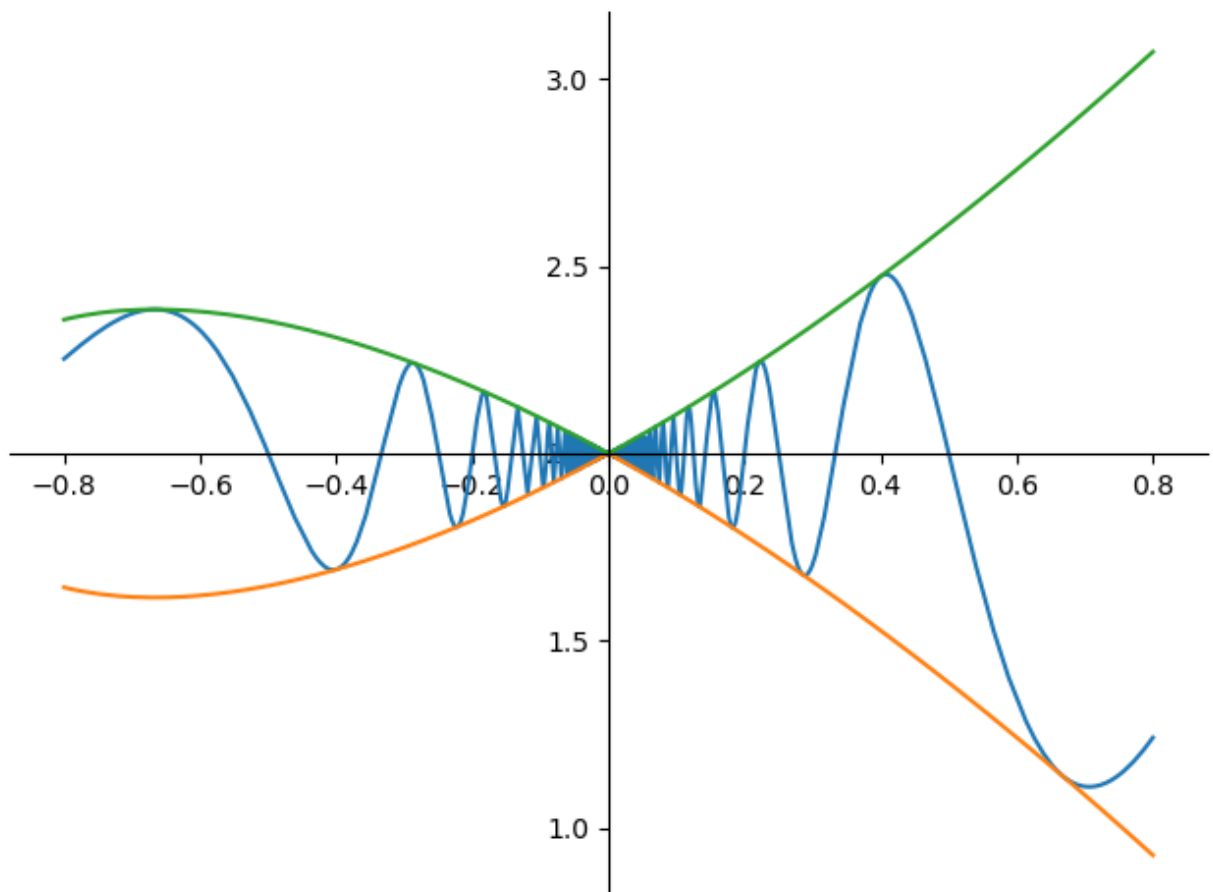


```
In [21]: print('Jaeheon Kim, Nico Bui, Alex Dao, Vishwa Kumaravel')
```

Jaeheon Kim, Nico Bui, Alex Dao, Vishwa Kumaravel

```
In [20]: from sympy import *
from sympy.plotting import (plot, plot_parametric)
x=symbols('x')
f = sqrt(x**3 + x**2) * sin(pi/x) + 2
g = -sqrt(x**3 + x**2) + 2
h = sqrt(x**3 + x**2) + 2
print('part 1a')
plot_parametric((x, f), (x, g), (x, h), (x, -0.8, 0.8))
print('part (1b)')
print("The squeeze theorem states that if  $g(x) \leq f(x) \leq h(x)$  for all  $x$  in some interval, a
print("then  $\lim_{x \rightarrow 0} f(x) = L$ ")
print("to solve  $\lim_{x \rightarrow 0} g(x)$  we plug in 0 into the  $x$  of  $-\sqrt{x^3 + x^2} + 2$  which become
print("to solve  $\lim_{x \rightarrow 0} h(x)$  we plug in 0 into the  $x$  of  $\sqrt{x^3 + x^2} + 2$  which becomes
print("lim $\rightarrow 0$   $g(x) = 2$  and lim $\rightarrow 0$   $h(x) = 2$  so lim $\rightarrow 0$   $f(x) = 2$ ")
print('part (1c)')
print ("lim $\rightarrow 0$   $g(x)$  is " , limit( g , x , 0) )
print ("lim $\rightarrow 0$   $h(x)$  is " , limit( h , x , 0) )
print ("lim $\rightarrow 0$   $f(x)$  is " , limit( f , x , 0) )
print("This shows that the limits match the reasoning of part B")
```

part 1a



part (1b)

The squeeze theorem states that if $g(x) \leq f(x) \leq h(x)$ for all x in some interval, and $\lim_{x \rightarrow 0} g(x) = \lim_{x \rightarrow 0} h(x) = L$

then $\lim_{x \rightarrow 0} f(x) = L$

to solve $\lim_{x \rightarrow 0} g(x)$ we plug in 0 into the x of $-\sqrt{x^3 + x^2} + 2$ which becomes $-0 + 2 = 2$

to solve $\lim_{x \rightarrow 0} h(x)$ we plug in 0 into the x of $\sqrt{x^3 + x^2} + 2$ which becomes $0 + 2 = 2$

$\lim_{x \rightarrow 0} g(x) = 2$ and $\lim_{x \rightarrow 0} h(x) = 2$ so $\lim_{x \rightarrow 0} f(x) = 2$

part (1c)

$\lim_{x \rightarrow 0} g(x)$ is 2

$\lim_{x \rightarrow 0} h(x)$ is 2

$\lim_{x \rightarrow 0} f(x)$ is 2

This shows that the limits match the reasoning of part B

```
In [19]: from sympy import *

x = symbols('x')
g_left = -2
g_middle1 = 2*x - 5
g_middle2 = (x**2 - 2*x - 8) / (x - 4)
g_right = E**((x - 4) * ln(7))
print('part 2(a)')
print("The left - hand limit is", limit ( g_left ,x , 1) )
print("The right - hand limit is", limit ( g_middle1 ,x , 1) )
print('because the left and right limit do not match, f(1) is not continuous')
print('part 2(b)')
print("The left - hand limit is", limit ( g_middle1 ,x , 3) )
print("The right - hand limit is", limit ( g_middle2 ,x , 3) )
print('because the left and right limit do not match, f(3) is not continuous')
print('part 2(c)')
print("The left - hand limit is", limit ( g_middle2 ,x , 5) )
print("The right - hand limit is", limit ( g_right ,x , 5) )
print('because the left and right limit do match, f(5) is continuous')
print('part 2(d)')
plot((g_left,(x,0,1)),
      (g_middle1,(x,1,3)),
      (g_middle2,(x,3,5)),
      (g_right,(x,5,6)))
```

