Lab #2 Report
DIGITAL SIMULATION

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Total: ___/50

1 STATE SPACE MODEL OF $H_1(s)$ ___/8

(Compare the plots of `y_dot` and `y` obtained in Part 1 of the lab with the plots previously made for the prelab. Why are they identical? Attach plots—if your prelab plot was wrong, fix it and attach the corrected plot.)

As expected, the plots from the prelab and Part 1 of the lab are identical; We can conclude that we are using the state-space model to represent the same system from the prelab. Moreover, the numbers in `A` and `B` match the coefficients of the transfer function from the prelab, which validates our hypothesis.

The plots of y and $\dot{y}$ from the prelab and lab are shown in Figures 1 and 2.

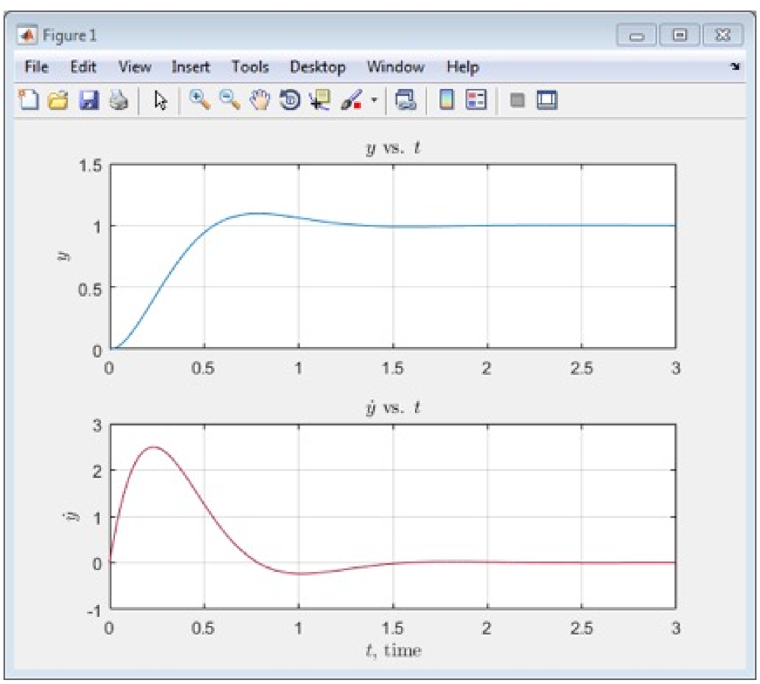

Figure 1: Plots of y and $\dot{y}$, from the prelab, for a step input.

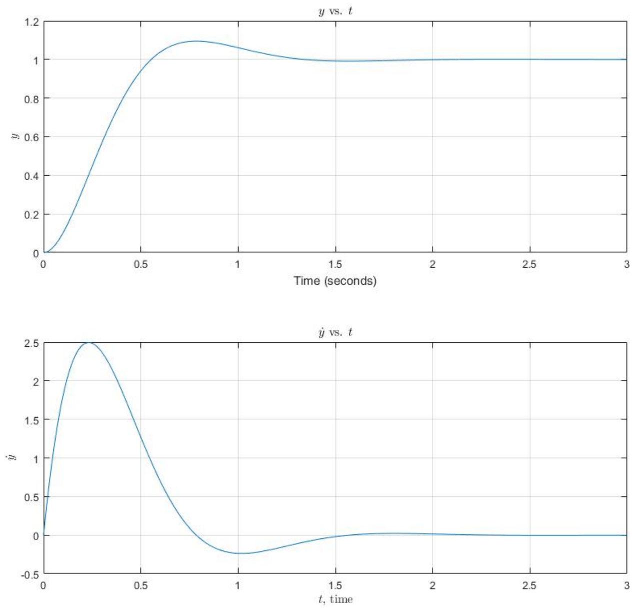

Figure 2: Plots of y and $\dot{y}$, from the lab, for a step input.

2 Effects of an extra Zero ___/22

The step responses after adding an extra zero are shown in Figure 3.

2.1 Effects of a Zero on M_p, t_r, and t_s ___/2

Fill the table of specs of time domain responses.

Table 1: Effects of Zero

Specs	No zero $H_1(s)$	$H_2(s)$ zero at $s = -30$	$H_2(s)$ zero at $s = -3$	$H_2(s)$ zero at $s = -1.5$	$H_2(s)$ zero at $s = 1.8$	$H_2(s)$ zero at $s = 18$
M_p [%]	0.094	0.096	0.411	1.147	0.195	0.098
t_r [s]	0.373	0.365	0.126	0.057	0.211	0.358
t_s [s]	1.073	1.012	0.847	1.376	1.338	1.099

2.2 Discuss the Effects of a LHP Zero ___/4

(Explain how M_p , t_r , and t_s are affected by the zero location. When can the zero be ignored?)

Generally, a zero in the LHP will have a greater effect on the step response when it is closer to the origin. Our results confirm this hypothesis because the percent overshoot increases as the pole location moves towards the origin. To illustrate, at $s = -30$, $M_p = 0.096$ while at $s = -1.5$, $M_p = 1.147$.

2.3 Effects of a Non-minimum Phase Zero ___/2

(What is unique in this situation?) Adding a zero in the RHP will result in stable yet suboptimal system. Because the dominant pole is in the LHP, the system will eventually reach the proper steady-state value; however, it will take a suboptimal path. As seen in Figure 3, adding an additional zero in the RHP will

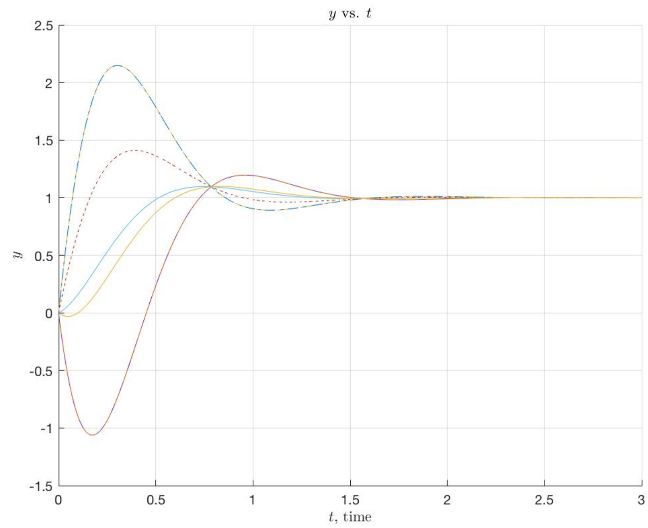

Figure 3: Effects of an extra zero on the step response.

cause the response to undershoot and curve downwards before approaching the steady-state. The specs are slightly deceiving in this case because the response curved downwards before increasing to the steady-state value; the undershoot in doing this is not accounted for in the M_p spec, of course.

2.4 Decomposition of $H_2(s)$ ___/14

(Take $H_2(s)$, set ζ to the value found in the prelab, separate the numerator into two terms so that $H_2(s)$ is a sum of 2 fractions. Discuss how this decomposition helps to explain the effect of the zero location. In particular, discuss what each term represents. Also discuss α's effect. Which term dominates as α approaches 0? As αapproaches ∞? What happens when α is negative?)

$$H_2(s) = \frac{25\left(1 + \frac{s}{\alpha\zeta}\right)}{s^2 + 10\zeta s + 25}$$
$$= \frac{25}{s^2+10\zeta s+25} + \frac{\frac{25s}{\alpha\zeta}}{s^2+10\zeta s+25}$$

The first term of decomposition represents the original system characterized by $H_1(s)$ while the second term represents the response with an additional zero. We can see from the transfer function that the zero occurs when $\frac{s}{0.6*\alpha} = -25$. The second term will dominate as α approaches 0, resulting in a large, adverse effect on the response. Conversely, the second term will disappear as α approaches ∞– this explains why placing the zero further away from from the origin has a marginal effect on our response. When α is negative, the zero will be in the RHP, resulting in non-minimum phase zero behavior described above. Specifically, the response will undershoot and then move towards the SS value.

3 Effects of an extra Pole ___/20

The step responses after adding an extra pole are shown in Figure 4.

3.1 Effects of a Pole on M_p, t_r, and t_s ___/2

(Fill out the table of specs of time domain responses.)

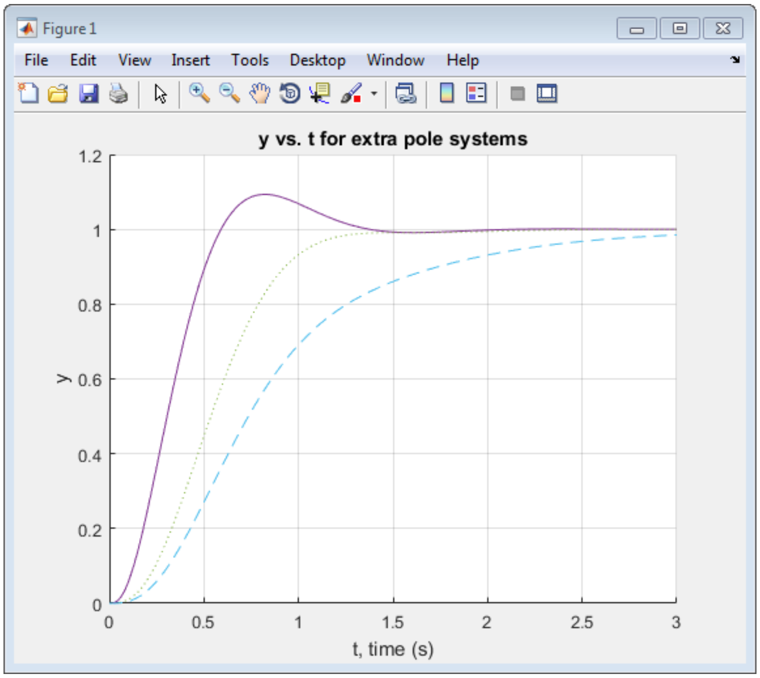

Figure 4: Effects of an extra pole on the step response.

Table 2: Effects of Pole

Specs	No pole $H_1(s)$	$H_2(s)$ pole at $s = -30$	$H_2(s)$ pole at $s = -3$	$H_2(s)$ pole at $s = -1.5$
M_p [%]	0.094	0.093	≈ 0	-0.015
t_r [s]	0.373	0.377	0.687	1.428
t_s [s]	1.073	1.078	1.052	2.214

3.2 Discuss the Effects of an Extra Pole ___ /4

(Explain how M_p , t_r , and t_s are affected by the location of the additional pole. When can the extra pole be ignored?)

As we observed in lab1, the pole closest to the origin dominates the step response; As expected, poles further into the LHP had a lesser effect of the response. To illustrate, at $s = -30$, $M_p = 0.093$ while at $s = -3$, $M_p \approx 0$. We observe that the extra affects the response more as we move it closer to the origin. Moreover, the addition of a pole in the RHP results in an unstable result.

3.3 Decomposition of $H_3(s)$ ___ /14

$$
\begin{aligned}
H_3(s) &= \frac{25}{\left(1+\frac{s}{\alpha\zeta}\right)(s^2+10\zeta s+25)} \qquad (\zeta = 0.6) \\
&= \frac{k_1}{1+\frac{5s}{3\alpha}} + \frac{k_2 s}{s^2+6s+25} + \frac{k_3}{s^2+6s+25}
\end{aligned}
$$

Using a partial fraction expansion,

$$
\begin{aligned}
k_1 &= \frac{25^2}{9\alpha^2 - 90\alpha + 25^2} \\
k_2 &= \frac{-375\alpha}{9\alpha^2 - 90\alpha + 25^2} \\
k_3 &= \frac{225\alpha\,(\alpha - 10)}{9\alpha^2 - 90\alpha + 25^2}
\end{aligned}
$$

(Discuss how this decomposition helps to explain the effect of the location of an additional pole. In particular, discuss what each term represents. Also discuss α's effect. Which term dominates as α approaches 0? As α approaches ∞?)

The first term of the decomposition represents the partial response from the extra pole while the second and third terms represent the response from the original two poles. We can see from the transfer function that the new pole occurs when $\frac{s}{0.6\alpha} = -1$. The first term will dominate the response as α approaches 0 while the second and third disappear; the transfer function will only have the k_1 term. Conversely, as α approaches ∞, the k_3 terms will dominate the response while k_1 and k_2 disappear.

MATLAB CODE

```
[Mp_1, tr_1, ts_1] = StepResponseMetrics(y2a,tout2a, 1, 1)
[Mp, tr, ts] = StepResponseMetrics(y(:,1),tout, 1, 1);
[Mp_3, tr_3, ts_3] = StepResponseMetrics(y2c,tout2c, 1, 1);
[Mp_4, tr_4, ts_4] = StepResponseMetrics(y2d,tout2d, 1, 1);
[Mp_5, tr_5, ts_5] = StepResponseMetrics(y2e,tout2e, 1, 1);
[Mp_6, tr_6, ts_6] = StepResponseMetrics(y6(:,2),y6(:,1), 1, 2);
[Mp_7, tr_7, ts_7] = StepResponseMetrics(y7(:,2),y7(:,1), 1, 2);
[Mp_8, tr_8, ts_8] = StepResponseMetrics(y8(:,2),y8(:,1), 1, 2);


zeta = [-10:.1:10]
p1 = -zeta + sqrt((zeta.^2) - 1);
p2 = -zeta - sqrt((zeta.^2) - 1);

plot(zeta, p1, 'b')
```

```
hold on
grid on
plot(zeta, p2, 'g')
xlabel('\zeta');
ylabel('poles');
title('\zeta vs pole locations');
```

```
a = -10;
zeta = 0.6;

plot(tout,y)
grid on
title('$y$ vs. $t$','interpreter','latex')
ylabel('$y$','interpreter','latex')
```

```
hold on
grid on
plot(tout2a, y2a, '-');
plot(tout2b, y2b, ':');
plot(tout2c, y2c, '-.');
plot(tout2d, y2d, '--');
plot(tout2e, y2e, '-');
title('$y$ vs. $t$','interpreter','latex')
xlabel('$t$, time','interpreter','latex')
ylabel('$y$','interpreter','latex')
```

```
A = [-6 -25; 1 0]
B = [25; 0]
C = [1 0; 0 1]
D = [0; 0]

subplot(2,1,1)
plot(tout,y)
grid on
title('$y$ vs. $t$','interpreter','latex')
ylabel('$y$','interpreter','latex')

subplot(2,1,2)
plot(tout,y_dot)
grid on
title('$\dot{y}$ vs. $t$','interpreter','latex')
xlabel('$t$, time','interpreter','latex')
ylabel('$\dot{y}$','interpreter','latex')

a = 0.5
zeta = 2
```