound is also affected by blowing of wind. In the direction along the wind blowing, the speed of sound increases whereas in the direction opposite to wind blowing, the speed of sound decreases.

EXAMPLE 11.9

The ratio of the densities of oxygen and nitrogen is 16:14. Calculate the temperature when the speed of sound in nitrogen gas at 17°C is equal to the speed of sound in oxygen gas.

Solution

From equation (11.25), we have

$$v = \sqrt{\frac{\gamma P}{\rho}}$$

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But $\rho = \frac{M}{V}$

Therefore,

$$v = \sqrt{\frac{\gamma PV}{M}}$$

Using equation (11.26)

$$v = \sqrt{\frac{\gamma RT}{M}}$$

Where, R is the universal gas constant and M is the molecular mass of the gas. The speed of sound in nitrogen gas at 17°C is

$$v_N = \sqrt{\frac{\gamma R(273K + 17K)}{M_N}} = \sqrt{\frac{\gamma R(290K)}{M_N}} \quad (1)$$

Similarly, the speed of sound in oxygen gas at temperature t

$$v_0 = \sqrt{\frac{\gamma R(273K + t)}{M_0}} \quad (2)$$

Given that the value of γ is same for both the gases, the two speeds must be equal. Hence, equating equation (1) and (2), we get

$$v_O = v_N$$

$$\sqrt{\frac{\gamma R(273 + t)}{M_0}} = \sqrt{\frac{\gamma R(290)}{M_N}}$$

Squaring on both sides and cancelling γR term and rearranging, we get

$$\frac{M_0}{M_N} = \frac{273 + t}{290} \quad (3)$$

Since the densities of oxygen and nitrogen is 16:14,

$$\frac{\rho_0}{\rho_N} = \frac{16}{14} \quad (4)$$

$$\frac{\rho_0}{\rho_N} = \frac{\frac{M_O}{V}}{\frac{M_N}{V}} = \frac{M_0}{M_N} \Rightarrow \frac{M_0}{M_N} = \frac{16}{14} \quad (5)$$

Substituting equation (5) in equation (3), we get

$$\frac{273 + t}{290} = \frac{16}{14} \Rightarrow 3822 + 14t = 4640$$

$$\Rightarrow t = 58.4\,°C$$

11.5 REFLECTION OF SOUND WAVES

When sound wave passes from one medium to another medium, the following things can happen

(a) **Reflection of sound:** If the medium is highly dense (highly rigid), the sound can be reflected completely (bounced back) to the original medium.

(b) **Refraction of sound:** When the sound waves propagate from one medium to another medium such that there can be some energy loss due to absorption by the second medium.

In this section, we will consider only the reflection of sound waves in a medium when it experiences a harder surface. Sound can also obey the *laws of reflection*, which state that

Figure 11.17 Reflection of sound in different surfaces

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(i) The angle of incidence of sound is equal to the angle of reflection.

(ii) When the sound wave is reflected by a surface then the incident wave, reflected wave and the normal at the point of incidence all lie in the same plane.

Similar to reflection of light from a mirror, sound also reflects from a harder flat surface, This is called as **specular reflection.**

Specular reflection is observed only when the wavelength of the source is smaller than dimensions of the reflecting surface, as well as smaller than surface irregularities.

11.5.1 Reflection of sound through the plane surface

Figure 11.18 Reflection of sound through the plane surface

When the sound waves hit the plane wall, they bounce off in a manner similar to that of light. Suppose a loudspeaker is kept at an angle with respect to a wall (plane surface), then the waves coming from the source (assumed to be a point source) can be treated as spherical wave fronts (say, compressions moving like a spherical wave front). Therefore, the reflected wave front from the plane surface is also spherical, such that its centre of curvature (which lies on the other side of plane surface) can be treated as the image of the sound source (virtual or imaginary loud speaker). These are shown in Figures 11.18, 11.19.

Figure 11.19 Common examples for reflection of sound in real situation

11.5.2 Reflection of sound through the curved surface

The behaviour of sound is different when it is reflected from different surfaces like convex or concave or plane. The sound reflected from a convex surface is spread out and so it is easily attenuated and weakened. Whereas, if it is reflected from the concave surface it will converge at a point and this can be easily amplified. The parabolic reflector (curved reflector) which is used to focus the sound precisely to a point is used in designing the parabolic mics which are known as high directional microphones.

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We know that any surface (smooth or rough) can absorb sound. For example, the sound produced in a big hall or auditorium or theatre is absorbed by the walls, ceilings, floor, seats etc. To avoid such losses, a curved sound board (concave board) is kept in front of the speaker, so that the board reflects the sound waves of the speaker towards the audience. This method will minimize the spreading of sound waves in all possible directions in that hall and also enhances the uniform distribution of sound throughout the hall. That is why a person sitting at any position in that hall can hear the sound without any disturbance.

Figure 11.20 Reflection of sound through the curved surface

Figure 11.21 Sound in a big auditorium

11.5.3 Applications of reflection of sound waves

(a) **Stethoscope:** It works on the principle of multiple reflections.

Figure 11.22 Stethoscope and multiple reflection of signal in a rubber tube

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It consists of three main parts:

(i) Chest piece

(ii) Ear piece

(iii) Rubber tube

(i) Chest piece: It consists of a small disc-shaped resonator (diaphragm) which is very sensitive to sound and amplifies the sound it detects.

(ii) Ear piece: It is made up of metal tubes which are used to hear sounds detected by the chest piece.

(iii) Rubber tube: This tube connects both chest piece and ear piece. It is used to transmit the sound signal detected by the diaphragm, to the ear piece. The sound of heart beats (or lungs) or any sound produced by internal organs can be detected, and it reaches the ear piece through this tube by multiple reflections.

Note

Scientists have estimated that *we can hear two sounds properly if the time gap or time interval between each sound is* $\left(\frac{1}{10}\right)^{th}$ *of a second (persistence of hearing)* i.e., 0.1 s. Then,

$$\text{velocity} = \frac{\text{Distance travelled}}{\text{time taken}} = \frac{2d}{t}$$

$$2d = 344 \times 0.1 = 34.4 \text{ m}$$

$$d = 17.2 \text{ m}$$

The minimum distance from a sound reflecting wall to hear an echo at 20°C is 17.2 meter.

(b) Echo: An echo is a repetition of sound produced by the reflection of sound waves from a wall, mountain or other obstructing surfaces. The speed of sound in air at 20°C is 344 m s^{-1}. If we shout at a wall which is at 344 m away, then the sound will take 1 second to reach the wall. After reflection, the sound will take one more second to reach us. Therefore, we hear the echo after two seconds.

(c) SONAR: SO*und* **NA***vigation* and **R***anging*. Sonar systems make use of reflections of sound waves in water to locate the position or motion of an object. Similarly, dolphins and bats use the sonar principle to find their way in the darkness.

(d) Reverberation: In a closed room the sound is repeatedly reflected from the walls and it is even heard long after the sound source ceases to function. The residual sound remaining in an enclosure and the phenomenon of multiple reflections of sound is called reverberation. The duration for which the sound persists is called reverberation time. It should be noted that the reverberation time greatly affects the quality of sound heard in a hall. Therefore, halls are constructed with some optimum reverberation time.

EXAMPLE 11.10

Suppose a man stands at a distance from a cliff and claps his hands. He receives an echo from the cliff after 4 second. Calculate the distance between the man and the cliff. Assume the speed of sound to be 343 m s^{-1}.

Solution

The time taken by the sound to come back as echo is $2t = 4 \Rightarrow t = 2$ s

$\therefore$ The distance is $d = vt = (343 \text{ m s}^{-1})(2 \text{ s}) = 686$ m.

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Note: Classification of sound waves: Sound waves can be classified in three groups according to their range of frequencies:

(1) ***Infrasonic waves:***
Sound waves having frequencies below 20 Hz are called infrasonic waves. These waves are produced during earthquakes. Human beings cannot hear these frequencies. Snakes can hear these frequencies.

(2) ***Audible waves:***
Sound waves having frequencies between 20 Hz to 20,000 Hz (20kHz) are called audible waves. Human beings can hear these frequencies.

(3) ***Ultrasonic waves:***
Sound waves having frequencies greater than 20 kHz are known as ultrasonic waves. Human beings cannot hear these frequencies. Bats can produce and hear these frequencies.

(1.) Supersonic speed:
An object moving with a speed greater than the speed of sound is said to move with a supersonic speed.

(2.) Mach number:
It is the ratio of the velocity of source to the velocity of sound.

$$\text{Mach number} = \frac{\text{velocity of source}}{\text{velocity of sound}}$$

11.6 PROGRESSIVE WAVES (OR) TRAVELLING WAVES

If a wave that propagates in a medium is continuous then it is known as progressive wave or travelling wave.

11.6.1 Characteristics of progressive waves

1. Particles in the medium vibrate about their mean positions with the same amplitude.
2. The phase of every particle ranges from 0 to 2π.
3. No particle remains at rest permanently. During wave propagation, particles come to the rest position only twice at the extreme points.
4. Transverse progressive waves are characterized by crests and troughs whereas longitudinal progressive waves are characterized by compressions and rarefactions.
5. When the particles pass through the mean position they always move with the same maximum velocity.
6. The displacement, velocity and acceleration of particles separated from each other by $n\lambda$ are the same, where n is an integer, and λ is the wavelength.

11.6.2 Equation of a plane progressive wave

Figure 11.23 Wave pulse moving with velocity v at two instants at $t = 0$ and at time t

Suppose we give a jerk on a stretched string at time $t = 0$ s. Let us assume that the wave pulse created during this disturbance moves along positive x direction with constant speed v as shown in Figure 11.23 (a).

We can represent the shape of the wave pulse mathematically as $y = y(x, 0) = f(x)$ at time $t = 0$ s. Assume that the shape of the wave pulse

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remains the same during the propagation. After some time t, the pulse moving towards the right and any point on it can be represented by x' (read it as x prime) as shown in Figure 11.23 (b). Then,

$$y(x, t) = f(x') = f(x - vt) \qquad (11.35)$$

Similarly, if the wave pulse moves towards left with constant speed v, then $y = f(x + vt)$. Both waves $y = f(x + vt)$ and $y = f(x - vt)$ will satisfy the following one dimensional differential equation known as the wave equation

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2} \qquad (11.36)$$

where the symbol ∂ represents partial derivative (read $\frac{\partial y}{\partial x}$ as partial y by partial x). Not all the solutions satisfying this differential equation can represent waves, because any physical acceptable wave must take finite values for all values of x and t. But if the function represents a wave then it must satisfy the differential equation. Since, in one dimension (one independent variable), the partial derivative with respect to x is the same as total derivative in coordinate x, we write

$$\frac{d^2 y}{dx^2} = \frac{1}{v^2}\frac{d^2 y}{dt^2} \qquad (11.37)$$

This can be extended to more than one dimension (two, three, etc.). Here, for simplicity, we focus only on the one dimensional wave equation.

EXAMPLE 11.11

Sketch $y = x - a$ for different values of a.

Solution

This implies, when increasing the value of a, the line shifts towards right side. For $a = vt$, $y = x - vt$ satisfies the differential equation. Though this function satisfies the differential equation, it is not finite for all values of x and t. Hence, it does not represent a wave.

EXAMPLE 11.12

How does the wave $y = \sin(x - a)$ for $a = 0$, $a = \frac{\pi}{4}$, $a = \frac{\pi}{2}$, $a = \frac{3\pi}{2}$ and $a = \pi$ look like?. Sketch this wave.

Solution

From the above picture we observe that $y = \sin(x-a)$ for $a = 0$, $a = \frac{\pi}{4}$, $a = \frac{\pi}{2}$, $a = \frac{3\pi}{2}$ and $a = \pi$, the function $y = \sin(x-a)$ shifts towards right. Further, we can take $a = vt$ and $v = \frac{\pi}{4}$, and sketching for different

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times $t = 0s$, $t = 1s$, $t = 2s$ etc., we once again observe that $y = \sin(x-vt)$ moves towards the right. Hence, $y = \sin(x-vt)$ is a travelling (or progressive) wave moving towards the right. If $y = \sin(x+vt)$ then the travelling (or progressive) wave moves towards the left. Thus, any arbitrary function of type $y = f(x-vt)$ characterising the wave must move towards right and similarly, any arbitrary function of type $y = f(x+vt)$ characterizing the wave must move towards left.

EXAMPLE 11.13

Check the dimensional of the wave $y = \sin(x-vt)$. If it is dimensionally wrong, write the above equation in the correct form.

Solution

Dimensionally it is not correct. we know that $y = \sin(x-vt)$ must be a dimensionless quantity but $x-vt$ has dimension. The correct equation is $y = \sin(k\,x-\omega t)$, where k and ω have the dimensions of inverse of length and inverse of time respectively. The sine functions and cosine functions are periodic functions with period 2π. Therefore, the correct expression is $y = \sin\left(\frac{2\pi}{\lambda}x - \frac{2\pi}{T}t\right)$ where λ and T are wavelength and time period, respectively. In general, $y(x,t)$=A $\sin(k\,x-\omega t)$.

11.6.3 Graphical representation of the wave

Let us graphically represent the two forms of the wave variation

(a) Space (or Spatial) variation graph

(b) Time (or Temporal) variation graph

(a) Space variation graph

Figure 11.24 Graph of sinusoidal function $y = A\sin(kx)$

By keeping the time fixed, the change in displacement with respect to x is plotted. Let us consider a sinusoidal graph, $y = A\sin(kx)$ as shown in the Figure 11.24, where k is a constant. Since the wavelength λ denotes the distance between any two points in the same state of motion, the displacement y is the same at both the ends

$y = x$ and $y = x + \lambda$, i.e.,

$$y = A\sin(kx) = A\sin(k(x+\lambda)) = A\sin(kx + k\lambda) \quad (11.38)$$

The sine function is a periodic function with period 2π. Hence,

$$y = A\sin(kx + 2\pi) = A\sin(kx) \quad (11.39)$$

Comparing equation (11.38) and equation (11.39), we get

$$kx + k\lambda = kx + 2\pi$$

This implies

$$k = \frac{2\pi}{\lambda}\ \text{rad m}^{-1} \quad (11.40)$$

where k is called wave number. This measures how many wavelengths are present in 2π radians.

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The spatial periodicity of the wave is

$$\lambda = \frac{2\pi}{k} \text{ in m}$$

Then,

At $t = 0$ s $\quad y(x, 0) = y(x + \lambda, 0)$ and

At any time t, $y(x, t) = y(x + \lambda, t)$

EXAMPLE 11.14

The wavelength of two sine waves are λ_1 = 1m and λ_2 = 6m. Calculate the corresponding wave numbers.

Solution

$$k_1 = \frac{2\pi}{1} = 6.28 \text{ rad m}^{-1}$$

$$k_2 = \frac{2\pi}{6} = 1.05 \text{ rad m}^{-1}$$

(b) Time variation graph

Figure 11.25 Graph of sinusoidal function $y = A \sin(\omega t)$

By keeping the position fixed, the change in displacement with respect to time is plotted. Let us consider a sinusoidal graph, $y = A \sin(\omega t)$ as shown in the Figure 11.25, where ω is angular frequency of the wave which measures how quickly wave oscillates in time or number of cycles per second.

The temporal periodicity or time period is

$$T = \frac{2\pi}{\omega} \Rightarrow \omega = \frac{2\pi}{T}$$

The angular frequency is related to frequency f by the expression $\omega = 2\pi f$, where the frequency f is defined as the number of oscillations made by the medium particle per second. Since inverse of frequency is time period, we have,

$$T = \frac{1}{f} \text{ in seconds}$$

This is the time taken by a medium particle to complete one oscillation. Hence, we can define the speed of a wave (wave speed, v) as the distance traversed by the wave per second

$$v = \frac{\lambda}{T} = \lambda f \text{ in m s}^{-1}$$

which is the same relation as we obtained in equation (11.4).

11.6.4 Particle velocity and wave velocity

In a plane progressive harmonic wave, the constituent particles in the medium oscillate simple harmonically about their equilibrium positions. When a particle is in motion, the rate of change of displacement at any instant of time is defined as velocity of the particle at that instant of time. This is known as particle velocity.

$$v_P = \frac{dy}{dt} \text{ m s}^{-1} \qquad (11.41)$$

But $\quad y(x, t) = A \sin(k x - \omega t) \qquad (11.42)$

Therefore, $\dfrac{dy}{dt} = -\omega A \cos(k x - \omega t) \qquad (11.43)$

Similarly, we can define velocity (here speed) for the travelling wave (or progressive wave). In order to determine the velocity of a progressive wave, let us consider a progressive wave (shown in Figure 11.23) moving towards right. This can be mathematically represented as a sinusoidal wave. Let P be any point on the phase of the wave and y_P be its displacement with respect

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to the mean position. The displacement of the wave at an instant t is

$$y = y(x,t) = A\sin(k\,x - \omega\,t)$$

At the next instant of time $t' = t + \Delta t$ the position of the point P is $x' = x + \Delta x$. Hence, the displacement of the wave at this instant is

$$y = y(x', t') = y(x + \Delta x, t + \Delta t)$$
$$= A\sin[k\,(x + \Delta x) - \omega\,(t + \Delta t)] \quad (11.44)$$

Since the shape of the wave remains the same, this means that the phase of the wave remains constant (i.e., the y- displacement of the point is a constant). Therefore, equating equation (11.42) and equation (11.44), we get

$$y(x',t') = y(x,t), \text{ which implies}$$

$$A\sin[k\,(x + \Delta x) - \omega\,(t + \Delta t)] = A\sin(k\,x - \omega\,t)$$

Or

$$k(x + \Delta x) - \omega(t + \Delta t) = kx - \omega t = \text{constant} \quad (11.45)$$

On simplification of equation (11.45), we get

$$v = \frac{\Delta x}{\Delta t} = \frac{\omega}{k} = v_p \quad (11.46)$$

where v_p is called wave velocity or phase velocity.

By expressing the angular frequency and wave number in terms of frequency and wave length, we obtain

$$\omega = 2\pi f = \frac{2\pi}{T}$$

$$k = \frac{2\pi}{\lambda}$$

$$v = \frac{\omega}{k} = \lambda f$$

EXAMPLE 11.15

A mobile phone tower transmits a wave signal of frequency 900MHz. Calculate the length of the waves transmitted from the mobile phone tower.

Solution

Frequency, $f = 900\,MHz = 900\times10^6\,Hz$

The speed of wave is $c = 3\times10^8 \text{m s}^{-1}$

$$\lambda = \frac{v}{f} = \frac{3\times10^8}{900\times10^6} = 0.33m$$

11.7 SUPERPOSITION PRINCIPLE

When a jerk is given to a stretched string which is tied at one end, a wave pulse is produced and the pulse travels along the string. Suppose two persons holding the stretched string on either side give a jerk simultaneously, then these two wave pulses move towards each other, meet at some point and move away from each other with their original identity. Their behaviour is very different only at the crossing/meeting points; this behaviour depends on whether the two pulses have the same or different shape as shown in Figure 11.26.

