AI1103 - Assignment 4

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Download latex codes from

https://github.com/vishwahurakadli/AI1103/tree/main/Assignment_4

1) CSIR-UGC-NET(mathA june 2015) Q.110 : Suppose X has density

$$f(x|\theta) = \frac{1}{\theta}e^{\frac{-x}{\theta}}, x > 0, \theta > 0 \tag{0.0.1}$$

Define Y as follows

$$Y = Kifk \leqslant X < k+1, k = 0, 1, 2, ... \tag{0.0.2}$$

Then the distribution of Y is

a) Normal
b) Binomial
c) Poisson
d) Geometric

Lemma 0.1. *PDF of X is*

$$\Pr(X) = \frac{1}{\theta}e^{-\frac{x}{\theta}} \tag{0.0.3}$$

Proof. The PDF of X will be given by

$$f(x|\theta) = \frac{1}{\theta}e^{-x/\theta}, x > 0, \theta > 0 \tag{0.0.4}$$

$$f(x) = \frac{1}{\theta}e^{-\frac{x}{\theta}}, \text{since x and } \theta \text{are independent} \tag{0.0.5}$$

Hence lemma 0.1 is proved. □

Lemma 0.2. *pdf of Y is*

$$p(Y = k) = e^{-\frac{k}{\theta}}\left(1 - e^{-\frac{k}{\theta}}\right) \tag{0.0.6}$$

is in the form of geometric distribution.

Proof. Also given that Y=k if $k \leqslant X < k+1$ k=0,1,2,...

$$p(Y = k) = \int_k^{k+1} p(()X = x)dx \tag{0.0.7}$$

$$= \int_k^{k+1} \frac{1}{\theta}e^{-\frac{x}{\theta}}dx$$

$$= e^{-\frac{k}{\theta}}\left(1 - e^{-\frac{k}{\theta}}\right) \tag{0.0.8}$$

□

Lemma 0.3. *When the pdf of X is in the form of*

$$p(X = k) = (1-p)^k p, \text{ where } (k = 0, 1, 2, ...) \tag{0.0.9}$$

Then the distribution is said to be in the form of geometric distribution.

Using Lemma 0.2 and Lemma 0.3, distribution of Y will be geometric.
So the correct option is (4)