

Assignment-2

Q.1) Explain the efficiency of an algorithm.

Ans- The efficiency of an algorithm is a measure of the amount of resources consumed in solving a problem of size n .

- The important resource is time, i.e., time complexity.
- To measure the efficiency of an algorithm requires to measure its time complexity using any of the following approaches:
 1. To run it and measure how much processor time is needed. [Empirical approach]
 2. Mathematically computing how much time is needed as a function of input [Theoretically]
- Running time of an algorithm depends upon,
 - (1.) Input size
 - (2.) Nature of Input.
- Generally time grows with the size of input, for example, sorting 100 numbers will take time than sorting of 10,000 numbers.
- So, running time of an algorithm is usually measured as a function of input size.
- In theoretical computation of time complexity, Running time is measured in terms of number of steps/primitive operations performed.

Q.2] Explain Asymptotic Notation in Detail.

Ans- Asymptotic Notations (Big O, O-Theta & Omega) allow us to analyze an algorithm's running time.

- This is also known as an algorithm's growth rate.

- Asymptotic Notations are used,

1. To characterize the complexity of an algorithm.
2. To compare the performance of two or more algorithms solving the same problem.

- O-Notation (Big O notation) (Upper Bound)

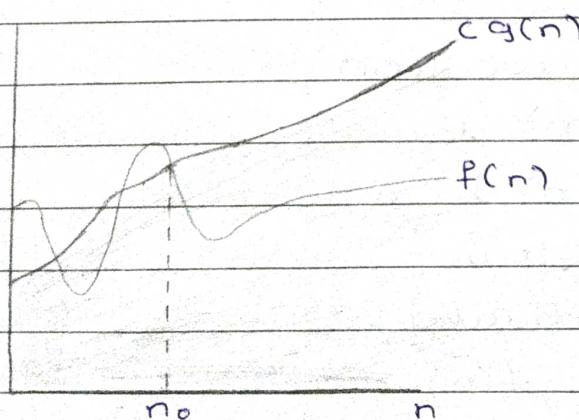
- For a given function $g(n)$, we denote by $O(g(n))$ the set of functions,

$O(g(n)) = \{f(n) : \text{there exist positive constants } c \text{ and } n_0 \text{ such that}$

$$0 \leq f(n) \leq cg(n) \text{ for all } n > n_0\}$$

- $g(n)$ is an asymptotic upper bound for $f(n)$

- $f(n) = O(g(n))$ implies : $f(n) \leq c \cdot g(n)$



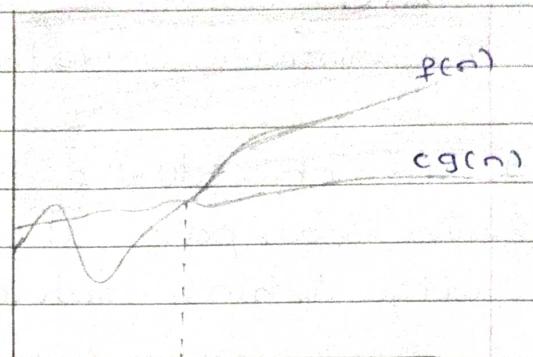
$$f(n) = O(g(n))$$

- Ω -Notation (Omega notation) (lower Bound)
- For a given notation $g(n)$, we denote by $\Omega(g(n))$ the set of functions,

$$\Omega(g(n)) = \{ f(n) : \text{there exist positive constants } c \text{ and } n_0 \text{ such that } 0 \leq c g(n) \leq f(n) \text{ for all } n \geq n_0 \}$$

- $g(n)$ is an asymptotically lower bound for $f(n)$
- $f(n) = \Omega(g(n))$ implies:

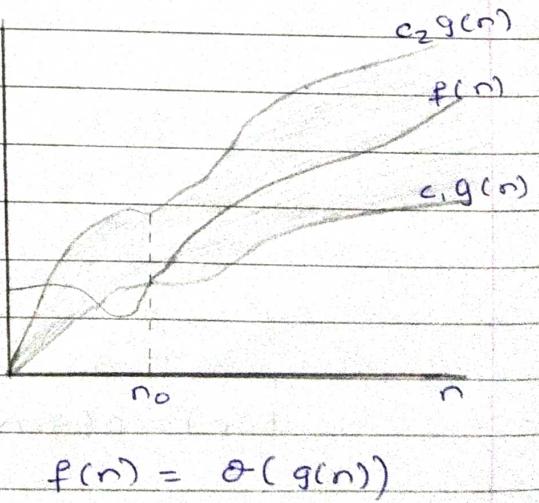
$$f(n) \geq c \cdot g(n)$$



- Θ -Notation (Theta notation) (same orders)
- For a given function $g(n)$, we denote by $\Theta(g(n))$ the set of functions,