Figure 11.26 Superposition of two waves

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When the pulses have the same shape, at the crossing, the total displacement is the algebraic sum of their individual displacements and hence its net amplitude is higher than the amplitudes of the individual pulses. Whereas, if the two pulses have same amplitude but shapes are 180° out of phase at the crossing point, the net amplitude vanishes at that point and the pulses will recover their identities after crossing. Only waves can possess such a peculiar property and it is called *superposition of waves*. This means that the principle of superposition explains the net behaviour of the waves when they overlap.

Generalizing to any number of waves i.e, if two or more waves in a medium move simultaneously, when they overlap, their total displacement is the vector sum of the individual displacements. We know that the waves satisfy the wave equation which is a linear second order homogeneous partial differential equation in both space coordinates and time. Hence, their linear combination (often called as linear superposition of waves) will also satisfy the same differential equation.

To understand mathematically, let us consider two functions which characterize the displacement of the waves, for example,

$$y_1 = A_1 \sin(kx - \omega t)$$

and

$$y_2 = A_2 \cos(kx - \omega t)$$

Since, both y_1 and y_2 satisfy the wave equation (solutions of wave equation) then their algebraic sum

$$y = y_1 + y_2$$

also satisfies the wave equation. This means, the displacements are additive. Suppose we multiply y_1 and y_2 with some constant then their amplitude is scaled by that constant Further, if C_1 and C_2 are used to multiply the displacements y_1 and y_2, respectively, then, their net displacement y is

$$y = C_1 y_1 + C_2 y_2$$

This can be generalized to any number of waves. In the case of n such waves in more than one dimension the displacements are written using vector notation.

Here, the net displacement $\vec{y}$ is

$$\vec{y} = \sum_{i=1}^{n} C_i \vec{y_i}$$

The principle of superposition can explain the following :

(a) Space (or spatial) Interference (also known as Interference)

(b) Time (or Temporal) Interference (also known as Beats)

(c) Concept of stationary waves

Waves that obey principle of superposition are called linear waves (amplitude is much smaller than their wavelengths). In general, if the amplitude of the wave is not small then they are called non-linear waves. These violate the linear superposition principle, e.g. laser. In this chapter, we will focus our attention only on linear waves.

We will discuss the following in different subsections:

11.7.1 Interference of waves

Figure 11.27 Interference of waves

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Interference is a phenomenon in which two waves superimpose to form a resultant wave of greater, lower or the same amplitude.

Figure 11.28 Interference of two sinusoidal waves

Consider two harmonic waves having identical frequencies, constant phase difference φ and same wave form (can be treated as coherent source), but having amplitudes A_1 and A_2, then

$$y_1 = A_1 \sin(kx - \omega t) \quad (11.47)$$

$$y_2 = A_2 \sin(kx - \omega t + \varphi) \quad (11.48)$$

Suppose they move simultaneously in a particular direction, then interference occurs (i.e., overlap of these two waves). Mathematically

$$y = y_1 + y_2 \quad (11.49)$$

Therefore, substituting equation (11.47) and equation (11.48) in equation (11.49), we get

$$y = A_1 \sin(kx - \omega t) + A_2 \sin(kx - \omega t + \varphi)$$

Using trigonometric identity $\sin(\alpha+\beta) = (\sin\alpha \cos\beta + \cos\alpha \sin\beta)$, we get

$$y = A_1 \sin(kx - \omega t) + A_2 [\sin(kx - \omega t) \cos\varphi + \cos(kx - \omega t) \sin\varphi]$$

$$y = \sin(kx - \omega t)(A_1 + A_2 \cos\varphi) + A_2 \sin\varphi \cos(kx - \omega t) \quad (11.50)$$

Let us re-define

$$A \cos\theta = (A_1 + A_2 \cos\varphi) \quad (11.51)$$

$$\text{and } A \sin\theta = A_2 \sin\varphi \quad (11.52)$$

then equation (11.50) can be rewritten as

$$y = A \sin(kx - \omega t) \cos\theta + A \cos(kx - \omega t) \sin\theta$$

$$y = A (\sin(kx - \omega t) \cos\theta + \sin\theta \cos(kx - \omega t))$$

$$y = A \sin(kx - \omega t + \theta) \quad (11.53)$$

By squaring and adding equation (11.51) and equation (11.52), we get

$$A^2 = A_1^2 + A_2^2 + 2A_1 A_2 \cos\varphi \quad (11.54)$$

Since, intensity is square of the amplitude ($I = A^2$), we have

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos\varphi \quad (11.55)$$

This means the resultant intensity at any point depends on the phase difference at that point.

(a) For constructive interference:

When crests of one wave overlap with crests of another wave, their amplitudes will add up and we get constructive interference. The resultant wave has a larger amplitude than the individual waves as shown in Figure 11.29 (a).

The constructive interference at a point occurs if there is maximum intensity at that point, which means that

$\cos\varphi = +1 \Rightarrow \varphi = 0, 2\pi, 4\pi, \ldots = 2n\pi$, where $n = 0, 1, 2, \ldots$

This is the phase difference in which two waves overlap to give constructive interference.

Therefore, for this resultant wave,

$$I_{\text{maximum}} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2 = (A_1 + A_2)^2$$

Hence, the resultant amplitude

$$A = A_1 + A_2$$

Figure 11.29 (a) Constructive interference (b) Destructive interference

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(b) For destructive interference:

When the trough of one wave overlaps with the crest of another wave, their amplitudes "cancel" each other and we get destructive interference as shown in Figure 11.29 (b). The resultant amplitude is nearly zero. The destructive interference occurs if there is minimum intensity at that point, which means $\cos\varphi = -1 \Rightarrow \varphi = \pi, 3\pi, 5\pi, \ldots = (2\,n\text{-}1)\,\pi$, where n = 0,1,2,…. i.e. This is the phase difference in which two waves overlap to give destructive interference. Therefore,

$$I_{\text{minimum}} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2 = \left(A_1 - A_2\right)^2$$

Hence, the resultant amplitude

$$A = |A_1 - A_2|$$

Let us consider a simple instrument to demonstrate the interference of sound waves as shown in Figure 11.30.

Figure 11.30 Simple instrument to demonstrate interference of sound waves

A sound wave from a loudspeaker S is sent through the tube P. This looks like a T-shaped junction. In this case, half of the sound energy is sent in one direction and the remaining half is sent in the opposite direction. Therefore, the sound waves that reach the receiver R can travel along either of two paths. The distance covered by the sound wave along any path from the speaker to receiver is called the path length . From the Figure 11.30, we notice that the lower path length is fixed but the upper path length can be varied by sliding the upper tube i.e., is varied. The difference in path length is known as path difference,

$$\Delta r = |r_2 - r_1|$$

Suppose the path difference is allowed to be either zero or some integer (or integral) multiple of wavelength λ. Mathematically, we have

$$\Delta r = n\lambda \qquad \text{where, } n = 0, 1, 2, 3, \ldots.$$

Then the two waves arriving from the paths r_1 and r_2 reach the receiver at any instant are in phase (the phase difference is 0° or 2π) and interfere constructively as shown in Figure 11.31.

Figure 11.31 Maximum intensity when the phase difference is 0°

Therefore, in this case, maximum sound intensity is detected by the receiver. If the path difference is some half-odd-integer (or half-integral) multiple of wavelength λ, mathematically, $\Delta r = n\dfrac{\lambda}{2}$

where, $n = 1, 3, \ldots$ (n is odd)

then the two waves arriving from the paths r_1 and r_2 and reaching the receiver at any instant are out of phase (phase difference of π or 180°). They interfere destructively as shown in Figure 11.32. They will cancel each other.

Figure 11.32 Minimum intensity when the phase difference is 180°

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Therefore, the amplitude is minimum or zero amplitude which means no sound. No sound intensity is detected by the receiver in this case. The relation between path difference and phase difference is

$$\text{phase difference} = \frac{2\pi}{\lambda}\,(\text{path difference}) \tag{11.56}$$

i.e., $\Delta\varphi = \frac{2\pi}{\lambda}\Delta r$ or $\Delta r = \frac{\lambda}{2\pi}\Delta\varphi$

EXAMPLE 11.16

Consider two sources A and B as shown in the figure below. Let the two sources emit simple harmonic waves of same frequency but of different amplitudes, and both are in phase (same phase). Let O be any point equidistant from A and B as shown in the figure. Calculate the intensity at points O, Y and X. (X and Y are not equidistant from A & B)

Solution

The distance between OA and OB are the same and hence, the waves starting from A and B reach O after covering equal distances (equal path lengths). Thus, the path difference between two waves at O is zero.

$$OA - OB = 0$$

Since the waves are in the same phase, at the point O, the phase difference between two waves is also zero. Thus, the resultant intensity at the point O is maximum.

Consider a point Y, such that the path difference between two waves is λ. Then the phase difference at Y is

$$\Delta\varphi = \frac{2\pi}{\lambda}\times\Delta r = \frac{2\pi}{\lambda}\times\lambda = 2\pi$$

Therefore, at the point Y, the two waves from A and B are in phase, hence, the intensity will be maximum.

Consider a point X, and let the path difference the between two waves be $\frac{\lambda}{2}$. Then the phase difference at X is

$$\Delta\varphi = \frac{2\pi}{\lambda}\frac{\lambda}{2} = \pi$$

Therefore, at the point X, the waves meet and are in out of phase, Hence, due to destructive interference, the intensity will be minimum.

EXAMPLE 11.17

Two speakers C and E are placed 5 m apart and are driven by the same source. Let a man stand at A which is 10 m away from the mid point O of C and E. The man walks towards the point O which is at 1 m (parallel to OC) as shown in the figure. He receives the first minimum in sound intensity at B. Then calculate the frequency of the source. (Assume speed of sound = 343 m s^{-1})

Solution

The first minimum occurs when the two waves reaching the point B are 180° (out of phase). The path difference $\Delta x = \frac{\lambda}{2}$.

In order to calculate the path difference, we have to find the path lengths x_1 and x_2.

In a right triangle BDC,

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$DB = 10\text{m and } OC = \frac{1}{2}\ (5) = 2.5 \text{ m}$

$CD = OC - 1 = (2.5 \text{ m}) - 1 \text{ m} = 1.5 \text{ m}$

$x_1 = \sqrt{(10)^2 + (1.5)^2} = \sqrt{100 + 2.25}$

$= \sqrt{102.25} = 10.1m$

In a right triangle EFB,

$DB = 10\text{m and } OE = \frac{1}{2}\ (5) = 2.5\text{m} = \text{FA}$

$FB = FA + AB = (2.5 \text{ m}) + 1 \text{ m} = 3.5 \text{ m}$

$x_2 = \sqrt{(10)^2 + (3.5)^2} = \sqrt{100 + 12.25}$

$= \sqrt{112.25} = 10.6m$

The path difference $\Delta x = x_2 - x_1 = 10.6$ m−10.1 m = 0.5 m. Required that this path difference

$$\Delta x = \frac{\lambda}{2} = 0.5 \Rightarrow \lambda = 1.0 \text{ m}$$

To obtain the frequency of source, we use

$$v = \lambda f \Rightarrow f = \frac{v}{\lambda} = \frac{343}{1} = 343 \text{ Hz}$$

$$= 0.3\ k\text{Hz}$$

Note If the speakers were connected such that already the path difference is $\frac{\lambda}{2}$. Now, the path difference combines with a path difference of $\frac{\lambda}{2}$. This gives a total path difference of λ which means, the waves are in phase and there is a maximum intensity at point B.

Figure 11.33: Two waves superimpose with different frequencies such that there is a time alternation in constructive and destructive interference i.e., they are periodically in and out of phase

11.7.2 Formation of beats

When two or more waves superimpose each other with slightly different frequencies, then a sound of periodically varying amplitude at a point is observed. This phenomenon is known as beats. The number of amplitude maxima per second is called beat frequency. If we have two sources, then their difference in frequency gives the beat frequency.

Number of beats per second

$$n = |f_1 - f_2| \text{ per second}$$

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Additional information (Not for examination): Mathematical treatment of beats

For mathematical treatment, let us consider two sound waves having same amplitude and slightly different frequencies f_1 and f_2, superimposed on each other.

Since the sound wave (pressure wave) is a longitudinal wave, let us consider $y_1 = A\sin(\omega_1 t)$ and $y_2 = A\sin(\omega_2 t)$ to be displacements of the two waves at a point $x = 0$ with same amplitude (region having high pressures) and different angular frequencies ω_1 and ω_2, respectively. Then when they are allowed to superimpose we get the net displacement

$$y = y_1 + y_2$$

$$y = A\sin(\omega_1 t) + A\sin(\omega_2 t)$$

But

$$\omega_1 = 2\pi f_1 \text{ and } \omega_2 = 2\pi f_2$$

Then

$$y = A\sin(2\pi f_1 t) + A\sin(2\pi f_2 t)$$

Using trigonometry formula

$$\sin C + \sin D = 2\cos\left(\frac{C-D}{2}\right)\sin\left(\frac{C+D}{2}\right)$$

$$y = 2A\cos\left(2\pi\left(\frac{f_1-f_2}{2}\right)t\right)\sin\left(2\pi\left(\frac{f_1+f_2}{2}\right)t\right)$$

Let $$y_p = 2A\cos\left(2\pi\left(\frac{f_1-f_2}{2}\right)t\right) \qquad (11.57)$$

and if f_1 is slightly higher value than f_2 then, $\left(\frac{f_1-f_2}{2}\right) \ll \left(\frac{f_1+f_2}{2}\right)$ means y_p in equation (11.57) varies very slowly when compared to $\left(\frac{f_1+f_2}{2}\right)$. Therefore

$$y = y_p \sin(2\pi f_{avg} t) \qquad (11.58)$$

This represents a simple harmonic wave of frequency which is an arithmetic average of frequencies of the individual waves, $f_{avg} = \left(\frac{f_1+f_2}{2}\right)$ and amplitude y_p varies with time t.

Case (A):

The resultant amplitude is maximum when y_p is maximum. Since $y_p \propto \cos\left(2\pi\left(\frac{f_1-f_2}{2}\right)t\right)$, this means maximum amplitude occurs only when cosine takes ±1,

$$\cos\left(2\pi\left(\frac{f_1-f_2}{2}\right)t\right) = \pm 1$$

$$\Rightarrow \quad 2\pi\left(\frac{f_1-f_2}{2}\right)t = n\pi,$$

$$\text{or, } (f_1 - f_2)t = n$$

$$\text{or, } t = \frac{n}{(f_1-f_2)} \qquad n = 0,1,2,3, \ldots.$$

Hence, the time interval between two successive maxima is

$$t_2 - t_1 = t_3 - t_2 = \ldots = \frac{1}{(f_1-f_2)}; \quad n = |f_1 - f_2| = \frac{1}{|t_1 - t_2|}$$

Therefore, the number of beats produced per second is equal to the reciprocal of the time interval between two consecutive maxima i.e., $|f_1 - f_2|$.

Case (B):

The resultant amplitude is minimum i.e., it is equal to zero when y_p is minimum. Since $y_p \propto \cos\left(2\pi\left(\frac{f_1-f_2}{2}\right)t\right)$, this means, minimum occurs only when cosine takes 0,

$$\cos\left(2\pi\left(\frac{f_1-f_2}{2}\right)t\right) = 0,$$

$$\Rightarrow \quad 2\pi\left(\frac{f_1-f_2}{2}\right)t = (2n+1)\frac{\pi}{2},$$

$$\Rightarrow \quad (f_1 - f_2)t = \frac{1}{2}(2n+1)$$

$$\text{or, } t = \frac{1}{2}\left(\frac{2n+1}{f_1-f_2}\right), \text{where } f_1 \neq f_2 \quad n = 0,1,2,3,\ldots..$$

Hence, the time interval between two successive minima is

$$t_2 - t_1 = t_3 - t_2 = \ldots = \frac{1}{(f_1-f_2)}; \quad n = |f_1 - f_2| = \frac{1}{|t_1 - t_2|}$$

Therefore, the number of beats produced per second is equal to the reciprocal of the time interval between two consecutive minima i.e., $|f_1 - f_2|$.

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EXAMPLE 11.18

Consider two sound waves with wavelengths 5 m and 6 m. If these two waves propagate in a gas with velocity 330 ms^{-1}. Calculate the number of beats per second.

Solution

Given $\lambda_1 = 5m$ and $\lambda_2 = 6m$

Velocity of sound waves in a gas is $v = 330\ ms^{-1}$

The relation between wavelength and velocity is $v = \lambda f \Rightarrow f = \dfrac{v}{\lambda}$

The frequency corresponding to wavelength

λ_1 is $f_1 = \dfrac{v}{\lambda_1} = \dfrac{330}{5} = 66\,Hz$

The frequency corresponding to wavelength

λ_2 is $f_2 = \dfrac{v}{\lambda_2} = \dfrac{330}{6} = 55\,Hz$

The number of beats per second is

$|f_1 - f_2| = |66 - 55| = 11$ beats per sec

EXAMPLE 11.19

Two vibrating tuning forks produce waves whose equation is given by $y_1 = 5\sin(240\pi t)$ and $y_2 = 4\sin(244\pi t)$. Compute the number of beats per second.

Solution

Given $y_1 = 5\sin(240\pi t)$ and $y_2 = 4\sin(244\pi t)$

Comparing with $y = A\sin(2\pi f_1 t)$, we get

$$2\pi f_1 = 240\pi \Rightarrow f_1 = 120Hz$$

$$2\pi f_2 = 244\pi \Rightarrow f_2 = 122Hz$$

The number of beats produced is $|f_1 - f_2| = |120 - 122| = |-2| = 2$ beats per sec

11.8 STANDING WAVES

11.8.1 Explanation of stationary waves

When the wave hits the rigid boundary it bounces back to the original medium and can interfere with the original waves. A pattern is formed, which are known as standing waves or stationary waves. Consider two harmonic progressive waves (formed by strings) that have the same amplitude and same velocity but move in opposite directions. Then the displacement of the first wave (incident wave) is

$$y_1 = A\sin(kx - \omega t) \quad (11.59)$$

(waves move toward right)

and the displacement of the second wave (reflected wave) is

$$y_2 = A\sin(kx + \omega t) \quad (11.60)$$

(waves move toward left)

both will interfere with each other by the principle of superposition, the net displacement is

$$y = y_1 + y_2 \quad (11.61)$$

Substituting equation (11.59) and equation (11.60) in equation (11.61), we get

$$y = A\sin(kx - \omega t) + A\sin(kx + \omega t) \quad (11.62)$$

Using trigonometric identity, we rewrite equation (11.62) as

$$y(x, t) = 2A\cos(\omega t)\sin(kx) \quad (11.63)$$

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This represents a stationary wave or standing wave, which means that this wave does not move either forward or backward, whereas progressive or travelling waves will move forward or backward. Further, the displacement of the particle in equation (11.63) can be written in more compact form,

$$y(x,t) = A' \cos(\omega t)$$

where, $A' = 2A\sin(kx)$, implying that the particular element of the string executes simple harmonic motion with amplitude equals to A'. The maximum of this amplitude occurs at positions for which

$$\sin(kx) = 1 \Rightarrow \text{kx} = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots = m\pi$$

where m takes half integer or half integral values. The position of maximum amplitude is known as *antinode*. Expressing wave number in terms of wavelength, we can represent the anti-nodal positions as

$$x_m = \left(\frac{2m+1}{2}\right)\frac{\lambda}{2}, \text{ where, } m = 0,1,2\ldots \quad (11.64)$$

For $m = 0$ we have maximum at

$$x_0 = \frac{\lambda}{4}$$

For $m = 1$ we have maximum at

$$x_1 = \frac{3\lambda}{4}$$

For $m = 2$ we have maximum at

$$x_2 = \frac{5\lambda}{4}$$

and so on.

