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① Karl - Pearson's Correlation co-efficient:-

$$\gamma(x,y) = \frac{Cov(x,y)}{\sigma_x \cdot \sigma_y}$$

② Spearman's Rank - correlation co-efficient:-

$$\rho(x,y) = 1 - \frac{6\Sigma d^2}{n(n^2-1)}$$

Note:-

$$-1 \leq \gamma(x,y) \text{ (or) } \rho(x,y) \leq 1$$

① Obtain the Rank Correlation co-efficient for the following data.

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| x : | 68 | 64 | 75 | 50 | 64 | 80 | 75 | 40 | 55 | 64 |
| y : | 62 | 58 | 68 | 45 | 81 | 60 | 68 | 48 | 50 | 70 |

Sol:-

| x | y | Rank by x | Rank by y | d = Rank by x − Rank by y | $d^2$ |
|---|---|---|---|---|---|
| 68 | 62 | 4 | 5 | −1 | 1 |
| 64 | 58 | 6 | 7 | −1 | 1 |
| 75 | 68 | 2.5 | 3.5 | −1 | 1 |
| 50 | 45 | 9 | 10 | −1 | 1 |
| 64 | 81 | 6 | 1 | 5 | 25 |
| 80 | 60 | 1 | 6 | −5 | 25 |
| 75 | 68 | 2.5 | 3.5 | −1 | 1 |
| 40 | 48 | 10 | 9 | 1 | 1 |
| 55 | 50 | 8 | 8 | 0 | 0 |
| 64 | 70 | 6 | 2 | 4 | 16 |
| | | | | | $\Sigma d^2 = 72$ |

WKT,

The Spearman's Rank Correlation coefficient is,

$$\rho(x,y) = 1 - \frac{6\Sigma d^2}{n(n^2-1)}$$

If the ranks are repeated then the Spearman's Rank Correlation coefficient becomes,

$$\rho(x,y) = 1 - \frac{6}{n(n^2-1)}\left(\Sigma d^2 + \frac{m_1(m_1^2-1)}{12} + \frac{m_2(m_2^2-1)}{12} + \cdots\right)$$

Since in Rank by x Column,

2.5 repeated 2 times. $\therefore$ $m_1 = 2$

6 repeated 3 times. $\therefore$ $m_2 = 3$

also in Rank by y Column,

3.5 repeated 2 times. $\therefore$ $m_3 = 2$

hence the correction Factors (C.F) are,

$$C.F_1 = \frac{m_1(m_1^2-1)}{12} = \frac{2(2^2-1)}{12} = \frac{2(3)}{12} = \frac{6}{12} = \frac{1}{2} = 0.5$$

$$C.F_2 = \frac{m_2(m_2^2-1)}{12} = \frac{3(3^2-1)}{12} = \frac{3(8)}{12} = \frac{24}{12} = 2$$

$$C.F_3 = \frac{m_3(m_3^2-1)}{12} = \frac{2(2^2-1)}{12} = \frac{2(3)}{12} = \frac{6}{12} = \frac{1}{2} = 0.5$$

WKT, $\rho(x,y) = 1 - \frac{6}{n(n^2-1)}(\Sigma d^2 + C.F_1 + C.F_2 + C.F_3)$

$= 1 - \frac{6(72 + 0.5 + 2 + 0.5)}{10(10^2-1)}$ here $n = 10$

$= 1 - \frac{6(75)}{10(99)}$

$\rho(x,y) = 0.545$

2) obtain the Rank correlation Co-efficient for the following data.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| x : | 65 | 66 | 67 | 67 | 68 | 69 | 70 | 72 |
| y : | 67 | 68 | 65 | 68 | 72 | 72 | 69 | 71 |