26/07/2021

UNIT-2: SET THEORY

Set: A set is a collection of well defined objects (distinct)

$A = \{1, 2, 3, 4\}$

$B = \{1, 2, 2, 3, 3, 3, 4, 4, 4\}$

$= \{1, 2, 3, 4\}$

Set Representation:

1) list Representation

$A = \{2, 4, 6, 8, 10\}$

$B = \{x / x$ in a possitive integers$\}$

$p(x) \rightarrow$ no. of +ve integers.

$B = \{x / p(x)\}$

$B = \{1, 2, 3, 4, 5, - - - \}$

$C = \{2, 4, 6, - - - \}$

$C = \{x / x$ is a +ve even integers$\}$

$B = \{x / x$ in an alphabet in English$\}$

$B = \{a, b, c, d, - - - \}$

Special sets:

Standard notations used to define some sets:

a) N-set of all natural numbers

b) Z-set of all integers

c) Q-set of all rational numbers

d) R-set of all real numbers

e) C-set of all complex numbers

Types of set:

1) Subset: If every element of a set A is also an element of set B, we say set A is a subset of set B $A \subseteq B$

ex: If $A = \{1, 2, 3, 4, 5, 6\}$ and $B = \{1, 2, 3, 4\}$

$\Rightarrow B \subseteq A$

2) Equal sets: 2 sets A and B are called equal if they have equal numbers and similar types of elements.

i.e $A \subseteq B$ and $B \subseteq A$. $\Rightarrow A = B$.

Eg: If $A = \{1, 3, 4, 5, 6\}$

$B = \{4, 1, 5, 6, 3\}$ the both set A and B are equal.

3) Empty sets: A set which does not contain any element is called as Empty set (or) Null set (or) void set. Denoted by ϕ (or) $\{\}$

Eg: a) The set of whole numbers less than 0.

b) clearly there is no whole number less than 0. Therefore, it is empty set.

c) $N = \{x : x \in N, 3 < x < 4\}$

let $A = \{x : 2 < x < 3, x$ is a natural number$\}$

Here A is an empty set because there is no natural number between 2 and 3.

let $B = \{x : x$ is a composite number less than $4\}$

Here B is an empty set because there is no composite number less than 4.

Singleton set: A singleton set is a set containing exactly one element.

eg: let $B = \{x : x$ is a even prime number$\}$

Here B is a singleton set because there is only one prime number which is even, i.e 2.

$A = \{x : x$ is neither prime nor composite$\}$

It is a singleton set containing one element, i.e 1.

Finite set: A set which contains a definite number of elements is called a finite set. Empty set is also called a finite set.

eg: The set of all colors in the rainbow

$N = \{x : x \in N, x < 7\}$

$P = \{2, 3, 5, 7, 11, 13, 17, --- 97\}$

Cardinal Number of a set: The number of distinct elements in a given set A is called the cardinal number of A. It is denoted by n(A).

eg: $A = \{x : x \in N, x < 5\}$

$A = \{1, 2, 3, 4\}$

$\therefore n(A) = 4$

B = set of letters in the word ALGEBRA.

Infinite set: the set whose elements cannot be listed, i.e set containing never-ending elements is called infinite set.

eg: Set of all points in a plane $A = \{x : x \in N, x > 1\}$

set of all prime numbers $B = \{x : x \in W, x = 2n\}$

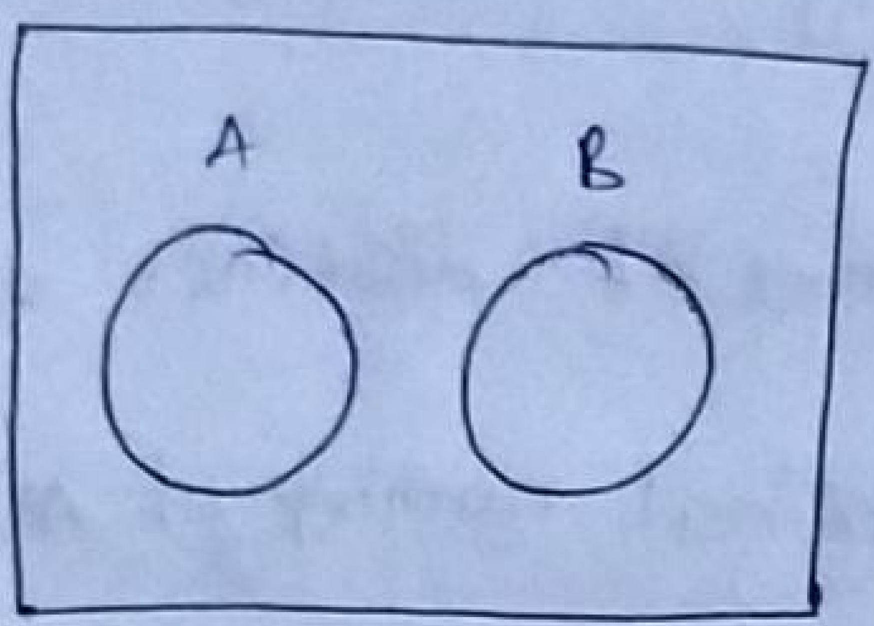
Note: All infinite sets cannot be expressed in roster form.

Disjoint sets: 2 sets A and B are said to be disjoint, if they do not have any element in common.

eg: $A = \{x : x$ is a prime number$\}$

$B = \{x : x$ is a composite number$\}$.

clearly, A and B do not have any element in common and are disjoint sets.



$A \cap B = \phi$

$A = \{1, 2, 3, 4\}$ $B = \{a, b, c, d\}$

power set: the collection of all subsets of set A is called the power set of A. denoted by $p(A)$. In $p(A)$ every element is a set.

eg: If $A = \{p, q\}$ then all the subsets of A will be

$p(A) = \{\phi, \{p\}, \{q\}, \{p, q\}\}$

No. of elements of $p(A) = n[p(A)] = 4 = 22$

In general, $n[p(A)] = 2m$ where m is the no. of elements in set A.

$A = \{1, 2, 3, 4, 5, 6\}$

$B = \{1\}, \{2\}, --- \{6\}, \{1, 2\}, \{2, 3\}$

$p(A) = \{$subsets of A$\}$ $A = \{1, 2, 3\}$

$p(A) = 2^n = 2^3 = 8$

$p(A) = \{\{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}, \phi\}$

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Universal set: A set which contains all the elements of other given sets is called a universal set. The symbol for denoting a universal set is U (or) ξ.

eg: 1) if $A = \{1,2,3\}$ $B = \{2,3,4\}$ $C = \{3,5,7\}$

then $U = \{1,2,3,4,5,7\}$

Here $A \subseteq U$, $B \subseteq U$, $C \subseteq U$ and $U \supseteq A$, $U \supseteq B$, $U \supseteq C$

2) If P is a set of ~~all~~ whole numbers and Q is a set of all negative numbers then the universal set is a set of all integers.

3) If $A = \{a,b,c\}$ $B = \{d,e\}$ $C = \{f,g,h,i\}$

then $U = \{a,b,c,d,e,f,g,h,i\}$ can be taken as universal set.

$A - B = \{x / x \in A$ & $x \notin B\}$

$A = \{1,2,3,4,5\}$

$B = \{4,5\}$

$A - B = \{1,2,3\} \neq A$

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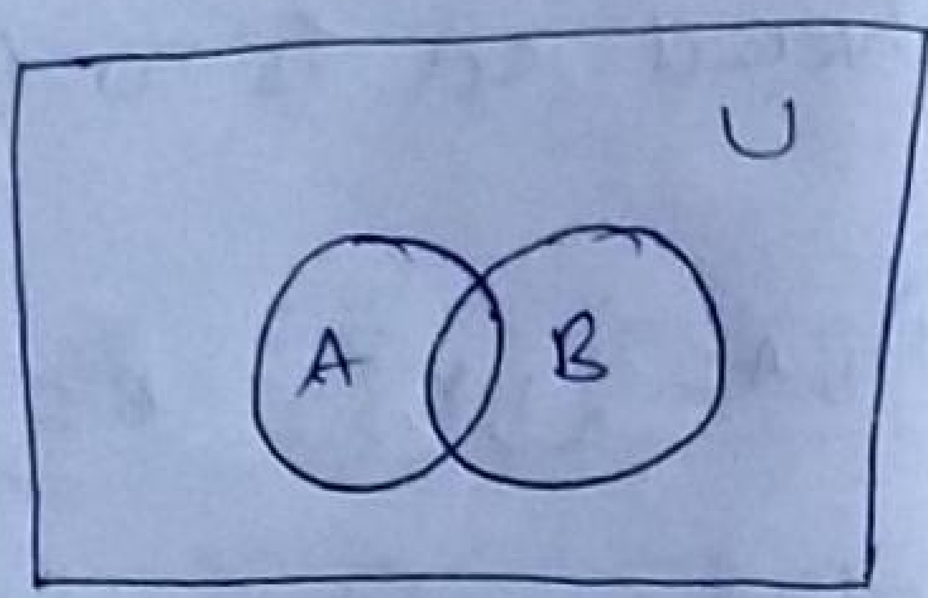
Set operations:

1) union: let A and B be sets. The union of the sets A and B, denoted by $A \cup B$, is the set: $\{x / x \in A \vee x \in B\}$

eg: what is $\{1,2,3\} \cup \{3,4,5\}$?

$\Rightarrow \{1,2,3,4,5\}$

venn diagram for $A \cup B$

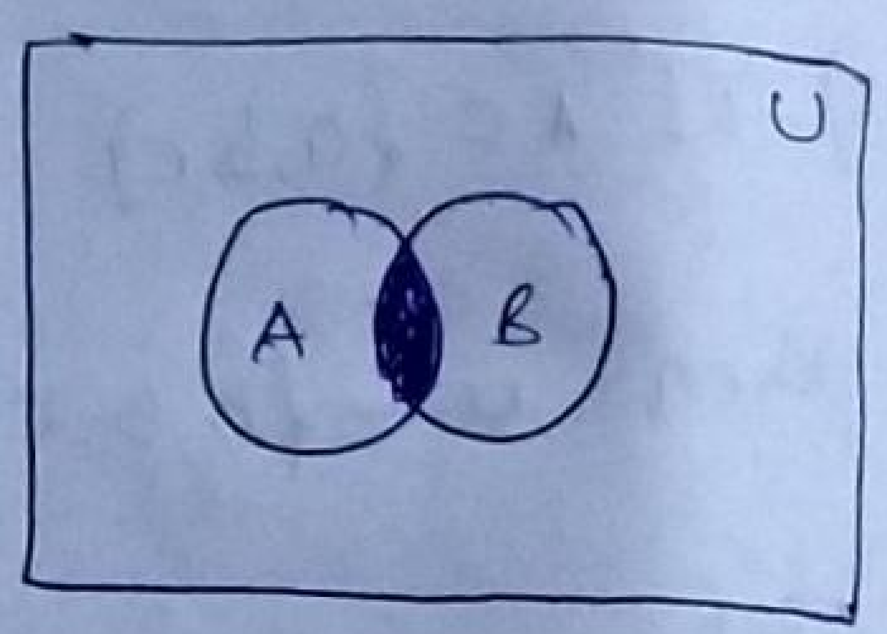


2) Intersection: The intersection of sets A and B, denoted by $A \cap B$ is $\{x / x \in A \wedge x \in B\}$

Note: If the intersection is empty, then A and B are said to be disjoint.

Example: 1) what is $\{1,2,3\} \cap \{3,4,5\}$?

$\Rightarrow \{3\}$



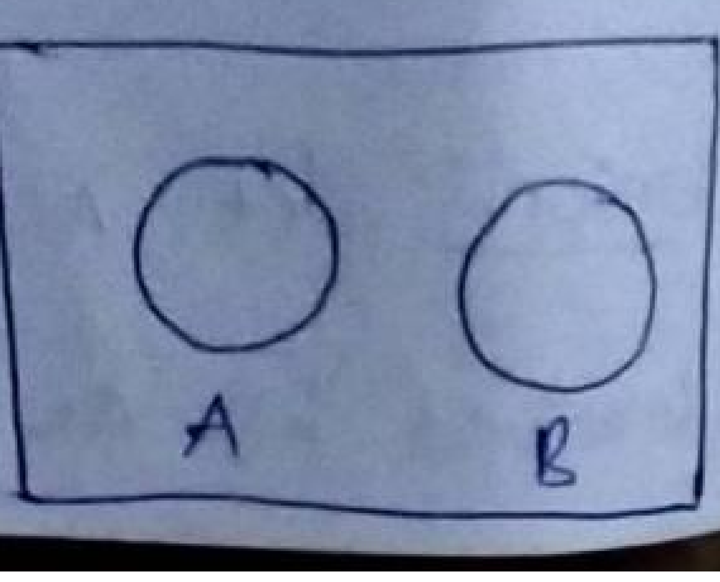
2) what is $\{1,2,3\} \cap \{4,5,6\}$?

