**Algorithm:**  $y := \text{MVMULT\_N\_UNB\_VAR1B}(A, x, y)$ 

#### Partition

$$A \to \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) ,$$

$$x \to \left(\begin{array}{c|c} x_T \\ \hline x_B \end{array}\right) , y \to \left(\begin{array}{c|c} y_T \\ \hline y_B \end{array}\right)$$

where  $A_{TL}$  is  $0 \times 0$ ,  $x_T$ ,  $y_T$  are  $0 \times 1$  while  $m(A_{TL}) < m(A)$  do

# Repartition

$$\begin{pmatrix}
A_{TL} & A_{TR} \\
A_{BL} & A_{BR}
\end{pmatrix} \rightarrow \begin{pmatrix}
A_{00} & a_{01} & A_{02} \\
a_{10}^T & \alpha_{11} & a_{12}^T \\
A_{20} & a_{21} & A_{22}
\end{pmatrix},$$

$$\begin{pmatrix}
x_T \\
x_B
\end{pmatrix} \rightarrow \begin{pmatrix}
x_0 \\
\hline
\chi_1 \\
x_2
\end{pmatrix}, \begin{pmatrix}
y_T \\
y_B
\end{pmatrix} \rightarrow \begin{pmatrix}
y_0 \\
\hline
\psi_1 \\
y_2
\end{pmatrix}$$
where  $\alpha_{11}, \chi_1$ , and  $\psi_1$  are scalars

$$\psi_1 := a_{10}^T x_0 + \alpha_{11} \chi_1 + a_{12}^T x_2 + \psi_1$$

## Continue with

$$\left(\begin{array}{c|c|c}
A_{TL} & A_{TR} \\
\hline
A_{BL} & A_{BR}
\end{array}\right) \leftarrow \left(\begin{array}{c|c|c}
A_{00} & a_{01} & A_{02} \\
\hline
a_{10}^T & \alpha_{11} & a_{12}^T \\
\hline
A_{20} & a_{21} & A_{22}
\end{array}\right),$$

$$\left(\begin{array}{c|c|c}
x_T \\
\hline
x_B
\end{array}\right) \leftarrow \left(\begin{array}{c|c}
\hline
x_0 \\
\hline
x_1
\end{array}\right), \left(\begin{array}{c|c}
y_T \\
\hline
y_B
\end{array}\right) \leftarrow \left(\begin{array}{c|c}
\hline
y_0 \\
\hline
y_1
\end{array}\right)$$

```
function [ y_out ] = Mvmult_unb_var1B( A, x, y )
  [ ATL, ATR, ...
   ABL, ABR ] = FLA_Part_2x2(A, ...
                                 0, 0, 'FLA_TL');
  [ xT, ...
   xB \mid = FLA_Part_2x1(x, ...
                          0, 'FLA_TOP');
  [ yT, ...
   yB ] = FLA_Part_2x1(y, ...
                          0, 'FLA_TOP');
 while (size(ATL, 1) < size(A, 1))
    [ A00, a01,
                      A02,
      a10t\;,\;\; alpha11\;,\;\; a12t\;,\;\; \dots
                      A22 ] = FLA_Repart_2x2_to_3x3 (ATL, ATR, ...
      A20, a21,
                                                        ABL, ABR, ...
                                                        1, 1, 'FLA_BR');
    [ x0, \dots
      chi1, ...
      x2 = FLA_Repart_2x1_{to_3}x1(xT, ...
                                      1, 'FLA_BOTTOM');
    [ y0, ...
      psi1, ...
      y2 = FLA_Repart_2x1_to_3x1(yT, ...
                                      yB, \dots
                                      1, 'FLA_BOTTOM');
    psi1 = laff_dots(a10t, x0, psi1);
    psi1 = laff_dots(alpha11, chi1, psi1);
    psi1 = laff_dots(a12t, x2, psi1);
                                                                     -%
    [ ATL, ATR, ...
      ABL, ABR ] = FLA_Cont_with_3x3_to_2x2(A00, a01,
                                                                A02, ...
                                                a10t\;,\;\; alpha11\;,\;\; a12t\;,\;\; \dots
                                                                A22, ...
                                                A20, a21,
                                                'FLA_TL');
    [ xT, ...
      xB = FLA_Cont_with_3x1_to_2x1 (x0, ...
                                          {\tt chil}\;,\;\;\ldots\;
                                          x2, ...
                                          'FLA_TOP');
    [ yT, ...
     yB = FLA_Cont_with_3x1_to_2x1(y0, ...
                                          psi1, ...
                                          y2, ...
                                          ^{\prime}FLA_{-}TOP^{\prime});
 end
 y_out = [yT]
            yB ];
return
```

Click to view .m file Mvmult\_unb\_var1B.m. Click for test script test\_Mvmult\_unb\_var1B.m. Then copy and paste it into PictureFlame to watch it in action.

