Assignment - 2

Unit Test - 1

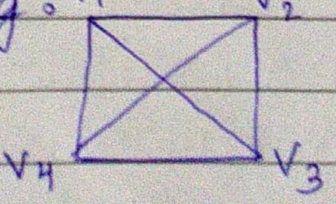
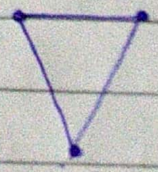
Q-1 (A) Do as directed

1. State any two application of matrix in computer science.

Ans:1 In ① Computer network
② Image processing

2. State any two differences between complete graph and regular graph?

Ans:2

	Complete graph	Regular graph
1.	A simple graph in which there is exactly one edge between each pair of vertices is known as complete graph.	1. If every vertex of simple graph has the same degree than the graph is known as regular graph.
2.	Eg: 	2. Eg: 

3. State one difference between symmetric and skew-symmetric matrix. Also give example.

Ans: 3

Symmetric Matrix	Skew symmetric Matrix
1. A square matrix $A = (a_{ij})$ is said to be a symmetric matrix if $A' = A$.	1. A square matrix $A = (a_{ij})$ is said to be a skew symmetric matrix if $A' = -A$

4. What is the sum of the degree of all the vertices of an undirected graph if number of edges are 13?

Ans: 4 $\sum_{v_i} = 2e$

$2 \times 13 = 26$

Sum of degree $= 26$

Scanned by CamScanner

Q-I (B) Answer the following in brief.

1. Define Matrix and Determinant. Also give any two difference between them.

Ans: 1. • A matrix is a collection of numbers arranged into a fixed number of rows and columns.

• In linear algebra, the determinant is a value that can be computed from the elements of a square matrix and encodes certain properties of the linear transformation described by the matrix.

	Matrix	Determinant
1.	A matrix is a rectangular grid of numbers or symbols that is represented in a row and column format.	1. A determinant is a component of a square matrix and it cannot be found in any other type of matrix.

Scanned by CamScanner

Matrix	Determinant
2. A matrix has no specific value. It is an arrangement of numbers.	2. A determinant has a specific value.

2. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 0 \\ 4 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 2 & 1 \\ 2 & 3 & 0 \end{bmatrix}$, find BA. Can we find AB also?

Ans: 2

$$BA = \begin{bmatrix} 1 & 2 \\ 3 & 0 \\ 4 & 1 \end{bmatrix} \times \begin{bmatrix} 0 & 1 & 0 \\ 0 & 2 & 1 \\ 2 & 3 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 2 & 1 \\ 2 & 3 & 0 \end{bmatrix} \times \begin{bmatrix} 1 & 2 \\ 3 & 0 \\ 4 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 0 \\ 10 & 1 \\ 11 & 4 \end{bmatrix}$$

- AB is not possible because

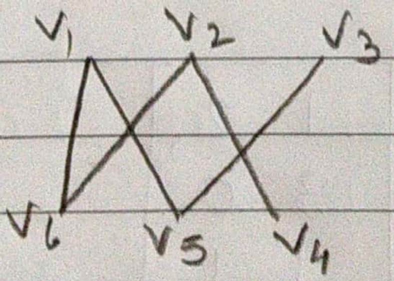
Scanned by CamScanner

the number of rows of matrix 1 does not matches numbers of column of matrix 2.

3. Define Bi-partite graph and complete bi-partite graph. Also draw each graph.

Ans:3 A graph $G = (V, E)$ is called bi-partite graph if vertex set $V(G)$ can be partition into two non-empty disjoint subset $V_1(G)$ and $V_2(G)$ in such way that each edge $e \in E$ has its one end point in $V_1(G)$ and another in $V = V_1 \cup V_2$

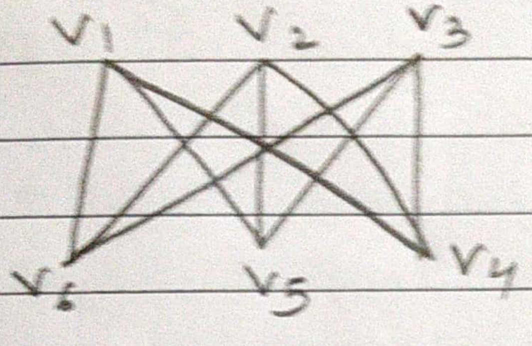
eg:



• Complete bi-partite graph
If each vertex of V_1 and is connected with every vertex of V_2 by an edge than G is called completely bi-partite graph.

Scanned by CamScanner

- It is denoted with K_{mn} where m is number of vertices in V_1 and n is number of vertices in V_2



Scanned by CamScanner

Q-2 Answer the following:

(A) If $A = \begin{bmatrix} 7 & 3 & -5 \\ 0 & 4 & 2 \\ 1 & 5 & 4 \end{bmatrix}$, $B = 3A$ and

$C = B + 2A - 5I$, find the matrix D such that $D = B + 2A - C$

Ans:

$D = \cancel{3A} + \cancel{2A} - \cancel{3A} - \cancel{2A} + 5I$

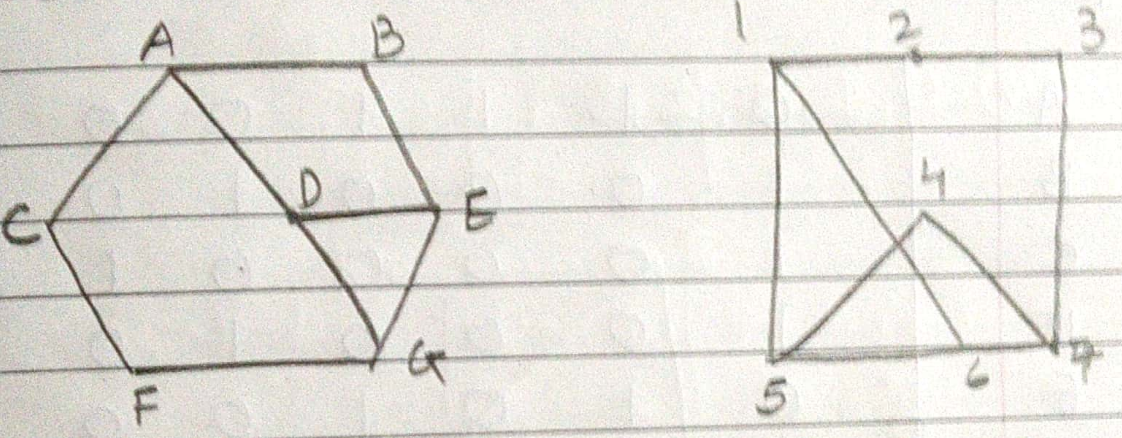
$= 5I$

$= 5 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$

Scanned by CamScanner

(B) Examine whether the following pair of graphs are isomorphic, give the reasons. Also verify that their adjacency matrices are same or not.



Ans: ① The no. of vertices in G1 = 7
The no. of vertices in G2 = 7
② The no. of edges in G1 = 9
The no. of edges in G2 = 9
③ The degree 2 in G1 = 3
The degree 3 in G2 = 4
The degree 2 in G1 = 3
The degree 3 in G2 = 4
④ Mapping

A - 7
B - 4
E - 5
G - 1
F - 2
C - 3
D - 6

Scanned by CamScanner

Thus, the given graph are isomorphic.

Adjacency Matrix:

Graph 1:

$$\begin{array}{c|c} & \begin{array}{ccccccc} A & B & C & D & E & F & G \end{array} \\ \begin{array}{c} A \\ B \\ C \\ D \\ E \\ F \\ G \end{array} & \begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

Graph 2:

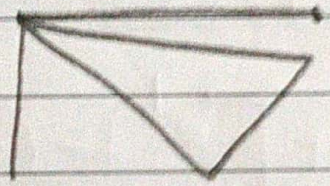
$$\begin{array}{c|c} & \begin{array}{ccccccc} 7 & 4 & 3 & 6 & 5 & 2 & 1 \end{array} \\ \begin{array}{c} 7 \\ 4 \\ 3 \\ 6 \\ 5 \\ 2 \\ 1 \end{array} & \begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

Scanned by CamScanner

(B) Does there exist a simple graph with 5 vertices of the given graph degree? If so draw such a graph, otherwise explain why no such graph exists?

