03. 01. 2022

GEOMETRIC DISTRIBUTION (G.D)

Defn:

A Discrete R.V 'x' is said to be Geometric Distribution if its p.m.f is,

$$P(X=x) = P(x) = q^{x-1}\cdot p\ ; \quad x = 1,2,3,\ldots\infty$$

Its Mean is q/p and variance is q/p^2.

PROBLEMS:

① Suppose that a trainee soldier shoots a target in an independent fashion if the prob. that the target is shoot on any one shot is 0.8

(i) What is the prob. that the target would be hit on 6^{th} attempt.

(ii) What is the prob. that it takes less than 5 attempts.

(iii) What is the prob. that it takes even number of shots.

Sol:

Given that a trainee soldier to shoot a target is 0.8.

ie) $p = 0.8$ (Prob. of success)

WKT, $q = 1-p = 1-0.8 = 0.2$

Since the p.m.f for a G.D is,

$$P(X=x) = P(x) = q^{x-1}\cdot p\ ; \quad x = 1,2,3,\ldots\infty$$

(i) What is the prob. that the target would be hit on 6^{th} attempt.

$$P(X=6) = q^{6-1}\cdot p = q^5\cdot p$$

$$= (0.2)^5(0.8)$$

$$P(X=6) = 0.000256$$

(ii) What is the prob. that it takes less than 5 attempts.

$$P(x<5) = P(x=1) + P(x=2) + P(x=3) + P(x=4)$$

$$= q^0 p + q^1 p + q^2 p + q^3 p$$

$$= p(1 + q + q^2 + q^3)$$

$$= (0.8)(1 + 0.2 + (0.2)^2 + (0.2)^3)$$

$$\Rightarrow P(x<5) = 0.9984$$

(iii) What is the prob. that it takes even number of shots.

$$P(\text{Even no. of Shots}) = P(x=2) + P(x=4) + P(x=6) + \cdots$$

$$= q^1 \cdot p + q^3 \cdot p + q^5 \cdot p + \cdots$$

$$= qp(1 + q^2 + q^4 + \cdots)$$

$$= qp(1 + q^2 + (q^2)^2 + \cdots)$$

$$= qp(1-q^2)^{-1}$$

$$\left(\because (1-x)^{-1} = 1 + x + x^2 + x^3 + \cdots\right)$$

$$= \frac{qp}{(1-q^2)^1}$$

$$= \frac{(0.2)(0.8)}{(1-(0.2)^2)}$$

$$\therefore P(\text{Even no. of Shots}) = 0.1667$$

2) A and B shoot independently until each has hits own target. The prob. of their hitting the target at each shot is $3/5$ and $5/7$ respectively. Find the prob. that B will require more shots than A.

(x)

Sol:

Let X = No. of trails required by A to get his first success.

In G.D the p.m.f is,

$$P(X=x) = P(x) = q_1^{x-1} \cdot p_1 ; \quad x = 1,2,3,\ldots\infty$$

$$P(X=x) = \left(\frac{2}{5}\right)^{x-1} \cdot \left(\frac{3}{5}\right) ; \quad x = 1,2,3,\ldots\infty$$

Let Y = No. of trails required by B to get his first success.

$$P(Y=x) = q_2^{x-1} \cdot p_2 ; \quad x = 1,2,3,\ldots\infty$$

$$P(Y=x) = \left(\frac{2}{7}\right)^{x-1} \cdot \left(\frac{5}{7}\right) ; \quad x = 1,2,3,\ldots\infty$$

P(B require more trails to get his 1st success than A requires his 1st success)

$$= \sum_{r=1}^{\infty} P(x = r \text{ and } y = r+1, r+2, \ldots)$$

$$= \sum_{r=1}^{\infty} P(X=r) \cdot P(Y = r+1, r+2, \ldots)$$

(∵ If A and B are indep. then $P(A \cap B) = P(A) \cdot P(B)$)

$$= \sum_{r=1}^{\infty} \left(\frac{2}{5}\right)^{r-1} \cdot \left(\frac{3}{5}\right) \cdot \sum_{k=1}^{\infty} \left(\frac{2}{7}\right)^{r+k-1} \cdot \left(\frac{5}{7}\right)$$

$$= \frac{3}{\cancel{5}} \cdot \frac{\cancel{5}}{7} \sum_{r=1}^{\infty} \left(\frac{2}{5}\right)^{r-1} \cdot \sum_{k=1}^{\infty} \left(\frac{2}{7}\right)^{r+k-1}$$

$$= \frac{3}{7} \sum_{r=1}^{\infty} \left(\frac{2}{5}\right)^{r-1} \cdot \sum_{k=1}^{\infty} \left(\frac{2}{7}\right)^{r-1} \cdot \left(\frac{2}{7}\right)^{k}$$

$$= \frac{3}{7} \sum_{r=1}^{\infty} \left(\frac{2}{5}\right)^{r-1} \cdot \left(\frac{2}{7}\right)^{r-1} \sum_{k=1}^{\infty} \left(\frac{2}{7}\right)^{k}$$

$$= \frac{3}{7} \sum \left(\frac{2}{5} \cdot \frac{2}{7}\right)^{r-1} \cdot \sum \left(\frac{2}{7}\right)^{k}$$

$$= \frac{3}{7}\sum_{r=1}^{\infty}\left(\frac{2}{5}\cdot\frac{2}{7}\right)^{r-1}\cdot\sum_{k=1}^{\infty}\left(\frac{2}{7}\right)^{k}$$

$$= \frac{3}{7}\sum_{r=1}^{\infty}\left(\frac{4}{35}\right)^{r-1}\cdot\sum_{k=1}^{\infty}\left(\frac{2}{7}\right)^{k}$$

$$= \frac{3}{7}\left[1+\left(\frac{4}{35}\right)+\left(\frac{4}{35}\right)^{2}+\cdots\right]\cdot\left[\left(\frac{2}{7}\right)+\left(\frac{2}{7}\right)^{2}+\left(\frac{2}{7}\right)^{3}+\cdots\right]$$

$$= \frac{3}{7}\left[1-\frac{4}{35}\right]^{-1}\cdot\left(\frac{2}{7}\right)\left[1+\left(\frac{2}{7}\right)+\left(\frac{2}{7}\right)^{2}+\cdots\right]$$

$$= \frac{6}{49}\left[\frac{31}{35}\right]^{-1}\cdot\left[1-\frac{2}{7}\right]^{-1}$$

$$= \frac{6}{49}\cdot\frac{35}{31}\cdot\left(\frac{5}{7}\right)^{-1}$$

$$= \frac{6}{\cancel{49}_{\,7}}\cdot\frac{\cancel{35}}{31}\cdot\frac{\cancel{7}}{\cancel{5}}$$

$$= \frac{6}{31}$$

$\therefore$ P(B require more trails to get his 1st success than A requires his 1st success) $= \frac{6}{31}$