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# ADA MID sem

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Q.1] (a) O(nlogn)

Q.2] (a) O(nlogn)

Q.3] (a) X will always be a better choice for large inputs

Q.4] (a) O(n^3)

Q.5] (a) Prim's Minimum Spanning Tree

Q.6] (d) Insertion sort

Q.7] (b) O(nlogn)

Q.8] (b) $O(n^2)$

Q.9] (d) 0

Q.10] (b) O(nlogn)

**Q.11]** coin denomination = 1, 2, 4, 6
Amount to pay = 10

**Ans**

| | $d_i$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $i=1$ | 1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| $i=2$ | 2 | 0 | 1 | 1 | 2 | 3 | 3 | 4 | 5 | 5 | 6 | 6 |
| $i=3$ | 4 | 0 | 1 | 1 | 2 | 1 | 2 | 3 | 3 | 2 | 3 | 4 |
| $i=4$ | 6 | 0 | 1 | 1 | 2 | 1 | 2 | 1 | 2 | 2 | 3 | $\boxed{2}$ |

**step 1]** $a[i][j] = 0$ when $j = 0$

$a[1][0] = a[2][0] = a[3][0] = a[4][0] = 0$

**step 2]** $j = 2$, $i = 2$; $d_2 = 2$

- $a[2][2] = \min(a[i-1][j], 1 + a[i][j-d_i])$
$= \min(a[1][2], 1 + a[2][0])$
$= \min(2, 1+0)$
$= 1$

- $a[2][10] = \min(a[1][10], 1 + a[2][8])$
$= \min(10, 1+5)$
$= 6$

- $a[2][7] = \min(a[1][7], 1 + a[2][5])$
$= \min(7, 1+4)$
$= 3$

- $a[2][5] = \min(a[1][5], 1 + a[2][3])$
$= \min(5, 1+2)$
$= 3$

step:3) $i = 3$ ; $j = 7$ ; $d_3 = 4$

- $a[3][7] = \min(a[2][7], 1 + a[3][3])$
  $= \min(5, 1+2)$
  $= 3$

- $a[3][9] = \min(a[2][9], 1 + a[3][5])$
  $= \min(6, 1+2)$
  $= 3$

- $a[3][10] = \min(6, 1+3)$
  $= 4$

step:4) $i = 4$, $j = 8$, $d_4 = 6$

- $a[4][8] = \min(a[3][8], 1 + a[4][2])$
  $= \min(2, 1+1)$
  $= 2$

- $a[4][9] = \min(a[3][9], 1 + a[4][3])$
  $= \min(3, 1+2)$
  $= 3$

- $a[4][10] = \min(a[3][10], 1 + a[4][4])$
  $= \min(4, 1+1)$
  $= 2$

- Denomination coin $= \{6, 4\}$

Q.12 let matrix $A(p \times q)$ & matrix $B(q \times r)$ are two given matrices for multiplication

- The resultanted matrix $= C = A \cdot B$
  so, $c_{ij} = \sum_{k=1}^{q} a_{ik} b_{kj}$

- $A_1[18 \times 4]$, $A_2[4 \times 13]$, $A_3[13 \times 7]$, $A_4[7 \times 15]$

- Here dimensions are $P_0 = 18$, $P_1 = 4$, $P_2 = 13$, $P_3 = 7$, $P_4 = 15$

Step: 1 If $i = j$ then $M[i][j] = 0$
so, $M[1][1] = M[2][2] = M[3][3] = M[4][4] = 0$

$$\begin{array}{c c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 0 & 936 & & \\ - & 0 & 364 & \\ - & - & 0 & 1365 \\ - & - & - & 0 \end{bmatrix} \end{array}$$

Step: 2 $M[1][2] = P_0 \times P_1 \times P_2 = 18 \times 4 \times 13$
$= \boxed{936}$

$M[2][3] = P_1 \times P_2 \times P_3 = 4 \times 13 \times 7$
$= \boxed{364}$

$M[3][4] = P_2 \times P_3 \times P_4 = 13 \times 7 \times 15$
$= \boxed{1365}$

Step: 3 $M[1][3]$ ; $i = 1$ & $j = 3$ ; $k = 1$ or $2$

For $k = 1$

- $M[1][3] = M[1][2] + M[3][3] + P_0 \times P_2 \times P_3$
$= 936 + 0 + (18 \times 13 \times 7)$
$= \boxed{2574}$

- $M[2][4]$ ; $i = 2$ & $j = 4$; $k = 2$ or $3$

For $k = 2$
$M[2][4] = M[2][2] + M[3][4] + P_1 \times P_3 \times P_4$
$= 0 + 1365 + (4 \times 13 \times 15)$
$= \boxed{2145}$

For $k = 3$
$M[2][4] = M[2][3] + M[4][4] + P_1 \times P_3 \times P_4$
$= 364 + 0 = (4 \times 7 \times 15)$
$= \boxed{784}$

- $M[1][4]$ ; $i = 1$ & $j = 4$ ; $k = 1$ or $2$ or $3$

For $k = 1$
$M[1][4] = M[1][1] + M[2][4] + P_0 \times P_1 \times P_4$
$= 0 + 784 + (18 \times 4 \times 15)$
$= \boxed{1864}$

For $k = 2$
$M[1][4] = M[1][2] + M[3][4] + P_0 * P_2 \times P_4$
$= 936 + 1365 + (18 \times 13 \times 15)$
$= \boxed{5811}$

For $k = 3$
$M[1][4] = M[1][3] + M[4][4] + P_0 \times P_3 \times P_4$
$= 868 + 0 + (18 \times 7 \times 15)$
$= \boxed{2758}$

Now,

$$\begin{array}{c|cccc|}
 & 1 & 2 & 3 & 4 \\
\hline
1 & 0 & 936 & 868 & 1864 \\
2 & - & 0 & 364 & 784 \\
3 & - & - & 0 & 1365 \\
4 & - & - & - & 0
\end{array}$$

- Parenthesize matrices is,

$$\boxed{(A_1 \cdot (A_2 \cdot A_3)) A_4}$$

**Q.13** Longest common sub-sequence
S1 = abbacdcba
S2 = bcdbbcaa

**Ans**

| | | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|---|---|
| | | $s_j$ | b | c | d | b | b | c | a | a |
| 0 | $s_i$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | a | 0 | ↑0 | ↑0 | ↑0 | ↑0 | ↑0 | ↑0 | ↖1 | ↖1 |
| 2 | b | $\boxed{0}$ | ↖1 | ←1 | ←1 | ↖1 | ↑1 | ←1 | ←1 | ←1 |
| 3 | b | 0 | $\boxed{↖1}$ | ↑1 | ↑1 | ↖2 | ↖2 | ←2 | ←2 | ←2 |
| 4 | a | 0 | $\boxed{↑1}$ | ↑1 | ↑1 | ↑2 | ↑2 | ↑2 | ↖3 | ↑3 |
| 5 | c | 0 | ↑1 | $\boxed{↖2}$ | ←2 | ↑2 | ↑2 | ↖3 | ↑3 | ↑3 |
| 6 | d | 0 | ↑1 | ↑2 | $\boxed{↖3}$ | $\boxed{←3}$ | $\boxed{←3}$ | ↑3 | ↑3 | ↑3 |
| 7 | c | 0 | ↑1 | ↖2 | ↑3 | ↑3 | ↑3 | $\boxed{↖4}$ | $\boxed{←4}$ | ←4 |
| 8 | b | 0 | ↖1 | ↑2 | ↑3 | ↖4 | ↖4 | ↑4 | $\boxed{↑4}$ | ↑4 |
| 9 | a | 0 | ↑1 | ↑2 | ↑3 | ↑4 | ↑4 | ↑4 | ↖5 | $\boxed{↖5}$ |

- Longest common subsequence is

$$\boxed{b\ c\ d\ c\ a}$$

**Q.14]** $W = 10$ (knapsack Problem)

| object | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| $v_i$ | 10 | 40 | 30 | 50 |
| $w_i$ | 5 | 4 | 6 | 3 |

**Ans**

$j = w \rightarrow$ knapsack capacity

| | $v_i$ | $w_i$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $i=0$ | | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $i=1$ | 10 | 5 | 0 | 0 | 0 | 0 | 0 | 10 | 10 | 10 | 10 | 10 | 10 |
| $i=2$ | 40 | 4 | 0 | 0 | 0 | 0 | 40 | 40 | 40 | 40 | 40 | 50 | 50 |
| $i=3$ | 30 | 6 | 0 | 0 | 0 | 0 | 40 | 40 | 40 | 40 | 40 | 50 | 70 |
| $i=4$ | 50 | 3 | 0 | 0 | 0 | 50 | 50 | 50 | 50 | 90 | 90 | 90 | $\boxed{90}$ |

$\Rightarrow$ Here $n$ = numbers of objects $= 4$
$W$ = capacity of knapsack $= 10$

**step:1]** $v[i][j] = 0$ where $j = 0$

**step:2]** If $j < w_i$ $v[i][j] = v[i-1][j]$

- $j = 4 < w = 6$

$v[3][4] = v[2][4] = 40$

**step:3]** If $j \geq w_i$ then
$v[i][j] = \max(v[i-1][j], v[i-1][j-w_i] + v_i)$

$j = 7 > w = 3$

$v[4][7] = \max(v[3][7], v[3][4] + 50)$
$= \max(40, 40 + 50)$
$= \max(40, 90)$
$= \boxed{90}$

- The knapsack carries two objects with total profit of 90
- The selected objects are 2 & 4