

11/03/23

### UNIT-3

## Clustering and Regression

### ⇒ Clustering:

- \* Clustering (or) cluster analysis is a machine learning techniques, which groups the unlabelled dataset. It can be defined as "A way of grouping the data points into different clusters, consisting of similar data points."
- \* The objects with the possible similarities remain in a group that has less (or) no similarities with another group.
- \* It does it by finding some similar patterns in the unlabelled dataset such as shape, size, color behaviour etc.... & divides them as per the presence & absence of the similar patterns.
- \* It is an Unsupervised learning method, hence no supervision is provided to the algorithm & it deals with the unlabeled dataset.
- \* After applying this clustering technique, each cluster (or) group is provided with a cluster id. ml system can use this id to simplify the processing of large & complex datasets.
- \* The clustering technique is commonly used for statistical data analysis. Some most common users of this technique are,
  1. Market Segmentation.
  2. Statistical data Analytics.
  3. Social network Analysis.

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→ K

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Ans

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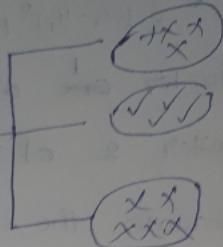
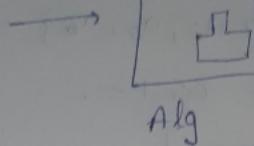
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4. Image Segmentation
5. Anomaly detection etc...

→ Raw data:

$\begin{matrix} x & x & v \\ o & o & o \\ \checkmark & x & o \\ x & x & \\ \checkmark & \checkmark & \checkmark \end{matrix}$



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→ K means clustering Alg to divide following data into  
2 cluster set

$x_1$	1	2	2	3	4	5
$x_2$	1	1	3	2	3	5

Ans: Choosing randomly 2 cluster sets

1. choosing randomly 2 cluster sets

$$v_1 = (2, 1)$$

$$v_2 = (2, 3)$$

2. finding the distance b/w the cluster center &  
each data points

Distance Table:-

Data point	Distance from $v_1 (2, 1)$	Distance from $v_2 (2, 3)$	Assigned Centre
$a_1 (1, 1)$	1	$\sqrt{5} = 2.236$	$v_1$
$a_2 (2, 1)$	0	2	$v_1$
$a_3 (2, 3)$	2	0	$v_2$
$a_4 (3, 2)$	$\sqrt{2} = 1.414$	$\sqrt{2} = 1.414$	$v_1$
$a_5 (4, 3)$	$2\sqrt{2} = 2.828$	2	$v_2$
$a_6 (5, 5)$	5	$\sqrt{13} = 3.605$	$v_2$

→ Euclidean distance

$$(x_1, x_2) \quad (y_1, y_2)$$

$$= \sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2}.$$

→ 3. cluster ~~one~~ of  $v_1 = \{a_1, a_2, a_4\}$

cluster 2 of  $v_2 = \{a_3, a_5, a_6\}$

4. Recalculate the cluster centres.

$$v_1 = \frac{1}{3} [(1, 1) + (2, 1) + (3, 2)]$$

$$= \frac{1}{3} [(6, 4)] = (2, 1.33)$$

$$v_2 = \frac{1}{3} [(2, 3) + (4, 3) + (5, 5)]$$

$$= \frac{1}{3} [(11, 11)] = (3.67, 3.67)$$

5. Repeat from step 2, until we get same cluster centre ( $v_1$ ) same cluster elements as in the previous iteration.

Data point	$v_1 (2, 1.33)$	$v_2 (3.67, 3.67)$	Assigned Centre
$a_1 (1, 1)$	1.05	3.78	$v_1$
$a_2 (2, 1)$	0.33	3.15	$v_1$
$a_3 (2, 3)$	1.67	1.8	$v_1$
$a_4 (3, 2)$	1.204	1.8	$v_1$
$a_5 (4, 3)$	2.605	0.75	$v_2$
$a_6 (5, 5)$	4.74	1.88	$v_2$

cluster 1 of  $v_1 = \{a_1, a_2, a_3, a_4\}$

cluster 2 of  $v_2 = \{a_5, a_6\}$

$$v_1 = \frac{1}{4} [(1,1) + (2,1) + (3,2) + (2,3)]$$

$$= \frac{1}{4} (8,7) = (2,1.75)$$

$$v_2 = \frac{1}{2} [(4,3) + (5,5)]$$

$$= \frac{1}{2} (9,8) = (4.5, 4)$$

→ so, cluster elements are not same as previous

iteration Elements of  $v_1$  are  $a_1, a_2, a_3, a_4$

Data point	$v_1 (2,1.75)$	$v_2 (4.5, 4)$	Assigned Centre
$a_1 (1,1)$	1.25	4.61	$v_1$
$a_2 (2,1)$	0.75	3.9	$v_1$
$a_3 (2,3)$	1.25	2.69	$v_1$
$a_4 (3,2)$	1.03	2.5	$v_2$
$a_5 (4,3)$	2.36	1.12	$v_2$
$a_6 (5,5)$	4.42	1.12	$v_2$

→ cluster 1 of  $v_1 = \{a_1, a_2, a_3, a_4\}$

cluster 2 of  $v_2 = \{a_5, a_6\}$

~~$\frac{1}{4}$~~  cluster elements are same as  
in the previous iteration. so, our clusters

cluster 1 =  $\{(1,1), (2,1), (2,3), (3,2)\}$

cluster 2 =  $\{(4,3), (5,5)\}$

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→ HAC:- Hierarchical Agglomerative clustering.

