Lab #6 Report
Lead Controller Design

Kush Gupta
Section AB6
Vishrut Dixit
Khaled Al-shehri

January 24, 2018

Total: ___ /90

1 Effects of Saturation Block ___ /10

(Discuss the effect of the saturation block in the simulations. Overlay plots for the saturated and plots for the non-saturated cases.)

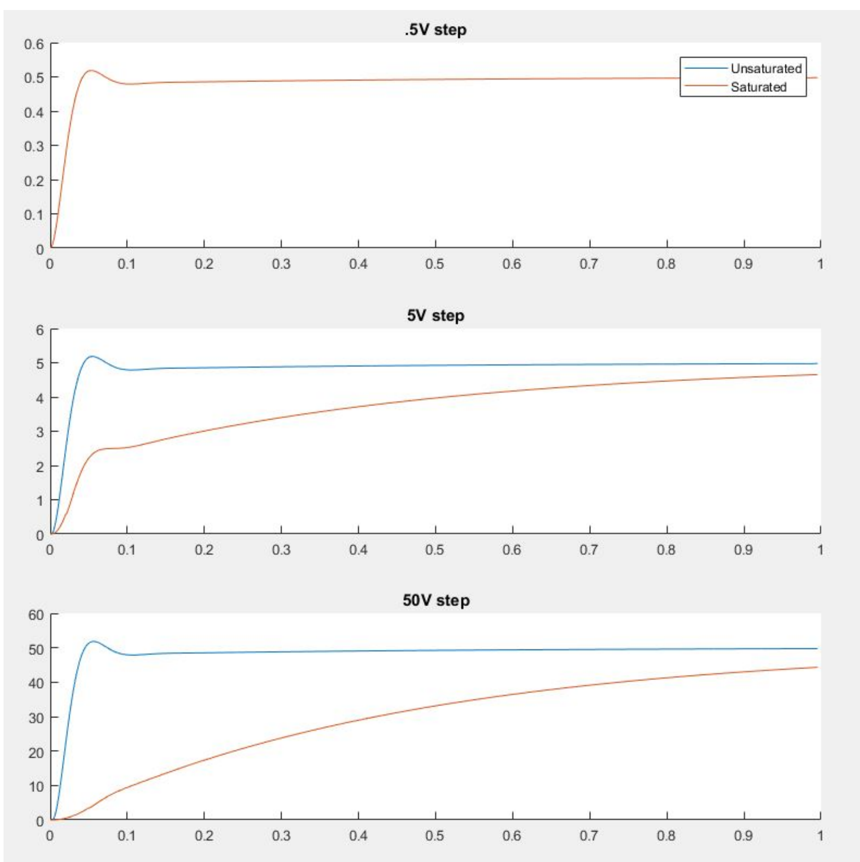

Figure 1: Step responses—with and without saturation block for $V_{\mathrm{ss}} = 0.5\mathrm{V}$, $V_{\mathrm{ss}} = 5\mathrm{V}$, and $V_{\mathrm{ss}} = 50\mathrm{V}$.

The purpose of the saturation block in simulation is to model the physical limits of the system that we use to control the motor in the lab. The practical limit of the control output in our lab system DAC is ±10V. This means that if the control function generates an output that is outside of these bounds, in the practical system it will be clipped to either of ±10V. This is an important distinction because the simulated system does not have this constraint while the real world system does. When we add the saturation block into the simulation diagram, we see different responses, especially in the cases where the steady-state value is higher. This is because higher steady-states require higher control output values, which may be too high for the DAC to generate. As a result, we can see that in the case of 0.5V, the saturation block has no effect on the output response, however in the other two graphs we can see an increasing effect on the response as the saturated one is severely dampened and takes longer to reach the steady-state value because the control effort cannot reach its intended value due to saturation.

2 Motor DC Gain ___/10

(Compute the transfer function of the system from the data set obtained from the DSA[system identification]. Then compute the DC gain K of the motor model. You need to include how you solve for K. Also compare the computed value with the value obtained in Lab 4 and Prelab 5-a. *Note: First, you need to rewrite the transfer function from the DSA in radians. Remember that the DC gain K obtained from the DSA includes the gains of the tachometer, amplifier and compensator. Hence you need to solve only for K.*)

$$K_{\text{DSA}} = 64.275$$
$$z[\text{Hz}] = \text{-1.23},\ p_1[\text{Hz}] = \text{-24.643},\ p_2[\text{Hz}] = \text{-0.785}$$

The transfer function is

$$\frac{V_{\text{tach}}(s)}{V_{\text{in}}(s)} = \frac{K_{\text{DSA}}\left(\frac{s[\text{rad}\,\text{s}^{-1}]}{2\pi} - z[\text{Hz}]\right)}{\left(\frac{s[\text{rad}\,\text{s}^{-1}]}{2\pi} - p[\text{Hz}]\right)\left(\frac{s[\text{rad}\,\text{s}^{-1}]}{2\pi} - p[\text{Hz}]\right)}$$
$$= \frac{64.275\,(0.1294s + 1)}{(0.0065s + 1)\,(0.2027s + 1)}$$

$$\boxed{K_{\text{lab4}} = 17.928,\ K_{\text{lab6}} = 18.66}$$

We solved for the value of K by substituting in the values from the DSA into the general form of the transfer function, converting the values to radians, and then rearranging the formula to match our previous one. By doing this, we were able to isolate the K from the system by dividing out by the various values of K_{tach}, K_{amp}, and K_c. After doing this, we obtained the transfer system shown above.

Based on the value that we obtained, we can see that he lead compensator is working as intended since the value of K that we found is very close to the original value from the motor that we found in an earlier lab. This means that the presence of the lead compensator is able to modify the phase of the overall transfer function without having a large effect on the magnitude. However, it still does change the value slightly and as we can see from the function, our new K is marginally higher than it was without the lead compensator.

3 Time Domain Specifications of Real/Simulated Responses of the Low/High DC Gain Compensators ___/30

3.1 Table of M_p, t_r, t_s, and e_{ss} ___/10

(Fill out Table 1 below.)

Table 1: Real and Simulated Responses for Low and High DC Gain Compensators

	Low DC Gain Lead Compensator				High DC Gain Lead Compensator			
	M_p [%]	t_r [ms]	t_s [ms]	e_{ss} [V][a]	M_p [%]	t_r [ms]	t_s [ms]	e_{ss} [V]
Actual[b]	3.7	30	80	0.11	na	610	610	0.07
Simulated Lab 4[c]	15	10	447	0	na	149	223	0
Simulated Lab 6[d]	4.22	27	35	0	na	153	226	0

[a] The unit is [V], not [%];
[b] Values calculated from data set obtained in Sections II and III of Lab 6;
[c] Values calculated from data set by simulating the closed loop system with both lead controllers and the parameters found in Lab 4;
[d] Values calculated from data set by simulating the closed loop system with both lead controllers and the parameters found in Lab 6 from the DSA.

3.2 Discussion of Results in Table 1 ___/10

(Compare the specs in Table 1 for low DC gain and high DC gain compensators and discuss the possible differences.)

As can be seen from the table, there are some significant differences between the results obtained from the various models of the motor and from the actual results. This is likely in part caused by different methods of creating the motor transfer function, and we can see that the DSA used in Lab 6 is generally more accurate than our Lab 4 method, likely because it swept across a whole range of frequency values in order to find the resulting output from the motor.

From this information, we can see a few things are present in the results that are only there because of physical constraints modeled in the simulation–namely saturation. For each of the high gain specs obtained, we can see that they are all overdamped (no M_p) because the control effort was saturated and could not meet the calculated specs as a result. When we ran the same simulations without the saturation block in place, we found that the resulting responses were much cleaner and closer to the expected ones. However, the physical system is bounded by the constraint of saturation and thus we need to compare it to the simulations running with the same physical model. In this case, the high gain lead controllers were producing a control effort so high that they were not performing in the way that was intended.

On the other hand, the low gain controllers did not have the issue of saturation, but instead were pretty close to the actual results found, especially using the DSA-calculated values of the motor transfer function. We can see slight differences in that the actual results are marginally dampened with slower rise times and a lower overshoot because of friction. Since we were looking to track a 1V step, it was not an issue for the low gain controller, however in the case of a larger step the same results may not be as easy to replicate.

3.3 Meeting the Specifications ___/10

(Fill out Table 2 below. Did your design meet the specification—$M_p \leqslant 15\%$, $t_r \leqslant 30$ms?)

Table 2: Overshoot and Rise Time for Lead Compensators and PD Controller

Experimental Values	M_p [%]	t_r [ms]
Low DC Gain Lead Compensator	3.7	30
High DC Gain Lead Compensator	na	610
PD Controller[e]	6.3	474

[e] Use the PD controller from Lab 5 with gains designed in Prelab 5-c.

We found in actuality that the controllers we designed would not meet the specs, either because of saturation in the case of the high gain lead controller, or because of friction increasing the rise time in the case of the low gain lead controller. If we were looking to further optimize these controllers, we could create a new one that had a gain in between these two, but not high enough to saturate the DAC as easily. This would help us overcome friction while also tracking the steady-state quickly.

In comparison with the PD controller from Lab 5, we can see that our new lead compensator designs are much better at tracking a step without unnecessary jerkiness and overshoot and an excessive rise time. Since we are no longer using the direct derivative values, our data free of noise and does not have unnecessary reactions to those derivative components. In sum, the lead controller is a better replacement for a direct observation of the derivative such as the one used in a PD controller.

4 Bode Plots ___/40

4.1 Bode Plots of $\dfrac{V_{\text{tach}}(s)}{V_{\text{in}}(s)}$ ___/10

(Plot the Bode plots (Magnitude and Phase) in dB and degrees respectively for the empirical data saved in `fresp.m`. And obtain Bode plots again using the transfer function estimated by the DSA (part II of this report). Overlay both plots. Use `semilogx` to scale the x-axis (frequency). **They should match well**.)

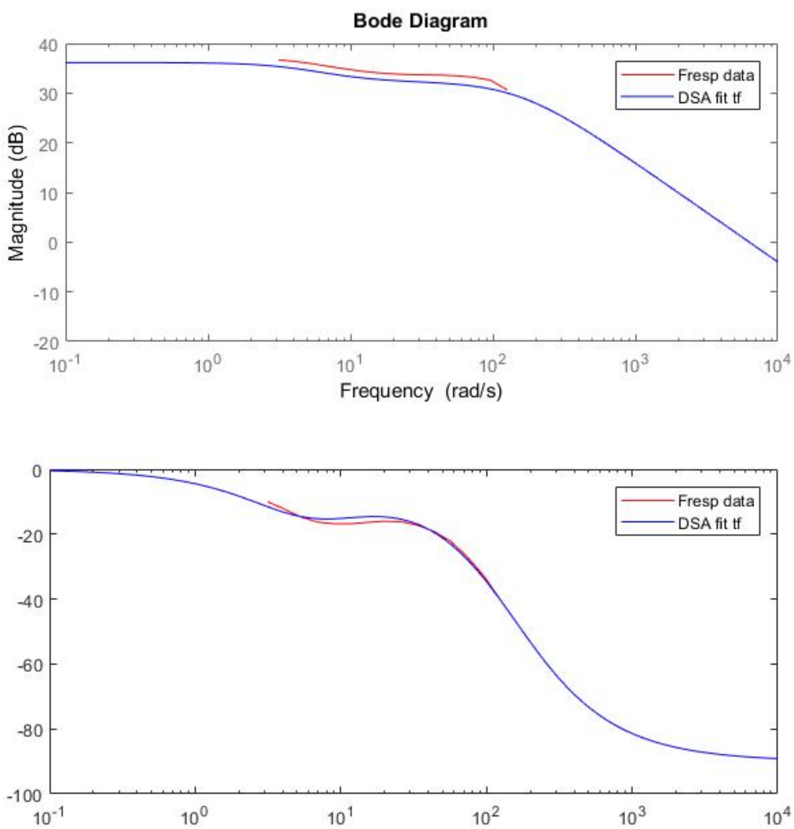

Figure 2: Overlaid Bode plots of open loop transfer function $V_{\text{tach}}(s)/V_{\text{in}}(s)$.

4.2 Bode Plots of $\frac{V_\theta(s)}{V_{\text{in}}(s)}$ ___/15

(Use the relation between Equations 6.2 and 6.3 (on page 57) of Lab Manual to graph the Bode plots for the transfer function $\frac{V_\theta(s)}{V_{\text{in}}(s)}$ (in dB and degrees) using the parameters identified by the DSA. Use `bode` command in MATLAB® to obtain magnitude and phase. Then overlay both plots. Follow hints in Lab Manual at the end of Lab 6 and overlay the *bode plot with the data* in `fresp.m` (after computing $\frac{V_\theta(s)}{V_{\text{in}}(s)}$). **They should match well**. Find the crossover frequency (ω_c) and phase margin (PM). Do they meet the design specifications from Prelab 6-d?)

