Lab #4 Report
Introduction to the DC Motor

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Total: ___ /90

1 Calibration of Tachometer ___ /5

1.1 Computing the Tachometer Gain ___ /3

(Briefly explain the procedure and the importance of computing the gain for the Tachometer. Why is it important to know K_{tach}?)

In order to measure the gain of the Tachometer, we recorded the time required for the motor to complete one rotation and the corresponding Tachometer output voltage (which correlates to the motor speed). This allowed us to calculate the angular velocity of the motor and also the gain of the Tachometer (volts/angular velocity). Repeating this procedure, we were able to collect several measures of gain values to generalize the behavior.

The Tachometer is an essential part of the system, and this it is important to calculate its gain. This allows us to more accurately model and simulate the system.

1.2 Experimental Parameters for the Tachometer ___ /2

Table 1: Measurement of K_{tach}

V_i (V)	Δt (ms)	V_{tach} (V)	ω (rad s^{-1})	K_{tach} (V s rad^{-1})
5	98.4	1.93	63.853	0.0302
10	50.8	4.676	153.999	0.0304
15	26.2	7.32	239.816	0.0305

The average value of K_{tach} is 0.0304 V s rad^{-1}.

2 Armature Resistance and Back-EMF ___ /10

2.1 Measuring the R_a and Back-EMF ___ /4

(Explain the procedure for obtaining the Armature Resistance and the torque gain ($K_v = K_\tau$). Why can we ignore L_a?)

We repeated a similar procedure to the first step in order to obtain the armature resistance and torque gain; we isolated the part of the system that involves these components and measured the responses to a few isolated inputs. A voltage was applied to the motor while measing the ss current and voltage on the Tachometer. From the prelab:

$$V_i = i_a R_a + K_v \omega$$

Using this relationship MATLAB, we found the values of R_a and K_v that satisfy the various data points that we collected. This equation ignores the effect of L_a because we are testing the system response at steady state, where the change in the current is 0 and thus the inductance is no longer affecting the circuit. The results are shown below.

2.2 Experimental Values ___/2

Table 2: Experimental values in Question 2

V_i (V)	$i_{a_{ss}}$ (A)	V_{tach} (V)	ω_{ss} (rad s^{-1})
5	0.309	2.029	66.743
6	0.315	2.517	82.796
7	0.327	3.106	102.171
8	0.339	3.64	119.739
9	0.358	4.176	137.37
10	0.367	4.702	154.67
11	0.391	5.23	172.039
12	0.412	5.766	189.67
-5	-0.311	-1.986	-65.329
-6	-0.318	-2.534	-83.355
-7	-0.334	-3.067	-100.888
-8	-0.348	-3.599	-118.388
-9	-0.366	-4.12	-135.526
-10	-0.381	-4.668	-153.552
-11	-0.399	-5.188	-170.658
-12	-0.422	-5.718	-188.092

2.3 Experimental Parameters ___/4

Table 3: Experimental parameters in Question 2

Parameter	Value
R_a (Ω)	5.077
K_V (V s rad^{-1})	0.0525

3 Friction Coefficients ___/10

3.1 Measuring the Coefficients ___/6

(Explain the procedure for obtaining the friction coefficients. Include the equations. Also plot the friction torque ($K_\tau i_a$) against ω. Use a different plot for each direction and estimate the Coulomb and viscous coefficients by using a linear fit.)

We used the data gathered from the isolated motor experiments to determine the coulomb and viscous friction coefficients based on the following equation from prelab:

$$K_t I_a = b\omega + c$$

The inertial component is not included because the angular velocity is constant in steady-state. We were able to graph the positive and negative components of our data and find the friction coefficients for the motor traveling in each direction. We took the slopes of the linear fits to calculate values for b and c.

Plots of frictional torque vs. angular velocity are shown in Figures 1 and 2.

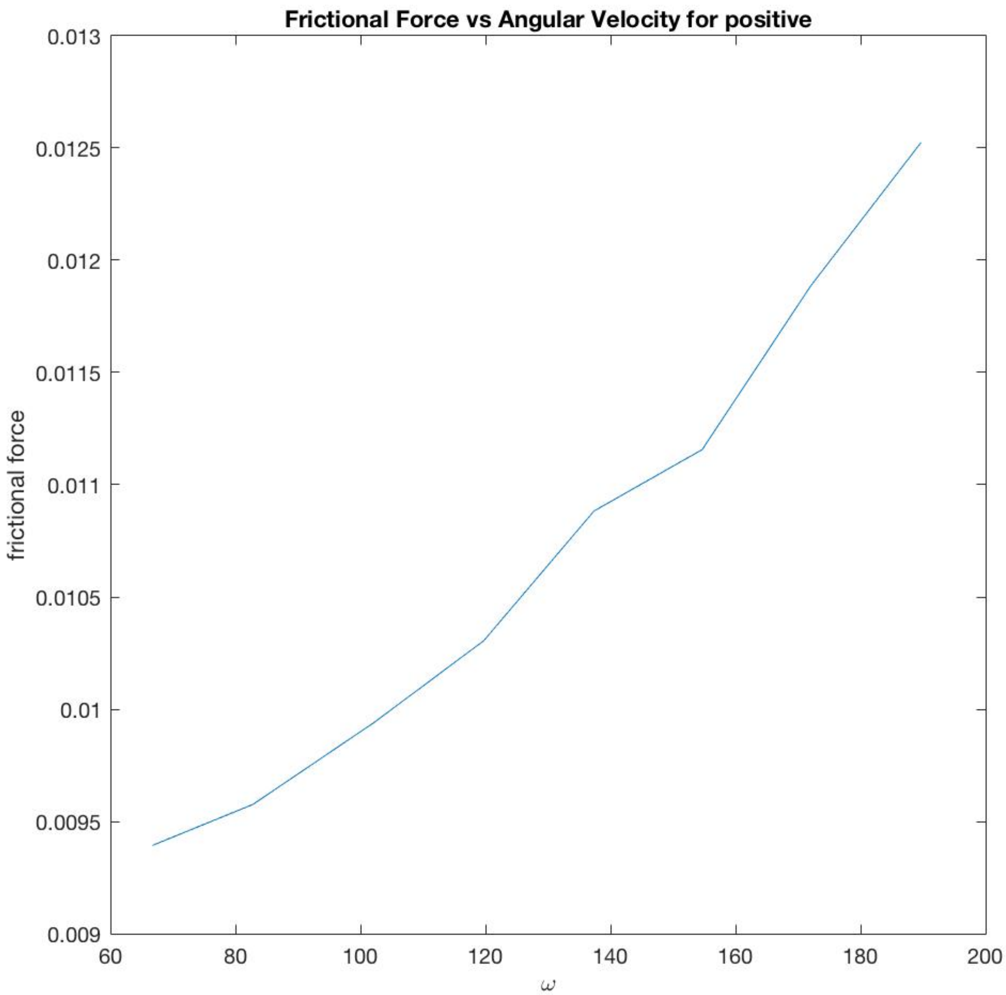


Figure 1: Frictional torque vs. angular velocity for positive ω.

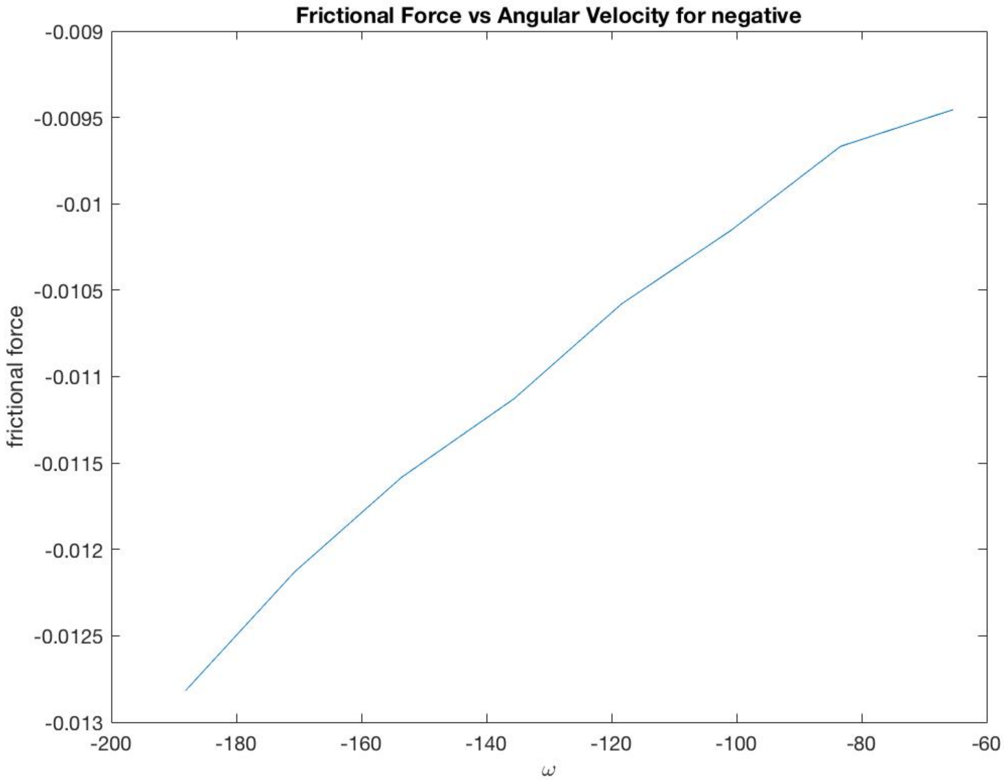


Figure 2: Frictional torque vs. angular velocity for negative ω.

Table 4: Experimental values of b and c

Viscous Coefficient ($\mathrm{N\,m\,s\,rad^{-1}}$)		Coulomb Coefficient ($\mathrm{N\,m}$)	
b^{+}	2.543E-5	c^{+}	0.0075
b^{-}	2.76E-5	c^{-}	-0.0074

3.2 Experimental Values ___/4

4 Armature Inductance ___/20

4.1 Procedure for Measuring R_s and L_a ___/14

(Explain the process of measuring both parameters. Explain how holding the motor still with the rotor-locking attachment allows us to more easily measure L_a. Include the equations (do not forget the logarithm fit). Also include two plots. In the first plot, overlay the linear region for the six data sets obtained after using the logarithm function (without plotting the linear fit). Then, take one (just one, as an example) of these plots, and do a linear fit showing the equation.)

In order to measure the inductance of the motor, we first locked the motor in place to ensure an isolated response. We tracked the response of the system as we repeatedly disconnected and reconnected the voltage source, allowing us to get a clear picture of the voltage as it dropped according to the time constant based on the inductance value we were looking to find.

We modified the data by taking a log of the modified equation as follows; this will allow us to solve for T_e:

$$\ln(V_{ss} - V_0) = \ln\left(\frac{R_a}{R_a + R_s}\right) + \frac{-t}{\tau_e}$$

In MATLAB, we were able to plot the responses that we obtained after they were modified and displayed in a logarithmic scale. We were then able to find a line of best fit which provided a slope value that we used to extrapolate τ_e. We know that this can be represented as such:

$$\tau_e = \frac{L_a}{R_a + R_s}$$

From this, we were able to use the values of τ_e to determine values of L_a for various inputs. We took the average of all of the trials to find the value of L_a that best matches the actual inductance of the motor.

Figure 3: Plot of linear regions (logarithm) of raw data $V_o(t)$ versus t

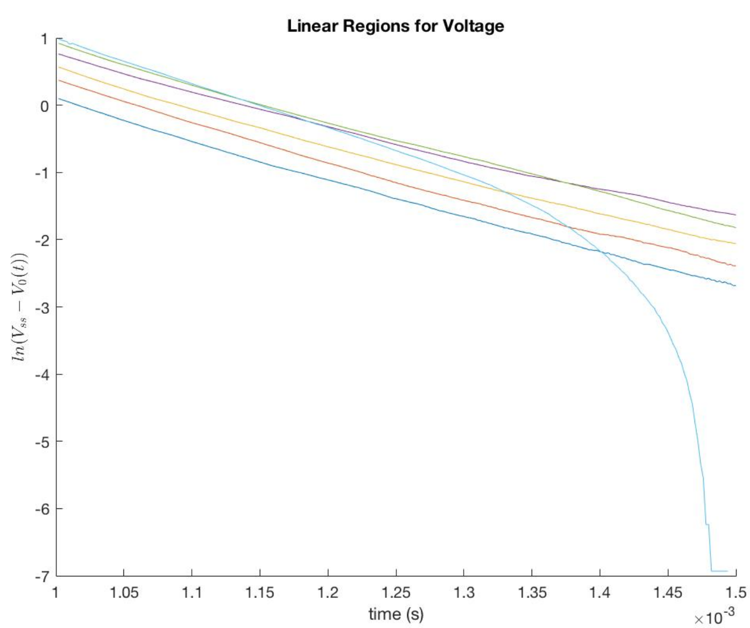
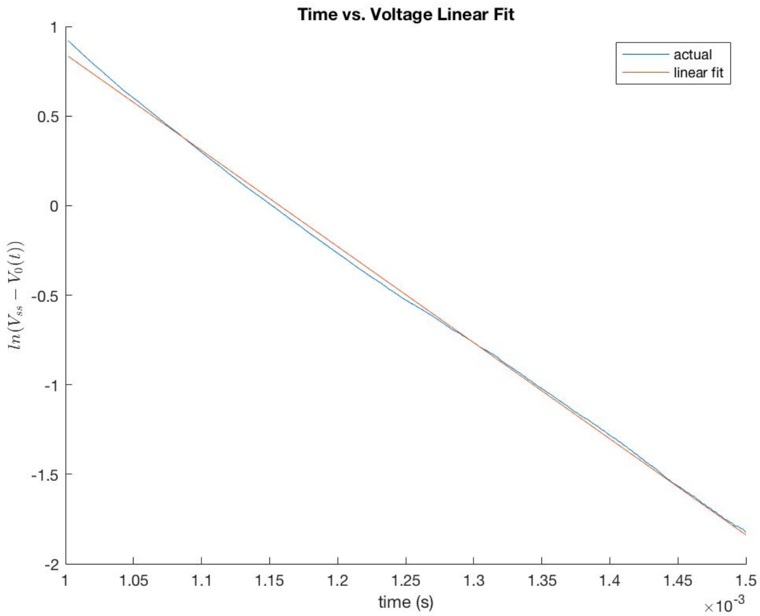


Figure 4: Plot of linear fit (logarithm) of raw data $V_o(t)$ versus t



4.2 Experimental Parameters ___/6

R_s was measured to be 2.524 Ω.

Table 5: Experimental parameter of τ_e and L_a

Trial	τ_e (ms)	L_a (mH)
1	1.814e-04	1.2
2	1.820e-04	1.4
3	1.914e-04	1.5
4	2.078e-04	1.6
5	1.864e-04	1.4
6	1.866e-04	1.5
Average	1.893e-04	1.45

5 Rotor Moment of Inertia ___/18

5.1 Procedure for Measuring J ___/14

(Explain how to obtain J. Include the equations for estimating J (use the natural logarithm function to obtain a linear relation between time and the angular velocity of the rotor). Also explain why we need to measure transient behavior to obtain J. Using the estimates found in part III, plot the linear region for the six sets of data on a single graph. Take one of these plots and do a linear fit showing the linear coefficients (i.e. an equation).)

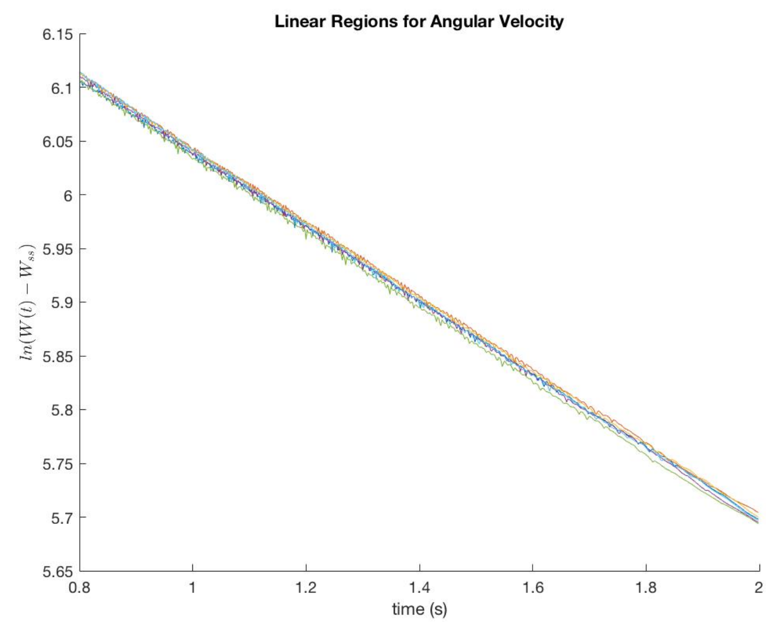
Similar to the previous experimental procedure, we represented the equation in a logarithmic scale to solve for the value of J. Recall the following equation from the prelab:

$$\ln(\omega_t - \omega_{ss}) = \ln(\omega(0) + \frac{c}{b}) - \frac{b}{J}t$$

Using MATLAB we were able to plot the responses that we obtained from the physical system determine the slope of the fit line which gave us the values of J for each trial. The average of these values is the typical inertial coefficient.

A logarithmic plot of angular velocity in time and its linear fit is shown in Figure 6.

Figure 5: Plot of linear regions (logarithm) of raw data $\omega(t)$ versus time



5.2 Experimental Values for J ___/4

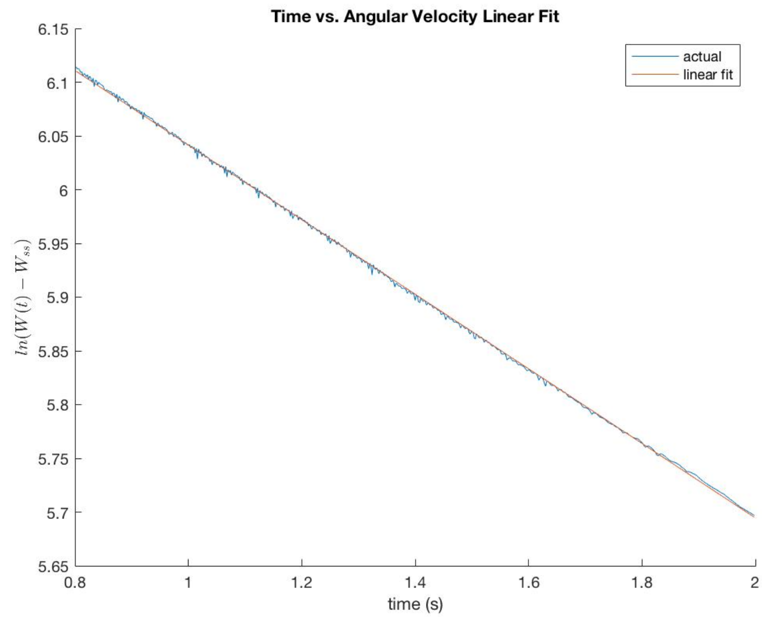
Table 6: Experimental values of J

Trial	Inertia ($\times 10^{-4}$ kg m^2)
1	0.746
2	0.741
3	0.741
4	0.735
5	0.735
6	0.733
Average	0.738

6 Conservation of Energy ___/12

(1. Ignoring losses such as friction and applying the conservation of energy law, show that ($K_v = K_\tau$) are identical. (*Hint*: electrical power is *voltage∗current* and mechanical power is *torque∗velocity*.); 2. Use unit

Figure 6: Plot of linear fit (logarithm) of raw data $\omega(t)$ versus time



conversions to show that their units in SI are equivalent. (units are on page 27 of the lab manual.))

1.
$$I_a V = \tau\omega$$
$$I_a(K_\tau \omega) = (K_v I_a)\omega$$
$$K_\tau = K_v$$

2.
$$\frac{Vs}{rad} = \frac{VsNm}{radJ} = \frac{VNm}{W} = \frac{NmV}{AV} = \frac{Nm}{A}$$

We have shown that the values of K_τ and K_v are equivalent.

7 System Transfer Function ___/10

7.1 Transfer Function $\Omega(s)/V_i(s)$ ___/5

(Find the second order transfer function (from part (e) of the prelab). Use the experimental parameter values and compute the pole locations.)

From the prelab, we know that the overall transfer function of the system is:

$$\frac{\Omega}{V_i} = \frac{K_v}{s^2(L_a J) + s(R_a J + L_a b) + K_v^2 + R_a b}$$

Plugging in the numerical values from above plugged in:

$$\frac{490608}{s^2 + 3501.72s + 26963.4}$$

Which has poles:

$$s \approx -3494, -7.717$$

7.2 Transfer Function $\Omega_{\text{approx}}(s)/V_i(s)$ ___/5

(Find the first order transfer function approximation of the system when ignoring L_a (set $L_a = 0$). Compute the pole location.)

When $L_a = 0$:

$$\frac{\Omega_{approx}}{V_i} = \frac{K_v}{s(R_a J) + K_v^2 + R_a b}$$

Numerically represented using our measured values:

$$\frac{140.119}{s + 7.701}$$

Which has a pole of:

$$s \approx -7.701$$

8 Steady-State Response of Non-Linear System ___/5

(Compute the steady-state angular velocity ω_{ss} for a 4 V input in V_i. Include the effect of Coulomb friction (the c term) in your computation.)

From the prelab, we know the following two relationships:

$$K_\tau I_a = b\omega + c$$

$$V_i = I_a R_a + K_v \omega$$

We can solve for ω in terms of the constants that we have determined experimentally to yield the following equation:

$$\omega = \frac{V_i K_\tau - cR_a}{K_\tau^2 + bR_a}$$

With our values used, we get the following angular velocity:

$$\omega_{ss} \approx 59.585$$

Attachments

- Friction torque vs. the angular velocity (to estimate the friction coefficients for positive and negative rotation) (two plots)
- Linear region of the six data sets for inductance L_a (one plot)
- One of the six sets of the inductance's data, with a linear fit approximation (one plot)
- Linear region of the six data sets for the inertia parameter J (one plot)
- One of the six sets of inertia data, with a linear fit approximation (one plot)
- MATLAB code