

AI1103 - Assignment 3

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Download latex codes from

https://github.com/vishwahurakadli/AI1103/blob/main/Assignment_3/Assignment_3.tex

1) Gate MA(2015) Q26:

Let X and Y be two random variables having the joint probability density function

$$f_{XY}(x, y) = \begin{cases} 2, & \text{if } 0 < x < y < 1 \\ 0, & \text{otherwise} \end{cases}$$

Then the conditional probability $\Pr(X \leq 2/3 | Y \leq 3/4)$ is equal to

- a) $\frac{5}{9}$ b) $\frac{2}{3}$ c) $\frac{7}{9}$ d) $\frac{8}{9}$

2) Solution

Given that X and Y are two random variables having joint PDF $f(x, y)$

The conditional PDF probability of two joint continuous random variable X and Y is given by-

$$f_{X|Y}(x|y) = \frac{f_{XY}(x, y)}{f_Y(y)} \quad (0.0.1)$$

where $f_Y(y)$ is marginal PDF given by

$$f_Y(y) = \int_{-\infty}^{\infty} f_{XY}(x, y) dx, \text{ for all } x \in \mathbf{R} \quad (0.0.2)$$

Here the marginal PDF of Y, lower limit of x is 0 and upper limit is y where function is equal to 2 (non zero value)

$$f_Y(y) = \int_0^y 2 dy = 2y \quad (0.0.3)$$

so the conditional PDF is

$$f_{X|Y}(x|y) = \begin{cases} \frac{2}{2y}, & \text{if } 0 < x < y < 1 \\ 0, & \text{otherwise} \end{cases} \quad (0.0.4)$$

the conditional probability X given Y=y is given by

$$\Pr(X \leq x | Y = y) = \int_{-\infty}^x f_{X|Y}(x|y) dx, \text{ for all } x \in \mathbf{R} \quad (0.0.5)$$

As $Y=3/4$ and lower limit of x is 0, so probability is

$$\Pr(X \leq x | Y = y) = \int_0^{2/3} \frac{4}{3} dx \quad (0.0.6)$$

$$\Pr(X \leq x | Y = y) = \frac{4}{3} \times \frac{2}{3} \quad (0.0.7)$$

$$\Pr(X \leq x | Y = y) = \frac{8}{9} \quad (0.0.8)$$

So the correct option is (d)