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Tutorial-4

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Q. 1) Show that the set of integers under ordinary multiplication is not group.

=> Here, set of integers given with ordinary multiplication.
i.e, ~~(I, o~~ $(I, \cdot)$

1) Closure property :- $\forall a, b \in I$
$a * b \in I$
e.g, $2, -2$
$2 * (-2) = -4 \in I$

2) Associative Property :- $\forall a, b, c \in I$
$\Rightarrow a * (b * c) = (a * b) * c$
e.g. $1, -2, 3$

$1 * (-2 * 3) = 1 * -6 = -6 \in I$
$(1 * -2) * 3 = -2 * 3 = -6 \in I$

3) Identity Property :- $\forall a \in I$
$a * e = a = e * a$
Here 1 is identity element for multiplication $e = 1$

4) Identity Property :- $\forall a, b \in I$
$a * e = \cancel{e} = \cancel{e} * a$
~~Here 1 is identity element for multiplication e = 1~~

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e.g., 1, -1

$1 * -1 = -1 \neq e$

∴ Therefore set of integers with ordinary multiplication is not group.

Q. 2) Show that the subset $\{1, -1, i, -i\}$ of the complex numbers is a group under complex multiplication.

=> Here, subset of complex number is $\{1, -1, i, -i\}$

Closure property: $\forall a, b \in C \Rightarrow a * b \in C$

$a = 1, \quad b = i$

$a * b = 1 * i = i \in C$

Associative Property :- $\forall a, b, c \in C$

$a * (b * c) = (a * b) * c$

Identity Element :- $\forall a \in C$

$a * e = a = e * a$

$e = 1$ is identity element for complex multiplication.

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Inverse Property :- $\forall$ a, b $\in$ C

$a * b = e$

$a * b = 1$

$a = \frac{1}{b}$

$\therefore$ Therefor subset of complex number is a group with complex multiplication

Q. 3) Show that the set Q_+ of positive rational is a group under ordinary multiplication. The inverse of any 'a' is $\frac{1}{a} = a^{-1}$

=> Here Q_+ = positive rational number
$(Q_+, \cdot)$

1) Closure property :- $\forall$ a, b $\in Q_+$
$a * b \in Q_+$
e.g., $2, 3 \in Q_+ \Rightarrow 2 * 3 = 6 \in Q_+$

2) Associative property :- $\forall$ a, b, c $\in Q_+$

$a * (b * c) = (a * b) * c$

e.g. 2, 3, 5

$2 * (3 * 5) = 2 * 15$

$= 30$

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3) Identity Element :- $\forall\ a \in Q_+$

$a * e = a = e * a$

e.g. $a = 2$

$2 * e = 2$

$e = 1$

4) Inverse Property :- $\forall\ a, b, \in Q_+$

$a * b = e$

$a * b = 1$

$\boxed{b = \frac{1}{a} = a^{-1}}$

$\therefore$ Therefor $(Q_+, \cdot)$ is group

Q.4) Show that the set S of positive rational number together with 1 under multiplication satisfies the three properties given in the definition of group but is not a group.
(hint : $\sqrt{2} . \sqrt{2} = 2$)

=> Here set S is positive irrational number is $(S, *)$

1) Closure Property : $\forall\ a, b \in I_+$
=> $a * b \in I_+$
e.g $\sqrt{2}, \sqrt{2}$
$\sqrt{2} . \sqrt{2} = 2 \in I_+$

2 does not belong to S as it is rational number.
so it isn't closed with respect to $*$

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2) Associative property :- $\forall a, b, c \in I_+$

$a * (b * c) = (a * b) * c$

e.g. $\sqrt{2}, \sqrt{3}, \sqrt{3}$

$a * (b * c) = \sqrt{2} * (\sqrt{3} * \sqrt{3}) = 3\sqrt{6} \notin I_+$

$(a * b) * c = (\sqrt{2} * \sqrt{3}) * \sqrt{3} = 3\sqrt{6} \notin I_+$

3) Identity element :- $\forall a \in I_+$

$a * e = a = e * a$

$\boxed{e = 1}$ identity element for multiplication

4) Inverse Property :- $\forall a, b \in I_+$

$a * b = e = b * a \Rightarrow a * b = e = 1$

$\boxed{b = \frac{1}{a}}$ $\quad$ $\boxed{\therefore e = 1}$

$\therefore$ Therefore, it is not a group.

Q.5] A rectangular array of the form $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is called 2×2 matrix. show the set of all 2×2 matrices with real entries is a group under component wise addition. That is

$$\begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix} + \begin{bmatrix} b_1 & b_2 \\ b_3 & b_4 \end{bmatrix} = \begin{bmatrix} a_1 + b_1 & a_2 + b_2 \\ a_3 + b_3 & a_4 + b_4 \end{bmatrix}$$

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=> Here, 2×2 matrix is $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ given.

Here, set is real number R.

1) Closure Property :-

$\forall\ a_1, b_1, c_1, d_1 \in R$ & $\forall\ a_2, b_2, c_2, d_2 \in R$

$\therefore \begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix} + \begin{bmatrix} a_2 & b_2 \\ c_2 & d_2 \end{bmatrix} = \begin{bmatrix} a_1 + a_2 & b_1 + b_2 \\ c_1 + c_2 & d_1 + d_2 \end{bmatrix} \in R$

2) Associative Property :-

$\forall\ a_1 b_1 c_1 d_1,\ a_2 b_2 c_2 d_2,\ a_3 b_3 c_3 d_3 \in R$

$\begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix} + \left\{ \begin{bmatrix} a_2 & b_2 \\ c_2 & d_2 \end{bmatrix} + \begin{bmatrix} a_3 & b_3 \\ c_3 & d_3 \end{bmatrix} \right\}$

$= \begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix} + \begin{bmatrix} a_2 + a_3 & b_2 + b_3 \\ c_2 + c_3 & d_2 + d_3 \end{bmatrix}$

$= \begin{bmatrix} a_1 + a_2 + a_3 & b_1 + b_2 + b_3 \\ c_1 + c_2 + c_3 & d_1 + d_2 + d_3 \end{bmatrix}$ — (1)

Similarly,

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$$\left\{\begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix} + \begin{bmatrix} a_2 & b_2 \\ c_2 & d_2 \end{bmatrix}\right\} + \begin{bmatrix} a_3 & b_3 \\ c_3 & d_3 \end{bmatrix}$$

$$= \begin{bmatrix} a_1 + a_2 & b_1 + b_2 \\ c_1 + c_2 & d_1 + d_2 \end{bmatrix} + \begin{bmatrix} a_3 & b_3 \\ c_3 & d_3 \end{bmatrix}$$

$$= \begin{bmatrix} a_1 + a_2 + a_3 & b_1 + b_2 + b_3 \\ c_1 + c_2 + c_3 & d_1 + d_2 + d_3 \end{bmatrix} \quad - \quad (2)$$

eq^n (1) & (2) same.

3) Identity matrix (2×2) :- $\forall\, a_1, b_1, c_1, d_1 \in R$

$$\begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix}$$

Here $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ is identity matrix

4) Inverse Property :- $\forall\, a_1, b_1, c_1, d_1, a_2, b_2, c_2, d_2 \in R$

$$\Rightarrow \begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix} + \begin{bmatrix} a_2 & b_2 \\ c_2 & d_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a_2 & b_2 \\ c_2 & d_2 \end{bmatrix} = -\begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix}$$

Here, $-\begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix}$ is inverse matrix

Therefore, 2×2 matrix with real entries is a group.

