

Assignment - 2

Vishnupradeepwar D

CO23BTECH11024

(Computer Aided Numerical
method 8 - 2)

1) The given PDEs are of form

$$AU_{xx} + BU_{xy} + CU_{yy} = 0$$

* Slope of their characteristic line ($2D$) is

$$\frac{dy}{dx} = \frac{B \pm \sqrt{B^2 - 4AC}}{2A}, \quad \Delta = B^2 - 4AC$$

a) $U_{xx} + 4U_{yy} = 0; A=1, B=0, C=4$

$$\Delta = B^2 - 4AC = -4$$

$\Delta < 0 \Rightarrow$ This PDE is Elliptic and hence no characteristic line exists

b) $U_{xx} + 2U_{xy} + U_{yy} = 0; A=1, B=2, C=1$

$$\Delta = 0 \Rightarrow$$
 Parabolic PDE

$$\Rightarrow \frac{dy}{dx} = \frac{2 \pm 0}{2 \times 1} \Rightarrow y = x + c \rightarrow \text{Characteristic line } (c \Rightarrow \text{const})$$

c) $U_{xx} - 4U_{xy} + 5U_{yy} = 0; A=1, B=-4, C=5$

$$\Delta = -4; \Delta < 0 \Rightarrow$$
 Elliptic PDE (No. Characteristic Line)

d) $xU_{xx} - yU_{xy} = 0; A=x, B=-y, C=0$

$$\Rightarrow \Delta = y^2$$

* Clearly for $y=0$, this PDE is parabolic. For all other real values of y , this PDE is Hyperbolic ($\Delta > 0$)

$$\Delta > 0$$

$$\Rightarrow \frac{dy}{dx} = -\frac{y \pm \sqrt{y^2}}{2x} = -\frac{y \pm |y|}{2x}$$

$$\frac{dy}{dx} = 0 \Rightarrow y = c$$

$$\frac{dy}{dx} = -\frac{y}{x} \Rightarrow \ln x + \ln y = c$$

$$\Delta = 0$$

$$\Rightarrow \frac{dy}{dx} = -\frac{0 \pm 0}{2x} = 0$$

where c, c_1 are constants

$$\Rightarrow \frac{dy}{dx} = c_1$$

Q) From the given finite difference expression,

$$u_{i,j+1} = u_{i,j} + \frac{\Delta t}{\Delta x^2} (u_{i+1,j+1} + u_{i-1,j+1})$$

$\frac{1+2\frac{\Delta t}{\Delta x^2}}{\Delta x^2}$

• Let's take $u(0,0) = 1$

• For given case,

$$\underline{i=0}$$

$$\underline{j=1}$$

$$u_{1,1} = u_{1,0} + \frac{\Delta t}{\Delta x^2} (u_{2,1} + u_{0,1})$$

$\frac{1+2\frac{\Delta t}{\Delta x^2}}{\Delta x^2}$

$$\Rightarrow u_{1,1} = 0 + \frac{\Delta t}{\Delta x^2} (1+0) = \frac{\frac{\Delta t}{\Delta x^2} (1+0)}{\frac{1+2\frac{\Delta t}{\Delta x^2}}{\Delta x^2}} = \frac{\frac{\Delta t}{\Delta x^2}}{\frac{1+2\frac{\Delta t}{\Delta x^2}}{\Delta x^2}}$$

$$\underline{i=1}$$

$$\underline{j=1}$$

~~$u_{1,2} - u_{2,0}$~~

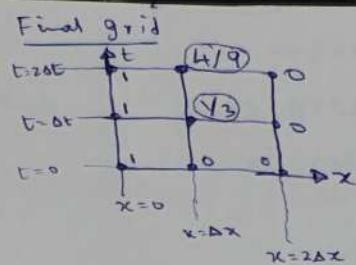
$$u_{1,2} = u_2 + \frac{\Delta t}{\Delta x^2} (u_{2,0} + u_{2,2})$$

$\frac{1+2\frac{\Delta t}{\Delta x^2}}{\Delta x^2}$

$$\Rightarrow u_{1,2} = \frac{\frac{\Delta t}{\Delta x^2} \left(\frac{1}{\frac{1+2\frac{\Delta t}{\Delta x^2}}{\Delta x^2}} + (1+0) \right)}{1+\frac{2\Delta t}{\Delta x^2}} = \frac{\frac{2\Delta t}{\Delta x^2} \left(1 + \frac{\Delta t}{\Delta x^2} \right)}{\left(1 + \frac{2\Delta t}{\Delta x^2} \right)^2}$$

* AS $t=1, \Delta x=1$

$$\boxed{U_{1,1} = \frac{1}{3}} \rightarrow \boxed{U_{1,2} = \frac{4}{9}}$$



* On generalising, we can form equation of type $A\vec{u} = \vec{b}$, hence solve for \vec{u} .

* This equation is in form of heat eqn. Heat eqn are stable when CFL number is less than $1/2$. Here CFL number is $\frac{1+1}{1}$ which is greater than $1/2$. Hence this scheme for given $\Delta t, \Delta x$ seems to be unstable.