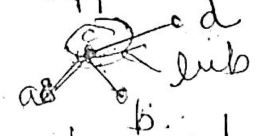
* Upper Bounds and lower bounds:-

→ Let $(A, \leq)$ be a poset for elements $a, b \in A$, an element $c \in A$ is called upper bound of a & b if $a \leq c$, $b \leq c$



→ c is known as least upper bound (lub) of a & b if c is an upper bound of a, b & if there is no other upper bound d of a & b such that $d \leq c$. Lub is known as supremum.

→ Illy, an element e is said to be lower bound of a & b if $e \leq a$ & $e \leq b$.

→ e is known as greatest lower bound (glb) of a & b if there is no other lower bound f such that $e \leq f$. Glb is known as infimum

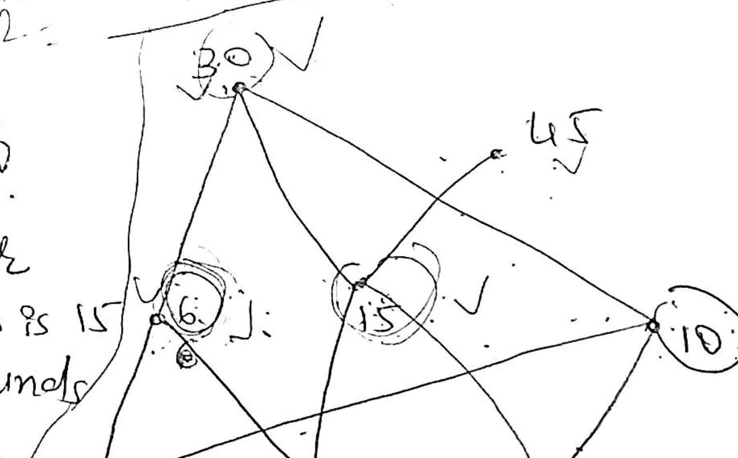
Ex:- $A = \{2, 3, 5, 6, 10, 15, 30, 45\}$, aRb iff $a | b$.
$(A, \leq)$ Hass diagram

Soln:- → 6 & 30 are upper bounds of 2 & 3 & lub is 6.

→ 15, 30, 45 are upper bounds of 3 & 5 & lub is 15

→ 10 & 30 are upper bounds of 2 & 5 & lub is 10.

→ 15, 3, 5 are lower bounds of 30 & 45 & glb is 15.



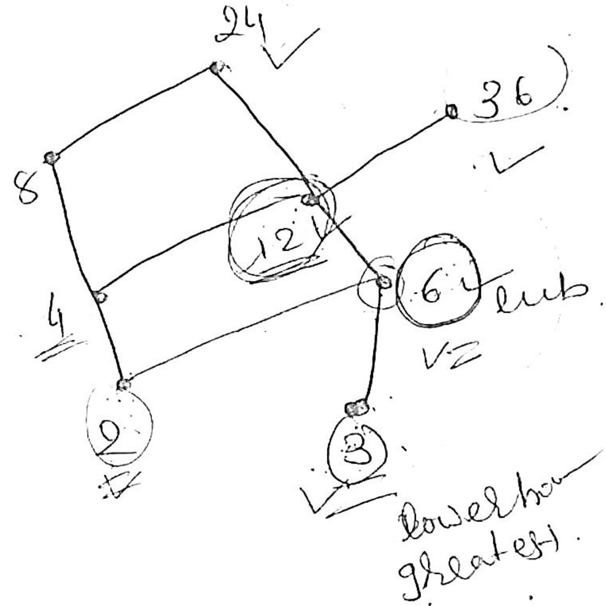
Ex:- $A = \{2, 3, 4, 6, 8, 12, 24, 36\}$, aRb iff $a|b$.

$(A, \leq)$ Hass diagram

→ 6, 12 are upper bounds of 2 & 3. ~~lubs~~

→ 24, 36 are upper bounds of 12, 6, 2, 3 & lub is 6.

→ lly, 2, 3, 4, 6, 12 are lower bounds of 24 & 36. & 12 is glb.



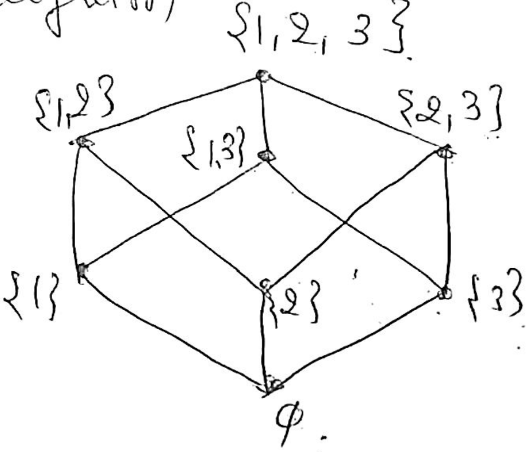
* Lattice:- A lattice is a poset $(A, \leq)$ $(P(S), \subseteq)$ in which every subset $\{a, b\}$ of A has a lub & glb.