$\Rightarrow \emptyset$

3) Disjoined sets: let A and B be 2 sets. They are said to be disjoined iff $A \cap B = \emptyset$ i.e A and B have no elements in common.

eg: $A = \{2,4,6\}$ $B = \{1,3,5\}$ here $A \cap B = \emptyset$

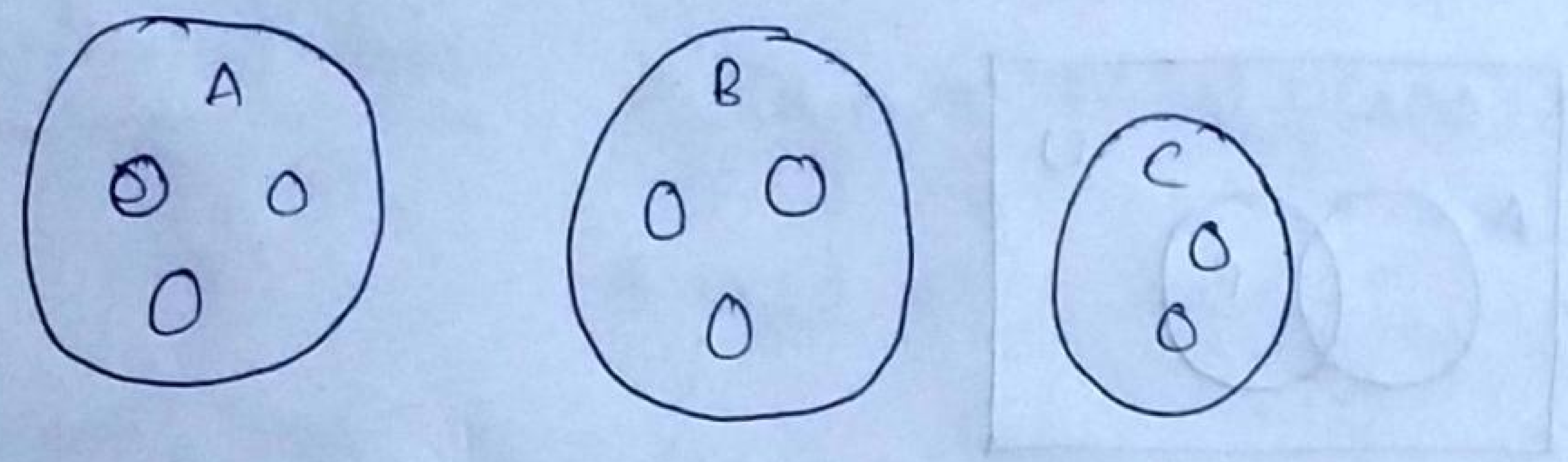
It means that A and B have no elements in common so they are disjoined sets



4) Disjoined collection of sets: let A_1, A_2, A_3 --- are disjoined sets

let $S = \{A_1, A_2, A_3 ---\}$ Here 'S is called the disjoined collection of sets iff $A_i \wedge A_j = \emptyset$, $i \neq j$

eg:



5) Complement: If A is a set, then the complement of the A (with respect to U) denoted by $\bar{A}$ is the set $U - A$

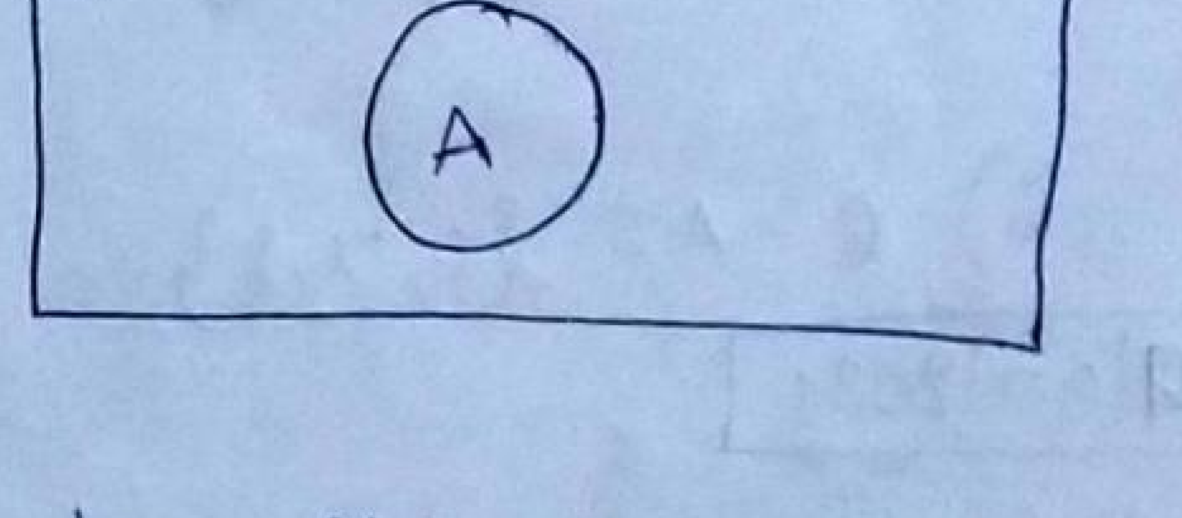
$$\bar{A} = \{x \in U \mid x \notin A\}$$

The complement of A is sometimes denoted by A^C.

eg: If U is the positive integers less than 100, what is the complement of $\{x \mid x > 70\}$

Venn diagram of complement

$\Rightarrow \{x \mid x \leq 70\}$

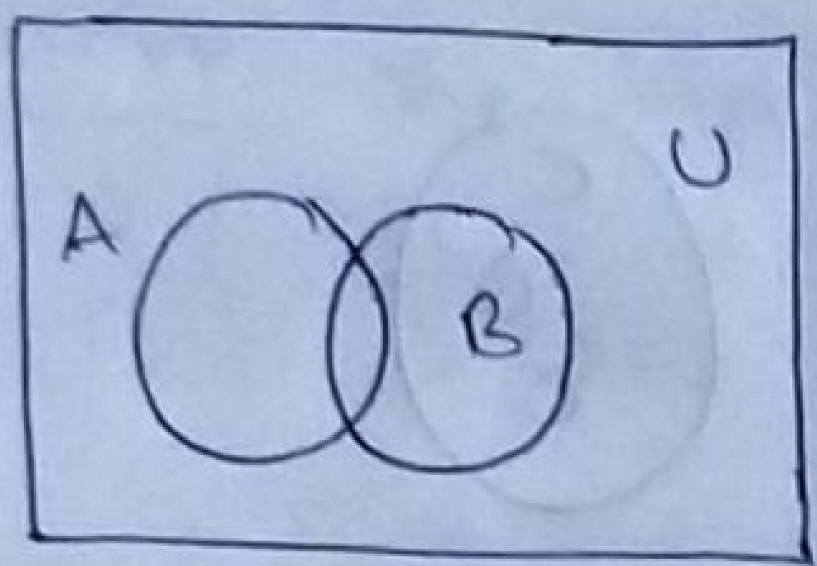


6) Difference: let A and B be sets. The diff of A and B, denoted by A - B, is the set containing the elements of A that are not in B.

The difference of A and B is also called the complement of B with respect to A.

$A - B = \{x \mid x \in A \wedge x \notin B\} = A \cap \overline{B}$

Venn diagram for A-B



Example: $U = \{0,1,2,3,4,5,6,7,8,9,10\}$ $A = \{1,2,3,4,5\}$

$B = \{4,5,6,7,8\}$

1) $A \cup B = \{1,2,3,4,5,6,7,8\}$

2) $A \cap B = \{4,5\}$

3) $\overline{A} = \{0,6,7,8,9,10\}$

4) $\overline{B} = \{0,1,2,3,9,10\}$

5) $A - B = \{1,2,3\}$

6) $B - A = \{6,7,8\}$

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Laws of set theory:

Set identities:

1) Commutative laws: $A \cup B = B \cup A$ $A \cap B = B \cap A$

2) Associative laws: $A \cup (B \cup C) = (A \cup B) \cup C$

$A \cap (B \cap C) = (A \cap B) \cap C$

3) Distributive laws: $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

4) Demorgan laws: $(A \cup B)^c = A^c \cap B^c$ and $(A \cap B)^c = A^c \cup B^c$

Commutative law:

1) $A \cup B = B \cup A$ 2) $A \cap B = B \cap A$

proof: $x \in (A \cup B)$

$\Leftrightarrow$ $x \in A$ (or) $x \in B$

$\Leftrightarrow$ $x \in B$ (or) $x \in A$

$\Leftrightarrow$ $x \in B \cup A$

$A \cup B = B \cup A$

Associative law:

1) $A \cup (B \cup C) = (A \cup B) \cup C$ 2) $A \cap (B \cap C) = (A \cap B) \cap C$

proof: let $x \in A \cup (B \cup C)$

$\Leftrightarrow$ $x \in A$ (or) $x \in B \cup C$

$\Leftrightarrow x \in A$ (or) $x \in B$ (or) $x \in C$

$\Leftrightarrow (x \in A$ or $x \in B)$ (or) $x \in C$

$\Leftrightarrow x \in (A \cup B)$ (or) $x \in C$

$\Leftrightarrow x \in (A \cup B) \cup C$

$\Leftrightarrow A \cup (B \cup C) = (A \cup B) \cup C$

Distributive law:

1) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

2) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

proof: let $x \in [A \cup (B \cap C)]$

$\Leftrightarrow x \in A$ (or) $x \in B \cap C$

$\Leftrightarrow x \in A$ (or) $x \in B$ and $x \in C$

$\Leftrightarrow (x \in A$ (or) $x \in B)$ and $x \in A$ (or) $x \in C$

$\Leftrightarrow x \in A \cup B$ and $x \in A \cup C$

$\Rightarrow A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Demorgan laws:

1) $(A \cup B)^c = A^c \cap B^c$

2) $(A \cap B)^c = A^c \cup B^c$

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proof: 1) $x \in (A \cup B)^c$

$\Leftrightarrow x \notin (A \cup B)$

$\Leftrightarrow x \notin A$ and $x \notin B$

$\Leftrightarrow x \in A^c$ and $x \in B^c$

$\Leftrightarrow x \in A^c \cap B^c$

$\Leftrightarrow (A \cup B)^c = A^c \cap B^c$

2) $x \in A$

$x \notin A^c$

$U = \{1, 2, 3, 4, 5\}$

$A = \{4, 5\}$

$A^c = \{1, 2, 3\}$

$4 \in A$

$4 \notin A^c$

$B = (1, 2)$

$A \cup B = \{1, 2, 4, 5\}$

3 of $A \cup B$

problems:

1) PT $A - B = A \cap B^c$

proof: let $x \in A - B$

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$\Rightarrow x \notin A$ and $x \notin B$

$\Rightarrow x \in A$ & $x \in B^c$

$\Rightarrow x \in A \cap B^c$

$\Rightarrow A - B = A \cap B^c$

2. P.T $A \cap (B - C) = (A \cap B) - (A \cap C)$

<u>proof:</u> R.H.S $A \cap B - A \cap C$

$= (A \cap B) \cap (A \cap C)^c$ $(\because A - B = A \cap B^c)$

$= \underbrace{(A \cap B)}_{1} \cap (\underbrace{A^c}_{2} \cup \underbrace{C^c}_{3})$ (Demorgan's law)

$= [(A \cap B) \cap A^c] \cup [(A \cap B) \cap C^c]$

(Associative law) (Distributive law)

$= (\phi \cap B) \cup (A \cap B \cap C^c)$

$= \phi \cup A \cap (B \cap C^c)$

$= A \cap (B \cap C^c)$

$= A \cap (B - C)$

R.H.S $\Rightarrow$ L.H.S

L.H.S = R.H.S

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3. If A, B and C are three sets such that $A \cup B = A \cup C$ & $A \cap B = A \cap C$ then P.T $B = C$

Sol Given i) $A \cup B = A \cup C$

2) $A \cap B = A \cap C$

To prove: $B = C$

i.e we can prove $B \subseteq C$ & $C \subseteq B$

$\Leftrightarrow B = C$

To prove $B \subseteq C$

let $x \in B \Rightarrow x \in A \cup B$

$\Rightarrow x \in A \cup C$

$\Rightarrow x \in A$ (or) $x \in C$ — ①

If $x \in A \Rightarrow x \in A \cap B$

$\Rightarrow x \in A \cap C$

$\Rightarrow x \in A$ and $x \in C$

$\Rightarrow x \in C$ — ②

sub $x \in A \Rightarrow x \in C$ in ①

$x \in B \Rightarrow x \in C$ (or) $x \in C$

$\Rightarrow x \in C$

$\left.\begin{array}{l} B \subseteq C \\ \text{III}^{ly}\ C \subseteq B \end{array}\right\} \rightarrow B = C$

∴ Hence proved.

4. let A, B and c be 3 sets and if $A \subseteq B$ and $B \cap C = \emptyset$

then prove that $A \cap C = \emptyset$

proof: Given $A \subseteq B$

$B \cap C = \emptyset$

To prove: $A \cap C = \emptyset$

let us prove this by contradiction method

i.e $A \cap C \neq \emptyset$

$\Rightarrow x \in A \cap C$

$\Rightarrow x \in A \;\&\; x \in C$

$\Rightarrow x \in B \;\&\; C \;(\because A \subseteq B)$

$x \in C$

$\Rightarrow x \in B \cap C$

this is a contradiction to $B \cap C = \emptyset$

$\Rightarrow$ our statement is wrong

$\therefore A \cap C = \emptyset$ Hence proved.

partition of a set:

1) A = {All the students in CSE II Year C_1 sec}

A_1 = { students got $\leq$ 50 marks}

A_2 = { " " 51 to 70 }

$A_3 = \{$ " " 71 to 90$\}$

$A_4 = \{$ " " more than 90$\}$

① $A_1 \cup A_2 \cup A_3 \cup A_4 = A$

② $A_i \cap A_j = \emptyset$, $i \neq j$

③ $A_i \neq \emptyset$

2) $A = \{a, b, c, d,\}$

$A_1 = \{a, c\}$

$A_2 = \{c\}$ $A_3 = \{d\}$

① $A_1 \cup A_2 \cup A_3 = A$

② $A_1 \cap A_2$, $A_1 \cap A_3$ & $A_2 \cap A_3 = \emptyset$

③ A_1 & A_2 & $A_3 \neq \emptyset$

Definition: let A be a non-empty set, then any set of subsets of A say A_1, A_2, A_3 --- A_n will form a partition if

1) $A_1 \cup A_2 \cup --- \cup A_n = A$

2) $A_i \cap A_j = \emptyset$, $i \neq j$ $\Big\}$ partition of A.

