20·12·2021

RANDOM VARIABLEMS (R.V)

DISCRETE R.V

If a values of x are either finite or Countable then the R.V is said to be Disc. R.V

Finite: $1, 2, 3, \ldots n$

Countable: $1, 2, 3, \ldots \infty$

Probability Mass Function (P.m.f)

(i) It is denoted by $P(\underbrace{X}_{R.V} = x)$ (or) $P(x)$

(ii) Its total probability is,

$$\sum_{x=1}^{n} P(X=x) \text{ (or) } \sum_{x=1}^{n} P(x) = 1$$

(iii) Cumulative Distribution Function (C.d.f)
(or)
Distribution Function:-

$$F(x) = \sum_{x=1}^{x} P(x)$$

CONTINUOUS R.V

If a values of x are either infinite (or) uncountable then the R.V is said to be Cont. R.V

Infinite: $0 < x < 1$ (Interval form)

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Uncountable: Stars in the Moon

Probability Density Function:- (P.d.f)

It is denoted by $f(x)$

Its total probability is,

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

Cumulative Distribution Function:-
(or)
Distribution Function:-

$$F(x) = \int_{-\infty}^{x} f(x)\,dx$$

Suppose if distribution function given then the density function is,

$$f(x) = \frac{d}{dx} F(x)$$

① A R.V X has the following probability distribution:

x:	0	1	2	3	4	5	6	7
$P(x)$:	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2+k$

(i) Find the value of k (ii) Find $P(2 \le x \le 5)$
(iii) Find $P(x \geq 6)$, $P(x < 6)$ (iv) Find $P(1.5 < x < 4.5 / x > 2)$
(v) The smallest value of λ for which $P(x \le \lambda) > \frac{1}{2}$.

Sol:- Since the values of x are finite.
$\therefore$ this R.V is said to be Discrete R.V

(i) Find the value of k
WKT, Its total probability is,

$$\sum P(x) = 1$$

$$\sum_{x=0}^{7} P(x) = 1$$

$$P(0) + P(1) + P(2) + \cdots + P(7) = 1$$

$$\Rightarrow 0 + k + 2k + 2k + 3k + k^2 + 2k^2 + 7k^2 + k = 1$$

$$\Rightarrow 10k^2 + 9k - 1 = 0$$

-10

9

10 -1

$$10k^2 + 10k - k - 1 = 0$$

$$10k(k+1) - 1(k+1) = 0$$

$$(k+1)(10k-1) = 0$$

$$k+1 = 0 \; ; \quad 10k - 1 = 0$$

$$k = -1 \; ; \quad 10k = 1 \Rightarrow k = \frac{1}{10}$$

Since $k=-1$ is not possible. If $k=-1$ then the prob. values are negative. It is contradiction to $0 \le P(x) \le 1$ (axioms of prob)

hence $k = \frac{1}{10}$ is only possible value.

(ii) Find $P(2 \le x \le 5)$

$$= P(x=2) + P(x=3) + P(x=4) + P(x=5)$$

$$= 2k + 2k + 3k + k^2$$

$$= 7k + k^2$$

$$= 7 \cdot \frac{1}{10} + \frac{1}{100} \quad \left(\because k = \frac{1}{10}\right)$$

$$= \frac{71}{100} = 0.71$$

(iii) Find $P(x \ge 6)$, $P(x < 6)$

$$P(x \ge 6) = P(x=6) + P(x=7)$$

$$= 2k^2 + 7k^2 + k$$

$$= 9k^2 + k$$

$$= 9 \cdot \frac{1}{100} + \frac{1}{10} \quad \left(\because k = \frac{1}{10}\right)$$

$$P(x \ge 6) = \frac{19}{100} = 0.19$$

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M①:- $P(x<6) = P(x=0)+P(x=1)+P(x=2)+P(x=3)+P(x=4)+P(x=5)$

$$= 0+k+2k+2k+3k+k^2$$

$$= 8k+k^2$$

$$= 8\cdot\frac{1}{10}+\frac{1}{100} \qquad \left(\because k=\frac{1}{10}\right)$$

$$P(x<6) = \frac{81}{100} = 0.81$$

M②:-

$$P(x<6) = 1-P(x\geq 6) \qquad (\because P(A)=1-P(\bar{A}))$$

$$= 1-0.19$$

$$P(x<6) = 0.81$$

(iv) Find $P(1.5<x<4.5/x>2)$

$$P(x>2) = 1-P(x\leq 2) \qquad (\because P(A)=1-P(\bar{A}))$$

$$= 1-(P(x=0)+P(x=1)+P(x=2))$$

$$= 1-(0+k+2k)$$

$$= 1-(3k)$$

$$= 1-3\cdot\frac{1}{10} \qquad \left(\because k=\frac{1}{10}\right)$$

$$P(x>2) = \frac{7}{10} = 0.7$$

$$P(1.5<x<4.5/x>2) = \frac{P(1.5<x<4.5 \cap x>2)}{P(x>2)} \qquad \left(\because P(A/B)=\frac{P(A\cap B)}{P(B)}\right)$$

$$= \frac{P(x=2,3,4 \cap x=3,4,5,6,7)}{P(x>2)}$$

$$= \frac{P(x=3,4)}{P(x>2)}$$

$$= \frac{P(x=3)+P(x=4)}{P(x>2)}$$

$$= \frac{2k+3k}{\ldots} = \frac{5k}{\ldots}$$

$$= \frac{2k+3k}{P(x>2)} = \frac{5k}{P(x>2)}$$

$$= \frac{5\cdot\frac{1}{10}}{7/10}$$

$\therefore P(1.5<x<4.5/x>2) = \frac{5}{7}$

(v) The Smallest value of λ for which $P(x \le \lambda) > \frac{1}{2}$

To find Cumulative Distribution Function:-

x	$P(x)$	$F(x)$	
0	0	0	$= 0$
1	k	k	$= \frac{1}{10}$
2	$2k$	$3k$	$= \frac{3}{10}$
3	$2k$	$5k$	$= \frac{5}{10}$
4	$3k$	$8k$	$= 8/10$
5	k^2	$8k+k^2$	$= 81/100$
6	$2k^2$	$8k+3k^2$	$= 83/100$
7	$7k^2+k$	$10k^2+9k$	$= 100/100 = 1$

from the above c.d.f table,

$$P(x \le 4) > \frac{1}{2}$$

$\therefore \lambda = 4$