Node and Mesh Analyses

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- It is not valid for ideal sources.
- Transform can be used for both dependent and independent sources.
- To obviate separating the branches consisting only of sources from other branches, it is useful to introduce the first two network transformations that allow us to relocate sources in the network without affecting the problem.

- A branch consisting of a current source alone is eliminated.

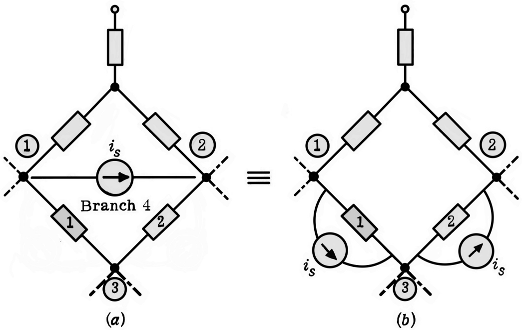

Figure 1: Source tranformation; A branch consisting of a current source alone is eliminated.

Exercise 1

- A branch consisting of a voltage source alone is eliminated.

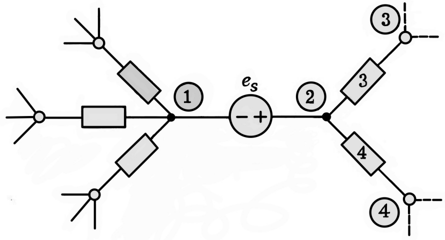

Figure 2: Source Transformation; A branch consisting of a voltage source alone is eliminated.

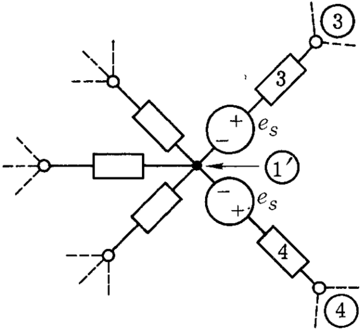

Figure 3: Circuit equivalent to Fig. 2

Exercise 2

Suppose that, in given figure, the non-source element is a linear time-invariant resistor of resistance R_k. Show that the branch equation is

$$v_k = v_{sk} - R_k j_{sk} + R_k j_k$$

Show that this branch can be further simplified to a resistive branch with an equivalent voltage source.

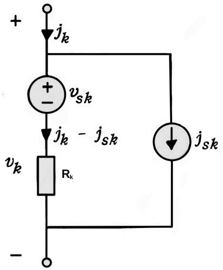

Figure 4: Branch k, including voltage and current sources.

Exercise 3

Suppose that, in given figure, the non-source element is a linear time-invariant resistor of conductance G_k. Show that the branch equation is

$$j_k = j_{sk} - G_k v_{sk} + G_k v_k$$

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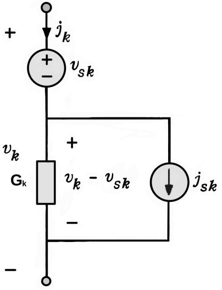

Figure 5: Branch k, including voltage and current sources.

Example

Apply source transformation

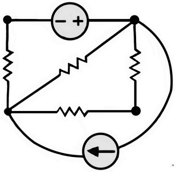

Figure 6: A network with voltage source and a current source.

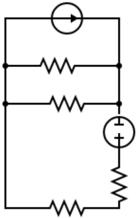

Figure 7: Equivalent network after source transformation

Node analysis

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- Let us consider any network $\mathscr{N}$ and let it have n_t nodes and b branches. Altogether there are b branch voltages and b branch currents to be determined.
- First, we pick arbitrarily a reference node. This reference node is usually called the **datum node**. We assign to the datum node the label (n_t) and to the remaining nodes the labels (1) , (2), ., (n), where $n \triangleq n_t - 1$.

Implications of KCL

- Consider the system of n linear algebraic equations that express KCL for all the nodes of $\mathscr{N}$ except the datum node. We assert that this system has the following matrix form:

$$\mathbf{Aj} = \mathbf{0} \quad \text{(KCL)} \tag{1}$$

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where $\mathbf{j}$ represents the branch current vector and is of dimension b;

that is, $\mathbf{j} = \begin{bmatrix} j_1 \\ j_2 \\ \vdots \\ j_b \end{bmatrix}$

and where $\mathbf{A} = (a_{ik})$ is an $n \times b$ matrix defined by

$$a_{ik} = \begin{cases} 1 \text{ if branch } k \text{ leaves node (i)} \\ -1 \text{ if branch } k \text{ enters node (i)} \\ 0 \text{ if branch } k \text{ is not incident with node (i)} \end{cases}$$

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- A is called the **reduced incidence matrix**.

Example

Consider the graph of Fig. 8, which contains four nodes and five branches ($n_t = 4, b = 5$). Let us number the nodes and branches, as shown in the figure, and indicate that node (4) is the datum node by the "ground" symbol used in the figure. The branch-current vector is

$$\mathbf{j} = \begin{bmatrix} j_1 \\ j_2 \\ j_3 \\ j_4 \\ j_5 \end{bmatrix}$$

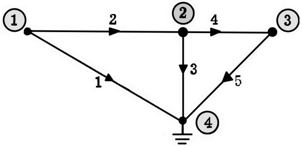

Figure 8:

The matrix **A** is obtained as

$$\begin{array}{rcl} & & \text{Node} \\ \mathbf{A} = & \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 1 & 0 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix} & \begin{array}{l} \leftarrow (1) \\ \leftarrow (2) \\ \leftarrow (3) \end{array} \\ & \begin{array}{ccccc} \uparrow & \uparrow & \uparrow & \uparrow & \uparrow \end{array} & \\ \text{Branch} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} & \end{array}$$

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\end{array}$$

Thus, Eq. (1) states that

$$\mathbf{Aj} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 1 & 0 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} j_1 \\ j_2 \\ j_3 \\ j_4 \\ j_5 \end{bmatrix} = \mathbf{0}$$

or

$$
\begin{aligned}
j_1 + j_2 &= 0 \\
-j_2 + j_3 + j_4 &= 0 \\
-j_4 + j_5 &= 0
\end{aligned}
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which are clearly the three node equations obtained by applying KCL to nodes (1), (2), and (3).

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- It is easy to see that the three equations are linearly independent, since each one contains a variable not contained in any of the other equations.
- In general , $Aj = 0$ is a set of n linearly independent equations.

Implications of KVL

- The branch voltages are obtained from the node voltages by the equation

$$\mathbf{v} = \mathbf{A}^T \mathbf{e} \tag{2}$$

where $\mathbf{A}^T$ is the $b \times n$ matrix which is the transpose of the **reduced incidence matrix A** defined in Eq. 1 and $\mathbf{v}$ is the vector of branch voltage, $v_1, v_2, \ldots, v_b$.
v_k is the k th branch voltage, $k = 1, 2, \ldots, b$.

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v_k is the k th branch voltage, $k = 1, 2, \ldots, b$.
- Let e be the vector whose components are e_1, e_2, ... , e_n. e_i is the node-to-datum voltage of node (i), $i = 1, 2, \ldots, n$. Thus if branch k connects the i th node to the datum node, we have

$$v_k = \begin{cases} e_i \text{ if branch } k\text{-leaves node (i)} \\ -e_i \text{ if branch } k \text{ enters node (i)} \end{cases}$$

Example

For Fig. 8

$$\mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{bmatrix} \quad \mathbf{e} = \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix}$$

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We have

$$\begin{array}{cc} & \text{Branch} \\ \mathbf{A}^T = \begin{bmatrix} 1 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} & \begin{array}{c} \leftarrow 1 \\ \leftarrow 2 \\ \leftarrow 3 \\ \leftarrow 4 \\ \leftarrow 5 \end{array} \\ \text{Node} \quad \begin{array}{ccc} \uparrow & \uparrow & \uparrow \\ 1 & 2 & 3 \end{array} & \end{array}$$

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\begin{aligned}
v_1 &= e_1 \\
v_2 &= e_1 - e_2 \\
v_3 &= e_2 \\
v_4 &= e_2 - e_3 \\
v_5 &= e_3
\end{aligned}
$$

These five scalar equations are easily recognized as expressions of the KVL.

Tellegen's theorem revisited

- From Eq. 1 and 2, we know $\mathbf{Aj} = \mathbf{0}$ and $\mathbf{v} = \mathbf{A}^T\mathbf{e}$ respectively.

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$$
\begin{aligned}
\sum_{k=1}^{b} v_k j_k &= \mathbf{v}^T\mathbf{j} \\
&= \left(\mathbf{A}^T\mathbf{e}\right)^T\mathbf{j} \\
&= \mathbf{e}^T\left(\mathbf{A}^T\right)^T\mathbf{j} \\
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- Hence, by Eq. 1,

$$\mathbf{v}^T\mathbf{j} = 0 \tag{3}$$

$$\sum_{k=1}^{b} v_k j_k = 0$$

Analysis of resistive networks

Consider a linear time-invariant resistive network with b branches, n_t nodes, and one separate part.

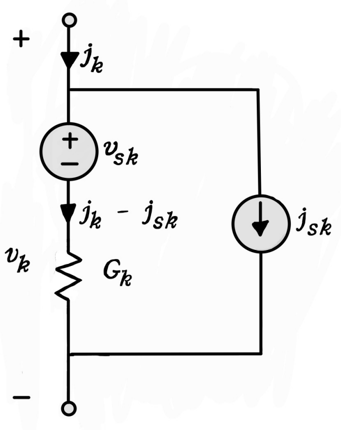

Figure 9: The k th branch.

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In matrix notation, we have from the above equation

$$\mathbf{j} = \mathbf{G}\mathbf{v} + \mathbf{j}_s - \mathbf{G}\mathbf{v}_s \tag{4}$$

where **G** is called the **branch conductance matrix**. It is a diagonal matrix of order b; that is,

$$\mathbf{G} = \begin{bmatrix} G_1 & 0 & \cdots\cdots\cdots & & \\ 0 & G_2 & & & \vdots \\ \vdots & \ddots & & \vdots & \\ \vdots & & \ddots & & \vdots \\ \vdots & & \ddots & \vdots & \\ \vdots & & & \ddots & 0 \\ 0 & \cdots\cdots\cdots & \ddots & 0 & G_b \end{bmatrix}$$

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The vectors $\mathbf{j}_s$ and $\mathbf{v}_s$ are source vectors of dimension b; that is,

$$\mathbf{j}_s = \begin{bmatrix} j_{s1} \\ j_{s2} \\ \vdots \\ j_{sb} \end{bmatrix} \quad \dot{\mathbf{v}}_s = \begin{bmatrix} v_{s1} \\ v_{s2} \\ \vdots \\ v_{sb} \end{bmatrix}$$

Premultiplying Eq. 4 by the matrix A, substituting v by $\mathbf{A}^T\mathbf{e}$, and using Eq. 1, we obtain

$$\mathbf{AGA}^T\mathbf{e} + \mathbf{Aj}_s - \mathbf{AGv}_s = \mathbf{0} \tag{5}$$

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In Eq. 6 $\mathbf{AGA}^T$ is an $n \times n$ square matrix, whereas $\mathbf{AGv}_s$ and $-\mathbf{Aj}_s$ are n-dimensional vectors. Let us introduce the following notations:

$$\mathbf{Y}_n \triangleq \mathbf{AGA}^T \tag{7}$$

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The set of equations (9) is usually called the node equations; $\mathbf{Y}_n$ is called the **node admittance matrix**, $\mathbf{i}_s$ is the node current source vector.

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$$\begin{bmatrix} y_{11} & y_{12} & \cdots & y_{1n} \\ y_{21} & y_{22} & \cdots & y_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ y_{n1} & y_{n2} & \cdots & y_{nn} \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_n \end{bmatrix} = \begin{bmatrix} i_{s1} \\ i_{s2} \\ \vdots \\ i_{sn} \end{bmatrix}$$

The following statements are easily verified in simple examples, and can be proved for networks without coupling elements.

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- Y_{ik} is the negative of the sum of the conductances of all branches connecting node (i) and node (k); Y_{ik} is called the **mutual admittance** between node (i) and node (k).
- If we convert all voltage sources into current sources, then i_{sk} is the algebraic sum of all source currents entering node (k): the current sources whose reference direction enter node (k) are assigned a positive sign, all others are assigned a negative sign.

- Consider now a linear time-invariant network containing resistors, inductors, capacitors, and independent sources.

Sinusoidal steady-state analysis

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- We shall use voltage phasors, current phasors, impedances, and admittances to write the branch equations and Kirchhoff 's laws.
- Thus,consider a typical branch contains an admittance, say Y_k (in the k_{th} branch), which is one of the forms G_k, $j\omega C_k$, or $\frac{1}{j\omega L_k}$, depending on whether the k_{th} branch is a resistor, capacitor, or inductor, respectively.

- The branch equation is

$$J_k = Y_k V_k + J_{sk} - Y_k V_{sk} \quad k = 1, 2, \dots, b \tag{10}$$

where J_k and V_k are the k_{th} branch current phasor.and voltage phasor, and J_{sk} and V_{sk} are the k_{th} branch phasors representing the current and voltage sources of branch k.

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- The matrix form of above equation is

$$\mathbf{J} = \mathbf{Y}_b \mathbf{V} + \mathbf{J}_s - \mathbf{Y}_b \mathbf{V}_s \tag{11}$$

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- The matrix $\mathbf{Y}_b$ is called the **branch admittance matrix**, and the vectors $\mathbf{J}$ and $\mathbf{V}$ are, respectively, **the branch-current phasor vector** and the **branch voltage phasor vector**.

- The analysis is exactly the same as that of the resistive network in the preceding section. The node equation is of the form

$$\mathbf{Y}_n \mathbf{E} = \mathbf{I}_s \tag{12}$$

where the phasor $\mathbf{E}$ represents the node-to-datum voltage vector, the phasor $\mathbf{I}_s$ represents the current-source vector, and $\mathbf{Y}_n$ is the node admittance matrix.

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- In terms of $\mathbf{A}$ and $\mathbf{Y}_b, \mathbf{Y}_n$ is given by

$$\mathbf{Y}_n = \mathbf{A}\mathbf{Y}_b\mathbf{A}^T \tag{13}$$

- The analysis is exactly the same as that of the resistive network in the preceding section. The node equation is of the form

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- Write the branch equations [from Eq. 11]

$$\mathbf{J} = \mathbf{Y}_b(j\omega)\mathbf{V} - \mathbf{Y}_b(j\omega)\mathbf{V}_s + \mathbf{J}_s$$

  where $\mathbf{Y}_b(j\omega)$ is the branch admittance matrix. Note that $\mathbf{Y}_b$ is evaluated at $j\omega$, where ω represents the angular frequency of the sinusoidal sources.

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Properties of the node admittance matrix $\mathbf{Y}_n(j\omega)$

From the basic equation

$$\mathbf{Y}_n(j\omega) = \mathbf{A}\mathbf{Y}_b(j\omega)\mathbf{A}^T \tag{15}$$

We obtain the following useful properties:

1. Whenever there are no coupling elements (i.e., neither mutual inductances nor dependent sources), the $b \times b$ matrix $\mathbf{Y}_b(j\omega)$ is diagonal, and consequently the $n \times n$ matrix $\mathbf{Y}_n(j\omega)$ is symmetric.

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2. Whenever there are no dependent sources and no gyrators (mutual inductances are allowed), both $\mathbf{Y}_b(j\omega)$ and $\mathbf{Y}_n(j\omega)$ are symmetric.

Duality

Planar Graphs, Meshes, Outer Meshes

- Graph is a basic skeleton of a network in which each element is represented by a line segment between 2 nodes and we are not concerned about nature of element.

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- A graph is said to be planar if it can be drawn on the plane in such a way that no two branches intersect at a point which is not a node.
- Graphs can be classified as hinged or unhinged.
- A hinged graph typically refers to a graph where certain edges (hinges) can rotate or move relative to each other.

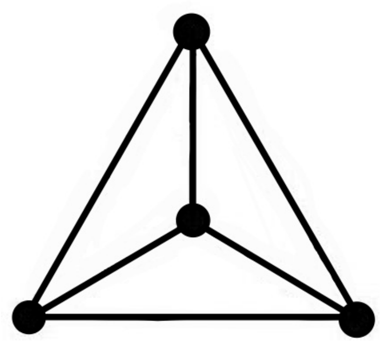
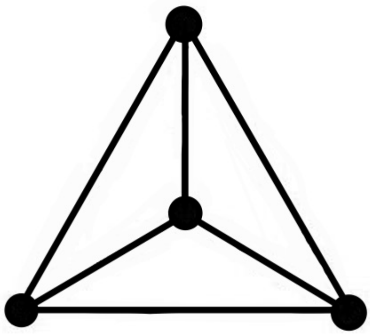


Figure 10: Planar graph



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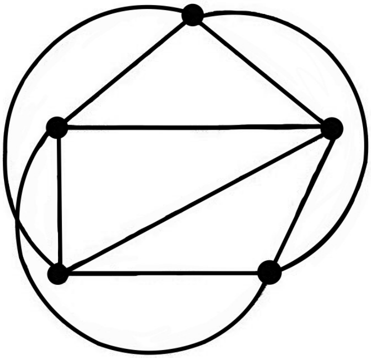

Figure 11: Nonplanar graph

The matrix $\mathbf{M}_a$

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- A fundamental property of a connected unhinged planar graph is that each branch of the graph belongs to exactly two meshes if we include the outer mesh.
- An oriented planar graph $\mathcal{G}$ that is connected and unhinged can be described analytically by a matrix $\mathbf{M}_a$.
- Let $\mathcal{S}$ have b branches and $l+1$ meshes (including the outer mesh); then $\mathbf{M}_a$ is defined as the rectangular matrix of $l+1$ rows and b columns whose (i,k) th element m_{ik} is defined by

$$m_{ik} = \begin{cases} 1 & \text{if branch } k \text{ is in mesh } i \text{ and if their reference directions coincide} \\ -1 & \text{if branch } k \text{ is in mesh } i \text{ and if their reference directions } \textit{do not} \text{ coincide} \\ 0 \text{ if branch } k \text{ does not belong to mesh } i \end{cases}$$

Dual graphs

A planar topological graph $\widehat{\mathscr{G}}$ is said to be a dual graph of a topological graph $\mathcal{G}$ if

- There is a one-to-one correspondence between the meshes of $\mathcal{G}$ (including the outer mesh) and the nodes of $\widehat{\mathcal{G}}$.

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- There is a one-to-one correspondence between the branches of each graph in such a way that whenever two meshes of one graph have the corresponding branch in common, the corresponding nodes of the other graph have the corresponding branch connecting these nodes.

ALGORITHM

Given a connected, planar, unhinged topological graph $\mathcal{G}$, we construct a dual graph $\widehat{\mathcal{G}}$ by proceeding as follows:

- To each mesh of $\mathcal{G}$, including the outer mesh, we associate a node of $\widehat{\mathcal{G}}$; thus, we associate node (1) to mesh 1 and draw node (1) inside mesh 1 ; a similar procedure is followed for nodes (2) , (3) , . . , including node $l + 1$, which corresponds to the outer mesh.

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- For each branch, say k, of $\mathcal{G}$ which is common to mesh i and mesh j, we associate a branch k of $\widehat{\mathscr{G}}$ which is connected to nodes ($\hat{i}$ and ($\hat{j}$. By its very construction, the resulting graph $\widehat{\mathscr{G}}$ is a dual of $\mathcal{G}$.

Dual networks

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- In this discussion we restrict ourselves to networks having the following properties: their graphs are connected, planar, and unhinged; and all their elements are one-port elements.
- It is fundamental to observe that the elements do not have to be linear and/or time-invariant.
- We say that a network $\widehat{\mathscr{R}}$ is the dual of the network $\mathscr{R}$ if the topological graph $\widehat{\mathscr{G}}$ of $\widehat{\mathscr{R}}$ is a dual of the topological graph $\mathcal{G}$ of $\mathscr{R}$.

Types of properties	$\widehat{\mathscr{R}}$	$\mathscr{R}$
Graph-theoretic properties	Node Cut set Datum node Tree branch Fundamental cut set Branches in series Reduced incidence matrix Fundamental cut-set matrix	Mesh Loop Outer mesh Link Fundamental loop Branches in parallel Mesh matrix Fundamental loop matrix
Graph-theoretic and electric properties	Node-to-datum voltages Tree-branch voltages KCL	Mesh currents Link currents KVL
Electric properties	Voltage Resistor Inductor Resistance Inductance	Current Resistor Capacitor Conductance Capacitance

Mesh analysis

- Consider any network which has n_t nodes and b branches; consequently it has $l = b - n_t + 1$ meshes, not counting the outer mesh.

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- We shall employ the concept of duality to develop the two basic facts of mesh analysis. Again it should be emphasized that the two facts are independent of the nature of the network elements.

Implications of KVL

- The l linear homogeneous algebraic equations in $v_1, v_2, \ldots, v_l$ obtained by applying KVL to each mesh (except the outer mesh) constitute a set of l linearly independent equations.

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- Analytically KVL may be expressed by the use of the mesh matrix

$$\mathbf{Mv} = \mathbf{0} \quad (\mathrm{KVL}) \tag{16}$$

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$$\mathbf{M}\mathbf{v} = \mathbf{0} \quad \text{(KVL)} \tag{16}$$

where $\mathbf{M} = (m_{ik})$ is an $l \times b$ matrix defined below by Eq. 18

- When we write that the i th component of $\mathbf{Mv}$ is zero, we merely assert that the sum of all branch voltages around the i th mesh is zero.

- Since this i th component is of the form

$$\sum_{k=1}^{b} m_{ik} v_k = 0 \tag{17}$$

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- We must have for $i = 1, 2, \ldots, l$ and $k = 1, 2, \ldots, b$,

$$m_{ik} = \begin{cases} 1 & \text{if branch } k \text{ is in mesh } i \text{ and if their reference directions coincide} \\ -1 & \text{if branch } k \text{ is in mesh } i \text{ and if their reference directions } \textit{do} \text{ not coincide} \\ 0 & \text{if branch } k \text{ does not belong to mesh } i \end{cases} \tag{18}$$

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- Let us call $i_1, i_2, \ldots, i_l$ the mesh currents. For convenience let us assign to each one a clockwise reference direction.

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- The branch currents can be calculated in terms of the mesh currents by the equation

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- Above equation means that every branch current can be expressed as a linear combination of mesh currents, and that the matrix which specifies these linear combinations is the transpose of the mesh matrix defined previously.

Summary

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- $\mathbf{j} = \mathbf{M}^T\mathbf{i}$ (KCL) Eq. 19 expresses KCL and relates the b branch currents $j_1, j_2, \ldots, j_b$ to the l mesh currents $i_1, i_2, \ldots, i_l$.

Sinusoidal steady-state analysis

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- Call $\mathbf{J}_s$ and $\mathbf{V}_s$ the b-vectors whose k th components are the phasors representing the sinusoidal sources in the k th branch. Similarly, $\mathbf{V}$ and $\mathbf{J}$ are the b-vectors whose k th components are the phasors representing the branch voltage v_k and the branch current j_k.

Mesh analysis of LTI networks

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$$\mathbf{V} = \mathbf{Z}_b(j\omega)\mathbf{J} - \mathbf{Z}_b(j\omega)\mathbf{J}_s + \mathbf{V}_s \tag{22}$$

The $b \times b$ matrix $\mathbf{Z}_b(j\omega)$ is called the **branch-impedance matrix**.

Sinusoidal Steady-state Analysis

The substitution gives

$$\left(\mathbf{M}\mathbf{Z}_b(j\omega)\mathbf{M}^T\right)\mathbf{I} = \mathbf{M}\mathbf{Z}_b(j\omega)\mathbf{J}_s - \mathbf{M}\mathbf{V}_s \tag{23}$$

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or

$$\mathbf{Z}_m(j\omega)\mathbf{I} = \mathbf{E}_s \tag{24}$$

where $\mathbf{Z}_m(j\omega)$ is an $l \times l$ matrix called the m**esh impedance matrix**, given by

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and $\mathbf{E}_s$, the mesh voltage source vector, is the l vector given by

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Equations (24) are called the mesh equations of network ; they constitute a system of l linear algebraic equations (with complex coefficients) in l unknowns, the phasors representing the mesh currents $I_1, I_2, \ldots, I_l$.

Properties of the mesh impedance matrix

- If the network π has no coupling elements, $\mathbf{Z}_b(j\omega)$ is diagonal and $\mathbf{Z}_m(j\omega)$ is symmetric; that is, $\mathbf{Z}_m(j\omega) = \mathbf{Z}_m{}^T(j\omega)$.

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    2. Call z_{ik} the (i, k) th element of $\mathbf{Z}_m$. z_{ik} is the negative of the sum of all the impedances of the branches which are in common with meshes i and k and is called the mutual impedance between mesh i and mesh k.

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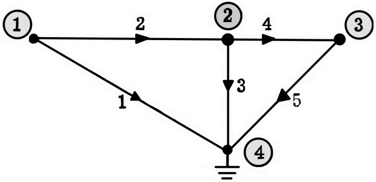
- In the case of resistive networks, if all resistances are positive, then det $(\mathbf{Z}_m) > 0$. The mesh equations (24) have a unique solution.
- In case network has coupling elements, the only general conclusions are that $\mathbf{Z}_b(j\omega)$ is no longer diagonal and $\mathbf{Z}_m(j\omega)$ is usually no longer symmetric.

Practice

Assume that all elements of the graph of below figure have conductance of 5 mhos and that a current source squirts $1\mathrm{amp}$ into node (1) and sucks it out of node (2). Write the node equations by inspection.



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