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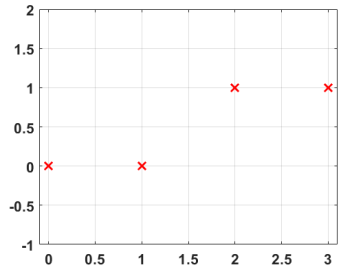
1. Consider interpolating the four data points $(0,0)$, $(1,0)$, $(2,1)$, $(3,1)$ shown below.



   (a) Use the monomial basis to write down a 4×4 system of equations to solve for the cubic polynomial, $p_3(x)$, which interpolates this data.

   Answer:
   Q + 0c1 + 0c2 + 0c3 = 0
   Q + 1c1 + 1c2 + 1c3 = 0
   Q + 2c1 + 4c2 + 8c3 = 1
   Q + 3c1 + 9c2 +27c3 = 1

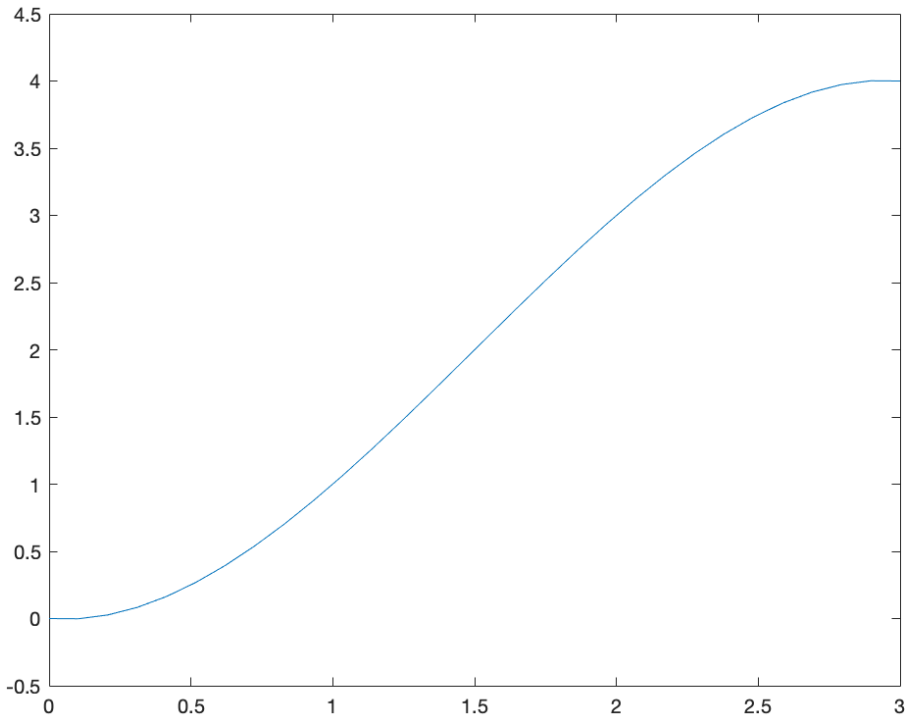
   (b) Solve the system in Matlab and write out the expression for $p_3(x)$. The `vander` function in Matlab may be helpful.

   Answer: MATLAB code
   A = [1 0 0 0; 1 1 1 1; 1 2 4 8; 1 3 9 27];
   y = [0;0;1;1];
   c = A \ y;
   c = [0;-1/6;3/2;-1/3];

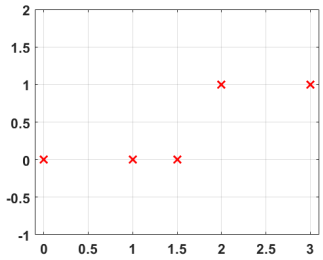
   $p_3(x) = 0 + (-1/6)x + (3/2)x^2 + (-1/3)x^3$

   (c) Plot the interpolating polynomial in Matlab on the interval $[0,3]$, using a spacing of 0.1 between each point. The `polyval` function in Matlab may be helpful. Sketch the interpolating polynomial on the plot above.
   Answer: MATLAB code

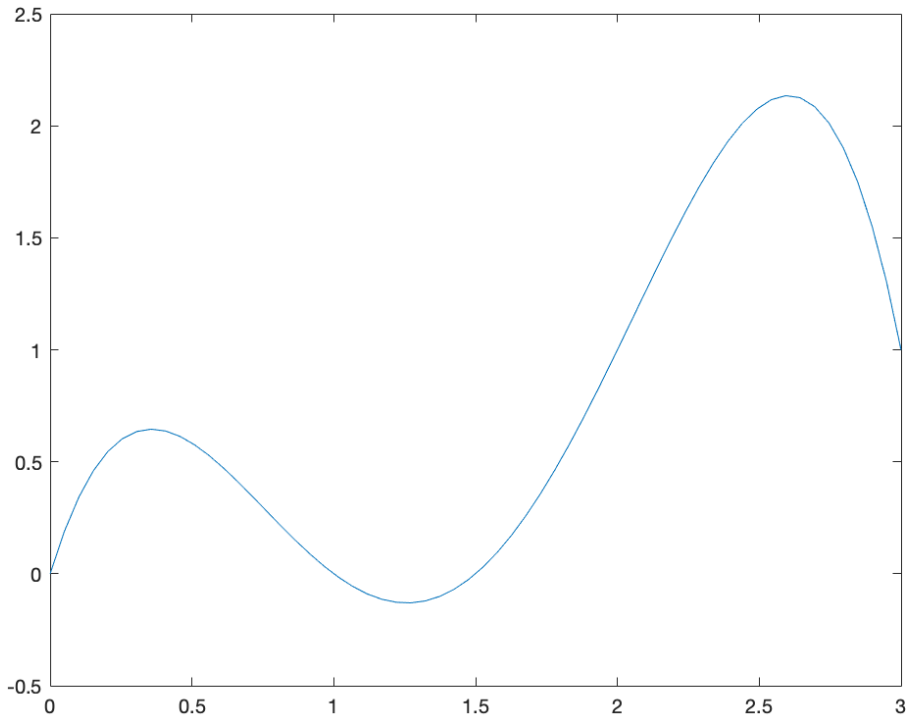
   x =linspace(0,3,30);
   c = [-1/3;3/2;-1/6;0];
   y = polyval(c,x);
   plot(x,y)



(d) Suppose that the point $(1.5, 0)$ is added to the data set, as shown below. Find the polynomial $p_4(x)$ which interpolates this data. Plot the interpolant below.



$$p_4(x) = (-0.8889)x^4 + 5x^3 + (-8.2778)x^2 + 4.1667x + 0$$



(e) What are the major differences between the two interpolants? Does this seem reasonable?

Answer: The major difference between two interpollants due to the degree of the polynomial being 4 it has almost zero error between given data points and the curve points whereas for polynomial of degree 3 it has errors between actual data points and the curve points. E.g. (1,0) for polynomial of degree 3 the Y value shoulds be 0 but it comes out to be 0.83 whereas for polynomial of degree 4 it exactly matches.