Control Systems Lab

Experiment 2

Mathematical modeling of Physical Systems

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2.1 Objective:

To mathematically model a DC motor with the given specifications using MATLAB Simulink.

2.2 DC Motor Model

A common actuator in control systems is the DC motor. It directly provides rotary motion and, coupled with wheels or drums and cables, can provide translational motion. The electric circuit of the armature and the free-body diagram of the rotor are shown in the following figure:

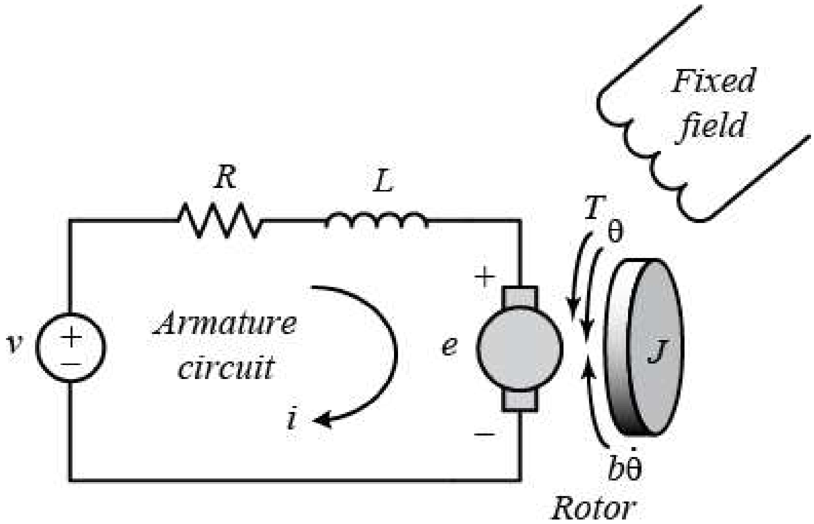

Figure 1: Equivalent circuit of DC Motor

For this circuit, we will assume that the input of the system is the voltage source (V_{dc}) applied to the motor's armature, while the output is the rotational speed of the shaft (Wm). The rotor and shaft are assumed to be rigid. We further assume a viscous friction model, that is, the friction torque is proportional to shaft angular velocity.

The physical parameters for our electromechanical system is given by the table given below,

Table 1: DC Motor specifications

DC motor Specifications		
(J)	Moment of inertia of the rotor	0.05 kg.m^2
(Bm)	Motor viscous friction constant	0.01 N-m.s
(Ka)	Motor constant	0.5 V.s/rad
(R)	Armature resistance	0.5 ohm
(L)	Armature inductance	100 mH
(Wm)	Rated motor speed	188 r.p.m
(V_{dc})	DC voltage source	100 V
(T_L)	Load torque	4 N-m

In general, the torque generated by a DC motor is proportional to the armature current and the strength of the magnetic field. In this case, we will assume that the magnetic field is constant and therefore the motor torque is proportional only to the armature current (Ia) by a constant factor (Ka) as shown in the equation given below. This type of motor is referred to as armature-controlled motor.

Based on the Newton's second law, the mathematical equations of DC motor can be given as follows,

$$\text{Electromechanical torque, } Tm = J\frac{d\mathrm{Wm}}{dt} + (\mathrm{Bm} \times \mathrm{Wm}) + \mathrm{T_L} \qquad (1)$$

$$\text{On solving equation (1), } \mathrm{Wm} = \frac{1}{J}\int\big((Tm - \mathrm{T}_L) - (\mathrm{Bm} \times \mathrm{Wm})\big)\mathrm{dt} \qquad (2)$$

$$\text{Where, } \mathrm{Wm} = \text{speed of the motor,}$$

$$\text{Effective torque, } \mathrm{Te} = Tm - \mathrm{T}_L = \mathrm{Ka} \times \mathrm{Ia} \qquad (3)$$

Based on the Kirchoffs voltage law (KVL), the mathematical model of the electrical circuit of the DC motor can be obtained as follows,

$$V_{dc} - (Ia \times Ra) - L\frac{dIa}{dt} = Eb \quad (4)$$

Back e.m.f, $E_b = K_a \times Wm$ (5)

On solving equation (3),

We get, $Ia = \frac{1}{L}\int(Vdc - (Ia \times Ra) - (\mathrm{Ka} \times \mathrm{Wm}))\mathrm{dt}$ (6)

By applying Laplace transform for equations (2,3 & 6), the SIMULINK model in section 2.3 is obtained.

