→ In a continuous r.v the probability density fun (pdf) is given by $f(x) = kx(2-x); 0<x<2$. 1)

i) find K, mean & variance. ii) find c.d.f

sol. If $f(x)$ is pdf, then $\int_{-\infty}^{\infty} f(x)\,dx = 1$.

$$\int_0^2 K\cdot x(2-x)\,dx = 1$$

$$K\int_0^2 (2x-x^2)\,dx = 1$$

$$K\left[\frac{\cancel{2}x^2}{\cancel{2}} - \frac{x^3}{3}\right]_0^2 = 1$$

$$K\left[\left((2)^2 - \frac{(2)^3}{3}\right) - 0\right] = 1$$

$$K\left[4 - \frac{8}{3}\right] = 1$$

$$\frac{4K}{3} = 1$$

$$\boxed{K = \frac{3}{4}}$$

∴ The p.d.f is $f(x) = \frac{3}{4}(2x-x^2); 0<x<2$.

i) <u>To find mean</u> $= E(x)$

$$E(x) = \int_{-\infty}^{\infty} x f(x)\,dx$$

$$= \int_0^2 x\cdot\frac{3}{4}(2x-x^2)\,dx$$

$$= \frac{3}{4}\int_0^2 (2x^2-x^3)\,dx$$

$$= \frac{3}{4}\left[\frac{2x^3}{3} - \frac{x^4}{4}\right]_0^2$$

$$= \frac{3}{4}\left[\left(\frac{16}{3} - \frac{16}{4}\right) - 0\right] = \frac{3}{4}\left[\frac{64-48}{12}\right]$$

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$$= \frac{\cancel{3}}{\cancel{4}}\left[\frac{\cancel{16}^{4}}{1\cancel{2}_4}\right] = \frac{4}{4} = 1$$

To find variance

$$E[x^2] = \int_{-\infty}^{\infty} x^2 f(x)\,dx$$

$$= \int_0^2 x^2 f(x)\,dx$$

$$= \frac{3}{4}\int_0^2 (2x^3 - x^4)\,dx$$

$$= \frac{3}{4}\left[\frac{2x^4}{4} - \frac{x^5}{5}\right]_0^2$$

$$= \frac{3}{4}\left[\frac{32}{4} - \frac{32}{5}\right]$$

$$= \frac{3}{4} \times 32\left[\frac{1}{20}\right]$$

$$= \frac{6}{5}$$

$$Var(x) = E(x^2) - [E(x)]^2 = \frac{6}{5} - 1 = \frac{1}{5}$$

ii) To find c.d.f $F(x)$

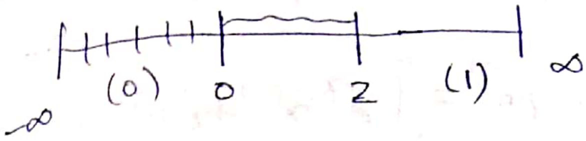
$$F(x) = P(X \le x)$$

$$= \int_{-\infty}^{x} f(x)\,dx$$

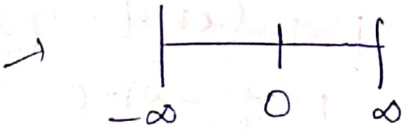
$$= \int_0^x \frac{3}{4}(2x - x^2)\,dx$$

$$= \frac{3}{4}\left[\frac{\cancel{2}x^2}{\cancel{2}} - \frac{x^3}{3}\right]_0^x$$

$$= \frac{3}{4}\left[x^2 - \frac{x^3}{3}\right]; \quad 0 < x < 2$$



$$F(x) = \begin{cases} 0 & ; x < 0 \\ \frac{3}{4}\left(x^2 - \frac{x^3}{3}\right) & ; 0 < x < 2 \\ 1 & ; x > 2 \end{cases}$$



$$F(-\infty) = 0$$

$$F(\infty) = 1$$

→ check whether $f(x) = 3x^2,\ 0 < x < 1$ is p.d.f or not.

Now $\int_{-\infty}^{\infty} f(x)dx = \int_0^1 3x^2 dx = \left[3 \times \frac{x^3}{3}\right]_0^1$

$$= 1 - 0$$

$$\int_{-\infty}^{\infty} f(x)dx = 1$$

$\Rightarrow f(x)$ is p.d.f.

→ A random variable x has the p.d.f $f(x)$ is given by

$$f(x) = \begin{cases} c\,x e^{-x} & ; x > 0 \\ 0 & ; x \le 0 \end{cases}$$

i) Find the value of c.

ii) Find the cummulative distribution function.

