Baye's theorem

$$P(X/y) = \frac{P(Y/x) \cdot P(x)}{P(Y)}$$

Q/ **Example:-** Suppose a disease has a prevalence rate (prior probability) of 1% (0.01). The rest for the disease has a sensitivity of 99% (0.99) and false positive rate of 5% (0.05)

Sol => given

$$P(D) = 0.01$$

$$P(\neg D) = 0.99$$

$$P(T/D) = 0.99$$

$$P(T/\neg D) = 0.05$$

Calculate $P(T)$:-

$$P(T) = P(T/D) \cdot P(D) + P(T/\neg D) \cdot P(\neg D)$$

$$P(T) = 0.99 \times 0.01 + 0.05 \times 0.99$$

$$= 0.0594$$

$$P(D/T) = \frac{P(T/D) \cdot P(D)}{P(T)}$$

$$= \frac{0.99 \times 0.01}{0.0594} = \frac{0.0099}{0.0594}$$

$$\approx 0.1667$$

2) Eigenvalues & Eigenvectors of a matrix

Let $A = \begin{pmatrix} 4 & 1 \\ 2 & 3 \end{pmatrix}$

$\Rightarrow \begin{pmatrix} 4-\lambda & 1 \\ 2 & 3-\lambda \end{pmatrix} = 0$

$\Rightarrow (4-\lambda).(3-\lambda) - 2 \times 1 = 0$

$\Rightarrow \lambda^2 - 7\lambda + 10 = 0$

$\lambda = \dfrac{7 \pm \sqrt{49-40}}{2}$

$\lambda = \cancel{7 \pm 3} \quad \dfrac{7 \pm 3}{2}$

$\lambda_1 = \dfrac{7+3}{2} \quad ; \quad \lambda_2 = \dfrac{7-3}{2}$

Eigenvalue $\Rightarrow \lambda_1 = 5 \quad ; \quad \lambda_2 = 2$

For Eigen Vector

$\lambda_1 = 5$

$\begin{pmatrix} 4-\lambda & 1 \\ 2 & 3-\lambda \end{pmatrix} = \begin{pmatrix} 4-5 & 1 \\ 2 & 3-5 \end{pmatrix} = \begin{pmatrix} -1 & 1 \\ 2 & -2 \end{pmatrix}$

Now, $\begin{pmatrix} -1 & 1 \\ 2 & -2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 0$

$-x + y = 0$

$y = x$

Eigenvector corresponding to $\lambda_1 = 5$ is $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$\lambda_2 = 2$

$\begin{pmatrix} 2 & 1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 0$

$2x + y = 0$

$y = -2x$

Eigen Vector $\lambda_2 = 2$ is $\begin{pmatrix} 1 \\ -2 \end{pmatrix}$

Q3 Determinant and inverse of a 3×3 Matrix

let $B = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 5 & 6 & 0 \end{pmatrix}$

$$\det(B) = 1(1\times0+6\times4) - 2(0\times0-4\times5) + 3(0\times6-1\times5)$$
$$= 1$$

$$B^{-1} = \frac{1}{\det(B)} \cdot \mathrm{adj}(B)$$

where adj(B) is the adjugete of B

$$C_{11} = \det\begin{pmatrix} 1 & 6 \\ 4 & 0 \end{pmatrix} = 1\times0 - 6\times4 = -24$$
$$C_{12} = -\det\begin{pmatrix} 2 & 6 \\ 3 & 0 \end{pmatrix} = -(2\times0-6\times3) = 18$$
$$C_{13} = \det\begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix} = (2\times4-1\times3) = 5$$
$$C_{21} = -\det\begin{pmatrix} 0 & 5 \\ 4 & 0 \end{pmatrix} = -(0\times0-5\times4) = 20$$
$$C_{22} = \det\begin{pmatrix} 1 & 0 \\ 3 & 4 \end{pmatrix} = 1\times0 - 5\times3 = -15$$
$$C_{23} = -\det\begin{pmatrix} 1 & 0 \\ 3 & 4 \end{pmatrix} = -(1\times4-0\times3) = -4$$
$$C_{31} = \det\begin{pmatrix} 0 & 5 \\ 1 & 6 \end{pmatrix} = 6\times0 - 5\times1 = -5$$
$$C_{32} = -\det\begin{pmatrix} 1 & 5 \\ 2 & 6 \end{pmatrix} = -(1\times6-5\times2) = 4$$
$$C_{33} = \det\begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} = 1\times1 - 0\times2 = 1$$

$$\mathrm{Cof}(B) = \begin{pmatrix} -24 & 20 & -5 \\ 18 & -15 & 4 \\ 5 & -4 & 1 \end{pmatrix}$$

$$B^{-1} = \frac{1}{\det(B)} \cdot adj(B) = 1 \cdot \begin{pmatrix} -24 & 20 & -5 \\ 18 & -15 & 4 \\ 5 & -4 & 1 \end{pmatrix}$$

$$B^{-1} = \begin{pmatrix} -24 & 20 & -5 \\ 18 & -15 & 4 \\ 5 & -4 & 1 \end{pmatrix}$$

Q4) Normal Distribution

Properties

- Symmetrical about the mean μ
- The mean, median and mode are equal
- The area under the curve is 1

Suppose :- the height of adult men are normally distributed with a mean $\mu = 70$ inches and a standard deviation $\sigma = 3$ inches. To find the probability that a randomly selected man is taller than 74 inches.

Solu

z - score $z = \frac{74-70}{3} = \frac{4}{3} \approx 1.33$

Using standard normal distribution table or a calculator, find $P(z > 1.33)$

$P(z > 1.33) \approx 0.0918$

The man is taller than 74 inches is appx. 9.18%.