An Accurate Method for Frequency Estimation of a Real Sinusoid

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Overview

We know that a signal having multiple frequency plots many sinc freq peaks in the freq domain graph and whatb happens is some side lobes of one freq interpolates with other frequency and we do not get the exact frequency on windowing and our aim is to get accurate frequency.

Thus we need to remove those interpolating frequencies, for example $2cos(2\pi f_0) = e^{i2\pi f_0} + e^{-i2\pi f_0}$ here $+ f_0\ and\ -f_0$ interpolates and by windowing we should have got f_0 but we get f_c a coarse frequency. $\delta f = |f_0 - f_c|$ so to remove this error we remove the negative frequency .

We do this by my multiplying the signal by $e^{2\pi f_c}$ (modulation) then or terms in signal would become $f_0 + f_c\ and\ f_c - f_0$ and the latter term is close to zero so we will filter that part completely by using a high pass filter   and all what is remaining in signal is one frequency of $f_0 + f_c$.

now we again do modulation but this time by $e^{-i2\pi f_c}$ this will now remove the coarse frequency and we will have a accurate frequency .

We have adopted a method to see the effect of various components like phase , SNR , no. of iteration etc. on determining the exact frequency and the algorithm is discussed in detail.

1. Abstract

It is generally understood that the positive and negative frequency components of a genuine sinusoid interact spectrally, resulting in bias in frequency estimate based on periodogram maximisation. We propose that the negative-frequency component be filtered out. To that purpose, a coarse frequency estimation is obtained using the windowing approach, which is known to reduce

estimation bias, and the negative-frequency component is then filtered away using modulation and a discrete Fourier transform bin excision approach. On the filtered signal, fine estimate is accomplished using precise frequency estimators established for complex sinusoids. In terms of additions/multiplications, the proposed technique has an O(N log2 N) complexity, while sine/cosine operations and comparisons have an O(N) complexity. Furthermore, it achieves the Cramer–Rao lower bound and is unaffected by sinusoid frequency or beginning phase, exceeding existing approaches.

2. *Important concepts and short discussion about the existing method*

2.1 Discrete Fourier Transformation(DFT)

Here we use the DFT of cosines and exponentials in the procedure after passing through a window function. We know the DFT's of cosine and exponentials from classes. Simple DFT expression looks like

DFT of x[n] is represented by X[k]

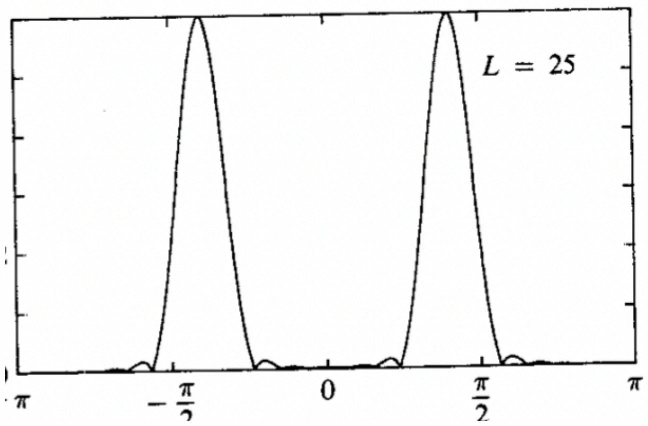
N-point DFT $X[k] = \sum_{n=0}^{N-1} x[n]e^{\frac{-2\pi kn}{N}}$

In this paper we are passing them through a window function w[n],so the $x_w[n]$ = x[n]w[n]. Now the DTFT of x[n] be X[e^{jw}] , w[n] be W[e^{jw}],

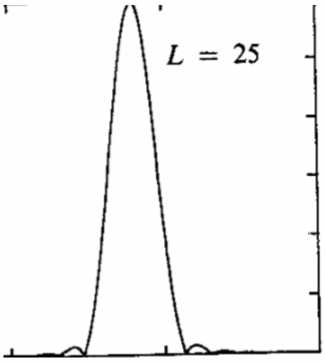
$$X_w[e^{jw}] = X[e^{jw}] * W[e^{jw}]$$

w[n] we used is kaiser window with $\beta = 5$;

Now the DFT for Cosine looks like below where the peaks are at f_0+f_b,-(f_0+f_b) they are not exactly at f_0 if the frequency of cosine is f_0



Now the DFT of a exponential function looks like the just as cosine but half part of removing the negative part or positive part depending upon the e^{2pift} where f is positive or negative.



