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UNIT-(2) PROBABILITY DISTRIBUTIONS

(1) BINOMIAL DISTRIBUTION:- (B.D)

Defn:- A Discrete R.V X is said to be B.D if its prob. mass function is,

$$P(X=x) = P(x) = nC_x \, p^x q^{n-x}; \quad x=0,1,2,\dots n$$

p = Prob. of success

q = Prob. of failure

$\therefore p+q=1$

$\Rightarrow p=1-q$ (or) $q=1-p$

Its Mean is np and variance is npq

① The mean of a B.D is 20 and standard deviation is 4. Find the parameters of the Distribution.

Sol:- WKT, The mean of a B.D is $np = 20$ — ①

S.D (Standard deviation) $= \sqrt{\text{Variance}}$

$S.D^2 = \text{variance} = npq$

$\therefore \quad npq = 4^2 = 16$ — ②

$\frac{②}{①} \Rightarrow \quad \frac{\cancel{np}q}{\cancel{np}} = \frac{\cancel{16}\,4}{\cancel{20}\,5}$

$$q = \frac{4}{5}$$

Since $p+q=1 \Rightarrow p = 1-q = 1-\frac{4}{5} = \frac{1}{5}$

Substitute p value in eqn ① we get,

① $\Rightarrow np = 20$

$n\left(\frac{1}{5}\right) = 20$

$n = 100.$

$\therefore$ The parameters of a B.D is,

$n=100, \; p=\frac{1}{5}, \; q=\frac{4}{5}.$

2) In a large number of electric bulbs 10% are

5

2) In a large number of electric bulbs 10% are defective, a random Sample of 20 is taken for inspection. Find the prob. that
(i) all are good bulbs
(ii) Almost 3 defective bulbs
(iii) exactly 3 defective bulbs.

Sol:- Let $p =$ Prob. of defective bulbs $= 10\% = \frac{10}{100} = 0.1$

$q =$ Prob. of good bulbs $= 1 - p = 1 - 0.1 = 0.9$

WKT, The P.m.f of a B.D is,

$$P(X=x) = nC_x\, p^x q^{n-x}; \quad x = 0, 1, 2, \ldots n$$

$$P(X=x) = 20C_x\,(0.1)^x (0.9)^{20-x}; \quad x = 0, 1, 2, \ldots 20$$

(i) P(All are good bulbs) = $P(X=0)$ (It means there is no defective)

$= 20C_0\,(0.1)^0 (0.9)^{20}$

$\therefore$ P(All are good bulbs) $= 0.121$

(ii) P(Almost 3 defective bulbs) $= P(X \le 3)$

$= P(X=0) + P(X=1) + P(X=2) + P(X=3)$

$= 20C_0\,(0.1)^0(0.9)^{20} + 20C_1\,(0.1)^1(0.9)^{19} + 20C_2\,(0.1)^2 + (0.9)^{18} + 20C_3\,(0.1)^3(0.9)^{17}$

$= 0.121 + 0.270 + 0.288 + 0.190$

$\therefore$ P(Almost 3 defective bulbs) $= 0.866$

(iii) P(Exactly 3 defective bulbs) = $P(X=3)$

$= 20C_3\,(0.1)^3(0.9)^{17}$

$= 0.1901$

3) Out of 800 families with 4 Children each. How many families would be expected to have

(i) 2 Boys & 2 Girls

Q: How many families would be expected to have

(i) 2 Boys & 2 Girls

(ii) Atleast one Boy

(iii) Atmost 2 Girls

(iv) children of both Boys and Girls.

Sol:

Consider each child as a trial, $n = 4$

Assuming birth of boy is success and birth of girl is failure.

$$\therefore p = P(\text{boy}) = \frac{1}{2} \text{ and } q = P(\text{girl}) = \frac{1}{2}$$

In B.D, the p.m.f is, $P(X=x) = nC_x p^x q^{n-x}$; $x = 0,1,2,\ldots n$

$$P(X=x) = 4C_x \left(\tfrac{1}{2}\right)^x \left(\tfrac{1}{2}\right)^{n-x} = 4C_x \left(\tfrac{1}{2}\right)^n$$

$$P(X=x) = 4C_x \left(\tfrac{1}{2}\right)^n = 4C_x \left(\tfrac{1}{2}\right)^4$$

(i) $P(2B \,\&\, 2G) = P(X=2)$ (consider p value as x)

$$= 4C_2 \left(\tfrac{1}{2}\right)^4 = 6 \times \frac{1}{16} = \frac{3}{8}$$

$$P(2B \,\&\, 2G) = \frac{3}{8}$$

$\therefore$ The No. of families having 2 Boys and 2 Girls $= \cancel{800}^{100} \times \frac{3}{\cancel{8}} = 300$

(ii) $P(\text{atleast one Boy}) = P(X \geq 1)$

$$= P(X=1) + P(X=2) + P(X=3) + P(X=4)$$

$$= 4C_1 \left(\tfrac{1}{2}\right)^4 + 4C_2 \left(\tfrac{1}{2}\right)^4 + 4C_3 \left(\tfrac{1}{2}\right)^4 + 4C_4 \left(\tfrac{1}{2}\right)^4$$

$$= (4+6+4+1)\left(\tfrac{1}{2}\right)^4$$

$$P(\text{Atleast one Boy}) = \frac{15}{16}$$

$\therefore$ The No. of families having Atleast one Boy $= 800 \times \frac{15}{16} = 750$

(iii) $P(\text{Atmost 2 Girls}) = P(0G \text{ (or) } 1G \text{ (or) } 2G)$

$$= P(4B \text{ (or) } 3B \text{ (or) } 2B)$$

$$= P(x=4) + P(x=3) + P(x=2)$$

$$= 4C_4\left(\frac{1}{2}\right)^4 + 4C_3\left(\frac{1}{2}\right)^3 + 4C_2\left(\frac{1}{2}\right)^4$$

$$= (1+4+6)\left(\frac{1}{2}\right)^4$$

$$P(\text{Atmost 2 Girls}) = \frac{11}{16}$$

$\therefore$ The No. of families having Atmost 2 Girls $= 800 \times \frac{11}{16} = 550$

(iv) $P(\text{children of both Boys and Girls}) = P(1B, 3G \text{ (or) } 2B, 2G \text{ (or) } 3B, 1G)$

$$= P(1B \text{ (or) } 2B \text{ (or) } 3B)$$

$$= P(x=1) + P(x=2) + P(x=3)$$

$$= 4C_1\left(\frac{1}{2}\right)^4 + 4C_2\left(\frac{1}{2}\right)^4 + 4C_3\left(\frac{1}{2}\right)^4$$

$$= (4+6+4)\left(\frac{1}{2}\right)^4$$

$$= 14 \times \frac{1}{16} = \frac{7}{8}$$

$$P(\text{children of both Boys and Girls}) = \frac{7}{8}.$$

$\therefore$ The No. of families having Children of both Boys and Girls $= \cancel{800}^{100} \times \frac{7}{\cancel{8}} = 700$