Lab #1 Report

SIMULATION USING THE ANALOG COMPUTER

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Total: ___ /40

Question 1 ___ /15

Theoretical and Experimental Results ___ /5

Table 1: Theoretical and Experimental Results

	M_p (%)		t_r (s)		t_s (s)	
ζ	Theory	Experiment	Theory	Experiment	Theory	Experiment
2.0	0.000	0.013	8.20	10.36	11.6	12.42
1.5	0.000	0.012	5.85	8.140	8.30	9.600
1.0	0.000	0.014	3.35	3.180	5.00	6.420
0.8	0.015	0.031	2.51	2.340	3.70	5.200
0.7	0.046	0.063	2.16	2.040	3.00	7.280
0.5	0.163	0.184	1.63	1.600	6.30	7.520
0.3	0.372	0.390	1.30	1.300	10.1	12.88
0.2	0.527	0.542	1.21	1.200	15.1	18.63

step_response_y7.jpg

Figure 1. The step response obtained from the system with $\zeta = 0.3$. Included on the graph are the values of M_p, t_r, and t_s. Using this methodology, we analyzed all of the responses and created the table shown above.

Comparison of Theoretical/Experimental Results ___/5

Analysis of the theoretical and experimental values that we obtained shows that our data generally matches the expected values. The experimental M_p values are all slightly higher than the theoretical ones, possibly due to imperfections or variance in the electrical system or in the equipment used to measure the system response. Likewise, the values we obtained for t_r and t_s are each marginally higher than the expected ones. This may be explained again by variances between the mathematical models and the physical system, as well as by slight delays introduced by the timing apparatus (oscilloscope) used in the experiment.

Discussion of Variation of ζ with M_p, t_r, and t_s ___/5

As ζ decreases from 2.0, the values obtained for M_p both theoretically and experimentally increase. The step response overshoots the steady-state value to a higher degree as ζ decreases. This result can be rationalized by the fact that as ζ decreases, the system shifts from being overdamped to being underdamped and as such the step response swings to larger values before returning to the steady state.

Unlike this, the value of t_r decreases as ζ decreases. This means that as ζ decreases, the time required for the step response to reach 90% of its steady-state value decreases as well. This is also likely the result of a decrease in the damping of the system as ζ decreases, which in turn causes the step response to have a stronger reaction to the input. Thus, the response increases more rapidly for lower values of ζ.

Finally, t_s reaches a minimum at $\zeta \approx 1$ and increases as ζ strays further from that value in either direction. This result can be rationalized by the fact that t_s measures the time until the step response has essentially reached its steady-state value and the system is no longer changing by a substantial amount. In cases with ζ much larger or smaller than 1, the system is either over- or under- damped meaning that the response will not follow the quickest path to the steady-state. In cases of overdamping the response will take a longer time to increase to the steady-state, while in cases of underdamping the response will reach and overshoot the steady-state, potentially multiple times, before settling. As such, the value of t_s should be minimized at the optimal level of damping, in this system found at $\zeta = 1$.

Question 2 ___ /15

lambda_vs_poles.jpg

Figure 2. A graph showing the two pole locations in the real plane as ζ changes. For values of $|\zeta| < 1$ the poles will be complex and only the real components are shown in this figure.

Effect of ζ on Pole Locations ___ /5

The poles of the transfer function $\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$ can be calculated by solving for when the expression in the denominator is equal to 0. This yields:

$$s = -\zeta \pm \sqrt{\zeta^2 - 1}$$

Thus, the poles of the original transfer function are complex if $\zeta < 1$, and real for $\zeta > 1$. Furthermore, for $\zeta > 1$ the poles are always in the LHP and as ζ increases, the location of one pole approaches 0 and the other decreases and moves further from the origin.

Effect of Pole Locations on M_p, t_r, and t_s for an Underdamped System ___ /5

This system becomes underdamped for $\zeta < 1$ since the values of the poles become complex. In this case, the values of M_p increase because the step response swings above the steady-state response before stabilizing. Additionally, t_r decreases since the underdamped system has a stronger reaction to the input and the step response reaches 90% of its value quickly when compared with other cases. Finally, t_s increases as ζ moves further from 1, because this is the point at which the system is critically damped. In an underdamped system, the response will take longer to reach the steady-state than if the system was critically damped. Thus, the values of t_s reach a minimum at $\zeta = 1$ and increase as the system becomes less damped.

Effect of Pole Locations on M_p, t_r, and t_s for an Overdamped/Critically Damped System ___ /5

Conversely in an overdamped system with $\zeta > 1$, M_p decreases to 0 while t_r increases and t_s increases. Since the system is more than critically damped, the step response will not increase beyond the steady-state value, making $M_p \to 0$. However, since the system reacts less to the input when overdamped, it will also take longer to reach the steady-state value, causing t_r to increase. Finally, t_s increases as we move further from the critically damped state ($\zeta = 1$) since it will take longer for an overdamped system to stabilize at the steady-state value.

If the system is critically damped ($\zeta = 1$) then the step response will follow the most optimal, quickest path to the steady-state value without overshooting. This means that theoretically the value of M_p should be 0, which matches with our observed reading (0.014). Additionally, t_s will be minimized since the step response will shift as quickly as possible towards the steady-state without overshooting at all; this means that the time taken to reach a steady-state will be at its lowest, which is backed up by our experimental data.

Question 3 ___ /10

Comparison of 2^{nd} Order System with 1^{st} Order System with Dominant Pole ___ /6

As can be seen from the graphs, the difference between an overdamped 2^{nd} order system and its 1^{st} order approximation using the dominant pole becomes almost negligible for larger values of ζ. Specifically in the case of $\zeta = 1.5$, the approximation was slightly higher than the true value at some points on the graph. However, for $\zeta = 5$ and for $\zeta = 40$ there was essentially no visible difference between the graphs. This implies that for overdamped systems, only the pole closer to the origin is primarily influencing the output of the system. Thus, the step response would be characterized by this pole alone even though the other pole is still present in the 2^{nd} order equation.

Effect of ζ on Accuracy of Approximation ___ /4

As was stated above, there was minimal difference between the approximation and the original function for values of $\zeta = 5$ and $\zeta = 40$. However it seems that for higher values of ζ, the first order approximation becomes better. It stands to reason that for these high values of drastic overdamping, the response is primarily driven by the stronger pole. As such, we can construct an approximation for the 2^{nd} order function using only its stronger pole.

1_5.jpg

5.jpg

40.jpg

Figures 3-5. Graphs of the 2^{nd} order step responses generated as well their 1^{st} order approximations using the dominant pole for values of $\zeta = 1.5$, 5, and 40.

MATLAB code for plotting the step responses to the transfer function for varying values of ζ, as well as pole-location analysis.

$[\mathrm{Mp}_2, tr_2, ts_2] = StepResponseMetrics(y2(:,2), y2(:,1), 1, 2);$
$[\mathrm{Mp}_3, tr_3, ts_3] = StepResponseMetrics(y3(:,2), y3(:,1), 1, 2);$
$[\mathrm{Mp}_4, tr_4, ts_4] = StepResponseMetrics(y4(:,2), y4(:,1), 1, 2);$
$[\mathrm{Mp}_5, tr_5, ts_5] = StepResponseMetrics(y5(:,2), y5(:,1), 1, 2);$
$[\mathrm{Mp}_6, tr_6, ts_6] = StepResponseMetrics(y6(:,2), y6(:,1), 1, 2);$
$[\mathrm{Mp}_7, tr_7, ts_7] = StepResponseMetrics(y7(:,2), y7(:,1), 1, 2);$
$[\mathrm{Mp}_8, tr_8, ts_8] = StepResponseMetrics(y8(:,2), y8(:,1), 1, 2);$
zeta = [-10:.1:10]
p1 = -zeta + sqrt((zeta.$^2) - 1$);
p2 = -zeta - sqrt((zeta.$^2) - 1$);
plot(zeta, p1, 'b')
hold on
grid on
plot(zeta, p2, 'g')
xlabel('ζ');
ylabel('poles');
title('$\zeta vspolelocations'); [Mp_1, tr_1, ts_1] = StepResponseMetrics(y1(:,2), y1(:,1), 1, 2);$
$[\mathrm{Mp}_2, tr_2, ts_2] = StepResponseMetrics(y2(:,2), y2(:,1), 1, 2);$
$[\mathrm{Mp}_3, tr_3, ts_3] = StepResponseMetrics(y3(:,2), y3(:,1), 1, 2);$
$[\mathrm{Mp}_4, tr_4, ts_4] = StepResponseMetrics(y4(:,2), y4(:,1), 1, 2);$
$[\mathrm{Mp}_5, tr_5, ts_5] = StepResponseMetrics(y5(:,2), y5(:,1), 1, 2);$
$[\mathrm{Mp}_6, tr_6, ts_6] = StepResponseMetrics(y6(:,2), y6(:,1), 1, 2);$
$[\mathrm{Mp}_7, tr_7, ts_7] = StepResponseMetrics(y7(:,2), y7(:,1), 1, 2);$
$[\mathrm{Mp}_8, tr_8, ts_8] = StepResponseMetrics(y8(:,2), y8(:,1), 1, 2);$
zeta = [-10:.1:10]
p1 = -zeta + sqrt((zeta.$^2) - 1$);
p2 = -zeta - sqrt((zeta.$^2) - 1$);
plot(zeta, p1, 'b')
hold on
grid on
plot(zeta, p2, 'g')
xlabel('ζ');
ylabel('poles');
title('$\zeta vspolelocations'$);