$\therefore$ By (2)

$$y_{4P} = \cancel{6.8}\ 3.234 + \frac{0.5}{24}\{9(4.4354) + 9(3.234) - 5(2.297) + (1.568)\}$$

$$y_{4,c} = 6.8731$$

$$\boxed{\text{Adam's Accuracy} > \text{Milen's Accuracy}}$$

$\boxed{\text{Cat-3}}$

08/10/2022.

Solution to Boundary valued problems (BVPs) by finite difference methods:-

Method:-

To solve $y'' + a(x)y' + b(x)y(x) = c(x)$

with $y(x_0) = \alpha \to (1)$ and $y(x_n) = \beta \to (2)$

we take,

$$y' = \frac{y_{i+1} - y_{i-1}}{2h} \qquad y'' = \frac{y_{i-1} + y_{i+1} - 2y_i}{h^2}$$

$\to$ (1) reads to

$$\frac{y_{i+1} + y_{i-1} - 2y_i}{h^2} + a(x_i)\frac{y_{i+1} - y_{i-1}}{2h} + b(x_i)y(x_i) = c(x_i) \longrightarrow (3)$$

On simplification, we get,

$$y_{i+1}\left(1 + \frac{h}{2}a_i\right) + y_i(h^2 b_i - 2) + y_{i-1}\left(1 - \frac{h}{2}a_i\right) = a_i h_i^2 \to (3)$$

for $i = 1, 2, \ldots n-1$ & $y_0 = \alpha$; $y_n = \beta$.

$a_i = a(x_i)$ $\quad b_i = b(x_i)$ $\quad c_i = c(x_i)$

Eqn (3) gives
$(n-1)$ eqn for $i=1,2,\dots,n-1$ together with
$y_0=\alpha$ & $y_n=\beta$,
we get $(n+1)$ equations $y_0, y_1 \dots y_n$
solving from these $(n+1)$ equations, we get
$y_0, y_1, \dots, y_{n+1}$.

(Qn) Using finite difference method solve for y.
Given: $\frac{d^2y}{dx^2} + y + 1 = 0$ where $x(0,1)$; & the boundary condition $y(0) = y(1) = 0$ taking
(i) $h = \frac{1}{2}$ (ii) $h = \frac{1}{4}$.

Soln:- Given:-
2nd ODE: $\frac{d^2y}{dx^2} + y + 1 = 0$
we divide the interval $(0,1)$ of x into 'n' equal parts such that $nh = 1$ (or) $h = \frac{1}{n}$

case (1): Suppose $n=2$ such that $h = \frac{1}{2}$

$$y' = \frac{y_{i+1} - y_{i-1}}{2h} \qquad y'' = \frac{y_{i-1} + y_{i+1} - 2y_i}{h^2}$$

we get,

$$\frac{y_{i+1} + y_{i-1} - 2y_i}{h^2} + y_i + 1 = 0.$$

$$y_{i+1} + y_{i-1} - 2y_i + y_i(h^2) + h^2 = 0.$$

$$y_{i+1} + y_{i-1} + y_i(h^2 - 2) + h^2 = 0$$

Taking $h = \frac{1}{2}$, $i = 1$

$$y_2 + y_1\left(\tfrac{1}{4} - 2\right) + y_0 + \left(\tfrac{1}{4}\right) = 0.$$

$$y_2 + y_1\left(\tfrac{7}{4}\right) + y_0 + \tfrac{1}{4} = 0.$$

$$\cancel{y_2 = -\tfrac{1}{4}}$$

$$y_1\left(\tfrac{7}{4}\right) + \tfrac{1}{4} = 0 \qquad y_1 = 0.1428.$$

x_0	x_1	x_2
0	1/2	1
0	?	0
y_0	y_1	y_2

x	x_0	x_1	x_2
0	0	1/2	1
y	0	0.1428	0.

Case 2 :: Taking $n=4$ $h=1/4$

x_0	x_1	x_2	x_3	x_4
0	$1/4$	$1/2$	$3/4$	1
0	?	?	?	0
y_0	y_1	y_2	y_3	y_4

Then 1,

$$y_{i+1} - \frac{31}{16} y_i + y_{i-1} = \frac{-1}{16}$$

Taking $i=1,2,3$ in this

$$y_2 - \frac{31}{16} y_1 + y_0 = \frac{-1}{16}$$

$$y_3 - \frac{31}{16} y_2 + y_1 = \frac{-1}{16}$$

$$y_4 - \frac{31}{16} y_3 + y_2 = \frac{-1}{16}$$

Solving for y_1, y_2, y_3 using any method, we get,

$$y_1 = \frac{4}{449} \quad y_2 = \frac{63}{449} \quad y_3 = \frac{47}{449}.$$

$\therefore$ The solution in this case is

x_0	x_1	x_2	x_3	x_4
0	$1/4$	$1/2$	$3/4$	1
0	$4/449$	$63/449$	$\frac{47}{449}$	0.
y_0	y_1	y_2	y_3	y_4

Note

By comparing $y(0.5)$ in cases 1 & 2, we get the accuracy depends on no. of subintervals chosen & the order of approximation

(Qn) Using finite difference method solve $\frac{d^2y}{dx^2} = y$ in $(0,2)$ $y(0) = 2$ $y(2) = 3.636$ by dividing interval into 4 equal parts