The distance between two successive anti-nodes can be computed by

$$x_m - x_{m-1} = \left(\frac{2m+1}{2}\right)\frac{\lambda}{2} - \left(\frac{(2m+1)+1}{2}\right)\frac{\lambda}{2} = \frac{\lambda}{2}$$

Similarly, the minimum of the amplitude A' also occurs at some points in the space, and these points can be determined by setting

$$\sin(kx) = 0 \Rightarrow k\,x = 0, \pi, 2\pi, 3\pi, \ldots = \text{n}\,\pi$$

where n takes integer or integral values. Note that the elements at these points do not vibrate (not move), and the points are called *nodes*. The nth nodal positions is given by,

$$x_n = n\frac{\lambda}{2} \text{ where, n = 0,1,2,...} \quad (11.65)$$

For $n = 0$ we have minimum at

$$x_0 = 0$$

For $n = 1$ we have minimum at

$$x_1 = \frac{\lambda}{2}$$

For $n = 2$ we have maximum at

$$x_2 = \lambda$$

and so on.

The distance between any two successive nodes can be calculated as

$$x_n - x_{n-1} = n\frac{\lambda}{2} - (n-1)\frac{\lambda}{2} = \frac{\lambda}{2}.$$

EXAMPLE 11.20

Compute the distance between anti-node and neighbouring node.

Solution

For n^{th} mode, the distance between anti-node and neighbouring node is

$$\Delta x_n = \left(\frac{2n+1}{2}\right)\frac{\lambda}{2} - n\frac{\lambda}{2} = \frac{\lambda}{4}$$

11.8.2 Characteristics of stationary waves

(1) Stationary waves are characterised by the confinement of a wave disturbance between two rigid boundaries. This means, the wave does not move forward or backward in a medium (does not advance), it remains steady at its place. Therefore, they are called "stationary waves or standing waves".

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Table 11.3: Comparison between progressive and stationary waves

S.No.	Progressive waves	Stationary waves
1.	Crests and troughs are formed in transverse progressive waves, and compression and rarefaction are formed in longitudinal progressive waves. These waves move forward or backward in a medium i.e., they will advance in a medium with a definite velocity.	Crests and troughs are formed in transverse stationary waves, and compression and rarefaction are formed in longitudinal stationary waves. These waves neither move forward nor backward in a medium i.e., they will not advance in a medium.
2.	All the particles in the medium vibrate such that the amplitude of the vibration for all particles is same.	Except at nodes, all other particles of the medium vibrate such that amplitude of vibration is different for different particles. The amplitude is minimum or zero at nodes and maximum at anti-nodes.
3.	These wave carry energy while propagating.	These waves do not transport energy.

(2) Certain points in the region in which the wave exists have *maximum amplitude, called as anti-nodes* and at certain points the *amplitude is minimum or zero, called as nodes*.

(3) The distance between two consecutive nodes (or) anti-nodes is $\frac{\lambda}{2}$.

(4) The distance between a node and its neighbouring anti-node is $\frac{\lambda}{4}$.

(5) The transfer of energy along the standing wave is zero.

11.8.3 Stationary waves in sonometer

Sono means *sound* related, and sonometer implies sound-related measurements. It is a device for demonstrating the relationship between the frequency of the sound produced in the transverse standing wave in a string, and the tension, length and mass per unit length of the string. Therefore, using this device, we can determine the following quantities:

Figure 11.34 Sonometer

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(a) the *frequency* of the tuning fork or frequency of alternating current

(b) the *tension* in the string

(c) the unknown hanging *mass*

Construction:

The sonometer is made up of a hollow box which is one meter long with a uniform metallic thin string attached to it. One end of the string is connected to a hook and the other end is connected to a weight hanger through a pulley as shown in Figure 11.34. Since only one string is used, it is also known as monochord. The weights are added to the free end of the wire to increase the tension of the wire. Two adjustable wooden knives are put over the board, and their positions are adjusted to change the vibrating length of the stretched wire.

Working :

A transverse stationary or standing wave is produced and hence, at the knife edges P and Q, nodes are formed. In between the knife edges, anti-nodes are formed.

If the length of the vibrating element is l then

$$l = \frac{\lambda}{2} \Rightarrow \lambda = 2l$$

Let f be the frequency of the vibrating element, T the tension of in the string and μ the mass per unit length of the string. Then using equation (11.13), we get

$$f = \frac{v}{\lambda} = \frac{1}{2l}\sqrt{\frac{T}{\mu}} \text{ in Hertz} \qquad (11.66)$$

Let ρ be the density of the material of the string and d be the diameter of the string. Then the mass per unit length μ,

$$\mu = \text{Area} \times \text{density} = \pi r^2 \rho = \frac{\pi \rho d^2}{4}$$

$$\text{frequency } f = \frac{v}{\lambda} = \frac{1}{2l}\sqrt{\frac{T}{\frac{\pi d^2 \rho}{4}}}$$

$$\therefore \quad f = \frac{1}{ld}\sqrt{\frac{T}{\pi\rho}} \qquad (11.67)$$

EXAMPLE 11.21

Let f be the fundamental frequency of the string. If the string is divided into three segments l_1, l_2 and l_3 such that the fundamental frequencies of each segments be f_1, f_2 and f_3, respectively. Show that

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3}$$

Solution

For a fixed tension T and mass density μ, frequency is inversely proportional to the string length i.e.

$$f \propto \frac{1}{l} \Rightarrow f = \frac{v}{2l} \Rightarrow l = \frac{v}{2f}$$

For the first length segment

$$f_1 = \frac{v}{2l_1} \Rightarrow l_1 = \frac{v}{2f_1}$$

For the second length segment

$$f_2 = \frac{v}{2l_2} \Rightarrow l_2 = \frac{v}{2f_2}$$

For the third length segment

$$f_3 = \frac{v}{2l_3} \Rightarrow l_3 = \frac{v}{2f_3}$$

Therefore, the total length

$$l = l_1 + l_2 + l_3$$

$$\frac{v}{2f} = \frac{v}{2f_1} + \frac{v}{2f_2} + \frac{v}{2f} \Rightarrow \frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3}$$

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11.8.4 Fundamental frequency and overtones

Let us now keep the rigid boundaries at $x = 0$ and $x = L$ and produce a standing waves by wiggling the string (as in plucking strings in a guitar). Standing waves with a specific wavelength are produced. Since, the amplitude must vanish at the boundaries, therefore, the displacement at the boundary must satisfy the following conditions

$$y(x = 0, t) = 0 \text{ and } y(x = L, t) = 0 \qquad (11.68)$$

Since the nodes formed are at a distance $\frac{\lambda_n}{2}$ apart, we have $n\left(\frac{\lambda_n}{2}\right) = L$, where n is an integer, L is the length between the two boundaries and λ_n is the specific wavelength that satisfy the specified boundary conditions. Hence,

$$\lambda_n = \left(\frac{2L}{n}\right) \qquad (11.69)$$

What will happen to wavelength if n is taken as zero? Why is this not permitted?

Therefore, not all wavelengths are allowed. The (allowed) wavelengths should fit with the specified boundary conditions, i.e., for n = 1, the first mode of vibration has specific wavelength $\lambda_1 = 2L$. Similarly for n = 2, the second mode of vibration has specific wavelength

$$\lambda_2 = \left(\frac{2L}{2}\right) = L$$

For n = 3, the third mode of vibration has specific wavelength

$$\lambda_3 = \left(\frac{2L}{3}\right)$$

and so on.

The frequency of each mode of vibration (called natural frequency) can be calculated.

We have,

$$f_n = \frac{v}{\lambda_n} = n\left(\frac{v}{2L}\right) \qquad (11.70)$$

The lowest natural frequency is called *the fundamental frequency*.

$$f_1 = \frac{v}{\lambda_1} = \left(\frac{v}{2L}\right) \qquad (11.71)$$

The second natural frequency is called *the first over tone*.

$$f_2 = 2\left(\frac{v}{2L}\right) = \frac{1}{L}\sqrt{\frac{T}{\mu}}$$

The third natural frequency is called *the second over tone*.

$$f_3 = 3\left(\frac{v}{2L}\right) = 3\left(\frac{1}{2L}\sqrt{\frac{T}{\mu}}\right)$$

and so on.

Therefore, the nth natural frequency can be computed as integral (or integer) multiple of fundamental frequency, i.e.,

$$f_n = nf_1, \quad \text{where } n \text{ is an integer} \qquad (11.72)$$

If natural frequencies are written as integral multiple of fundamental frequencies, then the frequencies are called harmonics. Thus, the first harmonic is $f_1 = f_1$ (the fundamental frequency is called first harmonic), the second harmonic is $f_2 = 2f_1$, the third harmonic is $f_3 = 3f_1$ etc.

EXAMPLE 11.22

Consider a string in a guitar whose length is 80 cm and a mass of 0.32 g with tension 80 N is plucked. Compute the first four lowest frequencies produced when it is plucked.

Solution

The velocity of the wave

$$v = \sqrt{\frac{T}{\mu}}$$

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The length of the string, L = 80 cm = 0.8 m
The mass of the string, m = 0.32 g
= 0.32 × 10^{-3} kg
Therefore, the linear mass density,

$$\mu = \frac{0.32\times10^{-3}}{0.8} = 0.4\times10^{-3}\,kg\,m^{-1}$$

The tension in the string, T = 80 N

$$v = \sqrt{\frac{80}{0.4\times10^{-3}}} = 447.2 \text{ m s}^{-1}$$

The wavelength corresponding to the fundamental frequency f_1 is $\lambda_1 = 2L = 2 \times 0.8 = 1.6$ m

The fundamental frequency f_1 corresponding to the wavelength λ_1

$$f_1 = \frac{v}{\lambda_1} = \frac{447.2}{1.6} = 279.5\,Hz$$

Similarly, the frequency corresponding to the second harmonics, third harmonics and fourth harmonics are

$$f_2 = 2f_1 = 559 \text{ Hz}$$

$$f_3 = 3f_1 = 838.5 \text{ Hz}$$

$$f_4 = 4f_1 = 1118 \text{ Hz}$$

11.8.5 Laws of transverse vibrations in stretched strings

There are three laws of transverse vibrations of stretched strings which are given as follows:

(i) The law of length :

For a given wire with tension T (which is fixed) and mass per unit length μ (fixed) the frequency varies inversely with the vibrating length. Therefore,

$$f \propto \frac{1}{l} \Rightarrow f = \frac{C}{l}$$

$$\Rightarrow l\times f = \text{C, where C is a constant}$$

(ii) The law of tension:

For a given vibrating length l (fixed) and mass per unit length μ (fixed) the frequency varies directly with the square root of the tension T,

$$f \propto \sqrt{T}$$

$$\Rightarrow f = A\sqrt{T}, \text{ where A is a constant}$$

(iii) The law of mass:

For a given vibrating length l (fixed) and tension T (fixed) the frequency varies inversely with the square root of the mass per unit length μ,

$$f \propto \frac{1}{\sqrt{\mu}}$$

$$\Rightarrow f = \frac{B}{\sqrt{\mu}}, \text{ where B is a constant}$$

11.9 INTENSITY AND LOUDNESS

Consider a source and two observers (listeners). The source emits sound waves which carry energy. The sound energy emitted by the source is same regardless of whoever measures it, i.e., it is independent of any observer standing in that region. But the sound received by the two observers may be different; this is due to some factors like sensitivity of ears, etc. To quantify such thing, we define two different quantities known as intensity and loudness of sound.

11.9.1 Intensity of sound

When a sound wave is emitted by a source, the energy is carried to all possible surrounding points. The average sound energy emitted or

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transmitted per unit time or per second is called sound power. Therefore, the *intensity of sound* is defined as *"the sound power transmitted per unit area taken normal to the propagation of the sound wave "*.

Figure 11.35 Intensity of sound waves

For a particular source (fixed source), the sound intensity is inversely proportional to the square of the distance from the source.

$$I = \frac{power\ of\ the\ source}{4\pi r^2} \Rightarrow I \propto \frac{1}{r^2}$$

This is known as inverse square law of sound intensity.

EXAMPLE 11.23

A baby cries on seeing a dog and the cry is detected at a distance of 3.0 m such that the intensity of sound at this distance is 10^{-2} W m^{-2}. Calculate the intensity of the baby's cry at a distance 6.0 m.

Solution

I_1 is the intensity of sound detected at a distance 3.0 m and it is given as 10^{-2} $W\ m^{-2}$. Let I_2 be the intensity of sound detected at a distance 6.0 m. Then,

r_1 = 3.0 m, r_2 = 6.0 m

and since, $I \propto \frac{1}{r^2}$

the power output does not depend on the observer and depends on the baby. Therefore,

$$\frac{I_1}{I_2} = \frac{r_2^2}{r_1^2}$$

$$I_2 = I_1 \frac{r_1^2}{r_2^2}$$

$$I_2 = 0.25 \times 10^{-2}\ \text{W m}^{-2}$$

11.9.2 Loudness of sound

Two sounds with same intensities need not have the same loudness. For example, the sound heard during the explosion of balloons in a silent closed room is very loud when compared to the same explosion happening in a noisy market. Though the intensity of the sound is the same, the loudness is not. If the intensity of sound is increased then loudness also increases. But additionally, not only does intensity matter, the internal and subjective experience of "how loud a sound is" i.e., the sensitivity of the listener also matters here. This is often called loudness. That is, loudness depends on both intensity of sound wave and sensitivity of the ear (It is purely observer dependent quantity which varies from person to person) whereas the intensity of sound does not depend on the observer. The *loudness of sound* is defined as *"the degree of sensation of sound produced in the ear or the perception of sound by the listener"*.

11.9.3 Intensity and loudness of sound

Our ear can detect the sound with intensity level ranges from 10^{-2} Wm^{-2} to 20 $W\ m^{-2}$.

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According to Weber-Fechner's law, "loudness (L) is proportional to the logarithm of the actual intensity (I) measured with an accurate non-human instrument". This means that

$$L \propto \ln I$$
$$L = k \ln I$$

where k is a constant, which depends on the unit of measurement. The difference between two loudnesses, L_1 and L_0 measures the relative loudness between two precisely measured intensities and is called as sound intensity level. Mathematically, sound intensity level is

$$\Delta L = L_1 - L_0 = k \ln I_1 - k \ln I_0 = k \ln\left[\frac{I_1}{I_0}\right]$$

If k = 1 bel, k = 10 decibel, then sound intensity level is measured in bel, in honour of Alexander Graham Bell. Therefore,

$$\Delta L = \ln\left[\frac{I_1}{I_0}\right] \text{ bel}$$

However, this is practically a bigger unit, so we use a convenient smaller unit, called decibel. Thus, decibel = $\frac{1}{10}$ bel. Therefore, by multiplying and dividing by 10, we get

$$\Delta L = 10\left(\ln\left[\frac{I_1}{I_0}\right]\right)\frac{1}{10} \text{ bel}$$

$$\Delta L = 10 \ln\left[\frac{I_1}{I_0}\right] \text{ decibel with } k = 10$$

For practical purposes, we use logarithm to base 10 instead of natural logarithm,

$$\Delta L = 10 \log_{10}\left[\frac{I_1}{I_0}\right] \text{ decibel} \qquad (11.73)$$

EXAMPLE 11.24

The sound level from a musical instrument playing is 50 dB. If three identical musical instruments are played together then compute the total intensity. Calculate the intensity of the sound from each instrument as the threshold of hearing is 10^{-12} W m^{-2}.

Solution

$$\Delta L = 10 \log_{10}\left[\frac{I_1}{I_0}\right] = 50\ dB$$

$$\log_{10}\left[\frac{I_1}{I_0}\right] = 5\ dB$$

$\frac{I_1}{I_0} = 10^5 \Rightarrow I_1 = 10^5\, I_0 = 10^5 \times 10^{-12}$ Wm^{-2}

$I_1 = 10^{-7}$ Wm^{-2}

Since three musical instruments are played, therefore, $I_{total} = 3I_1 = 3 \times 10^{-7}$ Wm^{-2}.

11.10 VIBRATIONS OF AIR COLUMN

Musical instruments like flute, clarinet, nathaswaram, etc are known as wind instruments. They work on the principle of vibrations of air columns. The simplest form of a wind instrument is the organ pipe. It is made up of a wooden or metal pipe which produces the musical sound. For example, flute, clarinet and nathaswaram are organ pipe instruments. Organ pipe instruments are classified into two types:

(a) Closed organ pipes:

Figure 11.36: Clarinet is an example of a closed organ pipe

Look at the picture of a clarinet, shown in Figure 11.36. It is a pipe with one end closed and the other end open. If one end of a pipe is closed, the wave reflected at this closed end is 180° out of phase with the incoming wave. Thus there is no

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Figure 11.37 No motion of particles which leads to nodes at closed end and antinodes at open end (fundamental mode) (N-node, A-antinode)

displacement of the particles at the closed end. Therefore, nodes are formed at the closed end and anti-nodes are formed at open end.

Let us consider the simplest mode of vibration of the air column called the fundamental mode. Anti-node is formed at the open end and node at closed end. From the Figure 11.37, let L be the length of the tube and the wavelength of the wave produced. For the fundamental mode of vibration, we have,

$$L = \frac{\lambda_1}{4} \text{ or } \lambda_1 = 4L \qquad (11.74)$$

The frequency of the note emitted is

$$f_1 = \frac{v}{\lambda_1} = \frac{v}{4L} \qquad (11.75)$$

which is called the fundamental note.

The frequencies higher than fundamental frequency can be produced by blowing air strongly at open end. Such frequencies are called overtones.

The Figure 11.38 shows the second mode of vibration having two nodes and two anti-nodes, for which we have, from example 11.20.

Figure 11.38 second mode of vibration having two nodes and two anti-nodes

$$4L = 3\lambda_2$$

$$L = \frac{3\lambda_2}{4} \quad \text{or} \quad \lambda_2 = \frac{4L}{3}$$

The frequency for this,

$$f_2 = \frac{v}{\lambda_2} = \frac{3v}{4L} = 3f_1$$

is called *first over tone*, since here, the frequency is three times the fundamental frequency it is called *third harmonic*.

The Figure 11.39 shows third mode of vibration having three nodes and three anti-nodes.

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Figure 11.39 Third mode of vibration having three nodes and three anti-nodes

We have, $4L = 5\lambda_3$

$$L = \frac{5\lambda_3}{4} \text{ or } \lambda_3 = \frac{4L}{5}$$

The frequency

$$f_3 = \frac{v}{\lambda_3} = \frac{5v}{4L} = 5f_1$$

is called *second over tone*, and since $n = 5$ here, this is called *fifth harmonic*. Hence, *the closed organ pipe has only odd harmonics and frequency of the* n^{th} *harmonic is* $f_n = (2n+1)f_1$. Therefore, *the frequencies of harmonics are in the ratio*

$$f_1 : f_2 : f_3 : f_4 : \ldots = 1 : 3 : 5 : 7 : \ldots \qquad (11.76)$$

(b) Open organ pipes:

Figure 11.40 Flute is an example of open organ pipe

Consider the picture of a flute, shown in Figure 11.40. It is a pipe with both the ends open. At both open ends, anti-nodes are formed. Let us consider the simplest mode of vibration of the air column called fundamental mode. Since anti-nodes are formed at the open end, a node is formed at the mid-point of the pipe.

Figure 11.41 Antinodes are formed at the open end and a node is formed at the middle of the pipe.

From Figure 11.41, if L be the length of the tube, the wavelength of the wave produced is given by

$$L = \frac{\lambda_1}{2} \text{ or } \lambda_1 = 2L \qquad (11.77)$$

The frequency of the note emitted is

$$f_1 = \frac{v}{\lambda_1} = \frac{v}{2L} \qquad (11.78)$$

which is called the *fundamental note*.

The frequencies higher than fundamental frequency can be produced by blowing air strongly at one of the open ends. Such frequencies are called overtones.

Figure 11.42 Second mode of vibration in open pipes having two nodes and three anti-nodes

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The Figure 11.42 shows the second mode of vibration in open pipes. It has two nodes and three anti-nodes, and therefore,

$$L = \lambda_2 \text{ or } \lambda_2 = L$$

The frequency

$$f_2 = \frac{v}{\lambda_2} = \frac{v}{L} = 2 \times \frac{v}{2L} = 2 f_1$$

is called ***first over tone***. Since $n = 2$ here, it is called the ***second harmonic***.

Figure 11.43 Third mode of vibration having three nodes and four anti-nodes

The Figure 11.43 above shows the third mode of vibration having three nodes and four anti-nodes

$$L = \frac{3}{2}\lambda_3 \;\; or \;\; \lambda_3 = \frac{2L}{3}$$

The frequency

$$f_3 = \frac{v}{\lambda_3} = \frac{3v}{2L} = 3 f_1$$

is called *second over tone*. Since $n = 3$ here, it is called the *third harmonic*.

Hence, the open organ pipe has all the harmonics and frequency of nth harmonic is $f_n = nf_1$. Therefore, the frequencies of harmonics are in the ratio

$$f_1 : f_2 : f_3 : f_4 : \ldots = 1 : 2 : 3 : 4 : \ldots \qquad (11.79)$$

EXAMPLE 11.25

If a flute sounds a note with 450*Hz*, what are the frequencies of the second, third, and fourth harmonics of this pitch?. If the clarinet sounds with a same note as 450*Hz*, then what are the frequencies of the lowest three harmonics produced ?.

Solution

For a flute which is an open pipe, we have

Second harmonics $f_2 = 2 f_1 = 900$ Hz

Third harmonics $f_3 = 3 f_1 = 1350$ Hz

Fourth harmonics $f_4 = 4 f_1 = 1800$ Hz

For a clarinet which is a closed pipe, we have

Second harmonics $f_2 = 3 f_1 = 1350$ Hz

Third harmonics $f_3 = 5 f_1 = 2250$ Hz

Fourth harmonics $f_4 = 7 f_1 = 3150$ Hz

EXAMPLE 11.26

If the third harmonics of a closed organ pipe is equal to the fundamental frequency of an open organ pipe, compute the length of the open organ pipe if the length of the closed organ pipe is 30 cm.

Solution

Let l_2 be the length of the open organ pipe, with $l_1 = 30\,cm$ the length of the closed organ pipe.

It is given that the third harmonic of closed organ pipe is equal to the fundamental frequency of open organ pipe.

The third harmonic of a closed organ pipe is

$$f_2 = \frac{v}{\lambda_2} = \frac{3v}{4l_1} = 3 f_1$$

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The fundamental frequency of open organ pipe is $f_1 = \frac{v}{\lambda_1} = \frac{v}{2l_2}$

Therefore,

$$\frac{v}{2l_2} = \frac{3v}{4l_1} \Rightarrow l_2 = \frac{2l_1}{3} = 20\,cm$$

11.10.1 Resonance air column apparatus

Figure 11.44: The resonance air column apparatus and first, second and third resonance

The resonance air column apparatus is one of the simplest techniques to measure the speed of sound in air at room temperature. It consists of a cylindrical glass tube of one meter length whose one end A is open and another end B is connected to the water reservoir R through a rubber tube as shown in Figure 11.44. This cylindrical glass tube is mounted on a vertical stand with a scale attached to it. The tube is partially filled with water and the water level can be adjusted by raising or lowering the water in the reservoir R. The surface of the water will act as a closed end and other as the open end. Therefore, it behaves like a closed organ pipe, forming nodes at the surface of water and antinodes at the open end. When a vibrating tuning fork is brought near the open end of the tube, longitudinal waves are formed inside the air column. These waves move downward as shown in Figure 11.44, and reach the surfaces of water and get reflected and produce standing waves. The length of the air column is varied by changing the water level until a loud sound is produced in the air column. At this particular length the frequency of waves in the air column resonates with the frequency of the tuning fork (natural frequency of the tuning fork). At resonance, the frequency of sound waves produced is equal to the frequency of the tuning fork. This will occur only when the length of air column is proportional to $\left(\frac{1}{4}\right)^{th}$ of the wavelength of the sound waves produced.

Let the first resonance occur at length L_1, then

$$\frac{1}{4}\lambda = L_1 \qquad (11.80)$$

But since the antinodes are not exactly formed at the open end, we have to include a correction, called end correction e, by assuming that the antinode is formed at some small distance above the open end. Including this end correction, the first resonance is

$$\frac{1}{4}\lambda = L_1 + e \qquad (11.81)$$

Now the length of the air column is increased to get the second resonance. Let L_2 be the length at which the second resonance occurs. Again taking end correction into account, we have

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$$\frac{3}{4}\lambda = L_2 + e \qquad (11.82)$$

In order to avoid end correction, let us take the difference of equation (11.82) and equation (11.81), we get

$$\frac{3}{4}\lambda - \frac{1}{4}\lambda = (L_2 + e) - (L_1 + e)$$

$$\Rightarrow \frac{1}{2}\lambda = L_2 - L_1 = \Delta L$$

$$\Rightarrow \lambda = 2\Delta L$$

The speed of the sound in air at room temperature can be computed by using the formula

$$v = f\lambda = 2f\,\Delta L$$

Further, to compute the end correction, we use equation (11.81) and equation (11.82), we get

$$e = \frac{L_2 - 3L_1}{2}$$

EXAMPLE 11.27

A frequency generator with fixed frequency of 343 Hz is allowed to vibrate above a 1.0 m high tube. A pump is switched on to fill the water slowly in the tube. In order to get resonance, what must be the minimum height of the water?. (speed of sound in air is 343 m s^{-1})

Solution

The wavelength, $\lambda = \frac{c}{f}$

$$\lambda = \frac{343\,ms^{-1}}{343\,Hz} = 1.0 \text{ m}$$

Let the length of the resonant columns be L_1, L_2 and L_3. The first resonance occurs at length L_1

$$L_1 = \frac{\lambda}{4} = \frac{1}{4} = 0.25 \text{ m}$$

The second resonance occurs at length L_2

$$L_2 = \frac{3\lambda}{4} = \frac{3}{4} = 0.75 \text{ m}$$

The third resonance occurs at length

$$L_3 = \frac{5\lambda}{4} = \frac{5}{4} = 1.25 \text{ m}$$

and so on.

Since total length of the tube is 1.0 m the third and other higher resonances do not occur. Therefore, the minimum height of water H_{min} for resonance is,

H_{min} = 1.0 m – 0.75 m = 0.25 m

EXAMPLE 11.28

A student performed an experiment to determine the speed of sound in air using the resonance column method. The length of the air column that resonates in the fundamental mode with a tuning fork is 0.2 m. If the length is varied such that the same tuning fork resonates with the first overtone at 0.7 m. Calculate the end correction.

Solution

End correction

$$e = \frac{L_2 - 3L_1}{2} = \frac{0.7 - 3(0.2)}{2} = 0.05\,\text{m}$$

EXAMPLE 11.29

Consider a tuning fork which is used to produce resonance in an air column. A resonance air column is a glass tube whose length can be adjusted by a variable piston. At room temperature, the two successive resonances observed are at 20 cm and 85 cm of the column length. If the frequency of the length is 256 Hz, compute the velocity of the sound in air at room temperature.

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Solution

Given two successive length (resonance) to be $L_1 = 20$ cm and $L_2 = 85$ cm

The frequency is $f = 256$ Hz

$$v = f\lambda = 2f\,\Delta L = 2f\,(L_2 - L_1)$$

$$= 2 \times 256 \times (85 - 20) \times 10^{-2}\ \text{m s}^{-1}$$

$$v = 332.8\ \text{m s}^{-1}$$

11.11 DOPPLER EFFECT

Imagine that you are standing on a railway platform and listening to the blowing whistle of a train moving past you, the pitch (or frequency) of the sound you listen as the train approaches you is higher than the pitch you listen as it moves away from you. This is an example of Doppler effect.

This effect occurs due to the relative motion between the source of sound and its listener. This motion–related frequency change was first observed and studied by Johann Christian Doppler (1803–1853), an Austrian Mathematician and Physicist.

Whenever there is a relative motion between the source of sound and the listener, the frequency of the sound observed by the listener is different from the frequency produced by the source. This is known as Doppler effect.

The Doppler effect is a wave phenomenon. Therefore, it occurs not only for sound waves but for any wave such as light and other electromagnetic waves. Here, we will discuss different cases of Doppler effect for sound and derive the expression for the frequency observed by the listener.

***Stationary observer* and *stationary source* means the observer and source are both at rest with respect to medium respectively**

i) Observed frequency: Stationary source and Moving listener

Consider a point source S of sound at rest with respect to the medium (air) in which it is kept. The medium is assumed to be uniform and is also at rest. The source emits sound waves of frequency f and wavelength λ.

Figure 11.45 Listener moves toward the stationary source

Sound waves travel with the same speed v in all directions radially away from the source in the form of spherical waves. The compressions (or wavefronts) of sound waves are represented by concentric circles in the Figure 11.45. The distance between two successive compressions is equal to its wavelength λ and the frequency of the wave is given by

$$f = \frac{v}{\lambda} \qquad (11.83)$$

When the listener L is stationary, there is no relative motion between the source and the listener. Since v and λ remain unchanged, the frequency of sound observed by the listener is the same as the source frequency f.

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Now the listener moves directly toward the stationary source (Figure 11.45). If v_L is the speed of the listener, then the relative speed of sound with respect to the listener becomes $v' = v + v_L$. Since the wavelength remains unchanged (because the source is stationary), the frequency of sound observed by the listener is changed and the observed frequency f' is given by

$$f' = \frac{v'}{\lambda} = \frac{v + v_L}{\lambda}$$

Using the equation (11.83),

$$f' = \left(\frac{v + v_L}{v}\right) f \qquad (11.84)$$

(listener moving toward the source)

Thus, the observed frequency is greater than the source frequency when the listener moves toward the stationary source.

If the listener is moving away from the stationary source, the observed frequency can be obtained from equation (11.84) by taking negative value for v_L. It is given by

$$f' = \left(\frac{v + (-v_L)}{v}\right) f$$

$$f' = \left(\frac{v - v_L}{v}\right) f \qquad (11.85)$$

(listener moving away from the source)

Thus, the observed frequency is less than the source frequency when the listener is moving away from the stationary source.

ii) Observed frequency: Moving source and stationary listener

Assume that both the source S and the listener L are at rest as shown in Figure 11.46a. Two successive compressions are also shown and are represented by two concentric circles. The second compression has just been emitted and is still near the source. The distance between two successive compressions is the wavelength λ of the sound. Since f is the frequency of the source, then the time between emissions of compressions is

$$T = \frac{1}{f} = \frac{\lambda}{v}$$

Figure 11.46 Source moves toward the stationary listener

Now the listener is stationary and the source moves directly toward the listener (Figure 11.46(b)). Let the speed of the source be v_S which is less than the speed of sound v.

In a time T, the first compression travels a distance $vT = \lambda$ and the source moves a distance $v_S T$. As a result, the distance between two successive compressions is decreased from λ to $\lambda' = \lambda - v_S T$. Therefore, the wavelength observed by the listener is given by

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$$\lambda' = \lambda - v_S T = \lambda - \left(\frac{v_S}{f}\right)$$

The observed frequency is then given by

$$f' = \frac{v}{\lambda'} = \frac{v}{\lambda - \left(\frac{v_S}{f}\right)}$$

$$= \frac{v}{\left(\frac{v}{f}\right) - \left(\frac{v_S}{f}\right)}$$

$$f' = \left(\frac{v}{v - v_S}\right) f \qquad (11.86)$$

(source moving toward the listener)

Thus, whenever the source moves toward the stationary listener, the observed frequency is greater than the source frequency.

If the source is moving away from the stationary listener, the observed frequency can be obtained from equation (11.86) by taking negative value for v_S. It is given by

$$f' = \left(\frac{v}{v - (-v_S)}\right) f$$

$$f' = \left(\frac{v}{v + v_S}\right) f \qquad (11.87)$$

(source moving away from the listener)

Thus, the observed frequency is less than the source frequency when the source is moving away from the stationary listener.

iii) Observed frequency: Both source and listener moving

When both source and listener are moving, the observed frequency is obtained by combining equations (11.84) and (11.86).

$$f' = \left(\frac{v + v_L}{v - v_S}\right) f \qquad (11.88)$$

The sign convention we used here is that both v_S and v_L take positive values if the source or the listener moves toward the other. Likewise, they are negative when the source or the listener moves away from the other.

The observed frequency for different situations of relative motion between the source and the listener is consolidated in Table 11.4.

Note

It is important to note that the change in frequency occurs either due to the change in speed of sound (when the listener moves and source at rest) or due to the change in wavelength of sound (when the source moves and observer at rest).

If both source and listener move, the change in frequency occurs due to both the change in speed of sound and the change in wavelength of sound wave.

DO YOU KNOW?

Suppose the source moves faster than sound (that is, the source is supersonic), the equations (11.84) and (11.86) for observed frequency will become invalid and a stationary listener in front of the source hears no sound as the sound waves are at the rear of the source.

At such speeds, the newly produced waves and the old waves interfere constructively which leads to very large amplitude of sound, called a 'sonic boom' or 'shock wave'.

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Table 11.4: Observed frequency for different situations

S.No.	Situation	Observed frequency
1	L moves toward the stationary S	$f' = \left(\frac{v + v_L}{v}\right) f$
2	L moves away from the stationary S	$f' = \left(\frac{v - v_L}{v}\right) f$
3	S moves toward the stationary L	$f' = \left(\frac{v}{v - v_S}\right) f$
4	S moves away from the stationary L	$f' = \left(\frac{v}{v + v_S}\right) f$
5	S and L move toward each other	$f' = \left(\frac{v + v_L}{v - v_S}\right) f$
6	S and L recede from each other	$f' = \left(\frac{v - v_L}{v + v_S}\right) f$
7	S chases the L	$f' = \left(\frac{v - v_L}{v - v_S}\right) f$
8	L chases the S	$f' = \left(\frac{v + v_L}{v + v_S}\right) f$
9	S and L move toward each other and the medium also moves in the direction of sound with speed v_m	$f' = \left(\frac{(v + v_m) + v_L}{(v + v_m) - v_S}\right) f$

Note

Doppler effect in sound is asymmetric while that in light is symmetric.

The observed frequency of sound when the source moves toward stationary listener and the observed frequency when the listener moves toward stationary source with the same speed are not equal. Although the relative speed is same in both the cases, observed frequency is different. Hence, we say that the Doppler effect in sound is asymmetric. The reason is that sound wave requires a medium for its propagation and it has its speed with respect to that medium.

But in the case of light and other electromagnetic radiations, the observed frequency is the same in both abovesaid cases. Therefore, Doppler effect in light and other electromagnetic waves is symmetric because the propagation of light is independent of the medium.

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EXAMPLE 11.30

A sound of frequency 1500 Hz is emitted by a source which moves away from an observer and moves towards a cliff at a speed of 6 ms^{-1}.

(a) Calculate the frequency of the sound which is coming directly from the source.

(b) Compute the frequency of sound heard by the observer reflected off the cliff. Assume the speed of sound in air is 330 m s^{-1}.

Solution

(a) Source is moving away and observer is stationary, therefore, the frequency of sound heard directly from source is

$$f' = \left[\frac{v}{v+v_s}\right] f = \left[\frac{330}{330+6}\right] \times 1500 = 1473\,\text{Hz}$$

(b) Sound is reflected from the cliff and reaches observer, therefore,

$$f' = \left[\frac{v}{v-v_s}\right] f = \left[\frac{330}{330-6}\right] \times 1500 = 1528\,\text{Hz}$$

EXAMPLE 11.31

An observer observes two moving trains, one reaching the station and other leaving the station with equal speeds of 8 m s^{-1}. If each train sounds its whistles with frequency 240 Hz, then calculate the number of beats heard by the observer.

Solution:

Observer is stationary

(i) Source (train) is moving towards an observer:

The observed frequency due to train arriving station is

$$f_{in} = \left[\frac{v}{v-v_s}\right] f = \left[\frac{330}{330-8}\right] \times 240 = 246\,\text{Hz}$$

(ii) Source (train) is moving away from an observer:

The observed frequency due to train leaving station is

$$f_{out} = \left[\frac{v}{v+v_s}\right] f = \left[\frac{330}{330+8}\right] \times 240 = 234\,\text{Hz}$$

So the number of beats = $|f_{in} - f_{out}|$ = (246–234) = 12

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SUMMARY

- A disturbance which carries energy and momentum from one point in space to another point in space without the transfer of medium is known as a wave.
- The waves which require medium for their propagation are known as mechanical waves.
- The waves which do not require medium for their propagation are known as non-mechanical waves.
- For a transverse wave, the vibration of particles in a medium is perpendicular to the direction of propagation of the wave.
- For a longitudinal wave, the vibration of particles in a medium is parallel to the direction of propagation of the wave.
- Elasticity and inertia are necessary properties of the medium for wave propagation.
- Waves formed in still water (ripples) are transverse and wave formed due to vibration of tuning fork is longitudinal.
- The distance between two consecutive crests or troughs is known as wavelength, λ.
- The number of waves which crossed a point per second is known as frequency, f.
- The time taken by one wave to cross a point is known as time period, T.
- velocity of the wave is $v = \lambda f$.
- Frequency is source dependent and wave velocity is medium dependent.
- The velocity of a transverse wave produce in a stretched string depends on tension in the string and mass per unit length. It does not depend on shape of the wave form.
- Velocity of transverse wave on a string is $v = \sqrt{\frac{T}{\mu}}\ ms^{-1}$.
- Velocity of longitudinal wave in an elastic medium is $v = \sqrt{\frac{E}{\rho}}\ ms^{-1}$.
- The minimum distance from a sound reflecting wall to hear an echo at 20°C is 17.2 meters.
- The wave equation is $\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$ in one dimension.
- Wave number is given by $k = \frac{2\pi}{\lambda} rad\, m^{-1}$.
- During interference the resultant intensity is $I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\varphi$, where the intensity is square of the amplitude $I = A^2$.

