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1) solve the following recursive relations.

a. $x(n)=2(n-1)+5$ for $x(1)=0$.

$x(n)=x(1)+5(n-1)$

let $x(1)=0$:

$x(n)=5(n-1)$

$\boxed{x(n)=5n-5}$

b. $x(n)=3x(n-1)$ for $x$ $n>1$ $x(1)=4$.

substituting:

$x(n)=3^{n-1}x(n)$

$x(1)=4$:

$x(n)=4\cdot 3^{n-1}$

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c. $x(n)=x(n/2)+n$ for $n>1$, $x(1)=1$ (solve for $n=2^k$).

$x(n)=n+n/2+n/4+\dots+1$

here $1+n/2+n/4+\dots n/2^{\log n}$ simplifies to $2n-1$.

$\therefore x(n)=2n-1$.

d. $x(n)=x(n/3)+1$ for $n>1$, $x(1)=1$ (solve fore $n=3^k$)

$x(n)=1+1+1+\dots$ (for $\log_3 n$ times)

$x(n)=\log_3 n$

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(2) Evaluate the following recurrences completely.

i) $T(n)=T(n/2)+1$, where $n=2$ for all $k<0$

here

$T(n)=T(n/2)+1$

$T(n/2)=T(n/4)+1$

$T(n/4)=T(n/8)+1$

$\Rightarrow T(n)=1+1+1+\ldots$ for $\log_2 n$ times.

$\therefore T(n)=T(n/2)+1$ for $n=2^k$ is:

$T(n)=\log_2 n$

$T(n)=O(\log n)$

ii) $T(n)=T(n/2)+T(2n/3)+(n)$, where 'c' is a constant and 'n' is the input size.

$T(n)=aT(n/b)+f(n)$

$a=1$ $b=2$, $f(n)=(n)$

i). calculate $n^{\log_b^a}$.

$n^{\log_2^1}=n^0=1$

ii) compare $f(n)$ with $n^{\log_b^a}$

$f(n)=cn$

$f(n)=O(n^{a-1})$

iii) Apply case 3 for master theorem.

if $f(n)=O(n\log_b^a \log^k n)$ for some $k\geq 0$, then $\Theta(n\log_b^a \log^{k+1} n)$

since $f(n)=\Theta(n)$

$T(n)=\Theta(n\log n)$

$\therefore T(n)=T(n/3)+T(2n/3)+cn$ is: $T(n)=\Theta(n\log n)$.

3) consider the following recursion algorithm.

min1 (A[0.... n-1])
if n=1 return A[0]
Else temp= min(A[0...n-12])
if temp <= A[n-1] return temp
Else return A(n-1)

a) what does this algorithm compute?

This algorithm is designed to find the minimum element in an array 'A' of size 'n'.

b) set up a recurrence relation for the algorithms basic operation count and solve it.

$T(n) = T(n-1) + 2$

$T(1) = 1$

$T(n) = T(n-1) + 2$

Expand:- $T(n) = T(n-2) + 2 + 2$

$T(n) = T(n-2) + 2 + 2 + 2$ [continue the pattern]

$T(n) = 1 + 2(n-1)$

$\boxed{T(n) = 2n-1}$ ⇒ Best case.

④ Analyze the order of growth.
$f(n) = 2n^2 + 5$ and $g(n) = 7$.
use the $\Omega(g(n))$ notation.

compute the limit:

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} = \lim_{n\to\infty} \frac{2n^2+5}{7^n}$$

simplify the fraction:-

$$\lim_{n\to\infty} \frac{2n^2+5}{7n} = \lim_{n\to\infty} \left(\frac{2n^4}{7n} + \frac{5}{7n}\right) = \lim_{n\to\infty} \left(\frac{2}{7}n + \frac{5}{7n}\right)$$

Evaluate the limit:-

$$\lim_{n\to\infty} \left(\frac{2}{7}n + \frac{5}{7n}\right) =$$

conclusion:-

$$f(n) = \Omega(g(n))$$

∴ $f(n)$ is asymtotically bounded below by $g(n)$, meaning f(n grows at least as fast a $g(n)$. In simplex terms f(n) a symtotically quadratic.