Sol:

Since $f(x)$ is p.d.f

$$\Rightarrow \int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\int_0^{\infty} c \cdot x e^{-x} dx = 1$$

$$c\int_0^\infty e^{-x} x^{2-1}\,dx = 1$$

$$\Gamma n = \int_0^\infty e^{-x} x^{n-1}\,dx.$$

$$c\,\Gamma 2 = 1$$

$$c\,\lfloor 1 = 1$$

<u>Note</u>: $e^{-\infty} = \frac{1}{e^{\infty}} = \frac{1}{\infty} = 0.$

$$c\times 1 = 1$$

$$\boxed{c=1}$$

(or)

$$c\int_0^\infty x e^{-x}\,dx = 1$$

<u>properties of c.d.f</u>

i) $f(-\infty)=0$

ii) $f(\infty)=1$

iii) $f(x)=F'(x)$

$u = x \quad | \quad v = e^{-x}$

$u' = 1 \quad | \quad v_1 = e^{-x}$

$u'' = 0 \quad | \quad v_2 = e^{-x}$

$$c\left[uv_1 - u'v_2\right] = 1$$

$$c\left[-xe^{-x} - 1\,e^{-x}\right]_0^\infty = 1$$

$$c\left[(0-0)-(0-1)\right] = 1$$

$$\boxed{c=1}$$

∴ The p.d.f is $f(x) = 1\times x\times e^{-x}$

$= xe^{-x};\ x>0$

To find c.d.f $F(x) = P(X\le x)$

$$= \int_{-\infty}^{\infty} f(x)\,dx$$

$$= \int_0^x xe^{-x}\,dx$$

$$= \left[uv_1 - u'v_2 + \cdots\right]_0^x$$

$$= \left[-xe^{-x} - e^{-x}\right]_0^x$$

$$= \left\{\left[xe^{-x} + e^{-x}\right] - (0+1)\right\}$$

$= 1 - e^{-x}(1+x) \; ; \; x \geq 0$

→ The distribution function of R.V x is given by

$f(x) = 1 - (1+x)e^{-x} \; ; \; x \geq 0$

i) Find the probability density fun (p.d.f).

ii) Find the mean and Variance.

Sol: Given $F(x) = 1 - (1+x)e^{-x} \; ; \; x \geq 0$

W.K.T, $f(x) = F'(x)$

$= 0 - \{-(1+x)e^{-x} + e^{-x}(0+1)\}$

$= (1+x)e^{-x} - e^{-x}$

$= e^{-x} + xe^{-x} - e^{-x}$

∴ p.d.f is $f(x) = xe^{-x} \; ; \; x \geq 0$

To find mean $= E(x)$

$$= \int_{-\infty}^{\infty} x f(x) dx$$

$$= \int_{0}^{\infty} x . x e^{-x} dx$$

$$= \int_{0}^{\infty} e^{-x} x^2 dx \quad \left(\because \int_{0}^{\infty} e^{-x} x^{n-1} dx = \Gamma n \right)$$

$$= \int_{0}^{\infty} e^{-x} x^{3-1} dx$$

$$= \Gamma 3 = \lfloor 2 = 1 \times 2 = 2$$

If x discrete $E(x) = \sum_{x=-\infty}^{\infty} x p(x)$

If x continuous $E(x) = \int_{-\infty}^{\infty} x f(x) dx$

To find Variance of $x = Var[x]$

$Var[x] = E[x^2] - [E(x)]^2$

$= 6-(2)^2$

$= 6-4 = 2$

$$E[x^2] = \int_{-\infty}^{\infty} x^2 f(x)\,dx$$

$$= \int_0^{\infty} x^2 \cdot x e^{-x}\,dx$$

$$= \int_0^{\infty} e^{-x} x^3\,dx = \int_0^{\infty} e^{-x} x^{4-1}\,dx = \Gamma 4$$

$$= \underline{|3} = 1\times 2\times 3 = 6$$

→ A continuous R.V x has the p.d.f $f(x) = \frac{K}{1+x^2}$; $-\infty < x < \infty$

i) find K, dist funcn of x.

ii) Evaluate $P(x \le 0)$

Sol. Given $f(x) = \frac{K}{1+x^2}$; $-\infty < x < \infty$.

Since $f(x)$ is p.d.f

$$\Rightarrow \int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$K\int_{-\infty}^{\infty} \frac{1}{1+x^2}\,dx = 1$$

$$K\left[\tan^{-1}(x)\right]_{-\infty}^{\infty} = 1$$

$$K\left[\frac{\pi}{2} - \tan^{-1}(-\infty)\right] = 1$$

$$K\left[\frac{\pi}{2} + \tan^{-1}(\infty)\right] = 1$$

$$K\left[\frac{\pi}{2} + \frac{\pi}{2}\right] = 1$$

$$K[\pi] = 1 \Rightarrow K = \frac{1}{\pi}$$

∴ p.d.f is $f(x) = \frac{1}{\pi}\left(\frac{1}{1+x^2}\right)$; $-\infty < x < \infty$

To find c.d.f

$$F(x) = P(X \le x)$$