  For constructive interference, $I_{maximum} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2 = \left(A_1 + A_2\right)^2$.

  For destructive interference, $I_{minimum} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2 = (A_1 - A_2)^2$.
- When we superimpose two or more waves with slightly different frequencies then a sound of periodically varying amplitude at a point is observed. This phenomenon is known as beats. The number of amplitude maxima per second is called beat frequency.

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SUMMARY (cont.)

- If natural frequencies are written as integral multiples of fundamental frequency, then the frequencies are said to be in harmonics. Thus, the first harmonic is $v_1 = v_1$, (the fundamental frequency is called first harmonics), the second harmonics is $v_2 = 2\ v_1$, the third harmonics is $v_3 = 3\ v_1$, and so on.
- *Loudness of sound* is defined as *"the degree of sensation of sound produced in the ear or the perception of sound by the listener"*.
- *The intensity of sound* is defined as *"the sound power transmitted per unit area placed normal to the propagation of sound wave "*.
- Sound intensity level, $\Delta L = 10 \log_{10} \frac{I_1}{I_0}\ decibel$.
- A closed organ pipe has only odd harmonics and the corresponding frequency of the nth harmonic is $f_n = (2n + 1)\ f_1$.
- In a closed organ pipe the frequencies of harmonics are in the ratio

  $f_1 : f_2 : f_3 : f_4 :... = 1 : 3 : 5 : 7 :...$
- The open organ pipe has all harmonics and frequency of the nth harmonic is $f_n = n\ f_1$.
- In the open organ pipe the frequencies of harmonics are in the ratio

  $f_1 : f_2 : f_3 : f_4 :... = 1 : 2 : 3 : 4 :...$
- Whenever there is a relative motion between the source of sound and the listener, the frequency of the sound observed by the listener is different from the frequency produced by the source. This is known as Doppler effect.

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CONCEPT MAP

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EVALUATION

I. Multiple Choice Questions:

1. A student tunes his guitar by striking a 120 Hertz with a tuning fork, and simultaneously plays the 4th string on his guitar. By keen observation, he hears the amplitude of the combined sound oscillating thrice per second. Which of the following frequencies is the most likely the frequency of the 4th string on his guitar?.

a) 130 b) 117
c) 110 d) 120

2. A transverse wave moves from a medium A to a medium B. In medium A, the velocity of the transverse wave is 500 ms^{-1} and the wavelength is 5 m. The frequency and the wavelength of the wave in medium B when its velocity is 600 ms^{-1}, respectively are

a) 120 Hz and 5 m
b) 100 Hz and 5 m
c) 120 Hz and 6 m
d) 100 Hz and 6 m

3. For a particular tube, among six harmonic frequencies below 1000 Hz, only four harmonic frequencies are given : 300 Hz, 600 Hz, 750 Hz and 900 Hz. What are the two other frequencies missing from this list?.

a) 100 Hz, 150 Hz
b) 150 Hz, 450 Hz
c) 450 Hz, 700 Hz
d) 700 Hz, 800 Hz

4. Which of the following options is correct?.

A	B
(1) Quality	(A) Intensity
(2) Pitch	(B) Waveform
(3) Loudness	(C) Frequency

Options for (1), (2) and (3), respectively are

a) (B),(C) and (A)
b) (C), (A) and (B)
c) (A), (B) and (C)
d) (B), (A) and (C)

5. Eqution of travelling wave on a stretched string of linear density 5 g/m is $y = 0.03 \sin(450t - 9x)$, where distance and time are measured in SI units. The tension in the string is

a) 5 N
b) 12.5 N
c) 7.5 N
d) 10 N

6. A sound wave whose frequency is 5000 Hz travels in air and then hits the water surface. The ratio of its wavelengths in water and air is

a) 4.30 b) 0.23
c) 5.30 d) 1.23

7. A person standing between two parallel hills fires a gun and hears the first echo

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after t_1 sec and the second echo after t_2 sec. The distance between the two hills is

a) $\frac{v(t_1 - t_2)}{2}$ b) $\frac{v(t_1 t_2)}{2(t_1 + t_2)}$

c) $v(t_1 + t_2)$ d) $\frac{v(t_1 + t_2)}{2}$

8. An air column in a pipe which is closed at one end, will be in resonance with the vibrating body of frequency 83Hz. Then the length of the air column is

a) 1.5 m (b) 0.5 m

(c) 1.0 m (d) 2.0 m

9. The displacement y of a wave travelling in the x direction is given by $y = (2 \times 10^{-3}) \sin(300t - 2x + \frac{\pi}{4})$, where x and y are measured in metres and t in second. The speed of the wave is

(a) 150 ms^{-1} (b) 300 ms^{-1}

(c) 450 ms^{-1} (d) 600 ms^{-1}

10. Consider two uniform wires vibrating simultaneously in their fundamental notes. The tensions, densities, lengths and diameter of the two wires are in the ratio 8 : 1, 1 : 2, $x : y$ and 4 : 1 respectively. If the note of the higher pitch has a frequency of 360 Hz and the number of beats produced per second is 10, then the value of $x : y$ is

(a) 36 : 35

(b) 35 : 36

(c) 1 : 1

(d) 1 : 2

11. Which of the following represents a wave

(a) $(x - vt)^3$ (b) $x(x+vt)$

(c) $\frac{1}{(x+vt)}$ (d) $\sin(x+vt)$

12. A man sitting on a swing which is moving to an angle of 60° from the vertical is blowing a whistle which has a frequency of 2.0 k Hz. The whistle is 2.0 m from the fixed support point of the swing. A sound detector which detects the whistle sound is kept in front of the swing. The maximum frequency the sound detector detected is

(a) 2.027 *k*Hz (b) 1.974 *k*Hz

(c) 9.74 *k*Hz (d) 1.011 *k*Hz

13. Let $y = \frac{1}{1+x^2}$ at t = 0 s be the amplitude of the wave propagating in the positive x-direction. At $t = 2\,s$, the amplitude of the wave propagating becomes $y = \frac{1}{1+(x-2)^2}$. Assume that the shape of the wave does not change during propagation. The velocity of the wave is

(a) 0.5m s^{-1} (b) 1.0m s^{-1}

(c) 1.5m s^{-1} (d) 2.0m s^{-1}

14. A uniform rope having mass m hangs vertically from a rigid support. A transverse wave pulse is produced at the lower end. Which of the following plots shows the correct variation of speed v with height h from the lower end?

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15. An organ pipe A closed at one end is allowed to vibrate in its first harmonic and another pipe B open at both ends is allowed to vibrate in its third harmonic. Both A and B are in resonance with a given tuning fork. The ratio of the length of A and B is

(a) $\frac{8}{3}$ (b) $\frac{3}{8}$

(c) $\frac{1}{6}$ (d) $\frac{1}{3}$

Answers:

1) b	2) d	3) b	4) a
5) b	6) a	7) d	8) c
9) a	10) a	11) d	12) a
13) b	14) d	15) c	

II. Short Answer Questions

1. What is meant by waves?.
2. Write down the types of waves.
3. What are transverse waves?. Give one example.
4. What are longitudinal waves?. Give one example.
5. Define wavelength.
6. Write down the relation between frequency, wavelength and velocity of a wave.
7. What is meant by interference of waves?.
8. Explain the beat phenomenon.
9. Define intensity of sound and loudness of sound.
10. Explain Doppler Effect.
11. Explain red shift and blue shift in Doppler Effect.
12. What is meant by end correction in resonance air column apparatus?
13. Sketch the function $y = x + a$. Explain your sketch.
14. Write down the factors affecting velocity of sound in gases.
15. What is meant by an echo?. Explain.

III. Long Answer Questions

1. Discuss how ripples are formed in still water.
2. Briefly explain the difference between travelling waves and standing waves.
3. Show that the velocity of a travelling wave produced in a string is $v = \sqrt{\frac{T}{\mu}}$
4. Describe Newton's formula for velocity of sound waves in air and also discuss the Laplace's correction.
5. Write short notes on reflection of sound waves from plane and curved surfaces.
6. Briefly explain the concept of superposition principle.
7. Explain how the interference of waves is formed.
8. Describe the formation of beats.
9. What are stationary waves?. Explain the formation of stationary waves and also write down the characteristics of stationary waves.
10. Discuss the law of transverse vibrations in stretched strings.
11. Explain the concepts of fundamental frequency, harmonics and overtones in detail.
12. What is a sonometer?. Give its construction and working. Explain how to determine the frequency of tuning fork using sonometer.

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13. Write short notes on intensity and loudness.

14. Explain how overtones are produced in a

(a) Closed organ pipe

(b) Open organ pipe

15. How will you determine the velocity of sound using resonance air column apparatus?

16. What is meant by Doppler effect?. Discuss the following cases

(1) Source in motion and Observer at rest

(a) Source moves towards observer

(b) Source moves away from the observer

(2) Observer in motion and Source at rest.

(a) Observer moves towards Source

(b) Observer resides away from the Source

(3) Both are in motion

(a) Source and Observer approach each other

(b) Source and Observer resides from each other

(c) Source chases Observer

(d) Observer chases Source

IV. Exercises

1. The speed of a wave in a certain medium is 900 m/s. If 3000 waves passes over a certain point of the medium in 2 minutes, then compute its wavelength?. **Answer** : $\lambda = 36$ m

2. Consider a mixture of 2 mol of helium and 4 mol of oxygen. Compute the speed of sound in this gas mixture at 300 K. **Answer** : 400.9 ms^{-1}

3. A ship in a sea sends SONAR waves straight down into the seawater from the bottom of the ship. The signal reflects from the deep bottom bed rock and returns to the ship after 3.5 s. After the ship moves to 100 km it sends another signal which returns back after 2s. Calculate the depth of the sea in each case and also compute the difference in height between two cases. **Answer** : $\Delta d = 1149.75$ m

4. A sound wave is transmitted into a tube as shown in figure. The sound wave splits into two waves at the point A which recombine at point B. Let R be the radius of the semi-circle which is varied until the first minimum. Calculate the radius of the semi-circle if the wavelength of the sound is 50.0 m. **Answer** : $R = 21.9\ m$

5. N tuning forks are arranged in order of increasing frequency and any two successive tuning forks give n beats per second when sounded together. If the last fork gives double the frequency of the first (called as octave), Show that the frequency of the first tuning fork is $f = (N-1)n$.

6. Let the source propagate a sound wave whose intensity at a point (initially) be I. Suppose we consider a case when the amplitude of the sound wave is

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doubled and the frequency is reduced to one-fourth. Calculate now the new intensity of sound at the same point ?.

Answer: $I_{new} \propto \frac{1}{4} I_{old}$.

7. Consider two organ pipes of same length in which one organ pipe is closed and another organ pipe is open. If the fundamental frequency of closed pipe is 250 Hz. Calculate the fundamental frequency of the open pipe.

Answer: 500Hz

8. A police in a siren car moving with a velocity 20 ms^{-1} chases a thief who is moving in a car with a velocity $v_0 ms^{-1}$. The police car sounds at frequency 300Hz, and both of them move towards a stationary siren of frequency 400Hz. Calculate the speed in which thief is moving. (Assume the thief does not observe any beat)

Answer: $v_{thief} = 10 \text{ m s}^{-1}$

9. Consider the following function

(a) $y = x^2 + 2\,\alpha\, t\, x$

(b) $y = (x + vt)^2$

which among the above function can be characterized as a wave ?.

Answer: (a) function is not describing wave
(b) satisfies wave equation.

BOOKS FOR REFERENCE

1. Vibrations and Waves – A. P. French, CBS publisher and Distributors Pvt. Ltd.
2. Concepts of Physics – H. C. Verma, Volume 1 and Volume 2, Bharati Bhawan Publisher
3. Halliday, Resnick and Walker, Fundamentals of Physics, Wiley Publishers, 10th edition
4. Serway and Jewett, Physics for scientist and engineers with modern physics, Brook/Coole publishers, Eighth edition

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ICT CORNER

Waves

Through this activity you will be able to learn about the wave motion.

STEPS:

- Use the URL or scan the QR code to open 'PhET' simulation on 'waves on a string'. Click the play button.
- In the activity window a diagram of string is given. Click the play icon to see the motion of wave.
- We can see the 'oscillations' and 'pulse' by selecting on the table given in the left side window and by changing the 'amplitude' and 'frequency' is given below.
- By selecting the 'end types' on the right side window and repeat the same as before.

Step1

Step2

Step3

Step4

URL:

https://phet.colorado.edu/en/simulation/wave-on-a-string

* Pictures are indicative only.

* If browser requires, allow **Flash Player** or **Java Script** to load the page.

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Higher Secondary First Year

PHYSICS

PRACTICAL

LIST OF PRACTICALS

1. Moment of Inertia of solid sphere of known mass using Vernier caliper
2. Non-uniform bending – verification of relation between the load and the depression using pin and microscope
3. Spring constant of a spring
4. Acceleration due to gravity using simple pendulum
5. Velocity of sound in air using resonance column
6. Viscosity of a liquid by Stoke's method
7. Surface tension by capillary rise method
8. Verification of Newton's law of cooling using calorimeter
9. Study of relation between the frequency and length of a given wire under constant tension using sonometer
10. Study of relation between length of a given wire and tension for constant frequency using sonometer
11. Verification of parallelogram law of forces (Demonstration only- not for examination)
12. Determination of density of a material of wire using screw gauge and physical balance (Demonstration only- Not for examination).

Note: Students should be instructed to perform the experiments given in ICT corner at the end of each unit of Volume 1. (Self study only- Not for examination)

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1. MOMENT OF INERTIA OF A SOLID SPHERE OF KNOWN MASS USING VERNIER CALIPER

AIM To determine the moment of inertia of a solid sphere of known mass using Vernier caliper

APPARATUS REQUIRED Vernier caliper, Solid sphere

FORMULA Moment of inertia of a solid sphere about its diameter $I_d = \frac{2}{5} MR^2$ kgm^2

Where M ← Mass of the sphere (known value to be given) in kg

R ← Radius of the sphere in metre

DIAGRAM

Vernier Caliper

(a) No error

(b) +ve error of +0.03 cm

(c) –ve error of -0.06 cm

(d) Vernier reading

A model reading
MSR = 2.2 cm ; VSC = 4 divisions;
Reading = [2.2 cm+(4x0.01cm)] = 2.24 cm

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PROCEDURE

- The Vernier caliper is checked for zero errors and error if found is to be noted.
- The sphere is kept in between the jaws of the Vernier caliper and the main scale reading (MSR)is noted.
- Vernier scale division which coincides with some main scale division (VSC) is noted. Multiply this VSC by least count (LC) gives Vernier scale reading (VSR).
- Add MSR with VSR. This will be the diameter of the sphere.
- Observations are to be recorded for different positions of the sphere and the average value of the diameter is found. From this value radius of the sphere R is calculated.
- Using the known value of the mass of the sphere M and calculated radius of the sphere R the moment of inertia of the given sphere about its diameter can be calculated using the given formula.

LEAST COUNT (LC)

One main scale division (MSD) = ... cm

Number of Vernier scale divisions = ...

$$\text{Least Count (LC)} = \frac{\text{1 Main Scale Division (MSD)}}{\text{Total Vernier scale divisions}}$$

= ..... cm

OBSERVATIONS

Zero error =

Zero correction (Z . C) = Zero error × LC

Sl.No.	MSR $\times 10^{-2}$m	Vernier coincidence VSC (div)	VSR = (VSC × LC) $\times 10^{-2}$m	TR = (MSR +VSR) $\times 10^{-2}$m	Diameter of the sphere = 2R correct reading = (TR ± Z.C) $\times 10^{-2}$m
1					
2					
3					
4					
5					
6					

Mean diameter 2R = ... m

Radius of the sphere R = ... m

R = ... m

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CALCULATION

Mass of the sphere M = ... kg

(Known value is given)

Radius of the sphere R =. . . . . .metre

Moment of inertia of a solid sphere

about its diameter $I_d = \frac{2}{5} MR^2$ = ... kg m^2

RESULT

The moment of inertia of the given solid sphere about its diameter using Vernier caliper I_d = …….. kg m^2

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2. NON – UNIFORM BENDING – VERIFICATION OF RELATION BETWEEN LOAD AND DEPRESSION USING PIN AND MICROSCOPE

AIM

To verify the relation between the load and depression using non-uniform bending of a beam.

APPARATUS REQUIRED

A long uniform beam (usually a metre scale), two knife – edges, mass hanger, slotted masses, pin and vernier microscope.

FORMULA

$$\frac{M}{s} = \text{a constant}$$

where $M \leftarrow$ Load applied (mass) (kg)

s $\leftarrow$ depression produced in the beam for the applied load(m)

DIAGRAM

EXPERIMENTAL SETUP OF NON - UNIFORM BENDING PIN AND MICROSCOPE

PROCEDURE

- Place the two knife – edges on the table.
- Place the uniform beam (metre scale) on top of the knife edges.
- Suspend the mass hanger at the centre. A pin is attached at the centre of the scale where the hanger is hung.
- Place a vernier microscope in front of this arrangement
- Adjust the microscope to get a clear view of the pin

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- Make the horizontal cross-wire on the microscope to coincide with the tip of the pin. (Here mass hanger is the dead load M).
- Note the vertical scale reading of the vernier microscope
- Add the slotted masses to the mass hanger one by one in steps of 0.05 kg (50 g) and corresponding readings are noted down.
- Repeat the experiment by removing masses one by one and note down the corresponding readings.
- Subtract the mean reading of each load from dead load reading. This gives the depressions for the corresponding load M.

OBSERVATIONS

To find M/s

LOAD (kg)	MICROSCOPE READINGS × 10^{-2} m								DEPRESSION FOR M (kg) (s)	M/s kg m $^{-1}$
	INCREASING LOAD			DECREASING LOAD			MEAN			
	MSR	VSR	TR	MSR	VSR	TR				
M								x_0		
M + 0.05								x_1	$x_1 - x_0 =$ _____	
M + 0.10								x_2	$x_2 - x_0 =$ _____	
M + 0.15								x_3	$x_3 - x_0 =$ _____	
M + 0.20								x_4	$x_4 - x_0 =$ _____	
M + 0.25								x_5	$x_5 - x_0 =$ _____	
									Mean	

MODEL GRAPH

Load (M) vs Depression (s)

A graph between M and s can be drawn by taking M along X- axis and s along Y – axis. This is a straight line.

Relation between Mass and depression

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CALCULATION

(i) $\frac{M}{s} =$

(ii) $\frac{M}{s} =$

(iii) $\frac{M}{s} =$

(iv) $\frac{M}{s} =$

(v) $\frac{M}{s} =$

RESULT

- The ratio between mass and depression for each load is calculated. This is found to be constant.
- Thus the relation between load and depression is verified by the method of non-uniform bending of a beam.

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3. SPRING CONSTANT OF A SPRING

AIM

To determine the spring constant of a spring by using the method of vertical oscillations

APPARATUS REQUIRED

Spring, rigid support, hook, 50 g mass hanger, 50 g slotted masses, stop clock, metre scale, pointer

FORMULA

Spring constant of the spring $k = 4\pi^2 \left(\frac{M_2 - M_1}{T_2^2 - T_1^2}\right)$

where $M_1, M_2 \leftarrow$ selected loads in kg

$T_1, T_2 \leftarrow$ time period corresponding to masses M_1 and M_2 respectively in second

DIAGRAM

PROCEDURE

- A spring is firmly suspended vertically from a rigid clamp of a wooden stand at its upper end with a mass hanger attached to its lower end. A pointer fixed at the lower end of the spring moves over a vertical scale fixed.

