AI1103 - Assignment 4

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Download latex codes from

https://github.com/vishwahurakadli/AI1103/tree/main/Assignment_4

1 Question

CSIR-UGC-NET(mathA june 2015)
Q.110: Suppose X has density $f\left(\frac{x}{\theta}\right) = \frac{1}{\theta}e^{-x/\theta}, x > 0,$ $\theta > 0$ Define Y as follows Y = K $k \leqslant X < k+1$, k= 0,1,2,... Then the distribution of Y is

1) Normal
2) Binomial
3) Poisson
4) Geometric

2 Solution

Lemma 2.1. *PDF of X is*

$$\Pr(X) = \frac{1}{\theta}e^{-\frac{x}{\theta}} \tag{2.0.1}$$

Proof. The PDF of X will be given by

$$f(x|\theta) = \frac{1}{\theta}e^{-x/\theta}, x > 0, \theta > 0 \tag{2.0.2}$$

$$f(x) = \frac{1}{\theta}e^{-\frac{x}{\theta}}, \text{since x and } \theta \text{are independent} \tag{2.0.3}$$

Hence lemma 2.1 is proved. □

Lemma 2.2. *pdf of Y is*

$$p(Y = k) = e^{-\frac{k}{\theta}}\left(1 - e^{-\frac{k}{\theta}}\right) \tag{2.0.4}$$

is in the form of geometric distribution.

Proof. given that Y=k if $k \leqslant X < k+1$ k=0,1,2,...

$$p(Y = k) = \int_k^{k+1} p(X = x)dx \tag{2.0.5}$$

$$= \int_k^{k+1} \frac{1}{\theta}e^{-\frac{x}{\theta}}dx$$

$$= e^{-\frac{k}{\theta}}\left(1 - e^{-\frac{k}{\theta}}\right) \tag{2.0.6}$$

□

Lemma 2.3. *When the pdf of X is in the form of*

$$p(X = k) = (1-p)^k p, \ \textit{where} \ (k = 0, 1, 2, ...) \tag{2.0.7}$$

Then the distribution is said to be in the form of geometric distribution.

Proof. the expected value is given by

$$E(X) = p(x = 0) + p(x = 1)2 + p(x = 3)3 + ... \tag{2.0.8}$$

$$= p + (1-p)^2 p2 + (1-p)^3 p3 + ... \tag{2.0.9}$$

solving these equations

$$E(X) = \frac{1}{p} \tag{2.0.10}$$

shows that p(X=k) is in the form of geometric distribution. □

Using Lemma 2.2 and Lemma 2.3, distribution of Y will be geometric. So the correct option is (4)