$$\Theta(g(n)) = \{ f(n) : \text{there exist positive constants } c_1, c_2 \text{ and } n_0 \text{ such that } 0 \leq c_1 g(n) \leq f(n) \leq c_2 g(n) \text{ for all } n \geq n_0 \}$$
- $\Theta(g(n))$ is a set, we can write $f(n) \in \Theta(g(n))$ to indicate that $f(n)$ is a member of $\Theta(g(n))$
- $g(n)$ is an asymptotically tight bound for $f(n)$
- $f(n) = \Theta(g(n))$ implies:

$$f(n) = c \cdot g(n)$$



Q.3) What is Amortized Analysis.

Ans- Amortized analysis is considered not just one operation, but a sequence of operation on a given data structure.

- The time required to perform a sequence of data structure operations is averaged over all operations performed.
- Amortized analysis can be used to show that the average cost of an operation is small even though a single operation might be expensive.
- These are three most common techniques of amortized analysis,

1. Aggregate method.

- A sequence of n operation takes worst case time $T(n)$
- Amortized cost per operation is $T(n)/n$.

2. The accounting method.

- Assign each type of operations an amortized cost.
- Overcharge some operations.
- Store the overcharges as credit on specific objects.
- Then use the credit for compensation for some later operations.

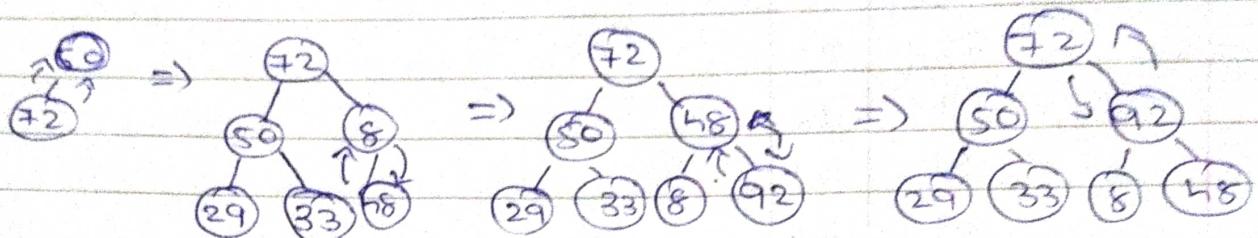
3. Potential method

- Same as accounting method.
- But store the credit as "potential energy" and as a whole.

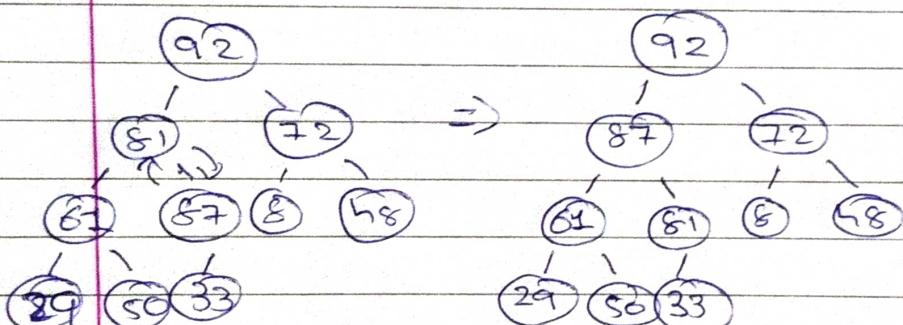
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Q.4) Using heap sort algorithm sort the array
50, 72, 8, 29, 33, 48, 92, 81, 61, 87.

