Poison Distribution:

A random Variable x is said to follow poisson distribution if it assume only non negative Values & its probability mass function is given by

$$P(X=x) = p(x) = \frac{e^{-\lambda}\lambda^{x}}{x!} \quad x=0,1,2,\ldots \ \lambda>0,$$

$\lambda \rightarrow$ parameter of the poisson distribution.
Its mean ~~&~~ and Variance is λ.

problem. 1:

① If X is a Poisson Variate such that $P(X=2) = 9\,P(X=4) + 90\,P(X=6)$, find the Variance.

Soln

The probability distribution for the Poisson R.V X is given by

$$P(X=x) = \frac{e^{-\lambda}\lambda^{x}}{x!}, \quad x=0,1,\ldots n \ \& \ \lambda>0$$

Given $P(X=2) = 9\,P(X=4) + 90\,P(X=6)$,

$$\frac{e^{-\lambda}\lambda^{2}}{2!} = 9\,\frac{e^{-\lambda}\lambda^{4}}{4!} + \frac{90\,e^{-\lambda}\lambda^{6}}{6!}$$

$\div$ throughout by $\lambda^{2}e^{-\lambda}$, we get

$$\frac{1}{2!} = \frac{9\lambda^{2}}{4!} + \frac{90\,\lambda^{4}}{6!}$$

$$\frac{1}{2} = \frac{\cancel{9}^{3}\lambda^{2}}{4\cdot\cancel{3}\cdot 2} + \frac{\cancel{90}\,\lambda^{4}}{\cancel{6}\cdot\cancel{5}\cdot 4\cdot\cancel{3}\cdot 2}$$

$$\tfrac{1}{2} = \frac{3\lambda^{2}}{8} + \frac{\lambda^{4}}{8} \Rightarrow \frac{\lambda^{4}}{8} + \frac{3\lambda^{2}}{8} = \frac{1}{2} \qquad \lambda^{4} + 3\lambda^{2} = 4$$

$\lambda^2+5\lambda-4=0$

-4

λ

$4 \quad -1$

$\lambda^4+4\lambda^2-\cancel{8}\lambda^2-4=0$

$\lambda^2(\lambda^2+4)-(\lambda^2+4)=0.$

$(\lambda^2-1)(\lambda^2+4)=0.$

$\lambda^2=-4 \quad [\lambda>0].$

$\lambda^2=1$

$\lambda=\pm1 \Rightarrow \boxed{\lambda=1}.$

For a poisson distribution

$Var(x)=\lambda=1.$

The probability of an item produced by a certain machine will be defective is 0.05. If the produced items are sent to the market in packets of 20 find the number of packets containing

1) atleast 2 defective items

2) exactly 2 ,,

3) atmost 2 defective items.

in a consignment of 1000 packets.

Soln. Let X denote the number of defective items produced by a certain machine.

P is the probability that an item to be defective is

$=0.05.$

$n=20. \quad N=1000$

Note:

In the limiting case as $n\to\infty$ then B.D becomes P.D.

$\because$ mean $\lambda=np$

$=20\times0.05$

$=1.$

$\boxed{\lambda=1}$

The pmf of PD is

$\therefore P(X=x)=\frac{e^{-\lambda}\lambda^x}{x!}=\frac{e^{-1}}{x!}$

1) Number of packets containing atleast 2 defective items $=N\,P(X\geq2).$

$=1000[1-P(X<2)]$

$=1000[1-(P(X=0)+P(X=1))]$

$=1000\left[1-\left(\frac{e^{-1}}{0!}+\frac{e^{-1}}{1!}\right)\right]$

$=1000[1-2e^{-1}]$

$=1000[1-2/e]$

$=1000(0.2642)$

$\simeq 264.$

(b) Number of packets containing exactly 2 defective item.

$= N \cdot P(X=2)$

$= 1000 \cdot \frac{e^{-1}}{2!}$

$= 500 \cdot e^{-1}$

$= 500\,(0.3679)$

$\simeq 184$ approx.

c) Number of packets containing atmost 2 defective items

$= NP(X \le 2)$

$= N\{P[X=0] + P[X=1] + P[X=2]\}$

$= 1000\left\{\frac{e^{-1}}{0!} + \frac{e^{-1}}{1!} + \frac{e^{-1}}{2!}\right\}$

$= 1000\{1/e + 1/e + 1/2e\}$

$= 1000\{2/e + 1/2e\}$

$= \frac{1000}{e}\{2 + 1/2\}$

$= \frac{\cancel{1000}^{500}}{e}\{5/\cancel{2}\} = \frac{2500}{e}$

$\simeq 920.$

The number of typing mistakes that a typist makes on a given page has a Poisson distribution with a mean of 3 mistakes. what is the probability that she makes

1) Exactly 7 mistakes

2) Fewer than 4 mistakes

3) No mistake on a given page.

Soln

Given Mean $(\lambda) = 3$

$P(X=x) = \frac{e^{-\lambda}\lambda^{x}}{x!}$

$= \frac{e^{-3}3^{x}}{x!}$

1) P[Exactly 7 mistakes]

$= P[X=7]$

$= \frac{e^{-3}3^{7}}{7!} = 0.0216.$

2) P[Fewer than 4 mistakes]

$P[X<4]$

$= P[X=0] + P[X=1] + P[X=2] + P[X=3]$

$= \frac{e^{-3}(3)^{0}}{0!} + \frac{e^{-3}\cdot 3}{1!} + \frac{e^{-3}3^{2}}{2!} + \frac{e^{-3}3^{3}}{3!}$

$e^{-3}\left[1 + 3 + 9/2 + 9/2\right].$

$= 13e^{-3} \simeq 0.6474$

(3) P[No mistake on a given page] i.e. P[X=0]

$$= \frac{e^{-3}(3)^0}{0!} = e^{-3}$$

$$= 0.0498$$

---

(4) From records of 10 Indian Army Corps kept over 20 years the following data were obtained showing the number of deaths caused by the horse. Calculate the theoretical Poisson frequencies.

No of deaths	0	1	2	3	4	Total
Frequency	109	65	22	3	1	200

Soln $\bar{x} = \frac{\sum f_i x_i}{N}$

$$= \frac{0.109+1.65+2.22+3.3+4.1}{200}$$

$$= \frac{65+44+9+4}{200}$$

$$= 0.61.$$

The theoretical frequencies are given by

$$f(x) = N\frac{e^{-\lambda}\lambda^x}{x!}$$

(1) $x=0$ $\lambda=0.61$

$$200 \cdot \frac{e^{-0.61}(0.61)^0}{0!}$$

$$\Rightarrow 200 \cdot e^{-0.61}$$ (0.5434)

$= 108.67$

$\simeq 109$

(2) $x=1$ $\lambda=0.61$

$$200\,\frac{e^{-0.61}(0.61)}{1!}$$

$= 200 \times (0.5434).(0.61)$

$\simeq 66.28$

$\simeq 66.$

(3)

Number of deaths	0	1	2	3	4	T
Observed frequency	109	65	22	3	1	20
Theoritical frequency	109	66	20	4	1	20