**Algorithm:**  $y := \text{MVMULT\_N\_UNB\_VAR2B}(A, x, y)$ 

## **Partition**

$$A \to \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right),$$
$$x \to \left(\begin{array}{c|c} x_T \\ \hline x_B \end{array}\right), y \to \left(\begin{array}{c|c} y_T \\ \hline y_B \end{array}\right)$$

where  $A_{TL}$  is  $0 \times 0$ ,  $x_T$ ,  $y_T$  are  $0 \times 1$  while  $m(A_{TL}) < m(A)$  do

## Repartition

$$\left(\begin{array}{c|c}
A_{TL} & A_{TR} \\
\hline
A_{BL} & A_{BR}
\end{array}\right) \rightarrow \left(\begin{array}{c|c}
A_{00} & a_{01} & A_{02} \\
\hline
a_{10}^T & \alpha_{11} & a_{12}^T \\
\hline
A_{20} & a_{21} & A_{22}
\end{array}\right),$$

$$\left(\begin{array}{c|c}
x_T \\
\hline
x_B
\end{array}\right) \rightarrow \left(\begin{array}{c|c}
x_0 \\
\hline
x_1 \\
\hline
x_2
\end{array}\right), \left(\begin{array}{c|c}
y_T \\
\hline
y_B
\end{array}\right) \rightarrow \left(\begin{array}{c|c}
y_0 \\
\hline
\psi_1 \\
\hline
y_2
\end{array}\right)$$

where  $\alpha_{11}$ ,  $\chi_1$ , and  $\psi_1$  are scalars

 $y_0 := \chi_1 a_{01} + y_0$   $\psi_1 := \chi_1 \alpha_{11} + \psi_1$  $y_2 := \chi_1 a_{21} + y_2$ 

## Continue with

$$\left(\begin{array}{c|c}
A_{TL} & A_{TR} \\
\hline
A_{BL} & A_{BR}
\end{array}\right) \leftarrow \left(\begin{array}{c|c}
A_{00} & a_{01} & A_{02} \\
\hline
a_{10}^T & \alpha_{11} & a_{12}^T \\
\hline
A_{20} & a_{21} & A_{22}
\end{array}\right),$$

$$\left(\begin{array}{c|c}
x_T \\
\hline
x_B
\end{array}\right) \leftarrow \left(\begin{array}{c|c}
x_0 \\
\hline
x_1 \\
\hline
x_2
\end{array}\right), \left(\begin{array}{c|c}
y_T \\
\hline
y_B
\end{array}\right) \leftarrow \left(\begin{array}{c|c}
y_0 \\
\hline
y_1 \\
\hline
y_2
\end{array}\right)$$

```
function [ y_out ] = Mvmult_unb_var2B( A, x, y )
  [ ATL, ATR, ...
   ABL, ABR ] = FLA_Part_2x_2(A, ...)
                                0, 0, 'FLA_TL');
  [ xT, ...
   xB = FLA_Part_2x1(x, ...
                         0, 'FLA_TOP');
  [ yT, ...
   yB = FLA_Part_2x1(y, ... 0, 'FLA_TOP');
 while ( size( ATL, 1 ) < size( A, 1 )
    [ A00, a01,
                     A02, ...
      a10t, alpha11, a12t, ...
                     A22 ] = FLA_Repart_2x_2_{to_3}x_3 (ATL, ATR, ...
      A20, a21,
                                                     ABL, ABR, ...
                                                     1, 1, 'FLA_BR');
    [ x0, ...
      chi1, ...
      x2 = FLA_Repart_2x1_{to_3}x1(xT, ...
                                     xB, ...
                                     1, 'FLA_BOTTOM');
    [ y0, ...
      psi1, ...
      y2 = FLA_Repart_2x1_to_3x1(yT, ...
                                     уB, ...
                                     1, 'FLA_BOTTOM');
   y0 = laff_axpy(chi1, a01, y0);
    psi1 = laff_axpy(chi1, alpha11, psi1);
   y2 = laff_axpy(chi1, a21, y2);
                                                                  -%
    ATL, ATR, ...
      ABL, ABR ] = FLA_Cont_with_3x3_to_2x2 ( A00, a01,
                                                              A02, ...
                                              a10t, alpha11, a12t, ...
                                              A20, a21,
                                                              A22\,,\quad\dots
                                              'FLA_TL');
    [ xT, ...
     xB = FLA_Cont_with_3x1_to_2x1(x0, ...
                                        {\tt chil} \; , \quad \dots
                                        x2, ...
                                        'FLA_TOP');
    [ yT, ...
     yB = FLA_Cont_with_3x1_to_2x1(y0, ...
                                        psi1, ...
                                        y2, ...
                                        'FLA_TOP');
 end
 y_out = [yT]
            yB ];
return
```

Click to view .m file Mvmult\_unb\_var2B.m. Click for test script test\_Mvmult\_unb\_var2B.m. Then copy and paste it into PictureFlame to watch it in action.

