Bonus Project: Probability Analysis in a Coin Toss Game

Mar. 20, 2024

Overview

In this project, you are tasked with developing a program that models a coin-tossing game between two players, Alice and Bob. The game operates under the following rules:

- The coin is tossed n times, and the outcomes are recorded as a sequence of Heads (H) and Tails (T).
- For every occurrence of "HH" in the sequence, Alice scores a point.
- For every occurrence of "HT" in the sequence, Bob scores a point.
- The player with the most points at the end of the game wins. The game is declared a tie if both players have the same number of points.

Your program should calculate the probabilities of Alice winning ($P(A$ wins)), Bob winning ($P(B$ wins)), and the game resulting in a tie (P(tie)).

Objectives

1. **Implement the Game Logic (5 pts)**: Write a program that simulates the coin-tossing game according to the rules specified. Your implementation should accurately track the scores of Alice and Bob throughout the game and determine the game's outcome.
2. **Probability Calculation (15 pts)**: Develop an algorithm that calculates $P(A$ wins), $P(B$ wins), and P(tie) for given values of n.
3. **Outcome Analysis**:
   - **(4 pts)** Calculate the probabilities for $n = 2, 3, 4$, and 5.
   - **(6 pts)** Extend your program to calculate the probabilities for $n = 1024$. Due to the computational complexity involved, you may need to devise an efficient approach or use approximations as necessary.

Requirements

- **Programming Language**: You can choose any programming language for this project. However, ensure that your chosen language can handle the computational tasks efficiently.
- **Documentation**: Provide detailed documentation of your algorithm and program. Explain how the game logic is implemented, how probabilities are calculated, and any optimizations or approximations used for large values of n.
- **Performance Analysis**: Discuss your algorithm's time and space complexity.

Hints

Use the decrease-and-conquer approach to calculate the distribution of $A(n) - B(n)$ where $A(n)$ and $B(n)$ are the points scored by Alice and Bob, respectively.

Submission Guidelines

- Submit your source code and a README file that includes installation and running instructions.
- Include a report detailing your approach, analysis, results (probabilities) for the specified values of n, and any observations or conclusions drawn from the project.
- Ensure that your code is well-commented and follows the coding standards and best practices for your chosen programming language.

Due Date

Apr. 3, 2024.