17.12.2021

Conditional Probability:-

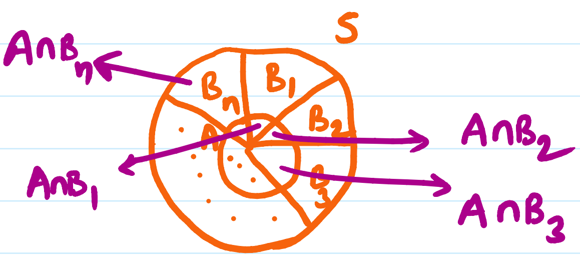
(i) $P(A/B) = P(A|B) = \dfrac{P(A\cap B)}{P(B)}$; $P(B) \neq 0$

Conditional along

(ii) $\|^{ly}$ $P(B/A) = \dfrac{P(B\cap A)}{P(A)}$ (OR) $\dfrac{P(A\cap B)}{P(A)}$; $P(A) \neq 0$

$$\Rightarrow \quad P(A\cap B) = P(B)\cdot P(A/B)$$
$$= P(A)\cdot P(B/A)$$

Theorem on Probability:-



The Smaller (inner) circle represents the event 'A'. It can occur along with $B_1, B_2, B_3, \ldots, B_n$ that are exhaustive and mutually exclusive events.

$$\therefore \quad A = (A\cap B_1)\cup(A\cap B_2)\cup(A\cap B_3)\cup\ldots\cup(A\cap B_n)$$

$$P(A) = P[(A\cap B_1)\cup(A\cap B_2)\cup(A\cap B_3)\cup\ldots\cup(A\cap B_n)]$$

Since $A\cap B_1, A\cap B_2, A\cap B_3, \ldots, A\cap B_n$ all are M.E.E's. hence by axioms on prob.

$$P(A) = P(A\cap B_1) + P(A\cap B_2) + P(A\cap B_3) + \cdots + P(A\cap B_n)$$

$$P(A) = \sum_{i=1}^{n} P(A\cap B_i) \quad \text{—} \ ①$$

By Conditional probability,

$$P(A/B_i) = \frac{P(A\cap B_i)}{P(B_i)} \quad ; \quad P(B_i) \neq 0$$

$$P(A) = \sum_{i=1} P(A \cap B_i) \quad \text{— ①}$$

By Conditional probability,

$$P(A/B_i) = \frac{P(A \cap B_i)}{P(B_i)} \quad ; \quad P(B_i) \neq 0$$

$$\Rightarrow \quad P(A \cap B_i) = P(B_i)\, P(A/B_i) \quad \text{— ②}$$

Substitute eqn. ② in eqn. ① we get,

$$P(A) = \sum_{i=1}^{n} P(B_i)\, P(A/B_i)$$

∴ The total probability is,

$$P(A) = \sum_{i=1}^{n} P(B_i)\, P(A/B_i)$$

BAYE'S THEOREM:-

Statement:-

If $B_1, B_2, \dots B_n$ are a set of exhaustive and M.E.E's associated with a random experiment and A is another event associated with B_i then Baye's thm. States that

$$P(B_i/A) = \frac{P(B_i) \cdot P(A/B_i)}{\sum_{i=1}^{n} P(B_i) \cdot P(A/B_i)}$$

Pf:-

First to prove theorem of total probability,

$$P(A) = \sum_{i=1}^{n} P(B_i)\, P(A/B_i)$$

WKS:-

$$P(B_i/A) = \frac{P(B_i \cap A)}{P(A)} \text{ (or) } \frac{P(A \cap B_i)}{P(A)}$$

$$= \frac{P(B_i) \cdot P(A/B_i)}{\sum_{i=1}^{n} P(B_i)\, P(A/B_i)} \quad (\because P(A \cap B_i) = P(B_i)\, P(A/B_i))$$

$$\therefore P(B_i/A) = \frac{P(B_i)\cdot P(A/B_i)}{\sum_{i=1}^{n} P(B_i)\cdot P(A/B_i)}$$

PROBLEMS:

① In a Bolt factory, Machines B_1, B_2, B_3 Produce 25%, 35%, 40% of the total output respectively. Of their outputs 5%, 4% and 2% are defective bolts respectively. If a bolt is chosen at random from the Combined output and it is defective. What is the prob. that it was produced by i) M/c B_1? (ii) M/c B_2? and (iii) M/c B_3?

Sol: Let 'A' be the defective bolt produced from the Combined output.

Given that,

$P(B_i)$	$P(A/B_i)$	$P(B_i)\cdot P(A/B_i)$
$P(B_1) = 0.25$	$P(A/B_1) = 0.05$	$P(B_1)P(A/B_1) = 0.0125$
$P(B_2) = 0.35$	$P(A/B_2) = 0.04$	$P(B_2)\cdot P(A/B_2) = 0.0140$
$P(B_3) = 0.40$	$P(A/B_3) = 0.02$	$P(B_3)\cdot P(A/B_2) = 0.0080$

$$\sum_{i=1}^{3} P(B_i)\,P(A/B_i) = 0.0345$$

By Baye's theorem,

(i) The prob. of defective bolt produced by M/c B_1 is,

$$P(B_1/A) = \frac{P(B_1)\cdot P(A/B_1)}{\sum_{i=1}^{3} P(B_i)\cdot P(A/B_i)} = \frac{0.0125}{0.0345} = 0.3623$$

(ii) The prob. of defective bolt produced by M/c B_2 is,

$$P(B_2/A) = \frac{P(B_2)\,P(A/B_2)}{\sum_{i=1}^{3} P(B_i)\,P(A/B_i)} = \frac{0.0140}{0.0345} = 0.4058$$

(iii) The prob. of defective bolt produced by M/C B_3 is,

$$P(B_3/A) = \frac{P(B_3)\,P(A/B_3)}{\sum_{i=1}^{3} P(B_i)\,P(A/B_i)} = \frac{0.0080}{0.0345} = 0.2318$$

2) A Bag contains 10 White and 3 Black balls, another bag Contains 3 White and 5 black balls. Two balls are drawn at random from the first bag and placed in a 2nd bag and then one ball is drawn at random from the 2nd bag. What is the prob. that it is a White ball?