Loop and Cut Set

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Tree

- Let $\mathcal{G}$ be a connected graph and T a subgraph of $\mathcal{G}$. We say that T is a **tree** of the connected graph $\mathcal{G}$ if
  1. T is a connected subgraph.
  2. It contains all the nodes of $\mathcal{G}$.
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  3. It contains no loops.
- Given a connected graph $\mathcal{G}$ and a tree T, the branches of T are called **tree branches**, and the branches of $\mathcal{G}$ not in T are called **links**.

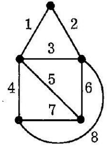
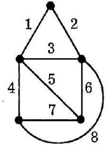


Figure 1: Planar graph G

Examples of trees of graph G .



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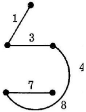
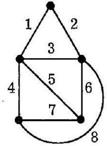


Figure 2: Tree T_1



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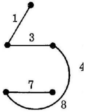

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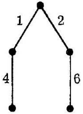
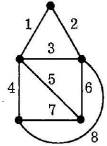


Figure 3: Tree T_2



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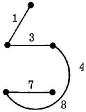

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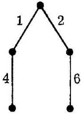


Figure 3: Tree T_2

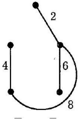
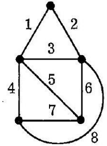


Figure 4: Tree T_3



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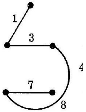

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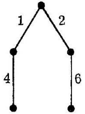

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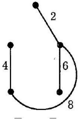

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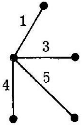

Figure 5: Tree T_4

Exercise

Draw all possible trees for the graph shown.

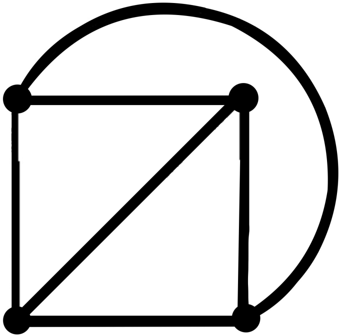

Figure 6: A connected graph with four nodes and six branches.

Fundamental theorem of graph theory

Given a connected graph $\mathcal{G}$ of n_t nodes and b branches, and a tree T of $\mathcal{G}$. There is a unique path along the tree between any pair of nodes.

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3. Every tree branch of T together with some links defines a unique cut set of $\mathcal{G}$. This cut set is called a **fundamental cut set** associated with the tree branch.

Examples of subgraphs of $\mathcal{G}$ in Fig.1 which are not trees.

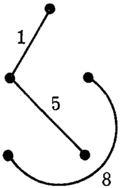

Figure 7: Subgraph 1

Violates property 1

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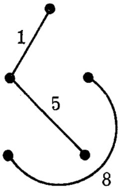

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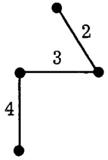

Figure 8: Subgraph 2

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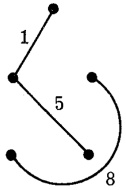

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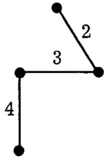

Figure 8: Subgraph 2

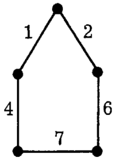

Figure 9: Subgraph 3

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- Suppose that $\mathcal{G}$ has n_t nodes, b branches, and s separate parts. Let $T_1,\ T_2, \ldots, T_s$ be trees of each separate part, respectively. The set $\{T_1,\ T_2, \ldots, T_s\}$ is called a forest of $\mathcal{G}$. Then the forest has $n_t - s$ branches, G has $b - n_t + s$ links, and the remaining statements of the theorem are true.

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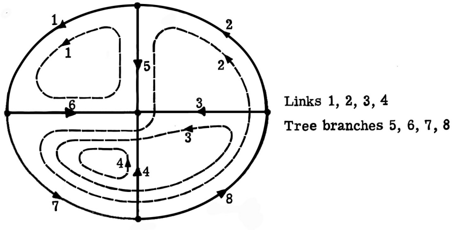

Figure 10: Fundamental loops for the chosen tree of a graph.

- The KVL equations can be written for the four fundamental loops in terms of the branch voltage from the above figure as follows:
  Loop 1: $v_1 - v_5 + v_6 = 0$
  Loop 2: $v_2 + v_5 - v_6 + v_7 + v_8 = 0$
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  Loop 4: $v_4 - v_6 + v_7 = 0$
- In matrix form, the equation gives

$$l\text{ loops}\left\{\left[\underbrace{\begin{matrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{matrix}}_{l\text{ links}}\quad\underbrace{\begin{matrix}-1 & 1 & 0 & 0\\ 1 & -1 & 1 & 1\\ 0 & -1 & 1 & 1\\ 0 & -1 & 1 & 0\end{matrix}}_{n\text{ tree branches}}\right]\right.\begin{bmatrix}v_1\\ v_2\\ v_3\\ v_4\\ v_5\\ v_6\\ v_7\\ v_8\end{bmatrix}=\begin{bmatrix}0\\ 0\\ 0\\ 0\end{bmatrix}$$

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- Furthermore, its (i, k) th element is defined as follows:

$$b_{ik} = \begin{cases} 1 & \text{if branch } k \text{ is in loop } i \text{ and their reference directions agree} \\ -1 & \text{if branch } k \text{ is in loop } i \text{ and their reference directions do not agree} \\ 0 & \text{if branch } k \text{ is not in loop } i \end{cases}$$

- Each fundamental loop includes one link only and since the orientations of the loop and the link are picked to be the same, it is clear that if we number the links $1, 2, \ldots, l$ and the tree branches $l+1, l+2, \ldots, b$, the matrix $\mathbf{B}$ has the form

$$\mathbf{B} = \left.\left[\begin{array}{c|c} \underbrace{\mathbf{1}_l}_{l \text{ links}} & \underbrace{\mathbf{F}}_{\substack{n \text{ tree} \\ \text{branches}}} \end{array}\right]\right\} l \text{ loops} \tag{2}$$

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- We have established the fact that the l fundamental loop equations written in terms of the branch voltages constitute a set of l linearly independent equations.

The second basic fact of loop analysis is as follows :

- Let $i_1, i_2, \ldots, i_l$ the currents in the l links of the tree T. Each tree branch current is the superposition of one or more loop currents. So,

$$\mathbf{j} = \mathbf{B^T i} \tag{3}$$

where $\mathbf{B}^T$ is the transpose of the fundamental loop matrix.

Example

Let us consider our example of Fig. 10. We can write the following equations :

$$
\begin{aligned}
j_1 &= i_1 \\
j_2 &= i_2 \\
j_3 &= i_3 \\
j_4 &= i_4 \\
j_5 &= -i_1 + i_2 \\
j_6 &= i_1 - i_2 - i_3 - i_4 \\
j_7 &= i_2 + i_3 + i_4 \\
j_8 &= i_2 + i_3
\end{aligned}
$$

In matrix form the equation is

$$\begin{bmatrix} j_1 \\ j_2 \\ j_3 \\ j_4 \\ j_5 \\ j_6 \\ j_7 \\ j_8 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 1 & 0 & 0 \\ 1 & -1 & -1 & -1 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix}$$

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- The branch equations are written in matrix form as follows:

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where

$$\mathbf{Z}_l \triangleq \mathbf{B}\mathbf{R}\mathbf{B}^T \tag{7}$$

$$\mathbf{e}_s \triangleq -\mathbf{B}\mathbf{v}_s + \mathbf{B}\mathbf{R}\mathbf{j}_s \tag{8}$$

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- The loop impedance matrix has properties similar to those of the mesh impedance matrix discussed in the previous lecture.
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- Let us rewrite Eq. 6 as follows:

$$\begin{bmatrix} z_{11} & z_{12} & \dots & z_{1l} \\ z_{21} & z_{22} & \dots & z_{2l} \\ \dots\dots & \dots & \dots & . \\ z_{l1} & z_{l2} & \dots & z_{ll} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ \vdots \\ i_l \end{bmatrix} = \begin{bmatrix} e_{s1} \\ e_{s2} \\ \vdots \\ e_{sl} \end{bmatrix}$$

Example

Let us consider the network of below . The graph of the network is that of Fig. 10.

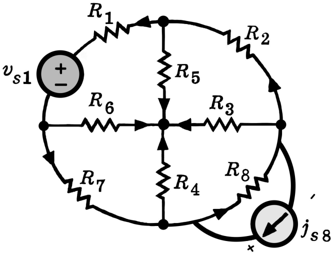

Figure 11: Example of loop analysis.

The branch equation is

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \\ v_8 \end{bmatrix} = \begin{bmatrix} R_1 & & & & & & & \\ & R_2 & & & 0 & & & \\ & & R_3 & & & & & \\ & & & R_4 & & & & \\ & & & & R_5 & & & \\ & 0 & & & & R_6 & & \\ & & & & & & R_7 & \\ & & & & & & & R_8 \end{bmatrix} \begin{bmatrix} j_1 \\ j_2 \\ j_3 \\ j_4 \\ j_5 \\ j_6 \\ j_7 \\ j_8 \end{bmatrix} + \begin{bmatrix} v_{s1} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ R_8 j_{s8} \end{bmatrix}$$

Using Eq. 7, we can obtain the loop impedance matrix

$$\mathbf{Z}_l = \mathbf{BRB}^T$$

$$= \begin{bmatrix} R_1 + R_5 + R_6 & -R_5 - R_6 & -R_6 & -R_6 \\ -R_5 - R_6 & R_2 + R_5 + R_6 + R_7 + R_8 & R_6 + R_7 + R_8 & R_6 + R_7 \\ -R_6 & R_6 + R_7 + R_8 & R_3 + R_6 + R_7 + R_8 & R_6 + R_7 \\ -R_6 & R_6 + R_7 & R_6 + R_7 & R_4 + R_6 + R_7 \end{bmatrix}$$

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$$\begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} = \begin{bmatrix} -v_{s1} \\ -R_8 j_{s8} \\ -R_8 j_{s8} \\ 0 \end{bmatrix}$$

Properties of the Loop impedance matrix

The following properties of the loop impedance matrix $\mathbf{Z}_l(j\omega)$ follow from the relation

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1. If the network has no coupling elements, the branch impedance matrix $\mathbf{Z}_b(j\omega)$ is diagonal, and the loop impedance matrix $\mathbf{Z}_l(j\omega)$ is symmetric.
2. Also, if the network has no coupling elements, the loop impedance matrix $\mathbf{Z}_l(j\omega)$ can be written by inspection.
    - The i th diagonal element of $\mathbf{Z}_l(j\omega), z_{ii}$, is equal to the sum of the impedances in loop i; z_{ii} is called the self-impedance of loop i.

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2. Also, if the network has no coupling elements, the loop impedance matrix $\mathbf{Z}_l(j\omega)$ can be written by inspection.
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   - The (i, k) element of $\mathbf{Z}_l(j\omega), z_{ik}$, is equal to plus or minus the sum of the impedances of the branches common to loop i and to loop k; the plus sign applies if, in the branches common to loop i and loop k, the loop reference directions agree, and the minus sign applies when they are opposite.

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Cut set analysis

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- That cut set is made up of links and of one tree branch, namely the tree branch which defines the cut set.

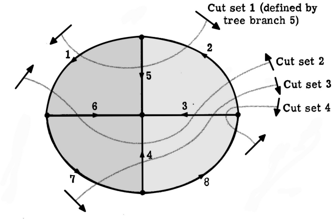

Figure 12: Fundamental cut sets for the chosen tree of a given graph.

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  Cut set 2: $-j_1 + j_2 + j_3 + j_4 + j_6 = 0$

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- In matrix form, the equation is

$$\begin{bmatrix} 1 & -1 & 0 & 0 & 1 & 0 & 0 & 0 \\ -1 & 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & -1 & -1 & -1 & 0 & 0 & 1 & 0 \\ 0 & -1 & -1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} j_1 \\ j_2 \\ j_3 \\ j_4 \\ j_5 \\ j_6 \\ j_7 \\ j_8 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

The first basic fact of cut-set analysis is as follows:

- In general, if we apply KCL to each one of the n fundamental cut sets, we obtain a system of n linear homogeneous equations in b unknowns $j_1, j_2, \cdots, j_b$.

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where $\mathbf{Q}$ is an $n \times b$ matrix defined by

$$q_{ik} = \begin{cases} 1 & \text{if branch } k \text{ belongs to cut set } (i) \text{ and has the } \textit{same} \text{ reference direction} \\ -1 & \text{if branch } k \text{ belongs to cut set } (i) \text{ and has the } \textit{opposite} \text{ reference direction} \\ 0 & \text{if branch } k \text{ } \textit{does not belong} \text{ to cut set } (i) \end{cases} \tag{11}$$

- $\mathbf{Q} = [q_{ik}]$ is called the fundamental cut-set matrix. As before we note that it is of the form

$$\mathbf{Q} = \left.\left[\begin{array}{c|c} \underbrace{\mathbf{E}}_{l \text{ links}} & \underbrace{\mathbf{I_n}}_{\substack{n \text{ tree} \\ \text{branches}}} \end{array}\right]\right\} n \text{ cut sets} \tag{12}$$

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- Hence, the n fundamental cut-set equations in terms of the branch currents are linearly independent.

