Fuel Efficiency Prediction: Linear Regression

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In this notebook, we will predict the fuel consumption rate of cars using the autompg dataset which is available on the popular UCI machine learning repository. To learn more about linear regression refer here.

Learning Outcome: By following the notebook you will be able to

- 1. Perform context inspired EDA to understand relationship between predictor variables and mpg (fuel consumption variable)
- 2. Implement & infer Simple Linear Regression
- 3. Plot OLS regression line & diagnostic plot
- 4. Implement & infer multiple linear regression model
- 5. Integrate interaction effects in the model
- 6. Implement non-linear transformation to the model

Setup

Here, we will load the libraries and packages.

```
library(MASS)
library(ISLR)
library(tidyverse)
library(ggplot2)
library(corrplot)
library(car)
```

The Auto dataset is present in the ISLR package and hence can be called directly

Exploratory Data Analysis

Glimpse of the data

Let us first look at the head of the dataset.

```
##
     mpg cylinders displacement horsepower weight acceleration year origin
                              307
                                                3504
                                                              12.0
                                                                     70
## 1
     18
                                         130
                  8
## 2
     15
                              350
                                         165
                                                3693
                                                              11.5
                                                                     70
                                                                              1
## 3 18
                  8
                              318
                                         150
                                                3436
                                                              11.0
                                                                     70
                                                                              1
                  8
                                         150
                                                              12.0
                                                                     70
                                                                              1
## 4
      16
                              304
                                                3433
                  8
                                                                     70
                                                                              1
## 5
      17
                              302
                                         140
                                                3449
                                                              10.5
## 6
     15
                  8
                              429
                                         198
                                                4341
                                                              10.0
                                                                     70
                                                                              1
##
                           name
## 1 chevrolet chevelle malibu
## 2
             buick skylark 320
## 3
            plymouth satellite
```

```
## 4      amc rebel sst
## 5      ford torino
## 6      ford galaxie 500

cat("The auto dataset shape is ", dim(Auto)[1], "x", dim(Auto)[2])
```

```
## The auto dataset shape is 392 \times 9
```

We notice tht there are 9 columns. Out of which a few look categorical such as model year and origin. Additionally, name is a string. Let's explore the summary of the data to undertand the data type of variables.

summary(Auto)

```
##
                       cylinders
                                        displacement
                                                          horsepower
         mpg
##
    Min.
           : 9.00
                             :3.000
                                              : 68.0
                                                               : 46.0
                     Min.
                                      Min.
                                                        Min.
                     1st Qu.:4.000
                                       1st Qu.:105.0
                                                        1st Qu.: 75.0
##
    1st Qu.:17.00
##
    Median :22.75
                     Median :4.000
                                      Median :151.0
                                                        Median: 93.5
##
    Mean
            :23.45
                     Mean
                             :5.472
                                      Mean
                                              :194.4
                                                                :104.5
                                                        Mean
    3rd Qu.:29.00
##
                     3rd Qu.:8.000
                                       3rd Qu.:275.8
                                                        3rd Qu.:126.0
            :46.60
                             :8.000
                                              :455.0
                                                                :230.0
##
    Max.
                     Max.
                                      Max.
                                                        Max.
##
##
        weight
                     acceleration
                                           year
                                                           origin
##
    Min.
           :1613
                    Min.
                            : 8.00
                                     Min.
                                             :70.00
                                                       Min.
                                                              :1.000
    1st Qu.:2225
                                                       1st Qu.:1.000
##
                    1st Qu.:13.78
                                     1st Qu.:73.00
##
    Median:2804
                    Median :15.50
                                     Median :76.00
                                                       Median :1.000
##
    Mean
            :2978
                    Mean
                            :15.54
                                     Mean
                                             :75.98
                                                       Mean
                                                              :1.577
                    3rd Qu.:17.02
##
    3rd Qu.:3615
                                     3rd Qu.:79.00
                                                       3rd Qu.:2.000
##
    Max.
            :5140
                    Max.
                            :24.80
                                     Max.
                                             :82.00
                                                       Max.
                                                              :3.000
##
##
                     name
##
    amc matador
                       :
                           5
##
    ford pinto
                           5
##
    toyota corolla
                           5
##
    amc gremlin
##
    amc hornet
                           4
                           4
##
    chevrolet chevette:
##
    (Other)
                        :365
```

From summary we notice that origin, and cylinder and year could be categorical variables as they seem to be whole numbers with a limited range. Looking at the data variable this is not evident as they are all numeric value.

