ASSIGNMENT 1

Vishwanath Hurakadli
AI20BTECH11023

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1 PROBLEM

(Vectors-2.20) If

$$\mathbf{P} = 3\mathbf{a} - 2\mathbf{b} \tag{1.0.1}$$

$$\mathbf{Q} = \mathbf{a} + \mathbf{b} \tag{1.0.2}$$

find $\mathbf{R}$ which divides PQ in the ratio $2:1$

1) internally
2) externally

2 SOLUTION

Vector $\mathbf{P}$ and $\mathbf{Q}$ can be represented using $\mathbf{a}$ and $\mathbf{b}$ as

$$\mathbf{P} = \begin{pmatrix} 3 & -2 \end{pmatrix}\begin{pmatrix} \mathbf{a} \\ \mathbf{b} \end{pmatrix} = 3\mathbf{a} - 2\mathbf{b} \tag{2.0.1}$$

$$\mathbf{Q} = \begin{pmatrix} 1 & 1 \end{pmatrix}\begin{pmatrix} \mathbf{a} \\ \mathbf{b} \end{pmatrix} = \mathbf{a} + \mathbf{b} \tag{2.0.2}$$

$\begin{pmatrix} \mathbf{P} \\ \mathbf{Q} \end{pmatrix}$ in terms of $\mathbf{a}$ and $\mathbf{b}$ is given by

$$\begin{pmatrix} \mathbf{P} \\ \mathbf{Q} \end{pmatrix} = \begin{pmatrix} 3 & -2 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} \mathbf{a} \\ \mathbf{b} \end{pmatrix} \tag{2.0.3}$$

$$= \begin{pmatrix} 3\mathbf{a} - 2\mathbf{b} \\ \mathbf{a} + \mathbf{b} \end{pmatrix} \tag{2.0.4}$$

1) section formula for internal division for ratio $m:n$ is given by

$$\mathbf{I} = \begin{pmatrix} \frac{m}{m+n} & \frac{n}{m+n} \end{pmatrix}\begin{pmatrix} \mathbf{P} \\ \mathbf{Q} \end{pmatrix} \tag{2.0.5}$$

$$= \begin{pmatrix} \frac{m}{m+n} & \frac{n}{m+n} \end{pmatrix}\begin{pmatrix} 3\mathbf{a} - 2\mathbf{b} \\ \mathbf{a} + \mathbf{b} \end{pmatrix} \tag{2.0.6}$$

$$\tag{2.0.7}$$

so for ratio $2:1$ R will be given by

$$\mathbf{R} = \begin{pmatrix} \frac{2}{2+1} & \frac{1}{2+1} \end{pmatrix}\begin{pmatrix} \mathbf{P} \\ \mathbf{Q} \end{pmatrix} \tag{2.0.8}$$

$$= \begin{pmatrix} \frac{2}{3} & \frac{1}{3} \end{pmatrix}\begin{pmatrix} 3\mathbf{a} - 2\mathbf{b} \\ \mathbf{a} + \mathbf{b} \end{pmatrix} \tag{2.0.9}$$

$$= \frac{2}{3}(3\mathbf{a} - 2\mathbf{b}) + \frac{1}{3}(\mathbf{a} + \mathbf{b}) \tag{2.0.10}$$

$$\mathbf{R} = \frac{7}{3}\mathbf{a} - \mathbf{b} \tag{2.0.11}$$

$\mathbf{R}$ will divide PQ internally

2) similarly section formula for external division for ration $m:n$ is given by

$$\mathbf{E} = \begin{pmatrix} \frac{m}{m-n} & \frac{n}{m-n} \end{pmatrix}\begin{pmatrix} \mathbf{P} \\ \mathbf{Q} \end{pmatrix} \tag{2.0.12}$$

$$= \begin{pmatrix} \frac{m}{m-n} & \frac{n}{m-n} \end{pmatrix}\begin{pmatrix} 3\mathbf{a} - 2\mathbf{b} \\ \mathbf{a} + \mathbf{b} \end{pmatrix} \tag{2.0.13}$$

so for ratio $2:1$ R will be given by

$$\mathbf{R} = \begin{pmatrix} \frac{2}{2-1} & -\frac{1}{2-1} \end{pmatrix}\begin{pmatrix} \mathbf{P} \\ \mathbf{Q} \end{pmatrix} \tag{2.0.14}$$

$$= \begin{pmatrix} 2 & -1 \end{pmatrix}\begin{pmatrix} 3\mathbf{a} - 2\mathbf{b} \\ \mathbf{a} + \mathbf{b} \end{pmatrix} \tag{2.0.15}$$

$$= 2(3\mathbf{a} - 2\mathbf{b}) - (\mathbf{a} + \mathbf{b}) \tag{2.0.16}$$

$$\mathbf{R} = 5\mathbf{a} - 5\mathbf{b} \tag{2.0.17}$$

$\mathbf{R}$ will divide PQ externally