Practice Problem Set 1 - Probability Basics

1. Prove that if X and Y are independent, the variance of $X + Y$ is the sum of the variances of X and Y.

2. (From Wasserman) A box contains 5 coins and each has a different probability of showing heads. Let $p_1, p_2, \ldots, p_5$ denote the probabilities of heads of each coin. Suppose that $p_1 = 0$, $p_2 = 1/4$, $p_3 = 1/2$, $p_4 = 3/4$, $p_5 = 1$. Let H denote "heads is obtained" and let C_i denote the event that coin i is selected.

   (a) Select a coin at random and toss it. Suppose a head is obtained. What is the posterior probability that coin i was selected ($i = 1, 2, \ldots, 5$)? In other words, find $\Pr(C_i \mid H)$ for $i = 1, 2, \ldots, 5$.

   (b) Toss the coin again. What is the probability of another head? In other words find $\Pr(H_2 \mid H_1)$ where $H_j =$ "heads on toss j."

   (c) Find $\Pr(C_i \mid B_4)$ where B_4 is "first head is obtained on toss 4."

3. Murphy Exercise 2.2, 2.8, 2.10

   Supplemental material

   - Murphy 2.8 [Mean, mode, variance for the beta distribution]

     A **Beta distribution** is a continuous distribution over the interval $[0, 1]$. It has two parameters, α and β, and the density takes the form

     $$p(x; \alpha, \beta) = x^{\alpha-1}(1-x)^{\beta-1}\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}$$

     where Γ is the **Gamma function** (a generalization of factorial). It might help to know that

     $$\Gamma(z+1) = z\Gamma(z)$$