SMTA1402 - Probability and Statistics

Unit-2 Probability Distribution

Discrete type

Binomial distribution:

A random variable X is said to follow binomial distribution if it assumes only non negative values and its probability mass function is given by $P(X=x)=p(x)=\begin{cases} nC_x p^x q^{n-x}, x=0,1,2,\ldots,n; q=1-p \\ 0, otherwise \end{cases}$

Notation: $X \sim B(n,p)$ read as X is following binomial distribution with parameter n and p.

Problem.1

Find m.g.f. of Binomial distribution and find its mean and variance.

Solution:

M.G.F.of Binomial distribution:-

$$M_X(t)=E\left[e^{tx}\right]=\sum_{x=0}^{n} e^{tx} P(X=x)$$

$$=\sum_{x=0}^{n} nC_x x\, P^x q^{n-x} e^{tx}$$

$$=\sum_{x=0}^{n} nC_x \left(pe^t\right)^x q^{n-x}$$

$$M_X(t)=\left(q+pe^t\right)^n$$

Mean of Binomial distribution

$$\text{Mean}=E(X)=M_X'(0)$$

$$=\left[n\left(q+pe^t\right)^{n-1} pe^t\right]_{t=0}=np \text{ Since } q+p=1$$

$$E\left(X^2\right)=M_X''(0)$$

$$=\left[n(n-1)\left(q+pe^t\right)^{n-2}\left(pe^t\right)^2+npe^t\left(q+pe^t\right)^{n-1}\right]_{t=0}$$

$$E\left(X^2\right)=n(n-1)p^2+np$$

$$=n^2p^2+np(1-p)=n^2p^2+npq$$

$$\text{Variance}=E\left(X^2\right)-\left[E[X]\right]^2=npq$$

$$Mean=np\,; Variance=npq$$

Problem.2

Comment the following: “The mean of a binomial distribution is 3 and variance is 4

Solution:

In binomial distribution, mean $>$ variance but Variance $<$ Mean

Since Variance $=4$ & Mean $=3$, the given statement is wrong.

Problem.3

If X and Y are independent binomial variates $B\left(5,\frac{1}{2}\right)$ and $B\left(7,\frac{1}{2}\right)$ find $P[X+Y=3]$

Solution:

$X+Y$ is also a binomial variate with parameters $n_1+n_2=12$ & $p=\frac{1}{2}$

$$\therefore P[X+Y=3]=12C_3\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^9=\frac{55}{2^{10}}$$

Problem.4

(i). Six dice are thrown 729 times. How many times do you expect atleast 3 dice show 5 or 6 ?

(ii) Six coins are tossed 6400 times. Using the Poisson distribution, what is the approximate probability of getting six heads 10 times?

Solution:

(i). Let X be the number of times the dice shown 5 or 6

$$P[5\ or\ 6]=\frac{1}{6}+\frac{1}{6}=\frac{1}{3}$$

$$\therefore P=\frac{1}{3} \text{ and } q=\frac{2}{3}$$

Here $n=6$

By Binomial theorem,

$$P[X=x]=6C_x\left(\frac{1}{3}\right)^x\left(\frac{2}{3}\right)^{6-x} \text{ where } x=0,1,2\ldots6.$$

$$P[X\geq 3]=P(3)+P(4)+P(5)+P(6)$$

$$=6C_3\left(\frac{1}{3}\right)^3\left(\frac{2}{3}\right)^3+6C_4\left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right)^2+6C_5\left(\frac{1}{3}\right)^5\left(\frac{2}{3}\right)+6C_6\left(\frac{1}{3}\right)^6$$

$$=0.3196$$

$\therefore$ Expected number of times atleast 3 dies to show 5 or 6 $=N\times P[X\geq 3]$

$$=729\times 0.3196=233.$$

(ii). Probability of getting six heads in one toss of six coins is $p=\left(\frac{1}{2}\right)^6$,

$\lambda=np=6400\times\left(\frac{1}{2}\right)^6=100$

Let X be the number of times getting 6 heads $P(X=10)=\dfrac{e^{-100}(100)^{10}}{10!}=1.025\times 10^{-30}$

Poisson distribution:

A random variable X is said to follow Poisson distribution if it assumes only non negative values and its probability mass function is given by

$$P(X=x)=\begin{cases}\dfrac{e^{-\lambda}\lambda^x}{x!}; x=0,1,2,\ldots;\lambda>0\\ 0, otherwise\end{cases}$$

Notation: $X \sim P(\lambda)$ read as X is following Poisson distribution with parameter λ.