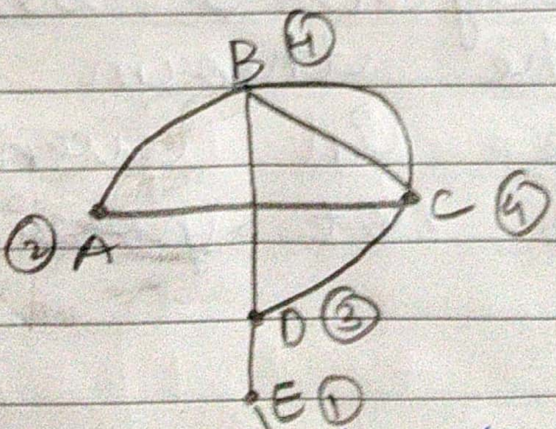
Ans:

(i) 1, 2, 3, 4, 5



It is not a simple graph because 5 degree is not possible as there are 5 vertices. The sum of degree is not even.

(ii) 1, 2, 3, 4, 4



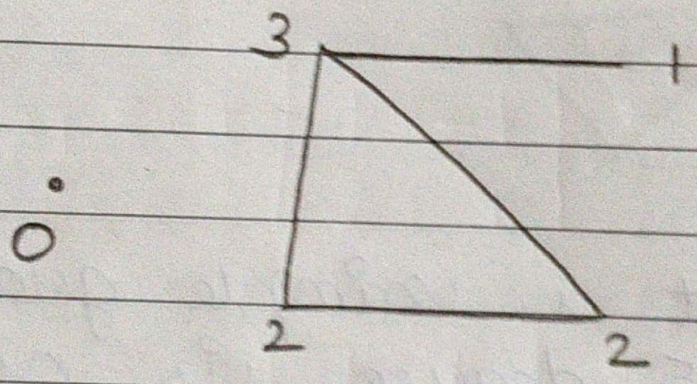
~~Here, there are 4 degree due to which one degree is not possible.~~

$\Sigma \deg(v_i) = 14$

(iii) 3, 4, 3, 4, 3

$\Sigma \deg(v_i) = 17$ and as per theorm sum must be in even so graph is not possible.

(iv) 0, 1, 2, 2, 3



(v) 1, 1, 1, 1, 1

$\Sigma \deg(v_i) = 5$ so, as per the theorm sum must be in even so, graph is not possible.

Scanned by CamScanner

Q-3 Answer the following in detail.

A) Solve the following system of equation by using matrix inversion method:
$x+y+z=6$; $x-y+z=2$; $2x+y-z=1$

Ans:

$AX = B$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 2 & 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 2 \\ 1 \end{bmatrix}$$

we know
$X = A^{-1}B$

$A^{-1} = \frac{1}{|A|} (\text{adj } A)$

$$|A| = \begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 2 & 1 & 1 \end{vmatrix}$$

$= 1(1-1) - 1(-1-2) + (1+2)$
$= 3 + 3 = 6$

Scanned by CamScanner

$\because$ $|A| \neq 0$, so it is consistent and have a unique solution & A^{-1} is also possible.

$$C_{11} = +\begin{vmatrix} -1 & 1 \\ 1 & -1 \end{vmatrix} = 1-1=0$$

$$C_{21} = -\begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix} = -1-2 = +3$$

$$C_{31} = +\begin{vmatrix} 1 & -1 \\ 2 & 1 \end{vmatrix} = 1-(-2) = 3$$

$$C_{21} = -\begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} = -1-1 = 2$$

$$C_{22} = \begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix} = -1-2 = -3$$

$$C_{23} = -\begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} = 1-2 = 1$$

$$C_{31} = +\begin{vmatrix} 1 & 1 \\ -1 & 1 \end{vmatrix} = 1+1 = 2$$

$$C_{32} = -\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = 1-1 = 0$$

Scanned by CamScanner

$$C_{33} = + \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} = -1-1 = -2$$

