05.01.2022

# Memoryless property in Geometric Distribution.

**Statement:-** If $x$ has a Geometric Distribution then for any two integers $m$ and $n$ such that

$$P(x > m+n / x > m) = P(x > n)$$

**Pf:-** Given that '$x$' is a Geometric Distribution.

WKT, $P(x = x) = q^{x-1} \cdot p \; ; \; x = 1, 2, 3, \dots \infty$

Now $P(x > k) = \sum_{x=k+1}^{\infty} P(x = x)$

$$= \sum_{x=k+1}^{\infty} q^{x-1} \cdot p$$

$$= p\left(q^{k} + q^{k+1} + q^{k+2} + \cdots\right)$$

$$= p \cdot q^{k}\left(1 + q + q^{2} + \cdots\right)$$

$$= p \cdot q^{k}(1-q)^{-1} \quad \left(\text{Binomial series, } (1+x+x^2+\cdots = (1-x)^{-1}\right)$$

$$= p \cdot q^{k} \cdot p^{-1} \quad (\because p+q = 1 \Rightarrow p = 1-q)$$

$$= \cancel{p} \cdot q^{k} \cdot \frac{1}{\cancel{p}}$$

$$\Rightarrow P(x > k) = q^{k} \quad \text{--- } (1)$$

**LHS:-** $P(x > m+n / x > m) = \dfrac{P(x > m+n \cap x > m)}{P(x > m)}$

$$\left(\because P(A/B) = \frac{P(A \cap B)}{P(B)}\right)$$

Let $m = 3$, $n = 5$

$m + n = 3 + 5 = 8$

$m+n = 3+5 = 8$

$>m+n = >8 = 9, 10, 11, 12, \dots$

w.k.t $>m = >3 = 4, 5, 6, 7, 8, 9, 10, 11, 12, \dots$

$>m+n \cap >m = 9, 10, 11, 12, \dots$

$\therefore >m+n \cap >m = >m+n$ — (2)

$$P(x>m+n/x>m) = \frac{P(x>m+n)}{P(x>m)} \quad (\text{by (2) eqn.})$$

(1) ⇒ $P(x>k) = q^k$

$$P(x>m+n/x>m) = \frac{q^{m+n}}{q^m} = \frac{\cancel{q^m} \cdot q^n}{\cancel{q^m}}$$

$$= q^n$$

$$\Rightarrow P(x>m+n/x>m) = P(x>n) \quad (\text{by (1) eqn.})$$

## UNIFORM DISTRIBUTION (OR) RECTANGULAR DISTRIBUTION

**Defn:-**

A cont. R.V $x$ is said to be uniform Distribution if its p.d.f is given by,

$$f(x) = \begin{cases} \frac{1}{b-a} & ; a < x < b \\ 0 & ; \text{otherwise} \end{cases}$$

Its mean is $\frac{b+a}{2}$

and variance is $\frac{(b-a)^2}{12}$

**PROBLEMS:-**

(1) Electric trains on a certain line run every half an hour between midnight and 6 in the morning. What is the prob. that a man entering into the station at a random time during this period will have to wait atleast 20 min?

**Sol:-** Let the R.V $x$ denotes the waiting time in mins for the next train.

Given that a man enter into the station at random.

the next train.

Given that a man enter into the station at random.

$\therefore$ 'x' is uniformly in the interval $(0, 30)$

$\therefore$ Its p.d.f is, $f(x) = \begin{cases} \frac{1}{30-0} ; & 0<x<30 \\ 0 ; & \text{otherwise} \end{cases}$

ie) $f(x) = \begin{cases} \frac{1}{30} ; & 0<x<30 \\ 0 ; & \text{otherwise} \end{cases}$

P(A man waiting atleast 20 min for the train) $= P(x \geq 20)$

$= P(20 \leq x < 30)$

$= \int_{20}^{30} f(x)\,dx$

$= \int_{20}^{30} \frac{1}{30}\,dx$

$= \frac{1}{30}\int_{20}^{30} dx$

$= \frac{1}{30}(x)_{20}^{30}$

$= \frac{1}{30}(30-20)$

$= \frac{1}{30} \times 10$

P(A man waiting atleast 20 min for the train) $= \frac{1}{3}$

2) If x is uniformly distributed with mean 1 and Variance is $4/3$. Find $P(x<0)$

Sol: Given that x is U.D

Its mean $= \frac{b+a}{2} = 1 \Rightarrow b+a = 2$ — ①

and variance $= \frac{(b-a)^2}{12} = \frac{4}{3}$

$$\Rightarrow (b-a)^2 = \frac{4 \times \cancel{12}^{4}}{\cancel{3}}$$

$$\Rightarrow (b-a)^2 = 4^2$$

$$\Rightarrow b-a = 4 \quad \text{—— (2)}$$

(1) + (2) $\Rightarrow$ $2b = 6 \Rightarrow b = 6/2 = 3$

$$\boxed{b=3}$$

Substitute b value in (1) eq.,

(1) $\Rightarrow$ $b+a = 2$

$3+a = 2$

$a = 2-3 = -1$

$$\boxed{a=-1}$$

WKT, The p.d.f of a U.D is,

$$f(x) = \begin{cases} \frac{1}{b-a} ; & a < x < b \\ 0 ; & \text{otherwise} \end{cases}$$

$\therefore$ The p.d.f of a U.D is,

$$f(x) = \begin{cases} \frac{1}{3-(-1)} ; & -1 < x < 3 \\ 0 ; & \text{otherwise} \end{cases}$$

$$f(x) = \begin{cases} \frac{1}{4} ; & -1 < x < 3 \\ 0 ; & \text{otherwise} \end{cases}$$

$\therefore$ $P(x<0) = P(-1<x<0)$

$$= \int_{-1}^{0} f(x)dx$$

$$= \int_{-1}^{0} \frac{1}{4} \cdot dx$$

$$= \frac{1}{4}(x)_{}^{0}$$

$-1$

$$= \frac{1}{4}\left(x\right)_{-1}^{0}$$

$$= \frac{1}{4}\left(0-(-1)\right) = \frac{1}{4}(1)$$

$$\boxed{P(x<0) = \frac{1}{4}}$$