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Q.6] show that $Z_6 = \{0, 1, 2, \ldots, 5\}$ is a group under addition modulo 6.

$\Rightarrow$ $(Z_6, +_6)$ is given

$+_6$	0	1	2	3	4	5
0	0	1	2	3	4	5
1	1	2	3	4	5	0
2	2	3	4	5	0	1
3	3	4	5	0	1	2
4	4	5	0	1	2	3
5	5	0	1	2	3	4

1) Closure property :-

If $a, b \in Z_6 \Rightarrow a +_6 b \in Z_6$, if $a+b < 6$
if $a+b \geq 6$ then divide the sum by 6 & the remainder will be less than 6.
Hence is an element of Z_6.

2) Associative Property :- $\forall\ a, b, c \in Z_6$

$a +_6 (b +_6 c) = (a +_6 b) +_6 c$

e.g. 2, 3, 4

$2 +_6 (3 +_6 4) = 2 +_6 1 = 3$

$(2 +_6 3) +_6 4 = 5 +_6 4 = 3$

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Hence $2 +_6 (3 +_6 4) = (2 +_6 3) +_6 4$

3) Identity Element :- $\forall a \in Z_6 \; \exists \; 0 \in Z_6$

such that $a +_6 e = a = e +_6 a$

$e = 0$

Hence, 0 is identity element for addition modulo 6

4) Inverse Properties :-

$\forall a \in Z_6 \; \exists \; b \in Z_6$

$a +_6 b = e = b +_6 a$

Inverse of 0 is 0
Inverse of 1 is 5
Inverse of 2 is 4
Inverse of 3 is 3
Inverse of 4 is 2
Inverse of 5 is 1

$\therefore$ Therefor $(Z_6, +_6)$ is a group

7) Show that $Z_5 = \{1, 2, 3, 4\}$ is a group under multiplication modulo 5

$\rightarrow Z_5 = \{1, 2, 3, 4\}$
multiplication modulo '5'
$\therefore (Z_5, *_5)$

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$*_5$	1	2	3	4
1	1	2	3	4
2	2	4	1	3
3	3	1	4	2
4	4	3	2	1

1) Closure property :- $\forall a, b \in Z_5$
$a *_5 b \in Z_5$

2) Associative Property :-
e.g. $1, 2, 3, \in Z_5$
$(1 *_5 2) *_5 3 = 2 *_5 3 = 1$
$1 *_5 (2 *_5 3) = 1 *_5 1 = 1$

3) Identify Element :- $\forall a \in Z_5 \ \exists \ 1 \in Z_5$

$\therefore a *_5 e = a = e *_5 a$
$\therefore \boxed{e = 1}$

Here, 1 is identify element for multiplication modulo '5'

4) Inverse Property :- $\forall a \in Z_5 \ \exists \ b \in Z_5$

such that $a *_5 b = e = b *_5 a$

$a *_5 b = 1 = b *_5 a$

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e.g. Inverse of 1 is 1
Inverse of 2 is 3
Inverse of 3 is 2
Inverse of 4 is 4

$\therefore$ Therefor $(z_5, *_5)$ is a group with multiplication modulo '5'.

Tutorial #5

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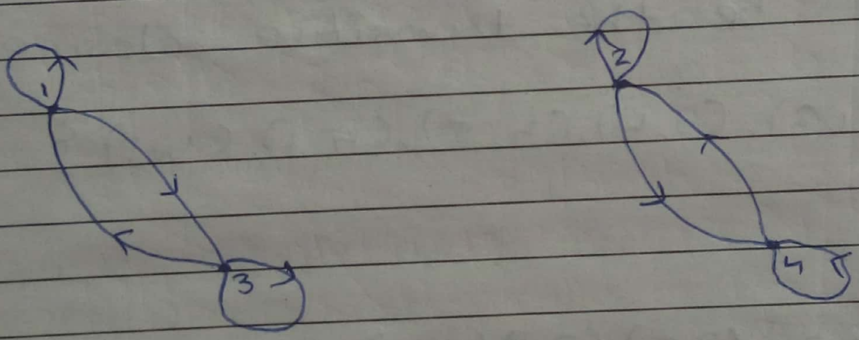
RELATIONS, PARTIAL ORDERING

Q.1 Find relation matrix M_R and draw diagraph of following relation set R:

(i) $A = \{1, 2, 3, 4\}$. If $R = \{(a, b) / (a-b)$ is an integral of $2\}$

$\Rightarrow R = \{(1,1)(2,2),(3,3),(4,4),(2,4),(4,2),(1,3)(3,1)\}$



$$M_R = \begin{array}{c c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} \end{array}$$

(ii) $A = \{1, 2, 3, 4\}$. If $R = \{(1,1),(1,3),(2,3),(2,4),(3,1)$
$(3,2),(3,3),(3,4),(4,2)(4,3)\}$

$$\Rightarrow M_R = \begin{array}{c c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

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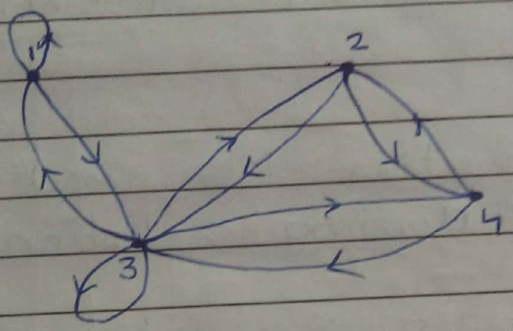

Q.2 Find the transitive closure of $R = \{(1,2),(3,4),(4,5),(4,1),(1,1)\}$ if $A = \{1,2,3,4,5\}$

⇒ Let R^* be the transitive closure of R.

$\therefore R = \{(1,2),(3,4),(4,5),(4,1),(1,1)\}$

Now,

$R^2 = R \circ R = \{(3,5),(3,1),(4,1),(4,2),(1,1),(1,2)\}$

$R^3 = \{(3,1),(4,2),(4,1),(1,2),(1,1),(3,2)\}$

$R^4 = \{(3,2),(3,1),(4,2),(4,1),(1,2),(1,1)\}$

$R^5 = \{(3,2),(3,1),(4,2),(4,1),(1,2),(1,1)\}$

Here, $R^3 = R^4 = R^5$

$\therefore R^* = R \cup R^2 \cup R^3$

$\therefore R^* = \{(1,1),(1,2),(3,4),(3,1),(3,2),(4,5),(4,1),(4,2)\}$

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Q.3 Find the transitive closure following R by Warshall's algorithm:

(i) $A = \{1,2,3,4,5,6\}$ and $R = \{(x,y) / |x-y| = 2\}$

$\Rightarrow$ $R = \{(1,3),(3,1),(2,4),(4,2),(3,5),(5,3),(4,6),(6,4)\}$

$$W_0 = M_R = \begin{array}{c|c} & \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 \end{matrix} \\ \hline \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{matrix} & \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

First we find W_1, so that $k=1$

p_1 : (3, 1)

q_1 : (1, 3)

add (p_i, q_i) i.e., (3, 3) in W_0 to get W_1

$$W_1 = \begin{array}{c|c} & \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 \end{matrix} \\ \hline \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{matrix} & \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

Now, we compute W_2, so that $k=2$

p_1 : (4, 2)

q_1 : (2, 4)

