3D Transformations

1. Basic Transformations
   1. Translational
   2. Rotational
   3. scaling.
2. Advance Transformations
   4. Reflection
   5. shearing.

1. Translation

+ Displacement of an object in given distance and a given direction from its original position

$$x' = x + t_x$$
$$y' = y + t_y$$
$$z' = z + t_z$$

Example

A $(0,3,1)$ B $(3,3,2)$ C $(3,0,0)$ D $(0,0,0)$

$t_x = t_y = 1 \quad t_z = 2$

$A' = (1,4,3)$

B' $(4,4,4)$

C' $(4,1,2)$

D' $(1,1,2)$

2. Rotation

* Rotation is applied to an object by Repositioning it along with any one axis.
* To generate the rotation we need pivot point (x, y, z) and rotation angle θ

along x axis:

$x' = x$; $y' = y\cos\theta - z\sin\theta$; $z' = y\sin\theta + z\cos\theta$

along y-axis:

$x' = z\sin\theta + x\cos\theta$; $y' = y$; $z' = y\cos\theta - x\sin\theta$

along z-axis

$x' = x\cos\theta - y\sin\theta$; $y' = x\sin\theta + y\cos\theta$; $z' = z$

Example: $(x, y, z) = (1, 2, 3)$ $\theta = 90^\circ$ towards x-axis

$x' = 1$; $y' = 2\cos 90^\circ - 3\sin 90^\circ$; $z' = 2\sin 90^\circ + 3\cos 90^\circ$

$= 2(0) - 3(1)$

$= -3$

3. scaling

* scaling alters the size of an object
  1. uniform scaling
  2. Non-uniform scaling

$x' = x * sx$

$y' = y * sy$

$z' = z * sz$

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Example A (0,3,1) B (3,3,1) C (3,0,0) D (0,0,0)

$S_z = S_y = 3$ ~~$S_z = 3$~~ $S_x = 2$

$A' = (0,9,3)$ $B'(6,9,\overset{6}{\cancel{18}})$ $C'(6,0,0)$ $D'(0,0,0)$

4. Reflection

- Reflection produces the mirror image of an object needed to an action of reflection

1. along xy plane :

$$x' = x$$
$$y' = y$$
$$z' = -z$$

2. along yz plane

$$x' = -x$$
$$y' = y$$
$$z' = z$$

3. Along zx plane

$$x' = x$$
$$y' = -y$$
$$z' = z$$

Example : A (3,4,1) B (6,4,2) C (5,6,3) on xy plane

$A' = (3,4,-1)$ $B'(6,4,-2)$ $C'(5,6,-3)$

5. **Shearing :**

→ A transformation that ~~slants~~ slants shape of an object is known as shearing.

Along x axis :

$x' = x$; $y' = y + sh_y * x$; $z' = z + sh_z * x$

Along y axis :

$x' = x + sh_x * y$; $y' = y$; $z' = z + sh_z * y$

Along z axis :

$x' = x + sh_x * z$; $y' = y + sh_y * z$; $z' = z$

Example :

A $(0, 0, 0)$ B $(1, 1, 2)$ C $(1, 1, 3)$ along y-axis

$sh_x = 2$ $sh_y = 2$ $sh_z = 3$

$A' = (0, 0, 0)$

$B' = (3, 1, 5)$

$C' = (3, 1, 6)$

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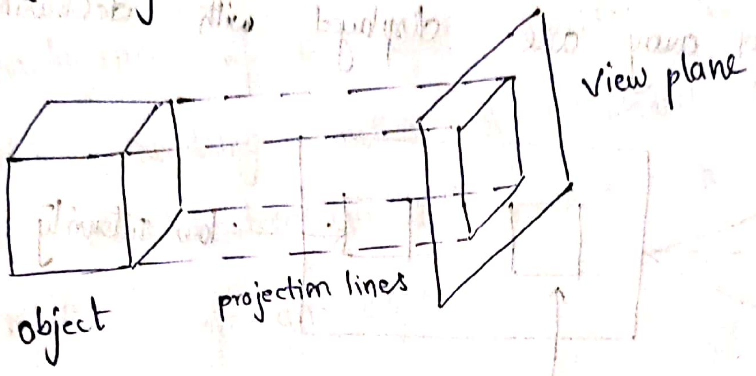
3D Concepts