2.3 Procedure to build the model with MATLAB Simulink

- To build the simulation model, open Simulink and open a new model window.
- The blocks can be fetched from the ***Library Browser*** in the top left corner of the Simulink window. The required blocks can be obtained from the search option in the top left corner of the Simulink library browser. Once the block is available in the right pane, double-click the block and insert the block in to the model window. The blocks can be labelled by selecting the block and editing the existing characters below the block. The connecting lines can be labelled in the same manner as well.
- In the equations (2,3 & 6), the mathematical operations such as Integration, addition / subtraction and multiplication are involved. We also have the constants, input and output variables.
- Integration can be performed by using the block named '***Integrator***', which can be obtained by either typing the word 'integrator' in the search option (or) by clicking **Simulink => continuous => Integrator** in the list of sub-category available in the left pane of the library browser window.
- Addition / Subtraction of the variables can be performed by using '**Add**' block. By double-clicking on the block, the list of symbols can be changed as per our requirement.
- The constant values can be introduced as the product of other variables using '**Gain**' block. By double-clicking on the block, the gain value can be entered in the block. Insert a Gain block below the "Inertia" block. Next right-click on the block and select **Rotate &**

Flip > Flip Block from the resulting menu to flip the block from left to right. You can also flip a selected block by holding down **Ctrl-I**.

- Add **'In1'** and **'Out1'** blocks from the Simulink/Ports & Subsystems library and respectively label them for input and output variables.
- Use the block '**Scope**' to display the graphical plot of the necessary output variables.
- After inserting all the above mentioned blocks, interconnect all the blocks as shown in the figure 2.