Ex:- $A = \{1, 2, 3\}$

$P(A) = \{\phi, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$

→ $(P(A), \subseteq)$ is poset & Hass diagram

Here every pair of elements has lub & glb. Hence $(P(A), \subseteq)$ is lattice.



Ex:- Above ex. of lub & glb.

→ Since the pair {2, 3} does not have glb & also {24, 36} does not have lub The poset is not lattice.

* Lattice Operators :-

1) lub is denoted by $a \vee b$ (a join b)

2) glb is denoted by $a \wedge b$ (a meet b)

Ex:- Let A be set of factor of positive integer m & relation is divisibility on A. i.e $R = \{(x,y) \mid x, y \in A,\ x \mid y\}$. For $m = 45$. S.T. Poset $(A, \leq)$ is lattice. Draw Hass diagram & give join & meet for the lattice.

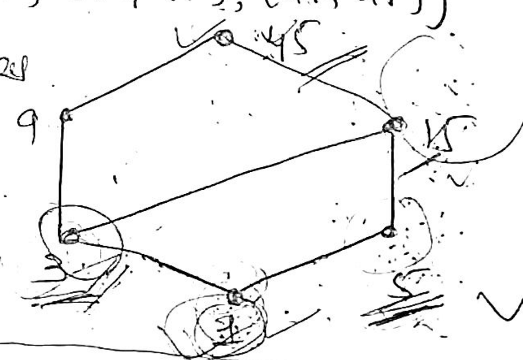
Sol^n:- A is the set of divisors of 45.

$\therefore A = \{1, 3, 5, 9, 15, 45\}$ & $R = \{(x,y) : x \mid y\ x, y \in A\}$

$\therefore R = \{(1,1), (1,3), (1,5), (1,9), (1,15), (1,45),$
$(3,3), (3,9), (3,15), (3,45), (5,5), (5,15),$
$(5,45), (9,9), (9,45), (15,15), (15,45), (45,45)\}$

Every pair of elements of A has glb & lub.

$\therefore (A, \leq)$ is lattice ξ



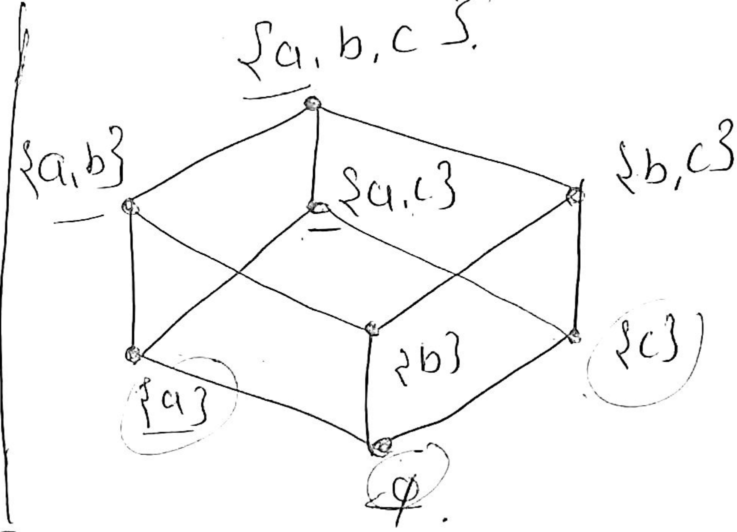
→ Join & meet of a & b as shown below:

$\vee$	1	3	5	9	15	45
1	1	3	5	9	15	45
3	3	3	15	9	15	45
5	5	15	5	45	15	45
9	9	9	45	9	45	45
15	15	15	15	45	15	45
45	45	45	45	45	45	45

$\wedge$	1	3	5	9	15	45
1	1	1	1	1	1	1
3	1	3	1	3	3	3
5	1	1	5	1	5	5
9	1	3	1	9	3	9
15	1	3	5	3	15	15
45	1	3	5	9	15	45

* **Sub lattice** :- Let $(A, \leq)$ be a lattice. A non empty subset S of A is called a Sub lattice of A if $a \vee b \in S$, $a \wedge b \in S$, whenever $a \in S$ & $b \in S$.

Ex :- $(P(S), \subseteq)$



→ $\{\{a\}, \{a,b\}, \{a,c\}, \{a,b,c\}\}$, $\{\{b\}, \{a,b\}, \{b,c\}, \{a,b,c\}\}$, $\{\{c\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}$ are sub lattice of $(P(S), \subseteq)$.

→ $A_1 = \{\phi, \{a\}, \{c\}, \{a,b,c\}\}$ is not sublattice as $a \vee c \notin A_1$.

→ $A_2 = \{\phi, \{a,b\}, \{a,c\}, \{a,b,c\}\}$ is not sublattice as $\{a,b\} \cap \{a,c\} = a \notin A_2$.