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- A suitable load M (eg; 100 g) is added to the mass hanger and the reading on the scale at which the pointer comes to rest is noted. This is the equilibrium position.
- The mass in the hanger is pulled downward and released so that the spring oscillates vertically on either side of the equilibrium position.
- When the pointer crosses the equilibrium position a stop clock is started and the time taken for 10 vertical oscillations is noted. Then the period of oscillation T is calculated.
- The experiment is repeated by adding masses in steps of 50 g to the mass hanger and period of oscillation at each time is calculated.
- For the masses M_1 and M_2 (with a difference of 50 g), their corresponding time periods are T_1 and T_2. Then the value $\frac{M_2 - M_1}{T_2^2 - T_1^2}$ is calculated and its average is found.
- Using the given formula the spring constant of the given spring is calculated.

OBSERVATIONS

Sl. No.	Mass M $\times 10^{-3}$ kg	Time taken for 10 oscillations (t) (s)			Period of oscillation $T = \frac{t}{10}$(s)	T^2 (s^2)	$\frac{M_2 - M_1}{T_2^2 - T_1^2}$ $\times 10^{-3} kg\ s^{-2}$
		Trial 1	Trial 2	Mean			
1	100						
2	150						
3	200						
4	250						
5	300						

Mean = ... kg s^{-2}

CALCULATION

Spring constant of the spring $k = 4\pi^2 \left(\frac{M_2 - M_1}{T_2^2 - T_1^2}\right)$

k = ... kg s^{-2}

RESULT

The spring constant of the given spring k is found to be = ... kg s^{-2}

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4 ACCELERATION DUE TO GRAVITY USING SIMPLE PENDULUM

AIM

To measure the acceleration due to gravity using a simple pendulum

APPARATUS REQUIRED

Retort stand, pendulum bob, thread, meter scale, stop watch.

FORMULA

Acceleration due to gravity $g = 4\pi^2\left(\frac{L}{T^2}\right)$ (m s^{-2})

where $T \leftarrow$ Time period of simple pendulum (second)

$g \leftarrow$ Acceleration due to gravity (metre sec^{-2})

$L \leftarrow$ Length of the pendulum (metre)

DIAGRAM

PROCEDURE

- Attach a small brass bob to the thread.
- Fix this thread on to the stand.
- Measure the length of the pendulum from top of the suspension hook to the middle of the bob of the pendulum. Record the length of the pendulum in the table given below.
- Note down the time (t) taken for 10 oscillations using stop watch.
- The period of oscillation $T = \frac{t}{10}$ is calculated.
- Repeat the experiment for different lengths of the pendulum 'L'. Find acceleration due to gravity g using the given formula.

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OBSERVATIONS To find the acceleration due to gravity 'g'

Length of the pendulum L (metre)	Time taken for 10 oscillations t (s)			Period of oscillation $T = \frac{t}{10}$ (s)	T^2 (s^2)	$g = \frac{4\pi^2 L}{T^2}$ m s^{-2}
	Trial 1	Trial 2	Average			
					Mean g =	

MODEL GRAPH

$$slope = \frac{\Delta y}{\Delta x} = \frac{T^2}{L}\ ;\ 1/\text{slope} = L/T^2$$

RESULT

The acceleration due to gravity 'g' determined using simple pendulum is found to be

i) By calculation = ... m s^{-2}

ii) By graph = ... m s^{-2}

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5. VELOCITY OF SOUND IN AIR USING RESONANCE COLUMN

AIM

To determine the velocity of sound in air at room temperature using the resonance phenomenon.

APPARATUS REQUIRED

Resonance tube, three tuning forks of known frequencies, a rubber hammer, one thermometer, plumb line, set squares, water in a beaker.

FORMULA

$$V = 2v\left(l_2 - l_1\right) \text{ m s}^{-1}$$

where V ← Speed of sound in air (m s^{-1})

l_1 and l_2 ←The length of the air column for the first and second resonance respectively (m)

v ← Frequency of the tuning fork (Hz)

DIAGRAM

PROCEDURE

- Adjust the position of the resonance tube, so that the length of air column inside the tube is very small.
- Take a tuning fork of known frequency and strike it with a rubber hammer. The tuning fork now produces longitudinal waves with a frequency equal to the natural frequency of the tuning fork.
- Place the vibrating tuning fork horizontally at the open end of the resonance tube. Sound waves pass down the total tube and reflect back at the water surface.
- Length of the water column in the tube is adjusted either by lowering or raising the reservoir or the tube, until a maximum sound(resonance) occurs.
- Measure the length of air column at this position. This is taken as the first resonating length, l_1

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- Then raise the tube approximately about two times the first resonating length. Excite the tuning fork again and place it on the open end of the tube.
- Adjust the height of the air column until the maximum sound is heard.
- Measure the length of air column at this position. This is taken as the second resonating length l_2
- We can now calculate the velocity of sound in air at room temperature by using the relation.

  $V = 2v(l_2 - l_1)$
- Repeat the experiment with tuning forks of different frequency and tabulate the corresponding values of l_1 and l_2.
- The mean of the calculated values will give the velocity of sound in air at room temperature.

OBSERVATIONS

Sl. No.	Frequency of tuning fork 'v' (Hz)	First resonating length l_1 $(\times 10^{-2} m)$			Second resonating length l_2 $(\times 10^{-2} m)$			$l_2 - l_1$ $(\times 10^{-2} m)$	Velocity of sound $V = 2v(l_2 - l_1)$ (m s^{-1})
		Trial 1	Trial 2	Mean	Trial 1	Trial 2	Mean		
1									
2									
3									
								Mean V =	

CALCULATION

Velocity sound in air at room temperature, $V = 2v(l_2 - l_1)$ = _____ m s^{-1}

RESULT

Velocity of sound in air at room temperature, is found to be (V) = _____ m s^{-1}

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6. VISCOSITY OF A LIQUID BY STOKE'S METHOD

AIM

To determine the co-efficient of viscosity of the given liquid by stoke's method

APPARATUS REQUIRED

A long cylindrical glass jar, highly viscous liquid, metre scale, spherical ball, stop clock, thread.

FORMULA

$$\eta = \frac{2r^2(\delta - \sigma)g}{9V} \text{ N s m}^{-2}$$

where η – Coefficient of viscosity of liquid (N s m^{-2})

$r \leftarrow$ radius of spherical ball (m)

$\alpha \leftarrow$ density of the steel sphere ($kg\ m^{-3}$)

$\pi \leftarrow$ density of the liquid (kg m^{-3})

$g \leftarrow$ acceleration due to gravity (9.8 m s^{-2})

$V \leftarrow$ mean terminal velocity (m s^{-1})

DIAGRAM

EXPERIMENTAL SET UP OF MEASURING VISCOSITY BY STOKE'S METHOD

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PROCEDURE

- A long cylindrical glass jar with markings is taken.
- Fill the glass jar with the given experimental liquid.
- Two points A and B are marked on the jar. The mark A is made well below the surface of the liquid so that when the ball reaches A it would have acquired terminal velocity V.
- The radius of the metal spherical ball is determined using screw gauge.
- The spherical ball is dropped gently into the liquid.
- Start the stop clock when the ball crosses the point A. Stop the clock when the ball reaches B and note down the time 't'.
- Note the distance between A and B and use it to calculate terminal velocity.
- Now repeat the experiment for different distances between A and B. Make sure that the point A is suitable for the ball to acquire terminal velocity.

OBSERVATIONS

To find Terminal Velocity:

S.No.	Distance covered by the spherical ball (d) (m)	Time taken (t) (s)	Terminal Velocity (V) d/t (m s^{-1})
		MEAN	

CALCULATION

Density of the spherical ball δ = _____ $kg\,m^{-3}$

Density of the given liquid π = _____ $kg\,m^{-3}$

Coefficient of viscosity of the liquid $\eta = \dfrac{2r^2 g(\delta - \sigma)}{9V}$ = _____ N s m^{-2}

RESULT

The coefficient of viscosity of the given liquid by stoke's method is found to be

η = _____ N s m^{-2}

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7. SURFACE TENSION BY CAPILLARY RISE METHOD

AIM

To determine surface tension of a liquid by capillary rise method.

APPARATUS REQUIRED

A beaker of Water, capillary tube, vernier microscope, double hole rubber stopper, a knitting needle, a short rubber tubing and retort clamp.

FORMULA

The surface tension of the liquid $T = \frac{hr\sigma g}{2}$ $N\,m^{-1}$

where $T \leftarrow$ Surface tension of the liquid ($N\,m^{-1}$)

$h \leftarrow$ height of the liquid in the capillary tube (m)

$r \leftarrow$ radius of the capillary tube (m)

$\sigma \leftarrow$ Density of water ($kg\,m^{-3}$) ($\sigma = 1000\ kg\,m^{-3}$)

$g \leftarrow$ Acceleration due to gravity ($g = 9.8\ m\,s^{-2}$)

DIAGRAM

PROCEDURE

- A clean and dry capillary tube is taken and fixed in a stand
- A beaker containing water is placed on an adjustable platform and the capillary tube is dipped inside the beaker so that a little amount of water is raised inside.

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- Fix a needle near the capillary tube so that the needle touches the water surface
- A Vernier microscope is focused at the lower meniscus of the water and the corresponding reading is taken after coinciding it with the horizontal line of the cross wire.
- Tip of the needle is focussed using vernier microscope after coinciding it with horizontal line of the cross wire
- The difference between the two readings of the vertical scale gives the height (h) of the liquid raised in the capillary tube.
- Now to find the radius of the tube, raise the capillary tube and remove the beaker. Carefully rotate the capillary tube so that the immersed lower end face towards you.
- Focus the capillary tube using Vernier microscope to clearly see the inner walls of the tube.
- Let the vertical cross wire coincide with the left side inner walls of the tube. Note down the reading (L_1)
- Turn the microscope screws in horizontal direction to view the right side inner wall of the tube. Note the reading (R_1). Thus the radius of the tube can be calculated as $\frac{1}{2}(L_1 - R_1)$.
- Finally calculate the surface tension using the given formula.

RADIUS OF THE CAPILLARY TUBE

OBSERVATIONS

To measure height of the liquid (h)

Least count of the microscope = _____ *cm*

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Trial No.	Microscope reading for the position of Lower meniscus of liquid			Microscope reading for the position of Lower tip of the needle			Height of the liquid h $\times 10^{-2}$ m a ~ b
	MSR	VSC × LC = VSR	TR (a) $\times 10^{-2}$ m	MSR	VSC × LC = VSR	TR (b) $\times 10^{-2}$ m	
						Mean h =	

Radius of the capillary tube

Tube	Microscope reading for the position of inner left wall of the tube L_1			Microscope reading for the position of inner right wall of the tube R_1			Radius of the capillary tube $r = \frac{1}{2}(L_1 \sim R_1)$ $\times 10^{-2}$ m
	MSR	VSC × LC = VSR	TR $\times 10^{-2}$ m	MSR	VSC × LC = VSR	TR $\times 10^{-2}$ m	

CALCULATION

Mean rise of the liquid in the capillary tube $h =$ _____ $\times 10^{-2}$ m

Diameter of the capillary tube $2r =$ _____ $\times 10^{-2}$ m

Radius of the capillary tube $r =$ _____ $\times 10^{-2}$ m

Density of the liquid $\sigma = 1000 \text{ kg m}^{-3}$

Acceleration due to gravity $g = 9.8 \text{ m s}^{-2}$

Surface tension $T = \frac{hr\sigma g}{2}$

$=$ _____ $N\,m^{-1}$

RESULT

Surface tension of the given liquid by capillary rise method $T =$ _____ $N\,m^{-1}$

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8. VERIFICATION OF NEWTON'S LAW OF COOLING USING CALORIMETER

AIM

To study the relationship between the temperature of a hot body and time by plotting a cooling curve.

APPARATUS REQUIRED

Copper calorimeter with stirrer, one holed rubber cork, thermometer, stop clock, heater / burner, water, clamp and stand

NEWTON'S LAW OF COOLING

Newton's law of cooling states that the rate of change of the temperature of an object is proportional to the difference between its own temperature and the ambient temperature. (i.e., the temperature of its surroundings)

$$\frac{dT}{dt} \propto (T - T_0)$$

where $\frac{dT}{dt} \leftarrow$ Rate of change of temperature (°C)

$T \leftarrow$ Temperature of water (°C)

$T_0 \leftarrow$ Room Temperature (°C)

DIAGRAM

EXPERIMENTAL SETUP OF NEWTON'S LAW OF COOLING

PROCEDURE

- Note the room temperature as (T_0) using the thermometer.
- Hot water about 90°C is poured into the calorimeter.
- Close the calorimeter with one holed rubber cork.
- Insert the thermometer into calorimeter through the hole in rubber cork.
- Start the stop clock and observe the time for every one degree fall of temperature from 80°C.

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- Take sufficient amount of reading, say closer to room temperature
- The observations are tabulated
- Draw a graph by taking time along the x axis and excess temperature along y axis.

MODEL GRAPH

RELATION BETWEEN TIME AND TEMPERATURE

ROOM TEMPERATURE (T_0) = _____°C

OBSERVATIONS

Measuring the change in temperature of water with time

Time (s)	Temperature of water (T) °C	Excess temperature ($T - T_0$) °C

RESULT

The cooling curve is plotted and thus Newton's law of cooling is verified.

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9. STUDY OF RELATION BETWEEN FREQUENCY AND LENGTH OF A GIVEN WIRE UNDER CONSTANT TENSION USING SONOMETER

AIM

To study the relation between frequency and length of a given wire under constant tension using a sonometer.

APPARATUS REQUIRED

Sonometer, six tuning forks of known frequencies, Metre scale, rubber pad, paper rider, hanger with half – kilogram masses, wooden bridges

FORMULA

The frequency n of the fundamental mode of vibration of a string

is given by $n = \frac{1}{2l}\sqrt{\frac{T}{m}}\ Hz$

a) For a given m and fixed T.

$n \propto \frac{1}{l}$ (or) nl = constant

where $n \leftarrow$ Frequency of the fundamental mode of vibration of the string (Hz)

$m \leftarrow$ Mass per unit length of the string ($kg\ m^{-1}$)

$l \leftarrow$ Length of the string between the wedges (m)

$T \leftarrow$ Tension in the string (including the mass of the hanger) = Mg (N)

$M \leftarrow$ Mass suspended, including the mass of the hanger (Kg)

DIAGRAM

SONOMETER - STUDY OF RELATION BETWEEN FREQUENCY AND LENGTH OF A GIVEN WIRE UNDER CONSTANT TENSION USING SONOMETER

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PROCEDURE

- Set up the sonometer on the table and clean the groove on the pulley to ensure minimum friction
- Stretch the wire by placing suitable mass in the hanger. Keep a small paper rider over the wire, between the two bridges.
- Set the tuning fork into vibrations by striking it against the rubber pad and place it over the sonometer,by its stem.
- Adjust the vibrating length of the wire by sliding the bridge B till the vibrating sound of the wire is maximum
- when the frequency of vibration is in resonance with the frequency of the tuning fork, the paper rider falls down.
- The length of the wire between the wedges A and B is measured using meter scale. It is called as resonant length.
- Repeat the above procedure for tuning forks of different frequencies by keeping the same load in the hanger.

OBSERVATIONS

Tension (constant) on the wire (mass suspended from the hanger including its own mass)
T = (mass suspended × 9.8) N

Variation of frequency with length		
Frequency of the tuning fork 'n ' (Hz)	Resonant length 'l' $\times 10^{-2}$ m	nl
n_1 =		
n_2 =		
n_3 =		
n_4 =		
n_5 =		
n_6 =		

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GRAPH:

Graph 1: Relation between frequency and length

Graph 2: Relation between frequency and inverse of length

Sonometer - study of relation between length of the given wire and tension for a consant frequency

CALCULATION

The product *nl* for all the tuning forks remain constant (last column in the table)

RESULT

- For a given tension, the resonant length of a given stretched string varies as reciprocal of the frequency (i.e., $n \propto \frac{1}{l}$)
- The product *nl* is a constant and found to be _____ (Hz m)

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10. STUDY OF RELATION BETWEEN LENGTH OF THE GIVEN WIRE AND TENSION FOR A CONSTANT FREQUENCY USING SONOMETER

AIM

To study the relationship between the length of a given wire and tension for constant frequency using a sonometer

APPARATUS REQUIRED

Sonometer, tuning fork of known frequency, meter scale, rubber pad, paper rider, hanger with half – kilogram masses, wooden bridges.

FORMULA

The frequency of the fundamental mode of vibration of a string is given by,

$$n = \frac{1}{2l}\sqrt{\frac{T}{m}}\ Hz$$

If n is a constant, for a given wire (m is constant)

$$\frac{\sqrt{T}}{l} \text{ is constant.}$$

where $n \leftarrow$ Frequency of the fundamental mode of vibration of a string (Hz)

$m \leftarrow$ Mass per unit length of string ($kg\ m^{-1}$)

$T \leftarrow$ Tension in the string (including the mass of the hanger) = Mg (N)

$l \leftarrow$ Length of the string between the wedges (metre)

$M \leftarrow$ Mass suspended, including the mass of the hanger (kg)

DIAGRAM

STUDY OF RELATION BETWEEN LENGTH OF THE GIVEN WIRE AND TENSION FOR A CONSTANT FREQUENCY USING SONOMETER

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PROCEDURE

- Set up the sonometer on the table and clean the groove on the pulley to ensure that it has minimum friction.
- Keep a small paper rider on the wire,between the bridges.
- Place a mass of 1 kg for initial reading in the mass hanger.
- Now, strike the tuning fork and place its shank stem on the bridge A and then slowly adjust the position of the bridge B till the paper rider is agitated violently and might eventually fall due to resonance.
- Measure the length of the wire between wedges at A and B which is the fundamental mode corresponding to the frequency of the tuning fork.
- Increase the load on the hanger in steps of 0.5 kg and each time find the resonating length as done before with the same tuning fork.
- Record the observations in the tabular column.

MODEL GRAPH

Relation between tension T (N) and length l^2 (cm²)

OBSERVATIONS

Frequency of the tuning fork = _____ Hz

Variation of resonant length with tension

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Sl.No.	Mass M (kg)	Tension T=Mg (N)	$\sqrt{T}$	Vibrating length l (m)	$\frac{\sqrt{T}}{l}$

CALCULATION

Calculate the value $\frac{\sqrt{T}}{l}$ for the tension applied in each case.

RESULT

- The resonating length varies as square root of tension for a given frequency of vibration of a stretched string.
- $\frac{\sqrt{T}}{l}$ is found to be a constant.

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SOME IMPORTANT CONSTANTS IN PHYSICS

Name	Symbols	Value
Speed of light in vacuum	c	2.9979×10^{8} m s^{-1}
Gravitational constant	G	6.67×10^{-11} N m^{2} kg^{-2}
Acceleration due to gravity (sea level, at 45° latitude)	g	9.8 m s^{-2}
Planck constant	h	6.626×10^{-34} J s
Boltzmann constant	k	1.38×10^{-23} J K^{-1}
Avogadro number	N_A	6.023×10^{23} mol^{-1}
Universal gas constant	R	8.31 J mol^{-1} K^{-1}
Stefan – Boltmann constant	σ	5.67×10^{-8} W m^{-2} K^{-4}
Wien's constant	b	2.898×10^{-3} m K
Permeability of free space	μ_0	$4\mu \times 10^{-7}$ H m^{-1}
Standard atmospheric pressure	1 *atm*	1.013×10^{5} Pa

SOLVED EXAMPLES

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(THE GREEK ALPHABET)

(கிரேக்க எழுத்துகள்)

The Greek Alphabet	Upper Case	Lower Case
Alpha	A	⊥
Beta	B	_
Gamma	Δ	δ
Delta	A	α
Epsilon	E	β
Zeta	Z	ζ
Eta	H	ε
Theta	N	ν
Iota	I	ϕ
Kappa	K	η
Lambda	I	ι
Mu	M	φ
Nu	N	κ
Xi	€	υ
Omicron	O	λ
Pi	©	μ
Rho	P	ο
Sigma	©	π
Tau	T	θ
Upsilon	Y	ρ
Phi	X	χ
Chi	≅	_
Psi	ς	ϖ
Omega	Ω	τ

COMPETITIVE EXAM CORNER

LOGARITHM TABLE

											Mean Difference								
	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	0.000	0.004	0.009	0.013	0.017	0.021	0.025	0.029	0.033	0.037	4	8	11	17	21	25	29	33	37
11	0.041	0.045	0.049	0.053	0.057	0.061	0.064	0.068	0.072	0.076	4	8	11	15	19	23	26	30	34
12	0.079	0.083	0.086	0.090	0.093	0.097	0.100	0.104	0.107	0.111	3	7	10	14	17	21	24	28	31
13	0.114	0.117	0.121	0.124	0.127	0.130	0.134	0.137	0.140	0.143	3	6	10	13	16	19	23	26	29
14	0.146	0.149	0.152	0.155	0.158	0.161	0.164	0.167	0.170	0.173	3	6	9	12	15	18	21	24	27
15	0.176	0.179	0.182	0.185	0.188	0.190	0.193	0.196	0.199	0.201	3	6	8	11	14	17	20	22	25
16	0.204	0.207	0.210	0.212	0.215	0.217	0.220	0.223	0.225	0.228	3	5	8	11	13	16	18	21	24
17	0.230	0.233	0.236	0.238	0.241	0.243	0.246	0.248	0.250	0.253	2	5	7	10	12	15	17	20	22
18	0.255	0.258	0.260	0.262	0.265	0.267	0.270	0.272	0.274	0.276	2	5	7	9	12	14	16	19	21
19	0.279	0.281	0.283	0.286	0.288	0.290	0.292	0.294	0.297	0.299	2	4	7	9	11	13	16	18	20
20	0.301	0.303	0.305	0.307	0.310	0.312	0.314	0.316	0.318	0.320	2	4	6	8	11	13	15	17	19
21	0.322	0.324	0.326	0.328	0.330	0.332	0.334	0.336	0.338	0.340	2	4	6	8	10	12	14	16	18
22	0.342	0.344	0.346	0.348	0.350	0.352	0.354	0.356	0.358	0.360	2	4	6	8	10	12	14	15	17
23	0.362	0.364	0.365	0.367	0.369	0.371	0.373	0.375	0.377	0.378	2	4	6	7	9	11	13	15	17
24	0.380	0.382	0.384	0.386	0.387	0.389	0.391	0.393	0.394	0.396	2	4	5	7	9	11	12	14	16
25	0.398	0.400	0.401	0.403	0.405	0.407	0.408	0.410	0.412	0.413	2	3	5	7	9	10	12	14	15
26	0.415	0.417	0.418	0.420	0.422	0.423	0.425	0.427	0.428	0.430	2	3	5	7	8	10	11	13	15
27	0.431	0.433	0.435	0.436	0.438	0.439	0.441	0.442	0.444	0.446	2	3	5	6	8	9	11	13	14
28	0.447	0.449	0.450	0.452	0.453	0.455	0.456	0.458	0.459	0.461	2	3	5	6	8	9	11	12	14
29	0.462	0.464	0.465	0.467	0.468	0.470	0.471	0.473	0.474	0.476	1	3	4	6	7	9	10	12	13
30	0.477	0.479	0.480	0.481	0.483	0.484	0.486	0.487	0.489	0.490	1	3	4	6	7	9	10	11	13
31	0.491	0.493	0.494	0.496	0.497	0.498	0.500	0.501	0.502	0.504	1	3	4	6	7	8	10	11	12
32	0.505	0.507	0.508	0.509	0.511	0.512	0.513	0.515	0.516	0.517	1	3	4	5	7	8	9	11	12
33	0.519	0.520	0.521	0.522	0.524	0.525	0.526	0.528	0.529	0.530	1	3	4	5	6	8	9	10	12
34	0.531	0.533	0.534	0.535	0.537	0.538	0.539	0.540	0.542	0.543	1	3	4	5	6	8	9	10	11
35	0.544	0.545	0.547	0.548	0.549	0.550	0.551	0.553	0.554	0.555	1	2	4	5	6	7	9	10	11
36	0.556	0.558	0.559	0.560	0.561	0.562	0.563	0.565	0.566	0.567	1	2	3	5	6	7	8	10	11
37	0.568	0.569	0.571	0.572	0.573	0.574	0.575	0.576	0.577	0.579	1	2	3	5	6	7	8	9	10
38	0.580	0.581	0.582	0.583	0.584	0.585	0.587	0.588	0.589	0.590	1	2	3	5	6	7	8	9	10
39	0.591	0.592	0.593	0.594	0.595	0.597	0.598	0.599	0.600	0.601	1	2	3	4	5	7	8	9	10
40	0.602	0.603	0.604	0.605	0.606	0.607	0.609	0.610	0.611	0.612	1	2	3	4	5	6	8	9	10
41	0.613	0.614	0.615	0.616	0.617	0.618	0.619	0.620	0.621	0.622	1	2	3	4	5	6	7	8	9
42	0.623	0.624	0.625	0.626	0.627	0.628	0.629	0.630	0.631	0.632	1	2	3	4	5	6	7	8	9
43	0.633	0.634	0.635	0.636	0.637	0.638	0.639	0.640	0.641	0.642	1	2	3	4	5	6	7	8	9
44	0.643	0.644	0.645	0.646	0.647	0.648	0.649	0.650	0.651	0.652	1	2	3	4	5	6	7	8	9
45	0.653	0.654	0.655	0.656	0.657	0.658	0.659	0.660	0.661	0.662	1	2	3	4	5	6	7	8	9
46	0.663	0.664	0.665	0.666	0.667	0.667	0.668	0.669	0.670	0.671	1	2	3	4	5	6	7	7	8
47	0.672	0.673	0.674	0.675	0.676	0.677	0.678	0.679	0.679	0.680	1	2	3	4	5	5	6	7	8
48	0.681	0.682	0.683	0.684	0.685	0.686	0.687	0.688	0.688	0.689	1	2	3	4	4	5	6	7	8
49	0.690	0.691	0.692	0.693	0.694	0.695	0.695	0.696	0.697	0.698	1	2	3	4	4	5	6	7	8
50	0.699	0.700	0.701	0.702	0.702	0.703	0.704	0.705	0.706	0.707	1	2	3	4	4	5	6	7	8
51	0.708	0.708	0.709	0.710	0.711	0.712	0.713	0.713	0.714	0.715	1	2	3	4	4	5	6	7	8
52	0.716	0.717	0.718	0.719	0.719	0.720	0.721	0.722	0.723	0.723	1	2	2	4	4	5	6	7	7
53	0.724	0.725	0.726	0.727	0.728	0.728	0.729	0.730	0.731	0.732	1	2	2	4	4	5	6	6	7
54	0.732	0.733	0.734	0.735	0.736	0.736	0.737	0.738	0.739	0.740	1	2	2	4	4	5	6	6	7

UNIT-12(XI-Physics_Vol-2) PRACTICAL FIRST YEAR.indd 310 17-03-2022 16:18:20

LOGARITHM TABLE

											Mean Difference								
55	0.740	0.741	0.742	0.743	0.744	0.744	0.745	0.746	0.747	0.747	1	2	2	4	4	5	5	6	7
56	0.748	0.749	0.750	0.751	0.751	0.752	0.753	0.754	0.754	0.755	1	2	2	4	4	5	5	6	7
57	0.756	0.757	0.757	0.758	0.759	0.760	0.760	0.761	0.762	0.763	1	2	2	4	4	5	5	6	7
58	0.763	0.764	0.765	0.766	0.766	0.767	0.768	0.769	0.769	0.770	1	1	2	4	4	4	5	6	7
59	0.771	0.772	0.772	0.773	0.774	0.775	0.775	0.776	0.777	0.777	1	1	2	3	4	4	5	6	7
60	0.778	0.779	0.780	0.780	0.781	0.782	0.782	0.783	0.784	0.785	1	1	2	3	4	4	5	6	6
61	0.785	0.786	0.787	0.787	0.788	0.789	0.790	0.790	0.791	0.792	1	1	2	3	4	4	5	6	6
62	0.792	0.793	0.794	0.794	0.795	0.796	0.797	0.797	0.798	0.799	1	1	2	3	3	4	5	6	6
63	0.799	0.800	0.801	0.801	0.802	0.803	0.803	0.804	0.805	0.806	1	1	2	3	3	4	5	5	6
64	0.806	0.807	0.808	0.808	0.809	0.810	0.810	0.811	0.812	0.812	1	1	2	3	3	4	5	5	6
65	0.813	0.814	0.814	0.815	0.816	0.816	0.817	0.818	0.818	0.819	1	1	2	3	3	4	5	5	6
66	0.820	0.820	0.821	0.822	0.822	0.823	0.823	0.824	0.825	0.825	1	1	2	3	3	4	5	5	6
67	0.826	0.827	0.827	0.828	0.829	0.829	0.830	0.831	0.831	0.832	1	1	2	3	3	4	5	5	6
68	0.833	0.833	0.834	0.834	0.835	0.836	0.836	0.837	0.838	0.838	1	1	2	3	3	4	4	5	6
69	0.839	0.839	0.840	0.841	0.841	0.842	0.843	0.843	0.844	0.844	1	1	2	2	3	4	4	5	6
70	0.845	0.846	0.846	0.847	0.848	0.848	0.849	0.849	0.850	0.851	1	1	2	2	3	4	4	5	6
71	0.851	0.852	0.852	0.853	0.854	0.854	0.855	0.856	0.856	0.857	1	1	2	2	3	4	4	5	5
72	0.857	0.858	0.859	0.859	0.860	0.860	0.861	0.862	0.862	0.863	1	1	2	2	3	4	4	5	5
73	0.863	0.864	0.865	0.865	0.866	0.866	0.867	0.867	0.868	0.869	1	1	2	2	3	4	4	5	5
74	0.869	0.870	0.870	0.871	0.872	0.872	0.873	0.873	0.874	0.874	1	1	2	2	3	4	4	5	5
75	0.875	0.876	0.876	0.877	0.877	0.878	0.879	0.879	0.880	0.880	1	1	2	2	3	3	4	5	5
76	0.881	0.881	0.882	0.883	0.883	0.884	0.884	0.885	0.885	0.886	1	1	2	2	3	3	4	5	5
77	0.886	0.887	0.888	0.888	0.889	0.889	0.890	0.890	0.891	0.892	1	1	2	2	3	3	4	4	5
78	0.892	0.893	0.893	0.894	0.894	0.895	0.895	0.896	0.897	0.897	1	1	2	2	3	3	4	4	5
79	0.898	0.898	0.899	0.899	0.900	0.900	0.901	0.901	0.902	0.903	1	1	2	2	3	3	4	4	5
80	0.903	0.904	0.904	0.905	0.905	0.906	0.906	0.907	0.907	0.908	1	1	2	2	3	3	4	4	5
81	0.908	0.909	0.910	0.910	0.911	0.911	0.912	0.912	0.913	0.913	1	1	2	2	3	3	4	4	5
82	0.914	0.914	0.915	0.915	0.916	0.916	0.917	0.918	0.918	0.919	1	1	2	2	3	3	4	4	5
83	0.919	0.920	0.920	0.921	0.921	0.922	0.922	0.923	0.923	0.924	1	1	2	2	3	3	4	4	5
84	0.924	0.925	0.925	0.926	0.926	0.927	0.927	0.928	0.928	0.929	1	1	2	2	3	3	4	4	5
85	0.929	0.930	0.930	0.931	0.931	0.932	0.932	0.933	0.933	0.934	1	1	2	2	3	3	4	4	5
86	0.934	0.935	0.936	0.936	0.937	0.937	0.938	0.938	0.939	0.939	1	1	2	2	3	3	4	4	5
87	0.940	0.940	0.941	0.941	0.942	0.942	0.943	0.943	0.943	0.944	0	1	1	2	2	3	3	4	5
88	0.944	0.945	0.945	0.946	0.946	0.947	0.947	0.948	0.948	0.949	0	1	1	2	2	3	3	4	4
89	0.949	0.950	0.950	0.951	0.951	0.952	0.952	0.953	0.953	0.954	0	1	1	2	2	3	3	4	4
90	0.954	0.955	0.955	0.956	0.956	0.957	0.957	0.958	0.958	0.959	0	1	1	2	2	3	3	4	4
91	0.959	0.960	0.960	0.960	0.961	0.961	0.962	0.962	0.963	0.963	0	1	1	2	2	3	3	4	4
92	0.964	0.964	0.965	0.965	0.966	0.966	0.967	0.967	0.968	0.968	0	1	1	2	2	3	3	4	4
93	0.968	0.969	0.969	0.970	0.970	0.971	0.971	0.972	0.972	0.973	0	1	1	2	2	3	3	4	4
94	0.973	0.974	0.974	0.975	0.975	0.975	0.976	0.976	0.977	0.977	0	1	1	2	2	3	3	4	4
95	0.978	0.978	0.979	0.979	0.980	0.980	0.980	0.981	0.981	0.982	0	1	1	2	2	3	3	4	4
96	0.982	0.983	0.983	0.984	0.984	0.985	0.985	0.985	0.986	0.986	0	1	1	2	2	3	3	4	4
97	0.987	0.987	0.988	0.988	0.989	0.989	0.989	0.990	0.990	0.991	0	1	1	2	2	3	3	4	4
98	0.991	0.992	0.992	0.993	0.993	0.993	0.994	0.994	0.995	0.995	0	1	1	2	2	3	3	4	4
99	0.996	0.996	0.997	0.997	0.997	0.998	0.998	0.999	0.999	1.000	0	1	1	2	2	3	3	3	4

UNIT-12(XI-Physics_Vol-2) PRACTICAL FIRST YEAR.indd 311 17-03-2022 16:18:20

ANTILOG TABLE

											Mean Difference								
	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
0.00	1.000	1.002	1.005	1.007	1.009	1.012	1.014	1.016	1.019	1.021	0	0	1	1	1	1	2	2	2
0.01	1.023	1.026	1.028	1.030	1.033	1.035	1.038	1.040	1.042	1.045	0	0	1	1	1	1	2	2	2
0.02	1.047	1.050	1.052	1.054	1.057	1.059	1.062	1.064	1.067	1.069	0	0	1	1	1	1	2	2	2
0.03	1.072	1.074	1.076	1.079	1.081	1.084	1.086	1.089	1.091	1.094	0	0	1	1	1	1	2	2	2
0.04	1.096	1.099	1.102	1.104	1.107	1.109	1.112	1.114	1.117	1.119	0	1	1	1	1	2	2	2	2
0.05	1.122	1.125	1.127	1.130	1.132	1.135	1.138	1.140	1.143	1.146	0	1	1	1	1	2	2	2	2
0.06	1.148	1.151	1.153	1.156	1.159	1.161	1.164	1.167	1.169	1.172	0	1	1	1	1	2	2	2	2
0.07	1.175	1.178	1.180	1.183	1.186	1.189	1.191	1.194	1.197	1.199	0	1	1	1	1	2	2	2	2
0.08	1.202	1.205	1.208	1.211	1.213	1.216	1.219	1.222	1.225	1.227	0	1	1	1	1	2	2	2	3
0.09	1.230	1.233	1.236	1.239	1.242	1.245	1.247	1.250	1.253	1.256	0	1	1	1	1	2	2	2	3
0.10	1.259	1.262	1.265	1.268	1.271	1.274	1.276	1.279	1.282	1.285	0	1	1	1	1	2	2	2	3
0.11	1.288	1.291	1.294	1.297	1.300	1.303	1.306	1.309	1.312	1.315	0	1	1	1	2	2	2	2	3
0.12	1.318	1.321	1.324	1.327	1.330	1.334	1.337	1.340	1.343	1.346	0	1	1	1	2	2	2	2	3
0.13	1.349	1.352	1.355	1.358	1.361	1.365	1.368	1.371	1.374	1.377	0	1	1	1	2	2	2	3	3
0.14	1.380	1.384	1.387	1.390	1.393	1.396	1.400	1.403	1.406	1.409	0	1	1	1	2	2	2	3	3
0.15	1.413	1.416	1.419	1.422	1.426	1.429	1.432	1.435	1.439	1.442	0	1	1	1	2	2	2	3	3
0.16	1.445	1.449	1.452	1.455	1.459	1.462	1.466	1.469	1.472	1.476	0	1	1	1	2	2	2	3	3
0.17	1.479	1.483	1.486	1.489	1.493	1.496	1.500	1.503	1.507	1.510	0	1	1	1	2	2	2	3	3
0.18	1.514	1.517	1.521	1.524	1.528	1.531	1.535	1.538	1.542	1.545	0	1	1	1	2	2	2	3	3
0.19	1.549	1.552	1.556	1.560	1.563	1.567	1.570	1.574	1.578	1.581	0	1	1	1	2	2	3	3	3
0.20	1.585	1.589	1.592	1.596	1.600	1.603	1.607	1.611	1.614	1.618	0	1	1	1	2	2	3	3	3
0.21	1.622	1.626	1.629	1.633	1.637	1.641	1.644	1.648	1.652	1.656	0	1	1	2	2	2	3	3	3
0.22	1.660	1.663	1.667	1.671	1.675	1.679	1.683	1.687	1.690	1.694	0	1	1	2	2	2	3	3	3
0.23	1.698	1.702	1.706	1.710	1.714	1.718	1.722	1.726	1.730	1.734	0	1	1	2	2	2	3	3	4
0.24	1.738	1.742	1.746	1.750	1.754	1.758	1.762	1.766	1.770	1.774	0	1	1	2	2	2	3	3	4
0.25	1.778	1.782	1.786	1.791	1.795	1.799	1.803	1.807	1.811	1.816	0	1	1	2	2	2	3	3	4
0.26	1.820	1.824	1.828	1.832	1.837	1.841	1.845	1.849	1.854	1.858	0	1	1	2	2	3	3	3	4
0.27	1.862	1.866	1.871	1.875	1.879	1.884	1.888	1.892	1.897	1.901	0	1	1	2	2	3	3	3	4
0.28	1.905	1.910	1.914	1.919	1.923	1.928	1.932	1.936	1.941	1.945	0	1	1	2	2	3	3	4	4
0.29	1.950	1.954	1.959	1.963	1.968	1.972	1.977	1.982	1.986	1.991	0	1	1	2	2	3	3	4	4
0.30	1.995	2.000	2.004	2.009	2.014	2.018	2.023	2.028	2.032	2.037	0	1	1	2	2	3	3	4	4
0.31	2.042	2.046	2.051	2.056	2.061	2.065	2.070	2.075	2.080	2.084	0	1	1	2	2	3	3	4	4
0.32	2.089	2.094	2.099	2.104	2.109	2.113	2.118	2.123	2.128	2.133	0	1	1	2	2	3	3	4	4
0.33	2.138	2.143	2.148	2.153	2.158	2.163	2.168	2.173	2.178	2.183	0	1	1	2	2	3	3	4	4
0.34	2.188	2.193	2.198	2.203	2.208	2.213	2.218	2.223	2.228	2.234	1	1	2	2	3	3	4	4	5
0.35	2.239	2.244	2.249	2.254	2.259	2.265	2.270	2.275	2.280	2.286	1	1	2	2	3	3	4	4	5
0.36	2.291	2.296	2.301	2.307	2.312	2.317	2.323	2.328	2.333	2.339	1	1	2	2	3	3	4	4	5
0.37	2.344	2.350	2.355	2.360	2.366	2.371	2.377	2.382	2.388	2.393	1	1	2	2	3	3	4	4	5
0.38	2.399	2.404	2.410	2.415	2.421	2.427	2.432	2.438	2.443	2.449	1	1	2	2	3	3	4	4	5
0.39	2.455	2.460	2.466	2.472	2.477	2.483	2.489	2.495	2.500	2.506	1	1	2	2	3	3	4	5	5
0.40	2.512	2.518	2.523	2.529	2.535	2.541	2.547	2.553	2.559	2.564	1	1	2	2	3	4	4	5	5
0.41	2.570	2.576	2.582	2.588	2.594	2.600	2.606	2.612	2.618	2.624	1	1	2	2	3	4	4	5	5
0.42	2.630	2.636	2.642	2.649	2.655	2.661	2.667	2.673	2.679	2.685	1	1	2	2	3	4	4	5	6
0.43	2.692	2.698	2.704	2.710	2.716	2.723	2.729	2.735	2.742	2.748	1	1	2	3	3	4	4	5	6
0.44	2.754	2.761	2.767	2.773	2.780	2.786	2.793	2.799	2.805	2.812	1	1	2	3	3	4	4	5	6
0.45	2.818	2.825	2.831	2.838	2.844	2.851	2.858	2.864	2.871	2.877	1	1	2	3	3	4	5	5	6
0.46	2.884	2.891	2.897	2.904	2.911	2.917	2.924	2.931	2.938	2.944	1	1	2	3	3	4	5	5	6
0.47	2.951	2.958	2.965	2.972	2.979	2.985	2.992	2.999	3.006	3.013	1	1	2	3	3	4	5	5	6
0.48	3.020	3.027	3.034	3.041	3.048	3.055	3.062	3.069	3.076	3.083	1	1	2	3	4	4	5	6	6
0.49	3.090	3.097	3.105	3.112	3.119	3.126	3.133	3.141	3.148	3.155	1	1	2	3	4	4	5	6	6