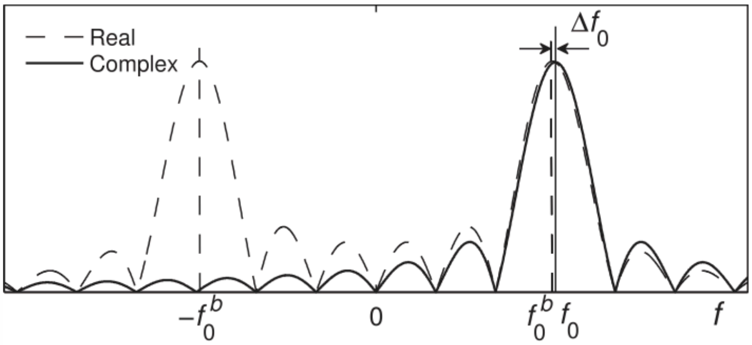
The graph looks some what as above for exponential function the peak will be at f_0

2.2 Expression for Input Signal

Here we took the input signal as $x[n] = Acos(2\pi f_0 t + \phi) + v[n]$ where v[n] is the gaussian random noise we get this noise when we are receiving the signal the important aspect here is that gaussian noise should not change the cosine with larger noises so we will give an amplitude to cosine we want such that there will be not much change after adding noise we will take SNRDB and $SNR = 10^{\frac{SNRDB}{10}}$ We will know a threshold SNRDB from that we will maintain A value as SNR = $\frac{A^2}{2\sigma^2}$ So we pass A to get less noise error or we will vary SNRDB value for the noise the noise is uncontrollable so we will pass A by changing its energy.

2.3 Existing Method To Find The Frequency

The existing method is so simple that we have to find the DFT of the $x_w[n]$ after passing through a window w[n] can be kaiser , blackmann which will decrease the spectral leakage. Then after finding the DFT of the obtained cosine we will find the position of the peak of it and after finding the peak its index will give us the frequency.but that frequency is not the exact f_0 its some f_0 + f_b because there is a spectrum coming from negative frequency also so both of them adding up we can't say that the frequency we have found is an exact f_0 .



The above picture will show the spectral distribution of positive and negative frequency. Now the the spectral leakage from negative one will give some spectral addition to the positive one and vice versa so we are expected to get an error of f_b which will be small because by using kaiser window we almost stopped spectral leakage. Now the frequency we found is f_0 +f_b or f_0-f_b but f_b is so low.

3 *Proposed method*

Step:1 finding coarse frequency:

The proposed method will make use of the existing method to estimate the frequency and remove the negative frequency component from the cosine signal we took.the estimated frequency from existing estimation is named as coarse frequency (f_0^c). Now we know that f_0^c is very close to f_0 from the 2.3.

Step:2 filtering the negative frequency component:

Here our main goal is to filter the negative frequency component for that we define a $x_m[n]$ as below

$$x_m[n] = x[n]e^{2j\pi f_0^c n}$$

By doing this the negative frequency $-f_0$ will be shifted to $f_0^c-f_0$ (which is too low) and the positive frequency will be shifted to $f_0^c+f_0$ now the negative frequency component fall under the low frequency medium so we use a filter that will remove the low frequencies and allows the high frequencies. First we will convert it to its DFT which we denote as $X_m[k]$ will have the peak of negative and positive frequency components.

Now we make

$$X_m[k] = 0 \; \forall \, k \; \in \{0,1,2,......K,N\text{-}K,N\text{-}K+1,.........N\text{-}1\}$$

Where it differs for several values K .K is a compromise between two opposing requirements: larger K gives stronger negative-frequency suppression, while smaller K suggests less positive-frequency modification.However, simulation findings demonstrate that using $K > 0$ does not improve performance over using $K= 0$.We simulated for other values of k there is no much difference in the value of RMSE.

Well this will remove the negative frequency component now we have to generate the positive frequency signal from the $X_m[k]$ for the value K we choose now the positive frequency signal can be generated as below

$$x_f[n] = IDFT[X_m[k]].\, e^{-j2\pi f_0^c n}$$

The $x_f[n]$ is just the simple positive frequency sample it can be interpreted as samples of $ae^{2j\pi f_0}$ Where f_0 is positive. We finally filtered out the negative frequency here.

Step3: Fine frequency estimation:

Now we got the samples of just positive frequency exponential. We will use the two algorithms proposed by scientists Aboutanios-Mulgrew (AM) method. Here we will find the frequency estimation of exponentials(which won't have any spectral error as cosine) proposes by Aboutanios-Mulgrew (AM) method. Where s is the complex sinusoid or exponential

Let $S = FFt(s)$ and $Y(n) = |S(n)|^2, n = 0,1,....N-1$

Find m = arg{max{Y(n)}}

Set $\hat{\delta}_0 = 0$

Loop : for each I from 1 to Q do

$X_p = \sum_{k=0}^{N-1} s(k) e^{-j2\pi k \frac{m+\hat{\delta}+p}{N}}$, p = +0.5,-0.5

$\hat{\delta}_i = \hat{\delta_{i-1}} + h(\hat{\delta_{i-1}})$

where

$h(\hat{\delta_{i-1}}) = \frac{1}{2} Re \frac{X_{0.5}+X_{-0.5}}{X_{0.5}-X_{-0.5}}$ -------> for algorithm 1

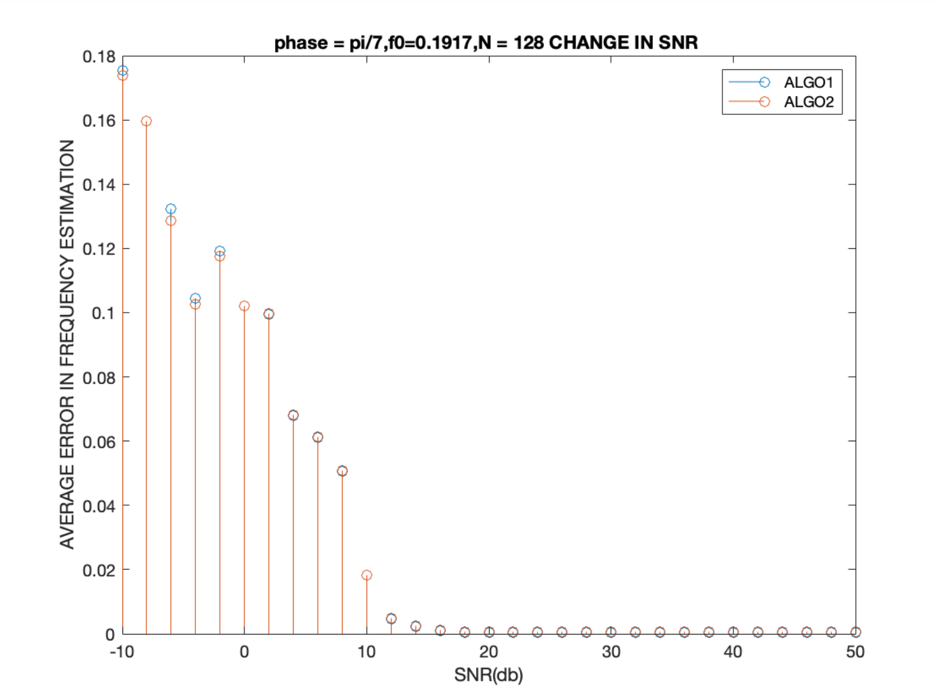
$h(\hat{\delta_{i-1}}) = \frac{1}{2} \frac{|X_{0.5}|-|X_{-0.5}|}{|X_{0.5}|+|X_{-0.5}|}$ ----------> for algorithm 2

Finally $\hat{f} = \frac{m+\hat{\delta_Q}}{N} f_s$

4 Simulations And Results

Variable SNR :

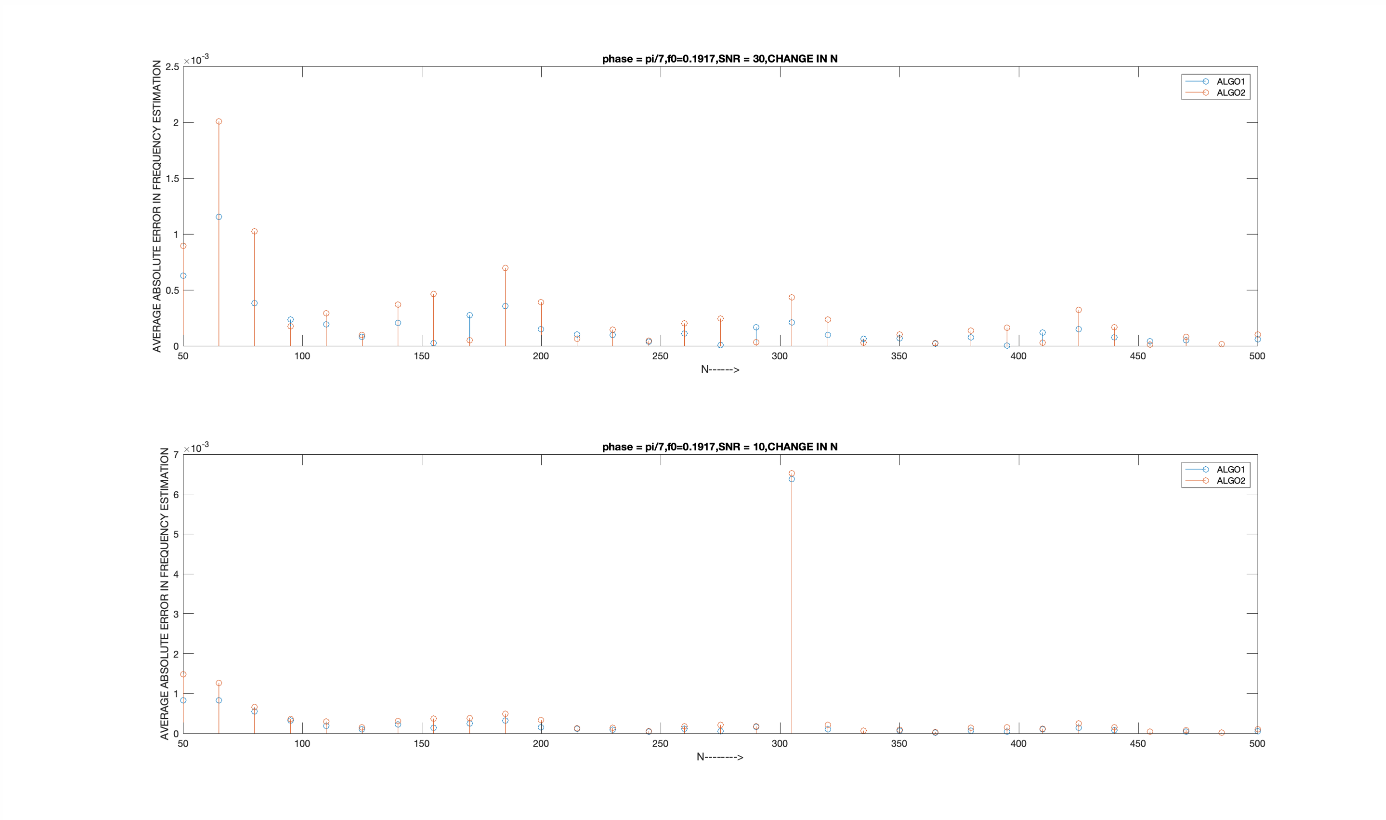
We first evaluate the performance of the proposed method versus SNR varied from -10 to 50 dB, in steps of 2dB. In addition f_0=0.1917,$\phi = \pi/7$,A=1and N = 128,K=0