**Algorithm:**  $y := \text{MVMULT\_T\_UNB\_VAR1}(A, x, y)$ 

Partition

$$A \to (A_L | A_R)$$
,  $y \to \left(\frac{y_T}{y_B}\right)$ 

where  $A_L$  is  $m \times 0$  and  $y_T$  is  $0 \times 1$ while  $m(y_T) < m(y)$  do

Repartition

$$(A_L | A_R) \rightarrow (A_0 | a_1 | A_2), \left(\frac{y_T}{y_B}\right) \rightarrow \left(\frac{y_0}{y_1}\right)$$

where  $a_1$  is a column

$$\psi_1 := a_1^T x + \psi_1$$

Continue with

$$(A_L | A_R) \leftarrow (A_0 | a_1 | A_2), \left(\frac{y_T}{y_B}\right) \leftarrow \left(\frac{y_0}{\psi_1}\right)$$

```
function [ y_out ] = Mvmult_t_unb_var1( A, x, y )
 [AL, AR] = FLA_Part_1x2(A, ...
                               0, 'FLA_LEFT');
 [ yT, ...
   yB ] = FLA_Part_2x1( y, ...
0, 'FLA_TOP');
 while (size(AL, 2) < size(A, 2))
   [ A0, a1, A2 ]= FLA_Repart_1x2_to_1x3 ( AL, AR, ...
                                         1, 'FLA_RIGHT');
   [ y0, ...
     psi1, ...
     y2 = FLA_Repart_2x1_{to_3}x1(yT, ...
                                    yB, \ldots
                                    1, 'FLA_BOTTOM');
   psi1 = laff_dots(a1, x, psi1);
   [ AL, AR ] = FLA_Cont_with_1x3_to_1x2 ( A0, a1, A2, ...
                                           'FLA_LEFT');
   [ yT, ...
     yB = FLA_Cont_with_3x1_to_2x1(y0, ...
                                       psi1, ...
                                       y2, ...
                                       'FLA_TOP');
 end
 y_out = [yT]
           yB ];
return
```

Click to view .m file Mvmult\_t\_unb\_var1.m. Click for test script test\_Mvmult\_t\_unb\_var1.m. Then copy and paste it into PictureFlame to watch it in action.

**Algorithm:**  $y := \text{MVMULT\_T\_UNB\_VAR2}(A, x, y)$ 

Partition

$$A \to \left(\frac{A_T}{A_B}\right), x \to \left(\frac{x_T}{x_B}\right)$$

where  $A_T$  is  $0 \times n$  and  $x_T$  is  $0 \times 1$ while  $m(A_T) < m(A)$  do

Repartition

$$\left(\frac{A_T}{A_B}\right) \to \left(\frac{A_0}{a_1^T}\right) \, , \, \left(\frac{x_T}{x_B}\right) \to \left(\frac{x_0}{\chi_1}\right)$$

where  $a_1$  is a row

 $y := \chi_1 a_1 + y$ 

Continue with

$$\left(\frac{A_T}{A_B}\right) \leftarrow \left(\frac{\frac{A_0}{a_1^T}}{A_2}\right), \left(\frac{x_T}{x_B}\right) \leftarrow \left(\frac{x_0}{x_1}\right)$$

```
function [ y_out ] = Mvmult_t_unb_var2( A, x, y )
  [ AT, ...
   AB \ ] \ = \ FLA\_Part\_2x1(\ A,\ \dots
                          0, 'FLA_TOP');
  [ xT, ...
   xB = FLA_Part_2x1(x, ...
                          0, 'FLA_TOP');
  while ( size(AT, 1) < size(A, 1))
    [ A0, ...
      alt, ...
      A2 = FLA_Repart_2x1_{to_3}x1(AT, \dots
                                     AB, ...
                                      1, 'FLA_BOTTOM');
    [ x0, ...
      chi1, ...
      x2 = FLA_Repart_2x1_{to_3}x1(xT, ...
                                     хВ, ...
                                      1, 'FLA_BOTTOM');
   y = laff_axpy(chil, alt, y);
    [ AT, ...
     AB ] = FLA_Cont_with_3x1_{to_2}x1 ( A0, ...
                                         a1t , ...
                                         A2, ...
                                         'FLA_TOP');
    [ xT, ...
     xB = FLA_Cont_with_3x1_to_2x1(x0, ...
                                         {\tt chi1}\;,\;\;\ldots\;
                                         x2, ...
                                         'FLA_TOP');
 end
 y_out = y;
return
```

Click to view .m file Mvmult\_t\_unb\_var2.m. Click for test script test\_Mvmult\_t\_unb\_var2.m. Then copy and paste it into PictureFlame to watch it in action.