UNIT-12(XI-Physics_Vol-2) PRACTICAL FIRST YEAR.indd 312 17-03-2022 16:18:21

ANTILOG TABLE

											Mean Difference								
0.50	3.162	3.170	3.177	3.184	3.192	3.199	3.206	3.214	3.221	3.228	1	1	2	3	4	4	5	6	7
0.51	3.236	3.243	3.251	3.258	3.266	3.273	3.281	3.289	3.296	3.304	1	2	2	3	4	5	5	6	7
0.52	3.311	3.319	3.327	3.334	3.342	3.350	3.357	3.365	3.373	3.381	1	2	2	3	4	5	5	6	7
0.53	3.388	3.396	3.404	3.412	3.420	3.428	3.436	3.443	3.451	3.459	1	2	2	3	4	5	6	6	7
0.54	3.467	3.475	3.483	3.491	3.499	3.508	3.516	3.524	3.532	3.540	1	2	2	3	4	5	6	6	7
0.55	3.548	3.556	3.565	3.573	3.581	3.589	3.597	3.606	3.614	3.622	1	2	2	3	4	5	6	7	7
0.56	3.631	3.639	3.648	3.656	3.664	3.673	3.681	3.690	3.698	3.707	1	2	2	3	4	5	6	7	8
0.57	3.715	3.724	3.733	3.741	3.750	3.758	3.767	3.776	3.784	3.793	1	2	3	3	4	5	6	7	8
0.58	3.802	3.811	3.819	3.828	3.837	3.846	3.855	3.864	3.873	3.882	1	2	3	4	4	5	6	7	8
0.59	3.890	3.899	3.908	3.917	3.926	3.936	3.945	3.954	3.963	3.972	1	2	3	4	5	5	6	7	8
0.60	3.981	3.990	3.999	4.009	4.018	4.027	4.036	4.046	4.055	4.064	1	2	3	4	5	6	6	7	8
0.61	4.074	4.083	4.093	4.102	4.111	4.121	4.130	4.140	4.150	4.159	1	2	3	4	5	6	7	8	9
0.62	4.169	4.178	4.188	4.198	4.207	4.217	4.227	4.236	4.246	4.256	1	2	3	4	5	6	7	8	9
0.63	4.266	4.276	4.285	4.295	4.305	4.315	4.325	4.335	4.345	4.355	1	2	3	4	5	6	7	8	9
0.64	4.365	4.375	4.385	4.395	4.406	4.416	4.426	4.436	4.446	4.457	1	2	3	4	5	6	7	8	9
0.65	4.467	4.477	4.487	4.498	4.508	4.519	4.529	4.539	4.550	4.560	1	2	3	4	5	6	7	8	9
0.66	4.571	4.581	4.592	4.603	4.613	4.624	4.634	4.645	4.656	4.667	1	2	3	4	5	6	7	9	10
0.67	4.677	4.688	4.699	4.710	4.721	4.732	4.742	4.753	4.764	4.775	1	2	3	4	5	7	7	9	10
0.68	4.786	4.797	4.808	4.819	4.831	4.842	4.853	4.864	4.875	4.887	1	2	3	4	5	7	8	9	10
0.69	4.898	4.909	4.920	4.932	4.943	4.955	4.966	4.977	4.989	5.000	1	2	3	4	5	7	8	9	10
0.70	5.012	5.023	5.035	5.047	5.058	5.070	5.082	5.093	5.105	5.117	1	2	3	4	5	7	8	9	11
0.71	5.129	5.140	5.152	5.164	5.176	5.188	5.200	5.212	5.224	5.236	1	2	4	5	6	7	8	10	11
0.72	5.248	5.260	5.272	5.284	5.297	5.309	5.321	5.333	5.346	5.358	1	2	4	5	6	7	9	10	11
0.73	5.370	5.383	5.395	5.408	5.420	5.433	5.445	5.458	5.470	5.483	1	3	4	5	6	8	9	10	11
0.74	5.495	5.508	5.521	5.534	5.546	5.559	5.572	5.585	5.598	5.610	1	3	4	5	6	8	9	10	12
0.75	5.623	5.636	5.649	5.662	5.675	5.689	5.702	5.715	5.728	5.741	1	3	4	5	7	8	9	10	12
0.76	5.754	5.768	5.781	5.794	5.808	5.821	5.834	5.848	5.861	5.875	1	3	4	5	7	8	9	11	12
0.77	5.888	5.902	5.916	5.929	5.943	5.957	5.970	5.984	5.998	6.012	1	3	4	5	7	8	10	11	12
0.78	6.026	6.039	6.053	6.067	6.081	6.095	6.109	6.124	6.138	6.152	1	3	4	6	7	8	10	11	13
0.79	6.166	6.180	6.194	6.209	6.223	6.237	6.252	6.266	6.281	6.295	1	3	4	6	7	9	10	11	13
0.80	6.310	6.324	6.339	6.353	6.368	6.383	6.397	6.412	6.427	6.442	1	3	4	6	7	9	10	12	13
0.81	6.457	6.471	6.486	6.501	6.516	6.531	6.546	6.561	6.577	6.592	2	3	5	6	8	9	11	12	14
0.82	6.607	6.622	6.637	6.653	6.668	6.683	6.699	6.714	6.730	6.745	2	3	5	6	8	9	11	12	14
0.83	6.761	6.776	6.792	6.808	6.823	6.839	6.855	6.871	6.887	6.902	2	3	5	6	8	9	11	13	14
0.84	6.918	6.934	6.950	6.966	6.982	6.998	7.015	7.031	7.047	7.063	2	3	5	6	8	10	11	13	15
0.85	7.079	7.096	7.112	7.129	7.145	7.161	7.178	7.194	7.211	7.228	2	3	5	7	8	10	12	13	15
0.86	7.244	7.261	7.278	7.295	7.311	7.328	7.345	7.362	7.379	7.396	2	3	5	7	8	10	12	13	15
0.87	7.413	7.430	7.447	7.464	7.482	7.499	7.516	7.534	7.551	7.568	2	3	5	7	9	10	12	14	16
0.88	7.586	7.603	7.621	7.638	7.656	7.674	7.691	7.709	7.727	7.745	2	3	5	7	9	10	12	14	16
0.89	7.762	7.780	7.798	7.816	7.834	7.852	7.870	7.889	7.907	7.925	2	4	5	7	9	11	12	14	16
0.90	7.943	7.962	7.980	7.998	8.017	8.035	8.054	8.072	8.091	8.110	2	4	6	7	9	11	13	15	17
0.91	8.128	8.147	8.166	8.185	8.204	8.222	8.241	8.260	8.279	8.299	2	4	6	8	9	11	13	15	17
0.92	8.318	8.337	8.356	8.375	8.395	8.414	8.433	8.453	8.472	8.492	2	4	6	8	10	12	14	15	17
0.93	8.511	8.531	8.551	8.570	8.590	8.610	8.630	8.650	8.670	8.690	2	4	6	8	10	12	14	16	18
0.94	8.710	8.730	8.750	8.770	8.790	8.810	8.831	8.851	8.872	8.892	2	4	6	8	10	12	14	16	18
0.95	8.913	8.933	8.954	8.974	8.995	9.016	9.036	9.057	9.078	9.099	2	4	6	8	10	12	15	17	19
0.96	9.120	9.141	9.162	9.183	9.204	9.226	9.247	9.268	9.290	9.311	2	4	6	8	11	13	15	17	19
0.97	9.333	9.354	9.376	9.397	9.419	9.441	9.462	9.484	9.506	9.528	2	4	7	9	11	13	15	17	20
0.98	9.550	9.572	9.594	9.616	9.638	9.661	9.683	9.705	9.727	9.750	2	4	7	9	11	13	16	18	20
0.99	9.772	9.795	9.817	9.840	9.863	9.886	9.908	9.931	9.954	9.977	2	5	7	9	11	14	16	18	20

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GLOSSARY
கலைச்சொற்கள்

1.	Altitude	-	குத்துயரம்
2.	Astronomy	-	வானியல்
3.	Angle of Contact	-	தொடுகோணம்
4.	Aerofoil lift	-	விமான இறக்கை உயர்த்தல்
5.	Adiabatic process	-	வெப்ப பரிமாற்றமில்லா நிகழ்வு
6.	Average (or) Mean speed	-	சராசரி வேகம்
7.	Angular Harmonic motion	-	கோணச்சீரிசை இயக்கம்
8.	Beats	-	விம்மல்கள்
9.	Buoyant force	-	மிதப்பு விசை
10.	Breaking of rupture point	-	முறிவுப்புள்ளி
11.	Compressive stress	-	அமுக்கத்தகைவு
12.	Calorimeter	-	வெப்ப அளவீட்டியல்
13.	Conduction	-	வெப்பக்கடத்தல்
14.	Change of state	-	நிலை மாற்றம்
15.	Capillary rise or fall	-	நுண்புழை ஏற்றம் அல்லது இறக்கம்
16.	Compress	-	அமுக்கம் / இறுக்கம்
17.	Compliance	-	இணக்கம்
18.	Coefficient of performance	-	செயல்திறன் குணகம்
19.	Degree of freedom	-	சுதந்திர இயக்கக்கூறு
20.	Damped oscillation	-	தடையுறு இயல்பு அதிர்வுகள்
21.	Elongate	-	நீட்சி
22.	Elastic limit	-	மீட்சி எல்லை
23.	Escape speed	-	விடுபடு வேகம்
24.	Epicycle	-	பெரு வட்டத்தில் அமையும் சிறு வட்டச் சுழற்சி
25.	Epoch	-	தொடக்கக்கட்டம்
26.	Equilibrium	-	சமநிலை
27.	Flexible constant	-	நெகிழ்வு தன்மை மாறிலி
28.	Free oscillations	-	தனி அலைவியக்கம்
29.	Force constant	-	விசை மாறிலி
30.	Forced Oscillation	-	திணிக்கப்பட்ட அலைவுகள்
31.	Frequency	-	அதிர்வெண்
32.	Geocentric model	-	புவிமையக் கோட்பாட்டு மாதிரி
33.	Gravitational field	-	ஈர்ப்பு புலச்செறிவு (அல்லது) ஈர்ப்புப்புலம்

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34.	Gravitational Potential	-	ஈர்ப்பழுத்தம்
35.	Gravitational Potential energy	-	ஈர்ப்பழுத்த ஆற்றல்
36.	Geo- Stationary satellite	-	புவி நிலைத் துணைக்கோள்
37.	Hydrostatic paradox	-	நீர்ம நிலையியல் முரண்பாடு
38.	Heat engine	-	வெப்ப இயந்திரம்
39.	Hydraulic lift	-	நீரியல் தூக்கி
40.	Harmonics	-	சீரிசை
41.	Heliocentric model	-	சூரிய மையக்கோட்பாடு
42.	Interference	-	குறுக்கீட்டு விளைவு
43.	Isothermal process	-	வெப்பநிலை மாறா நிகழ்வு
44.	Isobaric process	-	அழுத்தம் மாறா நிகழ்வு
45.	Isochoric process	-	பருமன் மாறா நிகழ்வு
46.	Lateral strain	-	பக்கவாட்டுத்திரிபு
47.	Longitudinal strain	-	நீளவாட்டுத்திரிபு
48.	Longitudinal stress	-	நீட்சித்தகைவு
49.	Loudness	-	உரப்பு / ஒலி உரப்பு
50.	Lunar Eclipse	-	சந்திர கிரகணம்
51.	Latent heat	-	உள்ளுறை வெப்பம்
52.	Latitude	-	குறுக்குக்கோடு / அட்சக்கோடு
53.	Law of equipartition of energy	-	ஆற்றல் சமபங்கீட்டு விதி
54.	Mean square speed	-	சராசரி இருமடிவேகம்
55.	Most probable speed	-	மிகவும் சாத்தியமான வேகம்
56.	Mean free path	-	சராசரி மோதலிடைத்தூரம்
57.	Maintained Oscillation	-	நிலைநிறுத்தப்பட்ட அதிர்வுகள்
58.	Mean position	-	நடுநிலை
59.	Number density	-	எண் அடர்த்தி
60.	Node	-	கணு
61.	Natural oscillation	-	இயல்பு அதிர்வுகள்
62.	Non-periodic motion	-	சீரலைவற்ற இயக்கம்
63.	Oscillatory motion	-	அலைவியக்கம்
64.	Orbital velocity	-	சுற்றியக்க திசைவேகம்
65.	Overtone	-	மேற்சுரம்
66.	Polar satellite	-	துருவ துணைக்கோள்
67.	Phase	-	கட்டம்
68.	Periodic motion	-	சீரலைவு இயக்கம்
69.	Phase difference	-	கட்ட வேறுபாடு
70.	Penumbra	-	பகுதி நிழல்
71.	Retrograde motion	-	பின்நோக்கு இயக்கம்
72.	Restoring force	-	மீள்விசை
73 .	Restoring Torque	-	மீள்திருப்பு விசை
74.	Random motion	-	ஒழுங்கற்ற இயக்கம் / சீரற்ற இயக்கம்
75.	Root mean square speed (Vrms)	-	சராசரி இருமடிமூல வேகம்
76.	Radiation	-	வெப்பக்கதிர்வீச்சு

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77.	Resonance	-	ஒத்ததிர்வு
78.	Ripples	-	சிற்றலைகள்
79.	Standard (or) Normal temperature and pressure	-	படித்தர (அல்லது) இயல்பு வெப்பநிலை மற்றும் அழுத்தம்
80.	Stethoscope	-	இதயத்துடிப்பு அறியும் கருவி
81.	Sonometer	-	சுரமானி
82.	Superposition	-	மேற்பொருந்துதல்
83.	Spring constant / force constant	-	சுருள்மாறிலி
84.	Stiffness constant	-	முறுக்கு மாறிலி
85.	Simple pendulum	-	தனி ஊசல்
86.	Satellite	-	துணைக்கோள்
87.	Stationary waves	-	நிலை அலைகள்
88.	Shearing Stress	-	சறுக்குப்பெயர்ச்சித் தகைவு
89.	Surface tension	-	பரப்பு இழுவிசை
90.	Streamlined flow	-	வரிச்சீர் ஓட்டம்
91.	Specific heat capacity	-	தன்வெப்ப ஏற்புத்திறன்
92.	Speed distribution function	-	வேகப் பகிர்வுச் சார்பு
93.	Specific heat capacity at constant pressure	-	அழுத்தம் மாறா தன்வெப்ப ஏற்புத்திறன்
94.	Specific heat capacity at constant volume	-	பருமன் மாறா தன்வெப்ப ஏற்புத்திறன்
95.	Simple Hormonic motion	-	தனிச்சீரிசை இயக்கம்
96.	Tensile stress	-	இழுவிசைத்தகைவு
97.	Terminal velocity	-	முற்றுத் திசைவேகம்
98.	Time period	-	அலைவுக்காலம்
99.	Turbulent flow	-	சுழற்சி ஓட்டம்
100.	Tsunami	-	ஆழிப்பேரலை
101.	Thermal conductivity	-	வெப்பக்கடத்துத்திறன்
102.	Thermometer	-	வெப்பநிலைமானி
103.	Triple Point	-	முப்புள்ளி
104.	Universal gas constant	-	பொது வாயுமாறிலி
105.	Umbra	-	கருநிழல்
106.	Viscosity	-	பாகுநிலை
107.	Water striders	-	நீர்தாண்டிப்பூச்சிகள்
108.	Wavicle	-	அலைத்துகள்
109.	Zig – Zag path	-	குறுக்கு – நெடுக்கானப் பாதை

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State Council of Educational Research and Training, Chennai-6

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This book has been printed on 80 G.S.M.
Elegant Maplitho paper.
Printed by offset at:

NOTE

PH XI Std Acknowledgement.indd 318 25-03-2022 20:40:38

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PH XI Std Acknowledgement.indd 319 25-03-2022 20:40:38

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PH XI Std Acknowledgement.indd 318 25-03-2022 20:40:38