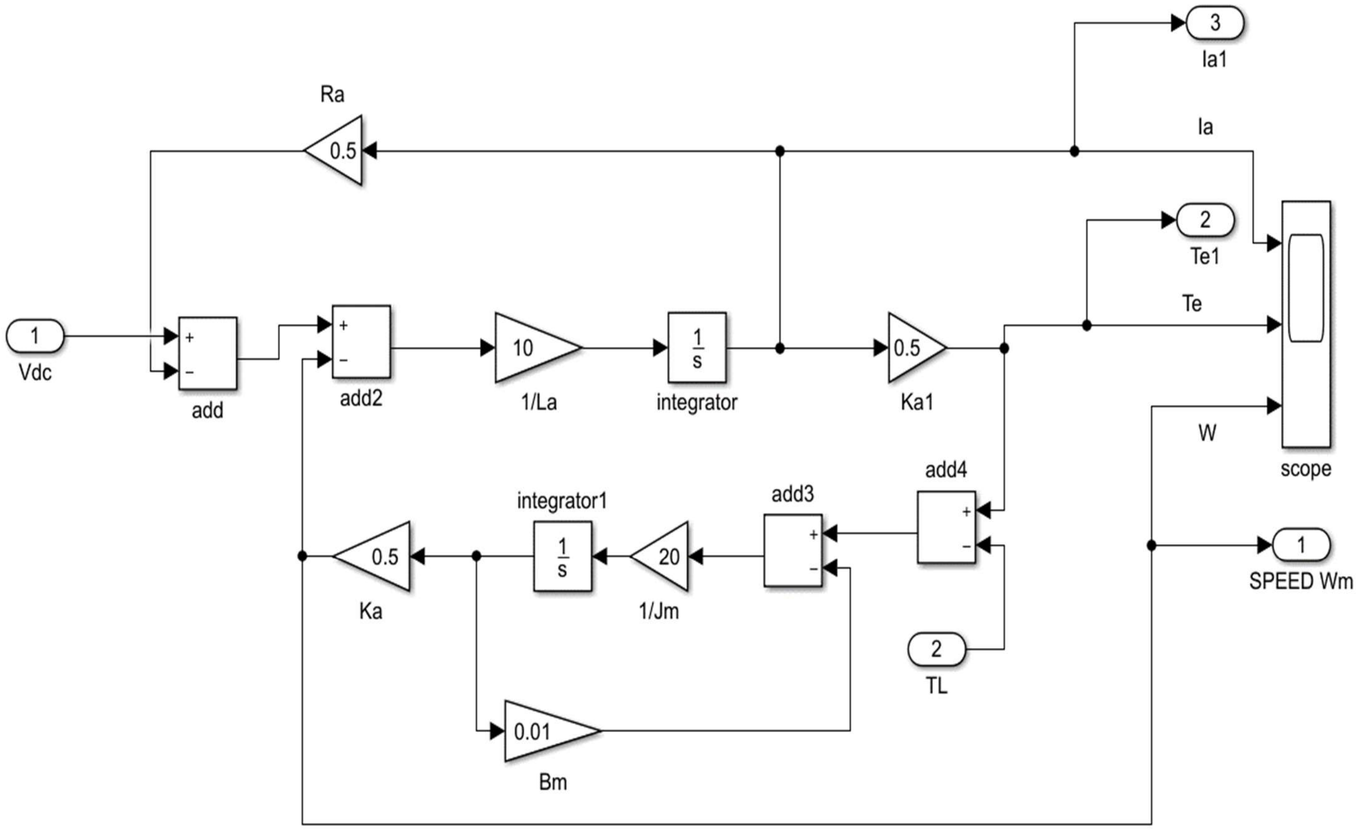

Figure 2: MATLAB/Simulink model of DC motor (Internal blocks)

2.4 Creating a subsystem

In order to save all of these components as a single subsystem block, first select all of the blocks, then select **Create Subsystem from Selection** after right-clicking on the selected portion. Name the subsystem and then save the model. Your model should appear as shown in Figure 3.

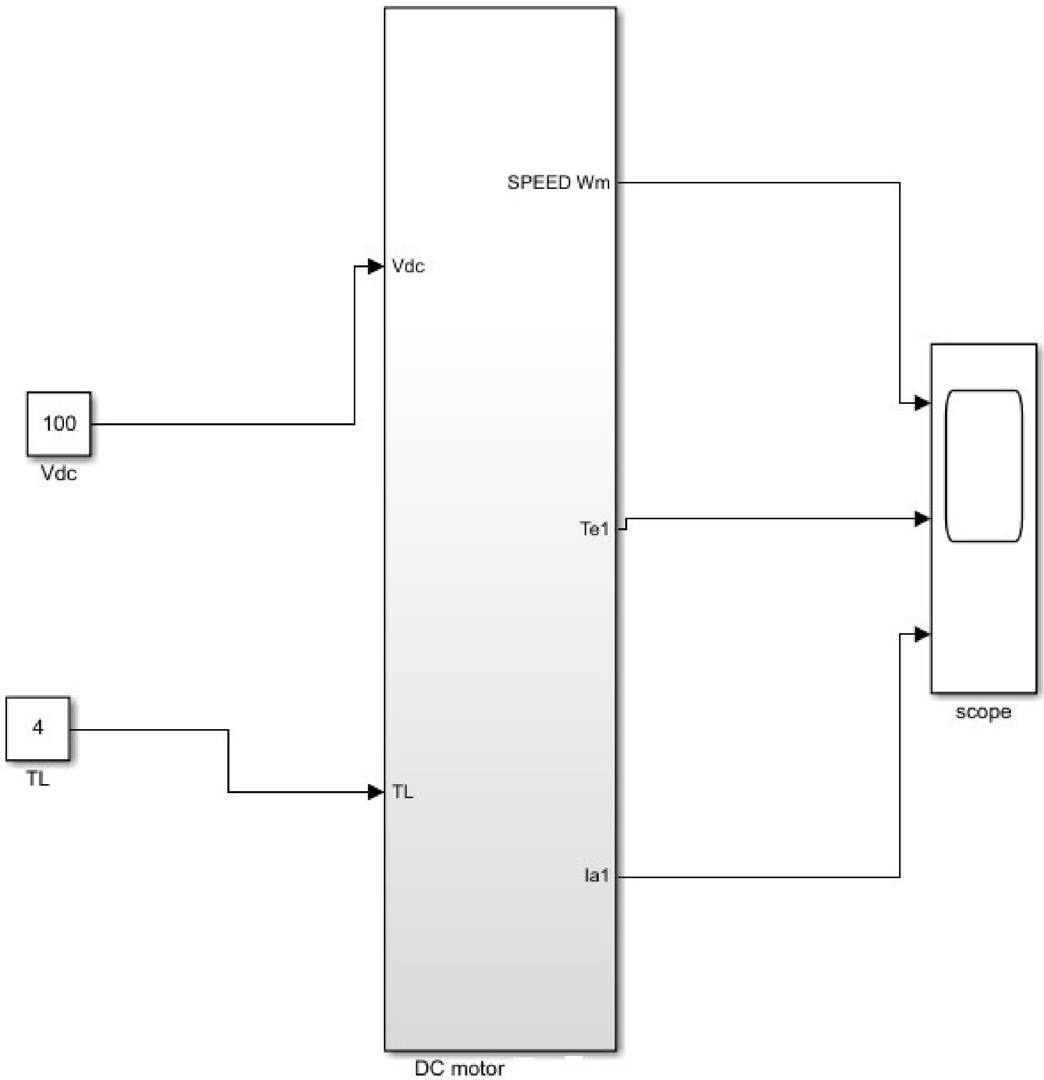

Figure 3: Single subsystem DC motor block

2.5 Result:

Figure 4 shows the expected responses of the given DC Motor.