As the SNR value increases the error is reducing rapidly that way at -10 the value is nearly at $6X10^{-3}$ after that the error 10 will be like 10^{-4} and at near 20 it is at 10^{-5} and after 30 it is at 10^{-6} which will ensure the accurate estimation. Algorithm 1 and algorithm 2 are most likely same

Variable N :

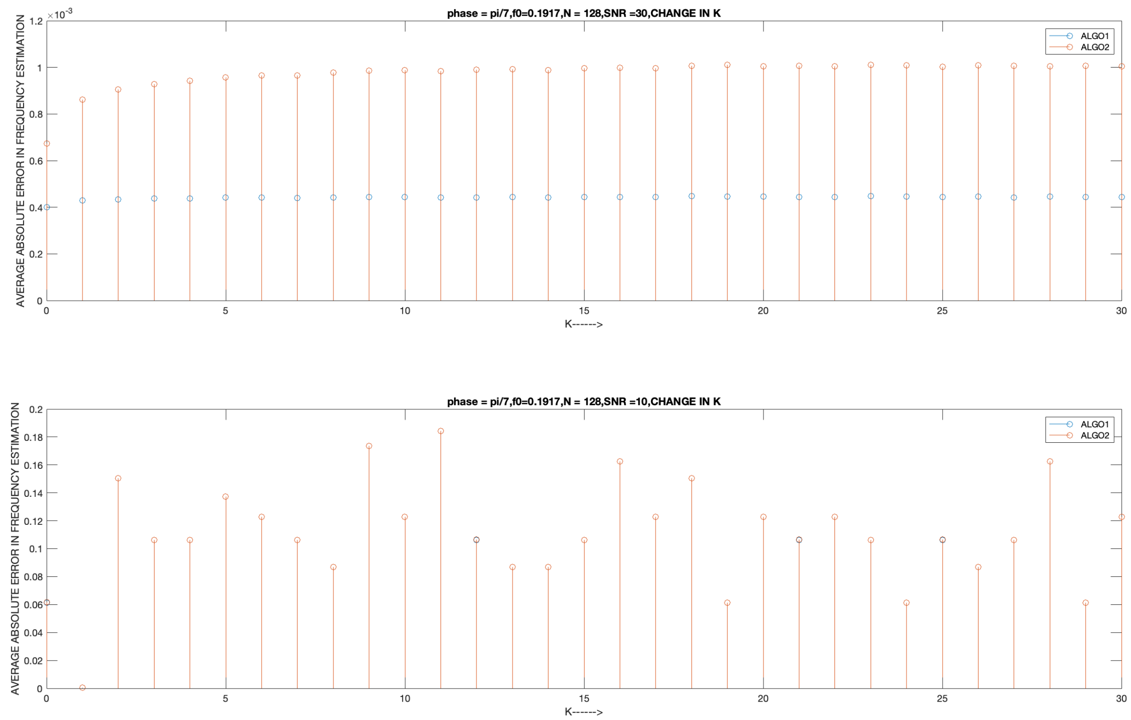
We first evaluate the performance the proposed method verses N varied from 50 to 500 in steps of 15. In addition A=1,f_0=0.1917,$\phi = \pi/7$,K=0 And we plot the graphs for SNR 30dB and 10dB



As the no.of samples taken increases the frequency error should be very low based on above depicted algorithm1 is better than algorithm2

