Unit 1 revision

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Random Variable

$S \to$ Sample Space H T

$P(S) = 1$ $0 < P < 1$

$P(\phi) = 0$

$P \to$ Success

$q \to 1 - p$

Conditional probability:

C.P of an event B, assuming that A has happened

~~P(A|B)~~ $P(B|A) = \frac{P(A \cap B)}{P(B)}$, $P(A) \neq 0$

Baye's theorem: & Theorem of Total probability.

Theorem of total probability:

If $B_1, B_2, \ldots B_n$ be a set of exhaustive & mutually exclusive events & A is another event associated with B_i then

$$P(A) = \sum_{i=1}^{n} P(B_i) \cdot P(A|B_i).$$

Problem:

In a coin tossing experiment, if the coin shows head, 1 dice is thrown & the result is recorded. But if the coin shows tail, 2 dice are thrown and their sum is recorded. What is the probability that the recorded number will be 2?

Soln.

When a single dice is thrown $p(2) = 1/6$.

when two dice are thrown, the sum will be 2, only if die shows 1.

$P(\text{getting 2 as sum with 2 dice}) = \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$ (since independent)

By theorem of total probability:

$$p(2) = P(H) \times P(2/H) + P(T) \cdot P(2/T)$$

$$= \left(\frac{1}{2} \times \frac{1}{6}\right) + \left(\frac{1}{2} \times \frac{1}{36}\right)$$

$$= \frac{1}{2}\left(\frac{1}{6} + \frac{1}{36}\right) = \frac{1}{2}\left(\frac{7}{36}\right) = \frac{7}{72}.$$

Baye's theorem:

Baye's Theorem:

If $B_1, B_2, \ldots B_n$ be a set of exhaustive & mutually exclusive events associated with a random experiment and A is another event associated with B_i, then

$$P(B_i|A) = \frac{P(B_i) \times P(A|B_i)}{\sum_{i=1}^{n} P(B_i) \times P(A|B_i)}$$

Problem:

There are 3 true coins and 1 false coin with head on both sides. A coin is chosen at random & tossed 4 times. If 'head' occurs all the 4 times, what is the probability that the false coin has been chosen and used?

Total number of coins $= 3+1 = 4$

Soln. $P(T) = 3/4 \qquad P(F) = 1/4$

Let A = Event of getting all heads in 4 tosses.

Then $P(A|T) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{16}$

$P(A|F) = 1$

By Baye's theorem $P(F|A) = \frac{P(F) \times P(A|F)}{P(F) \times P(A|F) + P(T) \cdot P(A|T)}$

$$= \frac{\frac{1}{4} \times (1)}{\left(\frac{1}{4} \times 1\right) + \left(\frac{3}{4} \times \frac{1}{16}\right)} = \frac{\frac{1}{4}}{\frac{1}{4} + \frac{3}{64}} = \frac{\cancel{\frac{1}{4}}}{\cancel{\frac{1}{4}}\left(1+\frac{3}{16}\right)}$$

$$= \frac{1}{\frac{19}{16}} = \frac{16}{19}$$

Problem 2

For a certain binary communication channel, the probability that a transmitted '0' is received as a '0' is 0.95 and the probability that transmitted '1' is received as '1' is 0.90. If the probability that a '0' is transmitted as '0.4' find the probability that

1) a '1' is received. and
2) a '1' was transmitted given that 1 was received.

Soln. Let A = event of transmitting 1

$\bar{A}$ = event of transmitting 0.

B = event of receiving 1

$\bar{B}$ = event of receiving 0.

Given $P(\bar{A}) = 0.4$ $\quad P(B|A) = 0.90$

$P(\bar{B}|\bar{A}) = 0.95$

$P(A) = 0.6$ $\quad P(B|\bar{A}) = 0.05$

By Baye's theorem $$P(A|B) = \frac{P(A)\,P(B|A)}{P(A)\cdot P(B|A) + P(\bar{A})\cdot P(B|\bar{A})}$$

$$= \frac{(0.6)\times(0.90)}{(0.6)(0.90) + (0.4)\times(0.05)} = \frac{27}{28}.$$