Tutorial-6

Q1. Ans.

Minimum spanning tree is a subset of edges of a connected edge-weighted undirected graph that connects all the vertices together without any cycles of width minimum possible edge weighted.

APPLICATIONS:

i) consider n stations are to be linked using a communication network and laying of communication link between any two stations involves a cost. The ideal solution would be to extract a subgraph termed as minimum cost spanning tree.

ii) Designing LAN

iii) suppose you want to construct highways and railroads spanning several cities, then we can use concept of MST.

iv) laying pipeline connecting offshore drilling sites, refineries of consumer market.

Ans 2: Time complexity of Prim's Algorithm : $O(|E| \log |V|)$

Space complexity of Prim's Algorithm : $O|V|$

Time complexity of Kruskal's Algorithm : $O|E| \log |E|$

Space complexity of Kruskal's Algorithm : $O|V|$

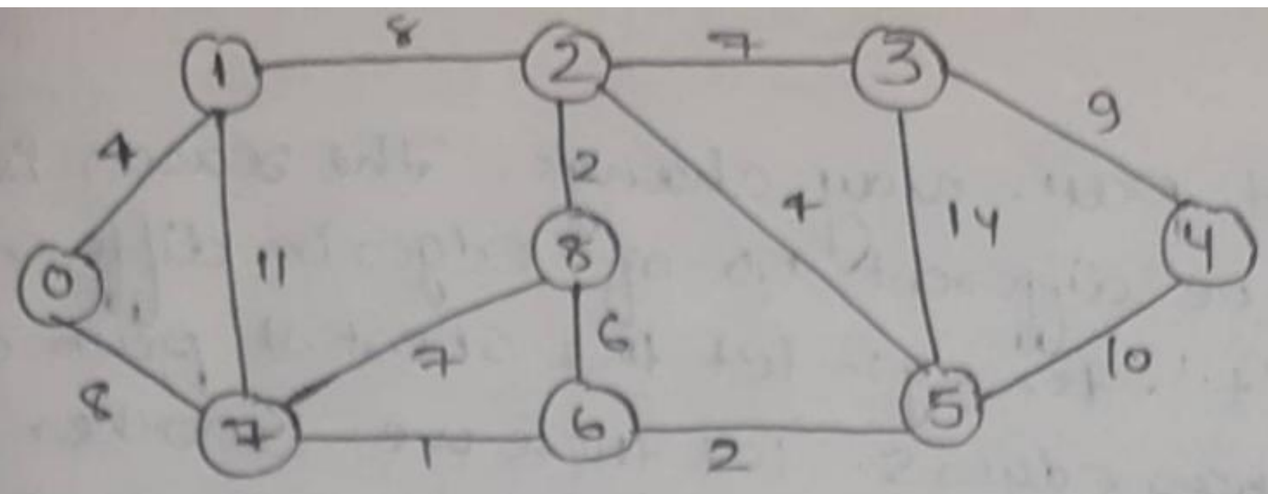
Time complexity of Dijkstra's Algorithm : $O(V)^2$

Space complexity of Dijkstra's Algorithm : $O(V^2)$

Time complexity of Bellman ford's Algorithm : $O(VE)$

Space complexity of Bellman ford's Algorithm : $O(E)$

Ans 3 :

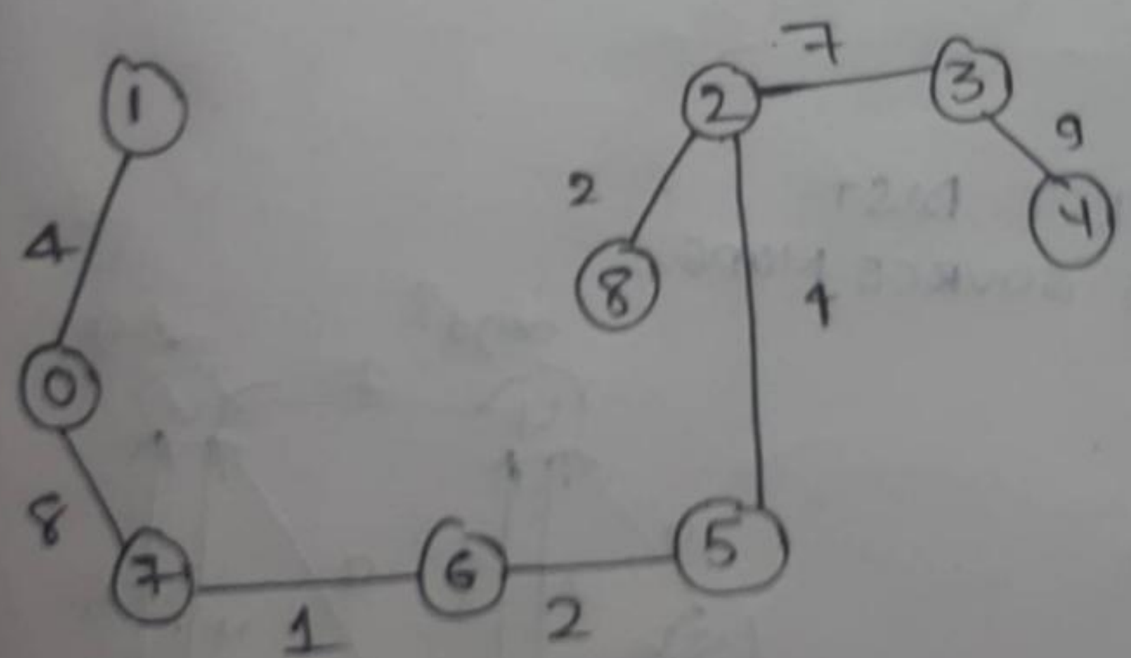


Kruskal's Algorithm :-

0	v	w	
6	7	1	✓
5	6	2	✓
2	8	2	✓
0	1	4	✓
2	5	4	✓
6	8	6	X
2	3	7	✓
7	8	7	X
0	7	8	✓
1	2	8	X
4	3	9	✓
4	5	10	X
1	7	11	X
3	5	14	X