Ans:-

x_0	x_1	x_2	x_3	x_4
0	0.5	1	1.5	2
0	0.5268	1.1853	2.1401	3.63
y_0	y_1	y_2	y_3	y_4

(Qn) Using finite difference method, solve $y'' - 64y + 10 = 0$ $x \in (0,1)$ given $y(0) = y(1) = 0$ subdividing the interval (i) 4 equal parts.
(ii) 2 equal parts

① Given:- 2nd ODE :- $\frac{d^2y}{dx^2} - y = 0$ ① ; we divide the interval $(0,2)$ of x into 4 equal parts.

$$y'' = \frac{y_{i-1} + y_{i+1} - 2y_i}{h^2}$$

x_0	x_1	x_2	x_3	x_4
0	0.5	1	1.5	2
2	?	?	?	3.636
y_0	y_1	y_2	y_3	y_4.

$$① \Rightarrow \frac{y_{i-1} + y_{i+1} - 2y_i}{h^2} - y = 0.$$

$$y_{i-1} + y_{i+1} - 2y_i - h^2 y_i = 0.$$

Taking $h = \frac{1}{2}$; $i = 1, 2, 3$; $y_0 = 2$; $y_2 = 3.636$.

$i=1$ $y_0 + y_2 - 2y_1 - \frac{1}{4}y_1 = 0$. $\quad y_0 + y_2 - \frac{9}{4}y_1 = 0 \rightarrow ①$

$i=2$ $y_1 + y_3 - 2y_2 - \frac{1}{4}y_2 = 0$ $\quad y_1 + y_3 - \frac{9}{4}y_2 = 0 \rightarrow ②$

$i=3$ $y_2 + y_4 - 2y_3 - \frac{1}{4}y_3 = 0$. $\quad y_2 + y_4 - \frac{9}{4}y_3 = 0 \rightarrow ③$

Solving for y_1, y_2, y_3, we get

$① \Rightarrow y_0 = \frac{9}{4}y_1 - y_2$ $\quad y_2 = \frac{9}{4}y_1 - y_0$

$② \Rightarrow y_3 = \frac{9}{4}y_2 - y_1$

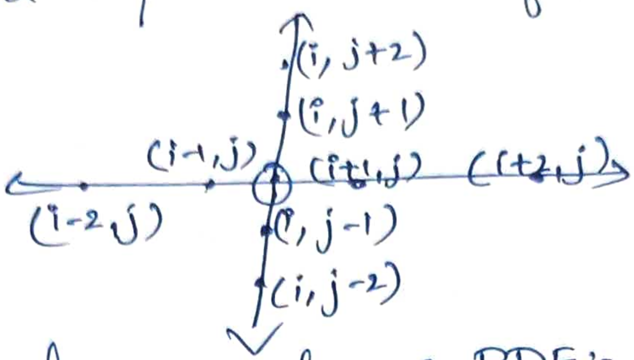
$③ \Rightarrow y_4 = \frac{9}{4}y_3 - y_2 = \frac{9}{4}\left(\frac{9}{4}y_2 - y_1\right) - y_2 = \frac{81}{16}y_2 - \frac{9}{4}y_1 - y_2$

$$= \frac{65}{16}y_2 - \frac{9}{4}y_1$$

10/10/2022

Numerical solution of partial Differential Equations:-

Graphical representation of partial quotients:-



General 2nd order linear PDE:-

This is given by

$$A\frac{\partial u^2}{\partial x^2} + B\frac{\partial^2 u}{\partial x \partial y} + C\frac{\partial^2 u}{\partial y^2} + D\frac{\partial u}{\partial x} + E\frac{\partial u}{\partial y} + FU(x, y) = 0.$$

(or)

$$Au_{xx} + Bu_{xy} + Cu_{yy} + Du_x + Eu_y + FU = 0. \rightarrow (1)$$

where A, B, C, D, E, F are functions of x & (

Equation (1) is said to be

(i) Elliptic:- if $S = B^2 - 4AC < 0$.

(ii) Parabolic if $S = B^2 - 4AC = 0$

(iii) Hyperbolic if $S = B^2 - 4AC > 0$.

Note:-

~~This~~ It is possible to have equation may be elliptic in one region & parabolic/hyperbolic in some other region.

For eg.

$x U_{xx} + U_{yy} = 0$ is $\rightarrow$ (1)

$A = x$ $\quad\quad \Delta = B - 4AC$

$B = 0$ $\quad\quad = 0 - 4(x)(1)$

$C = 1$ $\quad\quad = -4x$.

~~∴ Elliptic.~~

(i) If $x > 0$; δ is -ve & (1) is Elliptic.

(ii) If $x = 0$; δ is 0 & (1) is parabolic.

(iii) If $x < 0$; δ is +ve & (1) is hyperbolic.

Standard equations:-

(i) Elliptic (i) $\frac{\partial u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ (Laplace equation)

(ii) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = F(x,y)$ (Poisson Equation)

(ii) Parabolic (i) $\frac{\partial^2 u}{\partial x^2} = \frac{1}{\alpha^2}\frac{\partial u}{\partial t}$ (one-dimensional heat equation)

(iii) Hyperbolic:-

(i) $\frac{\partial^2 u}{\partial x^2} = \frac{1}{\alpha^2}\frac{\partial^2 u}{\partial t^2}$ (one-dimensional wave equation)

Qn)

Classify the following equations:-

(i) $\frac{\partial^2 u}{\partial x^2} + 2\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 u}{\partial y^2} = 0$

(ii) $x^2 f_{xx} + (1-y^2) f_{yy} = 0$.

Soln:-

(i) $A = 1 \quad B = 2 \quad C = 1$

$\delta = B^2 - 4AC = 4 - 4(1)(1) = 0$

∴ Parabolic for x, y.

(ii) $A = x^2$ $B = 0$ $C = (1-y^2)$

$$\delta = B^2 - 4AC$$
$$= 0 - 4(x^2)(1-y^2)$$
$$= -4x^2(1-y^2) = 4x^2(y^2-1)$$

(1) Now, for all $x \neq 0$; x is +ve.
& if $-1 < y < 1$; y is -ve.

$\therefore B^2 - 4AC$ is -ve if $-1 < y < 1$ & $x \neq 0$.

(i.e) For $-\infty < x < \infty$ $(x \neq 0)$; $-1 < y < 1$
the equation is Elliptic.

For $-\infty < x < \infty$ $(x \neq 0)$; $y < -1$ & $y > 1$, $\Delta > 0$
the equation is hyperbolic.

For $-\infty < x < \infty$ for $x = 0$; & for all values of
The equation is parabolic.

(Qn) Test the Nature of

(1) $U_{xx} + 4U_{yy} + (x^2 + 4y^2)U_{yy} = \sin(x+y)$

(2) $(x+1)U_{xx} - 2(x+2)U_{xy} + (x+3)U_{yy} = 0$

(3) $xF_{xx} + xF_{yy} = 0$; $x > 0$; $y > 0$

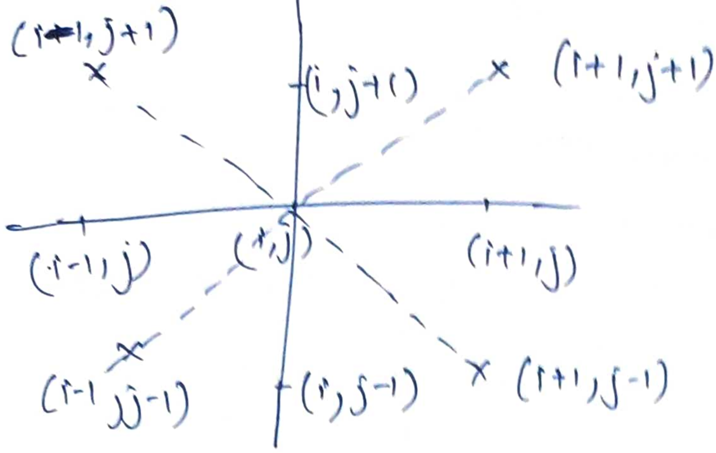
Solution to Laplace & Poisson equations.

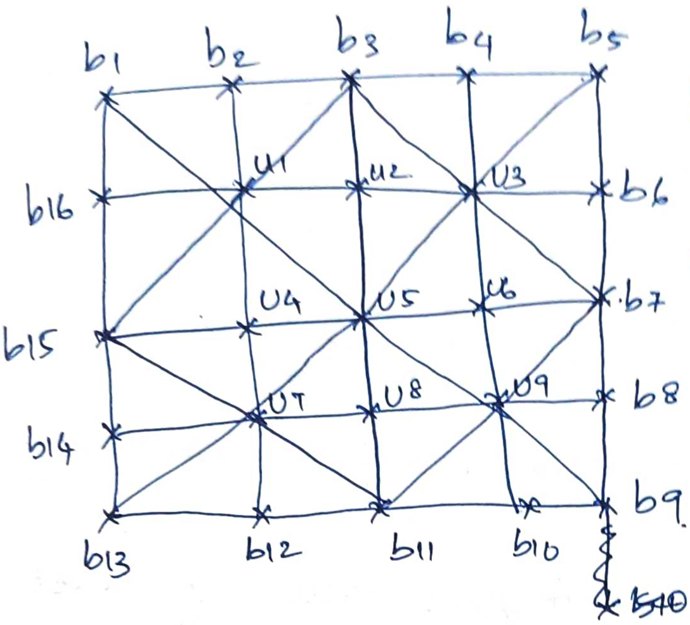
Using (i) SFPF

$$U_{i,j} = \frac{1}{4}\{U_{i-1,j} + U_{i+1,j} + U_{i,j-1} + U_{i,j+1}\}$$

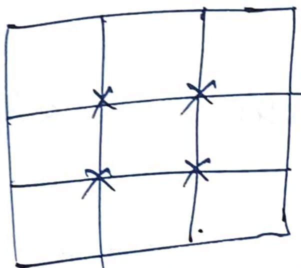
(ii) DFPF :-

$$U_{i,j} = \frac{1}{4}\{U_{i-1,j+1} + U_{i+1,j+1} + U_{i-1,j-1} + U_{i+1,j-1}\}$$





RV $\left\{\begin{array}{l} \text{Stg I} - U_5 \to \text{SFPF} \\ \text{II} \quad U_4, 3, 7, 9 \to \text{DFPF} \\ \text{III} \quad U_2, 4, 6, 8 \to \text{SFPF} \end{array}\right.$



→ modify & solve.
can't solve directly.

→ Solve:- $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = 0$

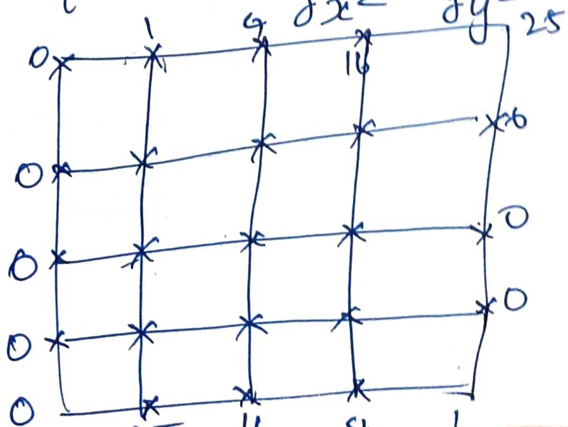
(i) $u(0, y) = 0$; $0 \le y \le 4$

(ii) $u(4, y) = 0$; $0 \le y \le 4$

(iii) $u(x, 0) = 0$; $0 \le x \le 4$

(iv) $u(x, 0) = 0$; $0 \le x \le 4$.

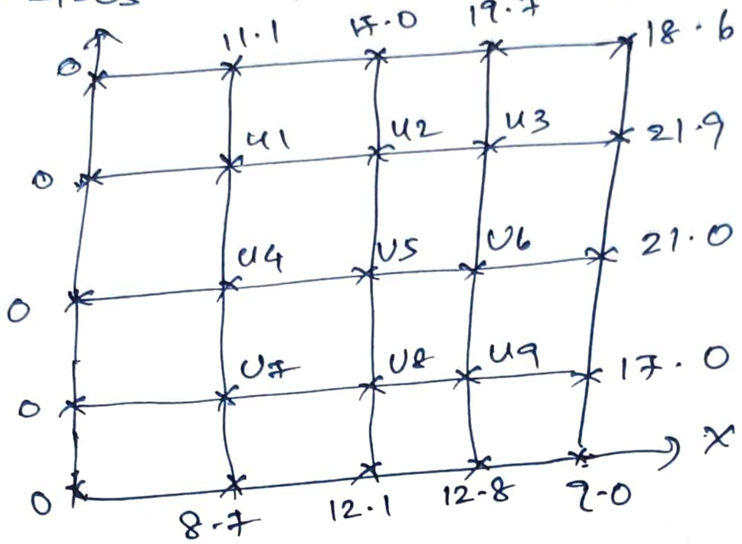
(Qn) solve the equation : $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = 0$ ~~$0 \le y \le 4$~~



#09
11/10/2022

(Qn) Solve the Laplace eqn $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ over Mesh of 4 units with boundary conditions

Given: Step sizes $h=1, k=1$ in x & y directions respective



Solution:-

Gn:- The LE, step sizes & the boundary condition.

To solve:- we need to find values of u at the interior grid points by $u_1, u_2 \dots u_9$

Stage ①:- Finding the rough values for u's

Step ①:-

$\boxed{SFDF}$ (i) To find $u_5 = \frac{1}{4}\{0+21+17+0+12.1\} = 12.52$

(ii) To find $u_1 = \frac{1}{4}\{0+17+12.525+0\}$

$= 7.38125$

$u_3 = \frac{1}{4}\{17+18.6+21+12.525\}$

$\boxed{DFPF}$ $= 17.28125.$

$u_7 = \frac{1}{4}\{0+12.525+12.1+0\}$

$= 6.15625.$

$u_9 = \frac{1}{4}\{12.525+21.0+9.0+12.1\}$

$= 13.65625$

(iii) $U_2 = \frac{1}{4}\{7.38125 + 17 + 17.28125 + 12.525\} = 13.546875$

$U_4 = \frac{1}{4}\{0 + 7.38125 + 12.525 + 6.15625\} = 6.515625$

SFPF

$U_6 = \frac{1}{4}\{\cancel{13.546875}\ 12.525 + 17.28125 + 21.0 + 13.65625\}$

$= 16.115625.$

$U_8 = \frac{1}{4}\{6.15625 + 12.525 + 13.65625 + 12.1\}$

$= 11.109375.$

(i.e) $u_1 = 7.3$ $\quad u_2 = 13.3$ $\quad U_3 = 17.2$ $\quad u_4 = 6.53$

$u_5 = 12.5$ $\quad U_6 = 16.1$ $\quad U_7 = 6.15$ $\quad U_8 = 11.12$ $\quad u_9 = 13.65$

Step (2) :-

(i) $u_1^{(1)} = \frac{1}{4}\{0 + 11.1 + 13.3 + 6.53\} = 7.73.$

SFPF

$u_2^{(1)} = \frac{1}{4}\{7.73 + 17.0 + 17.27 + 12.5\} = 13.61$

$u_3^{(1)} = \frac{1}{4}\{19.7 + 13.62 + 16.1 + 21.9\} = 17.83$

$u_4^{(1)} = \frac{1}{4}\{0 + 7.73 + 12.5\cancel{25} + 6.15\} = 6.59.$

$u_5^{(1)} = \frac{1}{4}\{6.59 + 13.61 + 16.1 + 11.12\} = 11.85$

$u_6^{(1)} = \frac{1}{4}\{11.82 + 17.83 + 13.65 + 21.0\} = 16.09.$

$u_7^{(1)} = \frac{1}{4}\{0 + 8.7 + 6.59 + 11.12\} = 6.6.$

$u_8^{(1)} = \frac{1}{4}\{6.6 + 12.1 + 11.85 + 13.65\} = 11.05.$

$u_9^{(1)} = \frac{1}{4}\{6.6 + 12.1 + 11.85 + 13.65\} = 11.05.$

$u_8^{(1)} = \frac{1}{4}\{6.6 + 12.1 + 11.85 + 13.65\} = 11.05.$

Step (3) :-

$u_1^{(2)} = \frac{1}{4}\{ \quad\quad \} = 7.9$

$u_2^{(2)} = \frac{1}{4}\{ \quad\quad \} = 13.7$

$u_3^{(2)} = \frac{1}{4}\{ \quad\quad \} = 17.9.$

$u_4^{(2)} = \frac{1}{4}\{ \quad\quad \} = 6.6$

$u_5^{(2)} = \frac{1}{4}\{ \quad\quad \} = 11.9$

$u_6^{(2)} = \frac{1}{4}\{ \quad\quad \} = 16.3$

$u_7^{(2)} = \frac{1}{4}\{ \quad\quad \} = 6.6$

$u_8^{(2)} = \frac{1}{4}\{ \quad\quad \} = 11.2$

$u_9^{(2)} = 1/4\{ \quad\quad \} = 14.3$

Conclusion:-

By observing the value of U's from Iteration ① & Iteration ② we conclude that the solution is given by

$u_1 = 7.9$ $u_2 = 13.7$ $u_3 = 17.9$

$u_4 = 6.6$ $u_5 = 11.9$ $u_6 = 16.3$

$u_7 = 6.6$ $u_8 = 11.2$ $u_9 = 14.3$.

(Qn) Solve LE $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ ~~with~~ over the given region

~~conclusion~~

(i) $u(0,y) = 0$; $0 \le y \le 4$

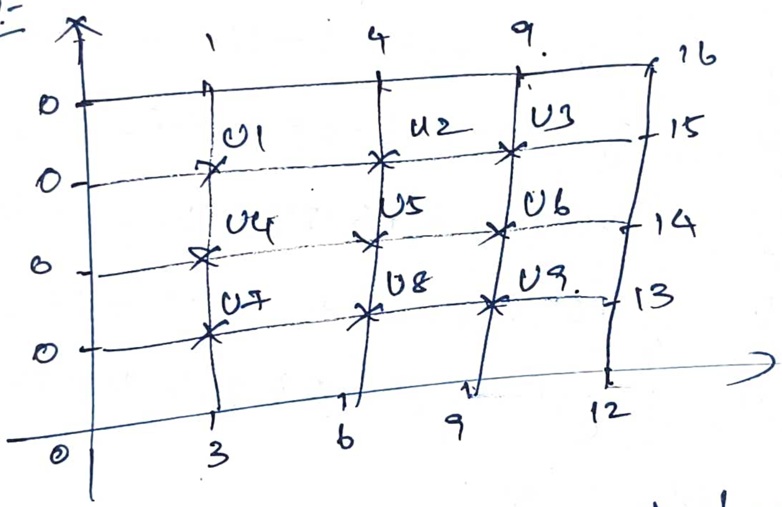
(ii) $u(4,y) = 12 + y$; $0 \le y \le 4$

(iii) $u(x,0) = 3x$; $0 \le x \le 4$

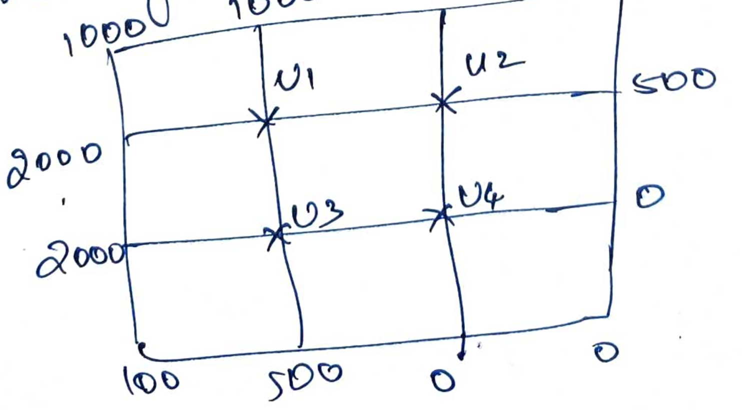
(iv) $u(x,4) = x^2$; $0 \le x \le 4$

$h = 1$ & $k = 1$ for x & y directions respectively

Soln:-



(Qn) Square mesh of size 3 units with boundary conditions given by:-



with step sizes $h=1$ & $k=1$ in x & y directions respectively.

$$U_1=\frac{1}{4}\{2000+1000+U_2+U_3\}$$

$$U_2=\frac{1}{4}\{U1+1000+500+U_4\}$$

$$U_3=\frac{1}{4}\{U1+U4+500+2000\}$$

$$U_4=\frac{1}{4}=\{U2+U3+0+0\}$$

14/10/2022

Solution to Poisson Equation:-

To solve:-

$$\nabla^2 u=f(x,y) \text{ or } \frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=f(x,y) \rightarrow (1)$$

