1

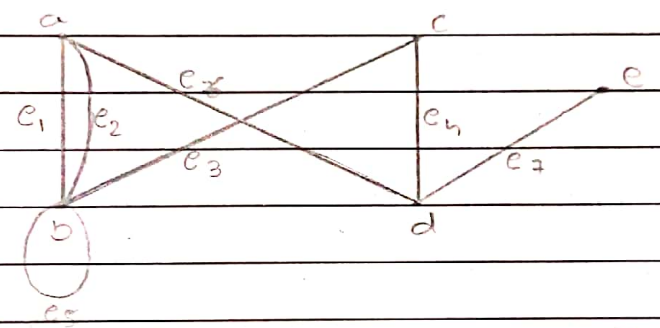
TUTORIAL #3

Date:

GRAPHS

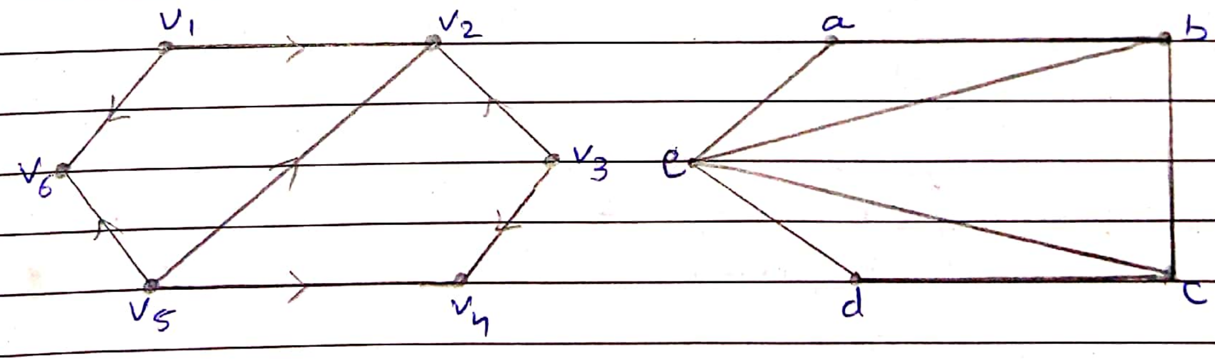
Q.1] Draw the undirected graph $G = (V, E)$, where $V = \{a, b, c, d, e\}$ and $E = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7\}$ and its incidence relation given as:
$e_1 = (a, b)$, $e_2 = (a, b)$, $e_3 = (b, c)$, $e_4 = (c, d)$, $e_5 = (b, b)$, $e_6 = (a, d)$; $e_7 = (e, d)$

=>



(Undirected graph)

Q.2] Consider the following graphs: Determine the degree of each node and verify handshaking lemma.



- As we can see it is directed graph so,

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Date:

(1) In-degree
(2) out degree

Now,

In-degree	Out-degree
$v_1 = 0$	$v_1 = 2$
$v_2 = 3$	$v_2 = 0$
$v_3 = 0$	$v_3 = 2$
$v_4 = 2$	$v_4 = 0$
$v_5 = 0$	$v_5 = 3$
$v_6 = 2$	$v_6 = 0$
7	7

Total number of degree = In-degree + out-degree
= 7 + 7
= 14

Now, Handshaking lemma

$\therefore \sum_{i=1}^{n} deg(v) = 2|E|$

$\therefore \sum_{i=1}^{6} deg(v) = 2|7|$

$\therefore \boxed{deg(v) = 14}$

Now, for undirected graph

Date:

degree of $a = 2$
$b = 3$
$c = 3$
$d = 2$
$e = 4$

Now,
Total Degree $= a + b + c + d + e$
$= 2 + 3 + 3 + 2 + 4$
$= 14$

Now, $\sum \deg(v) = 2|E|$

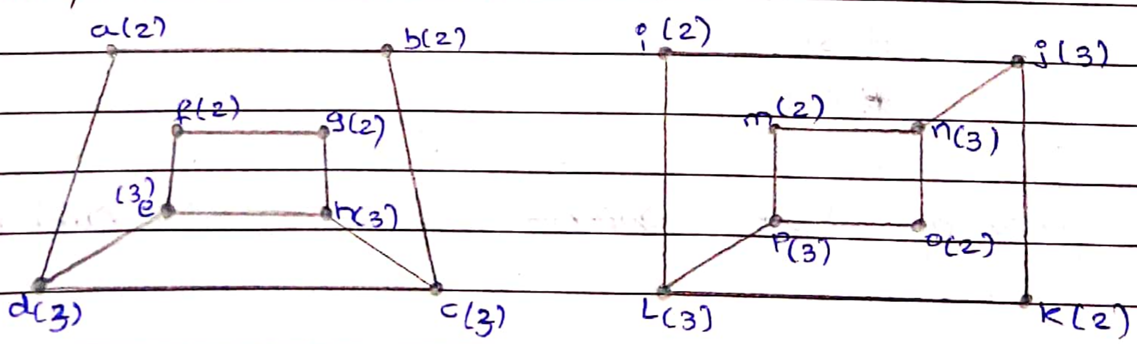
$\Rightarrow \deg(v) = 14$

Hence, $2|7| = 14$

$\therefore$ The given graph is an Handshaking lemma.

Q.3] Check whether the following graphs are isomorphic or not.

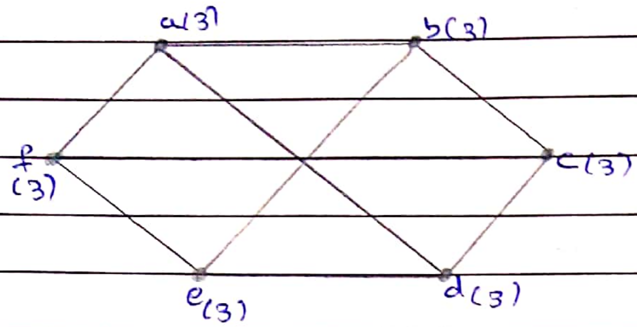
=> (A)

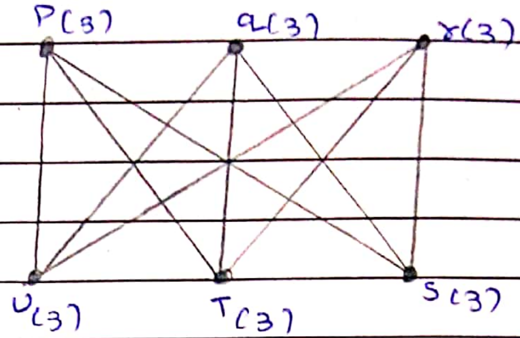


Here, $a \to j$ & $b \not\to j$ so graph is not ISOMORPHIC

Date:

(B)

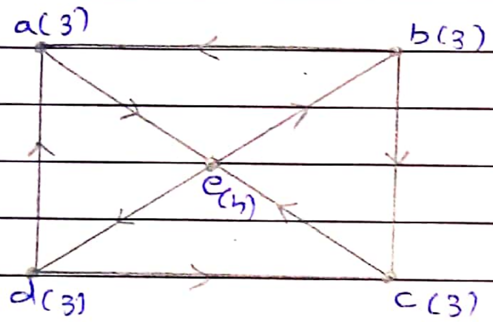


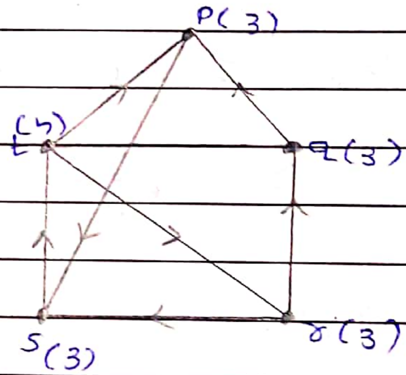


Here, $a \to p$, $b \to q$, $c \to r$, $d \to s$, $e \to t$, $f \to u$

Hence, this graph is ISOMORPHIC.

(C)





Here,

$a \to p$

$b \to q$

$c \to r$

$d \to s$

$e \to t$

Hence, This graph is ISOMORPHIC

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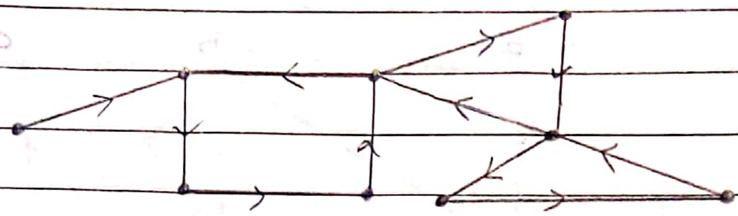
Page 5

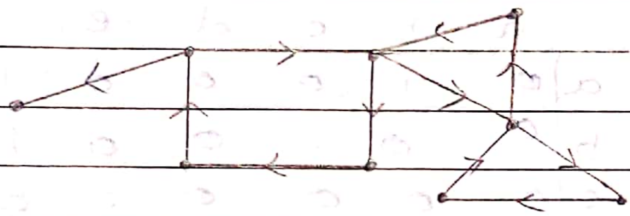
Q.4) Find converse of a given diagraph.

