DDA Line Drawing Algorithm:

- (Digital Differential Analyzer)

- In Screen Consist of pixel. Every pixel Consist of (x, y) values.

DDL Algm used to find the intermediate point.

$$y = mx + c$$

$m \rightarrow$ slope

replaced with

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad y_2 \rightarrow y_{k+1}, \quad y_1 \rightarrow y_k.$$

Slope $= (x_1, y_1)\ (x_2, y_2)$.

if the 2 points are,

(x_k, y_k) then next point is (x_{k+1}, y_{k+1}) the slope is

$$\boxed{m = \frac{y_{k+1} - y_k}{x_{k+1} - x_k}}$$

$$m = \frac{y_{k+1} - y_k}{x_{k+1} - x_k}$$

x_k → present point

x_{k+1} → next point

To find intermediate point,

In DDA algm to remember 3 cases:

<u>case: 1</u>

if $(m < 1)$

x — unit interval.

changes in

→ $x_{k+1} = x_k + 1$

suffix

$$m = \frac{y_{k+1} - y_k}{\cancel{x_{k+1}} - \cancel{x_k}}$$

1

$$m = \frac{y_{k+1} - y_k}{1}$$

$y_{k+1} = y_k + m$

$$\boxed{y_{k+1} = y_k + m}$$

Case : 2

$m > 1$

y — Unit interval

$$\boxed{y_{k+1} = y_k + 1}$$

$$m = \frac{y_{k+1} - y_k}{x_{k+1} - x_k}$$

$$m = \frac{1}{x_{k+1} - x_k}$$

$$x_{k+1} - x_k = 1/m$$

$$\boxed{x_{k+1} = x_k + 1/m}$$

Case : 3

$m = 1$

x & y → Unit interval.

$$x_{k+1} = x_k + 1$$

$$y_{k+1} = y_k + 1.$$

Base on those formula to find out remaining points.

DDA:

1. Calculate slope, m.

2. if $m<1$

x – changes in unit interval

y – moves with deviation.

$$(x_{k+1}, y_{k+1}) = (x_k+1, y_k+m).$$

3. if $(m>1)$

x moves with deviation

y moves in unit intervals

$$(x_{k+1}, y_{k+1}) = (x_k + 1/m, y_k+1)$$

4. if $m=1$

x & y moves in unit intervals. $(x_{k+1}, y_{k+1}) =$

$$(x_k+1, y_k+1).$$

Example:

(0, 0) & (4, 5) → end point.
↳ starting point.

$$m = \frac{5-0}{4-0} = 5/4.$$

$$5/4 > 1.$$

y → moves in unit interval.

$y \rightarrow x_k + 1/m$

$$(x_{k+1}, y_{k+1}) = (x_k + 1/m, y_k + 1)$$

$$\boxed{\begin{array}{l} x_{k+1} = x_k + 1/m. \\ y_{k+1} = y_k + 1. \end{array}}$$ → calcule 1/m.

$$m = 5/4.$$

$$1/m = \frac{4}{5} = \boxed{0.8}$$

x	y	x-plot	y-plot	(x,y)
0	0	0	0	(0,0)
0.8 (0+.8)	1	Round 1 0.8	1	(1,1)
1.6 (.8+.8)	2	Round 2 upper boundary	2	(2,2)
2.4	3	2	3	(2,3)
3.2	4	3	4	(3,4)
4	5	4	5	(4,5)

0,0 $\boxed{(1,1)\ (2,2)\ (2,3)\ (3,4)}$ (4,5)

Draw back:

* Every iteration apply the Round Function.
* Increase the Computation.

Bresenham Line Drawing Algorithm.

(i) calculate the Slope.