str(Auto)

```
##
  'data.frame':
                   392 obs. of 9 variables:
##
   $ mpg
                 : num
                        18 15 18 16 17 15 14 14 14 15 ...
##
   $ cylinders
                        888888888...
                 : num
   $ displacement: num
                        307 350 318 304 302 429 454 440 455 390 ...
   $ horsepower
                        130 165 150 150 140 198 220 215 225 190 ...
                 : num
   $ weight
                        3504 3693 3436 3433 3449 ...
                 : num
```

```
## $ acceleration: num 12 11.5 11 12 10.5 10 9 8.5 10 8.5 ...
## $ year : num 70 70 70 70 70 70 70 70 ...
## $ origin : num 1 1 1 1 1 1 1 1 1 1 ...
## $ name : Factor w/ 304 levels "amc ambassador brougham",..: 49 36 231 14 161 141 54 223 241 :
```

We notice that all the variables have the correct data type except name which can be converted to string.

Number of unique values in each column

Let's check out the number of unique values in each column.

```
no_unique_values <- function(column){
  unique_values_list <- unique(column)
  return(length(unique_values_list))
}
apply(Auto ,2,no_unique_values)</pre>
```

```
##
                    cylinders displacement
                                                                  weight
             mpg
                                               horsepower
             127
                                                                     346
##
                             5
                                          81
                                                        93
## acceleration
                                      origin
                          year
                                                      name
                                                       301
                            13
```

Thus, We have established that: * mpg: continuous

• cylinders: multi-valued discrete

• displacement: continuous

• horsepower: continuous

• weight: continuous

ullet acceleration: continuous

• model year: multi-valued discrete

• origin: multi-valued discrete

• car name: string (unique for each instance)

Number of missing values in the dataset

```
no_missing_values <- function(column){
  return(sum(is.na(column)))
}
cat("The number of missing values in each column\n\n\n")</pre>
```

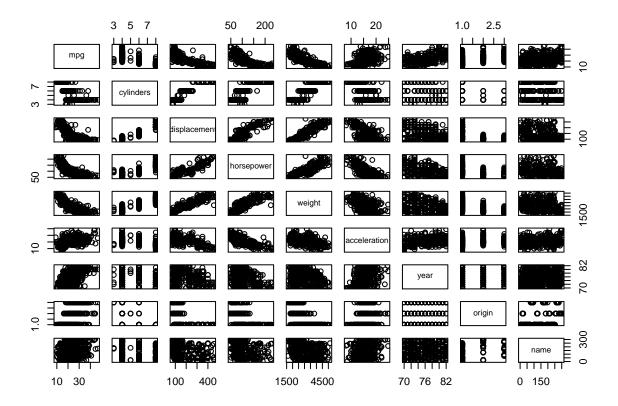
The number of missing values in each column

apply(Auto ,2,no_missing_values)

```
## mpg cylinders displacement horsepower weight
## 0 0 0 0 0 0
## acceleration year origin name
## 0 0 0 0
```

Scatterplot Matrix

```
plot(Auto)
```

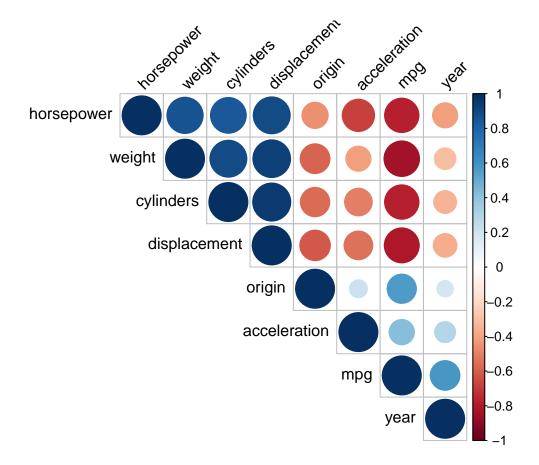


Vieweing the scatterplot along with the summary makes our observation about the nature of the variable more clear. From the scatterplot we can also notice the relationship trend between our response variable mpg and the other potential predictor variable.

We notive that horsepower, weight, displacement have a clear relationship with mpg though not necessarily linear

Correlation Matrix

```
corr_matrix <- cor(Auto[, -which(names(Auto)=="name")])
corrplot(corr_matrix, type = "upper", order = "hclust", tl.col = "black", tl.srt = 45)</pre>
```



We notice here that mpg has significant correlation with 1. horsepower

- 2. weight
- 3. cylinders
- 4. displacement
- 5. year

Clean Data

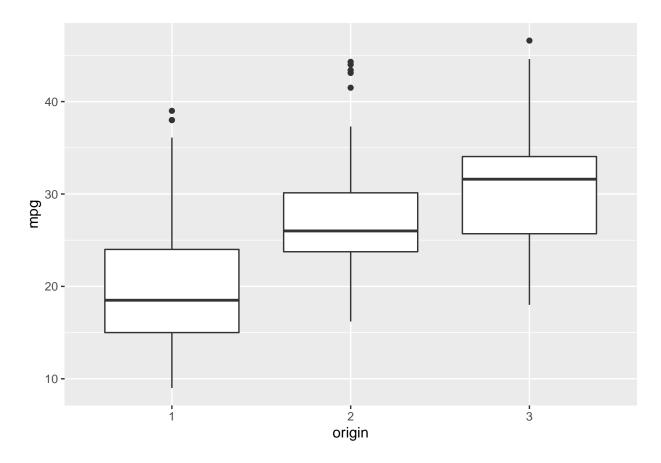
Convert to categorical form i.e. factor

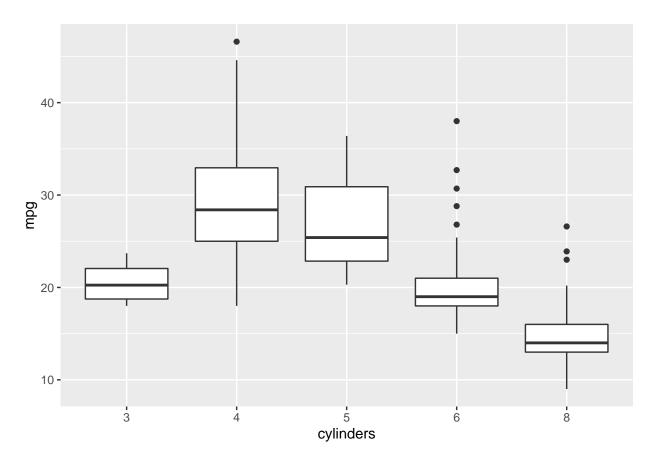
```
discrete_col <- c("origin", 'year', 'cylinders')
Auto[, discrete_col] <- lapply(Auto[, discrete_col], factor)
str(Auto[, discrete_col])

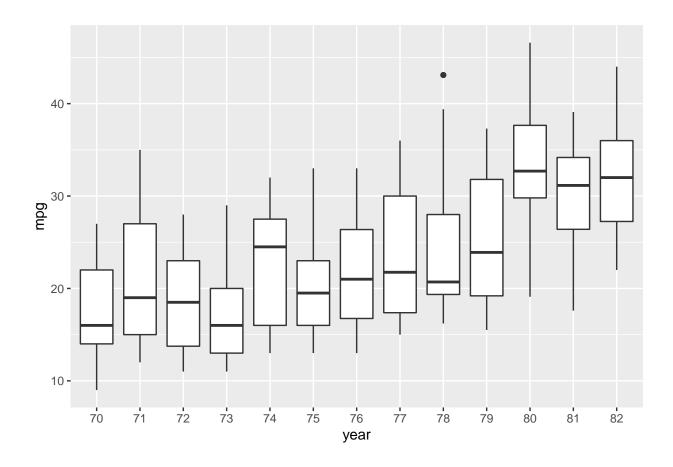
## 'data.frame': 392 obs. of 3 variables:
## $ origin : Factor w/ 3 levels "1","2","3": 1 1 1 1 1 1 1 1 1 1 1 1 ...
## $ year : Factor w/ 13 levels "70","71","72",..: 1 1 1 1 1 1 1 1 1 1 ...
## $ cylinders: Factor w/ 5 levels "3","4","5","6",..: 5 5 5 5 5 5 5 5 5 5 ...</pre>
```

Now, we can visualize the relationship between the response and other variables

Explore relationship of mpg with discrete variable







Simple Linear Regression

For building our simple linear regression model, using our findings from scatterplot and correlation matrix, we are using the following variables.