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So add path (4,4) in w_1 to get w_2

$$w_2 = \begin{array}{c|cccccc} & 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 1 & 0 & 0 \\ 3 & 1 & 0 & 1 & 0 & 1 & 0 \\ 4 & 0 & 1 & 0 & 1 & 0 & 1 \\ 5 & 0 & 0 & 1 & 0 & 0 & 0 \\ 6 & 0 & 0 & 0 & 1 & 0 & 0 \end{array}$$

=> Now compute w_3, so that $k=3$

=> P_1 : (1,3), (3,3), (5,3)

=> q_1 : (3,1), (3,3), (3,5)

=> So add path (P_i, q_i) i.e (1,1), (1,3), (1,5), (3,1), (3,3), (3,5), (5,1), (5,3), (5,5) in w_2 to get w_3

$$w_3 = \begin{array}{c|cccccc} & 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 2 & 0 & 0 & 0 & 1 & 0 & 0 \\ 3 & 1 & 0 & 1 & 0 & 1 & 0 \\ 4 & 0 & 1 & 0 & 1 & 0 & 1 \\ 5 & 1 & 0 & 1 & 0 & 1 & 0 \\ 6 & 0 & 0 & 0 & 1 & 0 & 0 \end{array}$$

Now, for w_4, $k=4$

P_1 : (2,4), P_2 : (4,4), P_3 : (6,4)

q_1 : (4,2), q_2 : (4,4), q_3 : (4,6)

∴ add path (P_i, q_i) i.e (2,2), (2,4), (2,6), (4,2), (4,4), (4,6), (6,2), (6,4), (6,4) in w_3 to get w_4

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$$W_4 = \begin{array}{c|cccccc} & 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 2 & 0 & 1 & 0 & 1 & 0 & 1 \\ 3 & 1 & 0 & 1 & 0 & 1 & 0 \\ 4 & 0 & 1 & 0 & 1 & 0 & 1 \\ 5 & 1 & 0 & 1 & 0 & 1 & 0 \\ 6 & 0 & 1 & 0 & 1 & 0 & 1 \end{array}$$

Now for W_5, $k=5$

$P_1 : (1,5)$, $P_2 : (3,5)$, $P_3 : (5,5)$

$q_1 : (5,1)$, $q_2 : (5,3)$, $q_3 : (5,5)$

$\therefore$ add path (P_i, q_i) i.e. (1,1), (1,3), (1,5), (3,1), (3,3), (3,5), (5,1), (5,3), (5,5) in W_4 to get W_5.

Here $W_4 = W_5$

Now for W_6 $k=6$

$P_1 : (2,6)$, $P_2 : (4,6)$, $P_3 : (6,6)$

$q_1 : (6,2)$, $q_2 : (6,4)$, $q_3 : (6,6)$

$\therefore$ add path (P_i, q_i) i.e (2,2), (2,4), (2,6), (4,2), (4,4), (4,6), (6,2), (6,4), (6,6) in W_5 to get W_6 and $W_5 = W_6$. Hence W_6 is R^* (i.e transitive closure of R)

$\therefore$ $R^* = \{(1,1), (1,3), (1,5), (2,2), (2,4), (2,6), (3,1), (3,3), (3,5), (4,2), (4,4), (4,6), (5,1), (5,3), (5,5), (6,2), (6,4), (6,6)\}$

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(ii) $A = \{1, 2, 3, 4, 5\}$ and $R = \{(1,4), (2,1), (2,5), (2,4), (4,3), (5,3), (3,2)\}$

$$\Rightarrow W_0 = M_R = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 0 & 0 & 0 & 1 & 0 \\ 2 & 1 & 0 & 0 & 1 & 1 \\ 3 & 0 & 1 & 0 & 0 & 0 \\ 4 & 0 & 0 & 1 & 0 & 0 \\ 5 & 0 & 0 & 1 & 0 & 0 \end{array}$$

First we find w_1 so that $k=1$
w_0 has 1's in location 2 of column 1. i.e. (2,1) and location 4 of row 1 i.e (1,4)
$P_1 : (2,1)$ $q_1 : (1,4)$
add (P_1, q_1) i.e $(2,4)$ in w_1

Thus w_1 is just w_0 with a new 1 in position (2,4) (1 is already at (2,4) position)

Hence,

$$w_1 = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 0 & 0 & 0 & 1 & 0 \\ 2 & 1 & 0 & 0 & 1 & 1 \\ 3 & 0 & 1 & 0 & 0 & 0 \\ 4 & 0 & 0 & 1 & 0 & 0 \\ 5 & 0 & 0 & 1 & 0 & 0 \end{array}$$

Now we compute $k=2$, column 2 has 1 in 3^{rd} location, i.e (3,2) row & have 1 in 4^{th}, 5^{th} & 6^{th} location

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i.e (2,1), (2,4), (2,5)
p_1 : (3,2)
q_1 : (3,1), (2,4), (2,5)
Hence (p_i, q_i) i.e (3,1), (3,4), (3,5) paths are added to w_1 to get w_2.
i.e. w_2 has 1 in location (3,1), (3,4) and (3,5).

$$W_2 = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 0 & 0 & 0 & 1 & 0 \\ 2 & 1 & 0 & 0 & 1 & 1 \\ 3 & 1 & 1 & 0 & 1 & 1 \\ 4 & 0 & 0 & 1 & 0 & 0 \\ 5 & 0 & 0 & 1 & 0 & 0 \end{array}$$

Now we compute $k=3$
p_1 : (4,3) , p_2 : (5,3)
q_1 : (3,1) , q_2 : (3,2) , q_3 : (3,4), q_4 : (3,5)
Hence, (p_i, q_i) i.e (4,1), (4,2), (4,4), (4,5), (5,1), (5,2), (5,4), (5,5) paths are added to w_2 to get w_3

$$W_3 = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 0 & 0 & 0 & 1 & 0 \\ 2 & 1 & 0 & 0 & 1 & 1 \\ 3 & 1 & 1 & 0 & 1 & 1 \\ 4 & 1 & 1 & 0 & 1 & 1 \\ 5 & 1 & 1 & 0 & 1 & 1 \end{array}$$

Now we compute $k=4$

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$p_1 : (1,4)$, $p_2 : (2,4)$, $p_3 : (3,4)$, $p_4 : (4,4)$, $p_5 : (5,4)$
$q_1 : (4,1)$, $q_2 : (4,2)$, $q_3 : (4,4)$, $q_5 : (4,5)$

Hence (p_i, q_i) i.e $(1,1), (1,2), (1,4), (1,5), (2,1), (2,2), (2,4), (2,5), (3,1), (3,2), (3,4), (3,5), (4,1), (4,2), (4,4), (4,5), (5,1), (5,2), (5,4), (5,5)$ paths are added to W_3 to get W_4.

$$W_4 = \begin{array}{c|c} & \begin{matrix} 1 & 2 & 3 & 4 & 5 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{bmatrix} 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

Hence, now, $W_4 = W_5 = M_{R^*}$

Hence,

$$R^* = \left\{ \begin{matrix} (1,1), (1,2), (1,4), (1,5) \\ (2,1), (2,2), (2,4), (2,5) \\ (3,1), (3,2), (3,4), (3,5) \\ (4,1), (4,2), (4,4), (4,5) \\ (5,1), (5,2), (5,4), (5,5) \end{matrix} \right\}$$

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(iii) $A = \{1,2,3,4,5\}$ & $R = \{(1,1),(1,2),(1,3),(1,4),(3,1),(3,2),(5,1),(5,2),(5,3),(5,5)\}$

$$\Rightarrow \quad M_R = W_0 = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 1 & 1 & 1 & 1 & 0 \\ 2 & 0 & 0 & 0 & 0 & 0 \\ 3 & 1 & 1 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 & 0 & 0 \\ 5 & 1 & 1 & 1 & 1 & 1 \end{array}$$

First we find w_1 so that $k=1$

$p_1 : (1,1)$, $p_2 : (3,1)$, $p_3 : (5,1)$

$q_1 : (1,1)$, $q_2 : (1,2)$, $q_3 : (1,3)$, $q_4 : (1,4)$,

$\therefore$ add path (p_i, q_i) i.e. $(1,1), (1,2), (1,3), (1,4), (3,1), (3,2), (3,3), (3,4), (5,1), (5,2), (5,3), (5,4)$ in w_0 to get w_1

$$w_1 = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 1 & 1 & 1 & 1 & 0 \\ 2 & 0 & 0 & 0 & 0 & 0 \\ 3 & 1 & 1 & 1 & 1 & 0 \\ 4 & 0 & 0 & 0 & 0 & 0 \\ 5 & 1 & 1 & 1 & 1 & 1 \end{array}$$

For w_2, $k=2$

$p_1 : (1,2)$, $p_2 : (3,2)$, $p_3 : (5,2)$

Hence no path is added to w_2

so $w_1 = w_2$

Now, for w_3, $k=3$

$p_1 : (1,3)$, $p_2 : (3,3)$, $p_3 : (5,3)$

$q_1 : (3,1)$, $q_2 : (3,2)$, $q_3 : (3,3)$, $q_4 : (3,4)$

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$\therefore$ add path (p_i, q_i) i.e. $(1,1), (1,2), (1,3), (1,4),$
$(3,1), (3,2), (3,3), (3,4), (5,1), (5,2), (5,3), (5,4)$
Again $w_2 = w_3$

Now, w_4 , $k = 4$
$p_1 : (1,4)$, $p_2 : (3,4)$, $p_3 : (5,4)$
$q_1 : -$

Hence no path is added to w_4
$\therefore w_3 = w_4$

Now, For w_5 , $k = 5$
$p_1 : (5,5)$
$q_2 : (5,1)$, $q_2 : (5,2)$, $q_3 : (5,3)$, $q_4 : (5,4)$, $q_5 : (5,5)$

Therefore add path (p_i, q_i) i.e
$(5,1), (5,2), (5,3) (5,4), (5,5)$, in w_4 to get
w_5
Again $w_4 = w_5$ $\therefore$ $w_5 = R^*$

$$w_1 = w_2 = w_3 = w_4 = w_5 = \begin{array}{c|c} & \begin{matrix} 1 & 2 & 3 & 4 & 5 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{bmatrix} 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} \end{array}$$

$\therefore R^* = \{(1,1), (1,2), (1,3), (1,4), (3,1), (3,2), (3,3), (3,4), (5,1), (5,2), (5,3), (5,4), (5,5)\}$

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Q.4) Show that the following R are an equivalence relation set.

(i) Let I be the set of all integers and the relation R be defined over the set I by aRb iff a-b is an even integers, where a, b ∈ I.

⇒ (i) Since $x-x=0$ and 0 is an even integer ⇒ xRx
Hence R is reflexive relation

(ii) If $x-y$ is an even integer then $y-x$ is also an even integer
Hence $xRy \Rightarrow yRx$.
Hence R is symm.

(iii) If xRy, yRz
⇒ $x-y$ is an even integer & also $y-z$ is an even integer
⇒ $(x-y)+(y-z)$ is even integer
⇒ $x-z$ is an even integer
⇒ xRz

Hence R is transitive relation Being reflixive symm. & transitive R is an equivalance relation

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Q.5) If R & S are equivalence relations on a set A, then $R \cap S$ is also an equivalence relation on set A.

=> To prove that $R \cap S$ is an equivalence relation, we have to prove that.
(i) $R \cap S$ is reflexive relation.
(ii) $R \cap S$ is symmetric relation.
(iii) $R \cap S$ is transitive relation.
Now, proving that

(i) As R is reflexive
then $(a, a) \in R$, $\forall a \in A$ also S is reflexive.
then $(a, a) \in S$, $\forall a \in A$
=> $(a, a) \in R \cap S$, $\forall a \in A$
Hence, $R \cap S$ is reflexive.

(ii) Now, R is symmetric then $(a, b) \in R$
=> $(b, a) \in R$ also is symmetric
Hence, $(a, b) \Rightarrow (b, a) \in R \cap S$.
Thus, $R \cap S$ is Symm. relation.

(iii) R & S are transitive relations. Hence
$(a, b) \in R$, $(b, c) \in R \Rightarrow (a, c) \in R$ & $(a, b) \in S$, $(b, c) \in S \Rightarrow (a, c) \in S$

=> $(a, b) \in R \cap S$, $(b, c) \in R \cap S$
=> $(a, c) \in R \cap S$

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Hence $R \cap S$ is transitive relation being reflexive, symmetric & transitive relation,

$R \cap S$ is an equivalence Relation.

Q.6] Let A be a non-empty set and R be an equivalence relation on set A. $a, b \in A$. Then prove that

Soln: (i) aRb iff $[a] = [b]$, i.e. $(a,b) \in R$ iff $[a] = [b]$

$\Rightarrow$ Suppose $(a) = (b)$
then to prove that aRb
Since R is reflexive
$\rightarrow aRa$
$\rightarrow a \in (a)$ also $(a) = (b)$
$\rightarrow a \in (b)$, aRb
$(a) = (b) \Rightarrow aRb$
So, to prove $aRb = (a) = (b)$

$x \in (a) \Rightarrow xRa$
xRa, $aRb \Rightarrow xRb$

$\Rightarrow x \in (b)$ — (3)
$\Rightarrow (a) \subseteq (b)$
Also, If $y \in (b) \Rightarrow yRb$
Also, $aRb \rightarrow bRa$
yRb, $bRa \Rightarrow yRa$

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$y \in (a) \rightarrow (b) \subseteq (a)$ - (4)

from eq^n (3) & (4)

$a = b$

(2) If no element is common to a and b then $(a) \cap (b) = \phi$ so nothing to prove

- Let one element is common to $a \cap b$, $x \in a \cap b$

$\Rightarrow x \in a$ and $x \in b$

$\Rightarrow xRa$ and xRb

$\Rightarrow aRx$ and xRb .. so R is symmetric

$\rightarrow aRb$ (as R is transitive)

$\rightarrow a = b$

Hence, two classes are either identical or disjoint.

Hence to show equality of two classes.

It is enough to show that one element is common to both the classes.

Q.7] Let I be the set of all integers and $R = \{(a,b) \mid 5 \mid a-b\}$ is an equivalence relation on the set I. By considering first five equivalence classes show that it is a partition of I.

=> $[0] = \{\ldots, -10, -5, 0, 5, 10, 15, \ldots\}$
$[1] = \{\ldots, -9, -4, 1, 6, 11, 16, \ldots\}$
$[2] = \{\ldots, -8, -3, 2, 7, 12, 17, \ldots\}$
$[3] = \{\ldots, -7, -3, 3, 8, 13, 18, \ldots\}$
$[4] = \{\ldots, -6, -1, 4, 9, 14, 19, \ldots\}$

0
1
2
3
4

- Here (i) (0)(1)(2)(3)(4) are non empty set

(ii) The set (0)(1)(2)(3)(4) are pairwise disjoint

(iii) $I = (0) \cup (1) \cup (2) \cup (3) \cup (4)$

Here (0)(1)(2)(3)(4) is partition.

Q.8] Let $S = \{1,2,3\}$ and $P(S) = \{\phi, \{1\}, \{2\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$. R is a relation defined on set P(S). ARB iff. $A \subseteq B$. Then show that R is a partial ordering on P(S).

=> R is reflexive
Every set is subset of itself hence
$ARA, \forall A \in P(S)$

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Hence, R is reflexive relation.

(ii) R is antisymmetric
Let ARB and BRA
$A \subseteq B$ and $B \subseteq A$
$A = B$
Hence R is anti-symmetric relation

(iii) R is transitive
Let ARB and BRC
$A \subseteq B$ and $B \subseteq C$
$A \subseteq C$, ARC
Hence, R is transitive. Being reflexive, antsymmetric and transitive relation. R is a partial ordering relation.

Q.9) Show that the following relations are partial ordering and draw the Hasse diagram and compare with diagraph.

(i) Let $A = \{5, 6, 8, 10, 28, 36, 48\}$ and
$R = \{(a, b) \mid a \text{ is divisor of } b\}$
(ii) Let $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 12, 18, 24\}$ be ordered by the relation x divides y.

=> (i) $A = \{5, 6, 8, 10, 28, 36, 48\}$
$R = (5,5), (6,6), (8,8), (10,10), (28,28), (36,36)$
$(48,48), (5,10), (6,36), (6,48), (8,48)$

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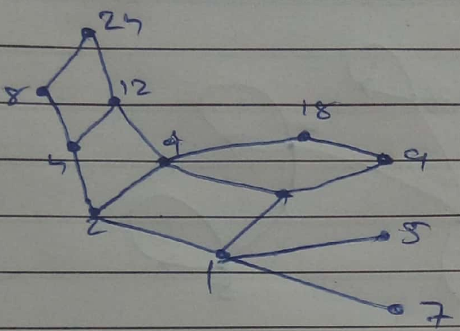
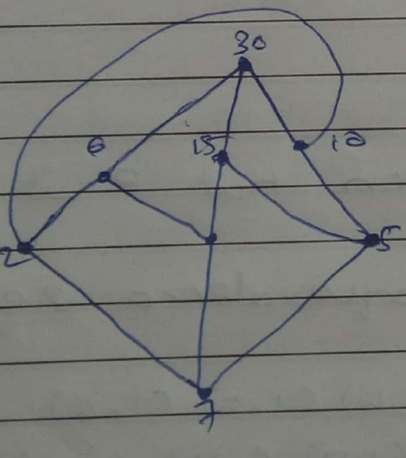


Q.10] Let A = {1, 2, 3, 5, 6, 10, 15, 30}, [Factors of 30]
aRb iff a|b (a divides b) then (A, R) is POSET. Draw the Hasse diagram. Find Chains and Antichains corresponding to the POSET.

=>



Chains:-

(1, 2, 6, 30) (1, 3, 6, 30), (1, 2, 10, 30), (1, 3, 15, 30)
(1, 5, 10, 30), (1, 5, 15, 30), (2, 6, 30), (1, 2, 6)
(3, 15, 30), (1, 3, 15), (1, 3, 6), (1, 5, 15), (1, 5, 10)
etc.

Antichains: (2, 3), (2, 5), (3, 5), (6, 10), (6, 15), (10, 15)

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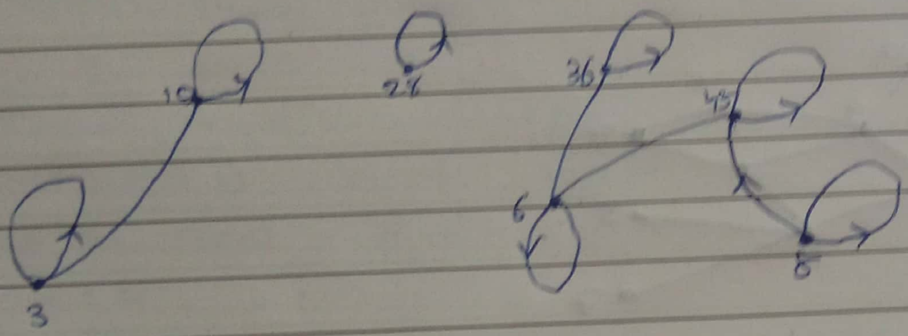
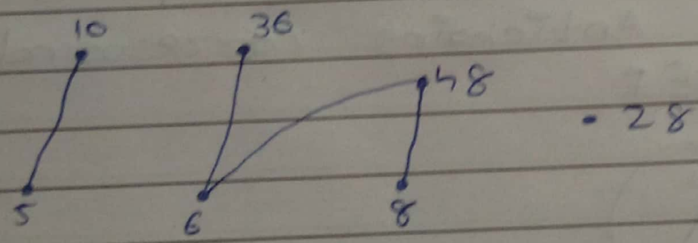


Hase diagram



∴ R is reflexive relation but R is not symmetric relation.
- Hence R is not equivalence relation.

∴ R = { (1,1), (1,2), (1,3), (1,4), (1,5), (1,6)
(1,5), (1,8), (1,3), (1,2), (1,18), (1,24)
(2,2), (2,4), (2,6), (2,8), (2,12), (2,16)
(2,26), (3,3), (3,6), (3,9), (3,12), (3,16)
(3,24), (4,4), (4,8), (4,12), (4,24), (5,5)
(6,6), (6,12), (6,18), (6,24), (7,7), (8,8)
(8,24), (9,9), (9,18), (12,12), (22,24), (26,24) }

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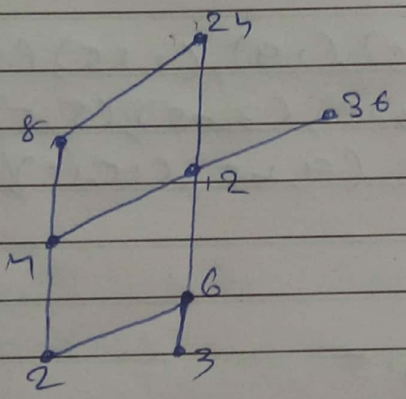
Q.11] Let $S = \{2,3,4,6,8,12,24,36\}$. Draw the Hasse diagram and find upper bounds and lower bounds of corresponding elements.

Sol^n



- Upper bounds of 2 and 3 are 6, 12 and 24, 36 in which 6 is the least upper bound.

- Similarly lower bounds of 24 and 36 are 12, 4, 6, 2, 3 in which 12 is the greatest lower bound.

Q.12] Let A be set of factor of positive integer m and relation is divisibility on A. i.e. $R = \{(x,y) / x, y \in A, x \text{ divides } y\}$. For $m = 45$ show that Poset $(A, \leq)$ is lattice. Draw Hasse diagram and give join and meet for the lattice.

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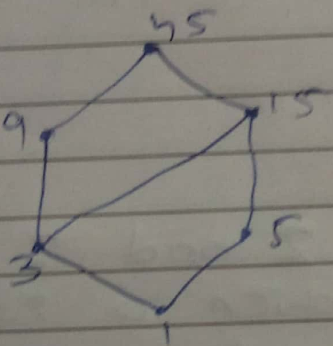
Solⁿ: A is set of divisor of 45

$A = \{1, 3, 5, 9, 15, 45\}$
$R = \{(x,y) ; x \mid y, \ x, y \in A\}$

$A = \begin{Bmatrix} (1,1), (1,3), (1,5), (1,9), (1,15), (1,45) \\ (3,3), (3,9), (3,15), (3,45), (5,5), (5,15) \\ (5,45); (9,9), (9,45), (15,15), (15,45), (45,45) \end{Bmatrix}$



Every pair of elements of A has glb and lub. Hence $(A, \leq)$ is lattice join of a and b is $a \vee b$ ~~Ex~~ meet of a and b is $a \wedge b$

⇒ Table for join & meet

$\vee$	1	3	5	9	15	45
1	1	3	5	9	15	45
3	3	3	15	9	15	45
5	5	15	5	45	15	45
9	9	9	45	9	45	45
15	15	15	15	45	15	45
45	45	45	45	45	45	45

∧	1	3	5	9	15	45
1	1	1	1	1	1	1
3	1	3	1	3	3	3
5	1	1	5	1	5	5
9	1	3	1	9	3	9
15	1	3	5	3	15	15
45	1	3	5	9	15	45

- Every pair has least upper bound and greatest lower bound.
- so it a lattice.

TUTORIAL #6

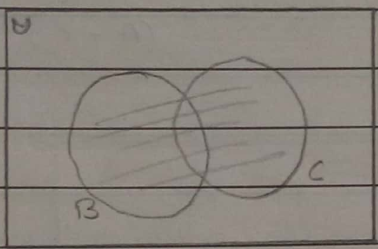
Date:

Q.1) Prove the following statement using Venn diagram.

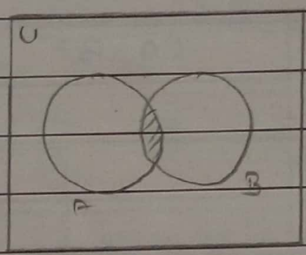
(a) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Sol^n $A \cap (B \cup C)$

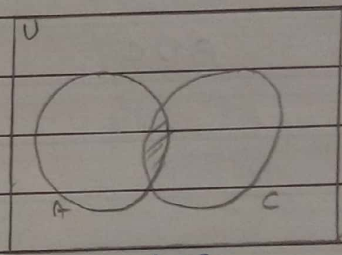
$(A \cap B) \cup (A \cap C)$



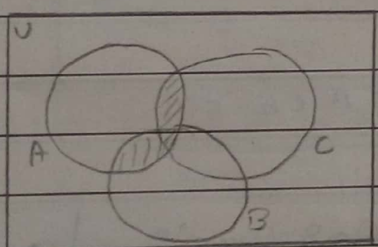
$B \cup C$



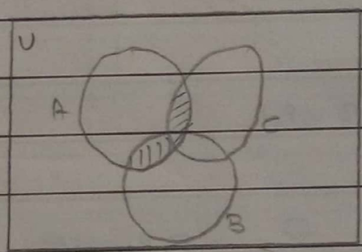
$A \cap B$



$A \cap C$

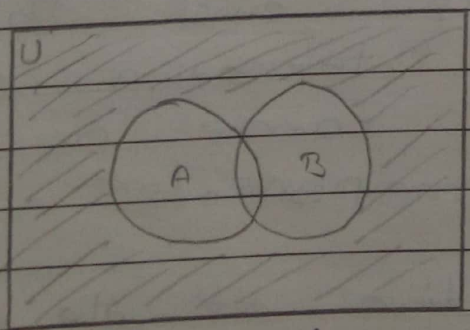


$A \cap (B \cup C)$

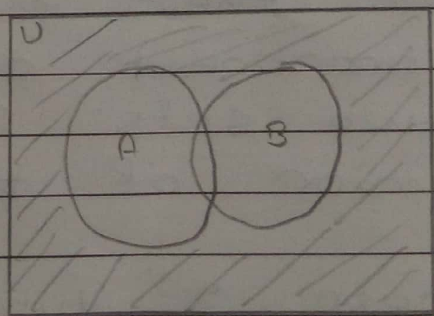


$(A \cap B) \cup (A \cap C)$

(B) $(A \cup B)' = A' \cap B'$



$(A \cup B)'$



$A' \cap B'$

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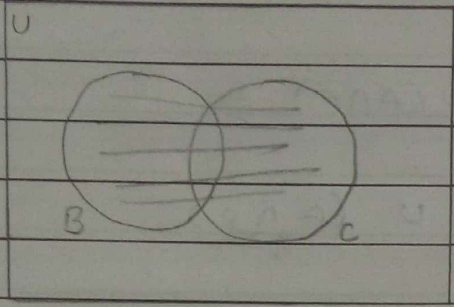
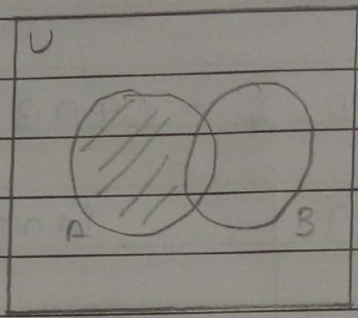
(c) $A - (B \cup C) = (A-B) \cap (A-C)$

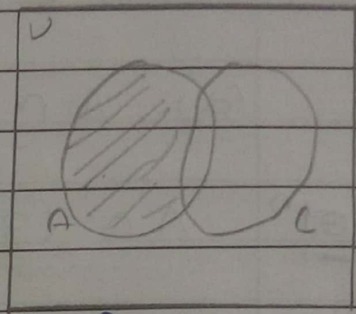
→



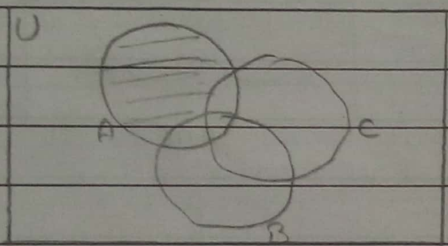
$B \cup C$



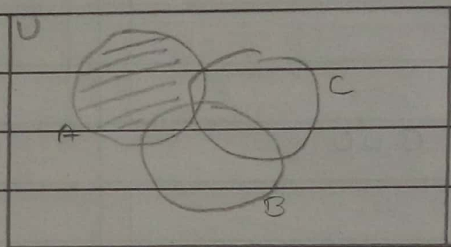
$(A-B)$



$(A-C)$



$A-(B \cup C)$



$(A-B) \cap (A-C)$

Q.2] State & prove De-Morgan's Law.

⇒ De-Morgan's Law :- "The component of the union of two sets is the same as the intersection of their complements" and "The component of the intersection of two sets is the same as the union of their components".

- These is two equation of de Morgan's law:

1) $(A \cup B)' = A' \cap B'$

2) $(A \cap B)' = A' \cup B'$

Date:

Proof of De-Morgan's Law:-

1) $(A \cup B)' = A' \cap B'$

Let x be an element of $(A \cup B)' = P$

$\Rightarrow x \notin (A \cup B)$

$\Rightarrow x \in A'$ and $x \in B'$

$\Rightarrow x \in A' \cap B'$

$\Rightarrow x \in Q$

Therefore $P \subset Q$ — (1)

Now, let y be element of Q then

$y \in Q$

$\Rightarrow y \in A'$ and $y \in B'$

$\Rightarrow y \notin A$ and $y \notin B$

$\Rightarrow y \notin (A \cup B)$

$\Rightarrow y \in (A \cup B)'$

$\Rightarrow y \in P$

Therefor $Q \subset P$ — (2)

Hence, $(A \cup B)' = A' \cap B'$

2) $(A \cap B)' = A' \cup B'$

Let $P = (A \cap B)'$ $Q = A' \cup B'$

Let x be an element of P then $x \in P$

$\Rightarrow x \notin (A \cap B)$

$\Rightarrow x \notin A$ or $x \notin B$

$\Rightarrow x \in A'$ or $x \in B'$

$\Rightarrow x \in A' \cup B'$

$\Rightarrow x \in Q$

$\Rightarrow P \subset Q$ — (1)

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Let y be an element of Q then
$y \in Q$

$\Rightarrow y \in A'$ or $y \in B'$
$\Rightarrow y \notin A$ or $y \notin B$
$\Rightarrow y \notin (A \cap B)$
$\Rightarrow y \in (A \cap B)'$
$\Rightarrow y \in P$
$\Rightarrow Q \subset P$ — (2)

Hence, $(A \cap B)' = A' \cup B'$

Q.3] Let $f(x) = x+2$, $g(x) = x-2$, $h(x) = 3x$ for $x \in R$ where R is the set of real numbers. Find (i) gof (ii) fog (iii) fof (iv) hog (v) gog (vi) foh (vii) hof (viii) fohog

$\Rightarrow$ Here, $f(x) = x+2$
$g(x) = x-2$
$h(x) = 3x$ & $x \in R$

(i) $gof = g(f(x))$ $\quad g : R \to R$
$= g(x+2)$
$= (x+2) - 2$
$\boxed{\therefore gof = x}$

(ii) $fog = f(g(x))$ $\quad f : R \to R$
$= f(x-2)$
$= (x-2) + 2$
$\boxed{fog = x}$

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(iii) $f \circ f = f(f(x))$
$= f(x+2)$
$\boxed{f \circ f = x+4}$

(iv) $h \circ g = h(g(x))$ $h: R \to R$
$= h(x-2)$
$= 3(x-2)$
$\boxed{h \circ g = 3x-6}$

(v) $g \circ g = g(g(x))$
$= g(x-2)$
$= (x-2) - 2$
$\boxed{\therefore g \circ g = x-4}$

(vi) $f \circ h = f(h(x))$
$= f(3x)$
$= 3x+2$

(vii) $h \circ f = h(f(x)) = h(x+2)$
$= -3x+6$

(viii) $f \circ h \circ g = f(h(g(x)))$
$= f(h(x-2))$
$= f(3x-6)$
$= (3x-6) + 2$
$\boxed{f \circ h \circ g = 3x-4}$

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Q.4] Find domain & range of the following functions

(i) $f(x) = |x-3|$

$\Rightarrow$ Here x can any value from R.
Hence, Domain is R
Also $\forall x \in R, \ |x-3| \geq 0$
Hence, Range $[0, \infty) = R^+ \cup \{0\}$,

$f : R \to [0, \infty)$

(ii) $f(x) = \sqrt{4-x^2}$

$\Rightarrow$ Here, $4-x^2 \geq 0$
$\Rightarrow (2-x)(2+x) \geq 0$
$\boxed{x = -2, 2}$

Hence, Domain $[-2, 2]$
$y = \sqrt{4-x^2}$
$y^2 = 4-x^2$
$x^2 = 4-y^2 \geq 0$
$(2-y)(2+y) \geq 0$

But $y \geq 0$
Hence, Range $= (0, 2)$
$f : [-2, 2] \to (0, 2)$

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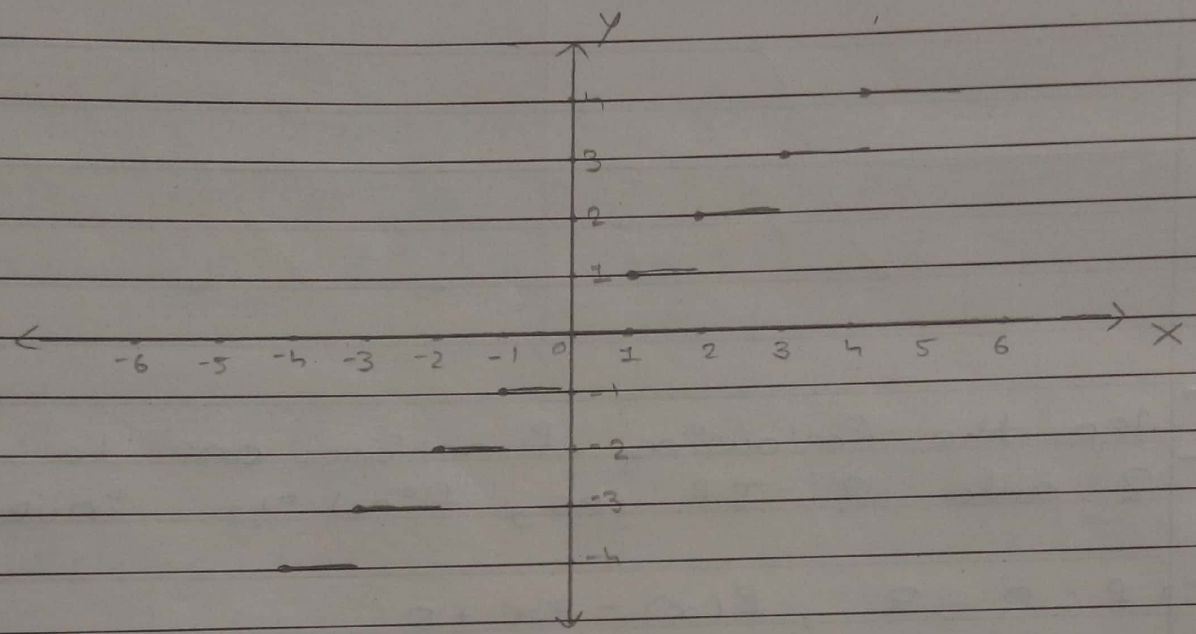
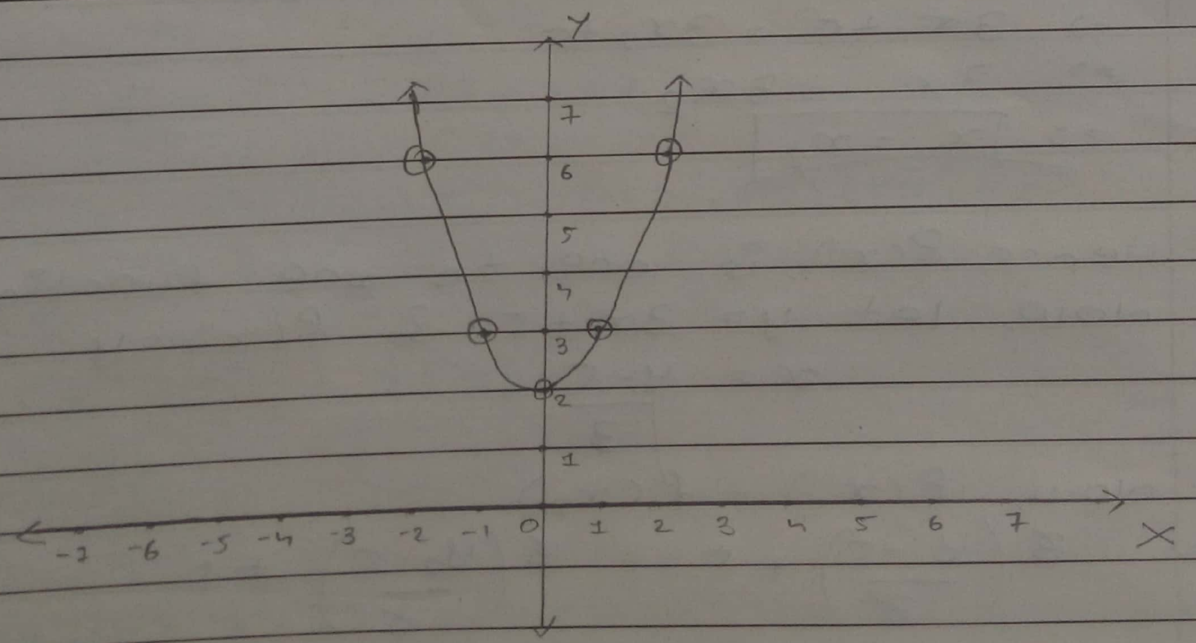
Q.5) Draw graph of the following function

(i) Floor Function.



(ii) $f(x) = x^2 + 2$



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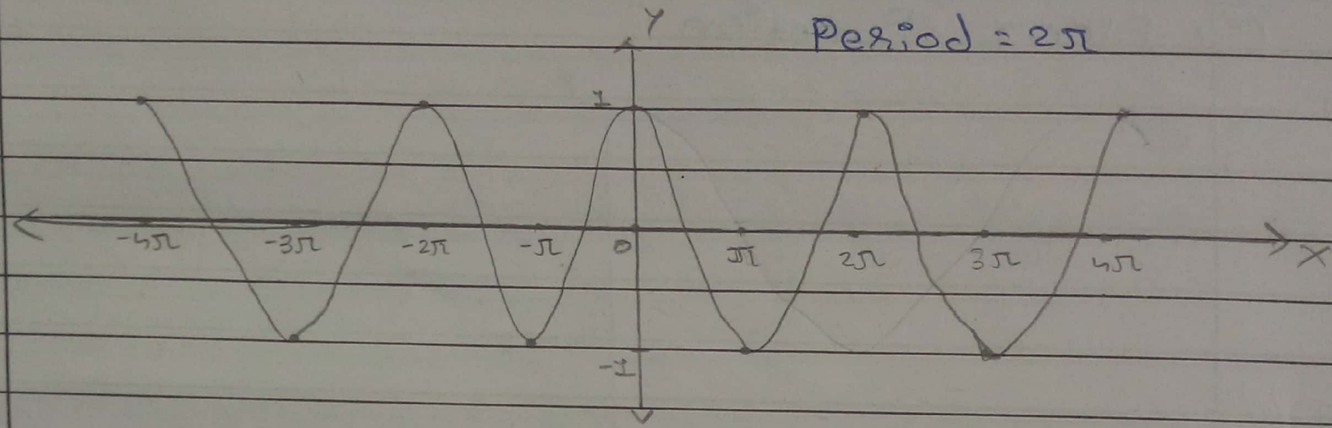
(iii) $f(x) = \cos x$

Period = 2π



Q.6] Are the following functions one to one & onto ? If yes, find its inverse

=>

(i) $f : R \to R$, $f(x) = 3x + 5$

->

$\forall\ x_1, x_2 \in R$, $f(x_1) = f(x_2)$

$\Rightarrow 3x_1 + 5 = 3x_2 + 5$

$\Rightarrow 3x_1 = 3x_2$

$\Rightarrow \boxed{x_1 = x_2}$

Hence, $f(x)$ is one to one function.

Now, let $y = 3x + 5$ & $f(x) = y$

$$x = \frac{y-5}{3}$$

Now, $f(x_1) = f(x_2)$

$\Rightarrow \cancel{3}\left(\frac{y_1 - 5}{\cancel{3}}\right) + 5 = \cancel{3}\left(\frac{y_2 - 5}{\cancel{3}}\right) + 5$

$\Rightarrow y_1 - 5 + 5 = y_2 - 5 + 5$

$\Rightarrow \boxed{y_1 = y_2}$

Hence, $f(x)$ is onto function

Date:

$f(x) = 3x+5$

$\therefore y = 3x+5$

$\therefore x = 3y+5$

$\therefore x-5 = 3y$

$\therefore y = \frac{x-5}{3}$

Therefor $\boxed{f^{-1}(x) = \frac{x-5}{3}}$

(iii) $f : R - \left\{-\frac{5}{2}\right\} \to R - \left\{+\frac{5}{2}\right\}$

$f(x) = \frac{5x+2}{2x+5}$

$\Rightarrow$ $f(x)$ is 1-1 function if $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$

Let $f(x_1) = f(x_2)$

$\Rightarrow \frac{5x_1+2}{2x_1+5} = \frac{5x_2+2}{2x_2+5}$

$\Rightarrow (5x_1+2)(2x_2+5) = (5x_2+2)(2x_1+5)$

$\Rightarrow \cancel{10x_1x_2} + 25x_1 + 4x_2 + \cancel{10} = \cancel{10x_1x_2} + 25x_2 + 4x_1 + \cancel{10}$

$\Rightarrow 25x_1 - 4x_1 = 25x_2 - 4x_2$

$\Rightarrow 21x_1 = 21x_2$

$\Rightarrow \boxed{x_1 = x_2}$

Hence, $f(x)$ is one to one function.

Now $y = \frac{5x+2}{2x+5}$

Date:

$$2xy + 5y = 5x + 2$$
$$x(2y - 5) = 2 - 5y$$
$$x = \frac{2 - 5y}{2y - 5}$$

Now $f(x_1) = f(x_2)$

$\Rightarrow \frac{2 - 5y_1}{2y_1 - 5} = \frac{2 - 5y_2}{2y_2 - 5}$

$\Rightarrow (2 - 5y_1)(2y_2 - 5) = (2 - 5y_2)(2y_1 - 5)$

$\Rightarrow 4y_2 - 10 - 10y_1y_2 + 25 = 4y_1 - 10 - 10y_1y_2 + 25$

$\Rightarrow \boxed{y_1 = y_2}$

Hence $f(x)$ is onto function

Now, $f(x) = \frac{5x + 2}{2x + 5}$

$$y = \frac{5x + 2}{2x + 5}$$

$$x = \frac{5y + 2}{2y + 5}$$

$$2xy + 5x = 5y + 2$$
$$(2x - 5)y = 2 - 5x$$
$$y = \frac{2 - 5x}{2x - 5}$$

Therefor $\boxed{f^{-1}(x) = \frac{2 - 5x}{2x - 5}}$

Date:

Q.7] In how many ways five boys & five girls are to be seated in a row if
(i) No two boys can be seated together
(ii) John & Mary must be seated together.

Soln: i) 5 boys & 5 girls

B	G	B	G	B	G	B	G	B	G

Here order is important.

$= 5! \times 5!$
$= \boxed{14,400}$

(ii) John & Mary must be seated together
i.e 1 boy & 1 girl seated together

John	Mary	B	G	B	G	B	G	B	G
		1	2	3	4	5	6	7	8

$= 8!$
$= 40320$

Q.8] Find the no. of distict permutation that can be formed from all the letter of:
(i) RADAR (ii) MATHEMATICS

YASH Royal ECO

Date:

→ (i) RADAR

Here order is important i.e. permutation

$= \frac{5!}{2!\,2!\,1!} = \frac{120}{4} = \boxed{30}$

(ii) MATHEMATICS

Here order is important i.e permutation

$= \frac{11!}{2!\,2!\,2!} = \frac{39916800}{8} = 4989600$

Q9) In Anand how many vehical no. plates are possible?

Soln

G	J	2	3	L	L	D	D	D	D

letter digit

→ Here, First 2 letter & digit are fixed

i.e

G	J	2	3

Here order is important is permutation

(i) without repetation,

$= {}^{26}P_2 \times {}^{10}P_4$

$= 650 \times 5040$

$= \boxed{3276000}$

(ii) With Repetation

$= 26^2 \times 10^4$

$= 676 \times 10000$

$= \boxed{6760000}$