Discrete Mathematics (that can be counted)

UNIT-1 : Logics

* logic → Declarative sentence - states T (or) F to sentence.

- logic is a study of the principles and methods that distinguish b/w a valid & invalid argument

- the's logics & the rules are being used in design of computer circuits the construction of computer programs.

* proposition: A declarative sentence either T (or) F but not both.

- A proposition (simple statement) may be denoted by a variable like p, q, r, --- called a proposition (statement) variable.

- the value of proposition is called truth value denoted by:

  * T (or) 1 is true

  * F (or) 0 is false

options, interogative & imperative are not propositions.

T T → P

0

1

* Compound propositions: 2 statements are connected with connectives

simple statement could be used to built a compound statements.

Ex: 3+2=5 & Lahore is a city in pakistan.

*Logical Connectives: connectives are used to create a compound proposition from 2 (or) more positions.

- Negation (or) Not ($\neg$ a (or) ! a (or) $\bar{a}$)
- conjuction (or) And ($\land$)
- Disjunction (or) or ($\lor$)
- implication ($\Rightarrow$ (or) $\rightarrow$)
- biconditional ($\Leftrightarrow$ (or) $\leftrightarrow$)

*Formal Name	NickName	~~Unitar~~	symbol
Negation operator	NOT	unary	$\neg$ ($\sim$)
conjunction opr	AND	Binary	$\land$
Disjunction opr	OR	Binary	$\lor$
Exclusive-OR opr	XOR	Binary	$\oplus$
Implication	Implies	Binary	$\rightarrow$
Biconditional	if & only if	Binary	$\leftrightarrow$
conditional	if then	Binary	$\rightarrow$

unary - one statement is enough

Binary - needed more than one statement.

*Precedence of logical operators:
(preference)

$\neg$	1
$\land$	2
$\lor$	3
$\rightarrow$	4
$\leftrightarrow$	5

*logical connective: Negation

- let p be a proposition. the negation of p denoted by $\neg p$, is the statement "It is not the case that p".

- the proposition $\neg p$ is "not p"

truth table

p	$\neg p$
T	F
F	T

Ex: find the negation of proposition "Today is Friday".

→ Today is not Friday.

*logical connective: ANd

- let p and q be the propositions. the conjunction of p & q denoted by $p \wedge q$.
- the conjunction $p \wedge q$ is true when both p and q are true & false otherwise

p	Q	$p \wedge q$	
T	T	1	T
F	F	0	F
F	T	0	F
F	F	1	F

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* logical connective : logical (or)

- The disjunction $p \vee q$ is true when any of p and q is true

p	Q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

* logical connective : implication

p	Q	$p \to Q$
T	T	T
T	F	F
F	T	T
F	F	T

p	Q	$p \Leftrightarrow Q$
T	T	T
T	F	F
F	T	F
F	F	T

If $p \to Q$

then $Q \to p$ is converse

$\neg Q \to \neg p$ is contrapositive

$\neg p \to \neg Q$ is invers of $p \to Q$

$p \to Q$ & $\neg Q \to \neg p$ are equivalent.

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p	Q	$\neg Q$	$\neg p$	$\neg Q \to \neg p$	$p \to Q$
T	T	F	F	T	T
T	F	T	F	F	F
F	T	F	T	T	T
F	F	T	T	T	T

* symbolise the following statements

1) It rains only if it is cold

If it is cold then it rains

$C \to R$

It rains – R

It is cold – C

condi $\boxed{} \to \boxed{}$ Result

2) whenever it rains it is cold

R - rains

c – cold

$c \to R$

If it is cold then it rain

*1. $p \wedge \neg p$

p	$\neg p$	$p \wedge \neg p$	$p \vee \neg p$
T	F	F	T
F	T	F	T

$p \wedge \neg p \to$ contradiction $\quad p \vee \neg p \longrightarrow$ Tautology

2. $p \vee (p \wedge Q) \Leftrightarrow p$

$\boxed{\begin{array}{c}\rightarrow \\ \leftrightarrow\end{array}}$ $\Rightarrow$ ✓ T
$\Leftrightarrow$ ✓

p / Q

if $p \rightarrow Q$ is Tautology

$p \Rightarrow Q$

if $p \leftrightarrow Q$ is Tautology

$p \Leftrightarrow Q$

(or) Truth value of p & Q

are same

p	Q	$p \wedge Q$	$p \vee (p \wedge Q)$	$p \vee (p \wedge Q) \leftrightarrow p$
T	T	T	T	T
T	F	F	T	T
F	T	F	F	T
F	F	F	F	T

since T.V of p & $p \vee (p \wedge Q)$ are

$$p \vee (p \wedge Q) \Leftrightarrow p$$

$\boxed{(p \vee p \wedge Q) \leftrightarrow p \text{ is Tautology}}$

* Equivalent: 2 statements formulae A and B are equivalent if

$A \leftrightarrow B$ is Tautology

P.T $p \rightarrow (Q \rightarrow p) \Leftrightarrow \neg p \rightarrow (p \rightarrow Q)$

$p \rightarrow Q$ is F only if T F→F

				A			B	
P	Q	P	$Q \to P$	$P \to (Q \to P)$	$\neg P$	$P \to Q$	$\neg P \to (P \to Q)$	$A \leftrightarrow B$
T	T	T	T	T	F	T	T	T
T	F	T	T	T	F	F	T	T
F	T	F	F	T	T	T	T	T
F	F	F	T	T	T	T	T	T

Thus $P \to (Q \to P) \Leftrightarrow \neg P \to (P \to Q)$

H.W

1. $(P \wedge (P \to Q)) \to Q$

2. $(P \to Q) \leftrightarrow (\neg P \vee Q)$

1. $(P \wedge (P \to Q)) \to Q$

P	Q	$P \to Q$	$P \wedge (P \to Q)$	$(P \wedge (P \to Q)) \to Q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

*Statement Formula: A stt formula is an expression which consists of variables, paranthesis & connectives.

*Tautology: A stt formula which is identically true is called tautology.

*contradiction: A stt formula which is identically false is called contradiction.

*contingency: A stt formula which is neither Tautology (or) contradiction.

*precedence of logical operation:

1. negation has precedence over all operators.

2. conjunction takes precedence over disjunction

eg: $\boxed{P \vee Q \wedge R}$ means $P \vee (Q \wedge R)$

$\boxed{a + b \cdot c}$

Construct T.T for

1. $[(P \to Q) \wedge (Q \to R)] \to (P \to R)$

$\therefore$ Given stt formula is Tautology.

