Lab #2 Report
Digital Simulation

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Total: ___ /50

1 State Space Model of $H_1(s)$ ___ /8

(Compare the plots of `y_dot` and `y` obtained in Part 1 of the lab with the plots previously made for the prelab. Why are they identical?)

The two plots are identical because both simulations represent the same system. While in the prelab we modeled the system with an all-integrator block diagram, the lab exercise involved using the state-space representation of the system. Since the inputs to the system remained constant, as did the system itself, the experiments resulted in identical graphs.

The plots of y and $\dot{y}$ from the prelab and lab are shown in Figures 1 and 2.

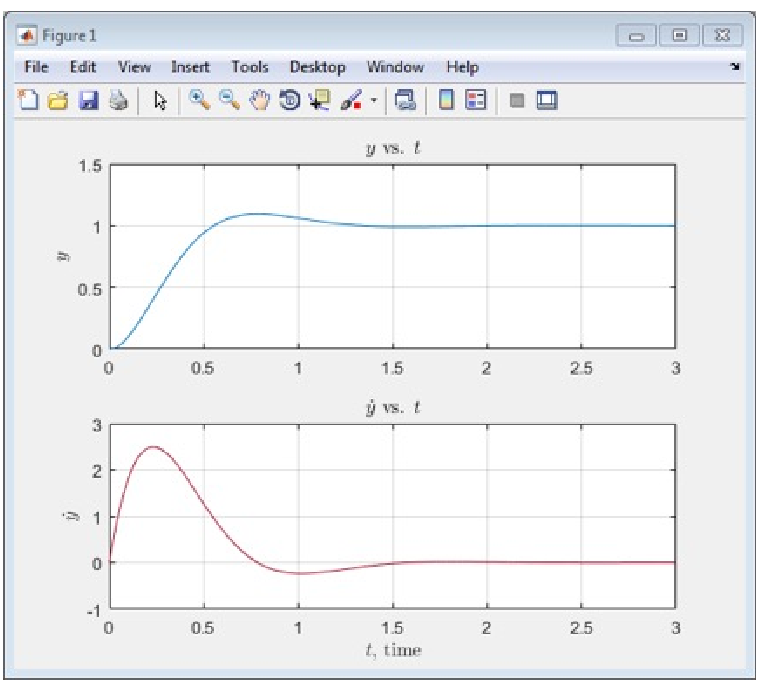

Figure 1: Plots of y and $\dot{y}$, from the prelab, for a step input.

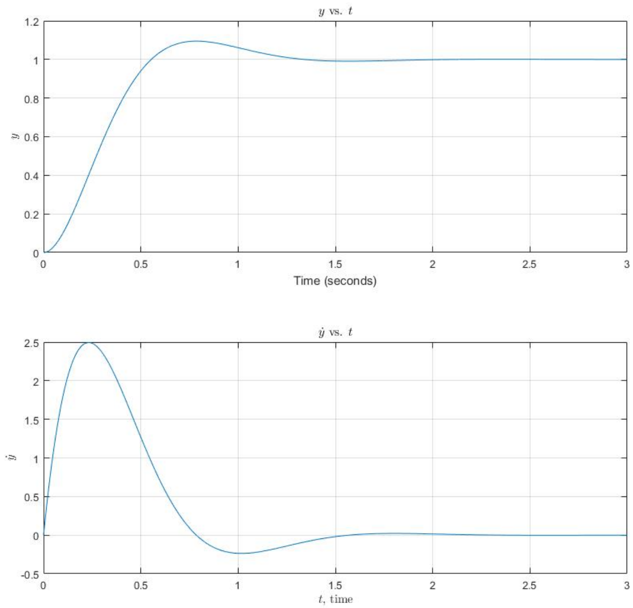

Figure 2: Plots of y and $\dot{y}$, from the lab, for a step input.

2 Effects of an extra Zero ___ /22

The step responses after adding an extra zero are shown in Figure 3.

2.1 Effects of a Zero on M_p, t_r, and t_s ___ /2

Fill the table of specs of time domain responses.

Table 1: Effects of Zero

Specs	No zero $H_1(s)$	$H_2(s)$ zero at $s = -30$	$H_2(s)$ zero at $s = -3$	$H_2(s)$ zero at $s = -1.5$	$H_2(s)$ zero at $s = 1.8$	$H_2(s)$ zero at $s = 18$
M_p [%]	0.094	0.096	0.411	1.147	0.195	0.098
t_r [s]	0.373	0.365	0.126	0.057	0.211	0.358
t_s [s]	1.073	1.012	0.847	1.376	1.338	1.099

2.2 Discuss the Effects of a LHP Zero ___ /4

(Explain how M_p , t_r , and t_s are affected by the zero location. When can the zero be ignored?)

The zero can have a profound effect on the resulting function depending on its location.

For example, while the zero is in the LHP and far from the origin (ie. $s = -30$), the value of M_p is essentially unchanged (0.096%), and the values of t_s and t_r basically match the values present without the extra zero. This is consistent with what we know about the transfer function in question, as the dominant pole primarily characterizes the behavior while the extra zero far from the origin does not affect its performance while it remains suitably distant.

