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6) Big omega notation Prove that $g(n) = n^3 + 2n^2 + 4n$ is $\Omega(n^3)$

$g(n) \geq c \cdot n^3$

$g(n) = n^3 + 2n^2 + 4n$

find constant and n_0

$n^3 + 2n^2 + 4n \geq c \cdot n^3$

Divide both sides with n^3

$$1 + \frac{2n^2}{n^3} + \frac{4n}{n^3} \geq c$$

$$1 + \frac{2}{n} + \frac{4}{n^2} \geq c$$

Here $\frac{2}{n}$ and $\frac{4}{n}$ approaches 0

$1 + 2/n + 4/n^2$

Example $c = 1/2$

$1 + 2/n + 4/n^2 \geq 1/2$

$1 + 2/n + 4/n^2 \geq 1$ $(1 \geq 1/2,\ n \geq 1)$

$1 + 2/n + 4/n^2 \geq 1/2$ $(n \geq 1, n_0 = 1)$

Thus, $g(n) = n^3 + 2n^2 + 4n$ is indeeded $\Omega(n^3)$.

7) Big thela notation: Determine whether $h(n) = 4n^2 + 3n$ is $\theta(n^2)$ or not.

$c_{12}\, n^2 \leq h(n) \leq c_2 n^2$

In upper bound $h(n)$ is $O(n^2)$

In lower bound $h(n)$ is $\Omega(n^2)$.

$h(n) = 4n^2 + 3n$

$h(n) \leq c_1 n^2$

$4n^2 + 3n \leq c_2 n^2$

$4n^2 + 3n \leq 5n^2$

lets $c_2 = 5$

Divide both sides by n^2.

$4 + 3/n \leq 5$

$h(n) = 4n^2 + 3n$ is $O(n^2)$ $(c_2 = 5, n_0 = 1)$

lower bound :-

$h(n) = 4n^2 + 3n$

$h(n) \geq c_1 n^2$

$4n^2 + 3n \geq c_1 n^2$

let's $c_1 = 4 \Rightarrow 4n^2 + 3n \geq 4n^2$

Divide both sides by n^2

$4 + 3/n \geq 4$

$h(n) = 4n^2 + 3n$ $(c_1 = 4, n_0 = 1)$

$h(n) = 4n^2 + 3n$ is $\theta(n^2)$

8) Let's $f(n) = n^3 - 2n^2 + n$ and $g(n) \leq n$ show wheather $f(n) = \Omega(g(n))$ is true or false and justify your answer.

$f(n) \geq c \cdot g(n)$

substuting $f(n)$ and $g(n)$ into this inequality we get

find c and n_0 hols $n \geq n_0$

$n^3 - 2n^2 + n \geq -cn^2$

$n^3 - 2n^2 + n + cn^2 \geq 0$

$n^3 + (c-2)n^2 + n \geq 0$

$n^3 + (c-2)n^2 + n \geq 0$ $(n^3 \geq 0)$

$n^3 + (1-2)n^3 + n = n^3 - n^2 + n \geq 0$ $(c = 2)$

$f(n) = n^3 - 2n^2 + n$ is $\Omega(g(n)) = \Omega(-n^2)$

therefore, the statement $f(n) = \Omega(g(n))$ is True

Q) Determine whether $h(n) = n \log n$ is in $\theta(n \log n)$ Prove a Rigorous Proof your conclusion.

Soln $c_1 n \log n < h(n) \le c_2 n \log n$

upper bound:

$h(n) \le c_2 n \log n$

$h(n) = n \log n + n$

$n \log n + n \le c_2 n \log n$

Divide both sides by $n \log n$

$1 + \frac{n}{n \log n} \le 2$

$1 + \frac{n}{\log n} \le 2$

$1 + \frac{1}{\log n} \le 2$ (simplify)

then $h(n)$ is $\theta(n \log n)$ $(c_2 = 2, n_0 = 2)$

lower bound:

$h(n) \ge c_1 n \log n$

$h(n) = n \log n + n$

$n \log n + n \ge c_1 n \log n$

divide both sides by $n \log n$

$1 + \frac{n}{n \log n} \ge c_1$

$1 + \frac{1}{\log n} \ge c_1$ (simplify)

$1 + \frac{1}{\log n} \ge 1$ $(c_1 = 1)$

$\frac{1}{\log n} \ge 0$ for all $n > 1$

$h(n)$ is $\Omega(n \log n)$ $(c_1 = 1, n_0 = 1)$

$h(n) = n \log n + n$ is $\theta(n \log n)$.

10) solve the following recurrence relations and find the order of growth for solutions $T(n)=4T(n/2)+n^2, T(1)=1$

$T(n)=4T(n/2)+n^2, T(1)=1$

$T(n)=aT(n/b)+f(n)$

$a=4, b=2, f(n)=n^2$

Applying master's theorem.

$T(n)=aT(n/b)+f(n)$ $\quad f>0$

$f(n)=O(n^{\log_b a-1})$ $\quad T(n)=\Theta(n^{\log_b a})$

$f(n)=\Theta(n^{\log_b a})$ then $T(n)=\Theta(n^{\log_b a}\log n)$

$f(n)=\Omega(n^{\log_b a+1})$, then $f(n)=f(n)$

calculating $\log_b a$:

$\log_b a=\log_2 4-2$

$f(n)=n^2=\Theta(n^2)$

$f(n)=\Theta(n^2)=\Theta(n^{\log_b a})$

$T(n)=4T(n/2)+n^2$

$T(n)=\Theta(n^{\log_b a}\log n)=\Theta(n^2\log n)$

order of growth

$T(n)=4T(n/2)+n^2$ with $T(1)=1$ is $\Theta(n^2\log n)$.