15. Find the truth set of each of these predicates where the domain is the set of integers.

(a) $P(x)$: "$x^2 < 3$" (b) $Q(x)$: "$x^2 > x$" (c) $R(x)$: "$2x + 1 = 0$"

(d) $P(x)$: "$x^3 \geq 1$" (e) $Q(x)$: "$x^2 = 2$" (f) $R(x)$: "$x^2 < x$"

16. Try to understand the following proof of a distributive law $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ and then prove it using membership table.

First we show that $A \cup (B \cap C) \subset (A \cup B) \cap (A \cup C)$.

Let $x \in A \cup (B \cap C)$.

$$\therefore x \in A \ \ or \ \ x \in B \cap C$$

Case 1: If $x \in A$, then $x \in A \cup B$ as well as $x \in A \cup C$.

$$\therefore x \in (A \cup B) \cap (A \cup C).$$

This proves that, in this case, $A \cup (B \cap C) \subset (A \cup B) \cap (A \cup C)$.

Case 2: If $x \notin A$, then x must belong to $B \cap C$.

$\therefore x \in B$ as well as $x \in C$.

$\therefore x \in A \cup B$ as well as $x \in A \cup C$.

This proves that, in this case also, $A \cup (B \cap C) \subset (A \cup B) \cap (A \cup C)$.

Thus, it is proved that $A \cup (B \cap C) \subset (A \cup B) \cap (A \cup C)$. (1)

Next, we have to show that $(A \cup B) \cap (A \cup C) \subset A \cup (B \cap C)$.

Let $x \in (A \cup B) \cap (A \cup C)$.

$\therefore x \in A \cup B$ as well as $x \in A \cup C$.

Case 1: If $x \in A$, then $x \in A \cup (B \cap C)$.

This proves that, in this case, $(A \cup B) \cap (A \cup C) \subset A \cup (B \cap C)$.

Case 2: If $x \notin A$, then x must belong to B as well as C.

So, $x \in B \cap C$ and hence must belong to $A \cup (B \cap C)$.

This proves that, in this case also, $(A \cup B) \cap (A \cup C) \subset A \cup (B \cap C)$.

Thus, it is proved that $(A \cup B) \cap (A \cup C) \subset A \cup (B \cap C)$. (2)

From (1) and (2), we can conclude that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

17. Draw the Venn diagrams for each of these combinations:

$(a)\ \bar{A}\cap\bar{B}\cap\bar{C}$ $(b)\ (A-B)\cup(A-C)\cup(B-C)$ $(c)\ (A\cap\bar{B})\cup(A\cap\bar{C})$

$(d)\ (A\cap B)\cup(C\cap D)$ $(e)\ A-(B\cap C\cap D)$

Use separate sheet of paper to answer this question.

18. In a recent survey, people were asked if they took a vacation in the summer, winter or spring in the last year. The results were: 73 took a vacation in the summer, 51 took a vacation in the winter, 27 took a vacation in the spring, and 2 had taken no vacation. Also, 10 had taken vacations at all three times, 33 had taken both a summer and a winter vacation, 18 had taken only a winter vacation and 5 had taken both a summer and a spring but not a winter vacation.

(a) How many people had been surveyed?

(b) How many people had taken vacations at exactly two times of the year?

(c) How many people had taken vacations during at most one time of the year?

(d) What percentage had taken vacations during both summer and winter but not spring?

$\boldsymbol{Ans}$: $(\boldsymbol{a})$ $\mathbf{103}$ $(\boldsymbol{b})$ $\mathbf{28}$ $(\boldsymbol{c})$ $\mathbf{67}$ $(\boldsymbol{d})$ $\mathbf{22.33}$ %

19. Find the sets A and B if $A-B=\{1,5,7,8\}$, $B-A=\{2,10\}$, and $A\cap B=\{3,6,9\}$.

20. Can you conclude that $A=B$ if A,B and C are sets such that

$(a)\ A\cup C=B\cup C$? $(b)\ A\cap C=B\cap C$? $(C)\ A\cup C=B\cup C\ and\ A\cap C=B\cap C$?

Give examples to justify your answer.

21. What can you say about the sets A and B if we know that
$(a)\ A \cup B = A$ $(b)\ A \cap B = A$ $(c)\ A - B = A$
$(d)\ A \cap B = B \cap A$ $(e)\ A - B = B - A$

22. Show that if A is a subset of a universal set U, then
$(a)\ A \oplus A = \varphi$ $(b)\ A \oplus \varphi = A$ $(c)\ A \oplus U = \bar{A}$ $(d)\ A \oplus \bar{A} = U.$

22. Show that if A and B are sets, then $(A \oplus B) \oplus B = A$.

23. What can you say about the sets A and B if $A \oplus B = A$?

24. If A, B and C are sets such that $A \oplus C = B \oplus C$, can we conclude that $A = B$?

25. Find

$$(a)\ \bigcup_{i=1}^{n} A_i \qquad (b)\ \bigcap_{i=1}^{n} A_i \qquad (c)\ \bigcup_{i=1}^{\infty} A_i \qquad (d)\ \bigcap_{i=1}^{\infty} A_i$$

if for every positive integer i,
$(a)\ \ A_i = \{1, 2, 3, \dots, i\}$ $(b)\ \ A_i = \{\dots, -2, -1, 0, 1, \dots, i\}$ $(c)\ \ A_i = \{0, i\}$
$(d)\ \ A_i = \{i, i+1, i+2, \dots\}$ $(e)\ \ A_i = (0, i)$ $(f)\ \ A_i = \{-i, i\}$
$(g)\ \ A_i = [-i, i]$ $(h)\ \ A_i = (i, \infty)$ $(i)\ \ A_i = [i, \infty]$
$(j)\ \ A_i = \{-i, -i+1, \dots, -1, 0, 1, \dots, i-1, i\}$

26. Suppose that the universal set is $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. Express the sets

$(a)\ \{3, 4, 5\}$ $\quad (b)\ \{1, 3, 6, 10\}$ $\quad (c)\ \{2, 3, 4, 7, 8, 9\}$

with bit strings where the ith bit in the string is 1 if i is in the set and 0 otherwise. Also, find the set specified by each of the bit strings

(a) 11 1100 1111 $\quad (b)$ 01 0111 1000 $\quad (c)$ 10 0000 0001

27. What subsets of a finite universal set do these bit strings represent?

(a) the string with all zeros (b) the string with all ones

28. What is the bit string corresponding to the difference of two sets?

29. What is the bit string corresponding to the symmetric difference of two sets?

30. Show how bitwise operations on bit strings can be used to find these combinations of

$$A = \{a, b, c, d, e\}, \qquad B = \{b, c, d, g, p, t, v\},$$
$$C = \{c, e, i, o, u, x, y, z\}, \qquad D = \{d, e, h, i, n, o, t, u, x, y\}$$

$(a)\ A \cup B$ $\quad (b)\ A \cap B$ $\quad (c)\ (A \cup D) \cap (B \cup C)$ $\quad (d)\ A \cup B \cup C \cup D$

31. How can the union and intersection of n sets that all are subsets of the universal set U be found using bit strings?