$$\text{adj } A = \begin{bmatrix} 0 & 3 & 3 \\ 2 & -3 & 1 \\ 2 & 0 & -2 \end{bmatrix} = \begin{bmatrix} 0 & 2 & 2 \\ 3 & -3 & 0 \\ 3 & 1 & -2 \end{bmatrix}$$

we know,

$$X = A^{-1}B = \frac{1}{|A|} (\text{adj } A) B.$$

$$X = \frac{1}{6} \begin{bmatrix} 0 & 2 & 2 \\ 3 & -3 & 0 \\ 3 & 1 & -2 \end{bmatrix} \begin{bmatrix} 6 \\ 2 \\ 1 \end{bmatrix}$$

$$= \frac{1}{6} \begin{bmatrix} 0+4+2 \\ 18-6+0 \\ 18+2-2 \end{bmatrix}$$

$$= \frac{1}{6} \begin{bmatrix} 6 \\ 12 \\ 18 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

Scanned by CamScanner

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$\therefore$ $x = 1$, $y = 2$ & $z = 3$

$x + y + z = 1 + 2 + 3 = 6$
$x - y + z = 1 - 2 + 3 = 2$
$2x + y - z = 2(1) + 2 - 3 = 1$

(B) State and prove the handshaking theorem by taking example of undirected graph.

Ans: If the $G = (V, E)$ is an undirected graph with e edges, then $\sum_i \deg(v_i) = 2e$.

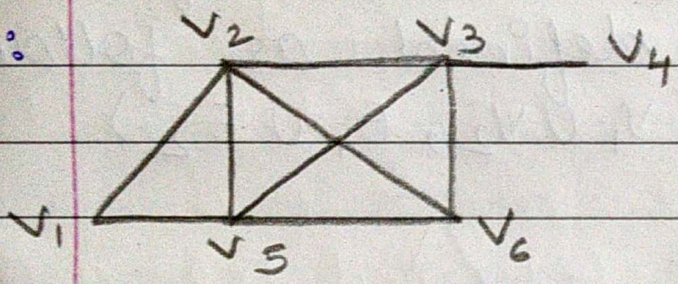
- The sum of degree of all the vertices of an undirected graph is twice the number of edges of the graph and hence even.

Proof: Since every edge is incident with exactly two vertices, every edge contributes 2 to the sum of the degree of vertices.

$\therefore$ All the e edges contribute (2e) to the sum of degree of vertices.

$\therefore \sum_i deg(v_i) = 2e$

eg:



$Deg(V_1) = 2$
$Deg(V_2) = 4$
$Deg(V_3) = 4$
$Deg(V_4) = 1$
$Deg(V_5) = 4$
$Deg(V_6) = 3$
$Deg(V_7) = 0$

$\sum Deg(V_i) = 18$ or $\sum Deg(v_i) = 2 \times 9$
$= 18$

$18 = 2e$
$e = 9$

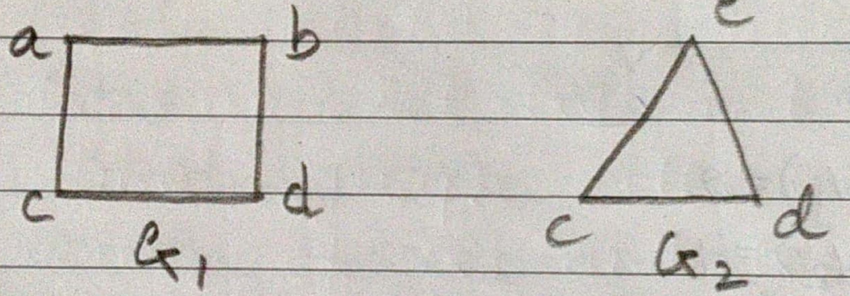
Thus, proof.

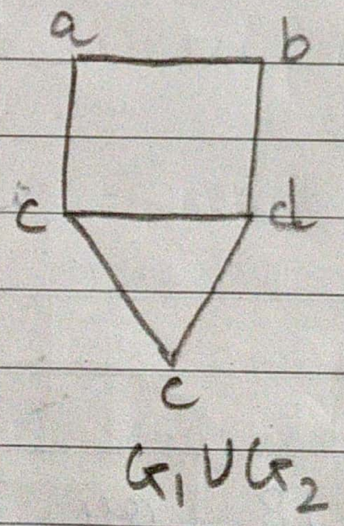
Scanned by CamScanner

(c) Explain following operation on graphs.
1) Union 2) Intersection 3) Complement
4) Sum 5) Ringsum.