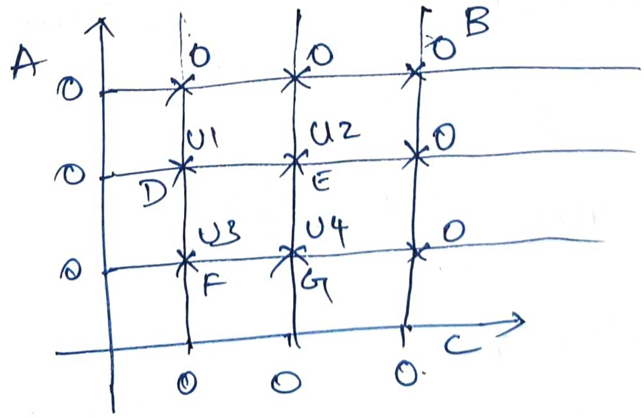
we use,

$$u_{i-1,j}+u_{i+1,j}+u_{i,j-1}+u_{i,j+1}-4u_{i,j}=h^2 f(ih,jh) \rightarrow (2)$$

By applying (2),

system of linear equations which can be solved for values of U at pivotal points/Grid points

Qn) $\nabla^2 u=-10(x^2+y^2+10)$ over the mesh with size ~~h~~: $x=0;x=3;y=0;y=3$ with $u=0$ on the boundary with mesh length one unit.



Solution:-

Given:- $\nabla u = -10(x^2+y^2-10) \rightarrow (1)$

$h=1 \quad k=1$

WKT,

$$U_{i-1,j} + U_{i+1,j} + U_{i,j-1} + U_{i,j+1} - 4U_{i,j} = h^2 f(ih, jh) = f(i,j)$$

$$U_{i-1,j} + U_{i+1,j} + U_{i,j-1} + U_{i,j+1} - 4U_{i,j} = -10(i^2+j^2+10) \rightarrow (2)$$

Using (2) at D $(i=1, j=2)$, we get

$$U_{0,2} + U_{2,2} + U_{1,1} + U_{1,3} - 4U_{1,2} = -10(15) = -150$$

$$0 + U_2 + U_3 + 0 - 4(U_1) = -150.$$

$$U_2 + U_3 - 4U_1 = -150 \rightarrow (3)$$

Using (2) at E $(i=2, j=2)$

$$U_{1,2} + U_{3,2} + U_{2,1} + U_{2,3} - 4U_{22} = -10(18) = -180.$$

$$U_1 + 0 + U_4 + 0 - 4U_2 = -180$$

$$U_1 + U_4 - 4U_2 = -180 \rightarrow (4)$$

Using (2) at F $(i=1, j=1)$

$$U_{0,1} + U_{2,1} + U_{1,0} + U_{1,2} - 4U_{1,1} = -10(12) = -120$$

$$0 + U_4 + 0 + U_1 - 4U_3 = -120.$$

$$U_4 + U_1 - 4U_3 = -120 \rightarrow (5)$$

Using (2) at G $(i=2, j=1)$

$$U_2 + U_3 - 4U_4 = -150 \rightarrow (6)$$

To find: U_1, U_2, U_3, U_4.

$(5) - (4) \Rightarrow U_4 + U_1 - 4U_3 - U_1 - U_4 + 4U_2 = -120+180$

$$U_2 - U_3 = 15 \rightarrow (7)$$

$(3)+(4) \Rightarrow -15U_2+U_3+7U_4=-870 \rightarrow (8)$.

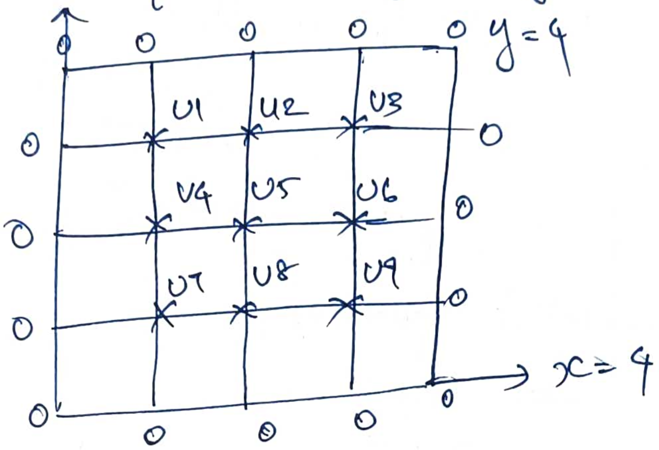
$(6)+(8) \Rightarrow -7U_2+U_3=-510 \rightarrow (9)$

(7) & $(9) \Rightarrow \boxed{U_2=82.5}$ & $\boxed{U_3=67.5}$

& using in $\boxed{U_1=75}$ & $\boxed{U_4=75}$.

(Qn) Solve $\nabla^2 u = 8x^2y^2$ for square mesh given $u=0$ on four boundaries dividing the square into 16 sub-squares of length 1 unit.

Soln:



Solution to parabolic equation:-

To solve:- $U_{xx} = aU_t \rightarrow (1)$

With respect to Boundary conditions, (one-dimensional wave equation)

$$U(0,t)=T_0$$
$$U(l,t)=T_l$$
$$\text{\& } U(x,0)=f(x);\quad 0 \le x < l.$$

We use the explicit formula,

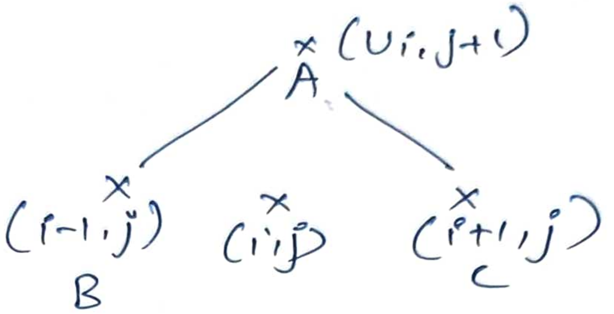
$$U_{i,j+1} = \lambda U_{i+1,j} + (1-2\lambda)U_{i,j} + \lambda U_{i-1,j} \rightarrow (2)$$

$$\text{where } \lambda^{1/2} = \frac{k}{ah^2}$$

where, $\lambda = \frac{1}{2}$ we get $\left(k=\frac{ah^2}{2}\right)$ we get,

$$U_{i,j+1} = \frac{1}{2}\left(U_{i+1,j} + U_{i-1,j}\right) \rightarrow (3)$$

(3) is called Bendes Schmidt recursive equation & is valid if $k = \frac{a}{2}h^2$. This is given schematically by,



Value of U at A = ½ {value of U at B + value of U at C}

(Qn) Solve $\frac{\partial^2 u}{\partial x^2} - 2\frac{\partial u}{\partial t} = 0$. Given $u(0,t) = 0$
$u(4,t) = 0$
$u(x,0) = x(4-x)$

By assuming $h = 1$, find values of u up to t=5.

Soln:-
Using Bendes schmidt equation,

~~k=2~~ $h = 1$

$$k = \frac{2 \cdot 1^2}{2} = 1$$

i ⟶

x	0	1	2	3	4
t 0	0	3	4	3	0
1	0	2	3	2	0
2	0	1.5	2	1.5	0
3	0	1	1.5	1	0
4	0	0.75	1	0.75	0
5	0	0.5	0.75	0.5	0

avg

0.7

(Qn) $\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$ $u(0,t)=0$ $u(4,t)=0$ $u(x,0)=x(2-x)$

Assuming $h=1$, $k=1/2$. Find using Bender Schmidt method.

Find values of u upto $t=5$

soln:- Using Bender Schmidt equation

$h=1$ $a=1$

$k = \frac{ah^2}{2} = \frac{1(1)}{2} = 1/2$

x / t	0	1	2	3	4
0	0	1	0	-3	0
1	0	0	-1	0	0
2	0	-0.5	0	-0.5	0
3	0	0	-0.5	0	0
4	0	-0.25	0	-0.25	0
5	0	0	-0.25	0.	0.

17/10/2022

Solve to parabolic equations by Crout Nicholson method.

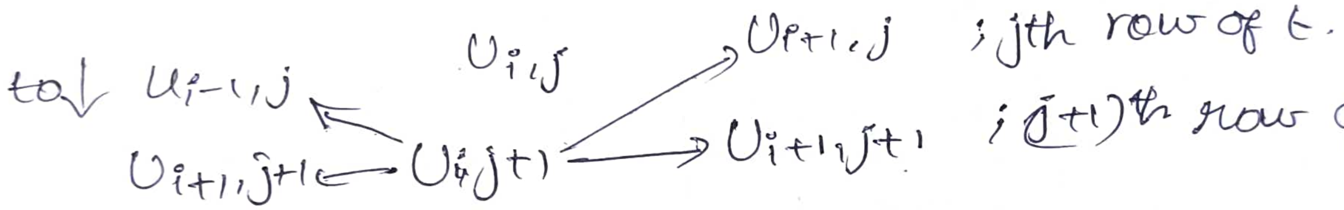
To solve : $U_{xx} = aU_t$.

Given:- $U(0,t) = T_0$, $U(l,t) = T_e$ & $U(x,0) = f(x)$

we use :- with $\lambda = \frac{k}{ah^2}$

$$\lambda(U_{i+1,j+1}) = 2(\lambda+1)U_{i,j+1} = 2(\lambda-1)U_{i,j} + \lambda(U_{i+1,j} + U_{i-1,j}) \longrightarrow (2)$$

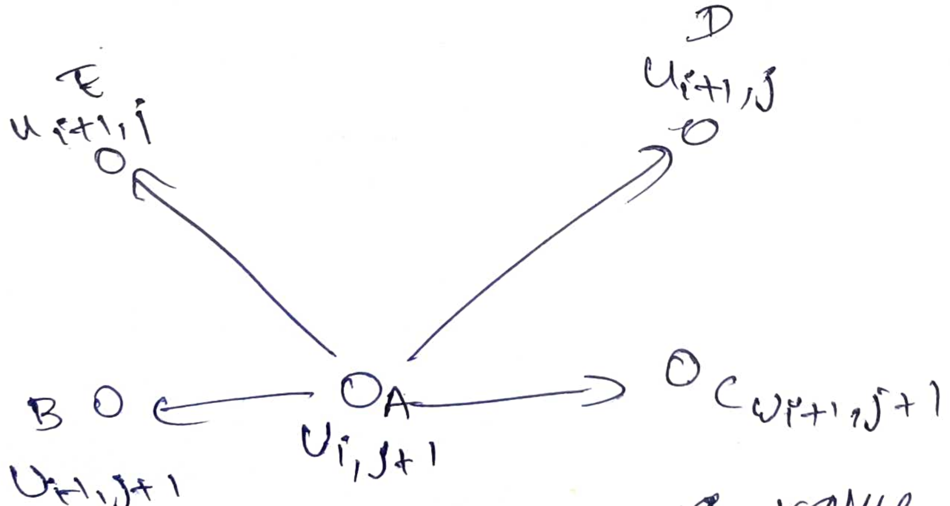
(2) is called the ~~C-N~~ ~~diff~~ Crout Nicholson method. Schematically this is given by.



For simplicity we use $\lambda = 1$, so that (2) reduces

$$U_{i,j+1} = \frac{1}{4}\{U_{i-1,j+1} + U_{i+1,j+1} + U_{i-1,j} + U_{i+1,j}\}$$

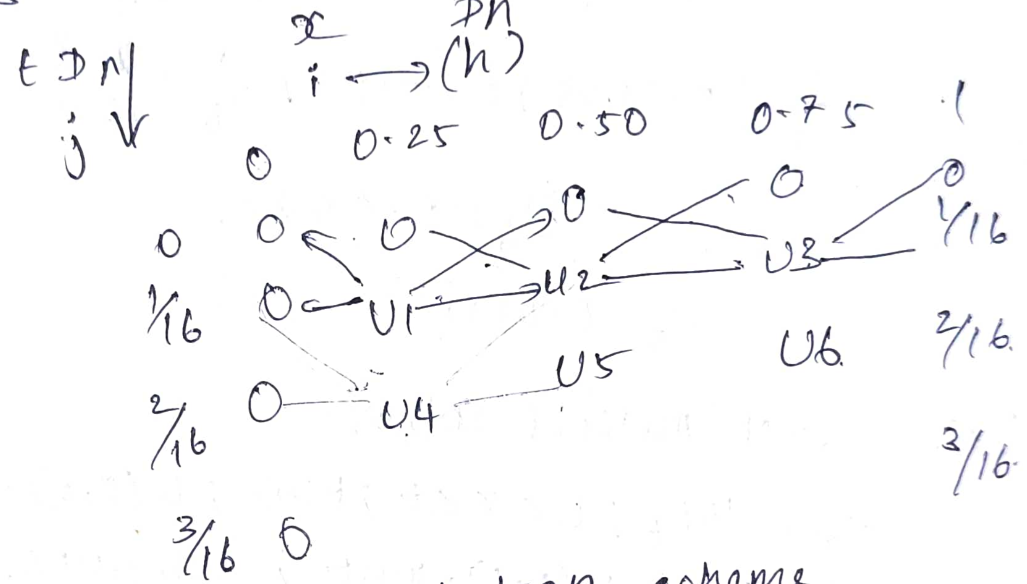
and this is represented by,



(i.e) The value u at A = Average of value of u at B, C, D, E.

Q.1) $U_{xx} = U_t$
Subject to $U(x,0)=0$; $U(0,t)=0$; $u(1,t)=t$ for 2 time steps.

solution:-
~~The~~ Here x ranges from $x=0$ to $x=1$, we get, $a=1$, & so to use Crout's Nicholson method we take $h=1/4$, subject to, $k=ah^2 \Rightarrow k=1/16$
we use.
$$u_{i,j+1} = \frac{1}{4}\{U_{i-1,j+1} + U_{i+1,j+1} + U_{i-1,j} + U_{i+1,j}\}$$
To get the values of u at difference pivotal points as shown below.



Using the Croaks Nicholson scheme,
$$U_1 = \frac{1}{4}\{0+0+0+U_2\} \Rightarrow U_1 = U_2/4$$
$$U_2 = \frac{1}{4}\{0+0+U_1+U_3\} \Rightarrow U_2 = \frac{1}{4}\{U_1+U_3\}$$
$$U_3 = \frac{1}{4}\{0+0+U_2+1/16\} \Rightarrow U_3 = \frac{1}{4}\{U_2 + \frac{1}{16}\}$$
$$4U_1 - U_2 = 0$$
$$U_1 - 4U_2 + U_3 = 0$$
$$U_2 - 4U_3 = -1/16.$$
solving for u_1, u_2 & u_3 we get
$u_1 = 0.0011$ $\quad u_2 = 0.0045$.
$U_3 = 0.0168.$

Using U_1, U_2 & U_3 and following similar steps we find U_4, U_5 & U_6 as follows,

$$U_4 = \frac{1}{4}\{0+0+U_2+U_5\}$$

$$U_5 = \frac{1}{4}\{U_1+U_3+U_4+U_6\}$$

$$U_6 = \frac{1}{4}\{\cancel{0}\, U_2+U_3+\frac{1}{16}+\frac{2}{16}\}$$

solving for U_4, U_5 & U_6, we get.

$$\frac{1}{4}\{U_2+U_5\} = U_4.$$

$$\frac{1}{4}\{0.0045+U_5\} = U_4. \rightarrow (1)$$

$$\frac{1}{4}\{0.0011+0.0168+U_4+U_6\} = U_5.$$

$$\frac{1}{4}\{0.0045+0.0168+\frac{1}{16}+\frac{2}{16}\} = U_6.$$

$$U_6 = 0.0522.$$

$$U_4 = 0.0058.$$

$$U_5 = 0.0191.$$

Q11) Using C-N method solve, $U_{xx} = 16U_t$; $0<x<t$; $t>0$; $u(0,0)=0$; $U(0,1)=0$; $U(1,t)=100t$; compute u for one time step m t on the $h = \frac{1}{4}$.

$\boxed{k = ah^2} \rightarrow$ parabolic.

$\boxed{k = h/a} \rightarrow$ hyperbolic