Unit II Revison

29 January 2022 09:14

Probability Distribution

- Discrete
  1) Binomial
  2) poisson
  3) Geometric
- Continuous
  1) Uniform
  2) Exponential
  3) normal

$p(x)$ = p.m.f — probability mass function

$f(x)$ = p.d.f — " density "

$F(x)$ = c.d.f — Cumilative distribution function

Mean $E[X] = \sum x_i \, p(x_i)$ $\qquad \sum_i p(x_i) = 1$

Variance $V[X] = E(X^2) - \{E(X)\}^2$

Std deviation $= \sqrt{\text{Var}X}$

① $E(aX+b) = aE(X)+b$

② $\text{Var}(aX+b) = a^2\,\text{Var}(X)$

Binomial distribution: p.m.f

p – success

$q = 1-p \rightarrow$ failure

$$P(X=x) = p(x) = \begin{cases} {}^nC_x \, p^x q^{n-x} & x = 0, 1, 2 \dots n \\ 0 & \text{otherwise} \end{cases}$$

Mean $= np$

Variance $= npq$

Std. Deviation $= \sqrt{npq}$

2) Poisson Distribution: p.m.f

$$p(X=x) = \begin{cases} \dfrac{e^{-\lambda}\lambda^x}{x!} & x = 0, 1, 2 \dots \\ 0 & \text{otherwise} \end{cases}$$

Mean $= E(x) = \lambda$

Variance $= V[X] = \lambda$

3) Geometric distribution:

$$P(X=x) = (1-p)^{x-1} p = q^{x-1} \cdot p \qquad x = 1, 2 \dots$$

Memoryless property of geometric distribution:

positive

... for integers

If X has a geometric distribution, then for any two integers m & n

$$P[X > m+n \mid X > m] = P[X > n].$$

4) <u>Uniform distribution</u> [Rectangular distribution].

X – RV ctus distribution (a, b)

$$f(x) = \begin{cases} \frac{1}{b-a} & , a < x < b \\ 0 & \text{otherwise} \end{cases}$$

(–a, a)
$$f(x) = \begin{cases} \frac{1}{2a} & -a < x < a \\ 0 & \text{otherwise} \end{cases}$$

$$\text{Mean} = \frac{a+b}{2}$$
$$\text{Variance} = \frac{(b-a)^2}{12}.$$

5) <u>Exponential Distribution:</u> p.d.f

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 , \lambda > 0 \\ 0 & \text{otherwise}. \end{cases}$$

$$\text{Mean} = 1/\lambda$$
$$\text{Variance} = 1/\lambda^2$$

Distribution function: $F(x) = \int_{-\infty}^{x} f(x)\,dx$

<u>Memoryless property of exponential distribution:</u>

If X is exponentially distributed, then

$$P(X > s+t \mid X > s) = P(X > t).$$

for any s, t > 0

6) <u>Normal Distribution:</u>

p.d.f
$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

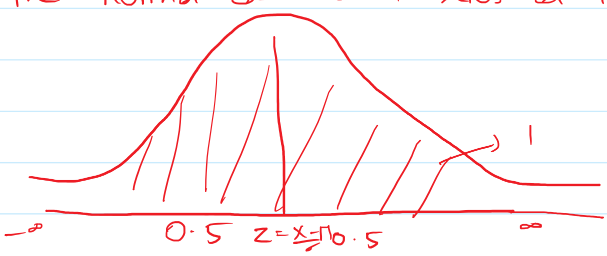
Mean – μ

Variance – σ

Standard normal variate $Z = \dfrac{X-\mu}{\sigma}$.

Properties:

1. The normal distribution is a symmetrical distribution and the graph is bell shaped
2. The mean of the normal distribution lies at the centre of the normal curve



Problem 1:

The number of monthly breakdown of a computer is a r.v having a poisson distribution with mean equal to 1.8.

Find the probability that this computer will function for a month

a) without a breakdown
b) with only one breakdown
c) with atleast 1 breakdown.

Soln Given mean $= \lambda = 1.8$

Let X denote the no. of breakdowns of a computer in a month.