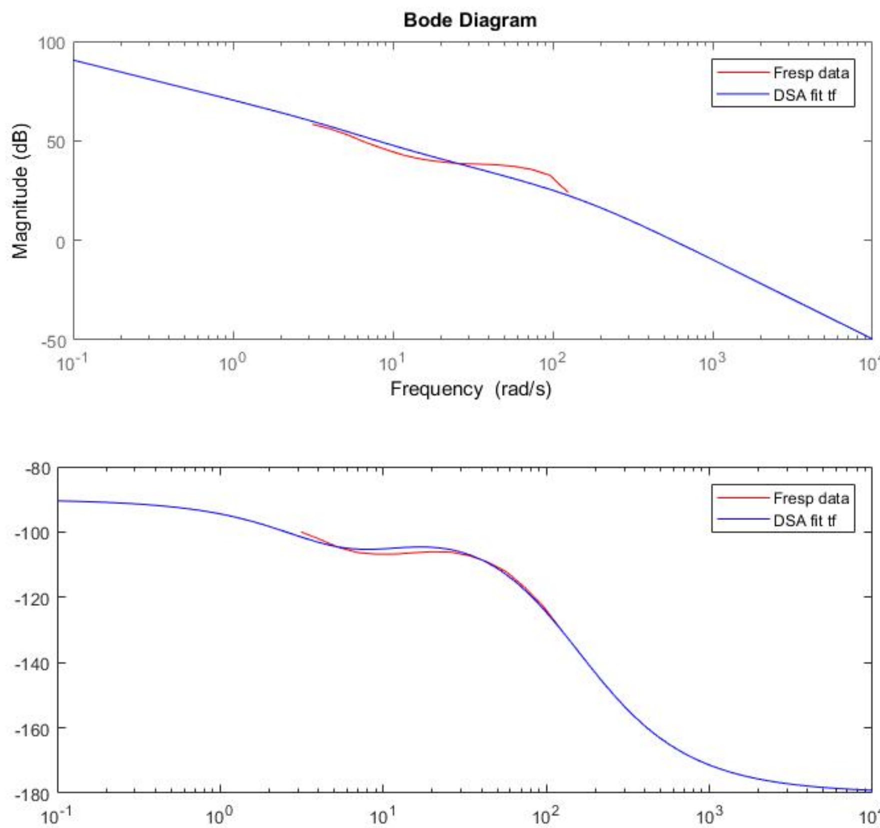

Figure 3: Overlaid Bode plots of open loop transfer function $V_\theta(s)/V_{\text{in}}(s)$.

Based on the bode plots that were able to get, from both the DSA-obtained data and the approximate transfer function, we can see that the resulting plots for the adjusted transfer function match reasonably well. We converted from the tachometer transfer function to the potentiometer one by carefully observing the differences between their individual transfer functions. We found that we needed to adjust for the presence of K_pot and remove the K_tach from our new function. Additionally, we had to include a new pole since the position of the motor, measured by the potentiometer, is the integral of its speed, measured by the tachometer. After doing this, we were able to create the bode plots shown above.

These show that the DSA-measured data and the transfer function parameters that were calculated were reasonably close, in both the case of the directly measured function as well as the calculated one. However, as shown below, the resulting function did not meet the specs that we were designing for based on the prelab. Our phase margin was almost a quarter of the needed value and the gain margin is not even present. This corroborates the earlier observation that we made, that the resulting step responses did not meet the specification criteria that we set out to match. Once again, the saturation of the motor prevented us from being able to accurately represent the control effort in the physical system and as a result we get a response that cannot meet the specs.

While the DSA is a useful tool for being able to measure and characterize the system in question, it also showed the result of over-saturation as an important factor to consider when designing lead controllers, where the control effort can be very large.

$$\boxed{\omega_c = 565\ \mathrm{rad\,s^{-1}},\ \mathrm{PM} = 15^\circ \text{ from Data Fit in DSA}}$$

$$\boxed{\omega_c = 560\ \mathrm{rad\,s^{-1}},\ \mathrm{PM} = 15^\circ \text{ from data in } \texttt{fresp.m}}$$

$$\boxed{\omega_c = 63.5\ \mathrm{rad\,s^{-1}},\ \mathrm{PM} = 51.9^\circ \text{ from specification in Prelab 6-d}}$$

4.3 Bode Plots of Closed-Loop $\frac{V_\theta(s)}{V_{\text{in}}(s)}$ ___/15

(Use the transfer function from the DSA fit (parameters identified by the DSA) and compute the closed-loop transfer function (assume negative unit feedback). Generate a Bode plot using MATLAB® . Also generate a bode plot for closed-loop transfer function $\frac{V_\theta(s)}{V_{\text{in}}(s)}$ *based on data set* `fresp.m`. Then overlay both Bode plots. **They should match well**. What is the closed-loop bandwidth? Is it reasonable? Give the reason why it is reasonable or discuss the possible reasons why it is not.)

As can be seen from the overlaid bode plots, our DSA-measured data matches the adjusted closed-loop transfer function to a reasonable degree. Both of these plots were generated by using the standard closed loop form of 'forward gain over one plus loop gain' to determine the frequency response along various points. However, it is also apparent that the frequency range that the DSA measured did not cover all of the break-points of the transfer function in question, meaning that additional testing would be required in order to fully reconstruct this transfer function purely from the `fresp.m` data.

From the plot of the generated transfer function, we can find the bandwidth, where the magnitude response reaches -3dB, to be at $\approx$ 882rad/sec. This is a reasonable value as it implies that for input frequencies up to that value, we would be able to have a suitable response from the system. However, if it was necessary to also respond to higher frequency input signals, then the system and compensators may need to be adjusted so that they function as needed.

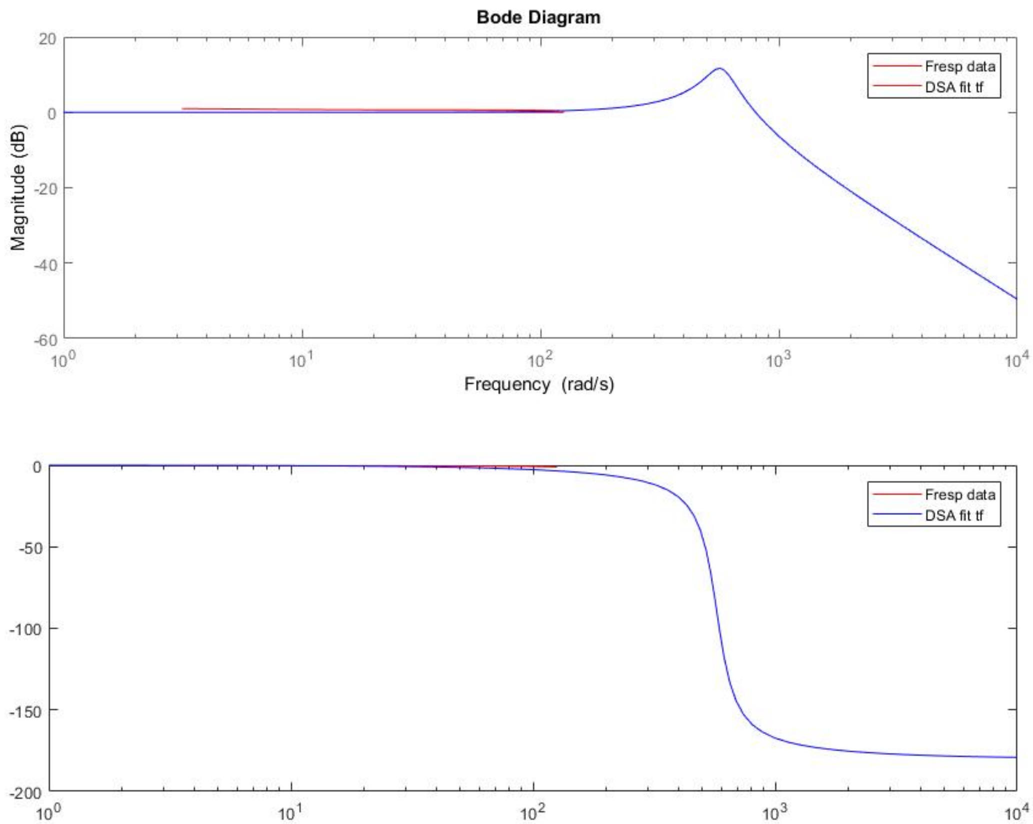

Figure 4: Overlaid Bode plots of closed-loop transfer function $V_\theta(s)/V_{\rm in}(s)$ with unit feedback.