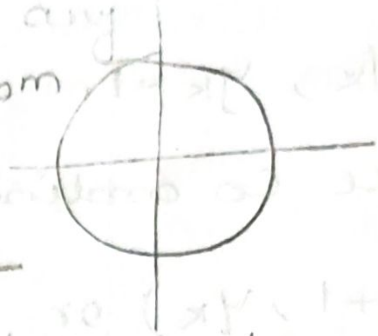
Mid-Point Circle Algm:

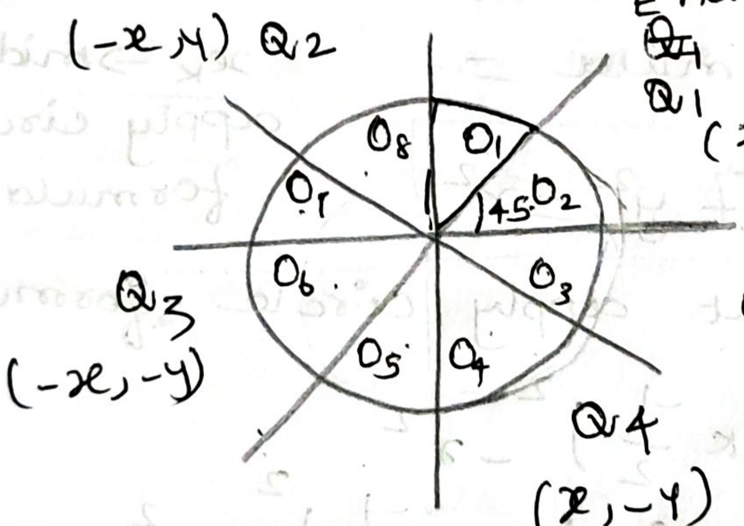
As we know, the distance is



* For a circle, at any point the distance from origin is radius.

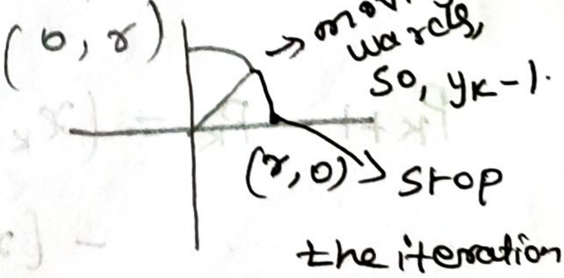
* That's why follow the symmetric property of a circle, Quadrant, Octant

as the radius is same, we can simply assume, if we calculate the first point, Remaining are same.



End Point of O_1 = $x \geq y$.

To find Quadrant of First Octant.



x - Unit intervals.

y - ?

y_k, $y_k - 1$.

Next co ordinate may be,

(x_k+1, y_k) or (x_k+1, y_k-1).

$\hookrightarrow$ depends upon the decision Parameter.

mid point $= \left(\frac{x_k+1 + x_k+1}{2}, \frac{y_k + y_k-1}{2} \right)$

$$\boxed{\text{mid Point} = x_k+1, y_k - \tfrac{1}{2}}$$

circle formula =

$$\boxed{x^2 + y^2 = r^2}$$

$x_k \Rightarrow$ mid point apply circle formula.

In mid point apply circle formula,

$P_k = (x_k+1)^2 + (y_k - \frac{1}{2})^2 - r^2$

$P_{k+1} = (x_{k+1}+1)^2 + (y_{k+1} - \frac{1}{2})^2 - r^2$.

$$P_{k+1} - P_k = (x_{k+1}+1)^2 + (y_{k+1} - \frac{1}{2})^2 - \cancel{r^2} - (x_k+1)^2 - (y_k - \tfrac{1}{2})^2 + \cancel{r^2}$$

x moves in unit interval. So x_{k+1} replace with x_k+1

$$=(\underbrace{(x_k+1)}_{a}+\underbrace{1}_{b})^2+(y_{k+1}-1/2)^2-(x_k+1)^2-(y_k-1/2)^2$$

Use $(a+b)^2$

↳ keep same. because whether we select y_{k+1} or y_k)

$$=\cancel{(x_k+1)^2}+2(x_k+1)+y_{k+1}^2+\cancel{1/4}-$$

$$y_{k+1}-\cancel{(x_k+1)^2}-y_k^2-\cancel{1/4}+y_k+1$$

$$=P_{k+1}-P_k=2(x_k+1)+y_{k+1}^2-y_k^2)-$$

$$(y_{k+1}-y_k)+1.$$

$$\boxed{P_{k+1}=P_k+2(x_k+1)+(y_{k+1}^2-y_k^2)-(y_{k+1}-y_k)+1.}$$

Initial Decision parameter,

$(0,r)$ → Starting point

0 ↳ x_k, r ↳ y_k.

$P_k=(x_k+1)^2+(y_k-\frac{1}{2})^2-r^2.$

↳ Substitute in P_k. calculate x_k & y_k value.

$$P_k=(0+1)^2+(r-1/2)^2-r^2$$

$$=1+\cancel{r^2}+1/4-r-\cancel{r^2}$$

$$P_k=5/4-r.$$

↳ consider only integer part. avoid fractional part.

$$\boxed{P_K = 1 - r.}$$

$$\boxed{P_{K+1} = P_K + 2(x_K + 1) + (y_{K+1}^2 - y_K^2) - (y_{K+1} - y_K) + 1.}$$

if $(P_K \geq 0) = y_{K+1} = y_K - 1.$

$NE = (x_K + 1,\ y_K - 1)$

if $(P_K < 0)$ $y_{K+1} = y_K.$

$NE = (x_K + 1,\ y_K).$

Stop the iteration $x \geq y$.

based on the first octant, we can fill remaing octant.

<u>Example:</u>

$r = 8.$

$P_0 = 1 - r = 1 - 8 = \boxed{-7.}$

$$P_{k+1} = P_k + 2(x_k+1) + (y_{k+1}^2 - y_k^2) - (y_{k+1} - y_k) + 1$$

Example:

K	(x_k, y_k)	P_k	x_{k+1}, y_{k+1}
0	(0, 8)	-7	(1, 8)

$y_{k+1} = 8 - y_k = 8 = 0$

$$\boxed{P_{k+1} = -7 + 2(\overset{x_k}{0}+1) + 0 - 0 + 1 = -7+2+1 = -4.}$$

(1, 8)	$(\overset{x_k}{1}, 8)$	-4	(2, 8).

$$\boxed{P_{k+1} = -4 + 2(1+1) + 0 - 0 + 1 = -4 + 4 + 1 = 1.}$$

2	(2, 8)	1	(3, 7).

$$P_{k+1} = 1 + 2(2+1) + (\overset{7^2}{49} - \overset{8^2}{64}) - (7-8) + 1$$

$$\boxed{= 1 + 6\cancel{8} - 15 + 1 + 1 = 9 - 15 = -6}$$

3	~~(7, 8)~~ (3, 7)	-6	$(4, \overset{7}{\cancel{8}})$.

$$\boxed{P_{k+1} = -6 + 2(3+1) + 0 - 0 + 1 = -6 + 8 + 1 = 3.}$$

4.	$(4, \overset{7}{\cancel{8}})$	3	(5, 6)

$$\boxed{P_{k+1} = 3 + 2(4+1) + (36 - 49) - (6-7) + 1 = 3 + 10 - 13 + 1 + 1 = 2}$$

5	(5, 6)	2	(6, 5) $(x \geq y)$

(0, 8), (1, 8) (2, 8) (3, 7) (4, 7),
(5, 6), (6, 5) First octant points.

Q_1	$Q(-x, y)$	$Q_3 (-x, -y)$	$Q_4 (x, -y)$
(0, 8)	(0, 8)	(0, 8)	(0, 8)
(1, 8)	(-1, 8)	(-1, -8)	(1, -8)
(2, 8)	(-2, 8)	(-2, -8)	(2, -8)
(3, 7)	(-3, 7)	(-3, -7)	(3, -7)
(4, 7)	(-4, 7)	(-4, -7)	(4, -7)
(5, 6) = O_1	(-5, 6)	(-5, -6)	(5, -6)
(6, 5.) Q_2	(-6, 5)	(-6, -5)	(6, -5)
(7, 4)	(-7, 4)	(-7, -4)	(7, -4)
(7, 3)	(-7, 3)	(-7, -3)	(7, -3)
(8, 2)	(-8, 2)	(-8, -2)	(8, -2)
(8, 1)	(-8, 1)	(-8, -1)	(8, -1)
(8, 0)	(-8, 0)	(-8, 0)	(8 - 0).