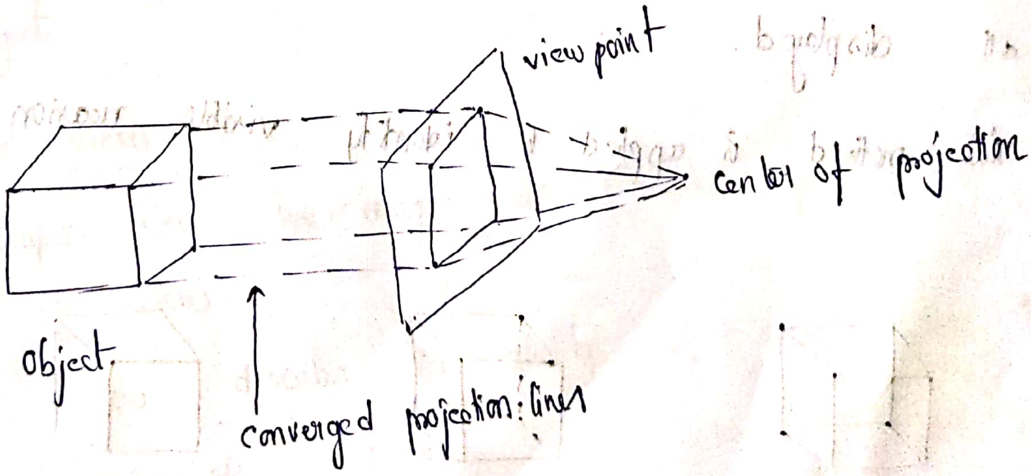
1. Parallel Projection

- In this method a view plane is used.
- Z co-ordinate is discarded.
- The 3D view is constructed by extending lines from each vertex on the object until they intersect the view point.



2. Perspective Projection

- In Perspective Projection the projection lines are not parallel instead they are convert at single point called center of projection



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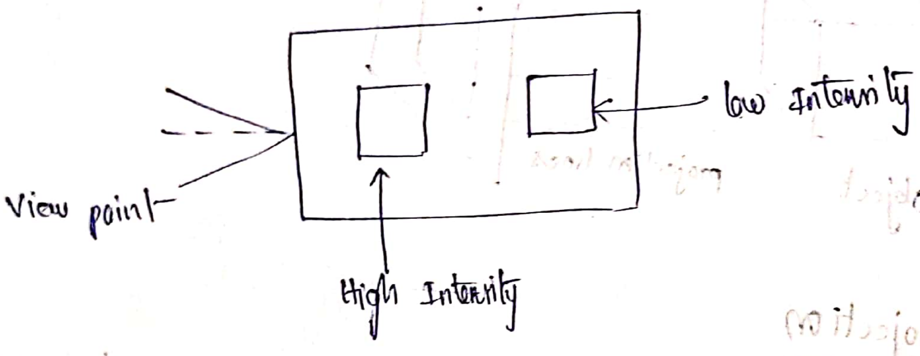
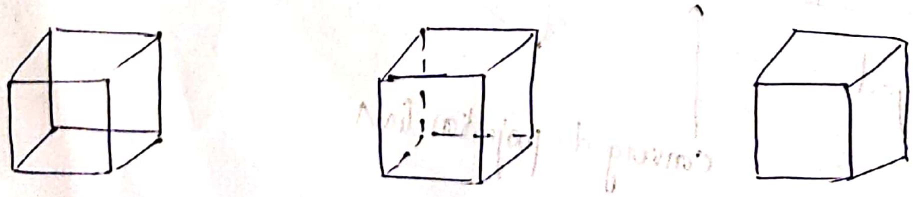
3. Depth Cueing

- In Depth Cueing an object is projection with Depth information
- The depth of an object can be represented by the intensity of the image.
- The parts of the objects closest to the viewing position are displayed with highest Intensities.
- Objects further away are displayed with decreasing Intensities



4. Visible Line and Surface Identification

- Visible lines are displayed in different color.
- Invisible lines either displayed in dashed lines or not at all displayed.
- This method is applied to identify visible reasion of the object



a) visible lines are highlighted

b) Non visible lines are shown by dashed lines

c) Non visible lines are not shown

5. Surface Rendering

* Surface rendering involves setting the surface intensity of the object according to two parameters they are

    1. Lighting Conditions
    2. Surface characteristics

* lighting condition specifies the intensity and position of light source
* surface characteristics of the object specifies the degree of transparency
* usually surface rendering method is combined with visible line and surface Identification (VLSI) method to generate high degree of realism of an object

6. Exploded and Cutaway view:

* In Exploded and cutaway view the internal property of an object is projected in hierarchical structures
* This method is used for detailed study of each subparts of an object.