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v_1 &= v_5 - v_6 = e_1 - e_2 \\
v_2 &= -v_5 + v_6 - v_7 - v_8 = -e_1 + e_2 - e_3 - e_4 \\
v_3 &= v_6 - v_7 - v_8 = e_2 - e_3 - e_4 \\
v_4 &= v_6 - v_7 = e_2 - e_3 \\
v_5 &= e_1 \\
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v_5 &= e_1 \\
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- By following the reasoning dual to that of the loop analysis,

$$\mathbf{v} = \mathbf{Q}^T \mathbf{e} \tag{13}$$

Cut-set analysis for LTI networks

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- The branch equations are written in matrix form as follows:

$$\mathbf{j} = \mathbf{G}\mathbf{v} + \mathbf{j}_s - \mathbf{G}\mathbf{v}\mathbf{v}_s \tag{14}$$

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- Combining Eqs. 10, 13, and 14, we obtain

$$\mathbf{Q}\mathbf{G}\mathbf{Q}^T\mathbf{e} = \mathbf{Q}\mathbf{G}\mathbf{v}_s - \mathbf{Q}\mathbf{j}_s \tag{15}$$

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- In scalar form, the cut-set equations are

$$\begin{bmatrix} y_{11} & y_{12} & \cdots & y_{1n} \\ y_{21} & y_{22} & \cdots & y_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ y_{n1} & y_{n2} & \cdots & y_{nn} \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_n \end{bmatrix} = \begin{bmatrix} i_{s1} \\ i_{s2} \\ \vdots \\ i_{sn} \end{bmatrix}$$

Properties of the cut-set admittance matrix

The cut-set admittance matrix $\mathbf{Y}_q$ has a number of properties based on the equation

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1. If all the voltage sources are transformed to current sources, then i_{sk} is the algebraic sum of all current sources in cut set k : the current sources whose reference direction is opposite to that of the k th cut set reference direction are assigned a positive sign, all others are assigned a negative sign.

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2. If the network is resistive and if all branch resistances are positive, then $\det(\mathbf{Y}_q) > 0$.

Example

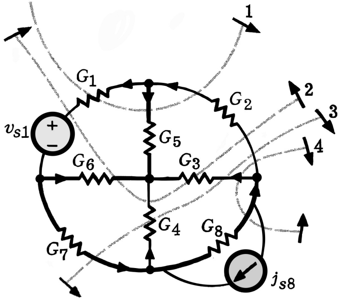

Figure 13: Example of cut-set analysis.

For the above network, the cut-set equations are

$$\begin{bmatrix} G_1+G_2+G_5 & -G_1-G_2 & G_2 & G_2 \\ -G_1-G_2 & G_1+G_2+G_3+G_4+G_6 & -G_2-G_3-G_4 & -G_2-G_3 \\ G_2 & -G_2-G_3-G_4 & G_2+G_3+G_4+G_7 & G_2+G_3 \\ G_2 & -G_2-G_3 & G_2+G_3 & G_2+G_3+G_8 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{bmatrix}$$

$$= \begin{bmatrix} G_1 v_{s1} \\ -G_1 v_{s1} \\ 0 \\ j_{s3} \end{bmatrix}$$

Some comments on loop and cut set analysis

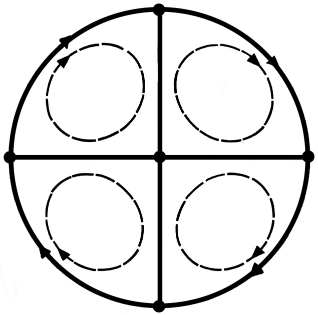

Figure 14: Fundamental loops for the chosen tree are identical with meshes.

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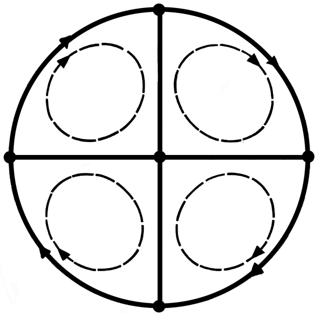

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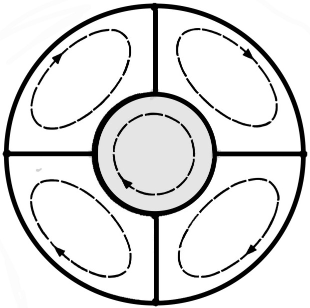

Figure 15: A graph showing that meshes are not special cases of fundamental loops.

