

RSA Algorithm

RSA \rightarrow Rivest - Shamir - Adleman

RSA developed in 1978

\rightarrow It is an asymmetric cryptography algorithm (2 keys)

① Public key ✓

② Private key ✓

Public key

Known to all the users in network

Private key

kept secret, not sharable to all

\rightarrow If the public key of user A is used for encryption we have to use the private key of same user for decryption

\rightarrow The RSA scheme is a block cipher in which plain text and cipher text are integers b/w 0 and $n-1$ for same value n .

RSA Algorithm

(i) Key Generation

(i) Select 2 large prime number 'p' and 'q'

(ii) ~~calculate~~ calculate $n = p \times q$

(iii) calculate euler totient function
 $\phi(n) = (p-1) \times (q-1)$

$\phi(n) \rightarrow$ euler totient function

(iv) choose value of e

(i) $1 < e < \phi(n)$ } condition

(ii) $\gcd(\phi(n), e) = 1$

(v) calculate

$$d \equiv e^{-1} \pmod{\phi(n)}$$

$$ed \equiv 1 \pmod{\phi(n)}$$

$$ed \pmod{\phi(n)} = 1$$

(vi) public key = $\{e, n\}$

(vii) private key = $\{d, n\}$

(viii) Encryption

$$C = M^e \pmod{n}$$

(ix) Decryption

$$M = C^d \pmod{n}$$

$$\text{let } p = 3, q = 11$$

Step 1

Two prime numbers p, q

$$p = 3, q = 11$$

Step 2

$$n = p \times q$$

$$n = 3 \times 11$$

$$\boxed{n = 33}$$

Step 3

$$\phi(n) = (p-1) \times (q-1)$$

$$\begin{aligned}\phi(n) &= (3-1) \times (11-1) \\ &= 2 \times 10\end{aligned}$$

$$\boxed{\phi(n) = 20}$$

Step 4

The value of e will be given in question

If not we need to choose

$$\boxed{e = 7} \quad \text{as } 1 < 7 < 20$$

$$\gcd(7, 20) = 1$$

Step 5

$$ed \bmod \phi(n) = 1$$

$$(7 \times d) \bmod 20 = 1$$

$$7 \times 3 \bmod 20 = 1$$

$$\boxed{d = 3}$$

hit & own method

Step 6

$$\text{public key} = \{e, n\}$$

$$= \{7, 33\}$$

Step 7

~~private~~
private

$$\text{private key} = \{d, n\}$$

$$= \{3, 33\}$$

Encryption

Abhi \rightarrow (4) \rightarrow As the letter in Abhi is (4) //

Encryption .

$$\text{plain text} = m < n$$

$c \rightarrow$ cipher text

$$\boxed{C = M^e \bmod n}$$

$$C = M^e \bmod n$$

$$C = 31^7 \bmod 33 = 4$$

$$\boxed{\text{Let } m = 31}$$

$$\boxed{C = 4}$$

Decryption

$$m = c^d \bmod n$$

$$= 4^3 \bmod 33$$

$$m = 31$$