MODULE 2 : NUMBER SYSTEM AND CODES

Introduction to Number systems, Binary Number systems, Signed Binary Numbers, Binary, Octal, Decimal and Hexadecimal number Systems and their conversion, Binary arithmetic using compliments, Gray Code, BCD Code, Excess-3 code, ASCII Code. inter-conversion of codes

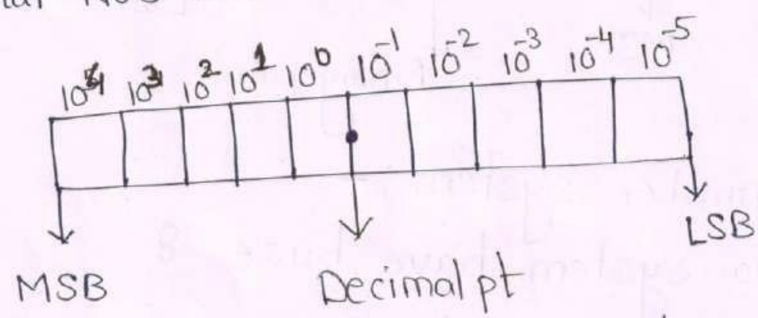
I Number Systems & Codes.

Number Systems.

1) Decimal Number System :-

→ Base or Radix is 10.

→ Decimal No's are 0 to 9



LSB = Right most digit having Lowest weight is called Least significant bit

MSB = Left most digit having highest weight is called most significant bit.

2) Binary Number System :-

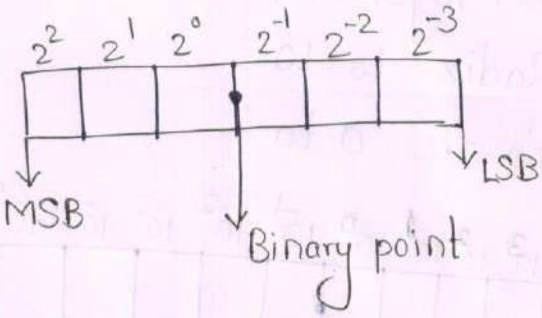
→ Binary no's are 0 and 1

→ ~~Bar~~ Binary no. has base of 2

→ 0 and 1 are also called as bits.

→ Group of four binary bits is known as Nibble

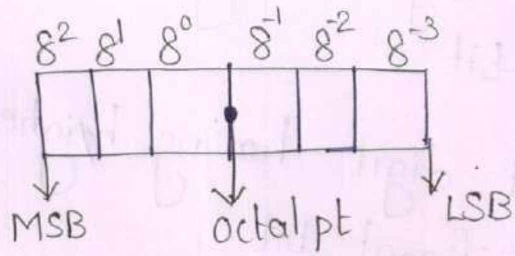
→ Group of eight binary bits is known as Bytes.



3) Octal Number system :-

→ Octal No. system have base 8

→ octal No's are 0 to 7



4) Hexadecimal Number System :-

→ Base is 16 & Numbers are 0 to 15

→ First 10 digit are represented as 0 to 9 & remaining 5 digit are represented as characters A to F.

Relation between Decimal, binary, octal, Hexadecimal.

Decimal (base 10)	Binary (Base 2)	Hexadecimal (base 16)	Octal (base 8)
0	0000	0	0
1	0001	1	1
2	0010	2	2
3	0011	3	3
4	0100	4	4
5	0101	5	5
6	0110	6	6
7	0111	7	7
8	1000	8	10
9	1001	9	11
10	1010	A	12
11	1011	B	13
12	1100	C	14
13	1101	D	15
14	1110	E	16
15	1111	F	17

Number System.