P	Q	R	P→Q (1)	Q→R (2)	(1) ∧ (2) (3)	P→R (4)	(3) → (4)
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	F	T	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

*Equivalent: Two st formula, A & B are equivalent iff $A \Leftrightarrow B$ is Tautology (i.e) $A \Leftrightarrow B$

*Implication: A stt for A is said to tautologically imply B iff $A \rightarrow B$ a tautology (or) if $A \rightarrow$ is Tautology then $A \Rightarrow B$

Eg: $P \wedge Q \Rightarrow P \rightarrow Q$

P	Q	P∧Q (1)	P→Q (2)	(1) → (2)
T	T	T	T	T
T	F	F	F	T
F	T	F	T	T
F	F	F	T	T

since $P \wedge Q \rightarrow (P \rightarrow Q)$ is Tautology

$(P \wedge Q) \Rightarrow (P \rightarrow Q)$

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*Basic equivalent formulas:

1. Idempotent law: $p \vee p \Leftrightarrow p$, $p \wedge p \Leftrightarrow p$

$\boxed{A \cup A = A}$

$\boxed{\begin{array}{l} \because a(b+c) = ab + ac \\ a+b = b+a \\ A \cup B = B \cup A, \\ A \cap B = B \cap A \end{array}}$

2. Identity law: $p \vee F \Leftrightarrow p$, $p \wedge T \Leftrightarrow p$

3. Inverse law: $p \wedge \neg p \Leftrightarrow F$, $p \vee \neg p \Leftrightarrow T$

4. Domination (or) Null law: $p \wedge F \Leftrightarrow F$, $p \vee T \Leftrightarrow T$

5. Commutative law: $p \vee Q \Leftrightarrow Q \vee p$, $p \wedge Q \Leftrightarrow Q \wedge p$

6. Distributive law: $p \vee (Q \wedge R) \Leftrightarrow (p \vee Q) \wedge (p \vee R)$

$p \wedge (Q \vee R) \Leftrightarrow (p \wedge Q) \vee (p \wedge R)$

7. Associative law: $p \vee (Q \vee R) \Leftrightarrow (p \vee Q) \vee R$

$p \wedge (Q \wedge R) \Leftrightarrow (p \wedge Q) \wedge R$

$\boxed{\because (A \cup B)' = A' \cap B'}$

8. Absorbtion law: $p \wedge (p \vee Q) \Leftrightarrow p$, $p \vee (p \wedge Q) \Leftrightarrow p$

9. Demorgon's law: $\neg(p \vee Q) \Leftrightarrow \neg p \wedge \neg Q$

$\neg(p \wedge Q) \Leftrightarrow \neg p \vee \neg Q$

$\boxed{\because (A')' = A}$

10. Double negation: $\neg(\neg p) \Leftrightarrow p$

*Conditional equivalence: $p \rightarrow (Q \rightarrow R) \Leftrightarrow (p \wedge Q) \rightarrow R$

$$\boxed{P \to Q \Leftrightarrow \neg P \vee Q}$$ ✓

* **Biconditional:**

$$P \leftrightarrow Q \Leftrightarrow (P \to Q) \wedge (Q \to P)$$

$$\Leftrightarrow (\neg P \vee Q) \wedge (\neg Q \vee P)$$

Prove without Truth table

1. P.T $\neg(P \to Q) \Leftrightarrow P \wedge \neg Q$

L.H.S $\neg(P \to Q) \Leftrightarrow \neg(\neg P \vee Q)$

$\Leftrightarrow \neg(\neg P) \wedge \neg Q$ (using De morgon's law)

$\Leftrightarrow P \wedge \neg Q$

H.W

1. $P \to Q \Leftrightarrow P \to (P \wedge Q)$

2. $P \to (Q \vee R) \Leftrightarrow (P \to Q) \vee (P \to R)$

H.W

1. $p \to Q \Leftrightarrow p \to (p \wedge Q)$

2. $p \to (Q \vee R) \Leftrightarrow (p \to Q) \vee (p \to R)$

1. $p \to Q \Leftrightarrow p \to (p \wedge Q)$

sol $p \to (p \wedge Q) \Leftrightarrow \neg \cancel{P}(p \wedge Q)$ (Result 1)

$\Leftrightarrow \neg p \vee (p \wedge Q)$ (Result 1)

$\Leftrightarrow (\neg p \vee p) \wedge (\neg p \vee Q)$ (distributive law)

$\Leftrightarrow T \wedge (\neg p \vee Q)$

$\Leftrightarrow \neg p \vee Q$

$\Leftrightarrow p \to Q$

2. $P \to (Q \vee R) \Leftrightarrow (P \to Q) \vee (P \to R)$

Sol $(P \to Q) \vee (P \to R) \Leftrightarrow (\neg P \vee Q) \vee (\neg P \vee R)$ (Result ①)

$\Leftrightarrow [\neg P \vee (Q \vee \neg P)] \vee R$ (Associative law)

$\Leftrightarrow [\neg P \vee (\neg P \vee Q)] \vee R$ (commutative law)

$\Leftrightarrow [(\neg P \vee \neg P) \vee Q] \vee R$ (Associative law)

$\Leftrightarrow (\neg P \vee Q) \vee R$ (Idempotent law)

$\Leftrightarrow \neg P \vee (Q \vee R)$ (Associative law)

$\Leftrightarrow P \to (Q \vee R)$ (Result ①)

$\boxed{10/07/2021}$

1. Using the laws of logic show that $(P \wedge Q) \to (P \vee Q)$ is a tautology.

Sol

~~$P \to Q \Leftrightarrow$~~

$\boxed{P \to Q \Leftrightarrow \neg P \vee Q}$

$(P \wedge Q) \to (P \vee Q) \Leftrightarrow \neg (P \wedge Q) \vee (P \vee Q)$

$\Leftrightarrow (\neg P \vee \neg Q) \vee (P \vee Q)$ $(\neg (P \wedge Q) \Leftrightarrow \neg P \vee \neg Q)$

$\boxed{P \wedge (P \vee Q) \Leftrightarrow P}$ $\Leftrightarrow (\neg P \vee \neg Q) \vee (Q \vee P)$ commutation

$\Leftrightarrow \neg P \vee (\neg Q \vee Q) \vee P$

$\Leftrightarrow \neg P \vee T \vee P$ $(\Leftrightarrow T)$

$\Leftrightarrow (\neg P \vee P) \vee T$

$\Leftrightarrow T \vee T$

$\Leftrightarrow T$

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2. without using Truth Table p.T

1) $(\neg p \vee Q) \wedge (p \wedge (p \wedge Q)) \Leftrightarrow p \wedge Q$

Sol $\neg(p \vee Q) \wedge (p \wedge (p \wedge Q))$

$\Leftrightarrow (\neg p \vee Q) \wedge ((p \wedge p) \wedge Q)$ (Associative)

$\Leftrightarrow (\neg p \vee Q) \wedge (p \wedge Q)$ (Idempotent law)

$\Leftrightarrow (p \wedge Q) \wedge (\neg p \vee Q)$ (commutative)

$\Leftrightarrow ((p \wedge Q) \wedge \neg p) \vee ((p \wedge Q) \wedge Q)$

$\Leftrightarrow (\neg p \wedge (p \wedge Q)) \vee (p \wedge (Q \wedge Q))$

$\Leftrightarrow ((\neg p \wedge p) \wedge Q) \vee (p \wedge Q)$

$\Leftrightarrow (F \wedge Q) \vee (p \wedge Q)$

$\Leftrightarrow F \vee (p \wedge Q)$ $(F \vee p \Leftrightarrow p)$

$\Leftrightarrow p \wedge Q$

$a(b+c)$
$= ab+bc$

$(b+c)a$
$= ab+bc$

$p \wedge F \Leftrightarrow F$
$p \vee F \Leftrightarrow p$
$p \wedge T \Leftrightarrow p$
$p \vee T \Leftrightarrow T$

2) $\neg(p \vee (\neg p \wedge Q)) \Leftrightarrow \neg p \wedge \neg Q$

Sol $\neg(p \vee (\neg p \wedge Q))$

$\Leftrightarrow [\neg p \wedge \neg(\neg p \wedge Q)]$ (Demorgon's)

$\Leftrightarrow [\neg p \wedge (p \vee \neg Q)]$

$\Leftrightarrow (\neg p \wedge p) \vee (\neg p \wedge \neg Q)$

$\Leftrightarrow F \vee (\neg p \wedge \neg Q)$ $(F \vee p \Leftrightarrow p)$

$\Leftrightarrow \neg p \wedge \neg Q.$

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3) $\neg p \to (Q \to R) \Leftrightarrow Q \to (p \vee R)$

sol $\neg p \to (Q \to R)$

$\Leftrightarrow \neg(\neg p) \vee (Q \to R)$

$\Leftrightarrow p \vee (\neg Q \vee R)$

$\Leftrightarrow (p \vee \neg Q) \vee R$ [Associative]

$\Leftrightarrow (\neg Q \vee p) \vee R$ [Commutative]

$\Leftrightarrow \neg Q \vee (p \vee R)$

$(p \to Q \Leftrightarrow \neg p \vee Q)$

$\Leftrightarrow Q \to (p \vee R)$

12/07/2021

*Normal forms: It will be convenient to use the word product in the place of conjunction and sum in the place disjunction.

*Elementary product: A product of the variables and their negation is called Elementary product.

eg: $P \wedge Q$, P, $P \wedge \neg Q$, $\neg P \wedge Q$, $\neg P \wedge \neg Q$.

*Elementary sum: sum of variable and their negation is called an Elementry sum

eg: P, Q, $P \vee \neg Q$, $\neg P \vee Q$.

*Disjunctive normal form: A compound preposition which consist of sum of products and which is equivalent to given proposition is called Disjunctive normal form.

$ab+bc$

eg: $(a \wedge b) \vee (b \wedge c)$

$a \vee (b \wedge c)$

* **Conjuctive normal form**: A compound proposition which consist of elementry sum is called conjuctive normal form

eg: $(a+b)(c+a)$

$(a \vee b) \wedge (c \vee a)$

* sum of products – DNF $\underline{ab+bc}$

* product of sum – CNF

procedure:

step-1: Remove conditional and biconditional symbol by writing equivalent formulae.

$p \rightarrow Q \Leftrightarrow \neg p \vee Q$, $p \leftrightarrow Q \Leftrightarrow (p \rightarrow Q) \wedge (Q \rightarrow p)$

step 2: if negation symbol present in the formula

* obtain CNF & DNF for $p \wedge (p \rightarrow Q)$

sol: $p \wedge (p \rightarrow Q) \Leftrightarrow p \wedge (\neg p \wedge Q)$

which is CNF

$p \wedge (p \rightarrow Q) \Leftrightarrow p \wedge (\neg p \vee Q)$

$\Leftrightarrow (p \wedge \neg p) \vee (p \wedge Q)$

Disjunctive normal form

step-1: Replace condition

step-2: Apply Demorgans

step-3: Discrete law.

*PCNF and PDNF:

min terms:

$P \wedge Q, P \wedge \neg Q, \neg P \wedge Q, \neg P \wedge \neg Q$

max terms:

$P \vee Q, P \vee \neg Q, \neg P \vee Q, \neg P \vee \neg Q$.

*principal normal forms:

minterm: If a given propositional expression contains n-component propositions $P_1, P_2, --- P_n$ then a minterm of the given proposition is a product $(x_1 \wedge x_2 \wedge, --- \wedge x_n)$ where $x_i \in \{P_i, \neg P_i\}$ is known as minterm.

minterms : $p \rightarrow (\neg p \wedge Q)$

$(p \wedge Q), (\neg p \wedge Q)$

* principal DNF (PDNF): An equivalent expression for the given propositional expression which is in the form of sum of minterms only is known as PDNF

→ PDNF is a unique expression.

maxterm: If $S = \langle P_1, P_2, P_3, ---, P_n \rangle$ is a proposition expression then a maxterm of S is of the form $(x_1 \vee x_2 \vee x_3 \vee \dots \vee x_n)$ where $x_i \in \{P_i, \neg P_i\}$

principal conjuctive normal form

product of maxterms.

problems on normal forms:

1. To get DNF of $p \wedge (p \to Q)$

Sol $p \wedge (p \to Q)$

$9 \geq 9$
$= 7+2$

$\Leftrightarrow p \wedge (\neg p \wedge Q)$ CNF

$\Leftrightarrow (p \wedge \neg p) \vee (p \wedge Q)$

$\Leftrightarrow F \vee (p \wedge Q)$ ($\because$ by Negation)

$\Leftrightarrow (p \wedge Q)$ (identity)

$\hookrightarrow$ DNF $[p \wedge Q \Leftrightarrow (p \wedge Q) \vee (p \wedge Q)]$

CNF $\to (p \vee p) \wedge (Q \vee Q)$

2. Find CNF of $p \to ((p \to Q) \wedge (\neg Q \vee \neg p))$

Sol $p \to ((p \to Q) \wedge (\neg Q \vee \neg p))$

$\Leftrightarrow \underbrace{p}_{A} \to \underbrace{((\neg p \vee Q) \wedge (\neg Q \vee \neg p))}_{B}$

$\Leftrightarrow \neg A \vee B$

$\Leftrightarrow \neg p \vee ((\neg p \vee Q) \wedge (\neg Q \vee \neg p))$

$\Leftrightarrow (\neg p \vee (\neg p \vee Q)) \wedge (\neg p \vee (\neg Q \vee \neg p))$ ($\because$ by distribution)

$\Leftrightarrow ((\neg p \vee \neg p) \vee Q) \wedge (\neg p \vee (\neg p \vee \neg Q))$ (by associative & commutative)

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$\Leftrightarrow (\neg p \vee Q) \wedge ((\neg p \vee \neg p) \vee \neg Q)$ (by idempotent & associative)

$\Leftrightarrow (\neg p \vee Q) \wedge (\neg p \vee \neg Q)$

$\hookrightarrow$ CNF form.

3. obtain PCNF and PDNF using T.T

i) $(\neg p \vee \neg Q) \rightarrow (p \leftrightarrow \neg Q)$

p	Q	$\neg p$	$\neg Q$	$\neg p \vee \neg Q$	$p \Leftrightarrow \neg Q$	① → ②
T	T	F	F	T	F	T
T	F	F	T	T	T	T
F	T	T	F	T	T	T
F	F	T	T	T	F	F

PDNF of $(\neg p \vee \neg Q) \rightarrow (p \leftrightarrow \neg Q)$

$\Leftrightarrow (p \wedge Q) \vee (p \wedge \neg Q) \vee (\neg p \wedge Q)$

1. obtain PDNF and PCNF of following statement using truth table. $(\neg p \vee \neg Q) \rightarrow (p \leftrightarrow \neg Q)$

sol

				a	b	
P	Q	$\neg P$	$\neg Q$	$(\neg P \vee \neg Q)$	$P \leftrightarrow \neg Q$	$a \rightarrow b$
T	T	F	F	F	F	T
T	F	F	T	T	T	T
F	T	T	F	T	T	T
F	F	T	T	T	F	F

PDNF of $(\neg p \vee \neg Q) \rightarrow (p \leftrightarrow \neg Q)$ is

$= (p \wedge Q) \vee (p \vee \neg Q) \vee (\neg p \vee \neg Q)$.

∴ the minterms corresponding to the 3T values of the last column are $p \wedge Q$, $p \wedge \neg Q$, $\neg p \wedge Q$.

Now, PDNF of $\neg(a \rightarrow b) = \neg p \wedge \neg Q$

∴ PCNF of $(a \rightarrow b) = \neg(\neg p \wedge \neg Q) = p \vee Q$.

2. obtain the PDNF and PCNF of following statement using truth table $[P \to (Q \wedge R)] \wedge [\neg P \to (\neg Q \wedge \neg R)]$

sol.

						a	b	c	d	
P	Q	R	$\neg P$	$\neg Q$	$\neg R$	$Q \wedge R$	$P \to a$	$\neg Q \wedge \neg R$	$\neg P \to c$	$b \wedge d$
T	T	T	F	F	F	T	T	F	T	T
T	T	F	F	F	T	F	F	F	T	F
T	F	T	F	T	F	F	F	F	T	F
T	F	F	F	T	T	F	F	T	T	F
F	T	T	T	F	F	T	T	F	F	F
F	T	F	T	F	T	F	T	F	F	F
F	F	T	T	T	F	F	T	F	F	F
F	F	F	T	T	T	F	T	T	T	T

PDNF of given statement = $(P \wedge Q \wedge R) \vee (\neg P \wedge \neg Q \wedge \neg R)$

PDNF of $\neg(b \wedge d) = (P \wedge Q \wedge \neg R) \vee (P \wedge \neg Q \wedge R) \vee (P \wedge \neg Q \wedge \neg R) \vee (\neg P \wedge Q \wedge R) \vee (\neg P \wedge Q \wedge \neg R) \vee (\neg P \wedge \neg Q \wedge R)$

$\therefore$ PCNF of $(b \wedge d) = (\neg P \vee \neg Q \vee R) \wedge (\neg P \vee Q \vee \neg R) \vee (\neg P \vee Q \vee R) \wedge (P \vee \neg Q \vee \neg R) \wedge (P \vee \neg Q \vee R) \wedge (P \vee Q \vee \neg R)$

procedure:

step-1: Replace the connectives $\rightarrow, \leftrightarrow$ by using $\wedge, \vee, \neg$

step-2: If negation is present before the given formula (or) a part of the given formula of demorgans law.

step-3: If necessary apply distributive (or) associative (or) and Idemponent law.

1. i) Find PDNF of $(\neg P \leftrightarrow Q) \wedge (Q \leftrightarrow P)$ without constructing the truth tables.

Sol. $(\neg P \leftrightarrow Q) \wedge (Q \leftrightarrow P)$

$\Leftrightarrow (P \vee Q) \wedge [(Q \wedge P) \vee (\neg Q \wedge \neg P)]$

$\Leftrightarrow (P \vee Q) \wedge [(P \wedge Q) \vee \neg(P \vee Q)]$

$\Leftrightarrow [(P \vee Q) \wedge (P \wedge Q)] \vee [(P \vee Q) \wedge \neg(P \vee Q)]$

$\Leftrightarrow [(P \vee Q) \wedge (P \wedge Q)] \vee F$

$\Leftrightarrow [P \wedge (P \wedge Q)] \vee [Q \wedge (P \wedge Q)]$

$\Leftrightarrow (P \wedge Q) \vee (P \wedge Q)$

$\Leftrightarrow P \wedge Q$

2) $[Q \vee (P \wedge R)] \wedge \neg[(P \vee R) \wedge Q]$

Sol $\Leftrightarrow [Q \vee (P \wedge R)] \wedge [\neg(P \vee R) \vee \neg Q]$

$\Leftrightarrow [Q \vee (P \wedge R)] \wedge [(\neg P \wedge \neg R) \vee \neg Q]$

$\Leftrightarrow [Q \wedge \neg P \wedge \neg R] \vee (Q \wedge \neg Q) \vee (P \wedge R \wedge \neg P \wedge \neg R) \vee (P \wedge R \wedge \neg Q)$ [∵ by distribution law]

$\Leftrightarrow (\neg P \wedge Q \wedge \neg R) \vee F \vee F \vee (P \wedge \neg Q \wedge R)$

$\Leftrightarrow (\neg P \wedge Q \wedge \neg R) \vee (P \wedge \neg Q \wedge R)$ [∵ deleting F's]

Obtain the principal disjunctive and conjunctive NF for the following formulas.

a. (~P v ~Q) → (P ↔ ~Q)

P	Q	~P	~Q	~P v ~Q	P ↔ ~Q	(~P v ~Q) → (P ↔ Q)
F	F	T	T	T	F	F
F	T	T	F	T	T	T
T	F	F	T	T	T	T
T	T	F	F	F	F	T

Minterms corresponding to 3T's

$P \wedge Q$, $P \wedge \neg Q$, $\neg P \wedge Q$

$(\neg P \vee \neg Q) \rightarrow (R \rightarrow Q)$

$\Leftrightarrow (P \wedge Q) \vee (P \wedge \neg Q) \vee (\neg P \vee Q)$

PCNF is the conjunction of maxterms corresponding to F

So, PCNF of

$(\neg P \vee \neg Q) \rightarrow (P \leftrightarrow \neg Q)$

$\Leftrightarrow P \vee Q$

(ii) We shall find the truth table for $(p \vee \neg q) \wedge (\neg p \vee q)$ and then find the PDNF.

p	q	$\neg p$	$\neg q$	$p \vee \neg q$	$\neg p \vee q$	$(p \vee \neg q) \wedge (\neg p \vee q)$
T	T	F	F	T	T	T
T	F	F	T	T	F	F
F	T	T	F	F	T	F
F	F	T	T	T	T	T

(iii)

Table 1.33

p	q	r	$\neg p$	$\neg q$	$\neg r$	$q \wedge r$ $\equiv a$	$p \to a$ $\equiv b$	$\neg q \wedge \neg r$ $\equiv c$	$\neg p \to c$ $\equiv d$	$b \wedge d$
T	T	T	F	F	F	T	T	F	T	T
T	T	F	F	F	T	F	F	F	T	F
T	F	T	F	T	F	F	F	F	T	F
T	F	F	F	T	T	F	F	T	T	F
F	T	T	T	F	F	T	T	F	F	F
F	T	F	T	F	T	F	T	F	F	F
F	F	T	T	T	F	F	T	F	F	F
F	F	F	T	T	T	F	T	T	T	T

PDNF of the given statement $\equiv (p \wedge q \wedge r) \vee (\neg p \wedge \neg q \wedge \neg r)$

PDNF of $\neg(b \wedge d) \equiv (p \wedge q \wedge \neg r) \vee (p \wedge \neg q \wedge r) \vee (p \wedge \neg q \wedge \neg r)$
$\vee (\neg p \wedge q \wedge r) \vee (\neg p \wedge q \wedge \neg r) \vee (\neg p \wedge \neg q \wedge r)$

$\therefore$ PCNF of $(b \wedge d) \equiv (\neg p \vee \neg q \vee r) \wedge (\neg p \vee q \vee \neg r) \wedge (\neg p \vee q \vee r)$
$\wedge (p \vee \neg q \vee \neg r) \wedge (p \vee \neg q \vee r) \wedge (p \vee q \vee \neg r)$

procedure:

step 1 :- Replace the connectives $\rightarrow, \leftrightarrow$ by using $\wedge, \vee, \neg$

step 2 : If negation is present before the given formula or a part of the formula of demorgan's Law

step 3 : If necessary apply distributive of associative and Idempotent Law

Example 1:

$\neg P \vee Q$ find PDNF

$\neg P \vee Q \Leftrightarrow (\neg P \wedge T) \vee (T \wedge Q)$ [$\because P \cdot 1 = P$]

$\Leftrightarrow (\neg P \wedge (Q \vee \neg Q)) \vee (P \vee \neg P \wedge Q)$ [Inverse law]

$\Leftrightarrow ((\neg P \wedge Q) \vee (\neg P \wedge \neg Q)) \vee ((P \wedge Q) \vee (\neg P \wedge Q))$ [$\because$ Distributive Law]

$\Leftrightarrow (\neg P \wedge Q) \vee (\neg P \wedge \neg Q) \vee (P \wedge Q)$

$\therefore$ PDNF of $\neg P \vee Q$ is $(\neg P \wedge Q) \vee (\neg P \wedge \neg Q) \vee (P \wedge Q)$

Note: If there are n minterms

PDNF = R

PCNF = n − R

2.) $(P \wedge Q) \vee (\neg P \wedge Q \wedge R)$ find PCNF

$\Leftrightarrow (P \wedge Q \wedge T) \vee (\neg P \wedge Q \wedge R)$

$\Leftrightarrow [(P \wedge Q) \wedge (R \vee \neg R)] \vee [\neg P \wedge Q \wedge R]$

$\Leftrightarrow [\underbrace{(P \wedge Q \wedge R) \vee (P \wedge Q \wedge \neg R)}_{\downarrow \text{ PDNF}} \vee (\neg P \wedge Q \wedge R)]$

$\therefore$ there are 3 variable $\rightarrow 2^n$

$\rightarrow 2^3 = 8$

PDNF $= 3$

PCNF $= 8 - 3$

$= 5$

Minterms :-

$\boxed{(P \wedge Q \wedge R)}$, $\boxed{(\neg P \wedge Q \wedge R)}$ $(P \wedge \neg Q \wedge R)$, $\boxed{(P \wedge Q \wedge \neg R)}$

$\boxed{(\neg P \wedge \neg Q \wedge R)}$, $(\neg P \wedge Q \wedge \neg R)$, $(P \wedge \neg Q \wedge \neg R)$,

$(\neg P \wedge \neg Q \wedge \neg R)$

The PCNF for $(P \wedge Q) \vee (\neg P \wedge Q \wedge R)$ is

$(P \vee \neg Q \vee R) \wedge (\neg P \vee Q \vee \neg R) \wedge$

$(\neg P \vee \neg Q \vee R) \wedge (P \vee \neg Q \vee \neg R) \wedge$

$(\neg P \vee \neg Q \vee \neg R)$

$\therefore$ Note: DNF $(P \wedge Q) \vee (\neg P \wedge Q \wedge R)$ which is not PDNF

17/07/2021

Inference theory:

1) Rule p : A premise can be introduced at any point in the derivation.

