ADA

Assignment-4

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Q.1) Solve make a change problem using dynamic programming.

- Coin denominations : 1, 4, 7
  Amount to pay : 9

Ans

		j → 0	1	2	3	4	5	6	7	8	9
i=1	$d_1=1$	0	1	2	3	4	5	6	7	8	9
i=2	$d_2=4$	0	1	2	3	1	2	3	4	2	3
i=3	$d_3=7$	0	1	2	3	1	2	3	1	2	3

step:1 If $a[i][j] = a[i][0] = 0$ [$\because j = 0$]
so, $a[1],[0] = a[4][0] = a[7][0] = 0$

step:2 If $i=1$ & $j=1$
$a[i][j] = 1 + a[i][j-d_1]$
$= 1 + 0$
$= 1$

step:3 Now, $d_2 = 4$
If $d_2 = 4 > j$ so, write value from above
Now $j = 4$

- $a[4][4] = \min [a[i-1][j], 1 + a[i][j-d_i]]$
  $= \min (a[1][4], 1 + a[2][0])$
  $= \min (4, 1+0)$
  $= 1$

- $a[4][5] = \min (a[1][5], 1 + a[2][1])$
  $= \min (5, 1+1)$
  $= 2$

- $a[4][6] = \min (6, 1+2)$
  $= \min (6, 3)$
  $= 3$

- $a[4][9] = \min(a[1][9], 1 + a[2][5])$
$= \min(9, 1+2)$
$= \min(9, 3)$
$= 3$

Step-4 If $d_3 = 7$ & $d_3 > j$. so write the value from above

Now,
$a[7][9] = \min(3, 1+2)$
$= \min(3, 3)$
$= 3$

- Therefore minimum coin for amount 9 is required 3
- Denomination coin = $\{1, 4, 4\}$
$= \{1, 1, 7\}$

Q.2) Solve a make a change problem using dynamic programming.

- Coin denomination = 1, 3, 5, 6
Amount to pay = 8

Ans

		j →							
	0	1	2	3	4	5	6	7	8
$i=1$ $d_1=1$	0	1	2	3	4	5	6	7	8
$i=2$ $d_2=3$	0	1	2	1	2	3	4	5	6
$i=3$ $d_3=5$	0	1	2	1	2	1	2	3	4
$i=4$ $d_4=6$	0	1	2	1	2	1	1	2	3

Step-1 If $a[i][j] = a[i][0] = 0$
so, $a[1][0] = a[2][0] = a[3][0] = a[4][0] = 0$

Step 2 $i = 2,\ j = 3,\ d_2 = 3$

- $a[2][3] = \min [a[i-1][j],\ 1 + a[i][j-d_i]]$
  $= \min (3, 1+0)$
  $= 1$

- $a[2][6] = \min (6, 1+3)$
  $= 4$

- $a[2][7] = \min (7, 1+4)$
  $= 5$

Step 3 $i = 4;\ j = 8;\ d_4 = 6$

$a[4][8] = \min (a[3][8],\ 1 + a[4][2])$
$= \min (4, 1+2)$
$= 3$

Denominations coin $= \{3, 5\}$
$= \{1, 11, 6\}$

Q.3) Solve make a change problem using dynamic programming.

- coin denomination $= 1, 2, 4, 6$
  Amount to pay $= 10$

Ans

		j →										
	d_i	0	1	2	3	4	5	6	7	8	9	10
$i=1$	1	0	1	2	3	4	5	6	7	8	9	10
$i=2$	2	0	1	1	2	3	3	4	5	5	6	6
$i=3$	4	0	1	1	2	1	2	3	3	2	3	4
$i=4$	6	0	1	1	2	1	2	1	2	2	3	2

step:1 $a[i][j] = 0$ when $j = 0$

$a[1][0] = a[2][0] = a[3][0] = a[4][0] = 0$

step:2 $j = 2$; $i = 2$; $d_2 = 2$

- $a[2][2] = \min(a[i-1][j], 1 + a[i][j-d_i])$
  $= \min(a[1][2], 1 + a[2][0])$
  $= \min(2, 1+0)$
  $= 1$

- $a[2][10] = \min(a[1][10], 1 + a[2][8])$
  $= \min(10, 1+5)$
  $= 6$

- $a[2][7] = \min(a[1][7], 1 + a[2][5])$
  $= \min(7, 1+4)$
  $= 5$

- $a[2][5] = \min(a[1][5], 1 + a[2][3])$
  $= \min(5, 1+2)$
  $= 3$

step:3 $i = 3$; $j = 7$; $d_8 = 4$

- $a[3][7] = \min(a[2][7], 1 + a[3][3])$
  $= \min(5, 1+2)$
  $= 3$

- $a[3][9] = \min(a[2][9], 1 + a[3][5])$
  $= \min(6, 1+2)$
  $= 3$

- $a[3][10] = \min(6, 1+3) = 4$

Step 4 $i = 4, \quad j = 8, \quad d_4 = 6$

- $a[4][8] = \min(a[3][8], 1 + a[4][2])$
  $= \min(2, 1+1)$
  $= 2$

- $a[4][9] = \min(a[3][9], 1 + a[4][3])$
  $= \min(3, 1+2)$
  $= 3$

- $a[4][10] = \min(a[3][10], 1 + a[4][4])$
  $= \min(4, 1+1)$
  $= 2$

- Denomination coin $= \{6, 4\}$

Q.4) Solve the following knapsack problem using dynamic programming $w = 10$

object	1	2	3	4
v_i	10	40	30	50
w_i	5	4	6	3

Ans

$j = w \longrightarrow$ knapsack capacity

	v_i	w_i	0	1	2	3	4	5	6	7	8	9	10
$i=0$		0	0	0	0	0	0	0	0	0	0	0	0
$i=1$	10	5	0	0	0	0	0	10	10	10	10	10	10
$i=2$	40	4	0	0	0	0	40	40	40	40	40	50	50
$i=3$	30	6	0	0	0	0	40	40	40	40	40	50	70
$i=4$	50	3	0	0	0	50	50	50	50	90	90	90	90

Here, n = number of objects = 4
w = capacity of knapsack = 10

step 1) $V[i][j] = 0$ where $j = 0$

step 2) If $j < w_i$ $V[i][j] = V[i-1][j]$

$j = 4 < w = 6$
$V[3][4] = V[2][4] = 40$

step 3) If $j \geq w_i$ then
$V[i][j] = \max(V[i-1][j], V[i-1][j-w_i] + v_i)$

$j = 7 > w = 3$
$V[4][7] = \max(V[3][7], V[3][4] + 50)$
$= \max(40, 40 + 50)$
$= \max(40, 90)$
$= \boxed{90}$

- The knapsack carries two objects with total profit of 90.
- The selected objects are 2 & 4

Q.5) Write equation for chained matrix multiplication using dynamic programming

Find out optimal sequence for multiplication $A_1[18 \times 4]$, $A_2[4 \times 13]$, $A_3[13 \times 7]$ & $A_4[7 \times 15]$

Also give the optimal parenthesization of matrices.

Ans- Let matrix A(p×q) & matrix B(q×r) are two given matrices for multiplication.

- The resultant matrix = C = A·B

so,

$$c_{ij} = \sum_{k=1}^{q} a_{ik} b_{kj}$$

- $A_1[18 \times 4]$, $A_2[4 \times 13]$, $A_3[13 \times 7]$, $A_4[7 \times 15]$

- Here dimensions are $P_0=18$, $P_1=4$, $P_2=13$, $P_4=7$, $P_5=15$

step:1 If $i=j$ then $M[i][j]=0$

so, $M[1][1] = M[2][2] = M[3][3] = M[4][4] = 0$

	1	2	3	4
1	0	936		
2	-	0	364	
3	-	-	0	1365
4	-	-	-	0

step:2 $M[1][2] = P_0 * P_1 * P_2 = 18 * 4 * 13$

$= \boxed{936}$

$M[2][3] = P_1 * P_2 * P_3 = 4 * 13 * 7$

$= \boxed{364}$

$M[3][4] = P_2 * P_3 * P_4 = 13 * 7 * 15$

$= 1365$

step:3 $M[1][3]$; $i=1$ & $j=3$; $k=1$ or 2

For $k=1$

- $M[1][3] = M[1][2] + M[3][3] + P_0 * P_2 * P_3$
$= 936 + 0 + (18 * 13 * 7)$
$= 2574$

- $M[2][4]$; $i=2$ & $j=4$; $k=2$ or 3

For $k=2$

$M[2][4] = M[2][2] + M[3][4] + P_1 * P_3 * P_4$
$= 0 + 1365 + (4 * 13 * 15)$
$= 2145$

For $k=3$

$M[2][4] = M[2][3] + M[4][4] + P_1 * P_3 * P_4$
$= 364 + 0 + (4 * 7 * 15)$
$= \boxed{784}$

→ $M[1][4]$; $i=1$ & $j=4$; $k=1$ or 2 or 3.

For $k=1$

$M[1][4] = M[1][1] + M[2][4] + P_0 * P_1 * P_4$
$= 0 + 784 + (18 * 4 * 15)$
$= \boxed{1864}$

For $k=2$

$M[1][4] = M[1][2] + M[3][4] + P_0 * P_2 * P_4$
$= 936 + 1365 + (18 * 13 * 15)$
$= 5811$

For $k=3$

$M[1][4] = M[1][3] + M[4][4] + P_0 * P_3 * P_4$
$= 868 + 0 + (18 * 7 * 15)$
$= 2758$

Now,

	1	2	3	4
1	0	936	868	1864
2	-	0	364	784
3	-	-	0	1365
4	-	-	-	0

- Parenthesize matrices is,
$(A_1 \cdot (A_2 \cdot A_3))\, A_4$

Q.6) Find any one longest common sub-sequence of given two strings using dynamic programming.

S1 = a b b a c d c b a S2 = b c d b b c a a

Ans

			0	1	2	3	4	5	6	7	8
		S_j	b	c	d	b	b	c	a	a
0	S_i	0	0	0	0	0	0	0	0	0
1	a	0	↑0	↑0	↑0	↑0	↑0	↑0	↖1	↖1
2	b	0	↖1	←1	←1	↖1	↖1	←1	←1	←1
3	b	0	↖1	↑1	↑1	↖2	↑2	←2	←2	←2
4	a	0	↑1	↑1	↑1	↑2	↑2	↑2	↖3	↖3
5	c	0	↑1	↖2	←2	↑2	↑2	↖3	↑3	↑3
6	d	0	↑1	↑2	↖3	←3	←3	↑3	↑3	↑3
7	c	0	↑1	↖2	↑3	↑3	↑3	↖4	←4	←4
8	b	0	↖1	↑2	↑3	↖4	↖4	↑4	↑4	↑4
9	a	0	↑1	↑2	↑3	↑4	↑4	↑4	↖5	↖5

- Largest common subsequence = bcdcaa

Q.7) Determine an LCS of
A = [0, 0, 1, 0, 1, 0, 1]
B = [1, 0, 1, 1, 0, 1, 1, 0]

Ans

	B	1	0	1	1	0	1	1	0
A	0	[0]	0	0	0	0	0	0	0
0	0	↑0	[↖1]	[←1]	[←1]	↖2	←2	←2	↑3
0	0	↑0	↖2	←2	←2	[↖3]	←3	←3	↑4
1	6	↖1	↑2	↖3	↖4	←4	[↖5]	↖5	←5
0	0	↑1	↑3	↑3	↑4	↑5	[↑5]	↑5	↑6
1	0	↖2	↑3	↖4	↖5	↑5	↖6	[↖6]	↑6
0	0	↑2	↖4	↑4	↑5	↖6	↑6	↑6	[↑7]
1	0	↖3	↑4	↖5	↖5	↑6	↖7	↖7	[↑7]

Largest common subsequence = 0 1 0 1 0