3) $A_i \neq \emptyset$

eg: 1) write all the partitions of $\{1,2,3,4\}$

① $\{\{1\},\{2\},\{3\},\{4\}\}$

② $\{\{1,2\},\{3,4\}\}$

③ $\{\{1,2,3\},\{4\}\}$

④ $\{\{1,2,4\},\{3\}\}$

⋮

2) write all the partitions of a set $\{a,b,c,d\}$

$A=\{1,2,3,4,5\}$

① $=\{\{1,2\},\{3,4\},\{2,5\}\}$

② $=\{\{1\},\{3,4\},\{5\}\}$

③ $=\{\{1,2\},\{3\},\{4,5\}\}$

cartesian product: A, B are 2 sets

$A=\{1,2\}$ $B=\{3,4\}$

$A\times B=\{(1,3),(1,4),(2,3),(2,4)\}$

let A & B be 2 sets. The c.p of A & B is denoted by $A \times B$, which is the set of all ordered occurs in which the 1st element belongs to A & 2nd $\in B$

i.e $\Rightarrow A \times B = \{(x, y) / x \in A \,\&\, y \in B\}$

$A \times B \times C = \{(x, y, z) / x \in A, y \in B \,\&\, z \in C\}$

eg: $A = \{1, 2, 3\}$ $B = \{4, 5\}$

$A \times B = \{(1,4), (1,5), (2,4), (2,5), (3,4), (3,5)\}$

properties:

1) commutative property:

$A \times B \neq B \times A$

2) Associative property:

$A \times (B \times C) = (A \times B) \times C$

proof: $A = \{1, 2\}$, $B = \{a, b\}$, $C = \{d\}$

$B \times C = \{(a, d), (b, d)\}$

$A \times (B \times C) = \{(1, a, d), (1, b, d), (2, a, d), (2, b, d)\}$

$A \times B = \{(1, a), (1, b), (2, a), (2, b)\}$

$(A \times B) \times C = \{(1, a, d), (1, b, d), (2, a, d), (2, b, d)\}$

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$\therefore$ A.L Holds C.P

3) Distributive property:

$A \times (B \cup C) = (A \times B) \cup (A \times C)$

$A \times (B \cap C) = (A \times B) \cap (A \times C)$

proof: $A \times (B \cap C) = \{(x,y) / x \in A \text{ and } y \in B \cap C\}$

$= \{(x,y) / x \in A \text{ and } [y \in B \text{ and } y \in C]\}$

$= \{(x,y) / (x \in A \text{ and } y \in B) \text{ \& } (x \in A) \text{ \& } (y \in C)\}$

$= \{(x,y) / (x,y) \in (A \times B) \text{ and } (x,y) \in (A \times C)\}$

$= \{(x,y) / (x,y) \in (A \times B) \cap (A \times C)\}$

$A \times (B \cap C) = (A \times B) \cap (A \times C)$

1. let $A = \{4,6,8,10\}$ $B = \{1,3,5,7,9\}$ find $A \times B$, $B \times A$, A^2, B^2

sol $A \times B = \{(4,1),(4,3),(4,5),(4,7),(4,9),(6,1),(6,3),(6,5),(6,7),$
$(6,9),(8,1),(8,3),(8,5),(8,7),(8,9),(10,1),(10,3),$
$(10,5),(10,7),(10,9)\}$

$B \times A = \{(1,4),(1,6),(1,8),(1,10),(3,4),(3,6),(3,8),(3,10),(5,4),$
$(5,6),(5,8),(5,10),(7,4),(7,5),(7,8),(7,10)(9,4),$

(9, 6), (9, 8), (9, 10)}

$A \times B \neq B \times A$ ∴ Commutative property is not satisfied.

$A^2 = A \times A = \{(4,4), (6,6), (8,8), (10,10)\}$

$B^2 = B \times B = \{(1,1), (3,3), (5,5), (7,7), (9,9)\}$

$A^2 = A \times A = \{(4,4), (4,6), (4,8), (4,10), (6,4), (6,6), (6,8), (6,10),$
$(8,4), (8,6), (8,8), (8,10), (10,4), (10,6), (10,8), (10,10)\}$

$B^2 = B \times B = \{(1,1), (1,3), (1,5), (1,7), (1,9), (3,1), (3,3), (3,5), (3,7),$
$(3,9), (5,1), (5,3), (5,5), (5,7), (5,9), (7,1), (7,3),$
$(7,5), (7,7), (7,9), (9,1), (9,3), (9,5), (9,7), (9,9)\}$

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Relations: let A & B be 2 sets, then the Relation A on B in a subset of a cartesian product (A×B) where 1st element ∈A & 2nd element ∈B. If A is related B then we can express it as ARB (or) (A,B) ∈ R

eg: let A = {1, 2, 3}, B = {2, 3, 4}, let R be the relation ≤ from A to B.

Types of Relation:

1) Reflexive Relation: A relation R on a set A is said

to be reflexive if $(a,a) \in R$ (or) aRa, where $a \in A$

eg: $A = \{1,2,3\} \rightarrow R$ in reflexive

$R_1 = \{(1,1), (2,2), (3,3)\} \rightarrow$ R.R

$R_2 = \{(1,1), (1,2), (1,3), (3,3)\}$

Here R_1 in reflexive and R_2 is not reflexive

because $(2,2) \notin R$ (or) $2 \not R 2$

Irreflexive: A Relation R on a set A is said to be irreflexive if $a \not R a$ (or) $(a,a) \notin R$ where $a \in A$.

Eg: $R_1 = \{(1,3), (2,1)\}$

$R_2 = \{(1,1), (1,2), (2,2), (3,3), (4,4)\}$

$A = \{1,2,3,4\}$

$R_3 = \{(1,1), (1,3), (2,3)\} \rightarrow$ neither Ref (or) irref

$A = \{a,b,c\}$

$R_1 = \{(a,a), (b,c), (b,b) (c,c)\} \rightarrow$ Ref $\rightarrow$ not irref

$R_2 = \{(a,a), (b,c), (b,b)\} \rightarrow$ not ref

$\hookrightarrow$ not irref

Symmetric Relation: A Relation R on a set A is said to be symmetric if whenever aRb then bRa $\forall a,b \in A$

cRd & dRc then the Rel R is not sym.

$A = \{a, b, c, d\}$

$R_1 = \{(a,b), (b,a), (c,d)\}$ × sym

$R_2 = \{(a,b), (a,a), (a,c), (c,a)\}$

$R_3 = \{(a,a), (a,b), (b,a), (c,c), (c,d), (d,c), (a,c), (c,a)\}$

- sym

Antisymmetric Relation: A Relation on a set A is said to be antisymmetric if when ever aRb then bRa only when a=b (or) A Relation R on a set A is said to be antisym if when aRb the bRa where $a \neq b$

① aRb then bRa provided $a=b$

② aRb the bRa provided $a \neq b$

$R = \{(1,3), (3,1), (2,3)\}$ on $A = \{1, 2, 3\}$

↳ not symm

$R = \{(1,1), (2,2), (3,3)\}$ → sym

aRb, bRa, $a \neq b$ Antisym

$\boxed{02/08/2021}$

Transitive Relation: A Relation R on a set A is said to be ~~transitvt~~ transitive when ever aRb, bRc ⇒ aRc

i.e $(a,b) \in R$ and $(b,c) \in R$

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then $(a,c) \in R$, $\forall a,b,c \in A$

equivalence relation: A Relation R on a set A is said to be E.R if it is sym Refl and transitive [RST]

eg: A Relation "parallel" on a set of st.lines in a plane.

partially ordering Relation [po set]: A Relation R on a set A is said to be partial ordering relation if it is reflexsive, Anti symmetric & transitive [RAT]

eg: $A = \{1,2,3,4\}$ " $\leq$ "

$R = \{(1,1),(2,2),(3,3),(4,4),(2,3),(2,4),(3,4),(1,2),(1,3),(1,4)\}$

problem:

1. let $A = \{1,2,3,6\}$ examine whether "Divides" is a partially ordering relation on A (poset) $\rightarrow$ (RAT)

sol $R = \{(1,1),(2,2),(3,3),(1,2),(1,3),(1,6),(2,6),(3,6),(6,6)\}$

Reflexsive: since $(1,1)$ $(2,2)$ $(3,3)$ $(6,6)$ $\forall (1,2,3,6) \in A$

$\therefore$ the Relation R is reflexisive.

Antisymmetric: since $(1,2) \in R$ but $(2,1) \notin R$

III^{ly} $(1,3) \in R$, $(3,1) \notin R$ - -

$(1,6) \in R$, $(6,1) \notin R$

$(2,6) \in R$, $(6,2) \notin R$

$(3,6) \in R$, $(6,3) \notin R$

$\therefore$ The condition for A.S is satisfied.

Transitive Relation: $\forall\ a,b,c \in A$ we have to prove aRb, $bRc \Rightarrow aRc$

$(1,3) \in R$ and $(3,6) \in R \Rightarrow (1,6) \in R$ $\rightarrow$ satisfy

$(1,6) \in R$ $(6,6) \in R \Rightarrow (1,6) \in R$ $\rightarrow$ satisfy

Hence R is a transitive

$\therefore$ since the Relation is refl, A.S, & transitive we can conclude that the Relation R on a set A is poset.

2. let xRy, iff $|x-y| \leq 3$ where R is defined on the set of all real no. R. Is the above relation is an equivalence relation?

sol Reflexisive: for any $a \in R$, consider $|a-a| = 0 \leq 3$

$\therefore$ The condition is satisfied

$\therefore$ R is reflexisive.

Symmetric: To check aRb then $bRa \ \forall a,b \in R$

consider $(a,b) \in R \Rightarrow |a-b| \leq 3$

$\Rightarrow |-(a-b)| \leq 3$

$\Rightarrow |b-a| \leq 3$

$\Rightarrow bRa$

$\therefore$ R is Symmetric.

Transitive: To check $aRb, bRc \Rightarrow aRc$

whenever

$aRb \Rightarrow |a-b| \leq 3$

$bRc \Rightarrow |b-c| \leq 3$

consider $|a-c| = |a-b+b-c|$

$= |a-b| + |b-c|$

$= 3+3$

$= 6$

$|a-c| = 6 \nleq 3 \Rightarrow R$ is not transitive

$\Rightarrow$ R is ref, sym, but not transitive.

$\therefore$ the relation R is not an equivalence relation.

3. let $A = \{1,2,3,4,5,6\}$ and let $R = \{(x,y) / x-y$ is divisible by $3\}$ be a relation on x. S.T R is a E.R.

Sol $R = \{(1,1)\ (2,2)\ (3,3)\ (4,4)\ (5,5)\ (1,4)\ (2,5)\ (3,6)\ (4,1)\ (5,$

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(6,3)}

since (1,1) (2,2) (3,3) (4,4) (5,5) → R is refle

(1,4) (4,1) (2,5) (5,2) (3,6) (6,3) → R is sym

(1,4) (4,1) ⇒ (1,1) → R is tran

the relation R is E.R on the given.

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congruence modulo m: x is said to be congruent to y modulo m if $x-y$ is divisible by m i.e if $x-y = k \cdot m$ for some integer k. we can denote it by $x \equiv y \mod m$.

1. show that the relation "congruence modulo m" given by $R = \{(x,y) | x-y$ is divisible by $m\}$ over a set of integers is an equivalence relation.

Sol Reflexive $\forall x \in Z$ consider $x-x=0$

$\Rightarrow x-x = 0 \cdot m$

$x-x$ is divisible by m $\Rightarrow (x,x) \in R$

R is reflexive

sym $\forall x,y \in Z$ $(x,y) \in R \Rightarrow (y,x) \in R$

consider $(x,y) \in Z$ $\Rightarrow x-y$ is divisible by m

$\Rightarrow -(x-y)$ " " "

$\Rightarrow (y-x)$ " " "

$\Rightarrow (y,x) \in R$

R is sym

Transitive : $x, y, z \in Z$ To prove : whenever

$(x,y) \in R$, $(y,z) \in R \Rightarrow (x,z) \in R$

consider $(x,y) \in R \Rightarrow x-y$ is divisible by m

$(y,z) \in R \Rightarrow y-z$ " " "

$\Rightarrow (x-y)+(y-z)$ " " "

$\Rightarrow (x-z)$ " " "

$\Rightarrow R$ is transitive.

$\Rightarrow R$ is an E.R.

2. If R and S are relations on a set A prove that

1) If R and S are Ref then $R \cup S$ & $R \cap S$ are ref

2) If R & S are sym then $R \cup S$ & $R \cap S$ are sym

3) If R & S are transitive then $R \cup S$ is not transitive &

$R \cap S$ is transitive.

