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Optimization Techniques and Algorithms Assignment 3

Problem 1: Maximizing the Area of a Rectangular Garden

A rectangular garden is to be constructed using a rock wall as one side of the garden, with the other three sides enclosed by wire fencing. The total length of the wire fencing available is 100 feet. The goal is to determine the dimensions of the garden that will maximize its area.

Mathematical Formulation:
Objective Function: Maximize the area A = x * y where x is the length parallel to the rock wall and y is the length perpendicular to the rock wall.
Constraints: The total fencing used is given by x + 2y = 100.
A(x) = x * ((100 - x) / 2)

MATLAB Function:

```
function A = garden_area(W)
  if W < 0 || W > 50
    error('Width W must be between 0 and 50.');
  end
  L = 100 - 2*W;
  A = L * W;
  fprintf('For width W = %.2f, Length L = %.2f, Area = %.2f\n', W, L, A);
end
```

Problem 2: Maximizing the Volume of an Open-Top Box

An open-top box is to be made from a 24-inch by 36-inch piece of cardboard by removing squares from each corner and folding up the flaps. The goal is to determine the size of the square to be cut out that maximizes the volume of the resulting box.

Mathematical Formulation:
Objective Function: Maximize the volume V = x(24 - 2x)(36 - 2x), where x is the side length of the square to be cut out.
Constraints: 0 <= x <= 12.

MATLAB Function:

```
function V = box_volume(x)

  if x < 0 || x > 12

    error('Side length x must be between 0 and 12.');

  end

  V = x * (24 - 2*x) * (36 - 2*x);

  fprintf('For square cut side x = %.2f, Volume = %.2f\n', x, V);

end
```

Problem 3: Minimizing Time for Travel

A visitor needs to travel from a cabin on the shore to an island. The cabin is 66 miles west of the point on the shore closest to the island, which is 22 miles north of this point. The visitor can run at 88 mph and swim at 33 mph. The objective is to minimize the time taken to reach the island by determining how far the visitor should run before swimming.

Mathematical Formulation:
Objective Function: Minimize the total time T = (r / 88) + (sqrt((66 - r)^2 + 22^2) / 33), where r is the distance run.
Constraints: 0 <= r <= 66.

MATLAB Function:

```
function T = travel_time(x)

  if x < 0 || x > 66

    error('Distance run x must be between 0 and 66.');

  end

  T = (x / 88) + (sqrt((66 - x)^2 + 22^2) / 33);

  fprintf('For distance run x = %.2f, Time = %.2f hours\n', x, T);

end
```

Problem 4: Maximizing Revenue for a Car Rental Company

A car rental company charges p dollars per day for renting a car. The number of cars rented per day n(p) can be modeled as n(p) = 1000 - 5p. The goal is to determine the price p that maximizes the revenue.

Mathematical Formulation:
Objective Function: Maximize revenue R(p) = p * n(p) = p * (1000 - 5p).
Constraints: 50 <= p <= 200.

MATLAB Function:

```
function R = rental_revenue(p)

  if p < 50 || p > 200

    error('Price p must be between 50 and 200.');

  end

  R = p * (1000 - 5*p);

  fprintf('For price p = %.2f, Revenue = %.2f\n', p, R);

end
```

Problem 5: Maximizing the Area of a Rectangle Inscribed in an Ellipse

A rectangle is to be inscribed in an ellipse given by (x^2 / a^2) + (y^2 / b^2) = 1. The goal is to determine the dimensions of the rectangle that maximize its area.

Mathematical Formulation:
Objective Function: Maximize the area A(x) = 4 * x * y where y = (b / a) * sqrt(a^2 - x^2).
Constraints: -a <= x <= a.

MATLAB Function:

```
function A = rectangle_area(x)

  if x < 0 || x > 2*sqrt(2)

    error('Value x must be between 0 and 2*sqrt(2).');

  end

  y = sqrt(1 - (x^2 / 4));

  A = 4 * x * y;

  fprintf('For x = %.2f, y = %.2f, Area = %.2f\n', x, y, A);

end
```

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MATLAB MAIN CODE:

```
% Problem 1: Rectangular Garden
W = 20;
area1 = garden_area(W);
% Problem 2: Open-Top Box
x = 3;
volume = box_volume(x)
% Problem 3: Minimize Travel Time
x = 30; % Example distance covered by running
time = travel_time(x);
% Problem 4: Maximize Revenue for Car Rental
p = 100; % Example charge per day
revenue = rental_revenue(p);
% Problem 5: Rectangle Inscribed in Ellipse
x = 1; % Example value for x
area = rectangle_area(x);
```

OUTPUT:

Constraint issue:

```
>> Assignment_3
For width W = 20.00, Length L = 60.00, Area = 1200.00
For square cut side x = 3.00, Volume = 1620.00
Error using Assignment_3>travel_time (line 20)
Distance run x must be between 0 and 66.

Error in Assignment_3 (line 52)
time = travel_time(x);
```

OUTPUT:

```
>> Assignment_3
For width W = 20.00, Length L = 60.00, Area = 1200.00
For square cut side x = 3.00, Volume = 1620.00
For distance run x = 30.00, Time = 1.62 hours
For price p = 100.00, Revenue = 50000.00
For x = 1.00, y = 0.87, Area = 3.46
>>
```