 ${\bf Predictor\ Variable: horsepower\ Response\ Variable: mpg}$

```
simple.lm <- lm(mpg ~ horsepower)
summary(simple.lm)</pre>
```

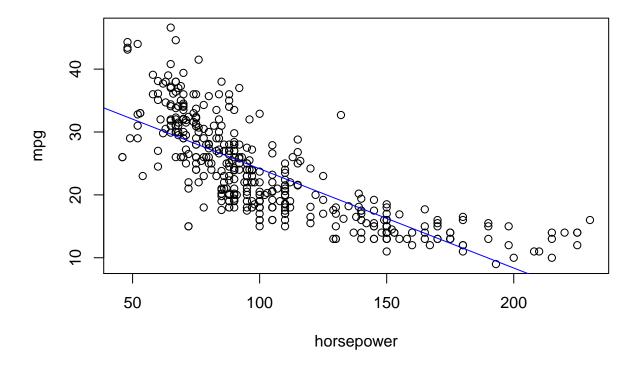
```
##
## Call:
## lm(formula = mpg ~ horsepower)
##
## Residuals:
##
                 1Q
                      Median
                                   3Q
                                           Max
  -13.5710 -3.2592 -0.3435
##
                               2.7630
                                       16.9240
##
## Coefficients:
               Estimate Std. Error t value Pr(>|t|)
## (Intercept) 39.935861
                          0.717499
                                     55.66
                                             <2e-16 ***
## horsepower -0.157845
                          0.006446
                                    -24.49
                                             <2e-16 ***
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
```

```
## Residual standard error: 4.906 on 390 degrees of freedom
## Multiple R-squared: 0.6059, Adjusted R-squared: 0.6049
## F-statistic: 599.7 on 1 and 390 DF, p-value: < 2.2e-16</pre>
```

Inference from Simple Linear Regression model: mpg ~ horsepower

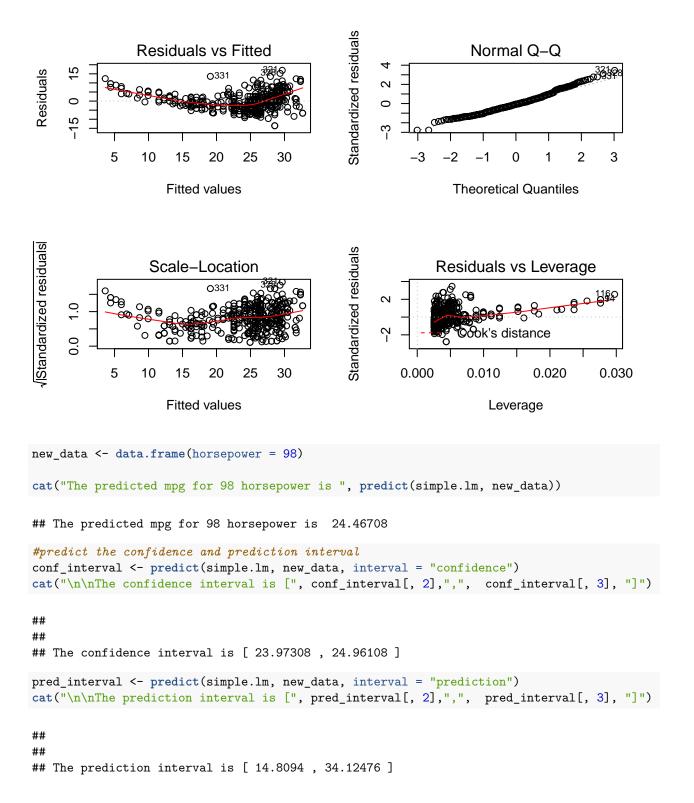
- 1. From the p-value associated with the F-statistic, we know that there is a strong association between horsepower & mpg.
- 2. As the coefficient estimate is negative, the relationship between mpg and horsepower is negative.
- 3. The fit can be observed below which shows a curved pattern which is missed by the straight line assumption of linear model

```
plot(horsepower, mpg)
abline(simple.lm, col = 'blue')
```



This non-linear relationship is highlighted in the residual vs fitted diagnostic plot.

```
diagnostic_plot <- function(model){
  par(mfrow=c(2,2))
  plot(model)
}
diagnostic_plot(simple.lm)</pre>
```



Multiple Linear Regression

For building our simple linear regression model, using our findings from scatterplot and correlation matrix, we are using all the predictor vairables expect name.

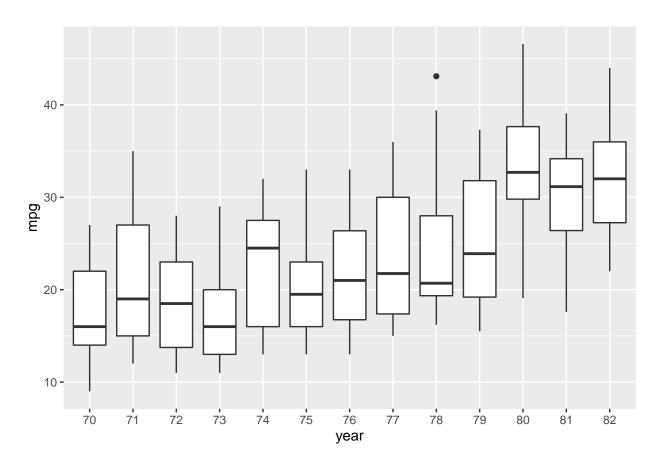
```
##
## Call:
## lm(formula = mpg ~ . - name, data = Auto)
## Residuals:
##
       Min
                1Q Median
                                3Q
                                       Max
##
  -7.9267 -1.6678 -0.0506
                            1.4493 11.6002
##
## Coefficients:
##
                  Estimate Std. Error t value Pr(>|t|)
## (Intercept) 30.9168415 2.3608985 13.095 < 2e-16 ***
## cylinders4
                6.9399216 1.5365961
                                        4.516 8.48e-06 ***
## cylinders5
                 6.6377310 2.3372687
                                        2.840 0.004762 **
## cylinders6
                 4.2973139
                           1.7057848
                                        2.519 0.012182 *
## cylinders8
                 6.3668129
                           1.9687277
                                        3.234 0.001331 **
## displacement 0.0118246 0.0067755
                                        1.745 0.081785 .
## horsepower
                -0.0392323 0.0130356
                                       -3.010 0.002795 **
## weight
                -0.0051802
                           0.0006241
                                       -8.300 1.99e-15 ***
## acceleration 0.0036080
                            0.0868925
                                        0.042 0.966902
## year71
                0.9104285
                           0.8155744
                                        1.116 0.265019
## year72
                -0.4903062
                           0.8038193
                                       -0.610 0.542257
## year73
                -0.5528934
                                       -0.766 0.443947
                            0.7214463
## year74
                 1.2419976
                           0.8547434
                                        1.453 0.147056
## year75
                0.8704016 0.8374036
                                        1.039 0.299297
## year76
                 1.4966598 0.8019080
                                        1.866 0.062782 .
## year77
                 2.9986967
                           0.8198949
                                        3.657 0.000292 ***
## year78
                 2.9737783
                           0.7792185
                                        3.816 0.000159 ***
## year79
                 4.8961763 0.8248124
                                        5.936 6.74e-09 ***
## year80
                 9.0589316
                           0.8751948 10.351 < 2e-16 ***
## year81
                 6.4581580
                            0.8637018
                                        7.477 5.58e-13 ***
## year82
                7.8375850 0.8493560
                                        9.228 < 2e-16 ***
## origin2
                 1.6932853 0.5162117
                                        3.280 0.001136 **
## origin3
                 2.2929268 0.4967645
                                        4.616 5.41e-06 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 2.848 on 369 degrees of freedom
## Multiple R-squared: 0.8744, Adjusted R-squared: 0.8669
## F-statistic: 116.8 on 22 and 369 DF, p-value: < 2.2e-16
```

multiple.lm <- lm(mpg ~ . -name, data = Auto)</pre>

summary(multiple.lm)

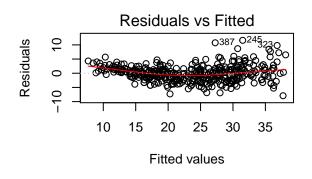
Inference from Multiple Linear Regression model : mpg \sim . - name

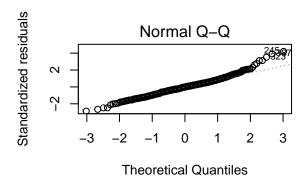
- 1. From the p-value associated with the F-statistic, we know that there is a relationship between mpg and the predictor variables.
- 2. Of all the predictors, displacement, weight, year and origin have statistically significant relationship to the response.
- 3. The coefficient of the year 0.75 suggest that later model have better mpg as shown in the figure below

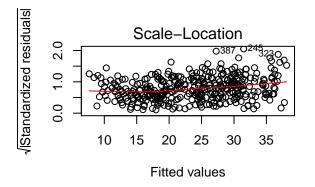


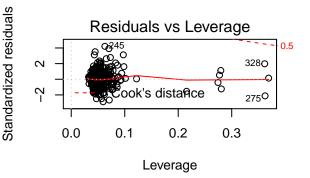
- 4. The residual vs fitted plot show evidence of non-linearity. There is a funnel shape hinting at heteroscedasticity.
- 5. Additionally, high leverage is noticed for point 14.

diagnostic_plot(multiple.lm)









vif(multiple.lm)

```
GVIF Df GVIF^(1/(2*Df))
##
## cylinders
                 17.980791
                                      1.434997
## displacement 24.240924
                                      4.923507
                            1
                                      3.484239
## horsepower
                 12.139922
## weight
                 13.551565
                                      3.681245
                            1
## acceleration
                  2.771138
                            1
                                      1.664674
                                      1.034966
## year
                  2.281543 12
## origin
                  2.527913
                                      1.260929
```

6. Multi collinearity: When VIF values lies in 5 - 10 range, a problematic amount of collinearity si presented. This was evident from the correlation plot too. The problem of collinearity is that it reduces the accuracy of the estimates of the regression coefficients and it causes the standard error to increase.

Interaction Term

```
inter.lm1 <- lm(mpg~displacement+origin+year*weight, data=Auto)
inter.lm2 <- lm(mpg~year+origin+displacement*weight, data=Auto)
summary(inter.lm1)</pre>
```

##

```
## Call:
## lm(formula = mpg ~ displacement + origin + year * weight, data = Auto)
## Residuals:
       Min
                1Q
                     Median
                                 3Q
                     0.0385
## -10.7913 -1.6923
                             1.6285 11.4805
## Coefficients:
##
                 Estimate Std. Error t value Pr(>|t|)
## (Intercept)
               34.7946118 2.3059551 15.089 < 2e-16 ***
## displacement
                0.0028271 0.0047578
                                     0.594 0.552747
## origin2
                2.2134814 0.5032141
                                      4.399 1.43e-05 ***
## origin3
                1.4619548 0.4872595
                                    3.000 0.002883 **
## year71
                2.0378249 2.7778352
                                    0.734 0.463665
               -1.0551438 2.9298669 -0.360 0.718957
## year72
## year73
               -5.2819682
                          2.7631468 -1.912 0.056718 .
                3.6795664 2.8416099
                                    1.295 0.196182
## year74
## year75
                0.1052171 3.1582296
                                    0.033 0.973442
## year76
                5.1245063 2.9259137
                                    1.751 0.080718 .
## year77
                7.3137550 2.9046368 2.518 0.012233 *
## year78
               14.5480420 3.1747764 4.582 6.33e-06 ***
               14.6067832 3.1845253 4.587 6.21e-06 ***
## year79
               ## year80
               15.7498499 3.4674095 4.542 7.59e-06 ***
## year81
## year82
               18.3543645 4.4689400 4.107 4.95e-05 ***
## weight
               ## year71:weight -0.0001164 0.0008262 -0.141 0.888073
                                    0.490 0.624412
## year72:weight 0.0004163 0.0008496
## year73:weight 0.0014515 0.0007880
                                    1.842 0.066282 .
## year74:weight -0.0004688 0.0008752 -0.536 0.592531
## year75:weight 0.0004605 0.0009436
                                     0.488 0.625816
## year76:weight -0.0009151 0.0008786 -1.042 0.298312
## year77:weight -0.0011901 0.0008751 -1.360 0.174663
## year78:weight -0.0038018 0.0010054 -3.782 0.000182 ***
                                     -2.939 0.003503 **
## year79:weight -0.0028298
                          0.0009628
## year80:weight -0.0046242 0.0014941 -3.095 0.002122 **
## year81:weight -0.0030900 0.0012098 -2.554 0.011053 *
## year82:weight -0.0036276  0.0017160 -2.114  0.035193 *
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 2.855 on 363 degrees of freedom
## Multiple R-squared: 0.8758, Adjusted R-squared: 0.8662
## F-statistic: 91.39 on 28 and 363 DF, p-value: < 2.2e-16
summary(inter.lm2)
##
## lm(formula = mpg ~ year + origin + displacement * weight, data = Auto)
## Residuals:
       Min
                1Q
                   Median
                                 3Q
## -10.8567 -1.5177 0.1011
                             1.4845 11.8388
```

```
##
## Coefficients:
##
                       Estimate Std. Error t value Pr(>|t|)
## (Intercept)
                      5.023e+01 1.658e+00 30.301 < 2e-16 ***
## year71
                      7.912e-01 7.516e-01
                                             1.053
                                                   0.29317
## year72
                     -3.950e-01 7.567e-01 -0.522 0.60202
## year73
                      -9.209e-01 6.818e-01 -1.351 0.17760
                      1.595e+00 7.836e-01
## year74
                                             2.035 0.04252 *
## year75
                      1.878e+00 7.496e-01 2.505 0.01266 *
## year76
                      2.298e+00 7.262e-01 3.164 0.00168 **
## year77
                       3.163e+00 7.617e-01 4.152 4.08e-05 ***
                       3.838e+00 7.214e-01
                                             5.321 1.79e-07 ***
## year78
## year79
                      5.928e+00 7.467e-01
                                           7.939 2.40e-14 ***
## year80
                      1.001e+01 7.957e-01 12.577 < 2e-16 ***
## year81
                                           9.397 < 2e-16 ***
                      7.349e+00 7.821e-01
## year82
                      8.734e+00 7.809e-01 11.184
                                                   < 2e-16 ***
                                             2.094 0.03692 *
## origin2
                      1.016e+00 4.852e-01
## origin3
                      3.832e-01 4.774e-01
                                             0.803 0.42262
                      -7.040e-02 8.897e-03 -7.913 2.88e-14 ***
## displacement
## weight
                      -1.018e-02 6.152e-04 -16.552 < 2e-16 ***
## displacement:weight 2.069e-05 2.072e-06
                                             9.984 < 2e-16 ***
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 2.732 on 374 degrees of freedom
## Multiple R-squared: 0.8828, Adjusted R-squared: 0.8775
## F-statistic: 165.7 on 17 and 374 DF, p-value: < 2.2e-16
```

displacement & weight have statistically significant interaction.

Non-linear transformations

Here we are exploring various non-linear transformations such as log, polynomial.

```
nonlinear.lm1 <- lm(mpg~displacement+I(log(weight))+year+origin, data=Auto)
nonlinear.lm2 <- lm(mpg~poly(displacement,3)+weight+year+origin, data=Auto)
nonlinear.lm3 <- lm(mpg~displacement+I(weight^2)+year+origin, data=Auto)
summary(nonlinear.lm1)</pre>
```

```
##
## Call:
## lm(formula = mpg ~ displacement + I(log(weight)) + year + origin,
       data = Auto)
##
##
## Residuals:
##
       Min
                  1Q
                       Median
                                    3Q
                                            Max
## -10.0601 -1.6555
                       0.1519
                               1.5370 11.3948
##
## Coefficients:
##
                    Estimate Std. Error t value Pr(>|t|)
## (Intercept)
                  185.394162 10.257706 18.074 < 2e-16 ***
## displacement
                                          2.310 0.021448 *
                    0.009309
                               0.004031
```

```
## I(log(weight)) -21.103367
                              1.385625 -15.230 < 2e-16 ***
## year71
                   1.198882
                              0.767482
                                         1.562 0.119108
## year72
                                         0.450 0.653049
                   0.346844
                              0.770952
                   -0.300029
                              0.696213 -0.431 0.666756
## year73
## year74
                   2.052366
                              0.797550
                                         2.573 0.010456 *
## year75
                   1.983609
                              0.771061
                                         2.573 0.010479 *
## year76
                   2.469789
                              0.745208
                                         3.314 0.001008 **
## year77
                   3.639869
                              0.778581
                                         4.675 4.11e-06 ***
## year78
                   3.853289
                              0.743565
                                         5.182 3.59e-07 ***
## year79
                   6.122424
                              0.769115
                                         7.960 2.06e-14 ***
## year80
                  10.474025
                              0.818927 12.790
                                                < 2e-16 ***
## year81
                   7.677553
                              0.804303
                                         9.546
                                                < 2e-16 ***
## year82
                   9.246233
                              0.802180 11.526 < 2e-16 ***
## origin2
                   1.865680
                              0.480531
                                         3.883 0.000122 ***
                                         2.779 0.005731 **
                              0.465406
## origin3
                   1.293261
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 2.823 on 375 degrees of freedom
## Multiple R-squared: 0.8746, Adjusted R-squared: 0.8692
## F-statistic: 163.4 on 16 and 375 DF, p-value: < 2.2e-16
```

