14.12.2021

① In a bag contains 5 Red balls, 3 White balls and 5 Black balls. What is the probability if i drawn 2 balls randomly they are (i) 1 Red ball & 1 black ball (ii) both are white balls (iii) 1 black ball and 1 white ball? (iv) both are Red balls or both are white balls?

Sol:- total balls in a bag = 5R + 3W + 5B = 13 balls

WKT,

$$P(\text{Event}) = \frac{\text{No. of favourable to the event}}{\text{total no. of exhaustive events}}$$

and $\times$

or $+$

(i) $$P(1R \,\&\, 1B) = \frac{5C_1 \times 5C_1}{13C_2} = \frac{5 \times 5}{78} = \frac{25}{78}$$

$$\therefore P(1R \,\&\, 1B) = \frac{25}{78}$$

(ii) $$P(\text{Both are white balls}) = \frac{3C_2}{13C_2} = \frac{3}{78} = \frac{1}{26}$$

$$\therefore P(\text{Both are white balls}) = \frac{1}{26}$$

(iii) $$P(1B \,\&\, 1W) = \frac{5C_1 \times 3C_1}{13C_2} = \frac{5 \times \cancel{3}^{1}}{\cancel{78}_{26}} = \frac{5}{26}$$

$$\therefore P(1B \,\&\, 1W) = \frac{5}{26}$$

(iv) $$P(\text{Both are black balls (or) Both are white balls}) = \frac{5C_2 + 3C_2}{13C_2}$$

$$= \frac{10+3}{78} = \frac{13}{78} = \frac{1}{6}$$

$$P(\text{Both are black balls (or) Both are white balls}) = \frac{1}{6}$$

2) What is the prob. that 53 Sundays in a leap year?

Sol:-

In a leap year, the total no. of days = 366 days

Out of 366 days = 52 weeks + 2 days

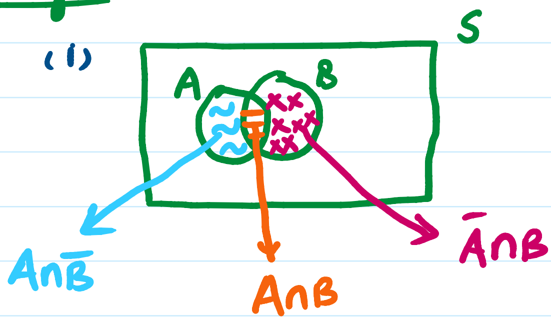
= 52 Sundays + 2 days

2 days = {M&T, T&W, W&Th, Th&F, F&Sa, Sa&S, S&M}

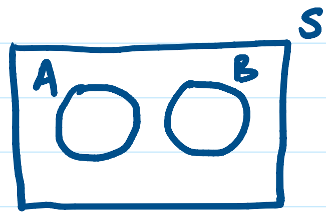
$\therefore$ $P(53 \text{ Sundays}) = \frac{2}{7}$

Venn diagram:-

(i)



(ii)



$A \cap B = \phi$ then A and B are M.E.E's

Independent events:-

A and B are independent events if and only if $P(A \cap B) = P(A) \cdot P(B)$

Addition Law on Prob:-

If A and B are any two events then

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Suppose A and B are M.E.E's then

$$P(A \cup B) = P(A) + P(B)$$

Theorem:-

If A and B are indep. events then Prove that

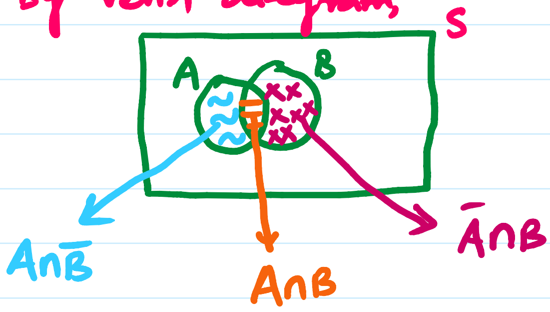
(i) $\bar{A}$ and B are indep.

(ii) A and $\bar{B}$ are indep.

(iii) $\bar{A}$ and $\bar{B}$ are also indep.

(iii) $\bar{A}$ and $\bar{B}$ are also indep.

Sol:- By venn diagram,



Given that A and B are indep events.

ie) $P(A\cap B) = P(A)\cdot P(B)$ — ①

(i) TP: $\bar{A}$ and B are indep :-

ETP: $P(\bar{A}\cap B) = P(\bar{A})\cdot P(B)$

from the venn diagram,

$$B = (A\cap B) \cup (\bar{A}\cap B)$$

$$P(B) = P[(A\cap B)\cup(\bar{A}\cap B)]$$

Since $A\cap B$ and $\bar{A}\cap B$ are M.E.E's

$\therefore$ $P(B) = P(A\cap B) + P(\bar{A}\cap B)$ (by axiom(iv))

$$P(B) - P(A\cap B) = P(\bar{A}\cap B)$$

$P(B) - P(A)\cdot P(B) = P(\bar{A}\cap B)$ (by using ①)

$$P(B)[1 - P(A)] = P(\bar{A}\cap B)$$

$P(B)\,P(\bar{A}) = P(\bar{A}\cap B)$ (by axiom (iii) total prob = 1 ie) $P(A) + P(\bar{A}) = 1$ $\Rightarrow P(\bar{A}) = 1 - P(A)$)

$\Rightarrow$ $P(\bar{A}\cap B) = P(B)\cdot P(\bar{A})$

$\therefore$ $\bar{A}$ and B are indep. events.

(ii) TP: A and $\bar{B}$ are indep. events

ETP: $P(A\cap\bar{B}) = P(A)\cdot P(\bar{B})$

By venn diagram,

$A = (A\cap B)\cup(A\cap\bar{B})$

$P(A) = P[(A\cap B)\cup(A\cap\bar{B})]$

Since $A\cap B$ and $A\cap\bar{B}$ are m.E.E's

$P(A) = P(A\cap B) + P(A\cap\bar{B})$ (by axiom (iv))

$P(A) - P(A\cap B) = P(A\cap\bar{B})$

$P(A) - P(A)\cdot P(B) = P(A\cap\bar{B})$ (by using (I))

$P(A)[1-P(B)] = P(A\cap\bar{B})$

$P(A)\cdot P(\bar{B}) = P(A\cap\bar{B})$ ($\because$ $P(B)+P(\bar{B}) = 1$ $\Rightarrow P(\bar{B}) = 1-P(B)$)

$\Rightarrow$ $P(A\cap\bar{B}) = P(A)\cdot P(\bar{B})$

$\therefore$ A and $\bar{B}$ are indep. events.

(iii) TP: $\bar{A}$ and $\bar{B}$ are indep events:-

ETP: $P(\bar{A}\cap\bar{B}) = P(\bar{A})\cdot P(\bar{B})$

WKT, $P(\bar{A}) = 1-P(A)$

Now $P(\bar{A}\cap\bar{B}) = P(\overline{A\cup B})$ (by Demorgan Law)

$= 1 - P(A\cup B)$

$= 1 - [P(A) + P(B) - P(A\cap B)]$

(by Addition Law on Prob.)

$= 1 - P(A) - P(B) + P(A\cap B)$

$$= P(\bar{A}) - \underbrace{P(B) + P(A)\cdot P(B)}\ (\text{by } ①)$$

$$= P(\bar{A}) - P(B)\underbrace{(1 - P(A))}$$

$$= P(\bar{A}) - P(B)\cdot P(\bar{A})$$

$$= P(\bar{A})\left[1 - P(B)\right]$$

$$\Rightarrow P(\bar{A} \cap \bar{B}) = P(\bar{A})\cdot P(\bar{B})$$

hence $\bar{A}$ and $\bar{B}$ are indep. events.