⇒ (a)



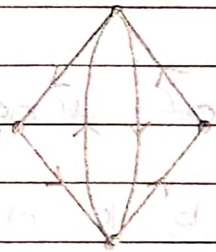
(P)

Now, Transpose of P will be P^T



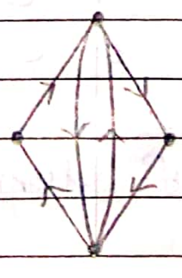
(P^T)

(b)



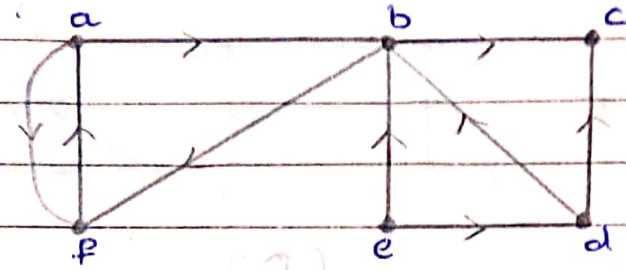
(q)

Now, Transpose of q will be q^T



(q^T)

Q.5) Find reachability matrix for the given graph.



=> Step-1 Path of length is 1

$$R^1 = \begin{array}{c|cccccc} & a & b & c & d & e & f \\ \hline a & 0 & 1 & 0 & 0 & 0 & 1 \\ b & 0 & 0 & 1 & 0 & 0 & 1 \\ c & 0 & 0 & 0 & 0 & 0 & 0 \\ d & 0 & 1 & 1 & 0 & 0 & 0 \\ e & 0 & 1 & 0 & 1 & 0 & 0 \\ f & 1 & 0 & 0 & 0 & 0 & 0 \end{array}$$

Step-2 Path of length is 2

$$R^2 = \begin{array}{c|cccccc} & a & b & c & d & e & f \\ \hline a & 0 & 0 & 1 & 0 & 0 & 1 \\ b & 1 & 0 & 0 & 0 & 0 & 0 \\ c & 0 & 0 & 0 & 0 & 0 & 0 \\ d & 0 & 0 & 1 & 0 & 0 & 1 \\ e & 0 & 1 & 1 & 0 & 0 & 1 \\ f & 0 & 1 & 0 & 0 & 0 & 0 \end{array}$$

Step-3 Path of length is 3

$$R^3 = \begin{array}{c} \\ a \\ b \\ c \\ d \\ e \\ f \end{array} \begin{array}{c} \begin{array}{cccccc} a & b & c & d & e & f \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

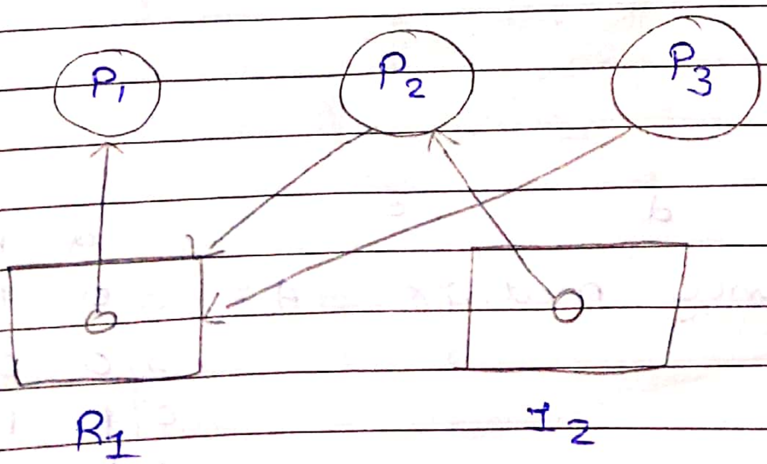
Now, Reachability matrix is $R = R^1 + R^2 + R^3$

$$R = \begin{array}{c} \\ a \\ b \\ c \\ d \\ e \\ f \end{array} \begin{array}{c} \begin{array}{cccccc} a & b & c & d & e & f \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 2 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 2 & 0 & 0 & 1 \\ 1 & 2 & 2 & 1 & 0 & 2 \\ 1 & 1 & 1 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

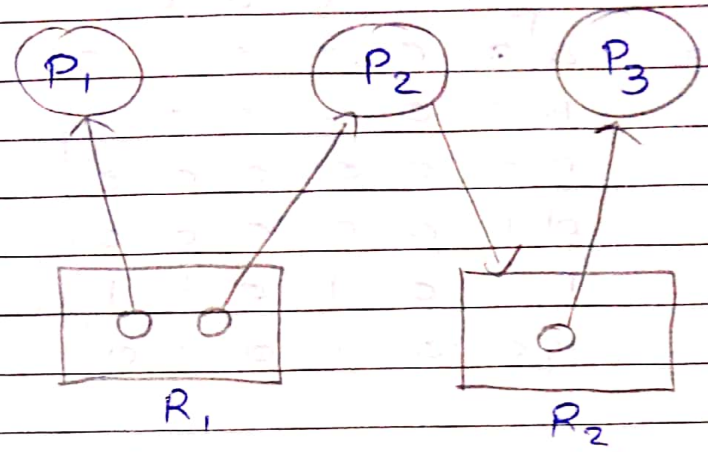
Q.6] Give three examples of resource allocation graph which does not have any deadlock

=> The example of resource allocation graph without deadlock.

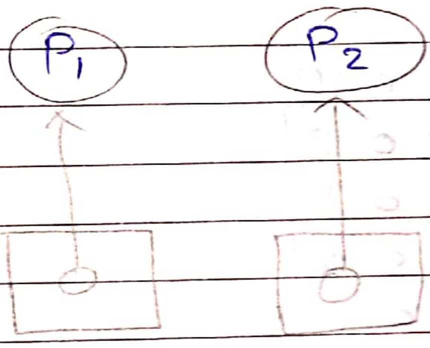
(A)



(B)

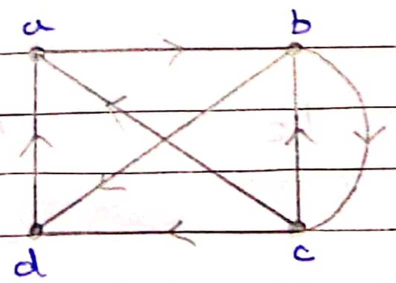


(C)



Hence, this are the three examples of Resource Allocation Graph.

Q.7] Obtain adjacency matrix A for the diagraph in the figure. Find A, A^2 without matrix multiplication



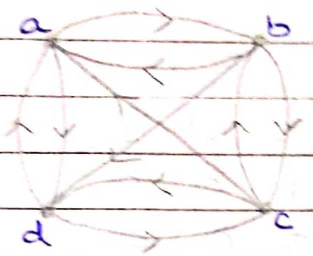
=> Adjacency matrix $A =$

$$\begin{array}{c|cccc|} & a & b & c & d \\ \hline a & 0 & 1 & 0 & 0 \\ b & 0 & 0 & 1 & 1 \\ c & 1 & 1 & 0 & 1 \\ d & 1 & 0 & 0 & 0 \end{array}$$

Now,

$$A^2 = \begin{array}{c|cccc} & a & b & c & d \\ \hline a & 0 & 0 & 2 & 2 \\ b & 2 & 0 & 0 & 2 \\ c & 2 & 2 & 0 & 2 \\ d & 0 & 2 & 0 & 0 \end{array}$$

Q.8] Find path of length 2 from the graph (using matrix)



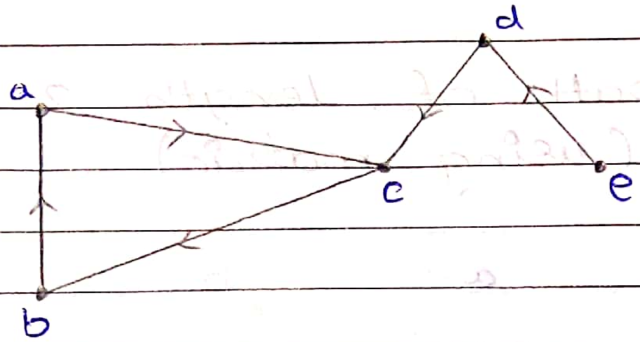
$$\Rightarrow \quad A = \begin{array}{c|cccc} & a & b & c & d \\ \hline a & 0 & 1 & 0 & 1 \\ b & 1 & 0 & 1 & 1 \\ c & 1 & 1 & 0 & 1 \\ d & 1 & 0 & 1 & 0 \end{array}$$

Now, path of length 2 using matrix
$A^2 = A \times A$

$$\therefore \quad A^2 = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} \times \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

$$\therefore \quad A^2 = \begin{bmatrix} 2 & 0 & 2 & 1 \\ 2 & 2 & 0 & 2 \\ 2 & 1 & 2 & 2 \\ 1 & 2 & 0 & 2 \end{bmatrix}$$

Q.9] Find the shortest path between each ~~path~~ pair of vertices for a simple diagraph using warshall's algorithm. Produce Path matrix.



$$\Rightarrow \quad A = \begin{array}{c|ccccc} & a & b & c & d & e \\ \hline a & 0 & 0 & 1 & 0 & 0 \\ b & 1 & 0 & 0 & 0 & 0 \\ c & 0 & 1 & 0 & 0 & 0 \\ d & 0 & 0 & 1 & 0 & 0 \\ e & 0 & 0 & 0 & 1 & 0 \end{array}$$

Now,

$$D_0 = \begin{array}{c|ccccc} & a & b & c & d & e \\ \hline a & 0 & 0 & 1 & 0 & 0 \\ b & 1 & 0 & \infty & \infty & \infty \\ c & 0 & 1 & 0 & \infty & \infty \\ d & 0 & \infty & 1 & 0 & \infty \\ e & 0 & \infty & \infty & 1 & 0 \end{array}$$

- $D_1[b,c] = \text{Min}[D_0(b,c), D_0(b,1)+D_0(1,c)]$
  $D_1[b,c] = \text{Min}[\infty, 1+1]$
  $D_1[b,c] = 2$
- $D_1[b,d] = \text{Min}[D_0(b,d), D_0(b,1)+D_0(1,d)]$
  $= \text{Min}[\infty, 1+0]$
  $= 1$

- $D_1[b,e] = Min[D_0(b,e), D_0(b,1) + D_0(1,e)]$
  $= 1$
- $D_1[c,d] = Min[D_0(c,d), D_0(c,1) + D_0(1,d)]$
  $= 0$
- $D_1[c,e] = Min[D_0(c,e), D_0(c,1) + D_0(1,e)]$
  $= 0$
- $D_1[d,b] = Min[D_0(d,b), D_0(d,1) + D_0(1,b)]$
  $= 0$
- $D_1[e,b] = Min[D_0(e,b), D_0(e,1) + D_0(1,b)]$
  $= 0$
- $D_1[e,c] = Min[D_0(e,c), D_0(e,1) + D_0(1,c)]$
  $= 1$

$$D_1 = \begin{array}{c|ccccc|} & a & b & c & d & e \\ \hline a & 0 & 0 & 1 & \infty & \infty \\ b & 1 & 0 & 2 & 1 & 1 \\ c & \infty & 1 & 0 & \infty & \infty \\ d & \infty & 0 & 1 & 0 & \infty \\ e & \infty & 0 & 1 & 1 & 0 \end{array}$$

- $D_2[a,d] = Min[D_1(a,d), D_1(a,2) + D_1(2,d)]$
  $= 1$
- $D_2[a,e] = Min[D_1(a,e), D_1(a,2) + D_1(2,e)]$
  $= 1$
- $D_2[c,a] = Min[D_1(c,a), D_1(c,2) + D_1(2,a)]$
  $= 2$
- $D_2[c,d] = Min[D_1(c,d), D_1(c,2) + D_1(2,d)]$
  $= 2$
- $D_2[c,e] = Min[D_1(c,e), D_1(c,2) + D_1(2,e)]$
  $= 2$
- $D_2[d,a] = Min[D_1(d,a), D_1(d,2) + D_1(2,a)]$
  $= 1$

- $D_2[d,e] = Min[D_1(d,e),\ D_1(d,2) + D_1(2,e)]$
  $= 1$
- $D_2[e,a] = Min[D_1(e,a),\ D_1(e,2) + D_1(2,a)]$
  $= 0$

$$D_2 = \begin{array}{c|ccccc} & a & b & c & d & e \\ \hline a & 0 & \infty & 1 & 1 & 1 \\ b & 1 & 0 & 2 & 1 & 1 \\ c & 2 & 1 & 0 & 2 & 2 \\ d & 1 & \infty & 1 & 0 & 1 \\ e & \infty & \infty & 1 & 1 & 0 \end{array}$$

$D_3[a,b] = Min[D_2(a,b),\ D_2(a,3) + D_2(3,b)]$
$= 2$

$D_3[d,b] = Min[D_2(d,b),\ D_2(d,3) + D_2(3,b)]$
$= 2$

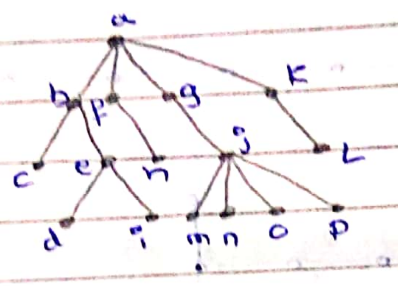
$D_3[e,b] = Min[D_2(e,b),\ D_2(e,3) + D_2(3,b)]$
$= 2$

$$D_4 = \begin{array}{c|ccccc} & a & b & c & d & e \\ \hline a & 0 & 2 & 1 & 1 & 1 \\ b & 1 & 0 & 2 & 1 & 1 \\ c & 2 & 1 & 0 & 2 & 2 \\ d & 1 & 2 & 1 & 0 & 1 \\ e & 2 & 2 & 1 & 1 & 0 \end{array}$$

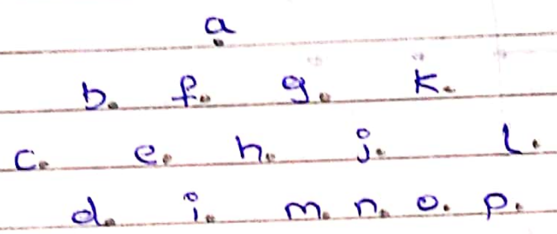
$$D_5 = \begin{array}{c|ccccc} & a & b & c & d & e \\ \hline a & 0 & 2 & 1 & 1 & 1 \\ b & 1 & 0 & 2 & 1 & 1 \\ c & 2 & 1 & 0 & 2 & 2 \\ d & 1 & 2 & 1 & 0 & 1 \\ e & 2 & 2 & 1 & 1 & 0 \end{array}$$

Hence,
D_5 is the path matrix

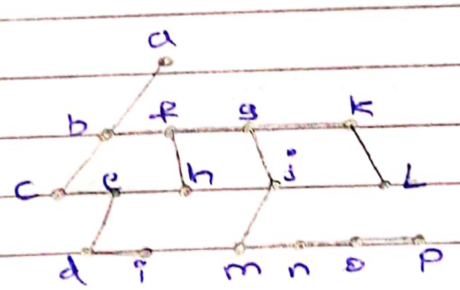
Q.10] Convert following m-ary trees into binary trees.



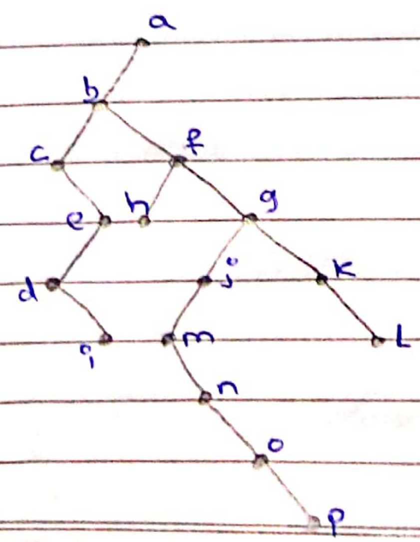
=> **Step-1** Removing all edges.



step 2 connecting left side vertices.



step.3 Representation in proper manner

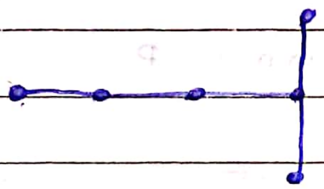


Date__/__/__
Page__

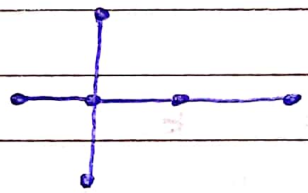
Q.11] Draw three non isomorphic trees with 6 vertices.

=> (A)

(B)



(C)



=> Hence, this are the required non-isomorphic trees with 6-vertices.