part a

The left - hand limit is -2

The right - hand limit is -3

because the left and right limit do not match, f(1) is not continuous

part b

The left - hand limit is 1

The right - hand limit is 5

because the left and right limit do not match, f(3) is not continuous

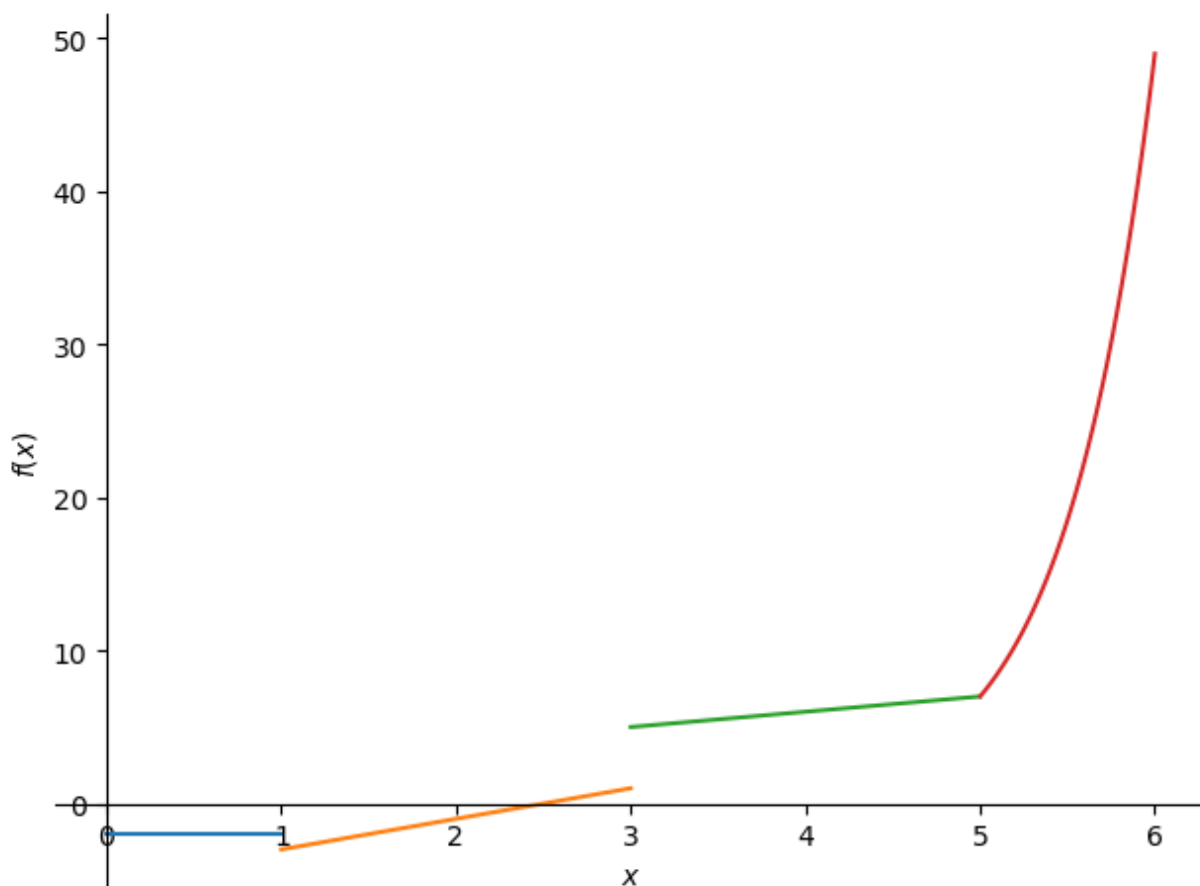
part c

The left - hand limit is 7

The right - hand limit is 7

because the left and right limit do match, f(5) is continuous

part d



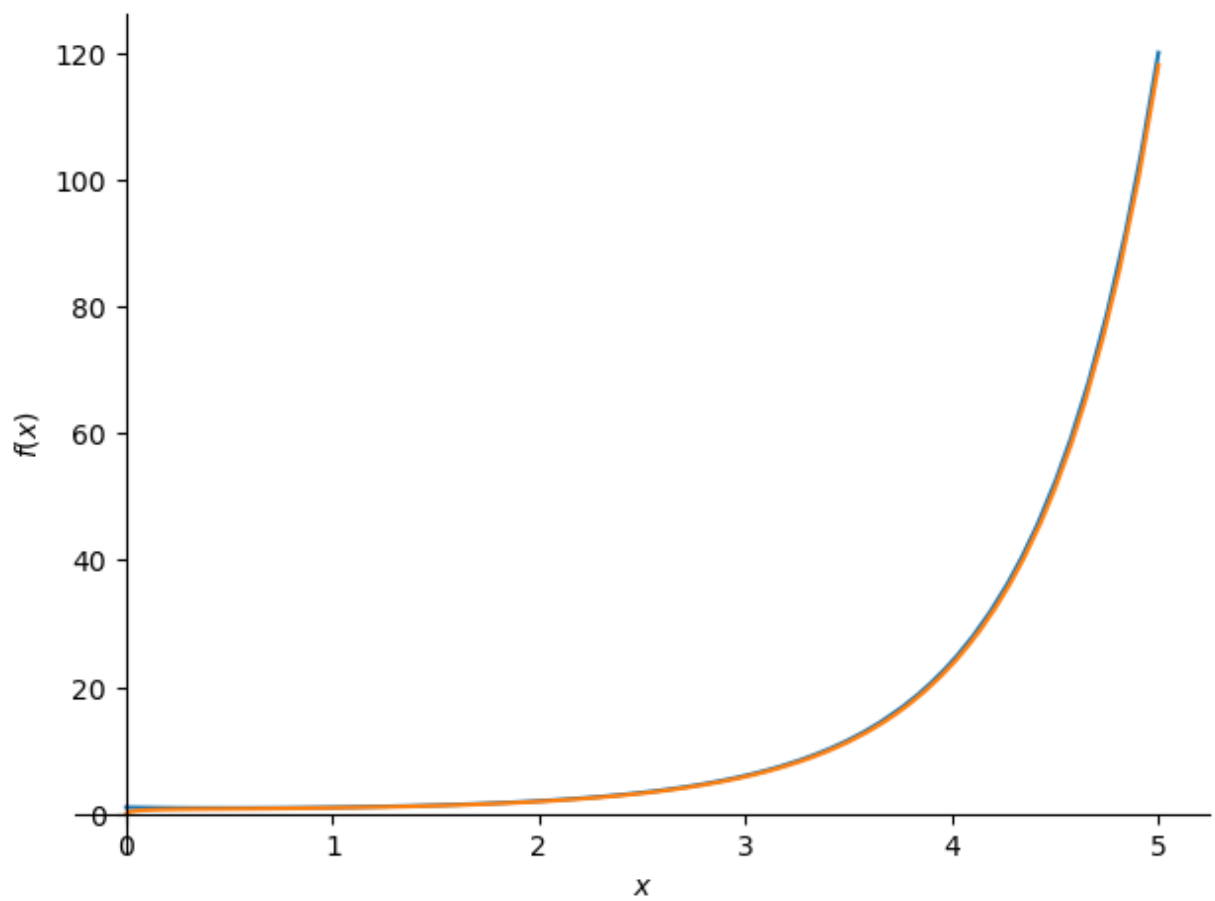
Out[19]: <sympy.plotting.plot.Plot at 0x1edd389f110>

```
In [13]: print('3(a)')
x = symbols('x')
f = factorial(x)
g = sqrt(2*pi*x)*(x/E)**x
proof = limit((g/f),x,oo)
print('g(x) is a good approximation since limit((g/f),x,oo) is', proof)
print('\3(b)')
plot(f,g,(x,0,5))
print('f(x) and g(x) have the same graph, so this is a nice approximation')
```

3(a)

g(x) is a good approximation since $\lim_{x \rightarrow \infty} (g/f)$ is 1

3(b)



$f(x)$ and $g(x)$ have the same graph, so this is a nice approximation

In []: