AI1103 - Assignment 2

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and latex codes from

https://github.com/vishwahurakadli/AI1103/blob/main/Assignment_2

1) Gate EC Q22:
   Let U and V be two independent zero mean Gaussian random variables of variances $\frac{1}{4}$ and $\frac{1}{9}$ respectively. The probability $\Pr(3V \geqslant 2U)$ is.

   a) $\frac{4}{9}$ b) $\frac{1}{2}$ c) $\frac{2}{3}$ d) $\frac{5}{9}$

2) Solution Given that random variables U and V are zero mean Gaussian random variables

$$i.e., E(U) = E(V) = 0 \tag{0.0.1}$$

also given that

$$Var(U) = \sigma_U^2 = \frac{1}{4} \neq 0 \tag{0.0.2}$$

$$Var(V) = \sigma_V^2 = \frac{1}{9} \neq 0 \tag{0.0.3}$$

As U and V are Gaussian random variable

$$f_u(u) = \frac{1}{\sqrt{2\pi} \times \sigma_u^2} e^{-u/2\sigma_u^2} \tag{0.0.4}$$

$$f_v(v) = \frac{1}{\sqrt{2\pi} \times \sigma_v^2} e^{-v/2\sigma_v^2} \tag{0.0.5}$$

As random variables are with zero mean and non zero variance by symmetry property imply that

$$\Pr(U \geqslant 0) = \frac{1}{2} \tag{0.0.6}$$

$$\Pr(V \geqslant 0) = \frac{1}{2} \tag{0.0.7}$$

To interpret with 2U and 3V random variable, Let X and Y are random variables with following distribution

$$X = \frac{v}{\sigma_v} = 3V \tag{0.0.8}$$

$$Y = \frac{u}{\sigma_u} = 2U \tag{0.0.9}$$

$$\tag{0.0.10}$$

and we have

$$E(X) = 3E(V) = 0 \tag{0.0.11}$$

$$E(Y) = 2E(U) = 0 \tag{0.0.12}$$

imply that random variable X-Y

$$E(X - Y) = 0 \tag{0.0.13}$$

and variance of X and Y

$$Var(X) = 9Var(V) = 1 \tag{0.0.14}$$

$$Var(Y) = 4Var(U) = 1 \tag{0.0.15}$$

imply that

$$Var(X - Y) \neq 0 \tag{0.0.16}$$

Hence we can say by symmetry property

$$\Pr(X - Y \geqslant 0) = \Pr(3V - 2U \geqslant 0) = \frac{1}{2} \tag{0.0.17}$$

So

$$\Pr(3V \geqslant 2U) = \frac{1}{2} \tag{0.0.18}$$

So the correct option is (B)