SrNo	Number System	Base	Allowed digits	Examples
1	Decimal	10	0, 1, --- 9	$(123)_{10}$, $(123)_D$
2	Binary	2	0, 1	$(011)_2$, $(011)_B$
3	octal	8	0, 1, --- 7	$(543)_8$, $(543)_O$
4	Hexadecimal	16	0, 1, --- 9, A, B, C, D, E, F	$(0AB67)_{16}$, $(0AB67)_H$

Decimal	Binary	BCD	Excess - 3	Gray
0	0000	0000	0011	0000
1	0001	0001	0100	0001
2	0010	0010	0101	0011
3	0011	0011	0110	0010
4	0100	0100	0111	0110
5	0101	0101	1000	0111
6	0110	0110	1001	0101
7	0111	0111	1010	0100
8	1000	1000	1011	1100
9	1001	1001	1100	1101
10	1010	0001 0000	1101	1111
11	1011	0001 0001	1110	1110
12	1100	0001 0010	1111	1010
13	1101	0001 0011	0000	1011
14	1110	0001 0100	0001	1001
15	1111	0001 0101	0010	1000

· Excess-3 Code (Non-Weighted Code)

This is a four bit code which can be derive from the BCD Code by adding $(3)_{10}$ i.e $(0011)_2$ to each coded no.

Ex :- $(246)_{10} = (?)_{Ex-3}$

```
   2     4     6
  0010  0100  0110
+ 0011  0011  0011
  0101  0111  1001
```

OR

```
   2   4   6
 + 3   3   3
   5   7   9
= 0101 0111 1001
```

$\therefore (246)_{10} = (0101\ 0111\ 1001)_{Ex-3}$

Gray Code :- (Non-Weighted Code)

→ It is not an arithmatic code.

→ Only one bit changes at a time, the decimal no. is incremented by 1. So also called as unit distance code.

A	B	Gray ($A \oplus B$)
0	0	0
0	1	1
1	0	1
1	1	0

i) To Convert Binary to Gray

ii) ——"—— Gray to Binary.

(1) Binary to Gray

1) Record MSB as it is
2) Add this bit to next position using 'x' or MOD2 addition. neglecting Carry.
3) Record sum until completed

Ex 1:- $(10010)_2$ into gray.

MSB $\qquad\qquad$ LSB

1 $\quad$ 0 $\quad$ 0 $\quad$ 1 $\quad$ 0

$\oplus$ $\quad$ $\oplus$ $\quad$ $\oplus$ $\quad$ $\oplus$

1 $\quad$ 1 $\quad$ 0 $\quad$ 1 $\quad$ 1

$= (11011)_{gray}$.

2] $(110100)_2 \rightarrow$ gray.

MSB $\qquad\qquad$ LSB

1 $\quad$ 1 $\quad$ 0 $\quad$ 1 $\quad$ 0 $\quad$ 0

$\oplus$ $\quad$ $\oplus$ $\quad$ $\oplus$ $\quad$ $\oplus$ $\quad$ $\oplus$

1 $\quad$ 0 $\quad$ 1 $\quad$ 1 $\quad$ 1 $\quad$ 0

$= (101110)_{gray}$.

(3) (1011) to gray.

(2) Gray to Binary

1) MSB of gray & Binary are same so write directly.
2) Add each binary digit generated to gray digit. Record result & ignore carry.
3) Continue the process till LSB.

Ex 1) $(11010)_{gray} = (?)_2$

MSB LSB

1 1 0 1 0

$\oplus$ $\oplus$ $\oplus$ $\oplus$

1 0 0 1 1

$= (10011)_2$

2] $(111011)_{gray} = (?)_2$

MSB LSB

1 1 1 0 1 1

$\oplus$ $\oplus$ $\oplus$ $\oplus$ $\oplus$

1 0 1 1 0 1

$= (101101)_2$

Conversions.

I i) Binary to Decimal

1> $(110110)_2 = 2^5\times1 + 2^4\times1 + 2^3\times0 + 2^2\times1 + 2^1\times1 + 2^0\times0$

$= 32 + 16 + 0 + 4 + 2 + 0$

$= (54)_{10}$

2> $(1011\cdot1011)_2 = 2^3\times1 + 2^2\times0 + 2^1\times1 + 2^0\times1 + 2^{-1}\times1 + 2^{-2}\times0 + 2^{-3}\times1 + 2^{-4}\times1$

$= 8+0+2+1+\frac{1}{2}+0+\frac{1}{8}+\frac{1}{16}$

$= 11+\frac{1}{2}+\frac{1}{8}+\frac{1}{16}$

$= \frac{11\times16+1\times8+1\times2+1}{16} = \frac{176+8+2+1}{16} = \frac{187}{16}$

$= (11\cdot6875)_{10}$

ii> Binary to Octal

1> $(1001010)_2 = (?)_8$

$= \underbrace{1}\ \underbrace{001}\ \underbrace{010}$

$= 001\ \ 001\ \ 010$

$= 1\ \ 1\ \ 2$

$= (112)_8$

2) $(1101.11011)_2 = (?)_8$

$$1101.11011 = 001\ 101 \cdot 110\ 110$$
$$= 1\ \ 5 \cdot 6\ \ 6$$
$$= (15.66)_8$$

iii) Binary to Hexadecimal

1) $(100111010)_2 = (?)_{16}$

$$100111010 = 0001\ 0011\ 1010$$
$$= 1\ \ 3\ \ A$$
$$= (13A)_{16}$$

2) $(100101110.11101)_2 = (?)_{16}$

$$100101110.11101 = 0001\ 0010\ 1110 \cdot 1110\ 1000$$
$$= 1\ \ 2\ \ 14 \cdot 14\ \ 8$$
$$= 1\ \ 2\ \ E \cdot E\ \ 8$$
$$= (12E.E8)_{16}$$

II i) Decimal to Binary

1) $(29)_{10} = (?)_2$

2	29	1
2	14	0
2	7	1
2	3	1
2	1	1
	0	

LSB ↑ MSB

$\therefore (29)_{10} = (11101)_2$

2) $(0.6875)_{10} = (?)_2$

MSB ↓ LSB

0.6875 × 2	1.375	1
0.375 × 2	0.75	0
0.75 × 2	1.5	1
0.5 × 2	1	1

$\therefore (0.6875)_{10} = (1011)_2$

ii) Decimal to octal

1) $(255)_{10} = (?)_8$

8	255	7
8	31	7
8	3	3
	0	

$\therefore (255)_{10} = (377)_8$

2) $(177.25)_{10} = (?)_8$

8	177	1
8	22	6
8	2	2
	0	

0.25×8	2.0	2
0.00×8	0.0	0

$\therefore (177.25)_{10} = (261.20)_8$

iii) Decimal to Hexadecimal

i) $(2001.43)_{10} = (?)_{16}$

16	2001	1
16	125	13 D
16	7	7
	0	

0.43×16	6.88	6	
0.88×16	14.08	14	E
0.08×16	1.28	1	
0.28×16	4.48	4	
0.48×16	7.68	7	
0.68×16	10.88	10	A
0.88×16	14.08	14	E

$(2001.43)_{10} = (7D1.6E147AE)_{16}$

III i) Octal to Binary

i) $(437.21)_8 = (?)_2$

4	3	7	.	2	1
100	011	111	.	010	001

$\therefore (437.21)_8 = (100011111.010001)_2$

i) Octal to Decimal

1) $(250)_8 = (?)_{10}$

$(250)_8 = 2\times8^2 + 5\times8^1 + 0\times8^0$

$= 2\times64 + 5\times8 + 0$

$= 128 + 40$

$\therefore (250)_8 = (168)_{10}$

2) $(35.7)_8 = (?)_{10}$ $(8^0 = 1)$

$(35.7)_8 = 3\times8^1 + 5\times8^0 + 7\times8^{-1}$

$= 24 + 5\times\cancel{8} + 7\times\frac{1}{8}$

$= 24 + 5 + \frac{7}{8} = 29 + \frac{7}{8}$

$= \frac{29\times8+7}{8} = \frac{232+7}{8} = \frac{239}{8}$

$(35.7)_8 = (29.875)_{10}$

iii) Octal to Hexadecimal

1) $(537)_8 = (?)_{16}$

① convert octal to binary $= (\overset{5}{101}\ \overset{3}{011}\ \overset{7}{111})_2$

② convert Binary to Hexadecimal

$(101011111)_2 = 0001\ 0101\ 1111 = (15F)_{16}$

- Rules for Binary Division

Example :-

```
11 ) 1100 ( 100
     -11
     ----
      000
     - 00
     ----
       00
```

* Binary Substraction

i) Using 1's Complement

1's Complement :- it can be obtained simply by changing all 1's to zero and all 0's to 1

Eg:- 10110 → 01001

Rules ⇒ 1) To subtract A-B
2) Find 1's complement of B.
3) Add 1's complement of B to A
4) if carry equal to 1 then add it to the result. it is called end around carry
5) if carry equal to 1 result is +ve & its in true form
6) if carry equal to 0 the result is -ve & is in 1's complement so convert it in true form

Ex 1) $(10110)_2 - (10000)_2$

→ 1's Complement of 10000 → 01111

Add 10110

$$
\begin{array}{r}
01111 \\
+\ 10110 \\
\hline
\boxed{1}\ 00101
\end{array}
$$

carry

carry = 1 ∴ Add 1 to Result

$$
\begin{array}{r}
00101 \\
+\ \ \ \ 1 \\
\hline
\boxed{00110}
\end{array}
$$

$\therefore (10110)_2 - (10000)_2 = (00110)_2$

Ex 2) $(33)_{10} - (64)_{10}$

2	33	1
2	16	0
2	8	0
2	4	0
2	2	0
	1	1

$= (100001)_2$

2	64	0
2	32	0
2	16	0
2	8	0
2	4	0
2	2	0
	1	1

$= (1000000)_2$

$\therefore (100001)_2 - (1000000)_2$

1's complement of $(1000000)_2 \rightarrow 0111111$

Add $(100001)_2$

$$\begin{array}{r} 0111111 \\ +\ 100001 \\ \hline 0\ \rfloor\ 1100000 \end{array}$$

if carry = 0 ∴ Not in true form

∴ 1's complement of answer and -ve sign

∴ 1's complement of $1100000 \rightarrow (-0011111)_2$

$$(-0011111)_2 = 0+0+1\times2^4+1\times2^3+1\times2^2+1\times2^1+1\times2^0$$
$$= 16+8+4+2+1$$
$$= (-31)_{10}$$

$\therefore (33)_{10}-(64)_{10} = (-31)_{10}$

ii) Using 2's Complement

2's Complement :- 2's complement of Binary no. can be obtained by adding 1 to 1's complement of that no.

Ex :- 1011

$$\begin{array}{rl} 0100 & \rightarrow \text{1's complement} \\ +\quad 1 & \rightarrow \text{Add 1} \\ \hline \boxed{0101} & \rightarrow \text{2's Complement.} \end{array}$$

Rules :- 1) To substract A - B.

2) Find 2's complement of B.

3) Add it to A

4) if carry is generated '1' then discard (ignore) carry.

5) if carry is 0 then answer will be -ve & in 2's complement form.

6) To get ans in true form take its 2's complement & give -ve sign.

Ex 1) $(50)_{10} - (2A)_{16}$

~~$(50)_{16}$ =~~

~~2~~	~~50~~	~~0~~
~~2~~	~~25~~	~~1~~
~~2~~	~~17~~	~~1~~
~~2~~	~~8~~	~~0~~
~~2~~	~~4~~	~~0~~
~~2~~	~~2~~	~~0~~
	~~1~~	~~1~~

$(50)_{10}$ =

2	50	0
2	25	1
2	12	0
2	6	0
2	3	1
	1	1

$= (110010)_2$

$(2A)_{16} = (0010\ 1010)_2$

$\therefore (110010)_2 - (00101010)_2$

2's complement of (00101010)

1's complement → 11010101

Add 1 + 1

11010110

Now 11010110

+ 110010

1 0000 1000

Discard carry.

∴ $(50)_{10} - (2A)_{16} = (00001000)_2$

Ex 2] $(7)_{10} - (15)_{10}$

$(7)_{10} = (0111)_2$, $(15)_{10} \rightarrow (1111)_2$

2's complement of $(1111)_2$

1's comp → 0000

+ 1

0001

Add
$$\begin{array}{r} 0001 \\ +\ 0111 \\ \hline 0\ \vert\ 1000 \end{array}$$

carry = 0 ∴ Take 2's complement of result & -ve sign

∴ For 2's complement of 1000

1's complement 0111
$$\begin{array}{r} 0111 \\ +\ \ \ \ 1 \\ \hline (-1000)_2 \end{array} = (-8)_{10}$$

∴ $(7)_{10} - (15)_{10} = (-8)_{10}$

IV i) Hexadecimal to Binary.

1) $(57EB \cdot AD)_{16} = (?)_2$

5 7 E B · A D

= 0101 0111 1110 1011 · 1010 1101

= $(0101011111101011 \cdot 10101101)_2$

2) $(D283)_{16} = (?)_2$

D 2 8 3

= 1101 0010 1000 0011

= $(1101001010000011)_2$

ii) Hexadecimal to Decimal

① $(268)_{16} = (?)_{10}$

$= 2\times16^2 + 6\times16^1 + 8\times16^0 = 512 + 96 + 8 = (616)_{10}$

② $(11A \cdot 62)_{16} = (?)_{10}$

$= 1\times16^2 + 1\times16^1 + 10\times16^0 + 6\times16^{-1} + 2\times16^{-2}$

$= 256 + 16 + 10 + \frac{6}{16} + \frac{2}{256}$

iii) Hexadecimal to Octal.

① $(5CB\cdot12E)_{16} = (?)_8$

i) Convert Hexadecimal to Binary

5 C B · 1 2 E

$= (0101\ 1100\ 1011 \cdot 0001\ 0010\ 1110)_2$

ii) Convert Binary to octal.

$\underline{010}\,\underline{111}\,\underline{001}\,\underline{011} \cdot \underline{000}\,\underline{100}\,\underline{101}\,\underline{110}$

$= 2\ 7\ 1\ 3 \cdot 0\ 4\ 5\ 6$

$= (2713\cdot0456)_8$

② $(78\cdot4B)_{16} = (?)_8$

i) Hexadecimal to Binary

7 8 · 4 B

$(0111\ 1000 \cdot 0100\ 1011)_2$

ii) Binary to octal

$\underline{01}\,\underline{111}\,\underline{000} \cdot \underline{010}\,\underline{010}\,\underline{11}$

$= 001\ 111\ 000 \cdot 010\ 010\ 110$

$= (170\cdot226)_8$

Binary Arithmatics

- Rules for Binary Addition

	Sum	Carry
0 + 0 =	0	0
0 + 1 =	1	0
1 + 0 =	1	0
1 + 1 =	0	1

Example :-

```
   1011
 + 1001
 ------
 1 0100
 ↓
 carry
```

- Rules for Binary Subtraction

	Subtraction	Borrow
0 − 0 =	0	0
0 − 1 =	1	1
1 − 0 =	1	0
1 − 1 =	0	0

Example :-

```
   1110
 − 1011
 ------
   0011
```

Decimal

```
  14
 -11
 ---
  03
```

- Rules for Binary Multiplication

0 × 0 = 0
0 × 1 = 0
1 × 0 = 0
1 × 1 = 1

Example :-

```
     101
   x 010
   -----
     000
 +  101+
 + 000++
 -------
   01010
```