Sol 1) R & S on a relation R are ref

$a \in A \Rightarrow (a,a) \in R$ and also $(a,a) \in S$

since R & S are ref

$\Rightarrow (a,a) \in R \cup S$ (since R & S are ref)

$\Rightarrow (a,a) \in R\cap S$ (since R&S are ref)

2) when ever $(a,b) \in R \Rightarrow (b,a) \in S$

when ever $(a,b) \in R\cup S \Rightarrow (b,a) \in R\cup S$

consider $(a,b) \in R\cup S \Rightarrow (a,b) \in R$ (or)

$(a,b) \in S$

$\Rightarrow (b,a) \in R$ (or) $(b,a) \in S$

($\because$ R&S are sym)

$\Rightarrow (b,a) \in R\cup S \Rightarrow R\cup S$ is sym

when ever $(a,b) \in R\cap S \Rightarrow (b,a) \in R\cap S$

$\Rightarrow (a,b) \in R\cap S \Rightarrow (a,b) \in R$ and $(a,b) \in S$

$\Rightarrow (b,a) \in R$ & $(b,a) \in S$

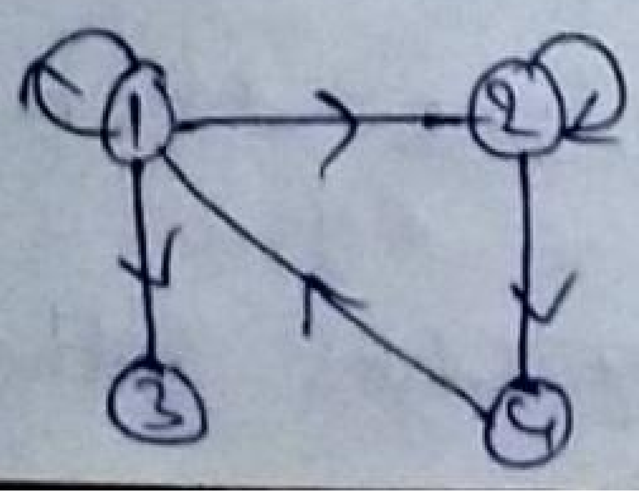
$\Rightarrow (b,a) \in R\cap S$

$R\cap S$ is sym

<u>Relation on graphs:</u> A graph is aclear representation of an information.

<u>eg</u>: on a set $A = \{1,2,3,4\}$

let $R = \{(1,1),(2,2),(1,2),(2,4),(4,1),(1,3)\}$



i.e, the elements of the set are called as the vertices and we represented them by dots (or) small circles. If $(1,2) \in R$ then we connect 1 to 2 by an arrow directed to be which is called as edge. we call these type of graphs as digraphs (or) directed graphs. vertices are called as nods (or) points.

1. let $A = \{1,2,3,4,5\}$. Define the relation R on A by xRy only if $y = x+1$. find R, R^2 & R^3

sol

$R = \{(1,2),(2,3),(3,4),(4,5)\}$

$R^2 = \{(1,3),(2,4),(3,5)\}$

$R^3 = \{(3,5)\}$

Hase Diagram: we use the following roles to Draw Hase diagram.

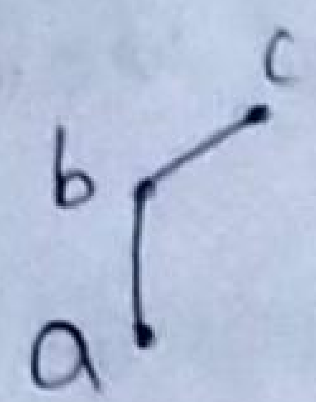
① each element is related to itself.

② If vertex 'b' appears above 'a' and a is connected b then aRb

③ If vertex 'c' is above 'a' and if 'c' is connected to 'a' by a sequence of edges then 'aRc'

④ vertices are denoted by dots rather than circles.

$R = \{(a,b) \in R,\ (b,c),\ (a,c)\}$

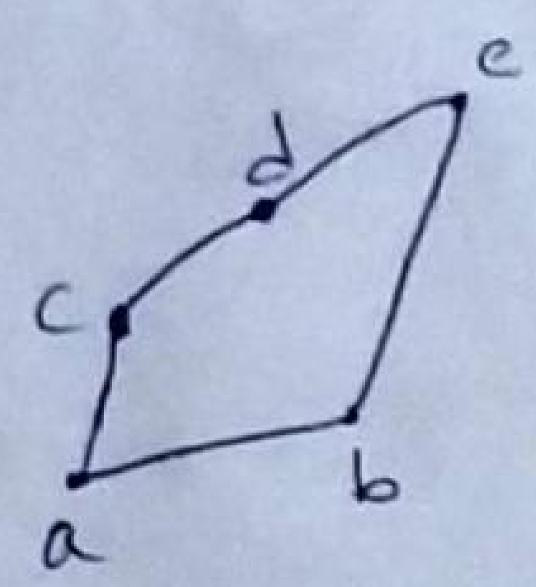


1. for the following relation draw Hase diagram

$R = \{(a,a), (b,b), (c,c), (d,d), (e,e), (a,c), (a,d), (a,e), (a,b), (c,d)$

$(c,e), (d,e), (b,e)\}$ on $A = \{(a,b,c,d,e)\}$

sol



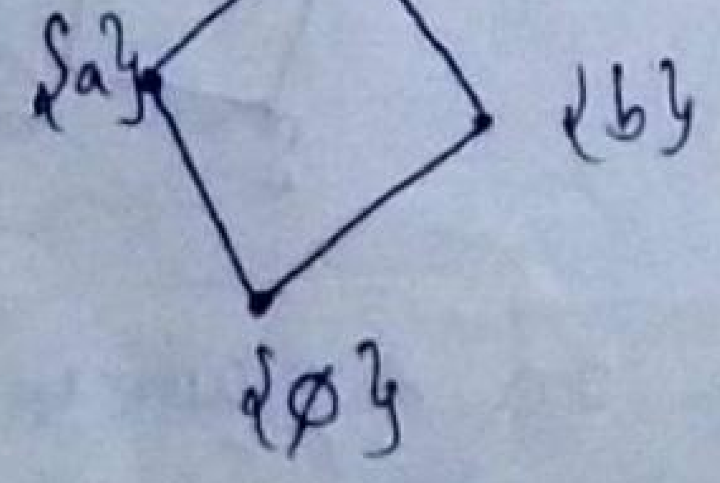
2. let $B = \{a,b\}$ let $A = p(B)$. If R is the relation $\subseteq$ on A,

draw Hase diagram

sol $A = p(B) = \{\{\emptyset\}, \{a\}, \{b\}, \{a,b\}\}$

$R = \{(\{\emptyset\}, \{\emptyset\}), (\{\emptyset\}, \{a\}), (\{\emptyset\}, \{b\}), (\{\emptyset\}, \{a,b\}), \ldots$

$(\{a\}, \{a,b\}), (\{b\}, \{a,b\})\}$



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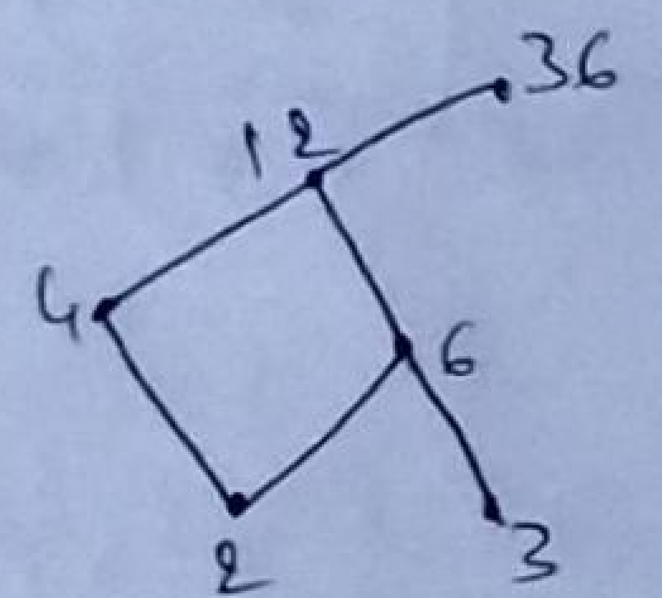
3. let 's' be the relation '|' on B where B = {2, 3, 4, 6, 12, 36}. Draw Hase Diagram for 's'.

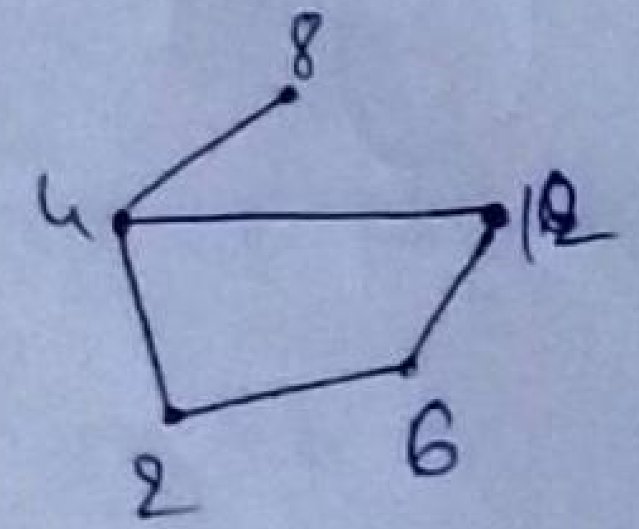
sol R = {(2,2), (2,4), (2,6), (2,12), (2,36), (3,3), (3,6), (3,12), (3,36), (4,4), (4,12), (4,36), (6,6), (6,12), (6,36), (12,12), (12,36), (36,36)}



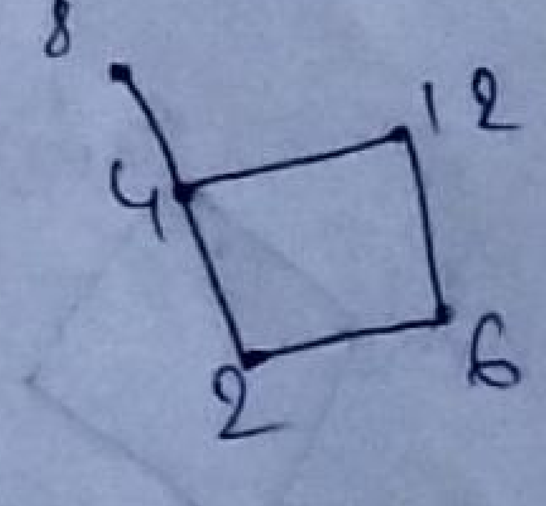
4. let R be a relation '|' on B where B = {2, 4, 6, 8, 12}

Draw Hase diagram.

sol R = {(2,2), (2,4), (2,6), (2,8), (2,12), (4,4), (4,8), (4,12), (6,6), (6,12), (8,8), (12,12)}



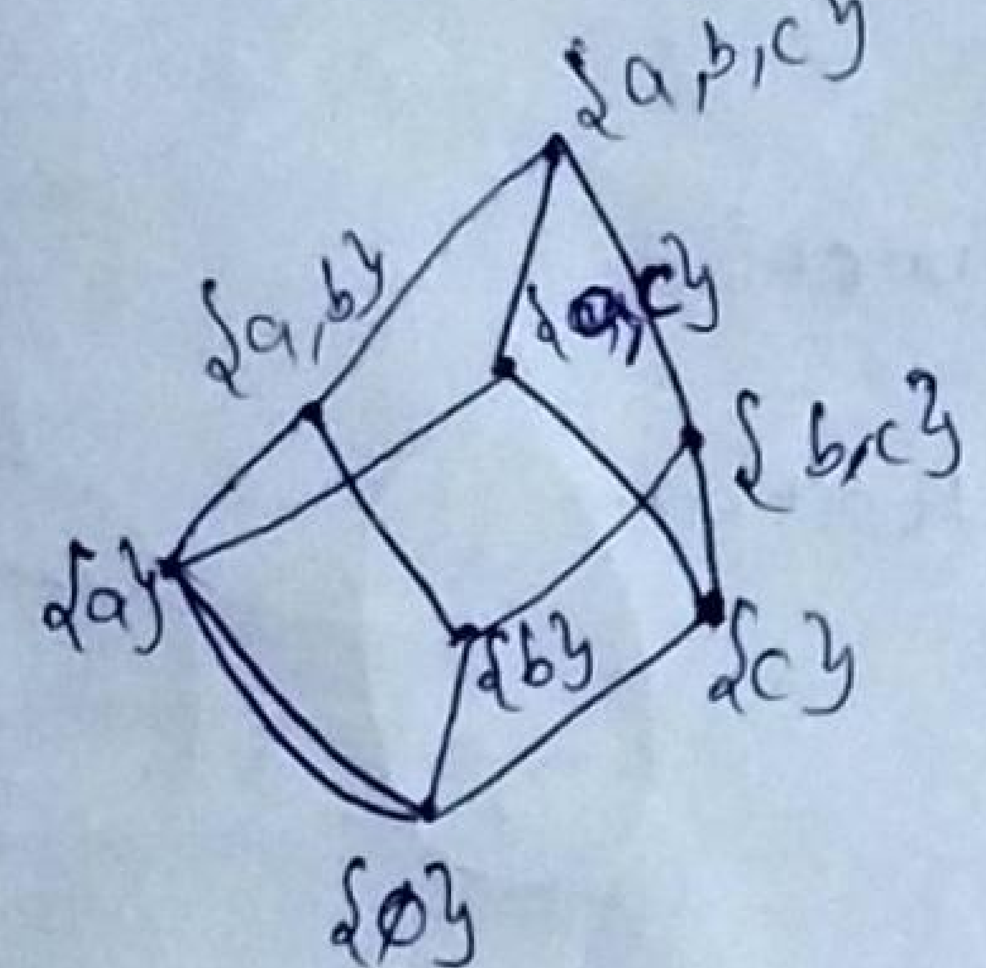
we can represent it so neatly as



5. let R be the relation $\leq$ on A where A = {2, 6, 8, 10, 12}

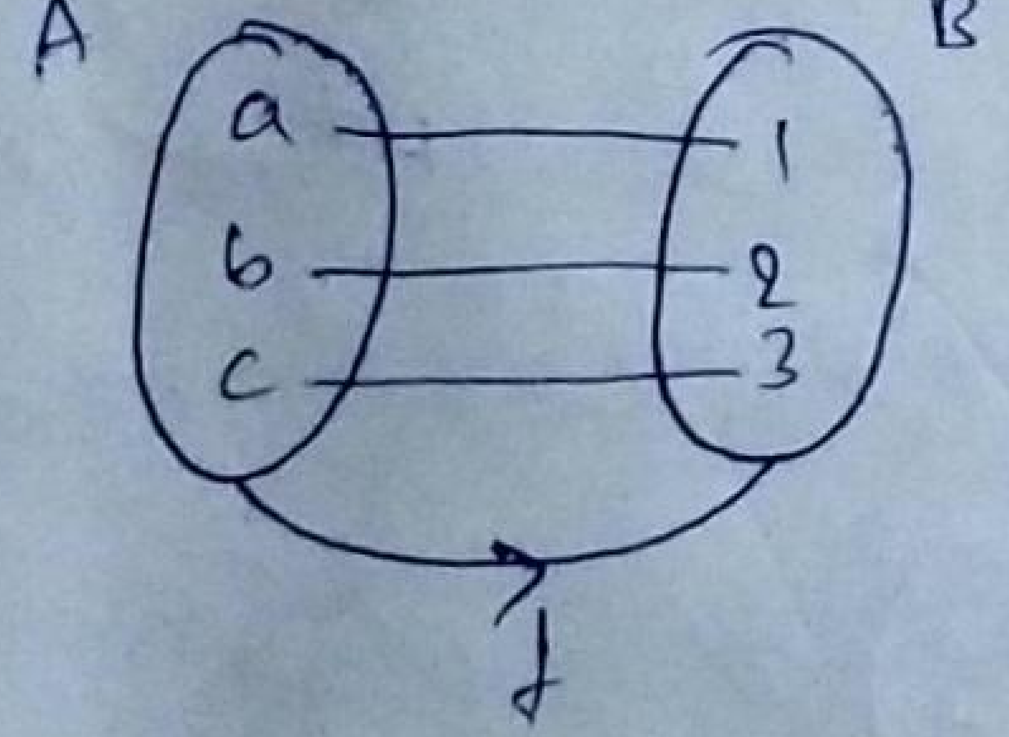
6. let $B = \{a, b, c\}$ and $P(B) = A$ be its power set. Draw the Hase Diagram for the relation $\subseteq$ on A.

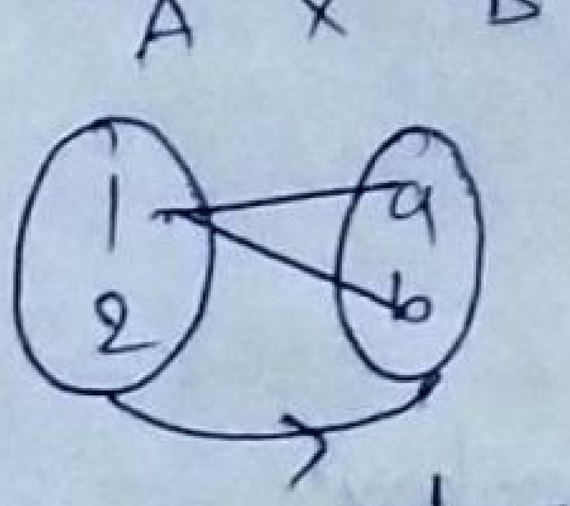
Sol $A = P(B) = \{\{\emptyset\}, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}$
$\{a,b,c\}\}$



Functions: let A and B be nonempty sets. A function f from A to B is an assignment of exactly one element of B to each element of A. We write $f(a) = b$ if b is the unique element of B assigned by the function f to the element a of A. If f is a function from A to B, we write $f: A \to B$.

Note: functions are sometimes also called mappings (or) transformations.





$f \to$ not a function.

What is a function?

A function relatives an input to an output. It is like a machine that has an input and an output.

$f(x) = ---$ is the classic way of writing a function. And there are other ways, as you will see

Examples of function:

$f(x) = x^2$ (squaring) is a function

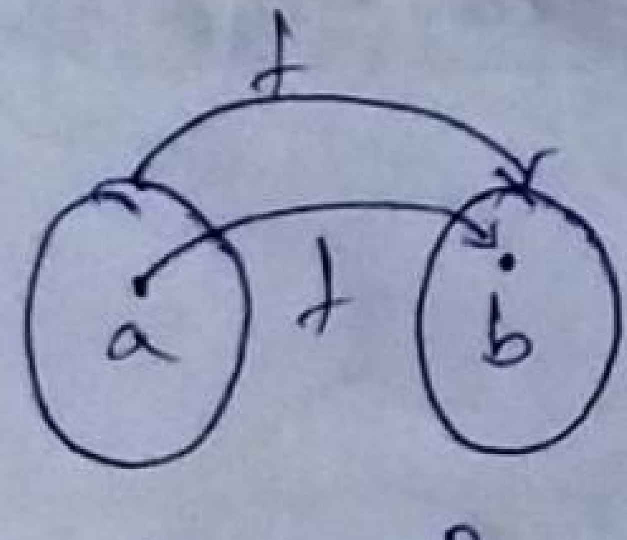
$f(x) = x^3 + 1$ is also a function.

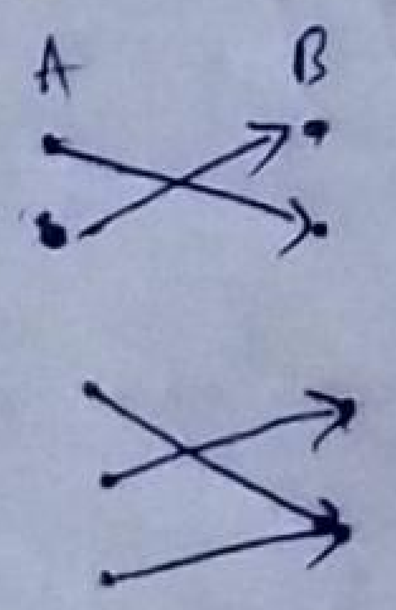
Notation:

$Y = f(x)$

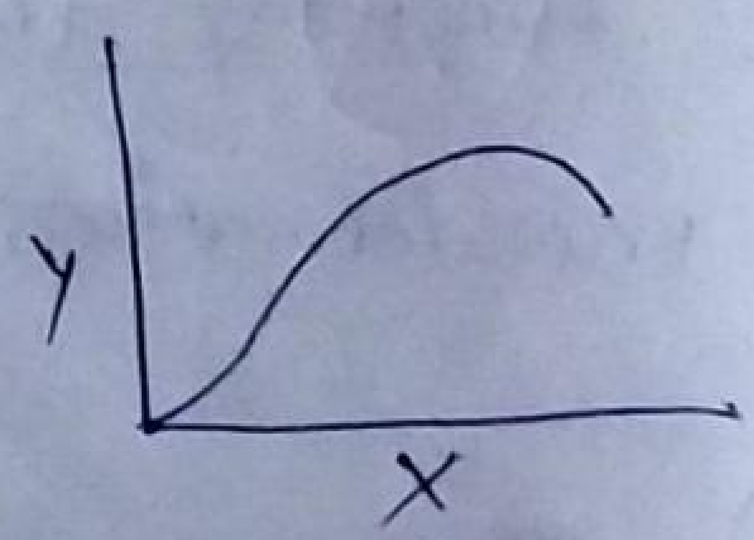
output ↓ , Name of function ↓ , $(x) \rightarrow$ input

Graphical Representations: Functions can be represented graphically in several ways.





Graph



plot

Function Teminology:

If $f: A \rightarrow B$ and $f(a) = b$ (where $a \in A$, & $b \in B$), then:

1) A is the domain of f

2) B is the codomain of f

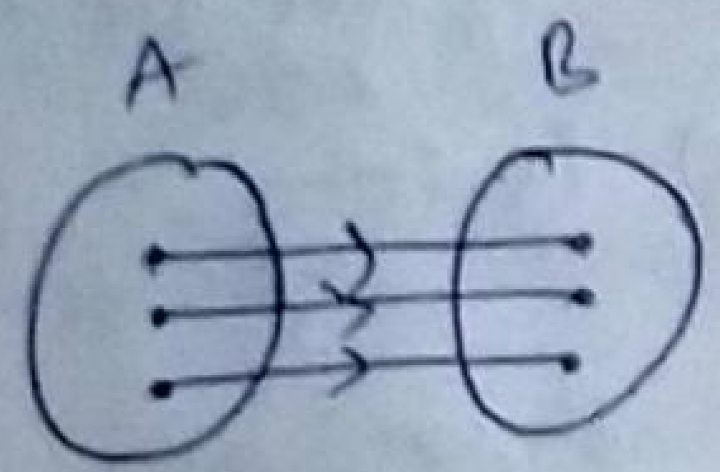
3) b is the image of a under f

4) a is a pre-image of b under f.

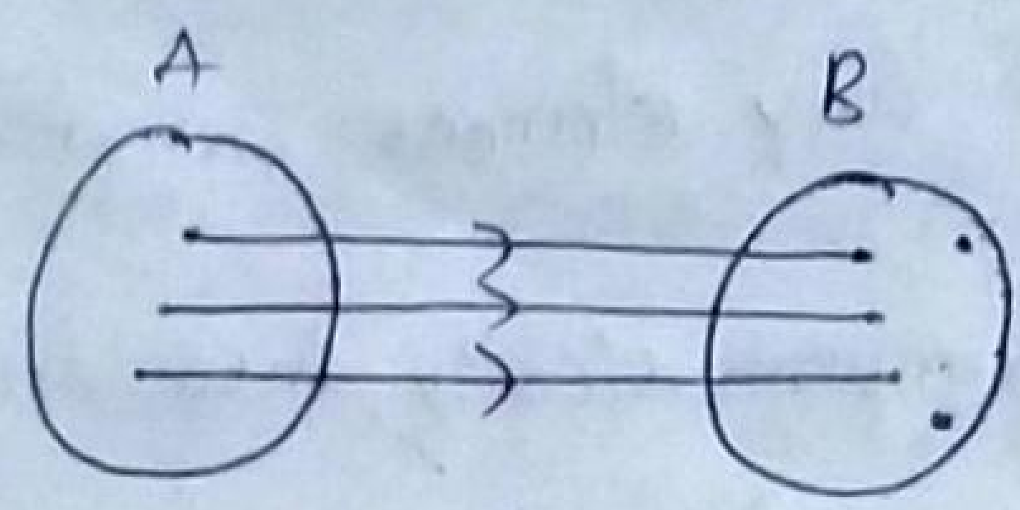
Note! In general, b may have more than one pre-image.

one-one function! A function $f: A \to B$ is said to be a one-one function (or) injective mapping if diff. elements of A have different f images in B. Thus for $x_1, x_2 \in A$ & $f(x_1), f(x_2) \in B$, $f(x_1) = f(x_2) \Leftrightarrow x_1 = x_2$ (or) $x_1 \neq x_2 \Leftrightarrow f(x_1) \neq f(x_2)$.

Diagrammatically an injective mapping can be shown as

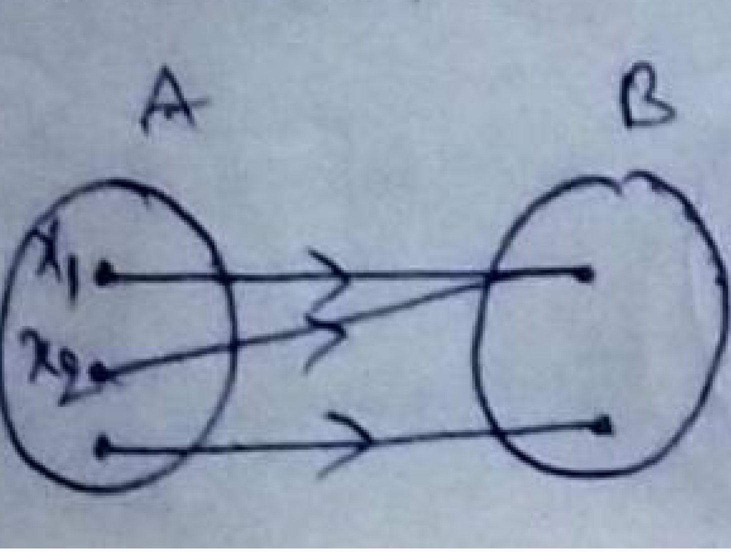


(or)



many-one function: A function $f: A \to B$ is said to be a many one function, if 2 (or) more elements of A have the same f image in B. Thus $f: A \to B$ is many one iff there exists atleast 2 elements $x_1, x_2 \in A$ such that $f(x_1) = f(x_2)$ but $x_1 \neq x_2$.

Diagrammatically a many one mapping can be shown as



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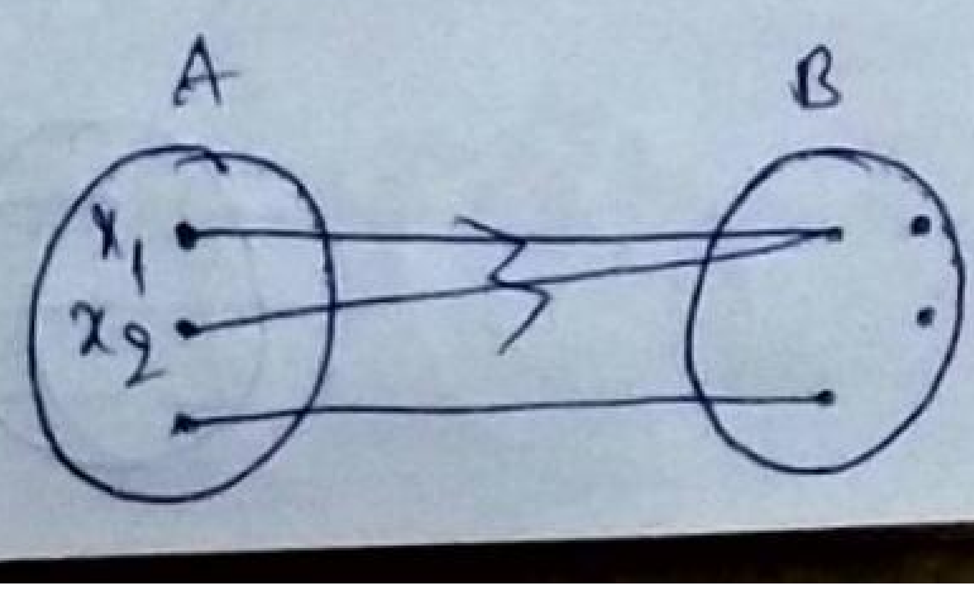
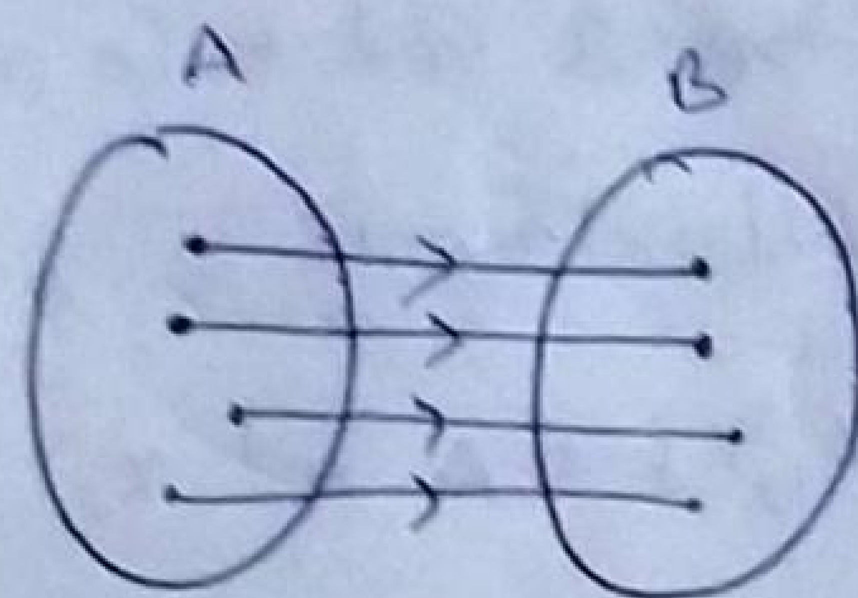


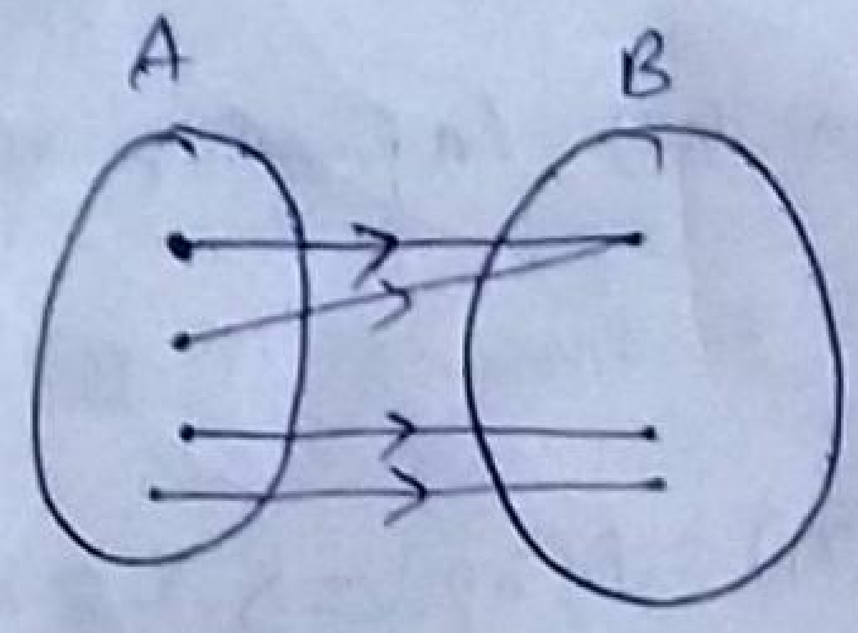
ONTO function: If the function $f: A \to B$ is such that each element in B (co-domain) must have atleast, one pre-image in A, then we say that f is a function of 'A' onto' B.

Thus. $f: A \to B$ is surjective iff $\forall\, b \in B$, there exists some $a \in A$ such that $f(a) = b$

Diagrammatically surjective mapping can be shown as

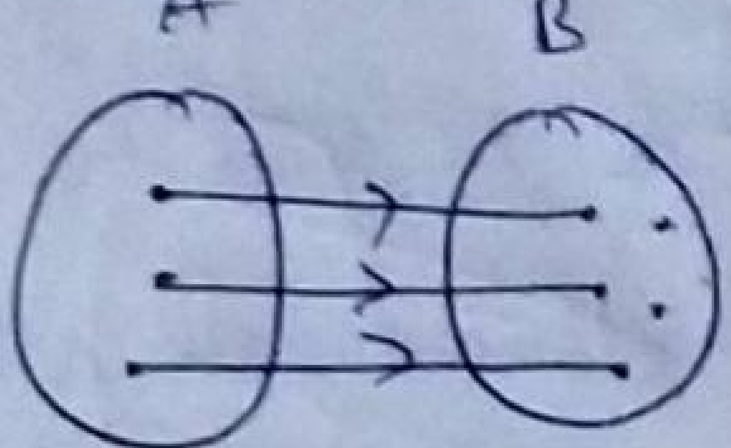


(or)

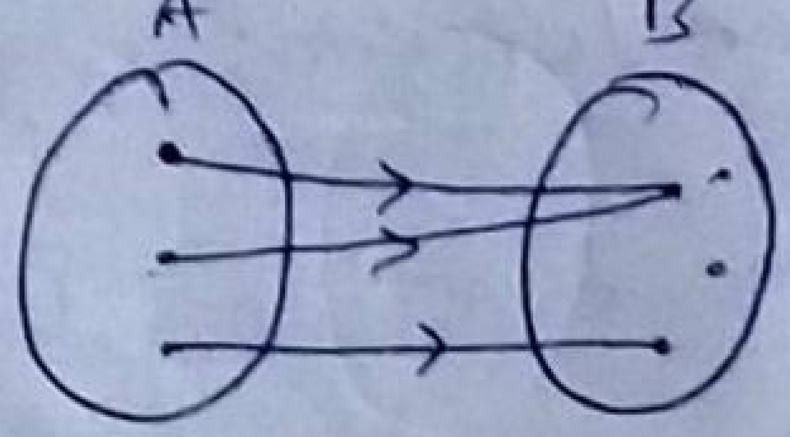


INTO function: If $f: A \to B$ is such that there exists atleast one element in co-domain which is not the image of any element in domain, then $f(x)$ is into.

Diagrammatically into function can be shown as



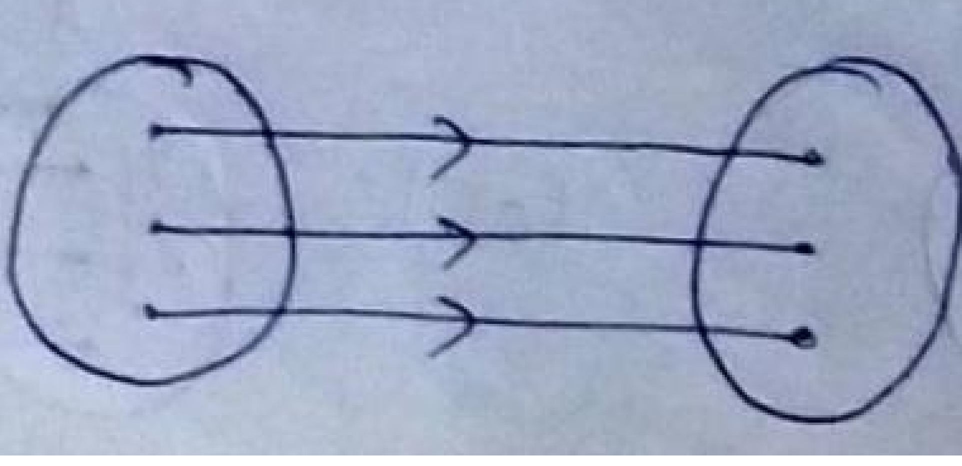
(or)



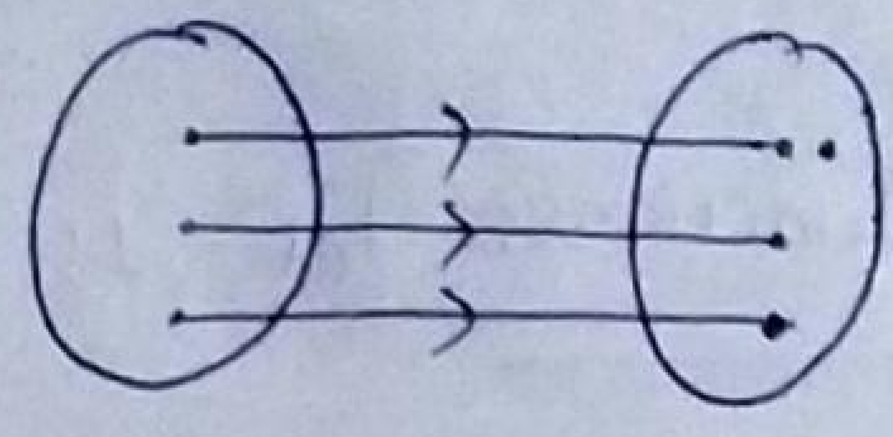
Note: If a function is onto, it cannot be into and vice versa

The function can be one of these 4 types:

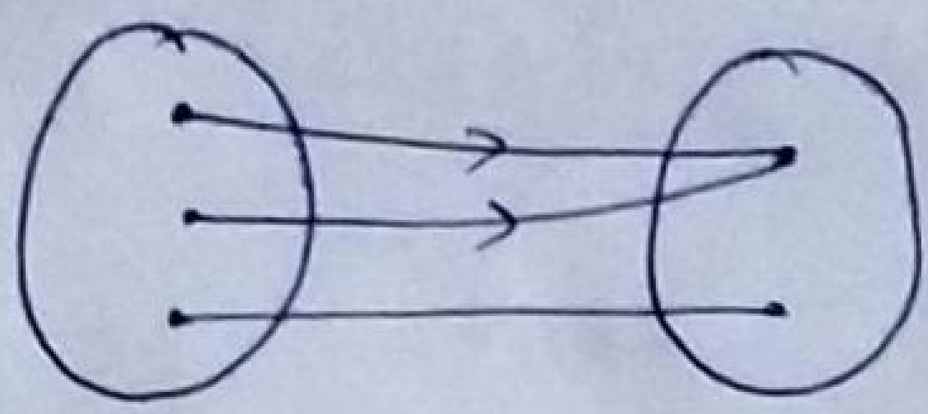
1. one - one onto (injective & surjective)



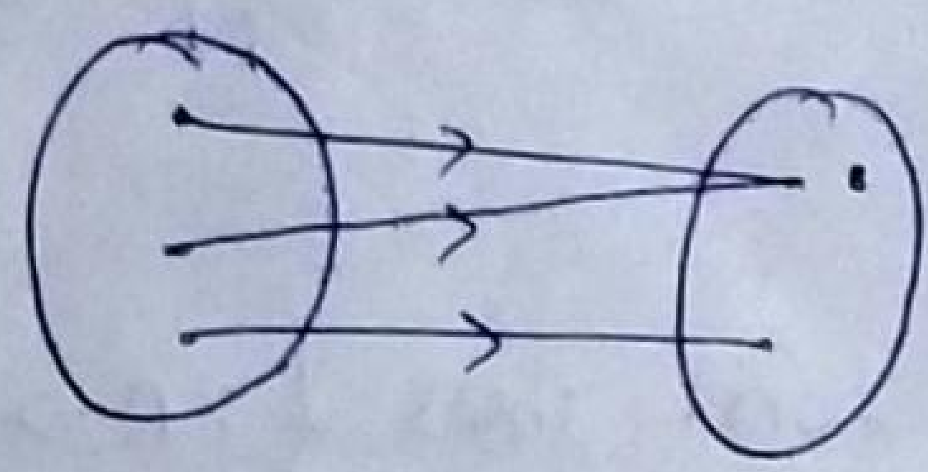
2. one-one into (injective but not surjective)



3. many-one onto (surjective but not injective)



4) many-one into (neither surjective nor injective)



If f is both injective & surjective, then it is called a bijective mapping. The bijective functions are also named as invertible, non singular (or) biuniform functions. If a set A contain 'n' distinct elements then the no. of diff functions defined from $A \to A$ is n^n and out of which $n!$ are one one.

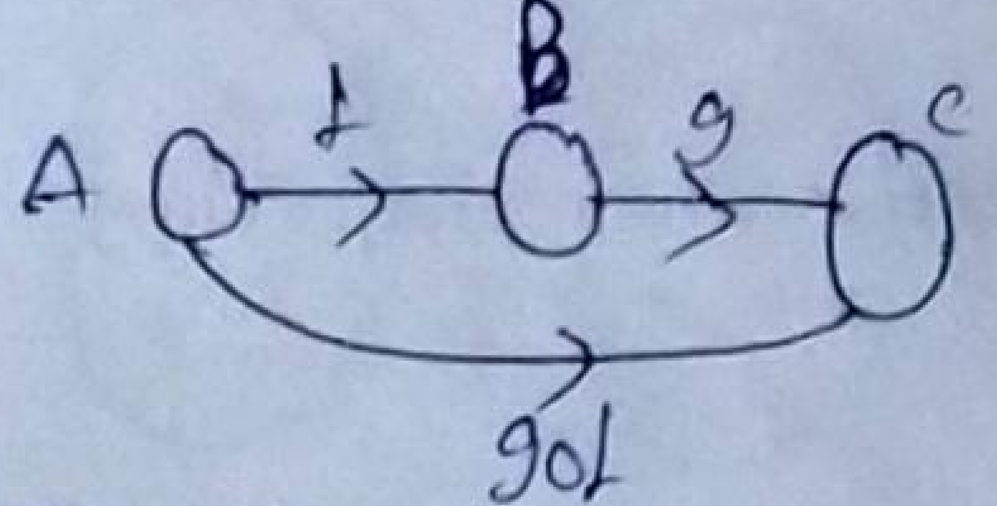
Identity function: The function $f: A \to A$ defined by $f(x) = x$ $\forall x \in A$ is called the identity function on A and is denoted by I_A. It is easy to observe that identity function is a bijection.

Constant function: A function $f: A \to B$ is said to be a constant function, if every element of A has the same f

image in B. thus $f: A \to B$, $f(x) = c$, $\forall x \in A$, $c \in B$ is a constant function.

product (or) composition of functions: let $f: A \to B$ & $g: B \to C$ be any 2 functions then the composition of f with g is denoted by $g \circ f$ is a function from $A \to C$ defined by

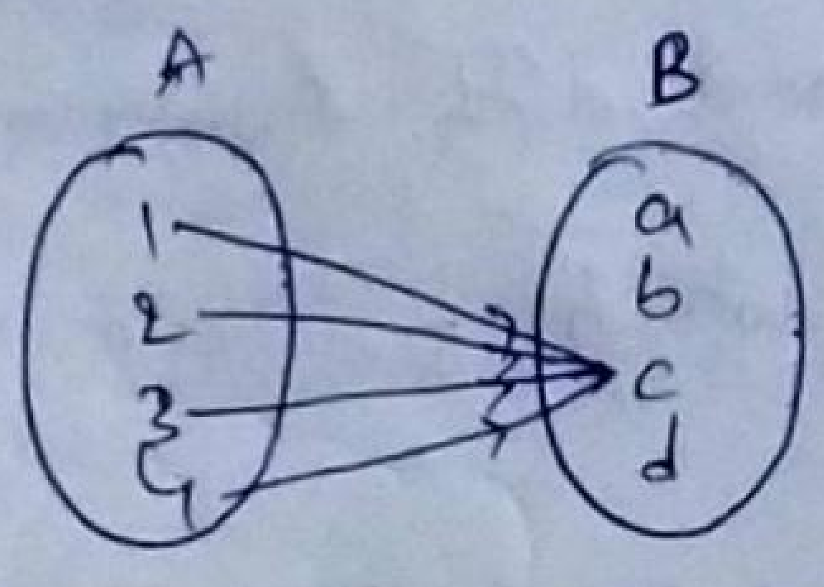
$g \circ f(x) = g[f(x)]$ $\forall x \in A$



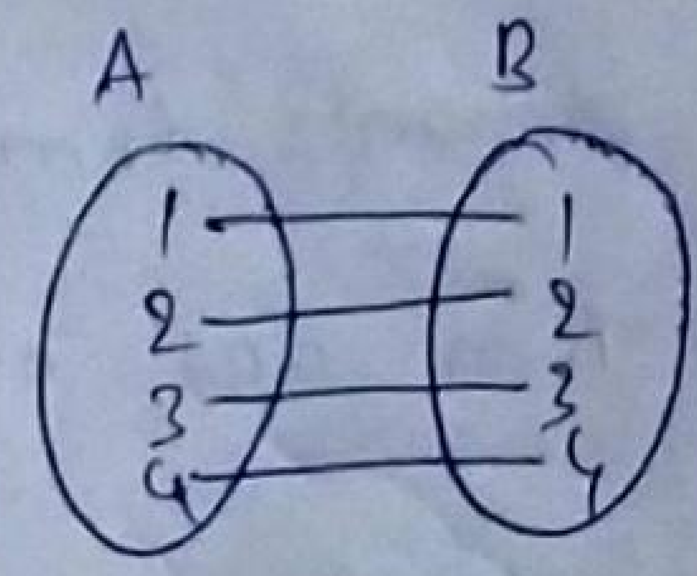
$\boxed{g \circ f: A \to C}$

Inverse function:

let $f: A \to B$ be a bijective fun. thus $f^{-1}: B \to A$ which associates with each element $b \in B$, a unique element $a \in A$ such that $f(a) = b$ is called the inverse function of "f".

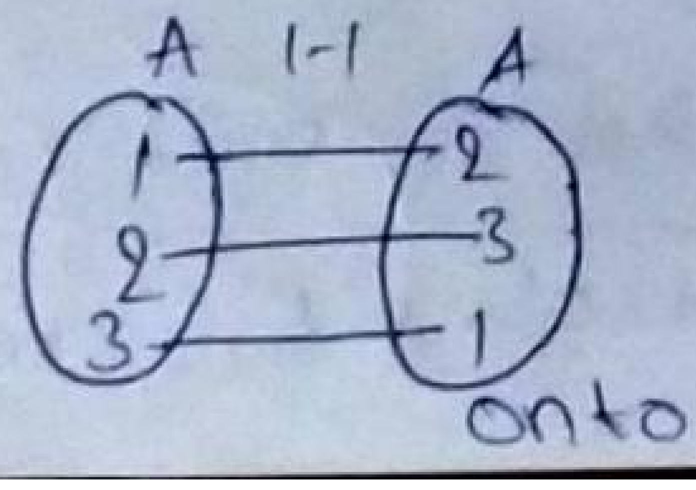


constant function.



1. let $A = \{1, 2, 3\}$ and $f: A \to A$ be defined by $f(1) = 2$, $f(2) = 3$, $f(3) = 1$ examine f^{-1} exists if so find f^{-1} & f^2

sol



since f is 1-1 & onto

$\Rightarrow f$ is bijective fun

$\therefore f^{-1}$ exist.

$f(1) = 2 \Rightarrow f^{-1}(2) = 1$

$f(2) = 3 \Rightarrow f^{-1}(3) = 2$

$f(3) = 1 \Rightarrow f^{-1}(1) = 3$

To find f^2:

$f^2 = f \circ f$

$f^2(1) = f \circ f(1) = f(2) = 3$

$f^2(2) = f \circ f(2) = f(3) = 1$

$f^2(3) = f \circ f(3) = f(1) = 2$

$\boxed{07/08/2021}$

1. consider $f(x) = 2x-1$, $g(x) = x^2-2$ where $f, g: R \to R$ then find fog & gof.

sol $f(x) = 2x-1$, $g(x) = x^2-2$

$fog(x) = f[g(x)] = f(x^2-2) = 2(x^2-2)-1$

$fog = 2x^2-5$

$gof = g[f(x)] = g[2x-1] = (2x-1)^2-2 = 4x^2-4x-1$

$gof = 4x^2-4x-1$

Note: since $fog \neq gof$, the commutative property does not hold for composition of 2 fun

2. Given $f(x) = x$, $g(x) = x+2$, & $h(x) = x^2$ verify whether $fo(goh) = (fog)oh$

Sol Given $f(x) = x$, $g(x) = x+2$, $h(x) = x^2$

$goh = g[h(x)] = g[x^2] = x^2+2$

$fo(goh) = f[x^2+2] = x^2+2$

consider $(fog) = f(g(x)) = f(x+2) = x+2$

$(fog)oh = (fog)(h(x))$

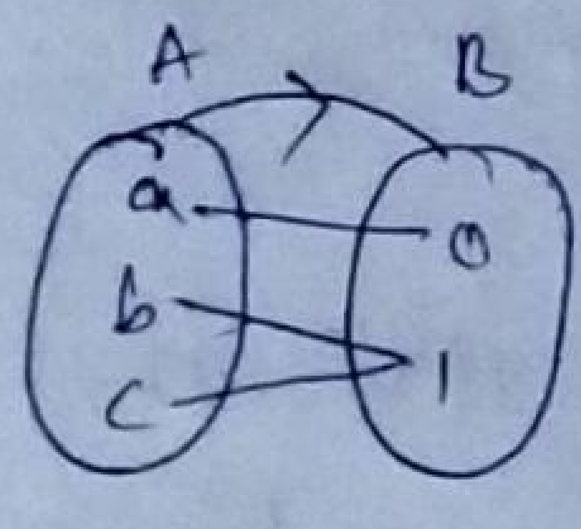
$= fog(x^2)$

$= x^2+2$

Note! Associative property will be satisfied for the compositio-n of fun

3. i) list all the possible fun from $A = \{a, b, c\}$ & $B = \{0, 1\}$ & examine in each case whether the fun is 1-1 and onto

Sol



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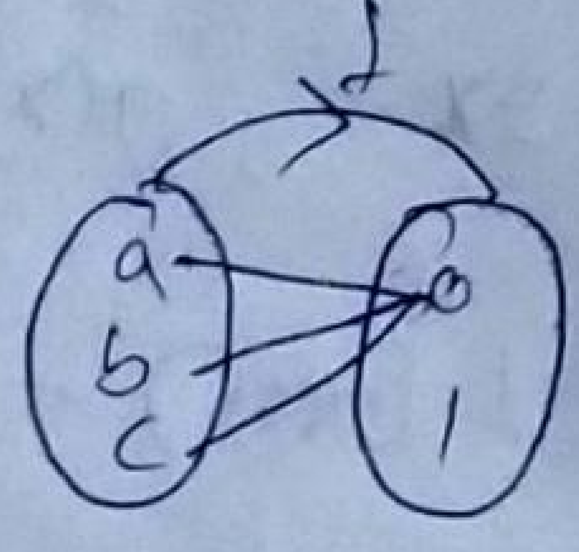
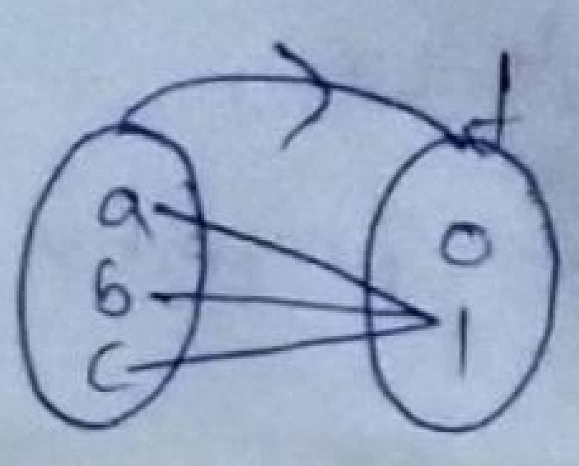
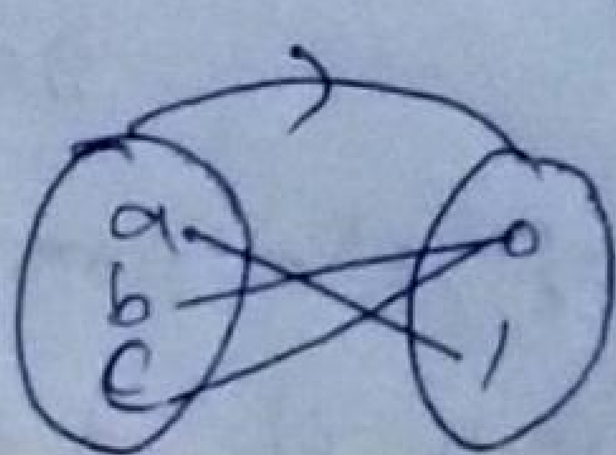


