21·12·2021

1) A R·V x has the following probability distribution

x :	-2	-1	0	1	2	3
$P(x)$:	0.1	k	0.2	$2k$	0.3	$3k$

(i) Find the value of k.

(ii) Evaluate $P(x<2)$ & $P(-2<x<2)$

(iii) Find c·d·f of x.

(iv) Find mean of x.

Sol:

Given that

x :	-2	-1	0	1	2	3
$P(x)$:	0.1	k	0.2	$2k$	0.3	$3k$

Since x values are finite.

∴ this R·V x is a discrete R·V.

WKT, Its total probability is,

$$\sum_{x=-2}^{3} P(x) = 1$$

$$P(-2)+P(-1)+P(0)+P(1)+P(2)+P(3)=1$$

$$0.1 + k + 0.2 + 2k + 0.3 + 3k = 1$$

$$6k + 0.6 = 1$$

$$6k = 1 - 0.6 = 0.4$$

$$k = \frac{0.4}{6}$$

$$k = 0.067$$

(ii) $P(x<2) = P(-2)+P(-1)+P(0)+P(1)$

$= 0.1 + k + 0.2 + 2k$

$= 0.3 + 3k$

$= 0.3 + 3(0.067)$

$P(x<2) = 0.501$

$$P(-2<x<2) = P(x=-1)+P(x=0)+P(x=1)$$
$$= P(-1)+P(0)+P(1)$$
$$= k+0.2+2k$$
$$= 3k+0.2$$
$$= 3(0.067)+0.2$$
$$= 0.401$$

(iii) To find c.d.f of x

x	$P(x)$	$F(x)$
-2	0.1	0.1
-1	k	$0.1+k = 0.1+0.067 = 0.167$
0	0.2	$0.167+0.2 = 0.367$
1	$2k$	$0.367+2k = 0.367+2(0.067) = 0.501$
2	0.3	$0.501+0.3 = 0.801$
3	$3k$	$0.801+3k = 0.801+3(0.067) = 1$

(iv) To find Mean of x:-

$$\text{Mean of } x = E(x) = \text{Expectation of } x$$
$$= \sum_{x=-2}^{3} x \cdot P(x)$$
$$= (-2)\cdot P(-2)+(-1)\cdot P(-1)+0\cdot P(0)+1\cdot P(1)+2\cdot P(2)+3\cdot P(3)$$
$$= (-2)\cdot(0.1)+(-1)\cdot(k)+0\cdot(0.2)+1\cdot(2k)+2\cdot(0.3)+3\cdot(3k)$$
$$= -0.2-k+0+2k+0.6+9k$$
$$= 10k+0.4$$
$$= 10(0.067)+0.4$$
$$= 0.67+0.4$$
$$E(x) = 1.07$$

$\therefore$ Mean value of $x = E(x) = \bar{x} = 1.07$

2) The density function of a Cont. R.V x is given by

$\{\ ax;\ 0\le x\le 1$

2) The density function of a Cont. R.V X is given by

$$f(x) = \begin{cases} ax; & 0 \le x \le 1 \\ a; & 1 \le x \le 2 \\ 3a - ax; & 2 \le x \le 3 \\ 0; & \text{otherwise} \end{cases}$$

(i) Find the value of a

(ii) Find c.d.f of X.

Sol: Given that

$$f(x) = \begin{cases} ax; & 0 \le x \le 1 \\ a; & 1 \le x \le 2 \\ 3a - ax; & 2 \le x \le 3 \\ 0; & \text{otherwise} \end{cases}$$

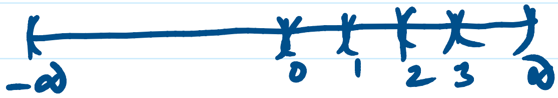
Since x values are in interval type.

∴ The values of x are infinite(or) uncountable.

hence the given R.V X has the Cont. R.V.

WKT, Its total probability is,

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$



$$\int_{-\infty}^{0} f(x)dx + \int_{0}^{1} f(x)dx + \int_{1}^{2} f(x)dx + \int_{2}^{3} f(x)dx + \int_{3}^{\infty} f(x)dx = 1$$

$$\cancel{\int_{-\infty}^{0} 0 \cdot dx} + \int_{0}^{1} ax\,dx + \int_{1}^{2} a\,dx + \int_{2}^{3} (3a - ax)\,dx + \cancel{\int_{3}^{\infty} 0 \cdot dx} = 1$$

$$a\left(\frac{x^2}{2}\right)_0^1 + a(x)_1^2 + \left(3ax - a\frac{x^2}{2}\right)_2^3 = 1$$

$$a\left(\frac{1}{2}\right) + a(2-1) + \left(9a - \frac{9a}{2} - 6a + \frac{4a}{2}\right) = 1$$

$$\frac{a}{2} + a + 3a - \frac{5a}{2} = 1$$

$$4a - \frac{4a}{2} = 1$$

$$\overline{2}$$

$$4a - 2a = 1$$
$$2a = 1$$
$$a = 1/2$$

(ii) To find c.d.f of x :-

Given that

$$f(x) = \begin{cases} ax ; & 0 \le x \le 1 \\ a ; & 1 \le x \le 2 \\ 3a - ax ; & 2 \le x \le 3 \\ 0 ; & \text{otherwise ie) } x > 3 \end{cases}$$

WKT, The c.d.f of x (or) distribution fn of x is,

$$F(x) = \int_{-\infty}^{x} f(x)\,dx$$

Case (i) :- $0 \le x \le 1$

$$F(x) = \int_{-\infty}^{0} f(x)dx + \int_{0}^{x} f(x)\,dx = \int_{0}^{x} ax\,dx = a\left(\frac{x^2}{2}\right)_0^x$$

$$= a\left(\frac{x^2}{2}\right)$$

$$F(x) = \frac{ax^2}{2} ; \quad 0 \le x \le 1$$

Case (ii) :- $1 \le x \le 2$

$$F(x) = \int_{-\infty}^{x} f(x)\,dx = \cancel{\int_{-\infty}^{0} f(x)dx}_{(0)} + \int_{0}^{1} f(x)dx + \int_{1}^{x} f(x)dx$$

$$= \int_{0}^{1} ax\,dx + \int_{1}^{x} a\,dx$$

$$= a\left(\frac{x^2}{2}\right)_0^1 + a(x)_1^x$$

$$= a\left(\frac{1}{2}\right) + a(x - 1)$$

$$= \frac{a}{} + ax - a$$

$$F(x) = ax - \frac{a}{2} \quad ; \quad 1 \le x \le 2$$

<u>Case (iii)</u> :- $2 \le x \le 3$

$$F(x) = \int_{-\infty}^{x} f(x)\,dx$$

$$= \cancel{\int_{-\infty}^{0} f(x)dx}_{(0)} + \int_{0}^{1} f(x)dx + \int_{1}^{2} f(x)dx + \int_{2}^{x} f(x)dx$$

$$= \int_{0}^{1} ax\,dx + \int_{1}^{2} a\,dx + \int_{2}^{x} (3a - ax)\,dx$$

$$= a\left(\frac{x^2}{2}\right)_0^1 + a(x)_1^2 + \left(3ax - a\frac{x^2}{2}\right)_2^x$$

$$= a\left(\frac{1}{2}\right) + a(2-1) + \left(3ax - \frac{ax^2}{2} - 6a + \frac{4a}{2}\right)$$

$$= \frac{a}{2} + a + 3ax - \frac{ax^2}{2} - 4a$$

$$F(x) = 3ax - \frac{ax^2}{2} - \frac{5a}{2} \quad ; \quad 2 \le x \le 3$$

<u>Case (iv)</u> :- $x > 3$ ie) $3 < x < \infty$

$$F(x) = \int_{-\infty}^{x} f(x)\,dx$$

$$= \cancel{\int_{-\infty}^{0} f(x)dx}_{(0)} + \int_{0}^{1} f(x)dx + \int_{1}^{2} f(x)dx + \int_{2}^{3} f(x)dx + \cancel{\int_{3}^{\infty} f(x)dx}_{(0)}$$

$$= \int_{0}^{1} ax\,dx + \int_{1}^{2} a\,dx + \int_{2}^{3} (3a - ax)\,dx$$

$$= a\left(\frac{x^2}{2}\right)\Big|_0^1 + a(x)\Big|_1^2 + \left(3ax - a\frac{x^2}{2}\right)\Big|_2^3$$

$$= a\left(\frac{1}{2}\right) + a(2-1) + \left(9a - \frac{9a}{2} - 6a + \frac{4a}{2}\right)$$

$$= \frac{a}{2} + a + 3a - \frac{5a}{2}$$

$$= 4a - \frac{4a}{2}$$

$$= 4a - 2a$$

$$F(x) = 2a \quad ; \quad x > 3$$

$\therefore$ The distribution function of x is,

$$F(x) = \begin{cases} \dfrac{ax^2}{2} & ; \ 0 \le x \le 1 \\ ax - \dfrac{a}{2} & ; \ 1 \le x \le 2 \\ 3ax - \dfrac{ax^2}{2} - \dfrac{5a}{2} & ; \ 2 \le x \le 3 \\ 2a & ; \ x > 3 \end{cases} \qquad \text{where } a = 1/2$$