Step-1 Building MAX HEAP

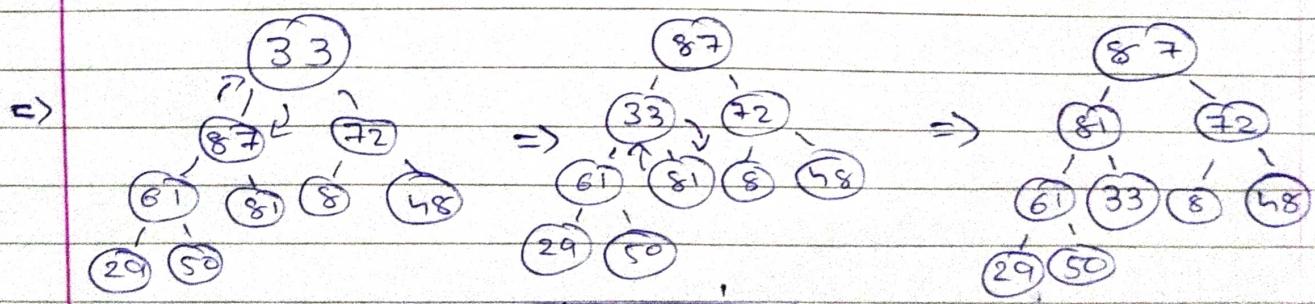


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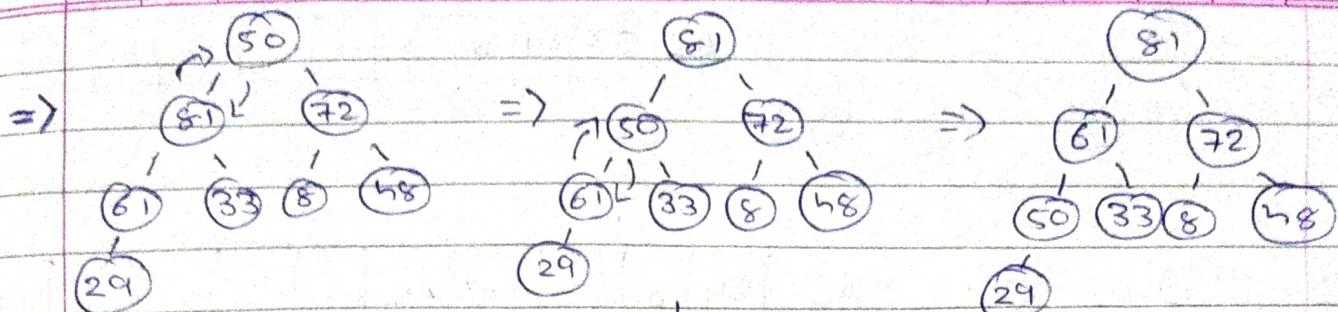


=> 92 87 72 61 81 8 48 29 50 33

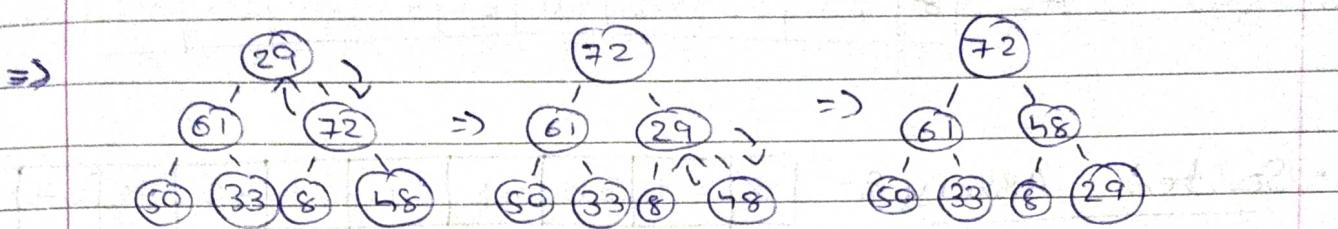
Step-2 Delete data from MAX HEAP



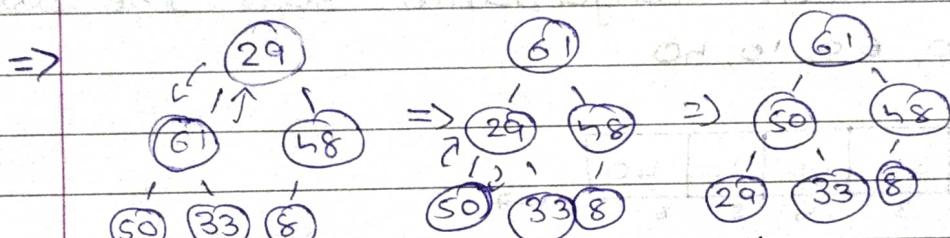
A = [33 87 72 61 81 8 48 29 50 92]



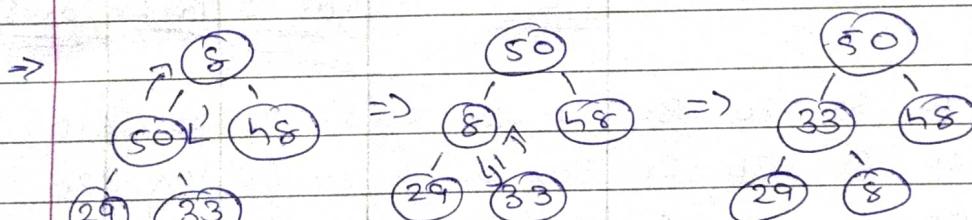
$$A = [50 \ 81 \ 72 \ 61 \ 33 \ 8 \ 48 \ 29 \ 87 \ 92]$$



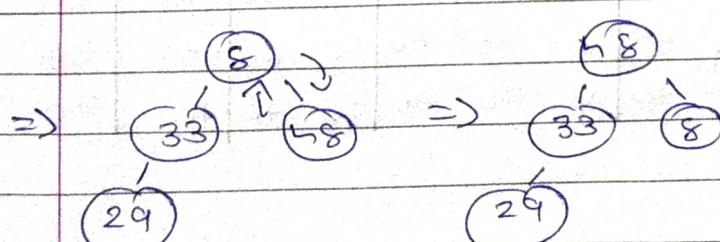
$$A = [29 \ 61 \ 72 \ 50 \ 33 \ 8 \ 48 \ 81 \ 87 \ 92] \Rightarrow [72 \ 61 \ 48 \ 50 \ 33 \ 8 \ 29 \ 81 \ 87 \ 92]$$



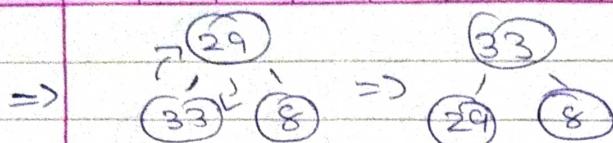
$$A = [61 \ 50 \ 48 \ 29 \ 33 \ 8 \ 72 \ 81 \ 87 \ 92]$$



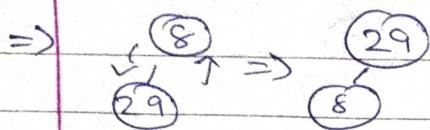
$$A = [50 \ 33 \ 48 \ 29 \ 8 \ 61 \ 72 \ 81 \ 87 \ 92]$$



$$A = [48 \ 33 \ 8 \ 29 \ 50 \ 61 \ 72 \ 81 \ 87 \ 92]$$



$$A = [33 \ 29 \ 8 \ 48 \ 50 \ 61 \ 72 \ 81 \ 87 \ 92]$$



$$A = [29 \ 8 \ 33 \ 48 \ 50 \ 61 \ 72 \ 81 \ 87 \ 92]$$

\Rightarrow (8)

$$A = [8 \ 29 \ 33 \ 48 \ 50 \ 61 \ 72 \ 81 \ 87 \ 92]$$

• Sorted Array :- $[8 \ 29 \ 33 \ 48 \ 50 \ 61 \ 72 \ 81 \ 87 \ 92]$

Q.5) Using Bubble sort algorithm sort the array.
70, 30, 20, 50, 60, 10, 40

Ans

$$\begin{array}{|c|c|c|c|c|c|c|c|} \hline & 70 & 30 & 20 & 50 & 60 & 10 & 40 \\ \hline & 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ \hline \end{array} \quad n=7$$

Pass:1

70 30 30 30 30 30
30 70 20 20 20 20
20 20 70 50 50 50
50 50 50 70 60 60
60 60 60 60 70 10
10 10 10 10 10 70
40 40 40 40 40 70

Pass:2

20 20 20 20 20 20
30 30 30 30 30 10
50 50 50 10 10 30
60 10 10 50 40 40
10 60 40 40 50 50
40 40 60 60 60 60
70 70 70 70 70 70

Pass:3

20 20 20 20 20 10
30 30 30 30 10 20
50 50 50 10 30 30
60 10 10 50 40 40
10 60 40 40 50 50
40 40 60 60 60 60
70 70 70 70 70 70

Pass:4

20 20 20 20 20 10
30 30 30 30 10 20
50 50 50 10 30 30
60 10 10 50 40 40
10 60 40 40 50 50
40 40 60 60 60 60
70 70 70 70 70 70

Pass:5

20 20 20 20 20 10
30 30 30 30 10 20
50 50 50 10 30 30
60 10 10 50 40 40
10 60 40 40 50 50
40 40 60 60 60 60
70 70 70 70 70 70

\Rightarrow Sorted Array :-

$$[10 \ 20 \ 30 \ 40 \ 50 \ 60 \ 70]$$