However, as the new zero moves closer to the origin, it has a larger effect on the resulting graph as is seen from the values in the third and fourth columns of the table, the result reaches a higher maximum value and

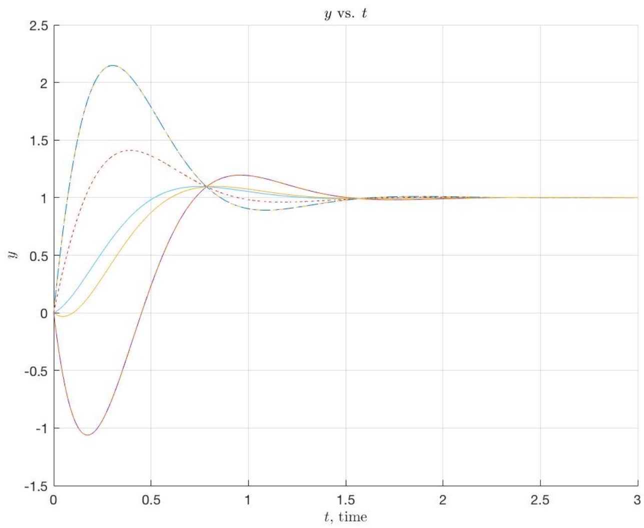

Figure 3: Effects of an extra zero on the step response.

it takes longer to stabilize to the steady-state. This implies that the extra zero nearer to the origin prevents the system from stabilizing quickly, resulting in a less efficient controller.

2.3 Effects of a Non-minimum Phase Zero ___/2

(What is unique in this situation?)

Extra zeroes placed on the RHP cause the response to undershoot before shifting back to the steady-state value. In the table, this is manifested as the maximum value reached lowering again because some of the energy is spent in traveling in the negative direction before becoming positive. Similarly, the time to the steady state is reduced because the response expends some energy moving negative first. Because of this, t_r is larger in this case as the RHP zero response does not near the steady-state value as quickly as a response that did not contain a RHP zero. Controllers with this schema will be stable because the dominant pole is on the LHP, however they will result in some erroneous behavior (ie. first moving in the direction opposite from a steady-state value) before stabilizing to the steady-state.

2.4 Decomposition of $H_2(s)$ ___/14

(Take $H_2(s)$, set ζ to the value found in the prelab, separate the numerator into two terms so that $H_2(s)$ is a sum of 2 fractions. Discuss how this decomposition helps to explain the effect of the zero location. In particular, discuss what each term represents. Also discuss α's effect. Which term dominates as α approaches 0? As αapproaches ∞? What happens when α is negative?)

$$H_2(s) = \frac{25\left(1 + \frac{s}{\alpha\zeta}\right)}{s^2 + 10\zeta s + 25}$$

$$= \frac{25}{s^2 + 6s + 25} + \frac{25\frac{s}{0.6\alpha}}{s^2 + 6s + 25}$$

The first fraction in the decomposition is simply the original transfer function that was used for the prelab and original lab experiment. The second component added in this section helps explain the zero location. In particular, the zero will be present at $s = -0.6\alpha$. As α approaches ∞, the pole moves further and further negative, meaning, based on our earlier observations, that the zero will have less of an effect on the resulting response. Based on the decomposition shown above, if α approaches ∞ then the second fraction will become smaller until eventually it becomes zero.

This corroborates our earlier observation and provides an explanation for why a zero closer to the origin (implying a smaller α) will have a larger effect on the response. For very small values of α, the effect of the second fraction is amplified and the overall response is characterized more by the zero than the poles.

Finally, for values of α that are negative, there is a zero on the RHP, resulting in a non-minimum phase response. The response in this scenario is affected in the opposite way from before and as such it actually moves away from the steady-state response before stabilizing because the second fraction is now acting against the first. In this case, α causes the system to respond to an input in a manner that does immediately converge to the steady-state value.

3 Effects of an extra Pole ___/20

The step responses after adding an extra pole are shown in Figure 4.

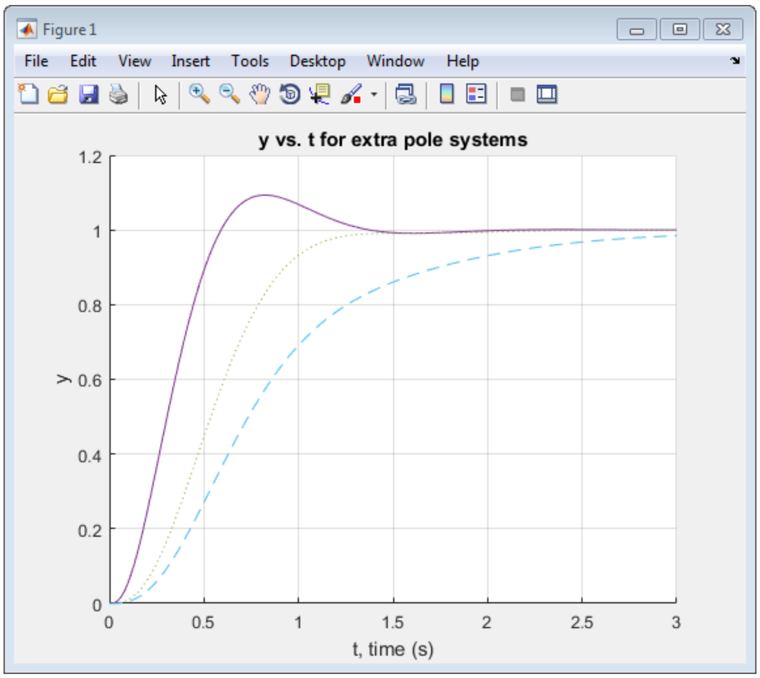

Figure 4: Effects of an extra pole on the step response.