Poisson distribution as limiting form of binomial distribution:

Poisson distribution is a limiting case of Binomial distribution under the following conditions:

(i). n the number of trials is indefinitely large, (i.e.) $n \to \infty$

(ii). p the constant probability of success in each trial is very small (i.e.) $p \to 0$

(iii). $np = \lambda$ is finite.

Proof:

$$P(X = x) = p(x) = nc_x \, p^x \, q^{n-x}$$

Let $np = \lambda$

$$\therefore \qquad p = \frac{\lambda}{n}, \; q = 1 - \frac{\lambda}{n}$$

$$\therefore \qquad p(x) = nc_x \left(\frac{\lambda}{n}\right)^x \left(1 - \frac{\lambda}{n}\right)^{n-x}$$

$$= \frac{n!}{x!\,(n-x)!}\left(\frac{\lambda}{n}\right)^x \left(1 - \frac{\lambda}{n}\right)^{n-x}$$

$$= \frac{n(n-1)\cdots(n-(x-1))\,\cancel{(n-x)!}}{x!\,\cancel{(n-x)!}}\left(\frac{\lambda}{n}\right)^x \left(1 - \frac{\lambda}{n}\right)^{n-x}$$

$$= \frac{1.\left(1-\frac{1}{n}\right)\cdots\left(1-\frac{x-1}{n}\right)}{x!}\cancel{n^x}\,\frac{\lambda^x}{\cancel{n^x}}\left(1 - \frac{\lambda}{n}\right)^{n-x}$$

$$p(x) = 1.\left(1-\frac{1}{n}\right)\cdots\left(1-\frac{x-1}{n}\right)\frac{\lambda^x}{x!}\left(1 - \frac{\lambda}{n}\right)^{n-x}$$

Taking limit $n \to \infty$ on both sides

$$\lim_{n\to\infty} p(x) = \frac{\lambda^x}{x!}\lim_{n\to\infty}\left[\left(1-\frac{1}{n}\right)\cdots\left(1-\frac{x-1}{n}\right)\left(1-\frac{\lambda}{n}\right)^{n-x}\right]$$

$$= \frac{\lambda^x}{x!}\lim_{n\to\infty}\left[\left(1-\frac{1}{n}\right)\cdots\left(1-\frac{x-1}{n}\right)\right]\lim_{n\to\infty}\left(1-\frac{\lambda}{n}\right)^{-x}\lim_{n\to\infty}\left(1-\frac{\lambda}{n}\right)^{n}$$

$$P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!}; x = 0,1,2,\ldots$$

Problem.1

Criticise the following statement: “The mean of a Poisson distribution is 5 while the standard deviation is 4”.

Solution:

For a Poisson distribution mean and variance are same. Hence this statement is not true.

Geometric distribution:

A random variable X is said to have a Geometric distribution if it assumes only non negative values and its probability mass function is given by

$$P(X=x)=\begin{cases} q^{x-1}p; x=1,2,...;0<p\le 1 \\ 0, otherwise \end{cases}$$

Problem.1

Find the Moment generating function of geometric distribution and find its Mean and Variance

Solution:

$$M_X(t)=E\left(e^{tX}\right)$$

$$=\sum_{x=1}^{\infty} e^{tx}q^{x-1}p$$

$$=\sum_{x=1}^{\infty} pe^t\left(qe^t\right)^{x-1}$$

$$=pe^t\left(1+qe^t+\left(qe^t\right)^2+\cdots\right)$$

$$=pe^t\left(1-qe^t\right)^{-1}$$

$$M_X(t)=\frac{pe^t}{1-qe^t}$$

$$\mu_1'=M_x'(0)=\left[\frac{d}{dt}\left(\frac{pe^t}{\left(1-qe^t\right)}\right)\right]_{t=0}=\left[\left(\frac{pe^t}{\left(1-qe^t\right)^2}\right)\right]_{t=0}=\frac{1}{p}$$

$$\mu_2'=M_x''(0)=\left[\frac{d^2}{dt^2}\left(\frac{pe^t}{\left(1-qe^t\right)}\right)\right]_{t=0}=\left[\frac{d}{dt}\left(\frac{pe^t}{\left(1-qe^t\right)^2}\right)\right]_{t=0}=\frac{1+q}{p^2}$$

Mean $=\mu_1'=\dfrac{1}{p}$

Variance $=\mu_2'-\left(\mu_1'\right)^2=\dfrac{1+q}{p^2}-\left(\dfrac{1}{p}\right)^2=\dfrac{q}{p^2}$

Problem.2

State and prove Memoryless property of geometric distribution.

Solution:

If X has a geometric distribution, then for any two positive integer $'s'$ and $'t'$ $P\left[X>s+t/X>s\right]=P[X>t]$.

The p.m.f of the geometric random variable X is $P(X=x)=q^{x-1}p$, $x=1,2,3,....$

$$P\left[X>s+t/X>s\right]=\frac{P[X>s+t\cap X>s]}{P[X>s]}=\frac{P[X>s+t]}{P[X>s]}\;------(1)$$

$$\therefore P[X>t]=\sum_{x=t+1}^{\infty}q^{x-1}p=q^{t}p+q^{t+1}p+q^{t+2}p+\ldots.=q^{t}p\left[1+q+q^{2}+q^{3}+\ldots.\right]$$

$$=q^{t}p(1-q)^{-1}=q^{t}p(p)^{-1}=q^{t}$$

Hence $P[X>s+t]=q^{s+t}$ and $P[X>s]=q^{s}$

$$(1)\Rightarrow P\left[X>s+t\Big/X>s\right]=\frac{P[X>s+t\cap X>s]}{P[X>s]}=\frac{q^{s+t}}{q^{s}}=q^{t}=P[X>t]$$

$$\Rightarrow P\left[X>s+t\Big/X>s\right]=P(X>t)$$

Problem.3

If the probability is $\frac{1}{4}$ that a man will hit a target what is the chance that he will hit the target for the first time in the 7th trial?

Solution:

The required probability is

$$P[FFFFFFS]=P(F)P(F)P(F)P(F)P(F)P(F)P(S)$$

$$=q^{6}p=\left(\frac{3}{4}\right)^{6}.\left(\frac{1}{4}\right)=0.0445.$$

Problem.4

A die is cast until 6 appears what is the probability that it must cast more then five times?

Solution:

Probability of getting six $=\frac{1}{6}$

$\therefore p=\frac{1}{6}$ & $q=1-\frac{1}{6}$

Let x: No of throws for getting the number 6. By geometric distribution $P[X=x]=q^{x-1}p, x=1,2,3\ldots.$

Since 6 can be got either in first, second……throws.

To find $P[X\geq 6]=1-P[X<6]$

$$=1-\sum_{x=1}^{5}\left(\frac{5}{6}\right)^{x-1}.\frac{1}{6}$$

$$=1-\left[\left(\frac{1}{6}\right)+\left(\frac{5}{6}\right)\left(\frac{1}{6}\right)+\left(\frac{5}{6}\right)^{2}\left(\frac{1}{6}\right)+\left(\frac{5}{6}\right)^{3}\left(\frac{1}{6}\right)+\left(\frac{5}{6}\right)^{4}\left(\frac{1}{6}\right)\right]$$

$$=1-\frac{\frac{1}{6}\left[1-\left(\frac{5}{6}\right)^{5}\right]}{1-\frac{5}{6}}=\left(\frac{5}{6}\right)^{5}=0.4019$$

Problem.5

Suppose that a trainee soldier shoots a target an independent fashion. If the probability that the target is shot on any one shot is 0.8.

(i) What is the probability that the target would be hit on 6th attempt?

(ii) What is the probability that it takes him less than 5 shots?

Solution:

Here $p = 0.8, q = 1 - p = 0.2$

$$P[X = x] = q^{x-1}p, x = 1,2...$$

(i) The probability that the target would be hit on the 6th attempt $= P[X = 6]$

$$= (0.2)^5 (0.8) = 0.00026$$

(ii) The probability that it takes him less than 5 shots $= P[X < 5]$

$$= \sum_{x=1}^{4} q^{x-1}p = 0.8\sum_{x=1}^{4}(0.2)^{x-1}$$

$$= 0.8[1 + 0.2 + 0.04 + 0.008] = 0.9984$$

Continuous type

Uniform (or) Rectangular distribution:

A continuous random variable X is said to have a uniform distribution over an interval (a,b) if its probability density function is given by

$$f(x) = \begin{cases} \dfrac{1}{b-a}, a < x < b \\ 0, otherwise \end{cases}$$

Problem.1

If X is uniformly distributed with Mean 1 and Variance $\dfrac{4}{3}$, find $P[X > 0]$

Solution:

If X is uniformly distributed over (a,b), then

$$E(X) = \frac{b+a}{2} \text{ and } V(X) = \frac{(b-a)^2}{12}$$

$$\therefore \frac{b+a}{2} = 1 \Rightarrow a + b = 2$$

$$\Rightarrow \frac{(b-a)^2}{12} = \frac{4}{3} \Rightarrow (b-a)^2 = 16$$

$\Rightarrow a + b = 2$ & $b - a = 4$ We get $b = 3, a = -1$

$\therefore a = -1$ & $b = 3$ and probability density function of x is

$$f(x) = \begin{cases} \dfrac{1}{4}; -1 < x < 3 \\ 0\,; Otherwise \end{cases}$$

$$P[x < 0] = \int_{-1}^{0} \frac{1}{4}dx = \frac{1}{4}[x]_{-1}^{0} = \frac{1}{4}.$$

Exponential distribution:

A continuous random variable X assuming non negative values is said to have an exponential distribution with parameter $\theta > 0$, if its probability density function is given by

$$f(x) = \begin{cases} \lambda e^{-\lambda x}, x \geq 0 \\ 0, otherwise \end{cases}$$

Problem.1

Find the moment generating function of Exponential distribution and find its mean and variance.

Solution:

We know that $f(x)=\begin{cases}\lambda e^{-\lambda x}, x\ge 0\\ 0, otherwise\end{cases}$

$$M_X(t)=E\left(e^{tx}\right)=\int_0^\infty e^{tx}f(x)dx=\int_0^\infty \lambda e^{-\lambda x}e^{tx}dx$$

$$=\lambda\int_0^\infty e^{-x(\lambda-t)}dx$$

$$=\lambda\left[\frac{e^{-x(\lambda-t)}}{-(\lambda-t)}\right]_0^\infty=\frac{\lambda}{\lambda-t}$$

$$\text{Mean}=\mu_1'=\left[\frac{d}{dt}M_X(t)\right]_{t=0}=\left[\frac{\lambda}{(\lambda-t)^2}\right]_{t=0}=\frac{1}{\lambda}$$

$$\mu_2'=\left[\frac{d^2}{dt^2}M_X(t)\right]_{t=0}=\left[\frac{\lambda(2)}{(\lambda-t)^3}\right]_{t=0}=\frac{2}{\lambda^2}$$

$$\text{Variance}=\mu_2'-\left(\mu_1'\right)^2=\frac{2}{\lambda^2}-\frac{1}{\lambda^2}=\frac{1}{\lambda^2}.$$

Problem.2

State and prove the memoryless property of exponential distribution.

Solution:

Statement:

If X is exponentially distributed with parameters λ, then for any two positive integers's' and't', $P[x>s+t/x>s]=P[x>t]$

Proof:

The p.d.f of X is $f(x)=\begin{cases}\lambda e^{-\lambda x}, x\ge 0\\ 0 \quad ,Otherwise\end{cases}$

$$\therefore P[X>k]=\int_k^\infty \lambda e^{-\lambda x}dx=\left[-e^{-\lambda x}\right]_k^\infty=e^{-\lambda k}$$

$$\therefore P[X>s+t/x>s]=\frac{P[x>s+t\cap x>s]}{P[x>s]}$$

$$=\frac{P[X>s+t]}{P[X>s]}=\frac{e^{-\lambda(s+t)}}{e^{-\lambda s}}=e^{-\lambda t}$$

$$=P[x>t]$$

Problem.3

A component has an exponential time to failure distribution with mean of 10,000 hours.

(i). The component has already been in operation for its mean life. What is the probability that it will fail by 15,000 hours?

(ii). At 15,000 hours the component is still in operation. What is the probability that it will operate for another 5000 hours.

Solution:

Let X denote the time to failure of the component then X has exponential distribution with $Mean = 1000$ hours.

$$\therefore \frac{1}{\lambda} = 10,000 \Rightarrow \lambda = \frac{1}{10,000}$$

The p.d.f. of X is $f(x) = \begin{cases} \frac{1}{10,000} e^{-\frac{x}{10,000}} & , x \geq 0 \\ 0 & , otherwise \end{cases}$

(i) Probability that the component will fail by 15,000 hours given it has already been in operation for its mean life $= P\left[x < 15,000 / x > 10,000\right]$

$$= \frac{P\left[10,000 < X < 15,000\right]}{P\left[X > 10,000\right]}$$

$$= \frac{\int_{10,000}^{15,000} f(x)\,dx}{\int_{10,000}^{\infty} f(x)\,dx} = \frac{e^{-1} - e^{-1.5}}{e^{-1}}$$

$$= \frac{0.3679 - 0.2231}{0.3679} = 0.3936.$$

(ii) Probability that the component will operate for another 5000 hours given that it is in operational 15,000 hours $= P\left[X > 20,000 / X > 15,000\right]$

$= P\left[x > 5000\right]$ [By memoryless prop]

$$= \int_{5000}^{\infty} f(x)\,dx = e^{-0.5} = 0.6065$$

Normal distribution:

A random variable X is said to have a Normal distribution with parameters μ (mean) and σ^2 (variance) if its probability density function is given by the probability law

$$f(x)=\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2},-\infty<x<\infty,-\infty<\mu<\infty,\sigma>0$$

Notation: $X \sim N\left(\mu,\sigma^2\right)$ read as X is following normal distribution with mean μ and variance σ^2 are called parameter.

Problem.1

Prove that "For standard normal distribution $N(0,1)$, $M_X(t)=e^{\frac{t^2}{2}}$.

Solution:

Moment generating function of Normal distribution

$$=M_X(t)=E\left[e^{tx}\right]$$

$$=\frac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}^{\infty}e^{tx}e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}dx$$

Put $z=\frac{x-\mu}{\sigma}$ then $\sigma dz=dx,\ -\infty<Z<\infty$

$$\therefore M_X(t)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}e^{t(\sigma z+\mu)-\frac{z^2}{2}}dz$$

$$=\frac{e^{\mu t}}{\sqrt{2\pi}}\int_{-\infty}^{\infty}e^{-\left(\frac{z^2}{2}-t\sigma z\right)}dz$$

$$=\frac{e^{\mu t}}{\sqrt{2\pi}}\int_{-\infty}^{\infty}e^{-\frac{1}{2}(z-t\sigma)^2+\left(\frac{\sigma^2t^2}{2}\right)}dz$$

$$=\frac{e^{\mu t}e^{\frac{\sigma^2t^2}{2}}}{\sqrt{2\pi}}\int_{-\infty}^{\infty}e^{-\frac{1}{2}(z-t\sigma)^2}dz$$

$\because$ the total area under normal curve is unity, we have $\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}e^{-\frac{1}{2}(z-t\sigma)^2}dz=1$

Hence $M_X(t)=e^{\mu t+\frac{\sigma^2t^2}{2}}$ $\therefore$ For standard normal variable $N(0,1)$

$$M_X(t)=e^{\frac{t^2}{2}}$$

Problem.2

State and prove the additive property of normal distribution.

Solution:

Statement:

If $X_1, X_2, ..., X_n$ are n independent normal random variates with mean $\left(\mu_1, \sigma_1^2\right)$, $\left(\mu_2, \sigma_2^2\right), ... \left(\mu_n, \sigma_n^2\right)$ then $X_1 + X_2 + ... + X_n$ also a normal random variable with mean $\left(\sum_{i=1}^{n} \mu_i, \sum_{i=1}^{n} \sigma_i^2\right)$.

Proof:

We know that. $M_{X_1+X_2+...+X_n}(t) = M_{X_1}(t) M_{X_2}(t) ... M_{X_n}(t)$

But $M_{X_i}(t) = e^{\mu_i t + \frac{t^2 \sigma_i^2}{2}}$, $i = 1, 2....n$

$$M_{X_1+X_2+...+X_n}(t) = e^{\mu_1 t + \frac{t^2 \sigma_1^2}{2}} e^{\mu_2 t + \frac{t^2 \sigma_2^2}{2}} ... e^{\mu_n t + \frac{t^2 \sigma_n^2}{2}}$$

$$= e^{(\mu_1 + \mu_2 + ... + \mu_n)t + \frac{\left(\sigma_1^2 + \sigma_2^2 + ... + \sigma_n^2\right)t^2}{2}}$$

$$= e^{\sum_{i=1}^{n} \mu_i t + \frac{\sum_{i=1}^{n} \sigma_i^2 t^2}{2}}$$

By uniqueness MGF, $X_1 + X_2 + ... + X_n$ follows normal random variable with parameter $\left(\sum_{i=1}^{n} \mu_i, \sum_{i=1}^{n} \sigma_i^2\right)$.

This proves the property.

Problem.3

X is a normal variate with $mean = 30$ and $S.D = 5$ Find the following $P[26 \le X \le 40]$

Solution:

$X \sim N\left(30, 5^2\right)$

$\therefore \mu = 30 \;\&\; \sigma = 5$

Let $Z = \dfrac{X - \mu}{\sigma}$ be the standard normal variate

$$P[26 \le X \le 40] = P\left[\frac{26-30}{5} \le Z \le \frac{40-30}{5}\right]$$

$$= P[-0.8 \le Z \le 2] = P[-0.8 \le Z \le 0] + P[0 \le Z \le 2]$$

$$= P[0 \le Z\ 0.8] + [0 \le z \le 2]$$

$$= 0.2881 + 0.4772 = 0.7653.$$

Problem.4

The average percentage of marks of candidates in an examination is 45 will a standard deviation of 10 the minimum for a pass is 50%.If 1000 candidates appear for the examination, how many can be expected marks. If it is required, that double that number should pass, what should be the average percentage of marks?

Solution:

Let X be marks of the candidates

Then $X \sim N\left(42, 10^2\right)$

Let $z = \dfrac{X - 42}{10}$

$P[X > 50] = P[Z > 0.8]$

$$= 0.5 - P[0 < z < 0.8]$$
$$= 0.5 - 0.2881 = 0.2119$$

Since 1000 students write the test, nearly 212 students would pass the examination.

If double that number should pass, then the no of passes should be 424.

We have to find z_1, such that $P[Z > z_1] = 0.424$

$$\therefore P[0 < z < z_1] = 0.5 - 0.424 = 0.076$$

From tables, $z = 0.19$

$$\therefore z_1 = \frac{50 - x_1}{10} \Rightarrow x_1 = 50 - 10z_1$$
$$= 50 - 1.9 = 48.1$$

The average mark should be 48 nearly.

Problem.5

Given that X is normally distribution with mean 10 and probability $P[X > 12] = 0.1587$. What is the probability that X will fall in the interval $(9,11)$.

Solution:

Given X is normally distributed with mean $\mu = 10$.

Let $z = \frac{x - \mu}{\sigma}$ be the standard normal variate.

For $X = 12, z = \frac{12 - 10}{\sigma} \Rightarrow z = \frac{2}{\sigma}$

Put $z_1 = \frac{2}{\sigma}$

Then $P[X > 12] = 0.1587$

$$P[Z > Z_1] = 0.1587$$
$$\therefore 0.5 - p[0 < z < z_1] = 0.1587$$
$$\Rightarrow P[0 < z < z_1] = 0.3413$$

From area table $P[0 < z < 1] = 0.3413$

$$\therefore Z_1 = 1 \Rightarrow \frac{2}{\sigma} = 1$$

To find $P[9 < x < 11]$

For $X = 9,\ z = -\frac{1}{2}$ and $X = 11,\ z = \frac{1}{2}$

$$\therefore P[9 < X < 11] = P[-0.5 < z < 0.5]$$
$$= 2P[0 < z < 0.5]$$
$$= 2 \times 0.1915 = 0.3830$$

31. In a normal distribution 31% of the items are under 45 and 8% are over 64.Find the mean and standard deviation of the distribution.

Solution:

Let μ be the mean and σ be the standard deviation.

Then $P[X \le 45] = 0.31$ and $P[X \ge 64] = 0.08$

When $X = 45$, $Z = \dfrac{45-\mu}{\sigma} = -z_1$

$\therefore z_1$ is the value of z corresponding to the area $\int_0^{z_1} \phi(z)dz = 0.19$

$\therefore z_1 = 0.495$

$45 - \mu = -0.495\sigma$ ---(1)

When $X = 64$, $Z = \dfrac{64-\mu}{\sigma} = z_2$

$\therefore z_2$ is the value of z corresponding to the area $\int_0^{z_2} \phi(z)dz = 0.42$

$\therefore z_2 = 1.405$

$64 - \mu = 1.405\sigma$ ---(2)

Solving (1) & (2) We get $\mu = 10$ (approx) & $\sigma = 50$ (approx)