Prim's Algorithm

weight :-
4+8+2+4+2+7+9+3+2
=37



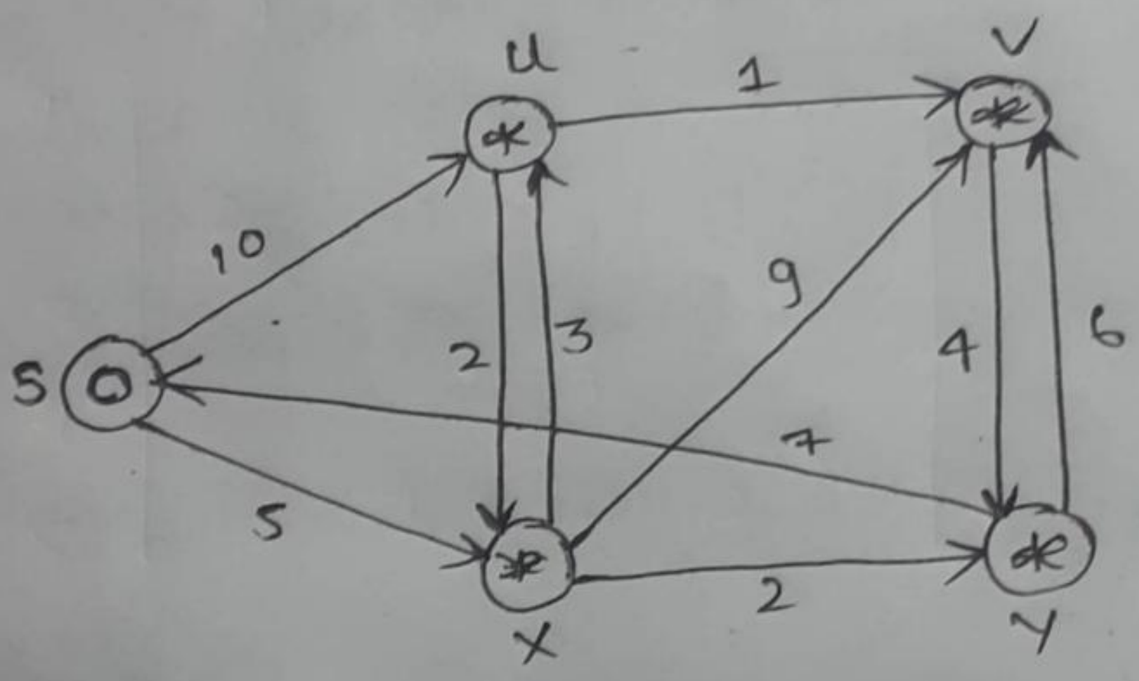
weight = 1+2+2+4+4+7+8+9
= 37

4 (i) T1

Ans :- The shortest path may change. The reason is that there may be different no. of edges in different paths from 's' to 't'. for eg :- let the shortest path of weight 15 and has edges 5. let these we another path with 2 edges and total weight 25. the weight of shortest path is increased by 5*10 and becomes 15+50 weight of other path is increased by 2*10 of becomes 26+20. so, the shortest path changes to other path with weight as 45.

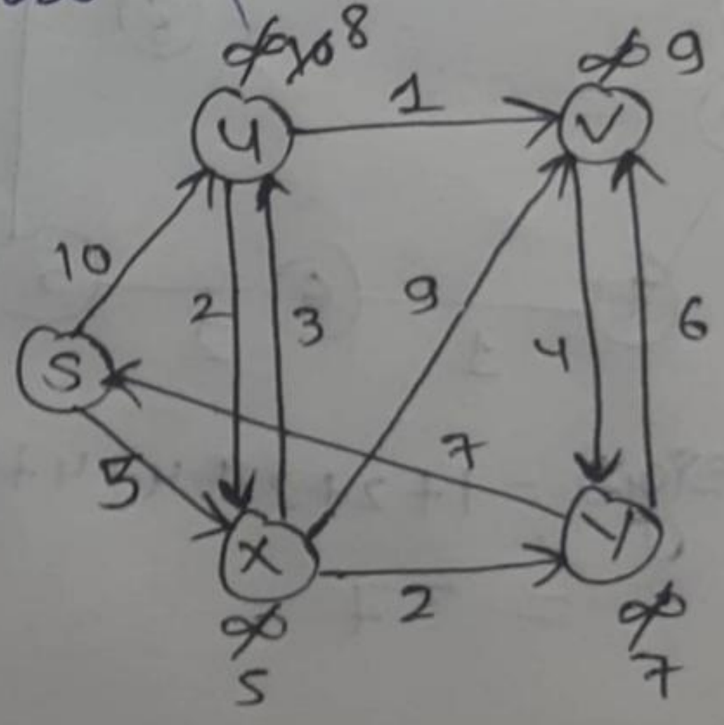
ii) If we multiply all edges weight by 10, the shortest path does not change. the reason is that weights of all paths from 's' to 't' gets multiplied by same unit. the number of edges or path doesn't matter.

5.



Dijkastra's Algorithm

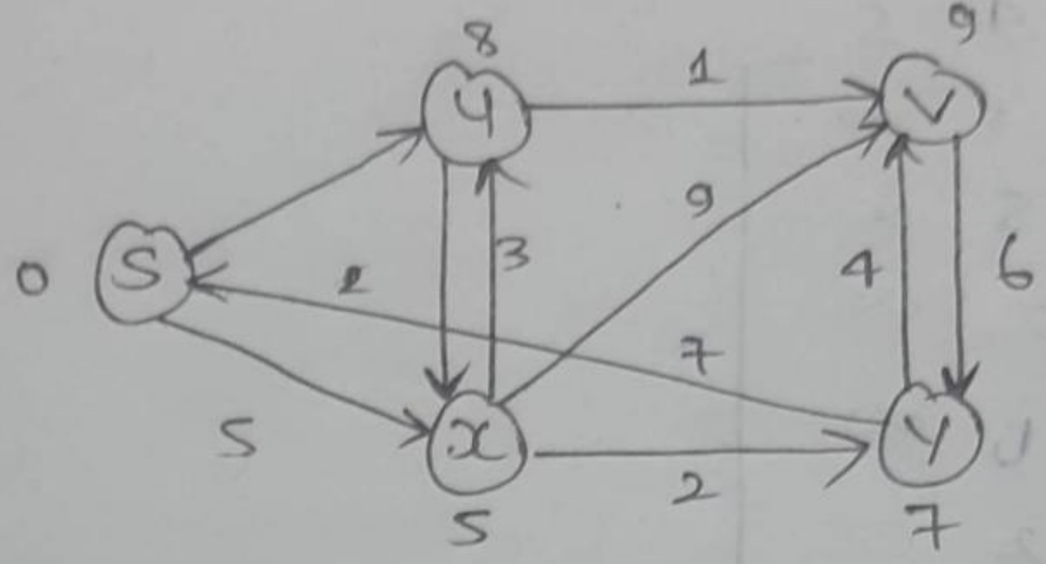
NODE	SHORTEST DIST FROM SOURCE NODE
u	8
x	5
v	9
y	7



