ASSIGNMENT 5

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Download all python codes from

https://github.com/vishwahurakadli/EE3900/blob/main/Assignment_5/EE3900_Assignment_5.ipynb

and latex-tikz codes from

https://github.com/vishwahurakadli/EE3900/blob/main/Assignment_5/EE3900_Assignment_5.tex

1 Quadratic forms 2.26

Find the coordinates of the foci, the vertices,the length of major axis,minor axis the eccentricity and the latus rectum of the ellipse of the ellipse

$$\mathbf{x}^T \begin{pmatrix} \frac{1}{25} & 0 \\ 0 & \frac{1}{9} \end{pmatrix} \mathbf{x} = 9 \tag{1.0.1}$$

2 SOLUTION

Given ellipse is

$$\mathbf{x}^T \begin{pmatrix} \frac{1}{25} & 0 \\ 0 & \frac{1}{9} \end{pmatrix} \mathbf{x} = 1 \tag{2.0.1}$$

$$\text{can also be written as} \tag{2.0.2}$$

$$\mathbf{x}^T \begin{pmatrix} 9 & 0 \\ 0 & 25 \end{pmatrix} \mathbf{x} = 225 \tag{2.0.3}$$

On comparing it with standard form we have,

$$\mathbf{V} = \begin{pmatrix} 9 & 0 \\ 0 & 25 \end{pmatrix}, \mathbf{u} = 0, f = -225 \tag{2.0.4}$$

$$\implies \mathbf{u}^T\mathbf{V}^{-1}\mathbf{u} - f = 225 \tag{2.0.5}$$

$$\implies \mathbf{c} = -\mathbf{V}^{-1}\mathbf{u} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{2.0.6}$$

$$\tag{2.0.7}$$

The eigen vector decomposition of

$$\mathbf{V} = \begin{pmatrix} 9 & 0 \\ 0 & 25 \end{pmatrix} \tag{2.0.8}$$

is given by

$$\mathbf{D} = \begin{pmatrix} 9 & 0 \\ 0 & 25 \end{pmatrix} \implies \lambda_1 = 9, \lambda_2 = 25 \tag{2.0.9}$$

$$\mathbf{P} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \implies \mathbf{p}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \mathbf{p}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{2.0.10}$$

Since

$$\lambda_1 < \lambda_2 \tag{2.0.11}$$

Eccentricity of the ellipse is,

$$e = \sqrt{1 - \frac{\lambda_1}{\lambda_2}} = \frac{4}{5} \tag{2.0.12}$$

Semi major and minor axes of ellipse are,

$$a = \sqrt{\frac{\mathbf{u}^\top\mathbf{V}^{-1}\mathbf{u} - f}{\lambda_1}} = 5 \tag{2.0.13}$$

$$b = \sqrt{\frac{\mathbf{u}^\top\mathbf{V}^{-1}\mathbf{u} - f}{\lambda_2}} = 3 \tag{2.0.14}$$

Length of major axis, minor axis and latus rectum is given by

$$M = 2a = 10 \tag{2.0.15}$$

$$m = 2b = 6 \tag{2.0.16}$$

$$L = 2\frac{\sqrt{\frac{\mathbf{u}^\top\mathbf{V}^{-1}\mathbf{u}-f}{\lambda_2}}^2}{\sqrt{\frac{\mathbf{u}^\top\mathbf{V}^{-1}\mathbf{u}-f}{\lambda_1}}} = \frac{2b^2}{a} = \frac{18}{5} \tag{2.0.17}$$

The co-ordinates of vertices are,

$$\pm\begin{pmatrix} 5 \\ 0 \end{pmatrix} \tag{2.0.18}$$

The co-ordinates of foci are given by,

$$\mathbf{F} = \frac{ce^2\mathbf{n} - \mathbf{u}}{\lambda_2} \tag{2.0.19}$$

Where,

$$\mathbf{n} = \sqrt{\lambda_2}\mathbf{p}_1 \tag{2.0.20}$$

$$c = \frac{e\mathbf{u}^\top \mathbf{n} \pm \sqrt{e^2 \left(\mathbf{u}^\top \mathbf{n}\right)^2 - \lambda_1 \left(e^2 - 1\right)\left(\|\mathbf{u}\|^2 - \lambda_1 f\right)}}{\lambda_1 e \left(e^2 - 1\right)} \tag{2.0.21}$$

Substituting we have,

$$\mathbf{n} = \begin{pmatrix} 5 \\ 0 \end{pmatrix} \tag{2.0.22}$$

$$c = \pm \frac{125}{4} \tag{2.0.23}$$

$$\mathbf{F} = \pm \begin{pmatrix} 4 \\ 0 \end{pmatrix}. \tag{2.0.24}$$

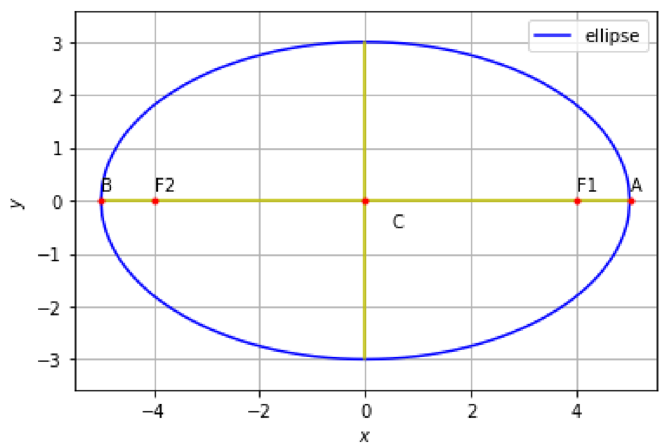

Fig. 0: Plot of ellipse