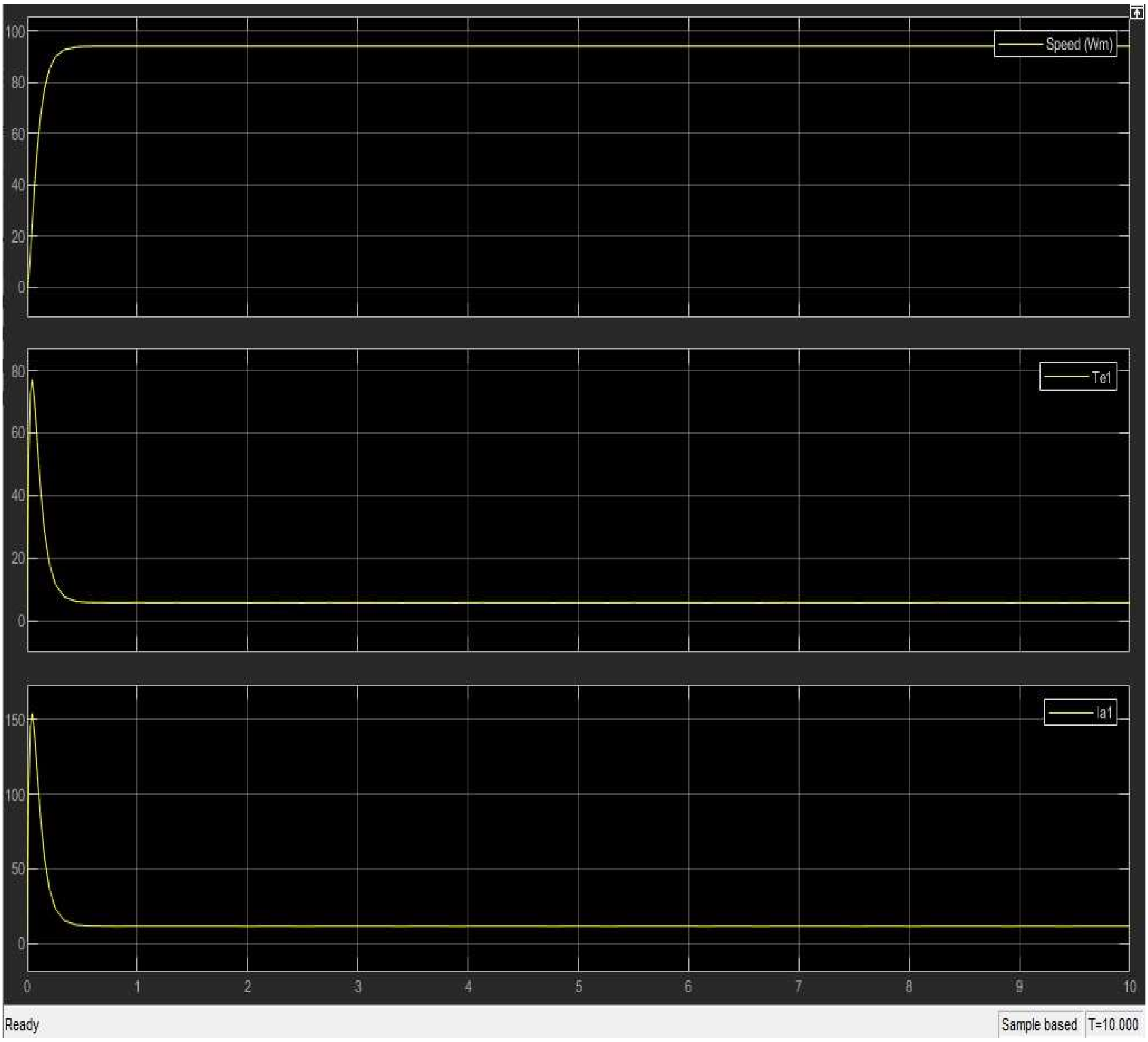

Figure 4: Expected responses of DC motor