7. Stereoscopic view

* In stereoscopic view two images are rendered from each eyes point of view
* stereoscopic view discribes the ability of visual plane to register a sense of 3D shape and form the visual inputs
* Resulted image will be display on the computer screen or other output devices like project wall, HMD etc.

Bezier Curve

→ The Concept of bezier curves was given by Pierre Bezier.

→ Bezier curve can be defined as:

⇒ Bezier are ends of the curve

⇒ ~~Oth~~ Two Points determine the shape of the curve.

→ The degree of a Bezier curve is defined by the control points n.

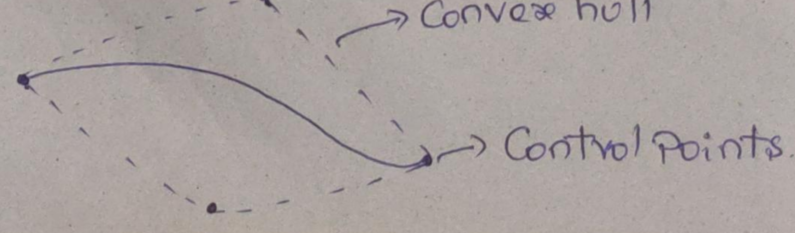
→ The curves passes through 1st and last control point called as Anchors and not passing is called hangers

→ All the Control Points are ~~moving~~ lies completly in Convex

→ If the Control Points are moving the shape of the curve changes.

→ The degree of the Polynomial defining the curve segment is one less than the total no. of ~~curves~~ Control Points.

Degree = Number of Control Points − 1



* Bezier curve is always contained within a polygon called as convex hull of its control points.

Derivation

degree quadratic

→ Q_0 and Q_1 are points on the lines $P_0 \to P_1$ and $P_1 \to P_2$

$$Q_0 = (1-t)P_0 + tP_1$$

$$Q_1 = (1-t)P_1 + tP_2$$

→ $c(t)$ is a point on the Bezier curve on the line $Q_0 \to Q_1$

$$c(t) = (1-t)Q_0 + tQ_1$$

$$c(t) = (1-t)[(1-t)P_0 + tP_1] + t[(1-t)P_1 + tP_2]$$

$$c(t) = (1-t)^2 P_0 + t(1-t)P_1 + t(1-t)P_1 + t^2 P_2$$

$$= (1-t)^2 P_0 + 2t(1-t)P_1 + t^2 P_2$$

degree cubic

A cubic bezier curve is defined by 4 control points.

$$c(t) = (1-t)^3 P_{0,0} + 3t(1-t^2)P_{1,0} + 3t^2(1-t)P_{2,0} + t^3 P_{3,0}$$

→ Degree of n Bezier curves

$$\cancel{c(t) = \sum_{i=0} P_t \cdot B_{t,n}}$$

$$\boxed{c(t) = \sum_{i=0}^{n} P_i \cdot B_{i,n} u}$$

where $(B_{i,n})u$ are called Bernstein Polynomials

→ Bernstein Polynomials can be defined as

$$\boxed{B_{i,n}(u) = \binom{n}{i} u^{i} (1-u)^{n-i}}$$

$$\boxed{\binom{n}{i} = \frac{n!}{i!(n-i)!}}$$

Given Problem,

$P_0(4,2)$

$P_1(8,8)$

$P_2(16,4)$

$Q(u) = \sum_{i=0}^{2} P_i B_{i,2}(u)$

$0 \le u \le 1$

$0, 0.2, 0.4, 0.6, 0.8, 1$

$Q(u) = P_0 B_{0,2}(u) + P_1 B_{1,2}(u) + P_2 B_{2,2}(u)$

$x(u) = x_0 B_{0,2}(u) + x_1 B_{1,2}(u) + x_2 B_{2,2}(u)$

$y(u) = y_0 B_{0,2}(u) + y_1 B_{(1,2)}(u) + y_2 B_{(2,2)}(u)$

$B_{0,2}(u) = \frac{n!}{i!(n-i)!} u^{(0)} (1-u)^{2-0}$

$= (1-u)^2$

$B_{1,2}(u) = \frac{n!}{i!(n-i)!} \times u^{(1)} (1-u)^{2-1}$

$= 2(u)(1-u)$

$B_{2,2}(u) = \frac{n!}{i!(n-i)!} \times u^{(2)} (1-u)^{2-2}$

$= u^2$

$x(u) = x_0 (1-u)^2 + x_1 2u(1-u) + x_2 \cdot u^2$

$y(u) = y_0 (1-u)^2 + y_1 2u(1-u) + y_2 \cdot u^2$

$x(u) = 4(1-u)^2 + 8(2u(1-u) + 16u^2$

$y(u) = 2(1-u)^2 + 8 \cdot 2u(1-u) + 4u^2$

$x(u) = 4(1-u)^2 + 16u \cancel{-} 16u^2 + 16u^2$

$= 4(1 + u^2 - 2u) + 16u$

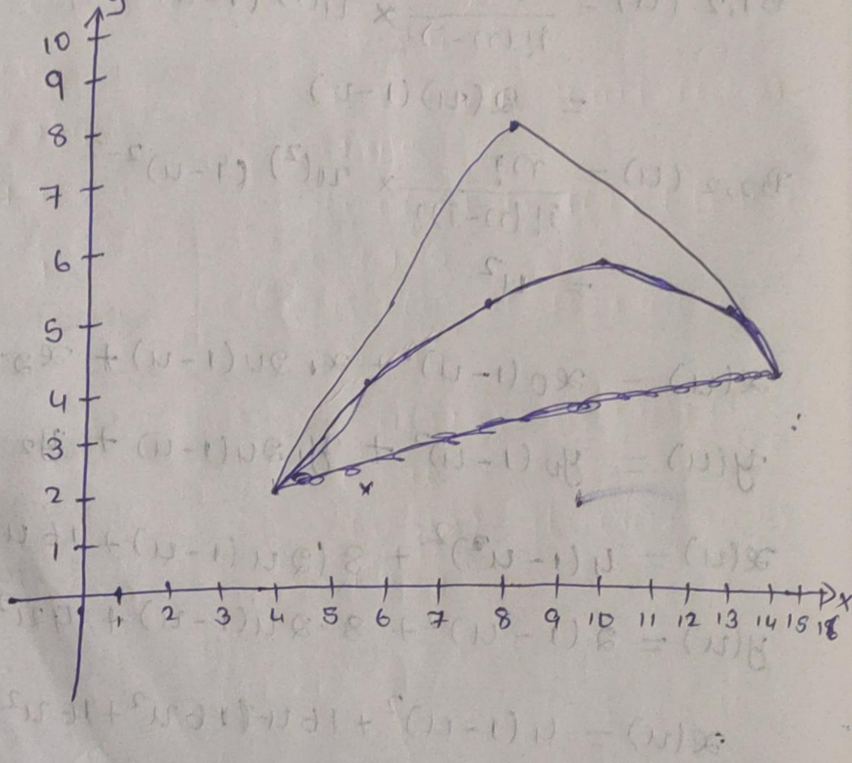
$= 4 + 4u^2 - 8u + 16u$

$$\boxed{x(u) = 4u^2 + 8u + 4}$$

$$y(u) = 2(1-u^2) + 16u - 16u^2 + 4u^2$$
$$= 2(1+u^2 - 2(1)) + 16u - 12u^2$$
$$= 2 + 2u^2 - 4u + 16u - 12u^2$$

$$\boxed{y(u = 2 + 12u - 10u^2}$$

	x	y
$u=0$	4	2
0.2	5.76	4
0.4	7.84	5.2
0.6	10.24	5.6
0.8	12.96	5.2
1	16	4



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Bezier Curve

Asha (1)

Construct Bezier curve for the control points:

(4, 2) (8, 8) (16, 4)

$$Q(u) = \sum_{i=0}^{2} P_i B_{i,2}(u) , \quad 0 \le u \le 1$$

$$Q(u) = P_0 B_{0,2}(u) + P_1 B_{1,2}(u) + P_2 B_{2,2}(u)$$

$$\Rightarrow x(u) = x_0 B_{0,2}(u) + x_1 B_{1,2}(u) + x_2 B_{2,2}(u)$$

$$y(u) = y_0 B_{0,2}(u) + y_1 B_{1,2}(u) + y_2 B_{2,2}(u)$$

$$B_{0,2}(u) = (1-u)^2$$

$$B_{i,n}(u) = {}^nC_i \, u^i (1-u)^{n-i}$$

$$B_{1,2}(u) = {}^2C_1 u^1 (1-u)^{2-1}$$

$$= \frac{2!}{1!(2-1)!} u(1-u) \qquad {}^nC_i = \frac{n!}{i!(n-i)!}$$

$$= \frac{2}{1.1} u(1-u)$$

$$= 2u(1-u)$$

$$B_{2,2}(u) = {}^2C_2 u^2 (1-u)^{2-2}$$

$$= u^2$$

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$$x(u) = x_0 (1-u)^2 + x_1 \cdot 2u(1-u) + x_2 u^2$$ ②

$$y(u) = y_0 (1-u)^2 + y_1 \cdot 2u(1-u) + x_2 u^2$$

For: $P_0 (4,2)$ $P_1 (8,8)$ $P_2 (16,4)$

$$x(u) = 4(1-u)^2 + 8 \cdot 2u(1-u) + 16u^2$$

$$y(u) = 2(1-u)^2 + 8 \, 2u(1-u) + 4u^2$$

$$x(u) = 4(1-u)^2 + 16u - 16u^2 + 16u^2$$

$$= 4(1+u^2-2u) + 16u$$

$$= 4 + 4u^2 - 8u + 16u$$

$$\boxed{x(u) = 4 + 4u^2 + 8u}$$

$$y(u) = 2(1-u)^2 + 16u - 16u^2 + 4u^2$$

$$= 2(1+u^2-2u) + 16u - 12u^2$$

$$= 2 + 2u^2 - 4u + 16u - 12u^2$$

$$\boxed{y(u) = 2 - 10u^2 + 12u}$$

③

$x(u) = 4 + 4u^2 + 8u$

$y(u) = 2 - 10u^2 + 12u$

$u = 0$:

$x(0) = 4$

$y(0) = 2$

$u = 0.2$:

$x(0.2) = 4 + 4(0.2)^2 + 8(0.2)$

$= 4 + 0.16 + 1.6$

$= 5.76$

$y(0.2) = 2 - 10(0.2)^2 + 12(0.2)$

$= 2 - 0.4 + 2.4$

$= 4$

$u = 0.4$:

$x(0.4) = 4 + 4(0.4)^2 + 8(0.4)$

$= 4 + 0.64 + 3.2$

$= 7.84$

$y(0.4) = 2 - 10(0.4)^2 + 12(0.4)$

$= 2 - 1.6 + 4.8$

$= 5.2$

$u = 0.6$:

$x(0.6) = 4 + 4(0.6)^2 + 8(0.6)$

$= 4 + 1.44 + 4.8$

$= 10.24$

$y(0.5) = 2 - 10(0.6)^2 + 12(0.6)$

$= 2 - 3.6 + 7.2$

$= 5.6$

4

$u = 0.8$:-

$$x(0.8) = 4 + 4(0.8)^2 + 8(0.8)$$
$$= 4 + 2.56 + 6.4$$
$$= 12.96$$

$$y(0.8) = 2 - 10(0.8)^2 + 12(0.8)$$
$$= 2 - 6.4 + 9.6$$
$$= 5.2$$

$u = 1$:-

$$x(1) = 4 + 4(1)^2 + 8(1)$$
$$= 4 + 4 + 8$$
$$= 16$$

$$y(1) = 2 - 10(1)^2 + 12(1)$$
$$= 2 - 10 + 12$$
$$= 4$$

u	$x(u)$	$y(u)$
0	4	2
0.2	5.76	4
0.4	7.84	5.2
0.6	10.24	5.6
0.8	12.96	5.2
1	16	4

Bezier Curve

Asha (5)

P0 : (4,2)

P1 : (8,8)

P2 : (16,4)