\* HAC starts with one cluster, individual item in its own cluster & iteratively merge clusters until all the items belong to one cluster.

\* Bottom up approach is followed to merge the clusters together.

\* Dendrogram (or) pictorially used to represent the HAC.

\* HAC represents 3 techniques:

1. single, nearest (or) single linkage.

2. Complete-farthest distance (or) complete linkage.

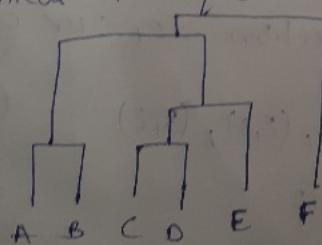
3. Average distance (or) Average linkage.

1. S.N:- This is the diff b/w the closest members of the two clusters.

2. C.L:- This is the diff b/w the members that are farthest apart.

3. A.L:- This Method involves looking at a distance b/w all pairs & averages all of these distance. this is also called unweighted pair group mean average

\* Dendrogram:- A tree like structure which represent hierarchical technique.



Problem:-

\* find the clusters using single link technique & draw the denrogram diagram.

	$\alpha$	$\gamma$
P <sub>1</sub>	0.40	0.53
P <sub>2</sub>	0.22	0.38
P <sub>3</sub>	0.35	0.32
P <sub>4</sub>	0.26	0.19
P <sub>5</sub>	0.08	0.41
P <sub>6</sub>	0.45	0.30

linkage.

Sol:-

1. Euclidean distance

$$(\alpha_1, \gamma_1) (\alpha_2, \gamma_2) = \sqrt{(\alpha_1 - \alpha_2)^2 + (\gamma_1 - \gamma_2)^2}$$

	P <sub>1</sub>	P <sub>2</sub>	P <sub>3</sub>	P <sub>4</sub>	P <sub>5</sub>	P <sub>6</sub>
P <sub>1</sub>	0					
P <sub>2</sub>	0.23	0				
P <sub>3</sub>	0.22	0.15	0			
P <sub>4</sub>	0.37	0.20	0.15	0		
P <sub>5</sub>	0.34	0.34	0.26	0.29	0	
P <sub>6</sub>	0.23	0.25	0.11	0.22	0.39	0

Distance Table

→ Smallest element = 0.11

P<sub>3</sub> P<sub>6</sub>

3 6

Recalculate the distance Matrix

(2)	P <sub>1</sub>	P <sub>2</sub>	P <sub>3</sub> P <sub>6</sub>	P <sub>4</sub>	P <sub>5</sub>
P <sub>1</sub>	0				
P <sub>2</sub>	0.23	0			
P <sub>3</sub> P <sub>6</sub>	0.22	0.15	0		
P <sub>4</sub>	0.37	0.20	0.15	0	
P <sub>5</sub>	0.34	0.34	0.26	0.29	0

Small value = 0.15

$\min(\text{dist}(x, y))$

$$\min(\text{dist}(P_3, P_6), P_1) = \min(\text{dist}((P_3, P_1), (P_6, P_1))).$$

$$= \min(\text{dist}(0.22, 0.23))$$

$$= 0.22$$

(3) updated dist Matrix.

	P <sub>1</sub>	P <sub>2</sub> P <sub>5</sub>	P <sub>3</sub> P <sub>6</sub>	P <sub>4</sub>
P <sub>1</sub>	0			
P <sub>2</sub> P <sub>5</sub>	0.23	0		
P <sub>3</sub> P <sub>6</sub>	0.22	0.15	0	
P <sub>4</sub>	0.37	0.20	0.15	0

small element = 0.15

$\Rightarrow$  New cluster  $[(P_2, P_5), (P_3, P_6)]$

	$P_1$	$P_2 P_5 P_3 P_6$	$P_4$
$P_1$	0	0.22	0.37
$P_2 P_5$	0.22	0	0.15
$P_3 P_6$	0.37	0.15	(0.22) b merge $(P_2 P_5)$ b
$P_4$			<small>small.</small>