Bell man ford Algorithm

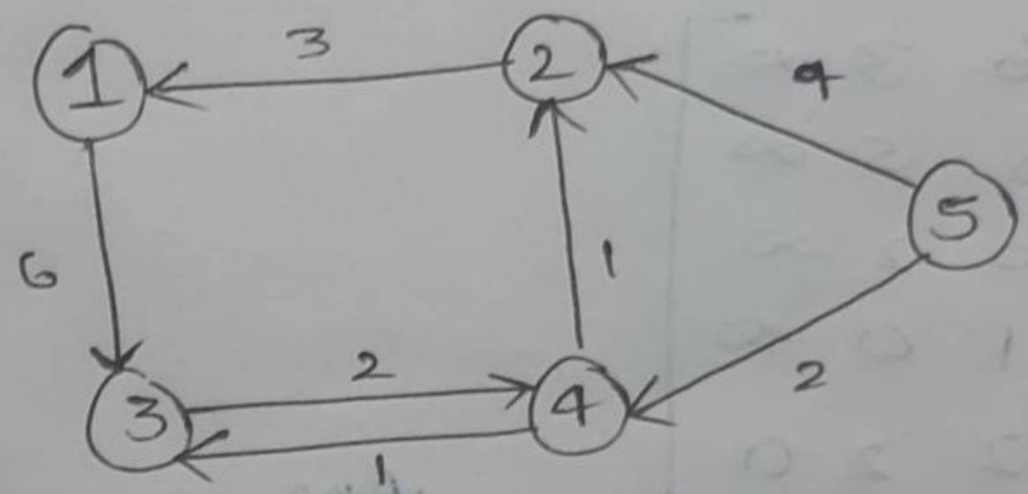
	S	U	V	X	Y
1st →	0	∞ 10	∞	∞ 5	∞
2nd →	0	10 6	∞ 11	5	∞
3rd →	0	10 8	11 9	5	∞ 7
4th →	0	8	9	5	7

graph doesnot have negative cycle.



→ final graph.

Ans 6



	1	2	3	4	5
1	0	∞	6	3	∞
2	2	0	∞	∞	∞
3	∞	∞	0	2	∞
4	∞	1	1	0	∞
5	∞	4	∞	2	0

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array}\begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 0 & \infty & 6 & 3 & \infty \\ 2 & 0 & 8 & 5 & \infty \\ \infty & \infty & 0 & 2 & \infty \\ \infty & 1 & 1 & 0 & \infty \\ \infty & 4 & \infty & 2 & 0 \end{bmatrix} \end{array}$$

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array}\begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 0 & \infty & 6 & 3 & \infty \\ 2 & 0 & 8 & 5 & \infty \\ \infty & \infty & 0 & 2 & \infty \\ 3 & 1 & 1 & 0 & \infty \\ 6 & 4 & 12 & 2 & 0 \end{bmatrix} \end{array}$$

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array}\begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 0 & \infty & 6 & 3 & \infty \\ 2 & 0 & 8 & 5 & \infty \\ \infty & \infty & 0 & 2 & \infty \\ 3 & 1 & 1 & 0 & \infty \\ 6 & 4 & 12 & 2 & 0 \end{bmatrix} \end{array}$$

Ans

Time complexity $\rightarrow O(|V|^3)$

Space complexity $\rightarrow O(|V|^2)$