Hass diagram.

Hass

- Reflexi
- Antisymm.
- Transitive

Poset can be represented by diagraph
A simpler way of representing Poset is Hass diagram. Partial order Relation set

→ Method to find Hass diagraph.

1. Omit loops as R is refl.
2. Omit all arrows.
3. Eliminate all edges that implied by transitive relation $(a,b) \in R$ & $(b,c) \in R \Rightarrow (a,c) \in R$.

Ex:- Let R be relation on set A
$A = \{5, 6, 8, 10, 28, 36, 48\}$.
Let $R = \{(a,b) \mid a \text{ is divisor of } b\}$.
Draw the Hass diagram & compare with digraph. Determine whether R is equivalence relation.

$A = \{5, 6, 8, 10, 28, 36, 48\}$

$R = \{(a,b),\ a \text{ is divisor of } b\}$

$R = \{(5,5), (6,6), (8,8), (10,10), (28,28), (36,36), (48,48), (5,10), (6,36), (6,48), (8,48)\}$

1) R is reflexive:- As aRa, i.e. $(a,a) \in R$.
R is reflexive.

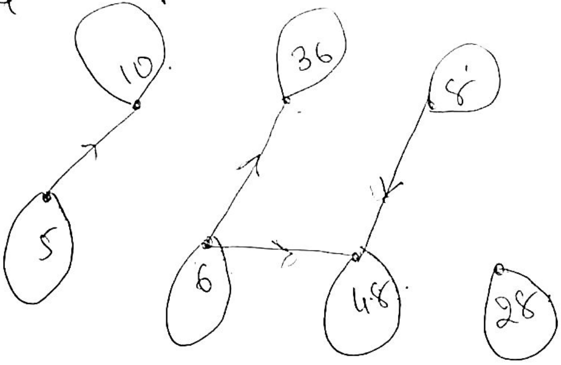
2) R is symmetric:- $aRb \Rightarrow bRa$
As $(5,10) \in R$ but $(10,5) \notin R$
$\therefore$ R is not symm.

3) R is transitive:- As $(a,b) \in R$ & $(b,c) \in R$ then $(a,c) \in R$

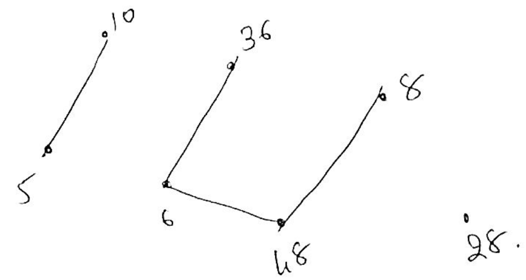
$\therefore$ R is transitive.

4) R is antisymm.:- $\therefore$ R is antisymm.
as aRb & $bRa \Rightarrow aRa$.

$\therefore$ R is not not equivalence relation.
& R is partial order relation.



Hass diagram.



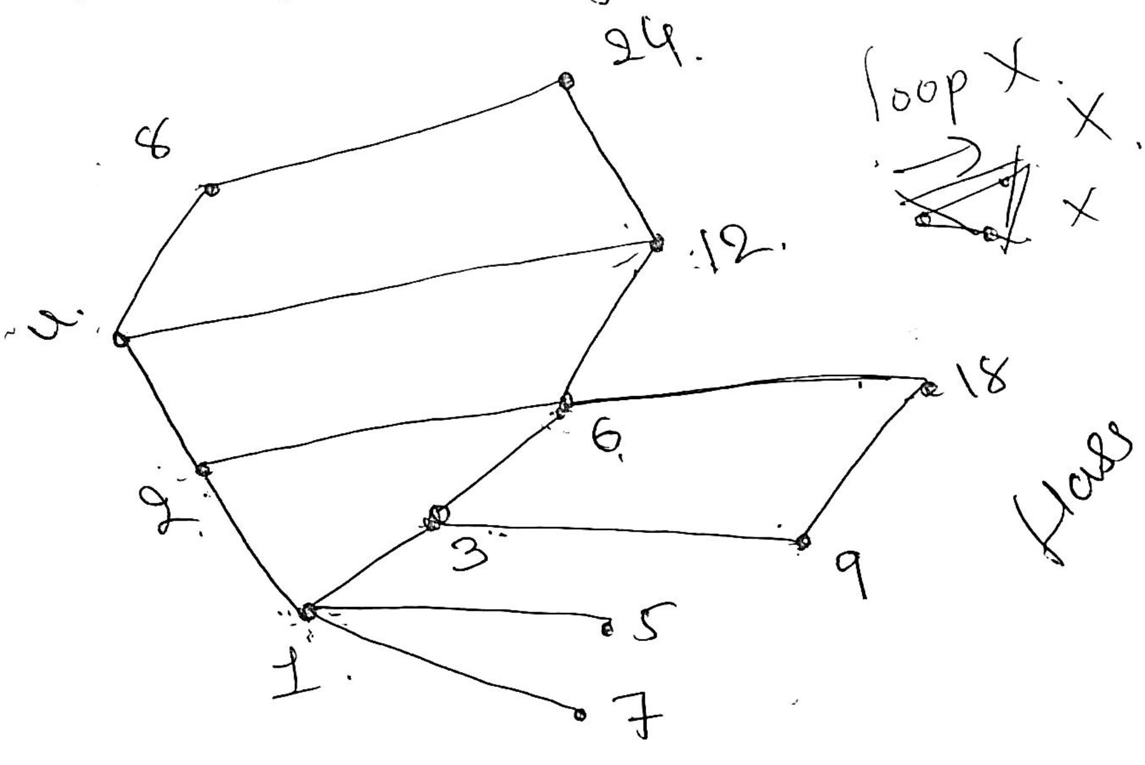
Ha

Ex:- Let $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 12, 18, 24\}$ be ordered by the relation x divides y. Show that R is partial ordering & draw the Hass diagram.

$R = \{(1,1), (1,2), (1,3), (1,4), (1,6), (1,7), (1,8), (1,9),$
$(1,12), (1,18), (1,24), (2,2), (3,3), (4,4), (5,5), (6,6),$
$(7,7), (8,8), (9,9), (12,12), (18,18), (24,24),$
$(2,4), (2,6), (2,8), (2,12), (2,18), (2,24), (3,6),$
$(3,9), (3,12), (3,18), (3,24), (4,8), (4,12), (4,24),$
$(6,12), (6,18), (6,24), (8,24), (9,18), (12,24)\}$

→ R is reflexive
→ R is antisymm.
→ R is transitive.

∴ R is partial ordered relation set.

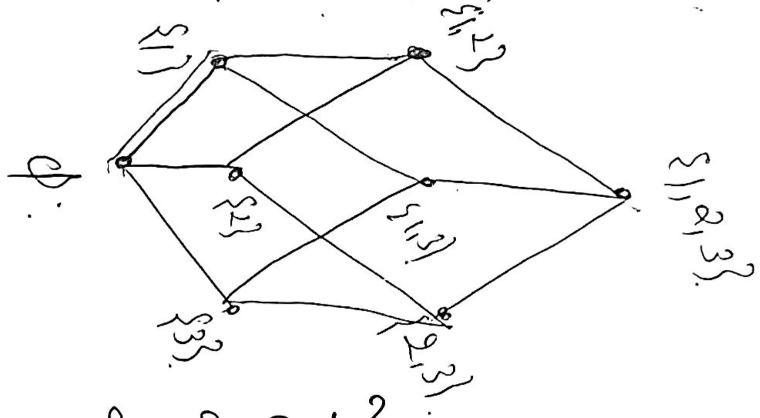


Hass diagram.

R X X R defi

Ex:-1) Let $S = \{1,2,3\}$. $(P(S), \subseteq)$ is a poset.
Draw the Hass diagram

$P(S) = \{\phi, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$



Chain:- $\{\phi, \{1\}, \{1,2\}, \{1,2,3\}\}$

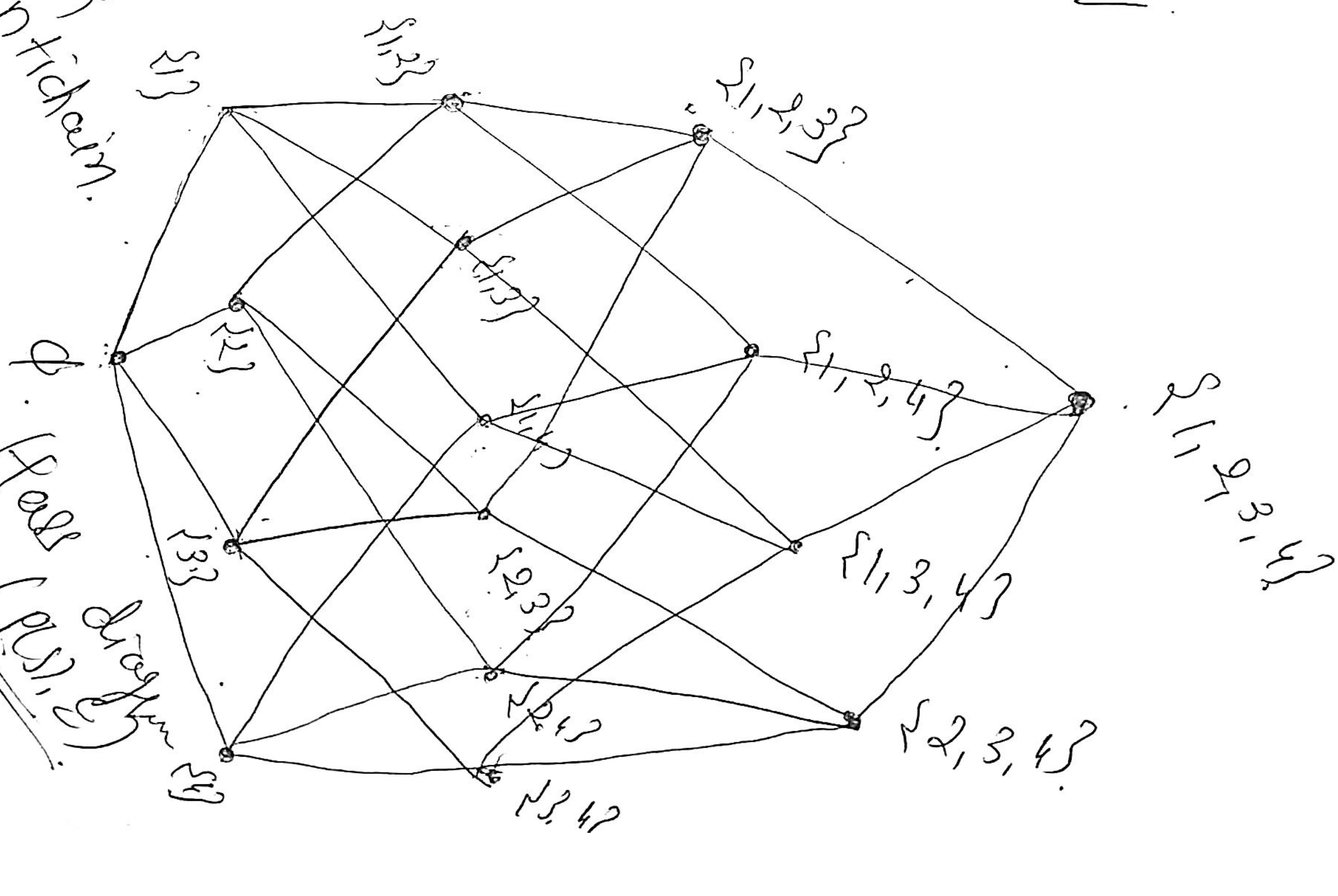
Antichain:- $\{\{1\}, \{2\}, \{3\}\}$

2) $S = \{1,2,3,4\}$.

$P(S) = \{\phi, \{1\}, \{2\}, \{3\}, \{4\}, \{1,2\}, \{1,3\}, \{1,4\}, \{2,3\}, \{2,4\}, \{3,4\}, \{1,2,3\}, \{2,3,4\}, \{1,2,4\}, \{1,3,4\}, \{1,2,3,4\}\}$.

Chain, Antichain.

Draw the Hass diagram $\subseteq$



Hass diagram of $(P(S), \subseteq)$.

* Chain & Antichains.

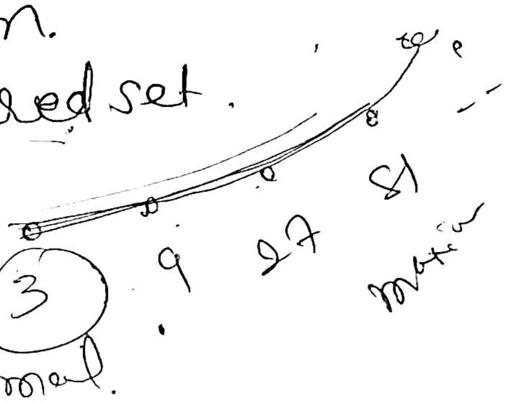
*. Totally ordered set or Linearly ordered set

→ If set A itself is a chain, the poset (A, ≤) is called totally ordered set.

Ex. $\boxed{A = \{3, 9, 27, 81, \dots\}}$ and aRb iff a|b then, (A, R) is totally ordered set.

→ Here A itself is a chain.

∴ (A, ≤) is totally ~~ordered set~~.



* Maximal & Minimal element :-

eg. A = {2, 3, 5, 6, 10, 15, 30} Hass diagram of Poset.

aRb ⇔ a|b.

(A, ≤) ⇒

maximal ⇒ 30

minimal ⇒ 2, 3, 5.