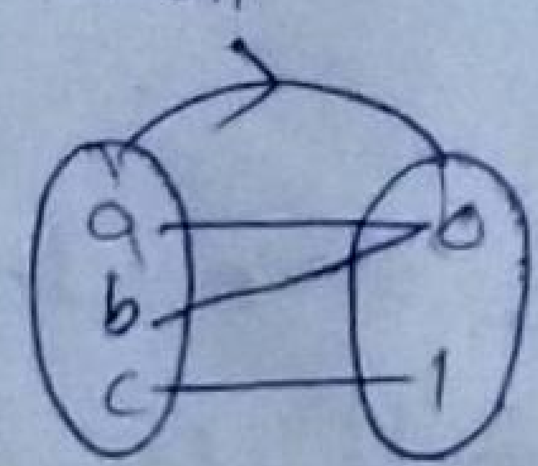
f is not a fun

f is not 1-1
not onto
many is 1 fun

f is not a 1-1, onto



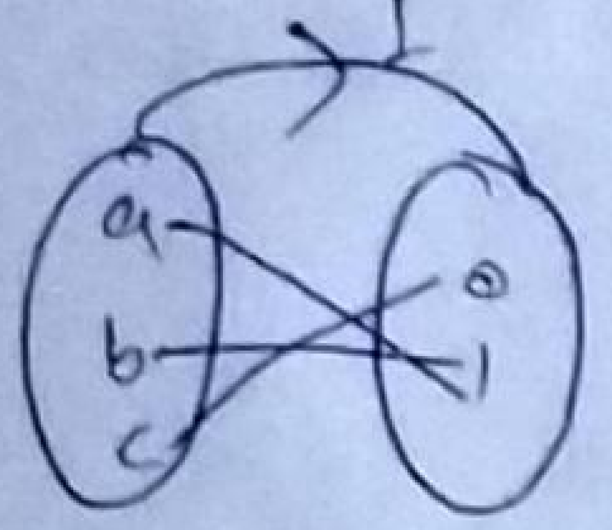


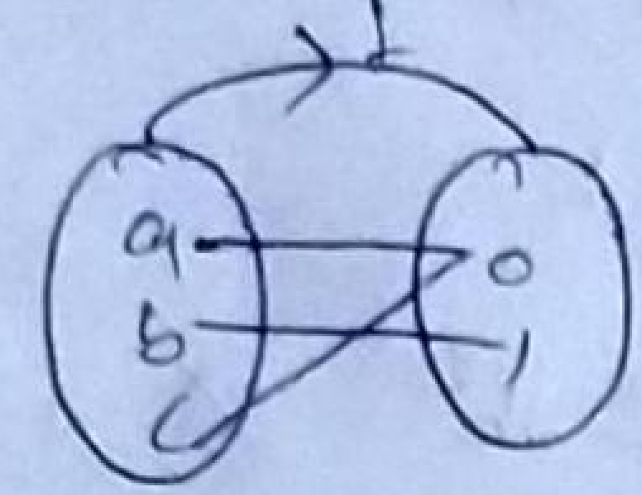


f is not 1-1, not onto

f is not 1-1, onto

f is not 1-1, onto



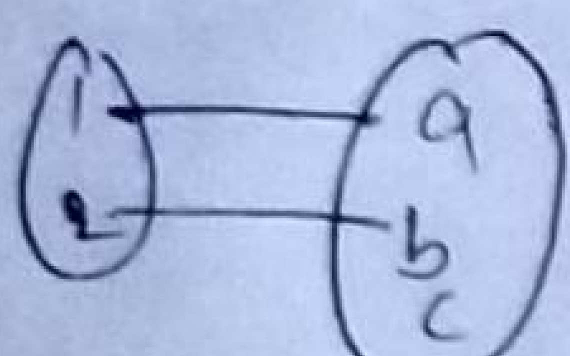


f is not 1-1 but onto

f is not 1-1 but onto

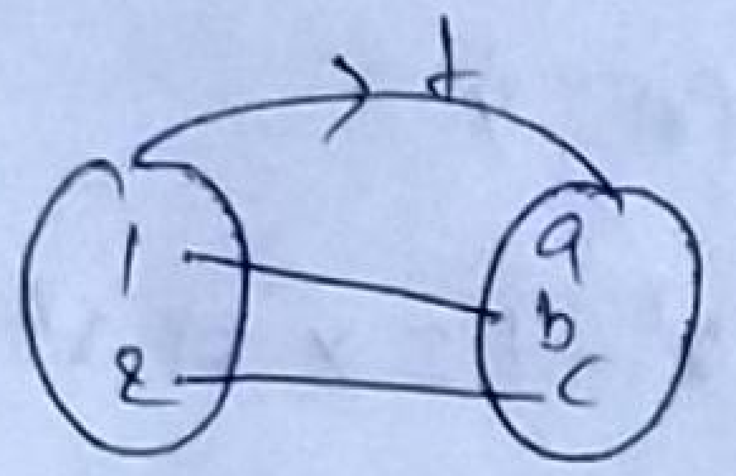
2) $A = \{1, 2\}$ $B = \{a, b, c\}$ → examine for 1-1 & onto.

①



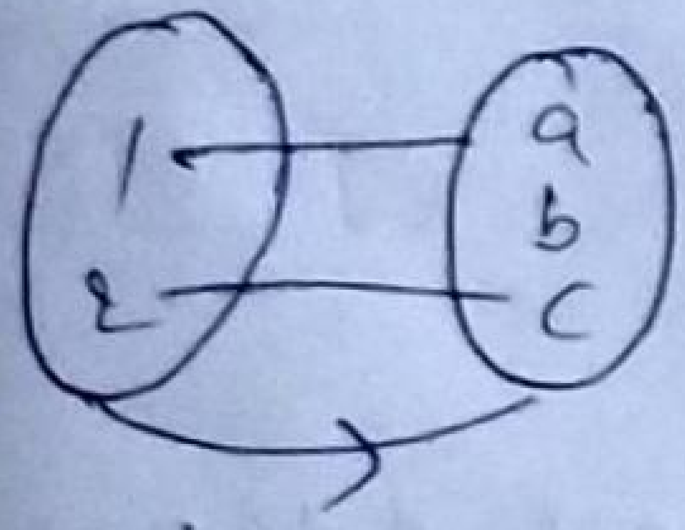
f is 1-1, not onto

②



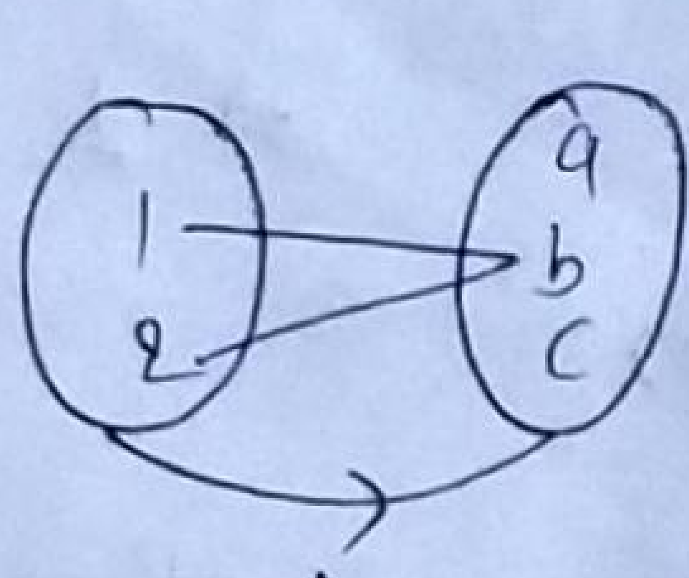
f is 1-1, not onto

③



f is 1-1, not onto

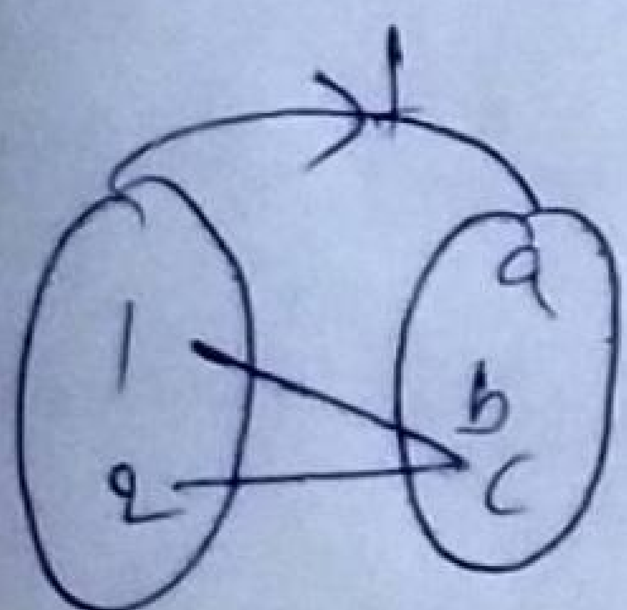
④



f

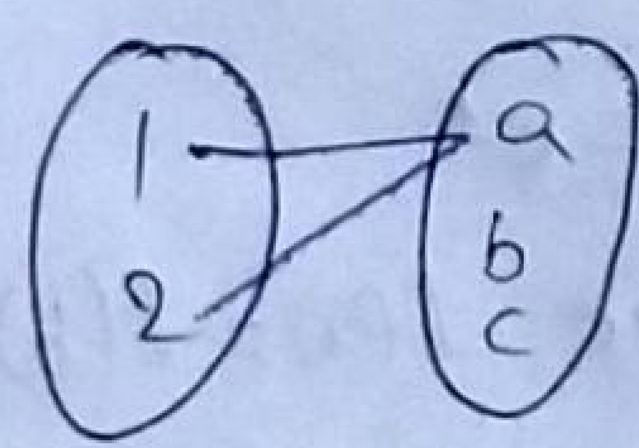
f is not 1-1, not onto

⑤



f is not 1-1 not, onto

⑥



f is not 1-1, onto

4. check for the bijective (1-1, onto), surjective (onto) (or) injective on R

① $f(x) = x^2 - 1$

$\forall a \in A$, $\exists b \in B$ P.T $f(a) = f(x)$

Assume $f(a) = f(b)$

$a^2 - 1 = b^2 - 1$

$a^2 = b^2$

$a = \pm\sqrt{b^2} \not\Rightarrow a = b$

$\Rightarrow a \neq b$

$\Rightarrow f$ is not 1-1

for onto: let $y \in R$ (codomain), $\exists\, x \in R$ (domain)

$f(x) = y$

$x^2 - 1 = y \Rightarrow x^2 = y + 1$

$x = \pm\sqrt{y+1}$ Here $y < 0$

then $x \notin R$

$\Rightarrow f$ is not onto

② $f(x) = \log x$

since the -ve values for $\log x$ is not defined

$f(x) = \log x$ is not a fun

③ $f(x) = e^x$

Assume $f(a) = f(b)$

$e^a = e^b$

$\log_e e^a = \log_e e^b$

$a = b$

$\therefore f$ is 1-1 fun.

for onto: let $y \in R(B)$ To prove that $\Rightarrow x \in R(A)$ P.T $f(x)=y$

consider $f(x)=y$

$e^x = y$

$\log e^x = \cancel{\log} \log y$

$x = \log y \notin R, y<0$

$\therefore$ f is not onto

(4) $f(x) = 5x-2$

consider $f(a)=f(b)$

$5a-2 = 5b-2$

$5a = 5b$

$\Rightarrow a = b \in R$

for onto: let $y \in R$ to find $x \in R$ P.T $f(x)=y$

$5x-2 = y \Rightarrow x = \frac{y+2}{5} \in R$

f is onto

Here diff angles give the same value.

(5) $f(x) = \cos x$

Assume $f(a)=f(b)$

$\cos a = \cos b$

$\not\Rightarrow a=b \Rightarrow$ f is not 1-1

$\boxed{\text{Real are from } (-\infty - \infty)}$

Here cos fun is defined is (-1 to 1)

$\therefore$ Range of f = range of $\cos x$

$= [-1, 1]$

$\neq$ codomain in R

$\therefore$ f is not onto

(6) $f(x) = \tan x$

$f(a) = f(b)$

$\tan a = \tan b$

$\not\Rightarrow a = b$

f is not 1-1

tan is defined from $(-\infty, \infty)$

$\therefore$ All the elem of codomain will here a pre-image in the domain

Range of f = range of $\tan x$

$= [-\infty, \infty]$

$=$ codomain in R

f is onto

Range of a function: let $f: A \to B$ be a fun. then the set of all the images of all the elem of A is called range of f (or) image of f.

5. Define the following fun on the integers $f(x) = x+1$

$g(x) = 3x$ which of these are 1-1 (or) onto.

Sol. $f(x) = x+1$

Assume $f(x) = f(y)$

$x+1 = y+1$

$x = y$

$\Rightarrow f$ is 1-1

Range of f = codomain of 'f'

i.e the set of all integers

$\therefore f \rightarrow$ onto

$g(x) = 3x$

Assume $f(a) = f(b)$

$3a = 3b \Rightarrow a = b \Rightarrow f$ is 1-1

Range of g = set of all integers multiple of 3 ($\because g(x) = 3x$)

$\neq$ codomain of f ($\because$ Domain & codomain $\in \mathbb{Z}$)

$\therefore$ g is not onto

$\therefore f$ is 1-1 & onto, g is 1-1 but not onto.