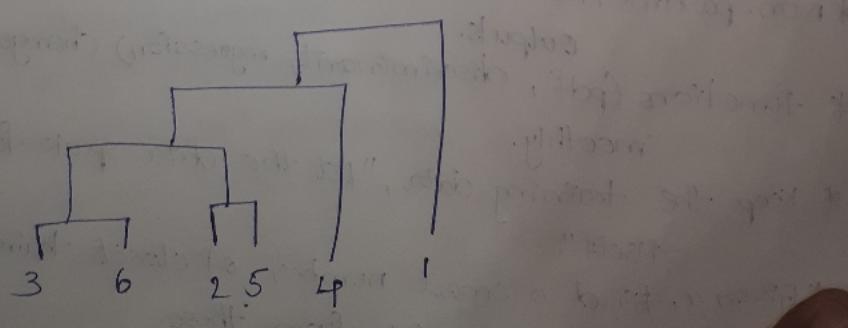
$\rightarrow \min(\text{dist}[(C P_2 P_5 P_3 P_6), P_1])$  and disjoint (a)  
 $= \min(\text{dist}[(C P_2 P_5), P_1] + (P_3 P_6), P_1])$  b  
 $= \min(\text{dist}(0.22) (0.22))$  disjoint - overlapping  
 $= 0.22$ . [distance between 2 elements]

$\rightarrow \min(\text{dist}[(C P_2 P_5 P_3 P_6), P_4])$  overlap & bellow  
 $= 0.15$ . b < 0.22 < 0.37 period with overlap

$\Rightarrow$  updated

	$P_1$	$P_2 P_5 P_3 P_6 P_4$
$P_1$	0	0.22
$P_2 P_5 P_3$	0.22	0
$P_6 P_4$		

Diagram :-



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### → Agglomerative clustering:

Distance b/w two groups  $G_i$  &  $G_j$

1) single link

$$d(G_i, G_j) = \min d(x^i, x^j)$$

2) Complete-link

$$d(G_i, G_j) = \max d(x^i, x^j)$$

Dendrogram :- Decompose data objects into several levels of nested partitioning (tree of clusters) called a dendrogram.

### ⇒ Density Estimation:

\* Given the training set  $\{x\}$  drawn iid from  $p(x)$

\* Divide data into bins of size  $h$

\* Histogram :-

$$p(x) = \frac{\#\{x-h < x \leq x+h\}}{2Nh}$$

### ⇒ Non-parametric methods:

\* parametric (single global model), semi-parametric (small number of local models).

\* Non-parametric : similar inputs have similar outputs.

\* functions (pdf, discriminant, regression) change smoothly.

\* Keep the training data, "let the data speak for itself".

\* Given  $x$ , find a small number of closest training instances & interpolate from these.

\* Aka Lazy/memory based / case-based / instance-based learning.

⇒ Kernal Estimator:

kernal function, eg:- Gaussian

Kernal:-

$$K(u) = \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{u^2}{2}\right]$$

Kernal Estimator: (Parzen window)

$$\hat{p}(x) = \frac{1}{Nh} \sum_{t=1}^n K\left[\frac{x-x_t}{h}\right]$$

→ K= Nearest Neighbour Estimator

$$\hat{p}(x) = \frac{1}{2N} d_k(x)$$

Generalisation of multivariate data:

1. Kernal density Estimator

2. multivariate Gaussian kernal density

↓  
1. Spherical

2. Ellipsoidal

→ Hamming distance:

$$H_0(x, x^t) = \sum_{j=1}^d I(x_j \neq x_j^t)$$

→ Non-parametric classification:

1. kernal estimator

2. KNN Estimator

→ Non-parametric Regression:

1. Running mean Smoother

2. Kernal smoothers.

$$1. \hat{g}(\alpha) = \frac{\sum_{t=1}^N w_t (\alpha - \alpha_t)^+ \cdot x_t^+}{\sum_{t=1}^N w_t (\alpha - \alpha_t)^+}$$

$$2. \hat{g}(\alpha) = \frac{\sum_{t=1}^N k_t (\alpha - \alpha_t)^+ \cdot x_t^+}{\sum_{t=1}^N k_t (\alpha - \alpha_t)^+}$$

$\Rightarrow$  Linear Discrimination:

$$g_i(\alpha_i / w_i) = w^T x + w = \sum w_j x_j + w.$$

\* likelihood vs discriminant based classification.

1. Assume model for  $P(c_i | \alpha)$ , use Baye's rule

to calculate  $p(c_i | \alpha) g_i(\alpha) = \log p(c_i | \alpha)$ .

2. D.B.C = Assume model for  $p(\alpha | d_i)$  NO

density estimation. Estimating the boundary is enough. No need to accurately estimate the densities inside the boundaries.

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$\rightarrow$  KNN Algorithm:

\* KNN is a Non-parametric classification Method.

\* It is used for classification & regression.

\* It is very simple & easy to implement supervised machine learning algorithm.

Advantages:-

\* more effective if training data is large.

→ problem :-

S.No	maths	CS	Result
1	4	3	Fail
2	6	7	Pass
3	7	8	Pass
4	5	5	Fail
5	8	8	Pass
6	6	8	Pass

$$x = (\text{math} - 6, \text{CS} - 8)$$

$$(1) \text{ Let } x = 3 \Rightarrow \text{prob} = 0$$

K=3

→ Euclidean Distance :-

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$x_2, y_2$  - Observed values

$x_1, y_1$  - Actual values.

$$1. = \sqrt{(6-4)^2 + (8-3)^2}$$

$$= \sqrt{4 + 25} = \sqrt{29} = 5.38$$

$$2. \sqrt{(6-6)^2 + (8-7)^2} = 1$$

$$3. \sqrt{(6-7)^2 + (8-8)^2} = 1$$

$$4. \sqrt{(6-5)^2 + (8-5)^2} = \sqrt{10} = 3.16$$

$$5. \sqrt{(6-8)^2 + (8-8)^2} = \sqrt{4} = 2$$

→ As per result K=3

2-pass

3-pass

5-pass

so, 3 greater than 0, probability of pass  
is high.

Hence,  $x = (\text{math} = 6, \text{CS} = 8)$  is pass.

seeks

→ Decision tree:

- \* Decision tree is a tree like a predictive model.
- used for classification & regression.
- \* it is a supervised learning method.

⇒ Learning Algorithm used:

① Cost → Regression → Gini index.

② ID3 Algorithm - Classification

↓ Entropy, Information Gain.

$$\rightarrow \text{Entropy} = \sum_{(D)}^C -P_i \log_2(P_i)$$

$$\rightarrow \text{Gain}(D, A) = \text{Entropy}(D) - \sum_{\text{Value}(A)} \frac{|D_v|}{|D|} \times \text{Entropy}(D_v)$$

problem:

It is used to generate Decision Tree from a Dataset.

→ Classify the given training set using ID3 Algorithm

Construct the decision tree.

<u>size</u>	<u>color</u>	<u>shape</u>	<u>class</u>
Small	Yellow	Round	A
Big	Yellow	Round	A
Big	Red	Round	A
Small	Red	Round	A
Small	Black	( <del>Round</del> ) Round	B
Big	Black	Cube	B
Big	Yellow	Cube	B
Big	Black	Round	B
Small	Yellow	Cube	B

mode)

Sol:-

$$A=4 \quad \checkmark \quad \left( \frac{4}{9} \log_2 \left( \frac{4}{9} \right) + \frac{5}{9} \log_2 \left( \frac{5}{9} \right) \right) H = 0.992$$
$$B=5 \quad \checkmark \quad \left( \frac{2}{5} \log_2 \left( \frac{2}{5} \right) + \frac{3}{5} \log_2 \left( \frac{3}{5} \right) \right) H$$
$$m=2$$

Calculate the Entropy (H).

$$\text{Entropy}(H) = - \sum_{i=1}^m p_i \log_2(p_i)$$

$$= - \sum_{i=1}^2 4/9 \log_2 (4/9) - 5/9 \log_2 (5/9)$$

$$= - \frac{4/9 \log_2 (4/9)}{2} - \frac{5/9 \log_2 (5/9)}{2}$$

$$= (-0.444 * -1.171) - (0.555 * -0.850)$$

$$= 0.520 + 0.472 H = 0.992$$

$$= 0.992$$

Identify the splitting attribute, calculate the gain.

\* Attribute - size:

	A	B	
Small	2	2	-4
Big	2	3	-5

$$\text{Gain}(D, \text{size}) = 0.992 - \left[ \frac{4}{9} \left( -\frac{2}{4} \log_2 \left( \frac{2}{4} \right) - \frac{2}{4} \log_2 \left( \frac{2}{4} \right) \right) + \frac{5}{9} \left( -\frac{3}{5} \log_2 \left( \frac{3}{5} \right) - \frac{2}{5} \log_2 \left( \frac{2}{5} \right) \right) \right]$$

$$= 0.992 - [0.444 + 0.539]$$

$$= 0.992 - 0.983$$

$$= 0.009$$

\* Attribute - color:

	A	B	
yellow	2	2	-4
red	2	0	-2
black	3	0	-3

$$\begin{aligned} \text{Gain}(D, \text{color}) &= 0.992 - \left[ \frac{4}{9} \left( -\frac{2}{4} \log_2 \left( \frac{2}{4} \right) - \frac{2}{4} \log_2 \left( \frac{2}{4} \right) \right) + \right. \\ &\quad \left. \frac{2}{9} \left( -\frac{2}{2} \log_2 \left( \frac{2}{2} \right) \right) + \right. \\ &\quad \left. \frac{3}{9} \left( -\frac{2}{3} \log_2 \left( \frac{2}{3} \right) - \frac{3}{3} \log_2 \left( \frac{3}{3} \right) \right) \right] \\ &= 0.992 - [0.4444 + 0.0] \\ &= 0.55. \end{aligned}$$

size	1
Small	
Big	
Big	
Small	

\* Attribute - shape

$$\begin{array}{ccc} A & B & C \\ \hline \text{Round} & 4 & 2 \\ \text{cube} & 3 & 0 \end{array}$$

$$\begin{aligned} \text{Gain}(D, \text{shape}) &= 0.992 - \left[ \frac{6}{9} \left( -\frac{4}{6} \log_2 \left( \frac{4}{6} \right) - \frac{2}{6} \log_2 \left( \frac{2}{6} \right) \right) + \right. \\ &\quad \left. \frac{3}{9} \left( -\frac{3}{3} \log_2 \left( \frac{3}{3} \right) \right) \right] \\ &= 0.992 - 0.612 \\ &= 0.38. \end{aligned}$$

→ Information Gain

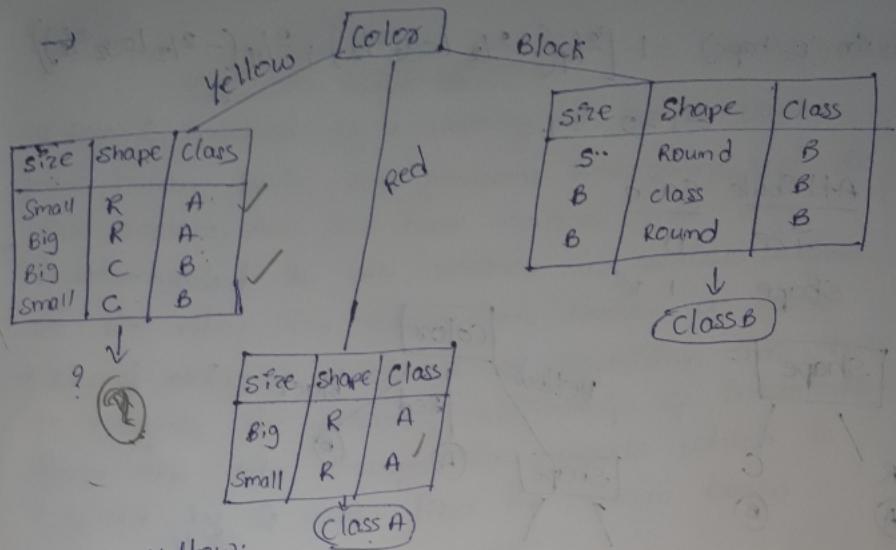
Attribute	Gain
size	0.009
color	0.55
shape	0.379

\* color has the largest gain, so it becomes the root node.

$$\frac{2}{4} \log_2(2/4) +$$

+

$$-\frac{3}{3} \log_2(3/3)]$$



yellow:

$$\text{Entropy}(D) = -\frac{2}{4} \log_2 \frac{2}{4} - \frac{2}{4} \log_2 \frac{2}{4}$$

$$= 1.$$

→ Attribute = size:

	A	B	
small	1	1	2
Big	1	1	2

$$\text{Gain}(D, \text{size}) = 1 - \left[ \frac{2}{4} \left( -\frac{1}{2} \log_2 \frac{1}{2} - \frac{1}{2} \log_2 \frac{1}{2} \right) + \frac{2}{4} \left( -\frac{1}{2} \log_2 \frac{1}{2} - \frac{1}{2} \log_2 \frac{1}{2} \right) \right]$$

$$= 1 - [0.5 + 0.5]$$

$$= 0.$$

→ Attribute : shape:

	A	B	
Round	2	0	2
Cube	0	2	2

becomes

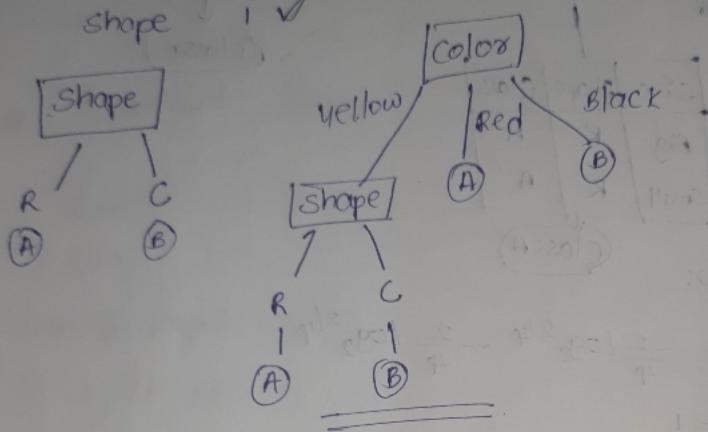
$$\text{Gain}(D, \text{shape}) = 1 - [2/4(-2/2 \log_2 2/2) + 2/4(-2/2 \log_2 2/2)]$$

