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① The two regression lines (or) equations are
$8x - 10y + 66 = 0$, $40x - 18y - 214 = 0$.
The variance of x is 9; (i) Find the mean values of x & y.
(ii) Correlation co-efficient between x and y.
(iii) to find the standard deviation of y.

Sol:

Given that the two regression lines are,
$8x - 10y + 66 = 0$ — ① & $40x - 18y - 214 = 0$ — ②

(i) To find the mean values of x and y:-

Since the mean values of x and y ie) $\bar{x}$ and $\bar{y}$ satisfies eqn. ① and ②.

$$8\bar{x} - 10\bar{y} = -66 \quad — ③$$
$$40\bar{x} - 18\bar{y} = 214 \quad — ④$$

③ × 5 ⇒ $\cancel{40\bar{x}} - 50\bar{y} = -330$

④ ⇒ $\underset{(-)}{\cancel{40\bar{x}}} \underset{(+)}{-} 18\bar{y} = \underset{(-)}{214}$

$$\cancel{-} 32\bar{y} = \cancel{-} 544$$

$$\bar{y} = \frac{\cancel{544}^{17}}{\cancel{32}}$$

$$\boxed{\bar{y} = 17}$$

Substitute $\bar{y}$ value in ③ eqn.,

③ ⇒ $8\bar{x} - 10\bar{y} = -66$

$$8\bar{x} - 10(17) = -66$$
$$8\bar{x} - 170 = -66$$
$$8\bar{x} = -66 + 170$$
$$8\bar{x} = 104$$
$$\bar{x} = \cancel{104}\ 13$$

$$8\bar{x} = 104$$

$$\bar{x} = \cancel{\frac{104}{8}}\ 13$$

$$\boxed{\bar{x} = 13}$$

$\therefore$ The mean value of x and y is,

$$\bar{x} = 13 \text{ and } \bar{y} = 17.$$

(ii) <u>To find the correlation co-efficient:-</u>

Let eqn. ① be x on y and
eqn. ② be y on x.

$$① \Rightarrow 8x - 10y + 66 = 0$$

$$8x = 10y - 66$$

$$x = \frac{10y}{8} - \frac{66}{8} \quad —⑤$$

WKT, The reg. line of x on y is,

$$x - \bar{x} = \frac{r\sigma_x}{\sigma_y}(y - \bar{y})$$

(or)

$$x - \bar{x} = b_{xy}(y - \bar{y})$$

$$x = b_{xy}\,y - b_{xy}\cdot\bar{y} + \bar{x} \quad —⑥$$

from eqn. ⑤ & ⑥,

$$\boxed{b_{xy} = \frac{10}{8}}$$

$||^{ly}$, $② \Rightarrow 40x - 18y - 214 = 0$

$$40x - 214 = 18y$$

$$\frac{40}{18}x - \frac{214}{18} = y$$

$$\frac{40}{18}x - \frac{214}{18} = y$$

$$y = \frac{40}{18}x - \frac{214}{18} \quad — (7)$$

WKT, The reg. line of y on x is,

$$y-\bar{y} = \frac{r\sigma_y}{\sigma_x}(x-\bar{x})$$

(or)

$$y-\bar{y} = b_{yx}(x-\bar{x})$$

$$y = b_{yx} \cdot x - b_{yx} \cdot \bar{x} + \bar{y} \quad — (8)$$

from eqn (7) and (8),

$$\boxed{b_{yx} = \frac{40}{18}}$$

WKT,
$$r = \pm\sqrt{b_{xy} \cdot b_{yx}}$$

$$= \pm\sqrt{\frac{\cancel{10}^{5}}{\cancel{8}} \cdot \frac{\cancel{40}^{5}}{\cancel{18}_{9}}}$$

$$= \pm\sqrt{\frac{25}{9}}$$

$$= \pm\frac{5}{3}$$

$$r = \pm 1.67$$

Since this is contradiction to correlation coefficient lies between −1 to 1.

ie) our assumption (1) be x on y and (2) be y on x is wrong.

$\therefore$ (1) be y on x and (2) be x on y is correct.

$$\therefore b_{yx} = \frac{8}{10}, \quad b_{xy} = \frac{18}{40}$$

WKT, $r = \pm\sqrt{b_{xy} \cdot b_{yx}}$

$= \pm\sqrt{\frac{\cancel{18}\,9}{\cancel{40}\,5} \cdot \frac{\cancel{8}}{\cancel{10}\,5}}$

$= \pm\sqrt{\frac{9}{25}}$

$= \pm\frac{3}{5}$

$\boxed{r = \pm 0.6}$

(iii) <u>To find the Standard deviation of y</u> :-

Since $b_{xy} = \frac{18}{40}$

$\frac{r \cdot \sigma_x}{\sigma_y} = \frac{18}{40}$

$\frac{(0.6)\,3}{\sigma_y} = \frac{18}{40}$ (Given that $Var(x) = 9$
$\sigma_x = \sqrt{Var(x)} = \sqrt{9}$
$\sigma_x = 3$)

$\frac{1.8}{\sigma_y} = \frac{18}{40}$

$\frac{\cancel{1.8}^{0.1} \times 40}{\cancel{18}} = \sigma_y$

$4 = \sigma_y$

$\therefore$ the Standard deviation of y is 4.

2) Heights of fathers and their Sons are given in Cm's. Find the two regression lines and Correlation co-efficient between heights of fathers and their Sons and hence find height of Son when height of father is 154 Cm.

Height of fathers (x)	:	150	152	155	157	160	161	164	166
Height of Sons (y)	:	154	156	158	159	160	162	161	164.

Height of Sons (y) : 154 156 158 159 160 162 161 164.

Soln:

X	Y	$X-\bar{X}$	$Y-\bar{Y}$	$(X-\bar{X})(Y-\bar{Y})$	$(X-\bar{X})^2$	$(Y-\bar{Y})^2$
150	154	-8	-5	40	64	25
152	156	-6	-3	18	36	9
155	158	-3	-1	3	9	1
157	159	-1	0	0	1	0
160	160	2	1	2	4	1
161	162	3	3	9	9	9
164	161	6	2	12	36	4
166	164	8	5	40	64	25
1265	1274			124	223	74

$$b_{XY} = \frac{\Sigma(X-\bar{X})(Y-\bar{Y})}{\Sigma(Y-\bar{Y})^2} = \frac{124}{74} = 1.675$$

$$b_{YX} = \frac{\Sigma(X-\bar{X})(Y-\bar{Y})}{\Sigma(X-\bar{X})^2} = \frac{124}{223} = 0.556$$

i) X on Y is,

$$X-\bar{X} = b_{XY}(Y-\bar{Y})$$

$$X-158 = 1.675(Y-159)$$

$$= 1.675Y - 266.325 + 158$$

$$X = 1.675Y - 108.325$$

(iii) Y on X is

$$Y - \bar{Y} = b_{YX}(X - \bar{X})$$

$$Y - 159 = 0.556(X - 158)$$

$$Y - 159 = 0.556X - 87.848$$

$$Y = 0.556X - 87.848 + 159$$

$$= 0.556X + 71.152.$$

(i) $r = \pm\sqrt{b_{XY} \cdot b_{YX}} = \pm\sqrt{1.675 \times 0.556}$

$$= \pm\sqrt{0.9313}$$

$$= \pm 0.965$$

(iii) $Y = 0.556(154) + 71.152$

$$Y = 156.776$$

$$\boxed{\therefore Y = 156.8 \text{ cm (approx)}}$$