Variable K

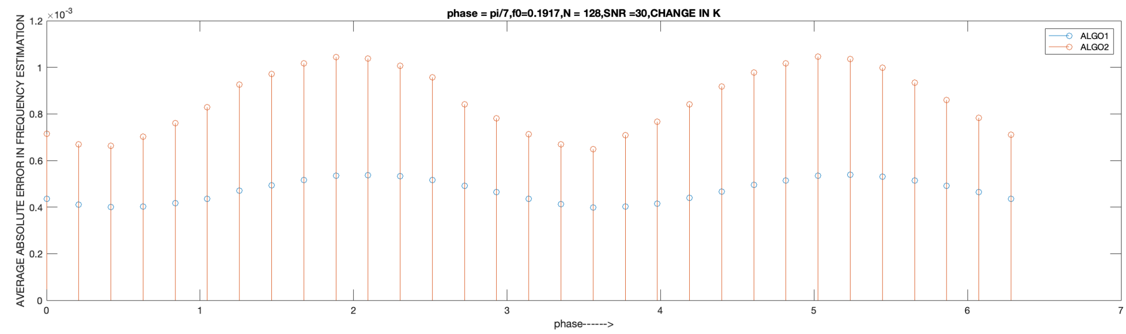
We first evaluate the performance the proposed method verses K varied from 0 to 30 in steps of 1. In addition A=1,f_0=0.1917,$\phi = \pi/7$,N=128, And we plot the graphs for SNR 30dB and 10dB

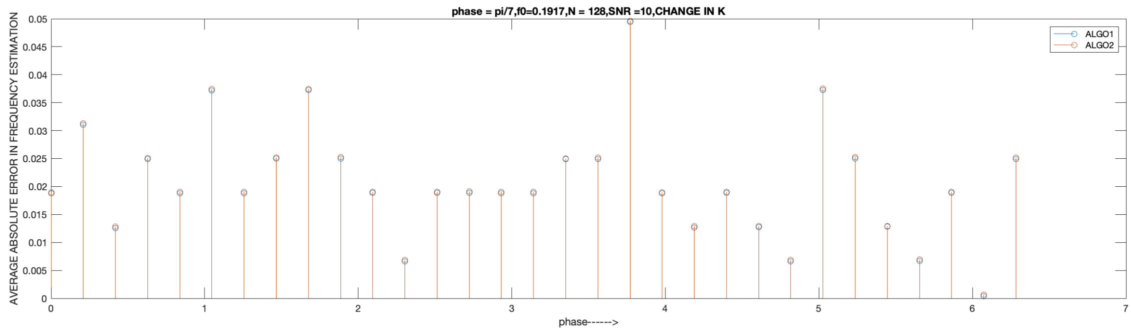


We can observe that for change in K didn't bring much change in the estimation process. and at SNR = 30 the algorithm's estimation error is very low implies algorithm1 is best than algorithm2

Variable phase

We first evaluate the performance the proposed method verses ϕ varied from 0 to 2π in steps of $\frac{\pi}{15}$. In addition A=1,f_0=0.1917,N=128, And we plot the graphs for SNR 30dB and 10dB,K =0





When SNR = 30, the algorithm 1 gives less estimation error than algorithm2 so algorithm 1 is better than algorithm 2.

5 Conclusion

In frequency estimation of a real sinusoid, estimate bias develops due to spectral superposition of the positive and negative frequency components.We propose filtering away the negative-frequency component and using high-precision estimators established for complex sinusoids with the filtered signal to solve this problem.A straightforward analytical manner of filtering out the negative-frequency component, or a complex sinusoid in general, has been presented, and numerical examples have been provided to support it.The proposed method has a low complexity, achieves the CRLB, and is unaffected by sinusoid frequency or beginning phase.

References

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[2] R. J. Kenefic and A. H. Nuttal, "Maximum likelihood estimation of the parameters of a tone using real discrete data," *IEEE J. Ocean. Eng.*, 1 vol. 12, no. 1, pp. 279–280, Jan. 1987.