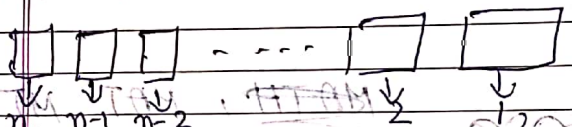
Permutations

Th^m : 1

* The no. of permutations of n different things taken all at a time is

$$n! = 1 \times 2 \times 3 \times \cdots \times n$$

e.g.

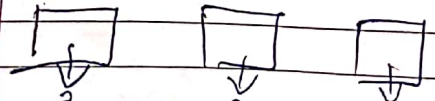


n-places

$\Rightarrow n \times (n-1) \times (n-2) \times \cdots 2 \times 1 = n!$

e.g. ~~If there are 10 different seats in a bar~~ arrange the Permutations of letters of word 'MAT'.

3 different letters



$3 \times 2 \times 1 = 3! = 6$

Th^m : 2

* When Things

* n-things

↓ divided into r classes

↓ such that

thing belonging to the same class are alike

↓

Then the no. of these things taken all at a time is,

$$\frac{n!}{n_1!\, n_2! \cdots n_r!}$$

e.g. In how many ways many letters of permuted!

R → 3
A → 2
E → 2
N → 1
G → 1

→ Order is important

Important ~~Unimportant~~

Th^m : 3

* The no. of different Permutations of n different things taken k at a time is

without repetition

$$n(n-1)(n-2)\cdots(n-k+1) = \frac{n!}{(n-k)!}$$

$$= {}^{n}P_{k}$$

& with repetitions is

n^k

* ~~are not different~~ things belonging to different classes are different

↓

no. of permutations taken all at a time

$n_1 + n_2 + \cdots + n_r = n$

n_i → no. of things in i^{th} class.

ways can the REARRANGE be

total → 9

$$\frac{9!}{3!\,2!\,2!\,1!\,1!}$$

$= 15,210$

without repetition	with repetition
e.g. In a coded telegram the letters are group of 5 letters called words,	arranged in
$\frac{26!}{(26-5)!}$	$26 \times 26 \times 26 \times 26 \times 26 = 26^5$
$= 7893600$	

Combinations :- Order is not important

★ **Combinations :-** A combination of given things means any selection of one or more things without regard to order.

- with repetitions
  - no. of combinations of n - different things taken k at a time
  - $n+k-1 C_k$
  - (Normally we don't use this)
  - e.g. Combinations of letters of MAT taken 2 letters at time
    - MA, MT, AT, MM, AA, TT,
    - ${}^4C_2 = \frac{4!}{2!(4-2)!} = \frac{4\times3\times2\times1}{2\times1\times2\times1} = 6$
- without repetitions
  - no. of combinations of n - different things taken k at a time
  - ${}^nC_k = \frac{n!}{k!(n-k)!}$
  - e.g. Combinations of letters of MAT taken 2 letters at time
    - MA, MT, AT
    - ${}^3C_2 = \frac{3!}{2!1!} = 3$

Important word → Selection

and → multiplication or intersection

OR → addition / or union

Th := $n_{P_k} = k! \, {}^{n}C_k$

$$n_{P_k} = \frac{n!}{(n-k)!}$$

$${}^{n}C_k = \frac{n!}{k!\,(n-k)!}$$

* CA student has seven books on his desk. In how many different ways can he select a set of three?

→ order is not important → Combination problem.

$${}^{7}C_3 = \frac{7!}{3!(7-3)!} = \frac{7\times6\times5\times4!}{3!\times4!}$$

$$= 35$$

* In how many ways can a committee of four be selected from a group of 10 people?

→ $${}^{10}C_4 = \frac{10!}{4!\,6!} = \frac{10\times9\times8\times7\times6!}{4\times3\times2\times1\times6!}$$

$$= 210$$

* A cafeteria offers a selection of 4 sweets, 6 vegetables, 5 desserts. In how many ways can you select a meal consisting of two different sweets, 3 different vegetables & 2 different deserts.

→ Meal

2S & 3V & 2D

↓ ↓ ↓

4S 6V 5D

$= {}^4C_2 \times {}^6C_3 \times {}^5C_2$

$= \frac{4!}{2!\,2!} \times \frac{6!}{3!\,3!} \times \frac{5!}{2!\,3!}$

$= 1200$

* Unsolved examples In book :-

Exercises 1.1

(1) Ten (10) students each submit one essay for competition. In how many ways can first, second, & third prizes be awarded?

→ Order is important.

10 → 9 → 8

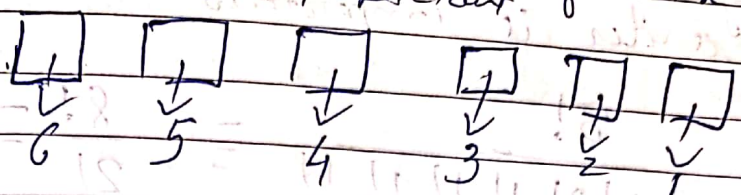
1st 2nd 3rd

$^{10}P_3 = \frac{10!}{7!}$

$\leftrightarrow 10 \times 9 \times 8 = 720$ ways

(2) How many different ways can six people be selected in a row of six chairs. order is important

→



$6! = 720$

(3) Many auto license plates have 3 letters followed by 3 digits, How many different license plates are possible if

(a) letters & digits are not repeated on a license plate?

(b) repetitions of letters & digits are allowed?

→ order is important

(a)

| 26 | 25 | 24 | 10 | 9 | 8 |

$^{26}P_3 \times {}^{10}P_3$

$26 \times 25 \times 24 \times 10 \times 9 \times 8$

$= 11232000$

(b)

| 26 | 26 | 26 | 10 | 10 | 10 |

$26^3 \times 10^3 = 17576000$

(4) How many different "words" are possible using all letters of POSSIBLE?

→ order is important

$$\frac{8!}{1!\,1!\,2!\,1!\,1!\,1!\,1!} = \frac{8!}{2!} = 8\times7\times6\times5\times4\times3$$

$$= 20,160$$

(5) How many 3 digit numbers can be formed using the digits from {2, 4, 6, 8} if

(a) repetitions are not allowed?

(b) " are allowed?

→ (a)

| 4 | 3 | 2 |

$4 \times 3 \times 2 = 24$

$^4P_3$

(b)

| 4 | 4 | 4 |

$4 \times 4 \times 4 = 64$

4^3

(6) A password to a computer consists of five characters: a letter, a digit, a letter, a digit, & a letter in that order, where the numbers from 1 through 9 are allowed for digits. How many different passwords are possible?

→ order is important

26	9	26	9	26
L	D	L	D	L

$26^3 \times 9^2 = 1423656$

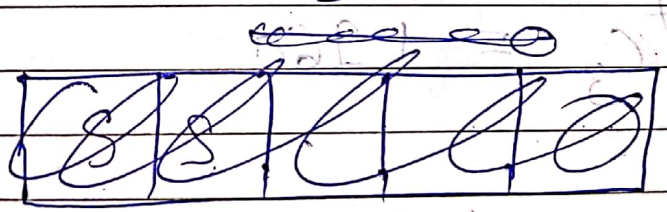
(7) A club has 14 male & 16 female members. A committee composed of 3 men & 3 women is formed. In how many ways can this be done?

→ order is not important.

$$^{14}C_3 \times {}^{16}C_3 = 203840$$

(8) 1 freshman, 3 sophomores, 4 juniors, & 6 seniors apply for five positions on an Honor council. If the council must have at least two seniors, in how many different ways can the council be selected?

→ order is not important



$$^{6}C_2 \; {}^{8}C_3 + {}^{6}C_3 \; {}^{8}C_2 + {}^{6}C_4 \; {}^{8}C_1 * {}^{6}C_5 \; {}^{8}C_0$$

$$= 840 + 560 + 120 + 6 = 1526$$

(9) A manufacturing firm forms a ~~six~~ 6 person advisory committee. The committee is composed of a chair, vice-chair, & secretary from the administrative staff & 3 members from the plant workers. 7 members from admin staff & 8 plant workers are elligible for the committee positions. In how many different ways can the committee be formed

→ order is not important

8 - plant workers → 3 - ~~member~~

7 - admin staff → 3 - ~~member~~ → chair, vice-chair & secretary

$${}^{8}C_3 \times {}^{7}C_3 = 1960$$

6 - person selection → 1960 ways

but 3 members from admin are selected in 3! way.

so $1960 \times 3! = 11760$

(10) Of a lot of 10 item, 2 are defective.
(a) Find the number of different samples of 4. Find the number of samples of 4 containing
(b) ~~no of~~ not defective.s

(c) 1 defectives (d) 2 defectives

→ Order is not important

(a) $^{10}C_4 = 210$

(b) $^{2}C_0 \times {}^{8}C_4 = 70$

(c) $^{2}C_1 \times {}^{8}C_3 = 112$

(d) $^{2}C_2 \times {}^{8}C_2 = 28$

(11) In how many different ways can you select 3 different letters from the word HISTORY?