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$$= 1 - 0 = 1.$$

Attribute Gain

size	Gain
shape	1 ✓



\* IPS:-

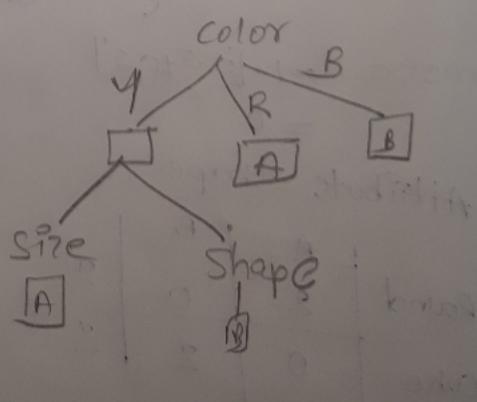
$$* \text{Entropy} = -\frac{P}{P+m} \log_2 \left( \frac{P}{P+m} \right) - \frac{m}{P+m} \log_2 \left( \frac{m}{P+m} \right)$$

\* Average Info.:

$$I(\text{Attribute}) = \sum \frac{P_i + m_i}{P+m} \text{Entropy}(s_i)$$

$$\text{Gain} = \text{Entropy}(s) - I(\text{Attribute}).$$

→ ~~Entropy~~.



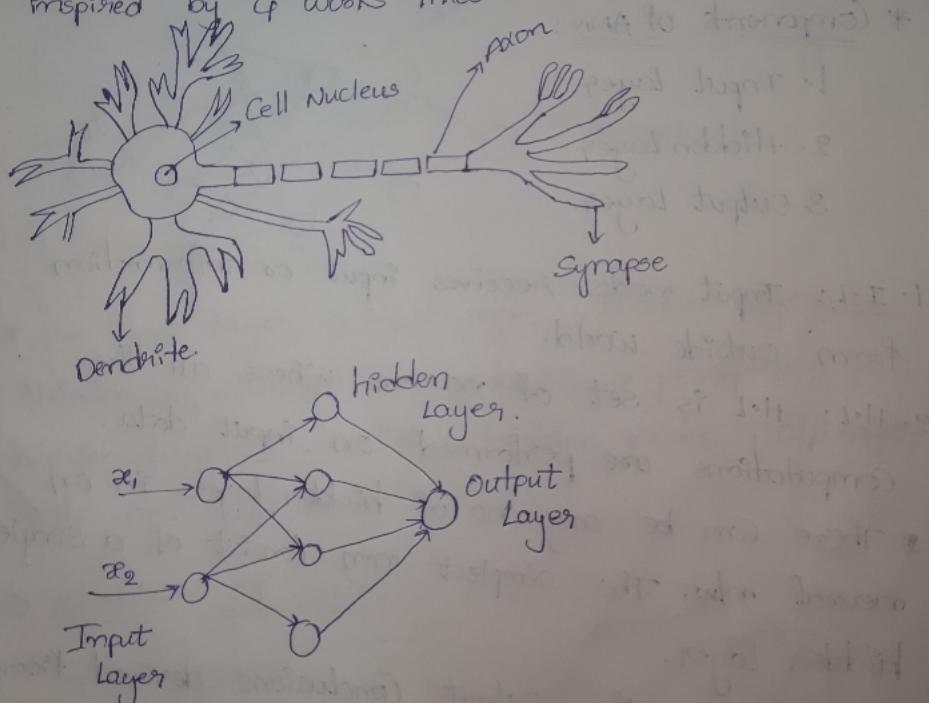
$\frac{2}{3} \log_2 \frac{2}{3}$ )

25/03/23.

Unit-4

### Artificial Neural Network

- \* ANN is inspired by a working of human brain.
- \* the human brain has neurons interconnected to one another, ANN also have neurons that are interconnected to one another in various layers of the N/w. These neurons are known as nodes.
- \* Neural networks are set of algorithm that tries to recognise the patterns, relationships & information from the data through the process which is inspired by & works like the human brain.



### Biological

- \* Dentists
- \* Cell Nucleus
- \* Synapse
- \* Axon

### Components of ANN:

1. Input Layer
2. Hidden Layer
3. Output Layer.

1. I.L: Input nodes receives input or information from outside world.

2. H.L: H.L is set of neurons where all the computations are performed on input data.

3. There can be any no. of hidden layers in an neural net. The simplest n.n consist of a single hidden layer.

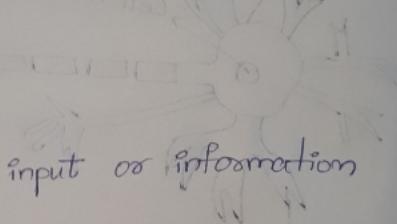
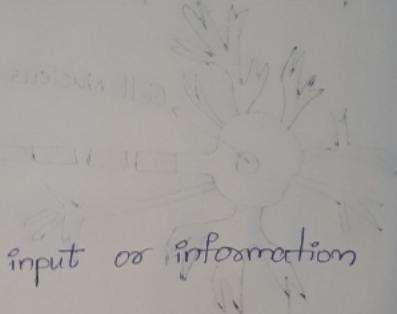
3. O.L: O.L is the output Conclusions derived from all the computations performed

\* There can be single or multiple nodes in the output layer. if we have a binary classification problem the output known is 1. But in case of multi class classification, the output nodes can be more than 1.

### Artificial

- \* Inputs
- \* Nodes.
- \* weights
- \* output.

but although the two calculate function is same i.e. product of input value with weight and sum of all the weights coming with neurons who are marked green and orange and total value is put before



\* perceptron

\* Com

\* Com

\* Mul

\* M

thom

mult

Input  
Layer

\* Ann

three

hid

with

=>

$x_1 \rightarrow v$

$x_2 \rightarrow w$

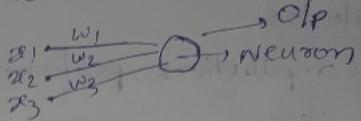
$y_1 \rightarrow$

$y_2 \rightarrow$

$y_3 \rightarrow$

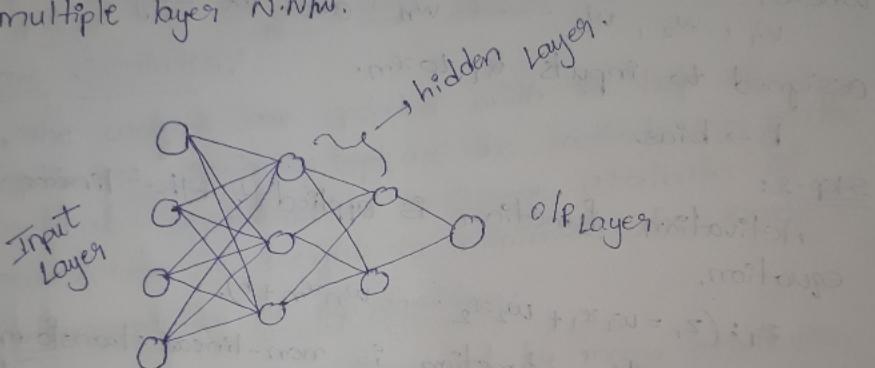
### \* perception:- perception:

\* perception is a simple form of N.N/w & consists of single layer where all the mathematical computations are performed.



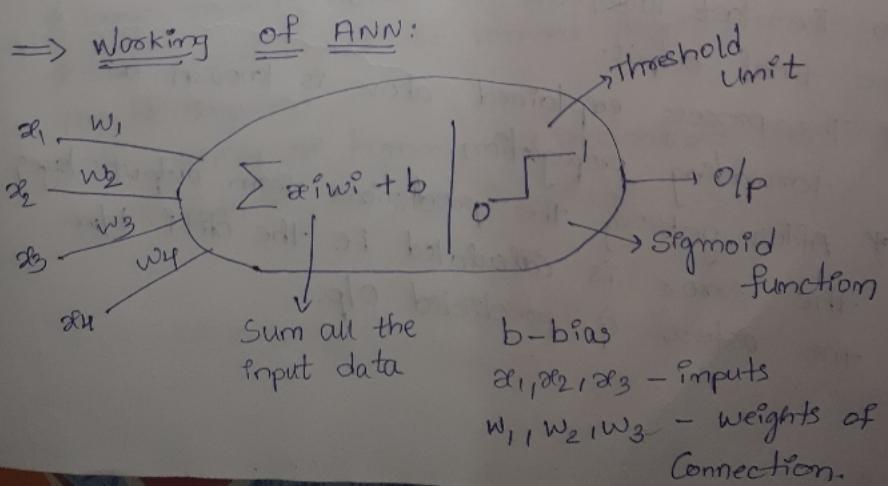
### \* Multilayer perception:

\* M.P is also known as A.N.N consists of more than 1 perception which is grouped together to form multiple layer N.N/w.



\* An input layer, 6 input nodes, Hidden Layer - 1, hidden Layer, with 4 hidden nodes (or) 4 perceptions, hidden Layer - 2 - with 4 hidden nodes, o/p Layer with one o/p node.

### $\Rightarrow$ Working of ANN:



\* bias is the constant that helps the model to fit in the best way possible.

27/03/23

\* Computation perform in hidden layers ~~is~~ done in

2 steps:

1. All the inputs are multiplied by the weight

$$\text{Step-1: } z_1 = w_1 x_1 + w_2 x_2 + \dots + w_n x_n + b$$

where,  $w_1, w_2, w_3, \dots, w_n$  are the weights assigned to inputs  $x_1$  to  $x_n$ .

$b \rightarrow$  bias.

Step-2:

Activation function is applied to the linear equation.

$$z_1 : (z_1 = w_1 x_1 + w_2 x_2 + \dots + w_n x_n + b)$$

\* the activation function is non-linear transformation that is applied to the input before sending it to the next layer of neurons.

Step-3:

The computation results from hidden layer passed to the last layer i.e output layer which gives as the final output.

\* The process explained above is known as forward propagation.

\* After getting the predictions from output layer the error is calculated i.e the diff b/w the actual & predicted op.

- \* If the error is large, then the steps are taken to minimize the error & for the same purpose, Back Propagation is performed.
- done in Back propagation:
- \* It is process of updating & finding the optimal values of weights (or) co-efficients which helps to minimize the error, i.e. diff b/w the actual & predicted value.
  - \* How the weights are updated & new weights are calculated?  
→ The weights are updated with the help of optimises, optimises (opt), are the methods (or) mathematical formulation to change attributes of neural n/w i.e. weight to minimises the error.
  - \* Characteristics of B.P Algorithm:
    - \* Instances are represented by many attributes.
    - \* The target function output (make) discrete value, real value, (or) a vector of several real (or) discrete value attributes.
    - \* The training examples may contain error.
    - \* Large training times are acceptable.
    - \* fast evalution of the learned target function may be required.
    - \* The ability of humans to understand the learned target function is not important.

27/05/93

### \* Perceptron learning Algorithm:

1. Initialise weights & threshold.
- \* Define  $w_i(t)$ , ( $i \in n$ ), to be the weight from input  $i$  at the time  $t$ , &  $\theta$  to be the threshold value in the output node.
- \* set  $w_0$  to be  $-\theta$ , the bias, &  $y$  to be always 1.
2. present input & desired output.  
\* present input  $x_0, x_1, x_2, \dots, x_n$  & desired o/p  $d(t)$ .
3. Calculate actual o/p.

$$y(t) = f_m \left[ \sum_{i=0}^n w_i(t)x_i(t) \right]$$

4. Adapt weights.  
\* if correct  $w_i(t+1) = w_i(t)$
- \* if o/p, 0, should be 1 (class A)  $w_i(t+1) = w_i(t) + \eta x_i(t)$ .
- \* if o/p 1, should be 0 (class B)  $w_i(t+1) = w_i(t) - \eta x_i(t)$ .
- Adapt weights - modified version.  
\* if correct,  $w_i(t+1) = w_i(t)$ .
- \* if output 0, should be 1 (class A),  $w_i(t+1) = w_i(t) + \eta x_i(t)$ .
- \* if output 1, should be 0 (class B),  $w_i(t+1) = w_i(t) - \eta x_i(t)$ .

where  $0 \leq \eta \leq 1$ , a positive gain term that controls the adoption rate.

\* The error term  $\Delta$  can be written

$$\Delta = d(t) - y(t).$$

→ Adapt weights - Widrow-Hoff delta rule.

$$\Delta = d(t) - y(t).$$

$$w_i(t+1) = w_i(t) + \eta \Delta w_i(t)$$

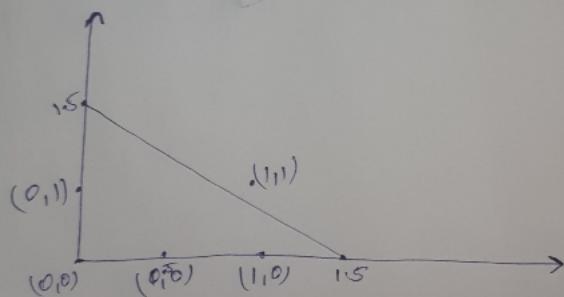
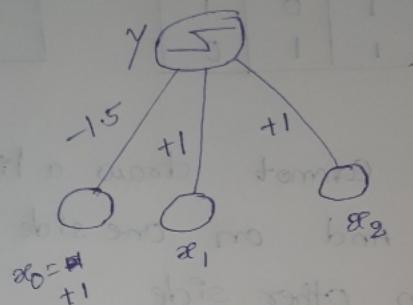
$$\Delta(t) = \begin{cases} +1, & \text{if input from class A} \\ 0, & \text{if input from class B.} \end{cases}$$

\* Input of  $\Delta p$  for the and function:

Learning Boolean function:

AND

$x_1$	$x_2$	$d$
0	0	0
0	1	0
1	0	0
1	1	1



\* In a Boolean function,

The perceptron that implements And and

its geometric interpretation.

\* In a Boolean function, that inputs are binary

& output is 1 if the corresponding function

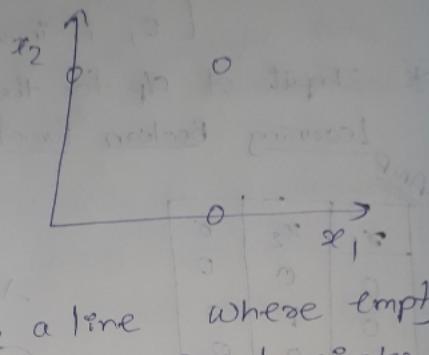
value is true (& the inputs are zero otherwise)

therefore, it can be seen as a 2 class classification problem.

\* Input of o/p for XOR function

XOR.

$x_1$	$x_2$	$\sigma$
0	0	0
0	1	1
1	0	1
1	1	0



\* we cannot draw a line where empty circle and on one side & filled circles are on other side.

\* XOR process is not linearly separable.