$m \rightarrow \frac{\text{Starting point}}{\text{End Point}}$

$\underline{m < 1}$

x - Unit Interval

$x_{k+1} = x_k + 1$

$y_{k+1} = ?$

$\underline{m > 1}$

y - Unit Interval

$y_{k+1} = y_k + 1$

$x_{k+1} = ?$

$\underline{m = 1}$

$x_{k+1} = x_k + 1$

$y_{k+1} = y_k + 1.$

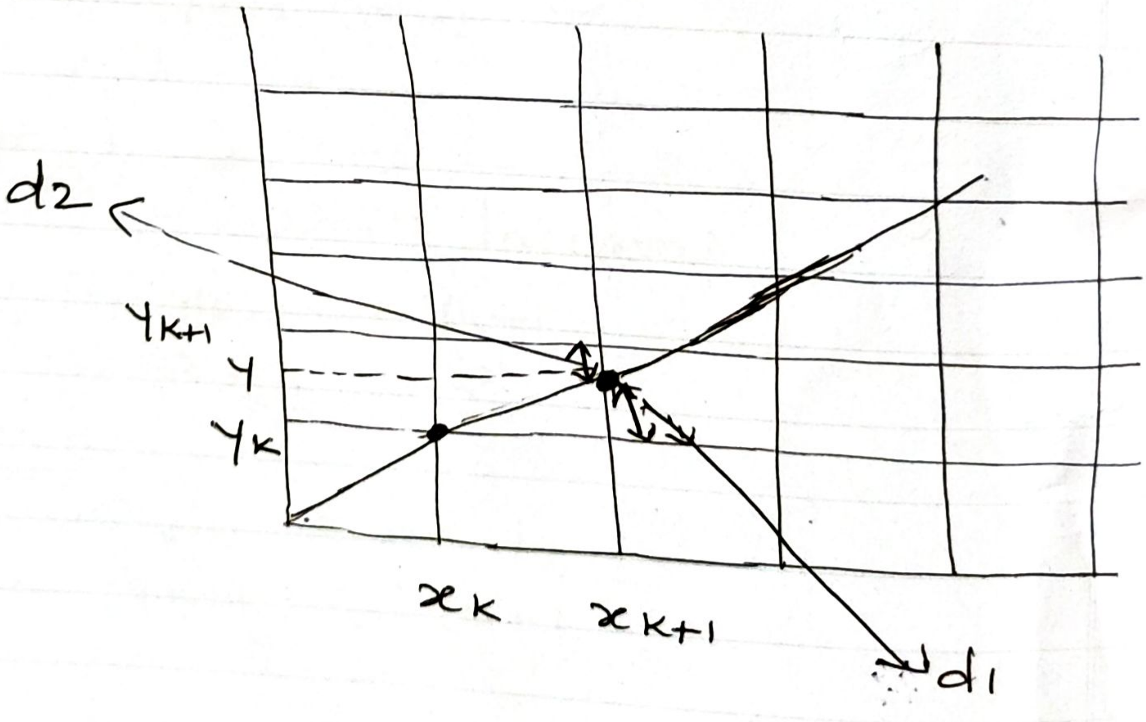
Case ($m<1$)

Line Formula is,

$$y = mx + c$$

$$\boxed{y = m(x_k + 1) + c} \quad \text{———} \quad (1)$$

Find the y value. x_{k+1}.



$$\boxed{\begin{array}{l}\text{Decision Paramater} \\ = \Delta x\,(d_1 - d_2)\end{array}}$$

d_1 = distance between y and y_k.

d_2 = Distance between y_{k+1} and y.

$$y = m\,(x_k + 1) + c)$$

$$P_K = \Delta x\,(d_1 - d_2)$$

To Find $\Delta x\,(d_1 - d_2)$

$$d_1 = y - y_k$$

$$d_1 = m(x_k - 1) + c - y_k.$$

$$d_2 = y_{k+1} - y.$$

substitute equation ① in d_2.

