AI1103 - Assignment 4

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Download latex codes from

https://github.com/vishwahurakadli/AI1103/tree/main/Assignment_4

1 Question

CSIR-UGC-NET(mathA june 2015)
Q.110: Suppose X has density $f\left(\frac{x}{\theta}\right) = \frac{1}{\theta}e^{-x/\theta}, x > 0$, $\theta > 0$ Define Y as follows $Y = K\ k \leqslant X < k+1$, $k = 0, 1, 2, cdots$
Then the distribution of Y is

1) Normal
2) Binomial
3) Poisson
4) Geometric

2 Solution

Lemma 2.1. *PDF of X is*

$$\Pr(X) = \frac{1}{\theta}e^{-\frac{x}{\theta}} \tag{2.0.1}$$

Proof. The PDF of X will be given by

$$f(x|\theta) = \frac{1}{\theta}e^{-x/\theta}, x > 0, \theta > 0 \tag{2.0.2}$$

$$f(x) = \frac{1}{\theta}e^{-\frac{x}{\theta}} \tag{2.0.3}$$

since x and θ are independent Hence lemma 2.1 is proved. □

Lemma 2.2. *pdf of Y is*

$$p(Y = k) = e^{-\frac{k}{\theta}}\left(1 - e^{-\frac{k}{\theta}}\right) \tag{2.0.4}$$

is in the form of geometric distribution.

Proof. given that $Y = k$ if $k \leqslant X < k+1$ $(k = 0, 1, 2, \cdots)$

$$p(Y = k) = \int_k^{k+1} p(X = x)dx \tag{2.0.5}$$

$$= \int_k^{k+1} \frac{1}{\theta}e^{-\frac{x}{\theta}}dx$$

$$= e^{-\frac{k}{\theta}}\left(1 - e^{-\frac{k}{\theta}}\right) \tag{2.0.6}$$

□

Lemma 2.3. *When the pdf of X is in the form of*

$$p(X = k) = (1-p)^k p, \tag{2.0.7}$$

where $(k = 0, 1, 2, \cdots)$
Then the distribution is said to be in the form of geometric distribution.

Proof. the expected value is given by

$$E(X) = \sum_{k=0}^{\infty}(1-p)^k k \tag{2.0.8}$$

by substituting p(x)

$$E(X) = p(x = 0) + 2p(x = 1) + 3p(x = 3) + ... \tag{2.0.9}$$

$$= p + 2(1-p)^2 p + 3(1-p)^3 p + ... \tag{2.0.10}$$

solving these equations

$$E(X) = \frac{1}{p} \tag{2.0.11}$$

shows that $p(X = k)$ is in the form of geometric distribution. □

Using Lemma 2.2 and Lemma 2.3, distribution of Y will be geometric. So the correct option is (4)