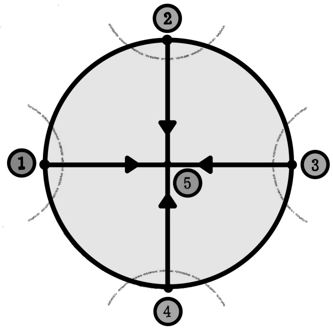

Figure 16: The four fundamental cut sets for the chosen tree coincide with the set of branches connected to nodes (1), (2), (3), and (4).

Relationship between B and Q

- **THEOREM** Call $\mathbf{B}$ the fundamental loop matrix and $\mathbf{Q}$ the fundamental cut-set matrix of the same oriented $\mathcal{G}$, and let both matrices pertain to the same tree T; then

$$\mathbf{B}\mathbf{Q}^T = \mathbf{0} \text{ and } \quad \mathbf{Q}\mathbf{B}^T = \mathbf{0} \tag{19}$$

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- If we number the links from 1 to l and number the tree branches from $l+1$ to b, then

$$\mathbf{B} = \begin{bmatrix} \mathbf{1}_l & \mathbf{F} \end{bmatrix} \quad \text{and} \quad \mathbf{Q} = \begin{bmatrix} -\mathbf{F}^T & \mathbf{1}_n \end{bmatrix} \tag{20}$$

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- Above equation tells us that the product of the $l \times b$ matrix **B** and the $b \times n$ matrix $\mathbf{Q}^T$ is the $l \times n$ zero matrix. In other words, the product of every row of **B** and every column of $\mathbf{Q}^T$ is zero.

Proof: For Eq. 19 i.e. $\mathbf{B}\mathbf{Q}^T = \mathbf{0}$ and $\mathbf{Q}\mathbf{B}^T = \mathbf{0}$

- From $\mathbf{B}\mathbf{v} = \mathbf{0}$ and $\mathbf{v} = \mathbf{Q}^T\mathbf{e}$, we have

$$\mathbf{B}\mathbf{Q}^T\mathbf{e} = \mathbf{0} \tag{21}$$

Proof: For Eq. 19 i.e. $\mathbf{BQ}^T = \mathbf{0}$ and $\mathbf{QB}^T = \mathbf{0}$

- From $\mathbf{Bv} = \mathbf{0}$ and $\mathbf{v} = \mathbf{Q}^T\mathbf{e}$, we have

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- Above means that whenever we multiply any n-vector $\mathbf{e}$ by $\mathbf{BQ}^T$, we get the zero vector. Therefore, Eq. 21 implies that the matrix $\mathbf{BQ}^T$ has all its elements equal to zero.

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- Therefore, we establish $\mathbf{BQ}^T = \mathbf{0}$ and $\mathbf{QB}^T = \mathbf{0}$

Proof: For Eq. 20 i.e. $\mathbf{B} = \begin{bmatrix} \mathbf{1}_l & \mathbf{F} \end{bmatrix}$ and $\mathbf{Q} = \begin{bmatrix} -\mathbf{F}^T & \mathbf{1}_n \end{bmatrix}$

- From eq. 12, $\mathbf{Q}$ was of the form
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- Noting that $\mathbf{I}_l$ has the same number of columns as $\mathbf{E}^T$ has rows, we conclude that

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- **Summary for Loop set Analysis** KVL is expressed by $\mathbf{Bv} = \mathbf{0}$, and KCL by $\mathbf{j} = \mathbf{B}^T\mathbf{i}$ where $\mathbf{i}$ is the loop current vector. As a result of our choice of reference directions, the fundamental loop matrix $\mathbf{B}$ is of the form (2). These equations are valid irrespective of the nature of the branches.

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- **Summary for Cut set Analysis** KCL requires that $\mathbf{Qj} = \mathbf{0}$. KVL is expressed by $\mathbf{v} = \mathbf{Q}^T\mathbf{e}$. As a result of our numbering convention, the fundamental cut-set matrix $\mathbf{Q}$ is of the form of (12). These equations are valid irrespective of the nature of the branches.

Practice

(1) Consider the graph $\mathcal{G}$ of below Figure. List all the fundamental loops and all the fundamental cut sets corresponding to tree T_1 shown in Fig. 2.

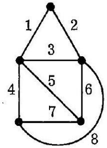

Figure 17: Planar graph

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