→ $^{7}C_3 = 35$

(12) What is the chance that a leap year selected at random will contain 53 sundays?

→

$$7 \overline{)366} \quad 52$$
$$35$$
$$16$$
$$14$$
$$2$$

Leap year = 366 days

2 days

total days = 7

$\Rightarrow \frac{2}{7}$

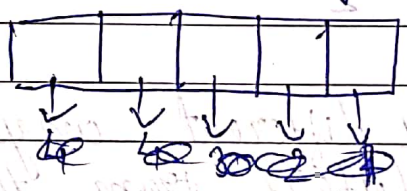
(13) A bag contains 7 white, 6 red & 5 black balls. Two balls are drawn at random. Find the probability that they will both be white.

→

$$\frac{{}^{7}C_{2} \times {}^{6}C_{0} \times {}^{5}C_{0}}{{}^{18}C_{2}} = \frac{\frac{7 \times 6 \times 5!}{2! \times 5!}}{\frac{18 \times 17 \times 16!}{2! \times 16!}} = \frac{7}{51}$$

(14) A five figure number is formed by the digits 0, 1, 2, 3, 4 without repetition. Find the probability that the number formed is divisible by 4.

→



4 4 3 2 1

$4 \times 4 \times 3 \times 2 \times 1 = 96$

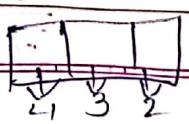
total no. of figure

Now for a number divisible by 4 last two digits has to be

04, 12, 24, 20, 32, 40

∴ this 6 way to select last 2 digits.

+



$\Rightarrow 4 \times 3 \times 2 = 24$

total $6 + 24 = 30$

(14) $\Rightarrow \frac{30}{96} = \frac{5}{16}$

$\frac{32}{16}$

(15) A bag contains 50 tickets numbered 1, 2, 3, ---, 50 of which ~~five~~ 5 are drawn at random & arranged in ascending order ($x_1 < x_2 < x_3 < x_4 < x_5$). What is the probability that $x_3 = 30$?

→ ~~$x_3 = 30$~~

~~total~~ selection of 5 from 50

$\rightarrow {}^{50}C_5$

$n(x_3 = 30) = {}^{29}C_2 \times {}^{20}C_2$

$\frac{{}^{29}C_2 \times {}^{20}C_2}{{}^{50}C_5} = \frac{551}{15134}$

(16)

(16) An urn contains 5 red & 10 black balls. Eight of them are placed in another urn. What is the chance that the latter then contains 2 red & 6 black balls?

→

$$\frac{{}^5C_2 \times {}^{10}C_6}{{}^{15}C_8} = \frac{140}{429}$$

(17) Four cards are drawn from a pack of cards. Find the probability that (i) all are diamonds (ii) there is one card of each suit, & (iii) there are two spades & two hearts.

→

(i) $$\frac{{}^{13}C_4}{{}^{52}C_4} = \frac{11}{17 \times 5 \times 49} = \frac{11}{4165}$$

(ii) $$\frac{{}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1}{{}^{52}C_4} = \frac{2197}{20825}$$

(iii) $$\frac{{}^{13}C_2 \times {}^{13}C_2}{{}^{52}C_4} = \frac{468}{20825}$$

(2⊗) A committee consists of 9 students, two of which are from 1st year, Three from 2nd year & four from 3rd year. 3 students are to be removed at random. What is the chance that

(i) The three students belong to different classes.

(ii) two belong to the same class & 3rd to the different class.

(iii) the 3 belong to the same class?

→

(i) $\frac{{}^2C_1 \; {}^3C_1 \; {}^4C_1}{{}^9C_3} = \frac{2}{7}$

(ii) $\frac{{}^2C_2 \; {}^3C_1 + {}^2C_2 \; {}^4C_1 + {}^3C_2 \; {}^2C_1 + {}^3C_2 \; {}^4C_1 + {}^4C_2 \; {}^2C_1 + {}^4C_2 \; {}^3C_1}{{}^9C_3}$

$= \frac{55}{84}$

(iii) $\frac{{}^3C_3 + {}^4C_3}{{}^9C_3} = \frac{1+4}{84} = \frac{5}{84}$