27.12.2021

1) The Distribution function of the R.V x is given by

$$F(x) = 1-(1+x)e^{-x} \; ; \; x>0$$

(i) <u>To find Probability Density function</u>:- (P.d.f)

WKT, $f(x) = \frac{d}{dx} F(x)$

$$f(x) = \frac{d}{dx}\left(1-(1+x)e^{-x}\right)$$

$$= \frac{d}{dx}\left(1-e^{-x}-xe^{-x}\right)$$

$\frac{d}{dx}(u.v) = u.\frac{dv}{dx}+v.\frac{du}{dx}$

$$= \frac{d}{dx}(1) - \frac{d}{dx}(e^{-x}) - \frac{d}{dx}(xe^{-x})$$

$$= 0 - e^{-x}.(-1) - \left(xe^{-x}.(-1) + e^{-x}.1\right)$$

$$= \cancel{e^{-x}} + xe^{-x} - \cancel{e^{-x}}$$

$$\Rightarrow f(x) = xe^{-x} \; ; \; x>0.$$

(ii) <u>To find Mean</u>:-

$$\text{Mean} = E(X) = \int_{-\infty}^{\infty} x\, f(x)\, dx$$

$$= \int_{-\infty}^{0} x\, f(x)\,dx + \int_{0}^{\infty} x\, f(x)\,dx$$

$$= \int_{-\infty}^{0} x.\cancel{\phi}.dx + \int_{0}^{\infty} x.\, xe^{-x}\,dx$$

(0)

$$E(x) = \int_{0}^{\infty} \underbrace{x^2 e^{-x} dx}$$

$$E(x) = \int_0^\infty \underbrace{x^2}_{u} \underbrace{e^{-x}dx}_{dv}$$

Apply Bernoulli's formula,

$$\int u\,dv = uv - u'v_1 + u''v_2 - u'''v_3 + \cdots$$

$u \to$ diff and $v \to$ Integ.

$$E(x) = \left[x^2\cdot\left(\frac{e^{-x}}{-1}\right) - 2x\cdot\left(\frac{e^{-x}}{1}\right) + 2\cdot\left(\frac{e^{-x}}{-1}\right) - 0\right]_0^\infty$$

$$= 0 - 2\left(\frac{1}{-1}\right) \qquad \left(\because e^{-\infty} = 0,\ e^{\infty} = \infty\right)$$

$$\text{Mean} = E(x) = 2$$

Note:-

$E(x) = \mu_1'$ (i) Mean $(\mu_1) = \mu_1' = E(x)$

$E(x^2) = \mu_2'$ Variance $(\mu_2) = \mu_2' - (\mu_1')^2$

$E(x^3) = \mu_3'$ (ii) Variance $= E(x^2) - (E(x))^2$

. . . .

$E(x^r) = \mu_r'$

TRANSFORMATION OF ONE DIMENSIONAL R.V:-

① Given the R.V x with density function

$$f(x) = \begin{cases} 2x ; & 0 < x < 1 \\ 0 ; & \text{otherwise.} \end{cases}$$

Find the P.d.f of $y = 8x^3$

Sol:- WKT, $f_y(y) = f_x(x)\left|\frac{dx}{dy}\right|$ — ①

$\therefore\ y = 8x^3 \quad x^3$

$\because \; y = 8x^3 = (2x)^3$

$$y^{1/3} = \left((2x)^3\right)^{1/3} = 2x$$

$$\frac{1}{2} \cdot y^{1/3} = x$$

diff w.r.to y

$$\frac{1}{2} \cdot \frac{1}{3} y^{\frac{1}{3}-1} = \frac{dx}{dy}$$

$$\frac{1}{6} y^{-2/3} = \frac{dx}{dy}$$

$$\Rightarrow \left|\frac{dx}{dy}\right| = \frac{1}{6} y^{-2/3}$$

$$① \Rightarrow f_y(y) = f_x(x)\left|\frac{dx}{dy}\right|$$

$$f_y(y) = \begin{cases} 2x \cdot \frac{1}{6} y^{-2/3} ; & 0 < \frac{1}{2} y^{1/3} < 1 \quad \left(\because x = \frac{1}{2} y^{1/3}\right) \\ 0 ; & \text{otherwise} \end{cases}$$

$$f_y(y) = \begin{cases} \cancel{2} \cdot \frac{1}{\cancel{2}} y^{1/3} \cdot \frac{1}{6} y^{-2/3} ; & 0 < y^{1/3} < 2 \\ 0 ; & \text{otherwise} \end{cases}$$

$$f_y(y) = \begin{cases} \frac{1}{6} y^{-1/3} ; & 0 < y < 8 \\ 0 ; & \text{otherwise} \end{cases}$$

PRACTICE PROBLEMS:-

1) A R.V X has a p.d.f $f(x) = \frac{k}{1+x^2}$; $-\infty < x < \infty$

Find (i) the value of k (ii) Distribution fn. of x.

2) A R.V x has the p.d.f $f(x) = \begin{cases} \lambda x e^{-x} & ; x > 0 \\ 0 & ; \text{o.w} \end{cases}$

Find (i) the value of λ (ii) Distribution fn.
(iii) Mean and variance.

3) A Discrete R.V x has a probability function

x:	0	1	2	3	4	5	6	7	8
$P(x)$:	a	$3a$	$5a$	$7a$	$9a$	$11a$	$13a$	$15a$	$17a$

Find (i) the value of a (ii) $P(x < 3)$, $P(x \geq 3)$
(iii) Distribution function.