ASSIGNMENT 1

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Download all latex-tikz codes from

https://github.com/vishwahurakadli/EE3900/blob/main/Assignment_1/Assignment_1.tex

1 Problem

(Vectors-2.20) If

$$\mathbf{P} = 3\mathbf{a} - 2\mathbf{b} \tag{1.0.1}$$

$$\mathbf{Q} = \mathbf{a} + \mathbf{b} \tag{1.0.2}$$

find $\mathbf{R}$ which divides PQ in the ratio 2 : 1

1) internally
2) externally

2 Solution

Vector $\mathbf{P}$ and $\mathbf{Q}$ can be represented using $\mathbf{a}$ and $\mathbf{b}$ as

$$\mathbf{P} = \begin{pmatrix}3 & -2\end{pmatrix}\begin{pmatrix}\mathbf{a}\\\mathbf{b}\end{pmatrix} \tag{2.0.1}$$

$$\mathbf{Q} = \begin{pmatrix}1 & 1\end{pmatrix}\begin{pmatrix}\mathbf{a}\\\mathbf{b}\end{pmatrix} \tag{2.0.2}$$

$\begin{pmatrix}\mathbf{P}\\\mathbf{Q}\end{pmatrix}$ in terms of $\mathbf{a}$ and $\mathbf{b}$ is given by

$$\begin{pmatrix}\mathbf{P}\\\mathbf{Q}\end{pmatrix} = \begin{pmatrix}3 & -2\\1 & 1\end{pmatrix}\begin{pmatrix}\mathbf{a}\\\mathbf{b}\end{pmatrix} \tag{2.0.3}$$

1) section formula for internal division for ratio m : n is given by

$$\mathbf{I} = \begin{pmatrix}\frac{m}{m+n} & \frac{n}{m+n}\end{pmatrix}\begin{pmatrix}\mathbf{P}\\\mathbf{Q}\end{pmatrix} \tag{2.0.4}$$

$$= \begin{pmatrix}\frac{m}{m+n} & \frac{n}{m+n}\end{pmatrix}\begin{pmatrix}3 & -2\\1 & 1\end{pmatrix}\begin{pmatrix}\mathbf{a}\\\mathbf{b}\end{pmatrix} \tag{2.0.5}$$

so for ratio 2 : 1 R will be given by

$$\mathbf{R} = \begin{pmatrix}\frac{2}{2+1} & \frac{1}{2+1}\end{pmatrix}\begin{pmatrix}\mathbf{P}\\\mathbf{Q}\end{pmatrix} \tag{2.0.6}$$

$$= \begin{pmatrix}\frac{2}{3} & \frac{1}{3}\end{pmatrix}\begin{pmatrix}3 & -2\\1 & 1\end{pmatrix}\begin{pmatrix}\mathbf{a}\\\mathbf{b}\end{pmatrix} \tag{2.0.7}$$

$$= \frac{2}{3}(3\mathbf{a} - 2\mathbf{b}) + \frac{1}{3}(\mathbf{a} + \mathbf{b}) \tag{2.0.8}$$

$$\mathbf{R} = \frac{7}{3}\mathbf{a} - \mathbf{b} \tag{2.0.9}$$

$\mathbf{R}$ will divide PQ internally

2) similarly section formula for external division for ration m : n is given by

$$\mathbf{E} = \begin{pmatrix}\frac{m}{m-n} & \frac{n}{m-n}\end{pmatrix}\begin{pmatrix}\mathbf{P}\\\mathbf{Q}\end{pmatrix} \tag{2.0.10}$$

$$= \begin{pmatrix}\frac{m}{m-n} & \frac{n}{m-n}\end{pmatrix}\begin{pmatrix}3 & -2\\1 & 1\end{pmatrix}\begin{pmatrix}\mathbf{a}\\\mathbf{b}\end{pmatrix} \tag{2.0.11}$$

so for ratio 2 : 1 R will be given by

$$\mathbf{R} = \begin{pmatrix}\frac{2}{2-1} & -\frac{1}{2-1}\end{pmatrix}\begin{pmatrix}\mathbf{P}\\\mathbf{Q}\end{pmatrix} \tag{2.0.12}$$

$$= \begin{pmatrix}2 & -1\end{pmatrix}\begin{pmatrix}3 & -2\\1 & 1\end{pmatrix}\begin{pmatrix}\mathbf{a}\\\mathbf{b}\end{pmatrix} \tag{2.0.13}$$

$$= 2(3\mathbf{a} - 2\mathbf{b}) - (\mathbf{a} + \mathbf{b}) \tag{2.0.14}$$

$$\mathbf{R} = 5\mathbf{a} - 5\mathbf{b} \tag{2.0.15}$$

$\mathbf{R}$ will divide PQ externally