$$d_2 = y_{k+1} - m(x_{k+1}) - c)$$

$$d_1 - d_2 = m(x_{k+1}) + c - y_k - y_k - 1 + m(x_{k+1}) + c$$

$$\boxed{d_1 - d_2 = 2m(x_k+1) - 2y_k + 2c - 1}$$

$$\Delta x (d_1 - d_2) =$$

$$\Delta x \left[2 \cdot \frac{\Delta y}{\Delta x}(x_k+1) - 2y_k + 2c - 1 \right]$$

$$\Delta x (d_1 - d_2) = 2\Delta y x_k + 2\Delta y - 2\Delta x y_k + 2\Delta x c - \Delta x$$

$$\boxed{P_k = 2\Delta y x_k + 2\Delta y - 2\Delta x y_k + \Delta x(2c-1)}$$

$$P_{k+1} = 2\Delta y x_{k+1} + 2\Delta y - 2\Delta x y_{k+1} + \Delta x(2c-1).$$

To Find the next decision parameter.

$$P_{k+1} - P_k = 2\Delta y\, x_{k+1} + \cancel{2\Delta y} - 2\Delta x\, y_{k+1} + \Delta x(2\cancel{c}-1)$$

$$- 2\Delta y\, x_k - \cancel{2\Delta y} + 2\Delta x\, y_k - \Delta x(2\cancel{c}-1)$$

$$P_{k+1} - P_k = 2\Delta y(x_{k+1} - x_k) - 2\Delta x(y_{k+1} - y_k)$$

$$P_{k+1} - P_k = 2\Delta y (x_{k+1} - x_k)$$
$$- 2\Delta x (y_{k+1} - y_k)$$

$$\boxed{P_{k+1} = P_k + 2\Delta y - 2\Delta x (y_{k+1} - y_k)}$$

$\therefore \quad y = m(x_k + 1) + c$ —— (I)

$P_k = \Delta x (d_1 - d_2)$.

$P_k = 2\Delta y x_k + 2\Delta y - 2\Delta x y_k +$
$\Delta x (2c - 1)$.

To Find P_K,

$$P_K = 2\Delta y x_K + 2\Delta y - 2\Delta x y_K +$$
$$\Delta x(2(y - m(x) - 1).$$

$$\boxed{\begin{array}{c} y = mx + c \\ c = y - mx \end{array}}$$

$$2\Delta y x_K + 2\Delta y - 2\Delta x y_K +$$
$$\Delta x\left(2y - 2 \cdot \frac{\Delta y}{\Delta x} \cdot x - 1\right)$$

$= 2\Delta y x_k + 2\Delta y - 2\Delta x y_k +$

$2\Delta x y - 2\Delta y x - \Delta x.$

Substitute x_k and y_k.

$P_k = 2\Delta y x_k + 2\Delta y - 2\Delta x y_k +$

$2\Delta x y_k - 2\Delta y x_k - \Delta x.$

$$\boxed{P_k = 2\Delta y - \Delta x} \quad \text{———— } (2)$$

$P_{k+1} = P_k + 2\Delta y - 2\Delta x (y_{k+1} - y_k)$

if $(P_k \geq 0)$,

$x_{k+1} = x_k + 1$

$y_{k+1} = y_k + 1$.

Next point $(x_k + 1, y_k + 1)$.

if $(P_k < 0)$

$x_k = x_k + 1$.

↳ Unit Interval.

$y_{k+1} = y_k$.

↳ y moves in previous value.

if ($P_k < 0$)
Select Previous value.
(Lower boundary).

if ($P_k \geq 0$)
Select upper value.

Summerize,

1. calculate the distance.
(i) Distance d_1
distance between
Y and Y_k.
(ii) Distance d_2
Distance between
Y and Y_{k+1}

2. Substitute both distance,

$$y = m(x_k+1) + c$$

3. To Find P_k

(i) Decision Parameter

(ii) Next Decision Parameter

4. To Solve the problem based on the decision parameter.

$P_k > 0$ and $P_k < 0$.

__Example:__

$$P_K = 2\Delta y - \Delta x$$

$$P_{K+1} = P_K + 2\Delta y - 2\Delta x\,(y_{K+1} - y_K)$$

if $(P_K \geq 0)$

$y_{K+1} = y_K + 1$.

NC $= (x_K + 1,\ y_K + 1)$.

if $(P_K < 0)$

$y_{K+1} = y_K$.

Next Coordinate $= (x_K + 1,\ y_K)$.

if $(m<1)$

$P_K = 2\Delta y - \Delta x$.

$P_{K+1} = P_K + 2\Delta y - 2\Delta x \, (y_{K+1} - y_K)$.

$P_K \geq 0 \Rightarrow y_{K+1} = y_K + 1$.

$NC = (x_{K+1}, y_{K+1})$

$P_K < 0 \Rightarrow y_{K+1} = y_K$.

$NC = (x_{K+1}, y_K)$.

if $(m > 1)$

$$P_K = 2\Delta x - \Delta y$$
$$P_{K+1} = P_K + 2\Delta x - 2\Delta y (x_{K+1} - x_K).$$

$$P_K \geq 0 \Rightarrow x_{K+1} = x_K + 1$$
$$P_K < 0 \Rightarrow x_{K+1} = x_K.$$

$$P_K \geq 0 \Rightarrow NC = (x_K + 1), (y_K + 1).$$
$$P_K < 0 \Rightarrow NC = (x_K, y_K + 1).$$

$(35, 40)$ $(43, 45)$.

$$m = \frac{5}{8} = 0.6 \quad \boxed{m < 1}$$

$$P_K = 2\Delta y - \Delta x$$
$$P_{K+1} = P_K + 2\Delta y - 2\Delta x \, (y_{K+1} - y_K)$$

$$P_K > 0 = (y_{K+1} = y_{K+1}).$$
$$Nc = (x_{K+1}, y_{K+1}).$$

$$P_K < 0 = y_{K+1} = y_K.$$
$$Nc = (x_{K+1}, y_K).$$

Initial Decision Parameter.

$P_K = 10 - 8 = 2$

$P_K > 0$

$\Delta x = 8$

$\Delta y = 5$

$2\Delta x = 16$

$2\Delta y = 10$

K	(x_K, y_K)	P_K	(x_{K+1}, y_{K+1})
0	(35, 40)	2	(36, 41)

$$\boxed{P_{K+1} = 2 + 10 - 16(1) = -4}$$

K	(x_K, y_K)	P_K	(x_{K+1}, y_{K+1})
1	(36, 41)	-4	(37, 41)

$$\boxed{P_{K+1} = -4 + 10 - 16(0) = 6}$$

2 (37, 41) 6 (38, 42)

$$\boxed{P_{k+1} = 6 + 10 - 16(1) = 0}$$

3 (38, 42) 0 (39, 43)

$$\boxed{P_{k+1} = 0 + 10 - 16(1) = -6}$$

4 (39, 43 -6 (40, 43)

$$\boxed{P_{k+1} = -6 + 10 - 16(0) = 4}$$

5 (40, 43) 4 (41, 44)

$$\boxed{P_{k+1} = 4 + 10 - 16(1) = 0}$$

6 (41, 44) 0 (42, 45)

$$\boxed{P_{k+1} = 0 + 10 - 16(1) = -6}$$

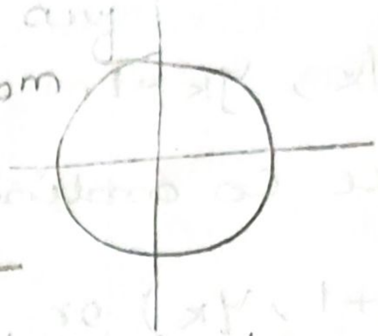
7 (42, 45) -6 (43, 45).

Points are,
(35, 40) (37, 41), (38, 42), (39, 43),
(40, 43) (41, 44), (42, 45), (43, 45).