Ans: ① Union: Let $G1 = (V_1 E_1)$ and $G_2 = (V_2 E_2)$ be the two graphs then union of two graph is defined as follow:
$G_1 \cup G_2 = (V_1 \cup V_2, E_1 \cup E_2)$.

Eg:

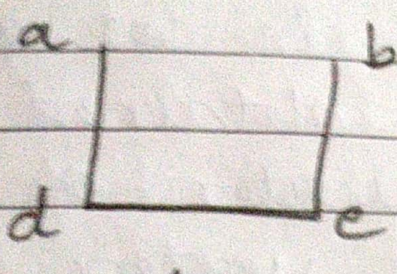




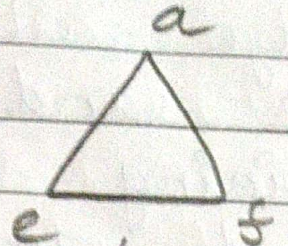
② Intersection: Let $G1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be the two graph then intersection of two graph is defined as follow $G_1 \cap G_2 = (V_1 \cap V_2, E_1 \cap E_2$

Scanned by CamScanner

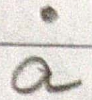
Eg:



G_1



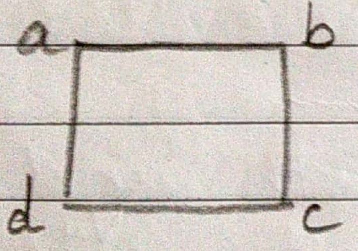
G_2



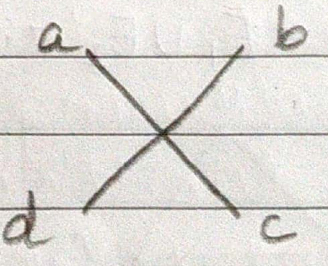
$G_1 \cap G_2$

③ Complement: The complement $\overline{G}$ of the graph G is the graph with same vertex set as G with two adjacent vertices u and v that are not adjacent in G.

Eg:



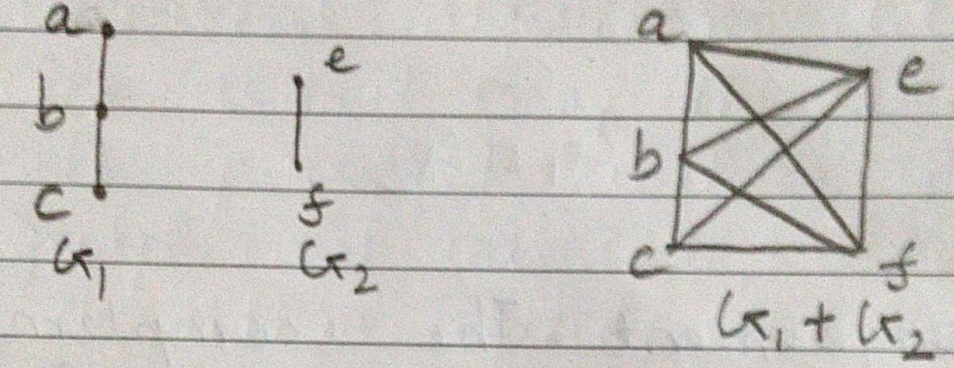
G



$\overline{G}$

④ Sum: If the graph G_1 and G_2 are given and $G_1 \cap G_2 = \phi$ than sum $G_1 + G_2$ is defined as the graph whose vertex set is $V_1 + V_2$ and the edge set is consisting those edge with are in G1 and G2 and edge obtained by joining each vertex of G1 and to each vertex of G2.

Eg:



⑤ Ringsum: Let G1 and G2 are two graph then ring sum of G1 and G_2 denoted $G_1 \oplus G_2$ is defined as

(i) $V_1 \cup V_2 = V$

(ii) $E = E_1 \cup E_2 - E_1 \cap E_2$

Eg: