TUTORIAL #2

Propositional logic and predicate logic

Q.1] Express the contrapositive, converse, inverse and negation forms of the following statements.

(i) If x is rational then x is real.

=> Contrapositive: If x is not real then, It is not rational

Converse: If x is real then, It is rational

Inverse: If x is not rational, then x is not real.

Negation Form: P = x is rational number.
q = x is real number.

$\therefore$ $\sim P$ = x is not rational number
$\therefore$ $\sim q$ = x is not real number.

(ii) If $3<b$ and $1+1=2$, then $\sin\frac{\pi}{3} = \frac{1}{2}$

=> Contrapositive: If $\sin\frac{\pi}{3} \neq \frac{1}{2}$, then, $3 \nless b$ or $1+1 \neq 2$

Converse: If $\sin\frac{\pi}{3} = \frac{1}{2}$, then $3<b$ & $1+1=2$

Inverse: If $3 \nless b$ or $1+1 \neq 2$, then $\sin\frac{\pi}{3} \neq \frac{1}{2}$

Negation form: $\sim p = 3 < b$ or $1+1 \neq 2$
$\sim q = \sin\frac{\pi}{2} \neq \frac{1}{2}$

Q.2] Construct truth table for the following questions to find if each of the following is a tautology, contradiction or contingency:

(i) $(p \to (q \to r)) \to ((p \to q) \to (p \to r))$

$A \to B$	p	q	r	$q \to r$	$p \to q$	$p \to r$	B: $(p \to q) \to (p \to r)$	A: $p \to (q \to r)$
T	T	T	T	T	F	T	T	T
T	T	T	F	T	F	T	T	T
T	T	F	T	F	T	F	F	F
T	T	F	F	T	F	F	T	T
T	F	T	T	T	T	T	T	T
T	F	T	F	T	T	T	T	T
T	F	F	T	F	T	T	T	T
T	F	F	F	T	T	T	T	T

=> Hence, this is Tautology.

(iii) $(p \to q) \leftrightarrow ((\sim p) \vee q)$

p	q	$p \to q$	$(\sim p)$	$(\sim p \vee q)$	$(p \to q) \leftrightarrow (\sim p) \vee q$
T	T	T	F	T	T
T	F	F	F	F	T
F	T	T	T	T	T
F	F	T	T	T	T

→ Hence given WFF is Tautology

(iii) $((p \to q) \wedge (q \to r)) \to (p \to r)$

A = $(p \to q) \wedge (q \to r)$, B = $(p \to r)$

p	q	r	$p \to q$	$q \to r$	$p \to r$	A	$A \to B$
T	T	T	T	T	T	T	T
T	T	F	F	T	T	F	T
T	F	T	T	F	F	F	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	T	T	T	T
F	F	T	T	F	T	F	T
F	F	F	T	T	T	T	T

⇒ This WFF is Tautology

(iv) $(p \wedge q) \wedge (\sim(p \vee q))$

p	q	$p \wedge q$	$p \vee q$	$(\sim(p \vee q))$	$(p \wedge q) \wedge (\sim(p \vee q))$
T	T	T	T	F	F
T	F	F	T	F	F
F	T	F	T	F	F
F	F	F	F	T	F

⇒ This given WFF is contradiction

Q.3] Prove the following:

(i) $(p \to (q \to r)) \equiv ((p \to q) \to (p \to r))$

⇒ Here, $(p \to (q \to r)) = A$

$((p \to q) \to (p \to r)) = B$

p	q	r	q → r	p → q	p → r	A	B
T	T	T	T	T	T	T	T
T	T	F	T	F	T	T	T
T	F	T	F	T	F	F	F
T	F	F	T	F	F	T	T
F	T	T	T	T	T	T	T
F	T	F	T	T	T	T	T
F	F	T	F	T	T	T	T
F	F	F	T	T	T	T	T

L.H.S R.H.S

=> This function's are equivalent.

(iii) $p \leftrightarrow q \equiv (p \to q) \wedge (q \to p) \equiv (\sim p \vee q) \wedge (\sim q \vee p)$

p	q	p ↔ q	(p → q) ∧	q → p	A	(~p ∨ q) ∧	(~q ∨ p)	B
T	T	T	T	T	T	T	T	T
T	F	F	F	T	F	F	T	F
F	T	F	T	F	F	T	F	F
F	F	T	T	T	T	T	T	T

1 2 3

$1 \equiv 2 \equiv 3$

=> This function's are equivalent.

Date:

(iii) Distributive law.

(i) $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

A B

p	q	r	$p \vee q$	$p \vee r$	$q \wedge r$	A	B
T	T	T	T	T	T	T	T
T	T	F	T	T	F	T	T
T	F	T	T	T	F	T	T
T	F	F	T	T	F	T	T
F	T	T	T	T	T	T	T
F	T	F	F	T	F	F	F
F	F	T	T	F	F	F	F
F	F	F	F	F	F	F	F

L.H.S R.H.S

(ii) $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

A B

p	q	r	$q \vee r$	$p \wedge q$	$p \wedge r$	A	B
T	T	T	T	T	T	T	T
T	T	F	T	F	T	T	T
T	F	T	T	T	F	T	T
T	F	F	F	F	F	F	F
F	T	T	T	F	F	F	F
F	T	F	T	F	F	F	F
F	F	T	T	F	F	F	F
F	F	F	F	F	F	F	F

L.H.S R.H.S

$\Rightarrow$ Hence, proved Distributive laws.

Date:

(iv) Absorption Laws

(i) $p \vee (p \wedge q) \equiv p$

p	q	$p \wedge q$	$p \vee (p \wedge q)$
T	T	T	T
T	F	F	T
F	T	F	F
F	F	F	F

↓ R.H.S ↓ L.H.S

(ii) $p \wedge (p \vee q) \equiv p$

p	q	$p \vee q$	$p \wedge (p \vee q)$
T	T	T	T
T	F	T	T
F	T	T	F
F	F	F	F

↓ R.H.S ↓ L.H.S

=> Proved Absorption Law

Q.4] obtain as follows:

(i) obtain conjuctive normal form of $\sim(p \vee q) \leftrightarrow (p \wedge q)$ by using laws.

=> $p \leftrightarrow q \equiv [(\sim p \vee q) \wedge (\sim q \vee p)]$

$\equiv [\sim(\sim(p \vee q)) \vee (p \wedge q)] \wedge [\sim(p \wedge q) \vee \sim(p \vee q)]$

$\equiv [(p \vee q) \vee (p \wedge q)] \wedge [(\sim p \vee \sim q) \vee (\sim p \wedge \sim q)]$

$\equiv [\{(p \vee p) \wedge (p \vee q)\} \vee \{(q \vee p) \wedge (q \vee p)\}] \wedge$

$[\{(\sim p \vee \sim p) \wedge (\sim p \vee \sim q)\} \vee \{(\sim q \vee \sim p) \wedge (\sim q \vee \sim q)\}]$

$\equiv [\{p \wedge (p \vee q)\} \vee \{(q \vee p) \wedge q\}] \wedge [\{\sim p \wedge (\sim p \vee \sim q)\}$
$\vee \{(\sim q \vee \sim p) \wedge (\sim q)\}]$

$\equiv (p \vee q) \wedge (\sim p \vee \sim q)$ ($\because$ Absorption law)

$\equiv$ CNF

(ii) Find disjunctive normal form of $(p \to (q \wedge r)) \wedge (\sim p \to (\sim p \wedge \sim r))$ by truth table method.

$(p \to (q \wedge r)) \wedge (\sim p \to (\sim p \wedge \sim r))$

$A = p \to (q \wedge r)$, $B = \sim p \to (\sim p \wedge \sim r)$

p	q	r	~p	~r	q∧r	(~p∧~r)	A	B	A∧B
T	T	T	F	F	T	F	T	T	T
T	T	F	F	T	F	F	F	T	F
T	F	T	F	F	F	F	F	T	F
T	F	F	F	T	F	F	F	T	F
F	T	T	T	F	T	F	T	F	F
F	T	F	T	T	F	F	T	F	F
F	F	T	T	F	F	T	T	T	T
F	F	F	T	T	F	T	T	T	T

$= (p \wedge q \wedge r) \vee (\sim p \wedge q \wedge \sim r) \vee (\sim p \wedge \sim q \wedge \sim r)$

$\equiv$ DNF

Date:

(iii) Find conjuctive normal form and disjunctive normal form for the following without using truth table : $(p \to q) \wedge (q \to p)$

$\equiv ((\sim p) \vee q) \wedge ((\sim q) \vee p)$

$\equiv$ cnf

$\equiv [\sim p \wedge (\sim q \vee p)] \vee [q \wedge (\sim q \vee p)]$

$\equiv [(\sim p \wedge \sim q) \vee (\sim p \wedge p)] \vee [(q \wedge \sim q) \vee (q \wedge p)]$

$\equiv [(\sim p \wedge \sim q) \vee F] \vee [F \vee (q \wedge p)]$

$\equiv (\sim p \wedge \sim q) \vee (q \wedge p)$

$\equiv$ dnf.

Q.5] Determine the validity of following argument:

S1 : All my friends are musicians

S2 : John is my friend

S3 : None of my neighbours are musicians

S : John is not my neighbor.

=> Assume that, $S1 = p$

$S2 = q$

$S3 = \sim r$

$S_1 \wedge S_2 \wedge S_3$, $p \wedge q \wedge \sim r$

Date:

Conclusion : John is not my neighbours

t_1 : John is musician
t_2 : John is my neighbours

$(q \wedge p) \rightarrow t_1$

i.e. If John is my friend and as my friends are musicians then John is musicians

so, $p \wedge q \wedge \sim r \rightarrow t_1 \wedge \sim r$

i.e. If John is my friend and my friends are musicians and none of my neighbours are musicians then John is musician and not my neighbours