Lab #3 Report
Digital Simulation of a Closed Loop System

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Total: ___ /45

1 Results ___ /15

1.1 Plots ___ /6

See Figures 1 and 2 for the responses to a unit step as the reference and disturbance, respectively.

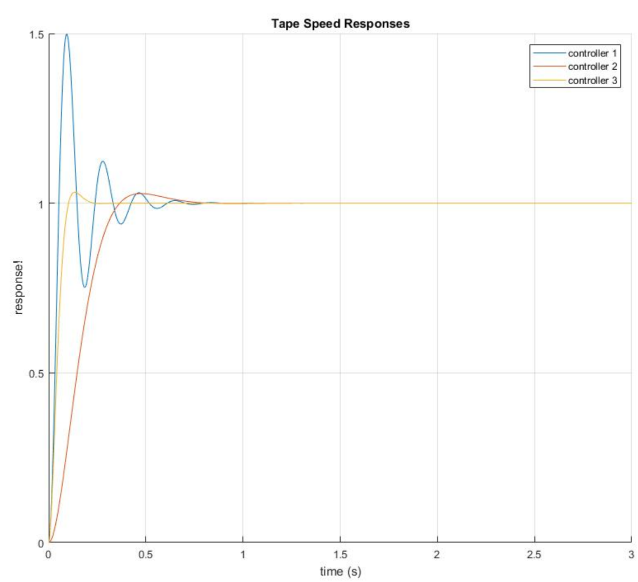


Figure 1: Response of three controllers due to unit step reference.

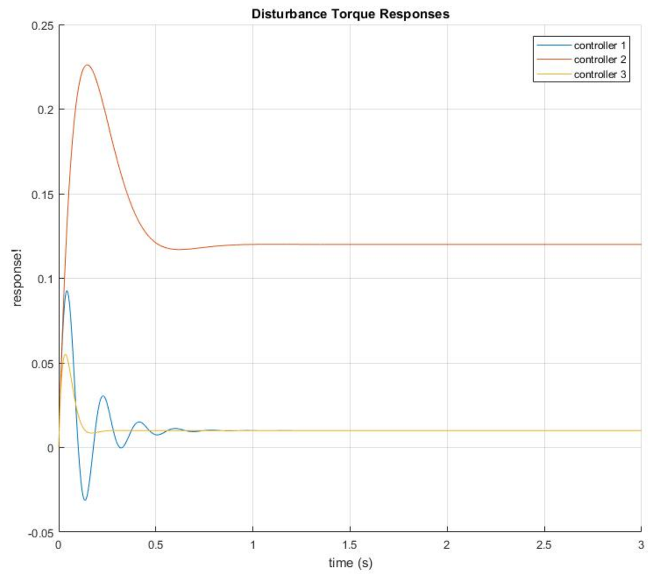


Figure 2: Response of three controllers due to unit disturbance.

Table 1: Time Response to a Unit Step for ω_r

parameters	Controller 1		Controller 2		Controller 3	
	Prelab	Lab	Prelab	Lab	Prelab	Lab
M_p [%]	49.8%	49.8%	2.8%	2.84%	2.8%	3.2%
t_r [s]	0.035	0.036	0.220	0.229	0.067	0.064
t_s [s]	0.402	0.390	0.320	0.312	0.097	0.088
K	19.4		1.067		19.4	
K_r	1.031		1.562		1.031	
K_d	0		0		0.031	

1.2 Step Response to ω_r ___/6

1.3 Comparison ___/3

(Compare M_p, t_r and t_s from Prelab with those from Lab. Are they close to each other? Which controllers met the specifications?)

Our experimental values from the simulation matched closely with those calculated in the prelab. This would make sense because the simulation does not introduce noise or other variables that would throw off the calculations. We observed that the efficacy of the controller varied:

Controller 1 achieved a disturbance response of < 0.01 with $K = 19.4$ as expected from the prelab calculations; however, the overshoot was quite large at $M_p = 49.8\%$. Comparing this to Controller 2, we can attribute the overshoot to our value of $\zeta = .217 << .75$.

Controller 2 performed better in terms of minimizing the overshoot, but had a high disturbance response at 0.12. This is not ideal because we would a good controller to respond well in the presence of disturbance.

Controller 3 introduced derivative feedback to better meet the specifications, and our data confirms that it performed the best. The tachometer allows this controller to better handle disturbance when compared to Controller 2 while keeping the overshoot low (better than Controller 1). We conclude that Controller performed the best!

2 Deriving e_{ss} Components ___/12

(For the system in Figure 3.1 in Lab Manual, derive the relationship between steady state error ($e_{ss} = \omega_r - \omega$) and natural frequency ω_n. Consider the error as a function of both ω_r and τ_d, and model these as step inputs. Since the system is linear, superposition allows the two components to be calculated separately and then summed. Notice that e_{ss} is not the same thing as "e" in the block diagram. ($e = K_r\omega_r - \omega$, this is the error signal.))

$$\omega_n^2 = 36 + 60K$$

$$e_{ss} = \Omega_r(s) - \Omega(s) = \omega_r - \frac{K_r(\omega_n^2 - 36)}{s^2 + 15s + \omega_n^2}\omega_r - \frac{4s + 12}{s^2 + 15s + \omega_n^2}\tau_d$$

Hint: Using the Final Value Theorem, solve the following subproblems:

- What is e_{ss} due to a step in ω_r ($\tau_d = 0$)? In order to minimize this error component, what value of ω_n should we choose?
- What is e_{ss} due to a step in τ_d ($\omega_r = 0$)? In order to minimize this error component, what value of ω_n should we choose?

By FVT, a step response in ω_r will result in the following error:

$$\omega_r(1 - K_r + \frac{36}{\omega_n^2})$$

Thus, $\omega_n^2 = \frac{36}{K_r - 1}$ will minimize the error term.

Similarly, modeling disturbance as a step response results in the following error:

$$-\frac{12}{\omega_n^2}\tau_d$$

Thus, a larger value of ω_n^2 will minimize the error in this case. This agrees with the prelab results showing that values of K that are larger will be more stable.

3 ζ and ω_n in Controller Three ___/18

3.1 Write down the expressions of ζ and ω_n ___/12

(For controller 3, derive the relationship between ζ, ω_n and the gains K and K_d.)

$$\zeta = \frac{15 + 60K_dK}{2\sqrt{36+60K}}$$
$$\omega_n = \sqrt{36+60K}$$

Discussion: If we increase K, what happens to ζ and ω_n? And at what rate does ζ change (*linearly, exponentially, as* K^2 etc)? How does ω_n change? What if we increase K_d? What happens to ζ and ω_n?

As K increases, ω_n and ζ will both increase by a factor of $\sqrt{K}$. The value of ω_n does not change with K_d, while ζ varies linearly with K_d.

3.2 Using these equations, show how the pole locations change as $K_d > 0$ increases in value ___/6

$$\text{Poles} \quad s_{1,2} = \frac{-2\zeta\omega_n \pm \sqrt{(2\zeta\omega_n)^2 - 4\omega_n^2}}{2}$$

(As K_d increases, what is the trajectory of poles? Again, you can use Matlab® to sketch a plot.)

The figure shows that the poles vary based on the value of K_d that is chosen. The poles diverge as K_d increases. The dominant pole converges towards the origin while the non-dominant pole moves further into the LHP. This is consistent with behavior we have observed in previous labs.

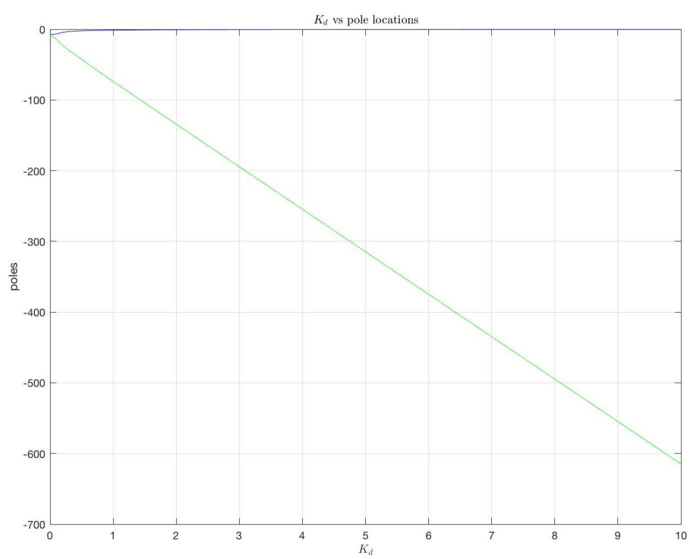
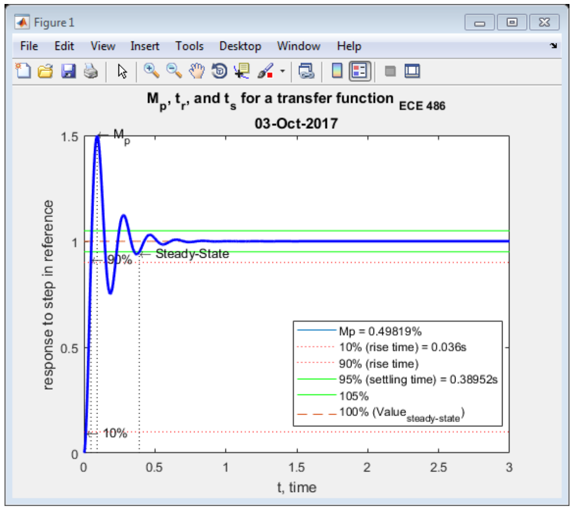


Figure 3: Pole locations for $0 < K_d <$ value.



Sample plot showing response specification values.

```
MATLAB Code used in Lab #3:

%
% Setup values for SIMULINK and then display results
%

K = 19.4;
Kr = (3/(5*K))+1;
J = 0.25;
B = 3;

%-------------------

PlotResponse(ctrl1_td_t, ctrl1_td_w, ctrl1_wr_t, ctrl1_wr_w);

%
% Plot the response for a particular controller
%

function [] = PlotResponse(tout1, w1, tout2, w2)
close all
hold on
grid on
plot(tout1, w1);
plot(tout2, w2);
legend('td_w', 'wr_w');
xlabel('time (s)');
ylabel('response!');
title('Controller 2 Responses');

%
% Calculate values of $M_p$, $t_r$, and $t_s$ for various responses and display results
%

[Mp_1, tr_1, ts_1] = StepResponseMetrics(ctrl1_wr_w,ctrl1_wr_t, 1, 1)
[Mp_2, tr_2, ts_2] = StepResponseMetrics(ctrl2_wr_w,ctrl2_wr_t, 1, 1);
[Mp_3, tr_3, ts_3] = StepResponseMetrics(ctrl3_wr_w,ctrl3_wr_t, 1, 1);
```

```
% ---- pole calculations
K = 1;
Kd = 0:.1:10;

zeta = (15+60*K*Kd)/(2*sqrt(36+60*K));
w = sqrt(36+60*K);

p1 = (-2*zeta*w + sqrt((2*zeta*w).^2 - 4*(w^2)))/2;
p2 = (-2*zeta*w - sqrt((2*zeta*w).^2 - 4*(w^2)))/2;

plot(Kd, p2, 'g')
hold on
grid on
plot(Kd, p1, 'b')

xlabel('$K_{d}$', 'interpreter','latex');
ylabel('poles');
title('$K_{d}$ vs pole locations','interpreter','latex');
```