

REGRESSION LINES

The two types of Regression lines are

(i) Reg. line of x on y :-

$$(x - \bar{x}) = \frac{r\sigma_x}{\sigma_y} (y - \bar{y})$$

$\nearrow b_{xy}$

(ii) Reg. line of y on x :-

$$(y - \bar{y}) = \frac{r\sigma_y}{\sigma_x} (x - \bar{x})$$

$\nearrow b_{yx}$

where \bar{x} = Mean of x

\bar{y} = Mean of y

σ_x = Standard deviation of x

σ_y = Standard deviation of y.

here $b_{xy} = \frac{r\sigma_x}{\sigma_y}$ and $b_{yx} = \frac{r\sigma_y}{\sigma_x}$

b_{xy} and b_{yx} are called regression
co-efficients.

Properties :-

(i) The Correlation Co-efficient is the Geometric mean between the regression Co-effts.

ie) $r = \pm \sqrt{b_{xy} \cdot b_{yx}}$

Prb:-

RHS:-

$$\begin{aligned} \pm \sqrt{b_{xy} \cdot b_{yx}} &= \pm \sqrt{\frac{r\sigma_x}{\sigma_y} \cdot \frac{r\sigma_y}{\sigma_x}} \\ &= \pm \sqrt{r^2} \end{aligned}$$

$$\Rightarrow \pm \sqrt{b_{xy} \cdot b_{yx}} = r \text{ (Correlation Coefft.)}$$

Property (2):-

If any one of the reg. coefficients is greater than unity then the other one must be less than unity.

ie) If $b_{xy} > 1$ then $b_{yx} < 1$

(or)

$b_{yx} > 1$ then $b_{xy} < 1$

PROBLEMS

① From the following data, Find

(i) Two regression lines (or) equations.

(ii) Correlation coefft. between marks in Economics and Statistics.

(iii) Most likely marks in Statistics when Economics marks are 30.

Marks in Economics (X): 25 28 35 32 31 36 29 38 34 32

Marks in Statistics (Y): 43 46 49 41 36 32 31 30 33 39

Sol:-

X	Y	$(x - \bar{x})$ ie) $(x - 32)$	$(x - \bar{x})^2$	$(y - \bar{y})$ ie) $(y - 38)$	$(y - \bar{y})^2$	$(x - \bar{x})(y - \bar{y})$
25	43	-7	49	5	25	-35
28	46	-4	16	8	64	-32
35	49	3	9	11	121	33
32	41	0	0	3	9	0
31	36	-1	1	-2	4	2
36	32	4	16	-6	36	-24
29	31	-3	9	-7	49	21
38	30	6	36	-8	64	-48
34	33	2	4	-5	25	-10
32	39	0	0	1	1	0
$\Sigma x = 320$	$\Sigma y = 380$		$\Sigma (x - \bar{x})^2 = 140$		$\Sigma (y - \bar{y})^2 = 398$	$\Sigma (x - \bar{x})(y - \bar{y}) = -93$

$$\bar{x} = \text{Mean of } x = \frac{\Sigma x}{n} = \frac{320}{10} = 32 \Rightarrow \bar{x} = 32$$

$$\bar{y} = \text{Mean of } y = \frac{\Sigma y}{n} = \frac{380}{10} = 38 \Rightarrow \bar{y} = 38$$

∴ The regression co-efficients are,

$$b_{xy} = \frac{\Sigma (x - \bar{x})(y - \bar{y})}{\Sigma (y - \bar{y})^2} = \frac{-93}{398} = -0.2337$$

$$\therefore b_{xy} = -0.2337$$

$$r_{xy} = \frac{\sum (x-\bar{x})(y-\bar{y})}{\sqrt{\sum (x-\bar{x})^2 \sum (y-\bar{y})^2}} = \frac{-93}{398} = -0.2337$$

$$\therefore b_{xy} = -0.2337$$

$$b_{yx} = \frac{\sum (x-\bar{x})(y-\bar{y})}{\sum (y-\bar{y})^2} = \frac{-93}{140} = -0.6643$$

$$\therefore b_{yx} = -0.6643$$

To find the Correlation Coefficient:-

$$\text{WKT, } r = \pm \sqrt{b_{xy} \cdot b_{yx}} = \pm \sqrt{(-0.2337)(-0.6643)}$$

$$r = \pm 0.394$$

To find the Regression Lines (or) Equations:-

Reg. Line of x on y:-

$$x - \bar{x} = b_{xy}(y - \bar{y})$$

$$x - 32 = (-0.2337)(y - 38)$$

$$x - 32 = -0.2337y + 8.8806$$

$$x = -0.2337y + 8.8806 + 32$$

$$x = -0.2337y + 40.8806$$

Reg. Line of y on x:-

$$y - \bar{y} = b_{yx}(x - \bar{x})$$

$$y - 38 = (-0.6643)(x - 32)$$

$$y = -0.6643x + 59.2576$$

To find Statistics Marks when Economics mark is 30:-

$$\text{ie) } x = 30, y = ?$$

$$y = (-0.6643)(30) + 59.2576$$

$$y \approx 39$$

\therefore the most likely marks of Statistics is 39
when Economics mark is 30.