summary(nonlinear.lm2)

```
##
## Call:
## lm(formula = mpg ~ poly(displacement, 3) + weight + year + origin,
##
       data = Auto)
##
## Residuals:
##
       Min
                  1Q
                       Median
                                    3Q
                                            Max
                       0.1376
                                1.6098
## -13.1739 -1.5085
                                       11.4039
##
## Coefficients:
##
                            Estimate Std. Error t value Pr(>|t|)
## (Intercept)
                           34.766016
                                       1.536021 22.634 < 2e-16 ***
## poly(displacement, 3)1 -16.625947
                                      10.186541 -1.632 0.10349
## poly(displacement, 3)2 25.946455
                                       3.482753
                                                  7.450 6.56e-13 ***
                                                 -3.191 0.00154 **
## poly(displacement, 3)3
                           -9.549773
                                       2.992799
## weight
                           -0.005054
                                       0.000535
                                                 -9.448 < 2e-16 ***
## year71
                            1.464949
                                       0.786550
                                                  1.862 0.06332 .
## year72
                           -0.160327
                                       0.793815
                                                -0.202 0.84005
## year73
                           -0.431942
                                       0.710701
                                                 -0.608
                                                         0.54371
                                                  2.351 0.01923 *
## year74
                            1.943962
                                       0.826763
## year75
                            2.112021
                                       0.788541
                                                  2.678 0.00772 **
                                                  3.234 0.00133 **
## year76
                            2.479230
                                       0.766511
## year77
                            3.422878
                                       0.799123
                                                  4.283 2.35e-05 ***
                            4.062263
                                                  5.353 1.52e-07 ***
## year78
                                       0.758887
                                       0.785820
                                                  7.774 7.47e-14 ***
## year79
                            6.109192
                                       0.831490 12.232 < 2e-16 ***
## year80
                           10.170661
                            7.659280
                                       0.816816
                                                  9.377
                                                         < 2e-16 ***
## year81
## year82
                            9.080391
                                       0.813725 11.159 < 2e-16 ***
                                                 1.239 0.21630
## origin2
                            0.657935
                                       0.531224
## origin3
                                                0.502 0.61594
                            0.260748
                                       0.519380
```

```
## Residual standard error: 2.85 on 373 degrees of freedom
## Multiple R-squared: 0.8728, Adjusted R-squared: 0.8667
## F-statistic: 142.2 on 18 and 373 DF, p-value: < 2.2e-16
summary(nonlinear.lm3)
##
## Call:
## lm(formula = mpg ~ displacement + I(weight^2) + year + origin,
##
      data = Auto)
##
## Residuals:
##
       Min
                 1Q
                      Median
                                   3Q
## -11.3824 -2.0885 -0.0761
                               1.7075 13.9065
## Coefficients:
##
                 Estimate Std. Error t value Pr(>|t|)
## (Intercept)
                2.767e+01 1.014e+00 27.286 < 2e-16 ***
## displacement -8.967e-03 5.293e-03 -1.694 0.091075 .
## I(weight^2) -6.635e-07
                           8.965e-08 -7.401 8.94e-13 ***
## year71
                1.471e+00 9.229e-01
                                       1.593 0.111918
## year72
               -2.879e-01 9.271e-01 -0.311 0.756302
## year73
               -5.384e-01 8.357e-01 -0.644 0.519808
## year74
                1.620e+00
                          9.640e-01
                                       1.680 0.093791
## year75
                7.272e-01 9.156e-01
                                       0.794 0.427592
## year76
                1.579e+00 8.901e-01
                                      1.774 0.076914 .
## year77
                3.049e+00 9.349e-01
                                       3.261 0.001211 **
## year78
                2.733e+00 8.805e-01
                                       3.104 0.002058 **
## year79
                5.348e+00 9.152e-01
                                       5.844 1.11e-08 ***
## year80
                9.348e+00 9.741e-01
                                       9.597 < 2e-16 ***
## year81
                6.835e+00 9.590e-01
                                       7.128 5.28e-12 ***
## year82
                                       8.943 < 2e-16 ***
                8.567e+00 9.580e-01
## origin2
                2.215e+00 5.816e-01
                                       3.808 0.000164 ***
## origin3
                2.580e+00 5.491e-01
                                       4.699 3.68e-06 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 3.354 on 375 degrees of freedom
## Multiple R-squared: 0.8228, Adjusted R-squared: 0.8153
## F-statistic: 108.9 on 16 and 375 DF, p-value: < 2.2e-16
```

Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1

From noticing the p-value associated with the different variables which have been transformed, we know that

- 1. displacement has a lower p-value for square as compared to cubic
- 2. weight² is statistically significant.