Mid-Point Circle Algm:

As we know, the distance is

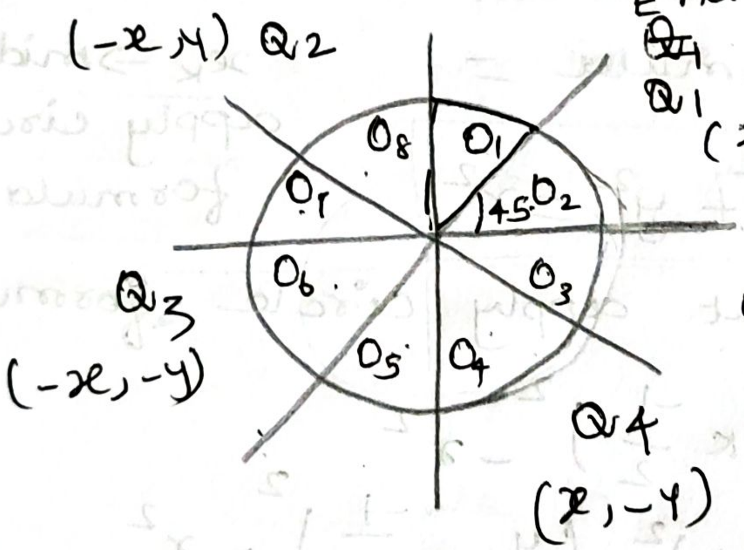
* For a circle, at any point the distance from Origin is radius.



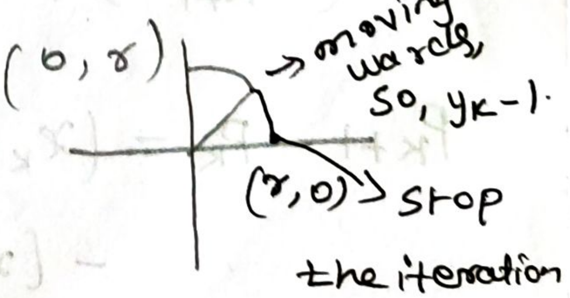
* That's why follow the symmetric property of a circle, Quadrant, Octant

as the radius is we can simply assume, if we calculate the first point, Remaining are same.

End point of O_1 $= x \geq y$.



To find Quartant of First Octant.



x - unit intervals.

y - ?

y_k, $y_k - 1$.

Next co ordinate may be,

(x_k+1, y_k) or (x_k+1, y_k-1).

↳ depends upon the decision Parameter.

$$\text{mid point} = \left(\frac{x_k+1 + x_k+1}{2}, \frac{y_k + y_k - 1}{2}\right)$$

$$\boxed{\text{mid Point} = x_k+1, y_k - \tfrac{1}{2}}$$

circle formula =

$$\boxed{x^2 + y^2 = r^2}$$

$x_k \rightarrow$ mid point apply circle formula.

In mid point apply circle formula,

$$P_k = (x_k+1)^2 + (y_k - \tfrac{1}{2})^2 - r^2$$

$$P_{k+1} = (x_{k+1}+1)^2 + (y_{k+1} - \tfrac{1}{2})^2 - r^2.$$

$$P_{k+1} - P_k = (x_{k+1}+1)^2 + (y_{k+1} - \tfrac{1}{2})^2 - \cancel{r^2} - (x_k+1)^2 - (y_k - \tfrac{1}{2})^2 + \cancel{r^2}$$

x moves in unit interval. So x_{k+1} replace with x_k+1

$$=\big(\underbrace{(x_k+1)}_{a}+\underbrace{1}_{b}\big)^2+\left(y_{k+1}-\tfrac{1}{2}\right)^2-(x_k+1)^2-\left(y_k-\tfrac{1}{2}\right)^2$$

Use $(a+b)^2$

↳ keep same. because whether we select y_{k+1} or y_k)

$$=\cancel{(x_k+1)^2}+2(x_k+1)+y_{k+1}^2+\cancel{1/4}-y_{k+1}-\cancel{(x_k+1)^2}-y_k^2-\cancel{1/4}+y_k+1$$

$$=P_{k+1}-P_k=2(x_k+1)+y_{k+1}^2-y_k^2)-(y_{k+1}-y_k)+1.$$

$$\boxed{P_{k+1}=P_k+2(x_k+1)+(y_{k+1}^2-y_k^2)-(y_{k+1}-y_k)+1.}$$

Initial Decision parameter,

$(0, r)$ → Starting point

0 ↳ x_k, r ↳ y_k.

$P_k=(x_k+1)^2+(y_k-\frac{1}{2})^2-r^2.$

↳ Substitute in P_k. calculate x_k & y_k value.

$$P_k=(0+1)^2+(r-1/2)^2-r^2$$

$$=1+\cancel{r^2}+1/4-r-\cancel{r^2}$$

$$P_k=5/4-r.$$

↳ consider only integer part. avoid fractional part.

$$\boxed{P_K = 1 - r.}$$

$$\boxed{P_{K+1} = P_K + 2(x_K + 1) + (y_{K+1}^2 - y_K^2) - (y_{K+1} - y_K) + 1.}$$

if $(P_K \geq 0) = y_{K+1} = y_K - 1.$

$Ne = (x_K + 1, y_K - 1)$

if $(P_K < 0)$ $y_{K+1} = y_K.$

$Ne = (x_K + 1, y_K).$

Stop the iteration $x \geq y$.

based on the first octant, we can fill remaing octant.

<u>Example:</u>

$r = 8.$

$P_0 = 1 - r = 1 - 8 = \boxed{-7.}$

Example:

$$P_{k+1} = P_k + 2(x_k+1) + (y_{k+1}^2 - y_k^2) - (y_{k+1} - y_k) + 1$$

K	(x_k, y_k)	P_k	x_{k+1}, y_{k+1}.
0	(0, 8)	-7	(1, 8)

$y_{k+1} = 8 - y_k = 8 = 0$

$$\boxed{\begin{aligned} P_{k+1} &= -7 + 2(\overset{x_k}{0}+1) + 0 - 0 + 1 \\ &= -7+2+1 = -4. \end{aligned}}$$

(1, 8)	$(\overset{x_k}{1}, 8)$	-4	(2, 8).

$$\boxed{\begin{aligned} P_{k+1} &= -4 + 2(1+1) + 0 - 0 + 1 \\ &= -4 + 4 + 1 = 1. \end{aligned}}$$

2	(2, 8)	1	(3, 7).

$$P_{k+1} = 1 + 2(2+1) + (\overset{7^2}{49} - \overset{8^2}{64}) - (7-8) + 1$$

$$\boxed{= 1 + 6 \cancel{8} - 15 + 1 + 1 = 9 - 15 = -6}$$

3	~~(7, 8)~~ (3, 7)	-6	$(4, \overset{7}{\cancel{8}})$.

$$\boxed{\begin{aligned} P_{k+1} &= -6 + 2(3+1) + 0 - 0 + 1 \\ &= -6 + 8 + 1 = 3. \end{aligned}}$$

4.	$(4, \overset{7}{\cancel{8}})$	3	(5, 6)

$$\boxed{\begin{aligned} P_{k+1} &= 3 + 2(4+1) + (36 - 49) - (6-7) + 1 \\ &= 3 + 10 - 13 + 1 + 1 = 2 \end{aligned}}$$

5	(5, 6)	2	(6, 5) $(x \geq y)$

(0,8), (1,8) (2,8) (3,7) (4,7),
(5,6), (6,5) First Octant points.

Q_1	$Q(-x, y)$	$Q_3(-x,-y)$	$Q_4 (x,-y)$
(0,8)	(0,8)	(0,8)	(0,8)
(1,8)	(-1,8)	(-1,-8)	(1,-8)
(2,8)	(-2,8)	(-2,-8)	(2,-8)
(3,7)	(-3,7)	(-3,-7)	(3,-7)
(4,7)	(-4,7)	(-4,-7)	(4,-7)
(5,6) ← O_1	(-5,6)	(-5,-6)	(5,-6)
(6,5.) Q_2	(-6,5)	(-6,-5)	(6,-5)
(7,4)	(-7,4)	(-7,-4)	(7,-4)
(7,3)	(-7,3)	(-7,-3)	(7,-3)
(8,2)	(-8,2)	(-8,-2)	(8,-2)
(8,1)	(-8,1)	(-8,-1)	(8,-1)
(8,0)	(-8,0)	(-8,0)	(8-0).