3.1 Effects of a Pole on M_p, t_r, and t_s ___/2

(Fill out the table of specs of time domain responses.)

Table 2: Effects of Pole

Specs	No pole $H_1(s)$	$H_2(s)$ pole at $s = -30$	$H_2(s)$ pole at $s = -3$	$H_2(s)$ pole at $s = -1.5$
M_p [%]	0.094	0.093	≈ 0	-0.015
t_r [s]	0.373	0.377	0.687	1.428
t_s [s]	1.073	1.078	1.052	2.214

3.2 Discuss the Effects of an Extra Pole ___ /4

(Explain how M_p , t_r , and t_s are affected by the location of the additional pole. When can the extra pole be ignored?)

Similar to the addition of a extra zero to the system, the result when an extra pole is added depends greatly on the position of the added pole.

In the case of a pole added far from the origin on the LHP, the result is essentially unchanged fom the original result. This can be seen through a comparison of the first two columns of Table 2. This result can be explained by the fact that the dominant pole still rests nearer to the origin, and as such the extra pole does not affect the system to the same degree that the original, dominant pole does.

However, as the pole nears the origin, its effect on the result becomes more apparent. Based on the results of our experiment, the system response seems to have been dampened by the presence of the extra pole. This is manifested as the response reaching a lower maximum value, or M_p, and having a longer rise and steady time, t_r and t_s respectively. This makes sense based on the adjusted transfer function because having a pole near the original, dominant one will cause the effect of having a pole in that region to be magnified. In this case, the result is a more dampened system.

Finally, pole added on the RHP has a tremendous effect on the result in that it causes the output to be unstable. For this reason, we did not calculate specification values for positive s poles and did not include them on our graph. Regardless of the original system, a pole added in the RHP will add terms to the response that do not converge and as such the output of such a function can only be unstable.

3.3 Decomposition of $H_3(s)$ ___ /14

$$
\begin{aligned}
H_3(s) &= \frac{25}{\left(1+\frac{s}{\alpha\zeta}\right)\left(s^2+10\zeta s+25\right)} \qquad (\zeta = 0.6) \\
&= \frac{k_1}{1+\frac{5s}{3\alpha}} + \frac{k_2 s}{s^2+6s+25} + \frac{k_3}{s^2+6s+25}
\end{aligned}
$$

Using a partial fraction expansion (and a computer),

$$
\begin{aligned}
k_1 &= \frac{25^2}{9\alpha^2-90\alpha+25^2} \\
k_2 &= \frac{-375\alpha}{9\alpha^2-90\alpha+25^2} \\
k_3 &= \frac{225\alpha\,(\alpha-10)}{9\alpha^2-90\alpha+25^2}
\end{aligned}
$$

(Discuss how this decomposition helps to explain the effect of the location of an additional pole. In particular, discuss what each term represents. Also discuss α's effect. Which term dominates as α approaches 0? As α approaches ∞?)

The decomposition of the modified transfer function helps explain why the results of this experiment occur.

In the specific case of large values of α, it is obvious that only the k_3 term will remain which corresponds to the original system. In our experiment, even a value of $\alpha = 50$ gave a result that matches this hypothesis. Conversely, as α approaches 0, we begin to see the other terms take precedence.

In this region, we see the effect of the k_2 term which dampens the response because it has a negative coefficient. As was described earlier, the response in this case is characterized as a dampened version of the original output.

Then, at $\alpha < 0$ the k_3 term. This term does not converge and as a result the response is unstable. Because the value of α directly determines the location of the extra pole, this forces a RHP pole which we know will cause this sort of result. As such, this is consistent with what we know about the transfer functions in play during this experiment.

MATLAB Code used in Lab #2:

```
%
% Init the state-space model and display results
%

A = [-6 -25; 1 0]
B = [25; 0]
C = [1 0; 0 1]
D = [0; 0]

a = 0.5
zeta = 2

subplot(2,1,1)
plot(tout,y)
grid on
title('$y$ vs. $t$','interpreter','latex')
ylabel('$y$','interpreter','latex')

%
% Calculate values of $M_p$, $t_r$, and $t_s$ for various responses and display results
%

[Mp_1, tr_1, ts_1] = StepResponseMetrics(y3a,tout3a, 1, 1)
[Mp_2, tr_2, ts_2] = StepResponseMetrics(y3b,tout3b, 1, 1);
[Mp_3, tr_3, ts_3] = StepResponseMetrics(y3c,tout3c, 1, 1);
```