

MATH 151 Lab 9

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```
In [45]: from sympy import *
from sympy.plotting import (plot,plot_parametric)
```

Question 1

1a

```
In [46]: from sympy import symbols, ln, limit, oo
print('part a')
# Define the variable
x = symbols('x')
# Given expression for y
y = (1 + (16 / x)) ** (2*x)
print(f'f(x) is 2ln(1+16/x) g(x) is x**-1 ')
f = 2*ln(1+16/x)
g = x**-1
```

part a
f(x) is 2ln(1+16/x) g(x) is x**-1

1b

```
In [47]: # Find the Limits of f and g as x → ∞
limit_f = limit(f, x, oo)
limit_g = limit(g, x, oo)
print(f'limit f: {limit_f}, limit g: {limit_g}')
```

limit f: 0, limit g: 0

1c

```
In [48]: from sympy import diff
# Apply L'Hopital's Rule
f_prime = diff(f, x)
g_prime = diff(g, x)
limit_lhopital = E**limit(f_prime/g_prime, x, oo)
print('f prime is:',f_prime, 'g prime is:',g_prime, ',and lim y:',limit_lhopital)
```

f prime is: -32/(x**2*(1 + 16/x)) g prime is: -1/x**2 ,and lim y: exp(32)

1d

```
In [49]: # Evaluate the Limit directly
limit_direct = limit(y, x, oo)
limit_direct
print(f'{exp(32)} from 1(c) and {limit_direct} from 1(d) are the same')
```

exp(32) from 1(c) and exp(32) from 1(d) are the same

Question 2

2a

```
In [50]: x = symbols('x')# x is theta
f = (20+20*cos(x))*10*sin(x)/2
ans = solve(diff(f))
print(f'possible theta values: {ans}')
print(f'volume when theta is {ans[0]}: {f.subs(x,ans[0])}')
print(f'volume when theta is {ans[1]}: {f.subs(x,ans[1])}')
print('Volume is maximum when thetat is ',ans[1], 'which is',float(ans[1]))
```

possible theta values: $[-\pi/3, \pi/3]$
 volume when theta is $-\pi/3$: $-75\sqrt{3}$
 volume when theta is $\pi/3$: $75\sqrt{3}$
 Volume is maximum when thetat is $\pi/3$ which is 1.0471975511965979

2b

```
In [51]: ddf = diff(f,x,2)
maxi = ddf.subs(x,ans[1])
print(f'You can verify the maximum by checking if the f double prime becomes negative
```

You can verify the maximum by checking if the f double prime becomes negative when $\pi/3$ is substituted. When $\pi/3$ is substituted, d double prime is $-150\sqrt{3}$, which is negative, so we can say $\pi/3$ is the maximum

Question 3

3a

```
In [52]: from sympy import *
from sympy.plotting import (plot,plot_parametric)
x = symbols('x')
c = symbols('c')
d = symbols('d')
fx_2_prime = 4 / (x + 1)**2
fx_prime = integrate(fx_2_prime, x)
c = 3 - fx_prime.subs(x, 0)
fx_prime = fx_prime + c
fx = integrate(fx_prime, x)
d = 9 - fx.subs(x, 0)
fx = fx + d
print(f"f'(x) = {fx_prime}")
print(f"f(x) = {fx}")
```

$f'(x) = 7 - 4/(x + 1)$
 $f(x) = 7x - 4\log(x + 1) + 9$

3b

```
In [53]: from sympy import *
from sympy.plotting import (plot,plot_parametric)
x = symbols('x')
c = symbols('c')
```

```
d = symbols('d')
fx_2_prime = 4 / (x + 1)**2
fx_prime = integrate(fx_2_prime, x)
fx = integrate(fx_prime, x)
print(f"f(x) = {fx} + cx + d")
print(f"c + d = {10 - fx.subs(x, 1)}")
print(f"4c + d = {10 - fx.subs(x, 4)}")
print("Solve the Systems of Equations")
c = -(4*(log(2)-log(5)))/3
d = 10 + ((16*log(2)-4*log(5))/3)
print("Plug vals into fx equation")
fx = fx + c*x + d
print(f"f(x) = {fx}")
```

$f(x) = -4*\log(x + 1) + cx + d$

$c + d = 4*\log(2) + 10$

$4c + d = 4*\log(5) + 10$

Solve the Systems of Equations

Plug vals into fx equation

$f(x) = x*(-4*\log(2)/3 + 4*\log(5)/3) - 4*\log(x + 1) - 4*\log(5)/3 + 16*\log(2)/3 + 10$

In []: