

# Assignment - 2

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Computer Aided Numerical  
methods - 2

1) The given PDEs are of form

$$AU_{xx} + BU_{xy} + CU_{yy} = 0$$

\* slope of their characteristic line (2D) is

$$\frac{dy}{dx} = \frac{B \pm \sqrt{B^2 - 4AC}}{2A}, \Delta = B^2 - 4AC$$

②  $U_{xx} + 4U_{yy} = 0$ ;  $A=1, B=0, C=4$

$$\Delta = B^2 - 4AC = -16$$

$\Delta < 0 \Rightarrow$  This PDE is Elliptic and hence no characteristic line exists

③  $U_{xx} + 2U_{xy} + U_{yy} = 0$ ;  $A=1, B=2, C=1$

$$\Delta = 0 \Rightarrow \text{Parabolic PDE}$$

$$\Rightarrow \frac{dy}{dx} = \frac{2 \pm 0}{2 \times 1} \Rightarrow y = x + C \rightarrow \text{Characteristic line (C} \Rightarrow \text{const)}$$

④  $U_{xx} - 4U_{xy} + 5U_{yy} = 0$ ;  $A=1, B=-4, C=5$

$$\Delta = -4; \Delta < 0 \Rightarrow \text{Elliptic PDE (No characteristic line)}$$

⑤  $xU_{xx} - yU_{xy} = 0$ ;  $A=x, B=-y, C=0$

$$\Rightarrow \Delta = y^2$$

\* clearly for  $y=0$ , this PDE is parabolic. For all other real values of  $y$ , this PDE is Hyperbolic ( $\Delta > 0$ )

$$\Delta > 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{-y \pm \sqrt{y^2}}{2x} = \frac{-y \pm |y|}{2x}$$

$$\frac{dy}{dx} = 0 \Rightarrow y = c$$

$$\frac{dy}{dx} = -\frac{y}{x} \Rightarrow \ln x + \ln y = c$$

$$\Delta = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{-0 \pm 0}{2x} = 0$$

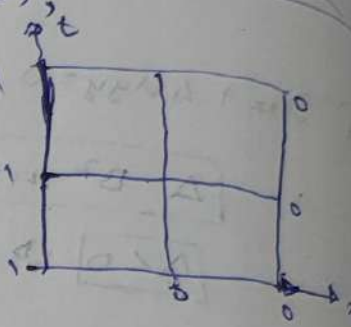
$$\Rightarrow \frac{dy}{dx} = 0$$

where  $c, c_1$  are constants

② From the given finite difference expression,

$$u_{i,j+1} = u_{i,j} + \frac{\Delta t}{\Delta x^2} (u_{i+1,j+1} + u_{i-1,j+1})$$

$$1 + \frac{2\Delta t}{\Delta x^2}$$



• Let's take  $u(0,0) = 1$

• For given case,

$$j=0$$

$$i=1$$

$$u_{1,1} = u_{1,0} + \frac{\Delta t}{\Delta x^2} (u_{2,1} + u_{0,1})$$

$$1 + \frac{2\Delta t}{\Delta x^2}$$

$$\Rightarrow u_{1,1} = 0 + \frac{\Delta t}{\Delta x^2} (1+0) = \frac{\Delta t}{\Delta x^2} \frac{1}{1 + \frac{2\Delta t}{\Delta x^2}}$$

$$j=1$$

$$i=1$$

$$u_{1,2} = u_{1,1} + \frac{\Delta t}{\Delta x^2} (u_{2,2} + u_{0,2})$$

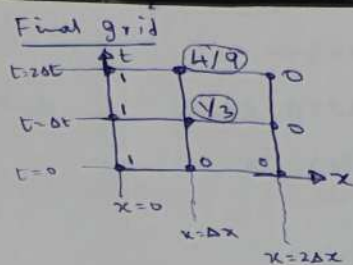
$$u_{1,2} = u_{1,1} + \frac{\Delta t}{\Delta x^2} (u_{2,2} + u_{0,2})$$

$$1 + \frac{2\Delta t}{\Delta x^2}$$

$$\Rightarrow u_{1,2} = \frac{\Delta t}{\Delta x^2} \left( \frac{1}{1 + \frac{2\Delta t}{\Delta x^2}} + (1+0) \right) = \frac{\frac{2\Delta t}{\Delta x^2} \left( 1 + \frac{\Delta t}{\Delta x^2} \right)}{\left( 1 + \frac{2\Delta t}{\Delta x^2} \right)^2}$$

\* AS  $t=1, \Delta x=1$

$$u_{1,1} = \frac{1}{3}, \quad u_{1,2} = \frac{1}{9}$$



On generalising, we can form equation of type  $A\hat{u} = \hat{b}$ , hence solve for  $\hat{u}$ .  
 \* This equation is in form of heat eq<sup>n</sup>. Heat eq<sup>n</sup> are stable when CFL number is less than  $1/2$ . Here CFL number is  $\frac{1}{1}$  which is greater than  $1/2$ . Hence this scheme for given  $\Delta t, \Delta x$  seems to be unstable.