2) Rule T : A formula S can be introduced in a derivation if S is tautologically implied by any one (or) more

of the preceding formulas in the derivatives.

3) Rule CP: To derive $P \to Q$ from the given set of premises, take P as an additional premise along with the given set of premises and prove Q as the conclution.

1. Determine whether the conclusion C follows logically from the premises H_1 and H_2 $H_1: P \to Q$ $H_2: P$ $C: Q$

sol

1. P Rule P
2. $P \to Q$ Rule P
3. Q Rule T $(P \wedge P \to Q \Rightarrow Q)$ or $P \wedge (P \to Q)$

2. $H_1: P \to Q$, $H_2: \neg(P \wedge Q)$, $C: \neg P$

sol

1. $\neg(P \wedge Q)$ Rule P
2. $\neg P \vee \neg Q$ Rule T (1) (Demorgan's law)
3. $P \to Q$ Rule P
4. $\neg P \vee Q$ Rule T (3) $(P \to Q \Leftrightarrow \neg P \vee Q)$

$\boxed{ab+ac = a(b+c)}$

5. $(\neg P \vee Q) \wedge (\neg P \vee \neg Q)$ (2) & (4). Rule T
6. $\neg P \vee (\neg Q \wedge Q)$ Rule T (5)

$\boxed{\neg Q \wedge Q = F}$

7. $\neg P \vee F$ Rule T (6)
8. $\neg P$ Rule T (7)

Note: $P \wedge P \to Q \Rightarrow Q$

$P \to Q \wedge Q \to R \quad P \to R$

$P \to Q \Leftrightarrow \neg P \vee Q$

$P \vee F \Leftrightarrow P$

3. Determine the validity of the arguments

$H_1 : P \to R \quad H_2 : \neg P \to Q \quad H_3 : Q \to S \quad C : \neg R \to S$

Sol ① $\neg P \to Q$ Rule P

② $Q \to S$ Rule P

③ $\neg P \to S$ Rule T ① & ② $P \Rightarrow Q, Q \Rightarrow R \Rightarrow P \to R$

④ $P \to R$ Rule P

⑤ $\neg R \to \neg P$ Rule T ④

⑥ $\neg R \to S$ Rule T $(P \to Q,\ Q \to R,\ P \to R)$ ③ & ⑤

4. PT $R \wedge (P \vee Q)$ is a valid conclusion from $P \vee Q$, $Q \to R$

$P \to M$, $\neg M$

Sol ① $P \to M$ Rule P

② $\neg M$ Rule P

③ $\neg P$ ① & ② Rule T

④ $P \vee Q$ Rule P $\quad P \to Q \Leftrightarrow \neg P \vee Q$

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(5) $\neg P \rightarrow Q$ Rule T $\neg P \rightarrow Q \Leftrightarrow P \vee Q$

(6) Q (3) & (5) Rule T

(7) $Q \rightarrow R$ Rule P

(8) R (6) & (7) Rule T

(9) $R \wedge (P \vee Q)$ (8) & (4) Rule T, $P, Q \Rightarrow P \wedge Q$

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Example: 1. show that $R \vee S$ follows logically from the premises

$C \vee D$, $(C \vee D) \rightarrow \neg H$, $\neg H \rightarrow (A \wedge \neg B)$ and $(A \wedge \neg B) \rightarrow (R \vee S)$.

Sol
(1) $C \vee D$ Rule P

(2) $(C \vee D) \rightarrow \neg H$ Rule P

(1)&(2) (3) $\neg H$ Rule T $P, P \rightarrow Q \Rightarrow Q$

(4) $\neg H \rightarrow (A \wedge \neg B)$ Rule P

(3)&(4) (5) $A \wedge \neg B$ Rule T $P, P \rightarrow Q \Rightarrow Q$

(6) $(A \wedge \neg B) \rightarrow (R \vee S)$ Rule P

(5)&(6) (7) $R \vee S$ Rule T $P, P \rightarrow Q \Rightarrow Q$

2. show that $S \vee R$ is tautologically implied by $(P \vee Q) \wedge (P \rightarrow R) \wedge (Q \rightarrow S)$

Sol
(1) $P \vee Q$ Rule P

(1) (2) $\neg P \rightarrow Q$ Rule T $P \rightarrow Q \Leftrightarrow \neg P \vee Q$ $\boxed{P \rightarrow Q \Leftrightarrow P \vee Q}$

(3) $Q \to S$ Rule P

(2) & (3) (4) $\neg P \to S$ Rule T $P \to Q, Q \to R \Rightarrow P \to R$

(4) (5) $\neg S \to P$ Rule T

(6) $P \to R$ Rule P

(5) & (6) (7) $\neg S \to R$ Rule T

(7) (8) $S \vee R$ Rule T $P \to Q \Leftrightarrow \neg P \vee Q$

3. $R \wedge (P \vee Q)$ is a valid conclusion from the premises $P \vee Q$, $Q \to R$, $P \to M$ and $\neg M$.

Sol

(1) $P \vee Q$ Rule P

(1) (2) $\neg P \to Q$ Rule T

(3) $Q \to R$ Rule P

(2) & (3) (4) $\neg P \to R$ Rule T

(4) (5) $\neg R \to P$ Rule T

(6) $P \to M$ Rule P

(5) & (6) (7) $\neg R \to M$ Rule T

(1) $P \to M$ Rule P

(2) $\neg M$ Rule P

(1) & (2) (3) $\neg P$ Rule T $(\neg Q, P \to Q \Rightarrow \neg P)$

(4) $P \vee Q$ Rule P

(3) & (4) (5) Q Rule T $(\neg P, P \vee Q \Rightarrow Q)$

(6) $Q \to R$ Rule P

(5) & (6) (7) R Rule T $(P, P \to Q \Rightarrow Q)$

(4) & (7) (8) $R \wedge (P \vee Q)$ Rule T $(P, Q \Rightarrow P \wedge Q)$

Here Rule CP:

4. S.T $R \to S$ can be derived from $P \to (Q \to S)$, $\neg R \vee P$

Sol

(1) $\neg R \vee P$ Rule P

(1) (2) $R \to P$ Rule T $(P \to Q \Leftrightarrow \neg P \vee Q)$

(3) $P \to (Q \to S)$ Rule P

(2) & (3) (4) $R \to (Q \to S)$ Rule T

(5) R Rule CP (Assume R as an additional premises.

(4) & (5) (6) $Q \to S$ Rule T $P, P \to Q \Rightarrow Q$

(7) Q Rule P

(6) & (7) (8) S Rule T

thus $R \to S$ is a valid conclusion

Rule CP: If we can derive S from R and given set of premises then we can derive $R \to S$ from the set of premise as (To prove $R \to S$, add R as additional premise and prove S).

Consistency of premises and Indirect method of proof:

Ex:

1. show that the following premises are inconsistent
   1) If Jack misses many classes through illness, then he fails high school $[E \to S]$
   2) If Jack fails high school then he is uneducated $[S \to H]$
   3) If Jack reach a lot of books then he is not uneducated $[A \to \neg H]$
   4) Jack missed many classes through illness and reads $[E \wedge A]$

lot of books

E : Jack missed many classes

S : Jack fails high school

A : Jack reads lot of books

H : Jack is un educated

Sol

∴ the premises are $E \to S$, $S \to H$, $A \to \neg H$, $E \wedge A$

1. $E \to S$ Rule P
2. $S \to H$ Rule P
3. $E \to H$ Rule T 1 & 2
4. $A \to \neg H$ Rule P
5. $H \to \neg A$ Rule T $(P \to Q \Leftrightarrow \neg Q \to \neg P)$ 4
6. $E \to \neg A$ Rule T 3 & 5
7. $E \wedge A$ Rule P
8. E Rule T $(P \wedge Q \Rightarrow P)$ 7
9. A Rule T $(P \wedge Q \Rightarrow Q)$ 7
10. $\neg A$ Rule T $(P, P \to Q \Rightarrow Q)$ 6 & 8
11. $A \wedge \neg A \Leftrightarrow F$ Rule T $(P, Q \Rightarrow P \wedge Q)$ 9 & 10

∴ the given set of premises is inconsistent.

2. Show that the following premises are inconsistent

1) If Rama gets his degree he will go for job [$P \to Q$]

2) If he goes for a job he will get married soon [$Q \to R$]

3) If he goes for higher study he will not get married [$S \to \neg R$]

4) Rama gets his degree and goes for higher study. [$P \wedge S$]

P: Rama gets his degree

Q: He goes for a job

R: He will get married soon

S: He goes for higher study.

Sol) ∴ The premises are ~~$P \to Q, S \to H, A$~~

$P \to Q,\ Q \to R,\ S \to \neg R,\ P \wedge S$

① $P \to Q$ Rule P

② $Q \to R$ Rule P

③ $P \to R$ Rule T ① & ②

④ $S \to \neg R$ Rule P

⑤ $R \to \neg S$ Rule T ($P \to Q \Leftrightarrow \neg Q \to \neg P$) ④

⑥ $P \to \neg S$ Rule T ③ & ⑤

⑦ $P \wedge S$ Rule P

⑧ P Rule T ($P \wedge Q \Rightarrow P$) ⑦

⑨ S Rule T ($P \wedge Q \Rightarrow Q$) ⑦

(10) $\neg S$ Rule T $(P, P \to Q \Rightarrow Q)$ (6) & (8)

(11) $S \wedge \neg S \Leftrightarrow F$ Rule T $(P, Q \Rightarrow P \wedge Q)$ (9) & (10)

$\therefore$ The given set of premises is inconsistent.

<u>Inference theory:</u>

<u>Implications:</u>

I_1 $P \wedge Q \Rightarrow P$

I_2 $P \wedge Q \Rightarrow Q$ (simplification)

I_3 $P \Rightarrow P \vee Q$

I_4 $Q \Rightarrow P \vee Q$ (addition)

I_5 $\neg P \Rightarrow P \to Q$

I_6 $Q \Rightarrow P \to Q$

I_7 $\neg(P \to Q) \Rightarrow P$

I_8 $\neg(P \to Q) \Rightarrow \neg Q$

I_9 $P, Q \Rightarrow P \wedge Q$

I_{10} $\neg P, P \vee Q \Rightarrow Q$ (disjunctive syllogism)

I_{11} $P, P \to Q \Rightarrow Q$ (modus ponens)

I_{12} $\neg Q, P \to Q \Rightarrow \neg P$ (modus tollens)

I_{13} $P \to Q, Q \to R \Rightarrow P \to R$ (hypothetical syllogism)

I_{14} $P \vee Q, P \to R, Q \to R \Rightarrow R$ (dilemma)

Equivalences:

E_1 $\neg\neg P \Leftrightarrow P$ (double negation)

E_2 $P \wedge Q \Leftrightarrow Q \wedge P$ (Commutative laws)

E_3 $P \vee Q \Leftrightarrow Q \vee P$

E_4 $(P \wedge Q) \wedge R \Leftrightarrow P \wedge (Q \wedge R)$ (associative laws)

E_5 $(P \vee Q) \vee R \Leftrightarrow P \vee (Q \vee R)$

E_6 $P \wedge (Q \vee R) \Leftrightarrow (P \wedge Q) \vee (P \wedge R)$ (distributive laws)

E_7 $P \vee (Q \wedge R) \Leftrightarrow (P \vee Q) \wedge (P \vee R)$

E_8 $\neg(P \wedge Q) \Leftrightarrow \neg P \vee \neg Q$ (De morgan's laws)

E_9 $\neg(P \vee Q) \Leftrightarrow \neg P \wedge \neg Q$

E_{10} $P \vee P \Leftrightarrow P$

E_{11} $P \wedge P \Leftrightarrow P$

E_{12} $R \vee (P \wedge \neg P) \Leftrightarrow R$

E_{13} $R \wedge (P \vee \neg P) \Leftrightarrow R$

E_{14} $R \vee (P \vee \neg P) \Leftrightarrow T$

E_{15} $R \wedge (P \wedge \neg P) \Leftrightarrow F$

E_{16} $P \rightarrow Q \Leftrightarrow \neg P \vee Q$

E_{17} $\neg(P \rightarrow Q) \Leftrightarrow P \wedge \neg Q$

E_{18} $P \rightarrow Q \Leftrightarrow \neg Q \rightarrow \neg P$

E_{19} $P \rightarrow (Q \rightarrow R) \Leftrightarrow (P \wedge Q) \rightarrow R$

E_{20} $\neg(P \rightleftarrows Q) \Leftrightarrow P \rightleftarrows \neg Q$

E_{21} $P \rightleftarrows Q \Leftrightarrow (P \rightarrow Q) \wedge (Q \rightarrow P)$

E_{22} $(P \rightleftarrows Q) \Leftrightarrow (P \wedge Q) \vee (\neg P \wedge \neg Q)$

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Indirect method: In order to prove a conclusion C follows from the premises $H1, H_2, ---- H_n$,

- Assume $\neg C$ as an additional premise
- prove contradiction or F for $H1, H_2, --- H_n, \neg C$

problem:

1. show that b can be derived from the premises $a \to b$ ①, $c \to b$ ②, $d \to (a \vee c)$ ③, d ④ by the indirective method

sol let us include $\neg b$ as an additional premise

To prove: contradiction

① $a \to b$ Rule P

② $c \to b$ Rule P

③ $(a \vee c) \to b$ Rule T $\left(\begin{matrix} P \to Q \\ R \to Q \end{matrix} \Rightarrow (P \vee R) \to Q\right)$ ① & ②

④ $d \to (a \vee c)$ Rule P

⑤ $d \to b$ Rule T $\left(\begin{matrix} P \to Q \\ Q \to R \end{matrix} \Rightarrow P \to R\right)$ ③ & ④

⑥ d Rule P

⑦ b Rule T $(P, P \to Q \Rightarrow Q)$ ⑤ & ⑥

⑧ $\neg b$ Rule P

(9) $b \land \neg b$ Rule T $(P, Q \Rightarrow P \land Q)$ (7) & (8)

(10) F   Rule T $(P \land \neg P \Leftrightarrow F)$ (9)

predicate calculus:

$p(x)$: x is studying in C_1 section

$p(\text{vasu})$: vasu is studing in C_1 section

$p(x, y)$: x is friend of y

$p(\text{melvin}, \text{vasu})$: melvin is friend of vasu.

Rule US universal specification:

- conclude $p(c)$ is true if for all x, $p(x)$ is true where c is arbitrary $\dfrac{\forall x\, p(x)}{\therefore p(y)}$

Rule ES Existential specification:

- conclude $p(c)$ is true if there exists x, $p(x)$ is true where c is not arbitrary but one for which $p(c)$ is true.

$\dfrac{P(c) \text{ for any } c}{\therefore \forall x\, p(x)}$

Rule UG Universal Generalisation:

- For all x, $p(x)$ is true if $p(c)$ is true where c is arbitrary member

$\dfrac{\exists x\, p(x)}{\therefore p(c) \text{ for any } c}$

Rule EG Existential Generalisation:

$p(c)$ for any c $\therefore \exists x\, p(x)$

- There exists x, $p(x)$ is true

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$U = \{1, 2, 3, --- 10\}$

$A = \{1, 3, 5, 7, 9\}$

$A' = \{2, 4, 6, 8, 10\}$

$p(x)$: x got pass mark in DM

U : {c, section}

$[\forall x, p(x), \quad (\boxed{\exists} x)\ p(x)]$ Quantifiers

Symbolize :

1. All men are mortal $\quad$ [$\forall x$: All Every anyone]

$p(x)$: x is a man

$Q(x)$: x is mortal $\quad$ [$\exists$: atleast]

① $\forall x\ p(x) \to Q(x)$

② $\forall x \quad p(x) \wedge Q(x)$

2. Every CS student needs a course in Mathematics

$M(x)$: x needs course in mathematics

where x consists of all CS students

$(\forall x)\ m(x)$

3. there is a student in this class who owns a personal computer

$p(x)$: x owns a personal computer

$(\exists x)\ p(x)$ – Symbolic Representation

4. Every student in this class has taken atleast one mathematics course

$Q(x,y)$: x has taken y

x – students in this class

y – all mathematics courses

$(\forall x)(\exists y)\ Q(x,y)$

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Eg: 1) $p(x)$: x got pass mark

$(\forall x)\, p(x)$ _ Everyone got pass mark in this class

some students got fail mark

$-(\exists x)\, F(x)$

2) prove

All men are mortal $-(\forall x)\, p(x) \rightarrow m(x)$

socrates is a man - Socrates ~~on~~ $p(s)$

so socrates is mortal conclusion $m(s)$

$p(x)$: x is a man

$m(x)$: x is mortal

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① $(\forall x)\, p(x) \rightarrow m(x)$ Rule p

② $p(s) \rightarrow m(s)$ US ①

③ $p(s)$ Rule p

④ $m(s)$ Rule T $(P, P \rightarrow Q \Rightarrow Q)$

m(s): socrates is mortal

3. Show that the presmises, "one student in this class knows how to write Java programming and Everyone ~~who~~ who knows Java programming can get high paying job imply the conclussion some in this class can get high paying job".

Sol C(x) – x is in this class

J(x) – x knows Java

H(x) – x can get high paying job

① $(\exists x)\; c(x) \wedge J(x)$

② $(\forall x)\; J(x) \rightarrow H(x)$

③ $(\exists x)\; c(x) \wedge H(x)$

step.No	st	Reason
①	$(\exists x)[C(x) \wedge J(x)]$	Rule p
②	$(C(a) \wedge J(a))$	ES

(3) $C(a)$ Rule T, $P \wedge Q \Rightarrow P$

(4) $J(a)$ Rule T, $P \wedge Q \Rightarrow Q$

(5) $(\forall x)\ J(x) \rightarrow H(x)$ Rule P

(6) $J(a) \rightarrow H(a)$ US

(7) $H(a)$ Rule T $(P, P \rightarrow Q \Rightarrow Q)$ (4) & (6)

(8) $C(a) \wedge H(a)$ Rule T, $(P, Q \Rightarrow P \wedge Q)$

(9) $(\exists x)\ (C(x) \wedge H(x))$ Rule EG

4. PT $(\forall x)\ P(x) \longrightarrow Q(x), (\forall x)\ Q(x) \rightarrow R(x)$

$\Rightarrow (\forall x)\ P(x) \longrightarrow R(x)$

Sol (1) $(\forall x)\ P(x) \rightarrow Q(x)$ Rule P

(2) $P(y) \longrightarrow Q(y)$ Rule US

(3) $(\forall x)\ Q(x) \longrightarrow R(x)$ Rule P

(4) $Q(y) \longrightarrow R(y)$ Rule US

(5) $P(y) \rightarrow R(y)$ Rule T (2) & (4)

(6) $(\forall x)\ P(x) \rightarrow R(x)$ Rule UG