$\therefore$ probability distribution $P(X=x) = \dfrac{e^{-\lambda}\lambda^{x}}{x!} = \dfrac{e^{-1.8}(1.8)^{x}}{x!}$

a) $P(\text{Without breakdown}) = P(X=0) = \dfrac{e^{-1.8}(1.8)^{0}}{0!} = e^{-1.8} = 0.1653$.

b) $P(\text{With one breakdown}) = P(X=1) = \dfrac{e^{-1.8}(1.8)^{1}}{1!} = e^{-1.8} \times 1.8$

$= 0.2975$.

c) $P(\text{with atleast 1 breakdown}) = P(X \geq 1) = 1 - P(X<1)$

$= 1 - P(X=0)$

$= 1 - 0.1653 = 0.8347$.

Problem 2:

Electric trains on a certain line run every half an hour between midnight & 6in in the morning. what is the probability that a man entering the station at a random time during this period will have to wait atleast 20 minutes?

Soln. Let the r.v X denote the waiting time (in minutes) for the next train.

Given that a man arrives at the station at random X is distributed Uniformly on (0, 30) with

$$\text{density } f(x) = \frac{1}{30} \qquad 0 < x < 30$$
$$= 0 \qquad \text{otherwise}$$

Thus the probability that he has to wait for atleast 20 mins

$$P(X \geq 20) = \int_{20}^{30} f(x)\,dx = \int_{20}^{30} \frac{1}{30}\,dx = \frac{1}{30}[x]_{20}^{30} = \frac{1}{30}[30-20] = \frac{10}{30} = \frac{1}{3}$$

Problem 3:

The time in hours required to repair a machine is exponentially distributed with parameter $\lambda = 1/2$.

1) what is the probability that the repair time exceeds 2h.
2) what is the conditional probability that a repair takes atleast 10h given that its duration exceeds 9h?

Solution: $\lambda = 1/2$

Let x denotes the time to repair the machine.

$$\text{density function } f(x) = \lambda e^{-\lambda x} = \frac{1}{2} e^{-x/2} \qquad x \geq 0$$

$$1)\ P(X>2) = \int_2^\infty f(x)\,dx = \int_2^\infty \frac{1}{2} e^{-x/2}\,dx = \frac{1}{2}\left[\frac{e^{-x/2}}{-1/2}\right]_2^\infty$$

$$= \left[-e^{-x/2}\right]_2^\infty = e^{-1} = 1/e = 0.3679.$$

2) $P(X \geq 10 \mid X > 9) = P(X \geq 9+1 \mid X > 9) = P(X > 1)$.

Memoriless $P(X > s+t \mid X > s) = P(X > t)$

$$= \int_1^\infty f(x)\,dx$$

$$= \int_1^\infty \frac{1}{2} e^{-x/2}\,dx$$

$$= \frac{1}{2}\left[\frac{e^{-x/2}}{-1/2}\right]_1^\infty$$

$$= \left[-e^{-x/2}\right]_1^\infty$$

$$= e^{-1/2} = 0.6065.$$

Problem 4:

If the density function of a cts r.v X is given by

$$f(x) = \begin{cases} ax & 0 \leq x \leq 1 \\ a & 1 \leq x \leq 2 \\ 3a - ax & 2 \leq x \leq 3 \\ 0 & \text{otherwise}. \end{cases}$$

1) Find the value of a.

Since $f(x)$ is a p.d.f then

$$\int_{-\infty}^{\infty} f(x)\,dx = 1 \quad \text{(ie)} \quad \int_0^3 f(x)\,dx = 1$$

$$\text{(ie)} \int_0^3 f(x)\,dx = 1$$

$$= \int_0^1 f(x)\,dx + \int_1^2 f(x)\,dx + \int_2^3 f(x)\,dx = 1$$

$$= \int_0^1 ax\,dx + \int_1^2 a\,dx + \int_2^3 (3a - ax)\,dx = 1$$

$$\Rightarrow a\left[\frac{x^2}{2}\right]_0^1 + a[x]_1^2 + 3a[x]_2^3 - a\left[\frac{x^2}{2}\right]_2^3 = 1$$

$$\Rightarrow \frac{a}{2} + a + \left[3a - \left(\frac{9a}{2} - 2a\right)\right] = 1$$

$$\Rightarrow \quad 2a = 1 \Rightarrow a = 1/2$$