**Algorithm:**  $y := \text{Trmvp\_un\_unb\_var1}(U, x, y)$ 

## Partition

$$U \to \left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) ,$$
$$x \to \left(\begin{array}{c|c} x_T \\ \hline x_B \end{array}\right) , y \to \left(\begin{array}{c|c} y_T \\ \hline y_B \end{array}\right)$$

where  $U_{TL}$  is  $0 \times 0$ ,  $x_T$ ,  $y_T$  are  $0 \times 1$  while  $m(U_{TL}) < m(U)$  do

## Repartition

$$\begin{pmatrix}
U_{TL} & U_{TR} \\
U_{BL} & U_{BR}
\end{pmatrix} \rightarrow \begin{pmatrix}
U_{00} & u_{01} & U_{02} \\
u_{10}^T & v_{11} & u_{12}^T \\
U_{20} & u_{21} & U_{22}
\end{pmatrix},$$

$$\begin{pmatrix}
x_T \\
x_B
\end{pmatrix} \rightarrow \begin{pmatrix}
\frac{x_0}{\chi_1} \\
\frac{\chi_1}{x_2}
\end{pmatrix}, \begin{pmatrix}
\frac{y_T}{y_B}
\end{pmatrix} \rightarrow \begin{pmatrix}
\frac{y_0}{\psi_1} \\
\frac{\psi_1}{y_2}
\end{pmatrix}$$

where  $v_{11}$ ,  $\chi_1$ , and  $\psi_1$  are scalars

$$\psi_1 := v_{11}\chi_1 + u_{12}^T x_2 + \psi_1$$

## Continue with

$$\left(\begin{array}{c|c|c}
U_{TL} & U_{TR} \\
\hline
U_{BL} & U_{BR}
\end{array}\right) \leftarrow \left(\begin{array}{c|c|c}
U_{00} & u_{01} & U_{02} \\
\hline
u_{10}^T & v_{11} & u_{12}^T \\
\hline
U_{20} & u_{21} & A_{22}
\end{array}\right),$$

$$\left(\begin{array}{c|c|c}
x_T \\
\hline
x_B
\end{array}\right) \leftarrow \left(\begin{array}{c|c|c}
x_0 \\
\hline
\chi_1 \\
\hline
x_2
\end{array}\right), \left(\begin{array}{c|c|c}
y_T \\
\hline
y_B
\end{array}\right) \leftarrow \left(\begin{array}{c|c|c}
y_0 \\
\hline
\psi_1 \\
\hline
y_2
\end{array}\right)$$

```
function [ y_out ] = Trmvp_un_unb_var1( U, x, y )
  [ UTL, UTR, ...
   UBL, UBR ] = FLA_Part_2x2(U, ...)
                                0, 0, 'FLA_TL');
  [ xT, ...
   xB = FLA_Part_2x1(x, ...
                         0, 'FLA_TOP');
  [ yT, ...
   yB ] = FLA_Part_2x1( y, ...
0, 'FLA_TOP');
  while (size(UTL, 1) < size(U, 1))
                       U02, ...
    [ U00, u01,
      u10t, upsilon11, u12t, ...
                       U22 ] = FLA_Repart_2x_2_{to_3}x_3 (UTL, UTR, ...
      U20, u21,
                                                       UBL, UBR, ...
                                                        1, 1, 'FLA_BR');
    [ x0, ...
      chi1, ...
      x2 = FLA_Repart_2x1_{to_3}x1(xT, ...
                                     xB, ...
                                     1, 'FLA_BOTTOM');
    [ y0, ...
      psi1, ...
      y2 = FLA_Repart_2x1_to_3x1(yT, ...
                                     уB, ...
                                     1, 'FLA_BOTTOM');
    psi1 = laff_dots( upsilon11, chi1, psi1);
    psi1 = laff_dots(u12t, x2, psi1);
                                                                  -%
    [ UTL, UTR, ...
      UBL, UBR ] = FLA_Cont_with_3x3_to_2x2 ( U00, u01,
                                                                U02, ...
                                              u10t\;,\;\;upsilon11\;,\;\;u12t\;,\;\;\ldots
                                              U20, u21,
                                                                U22, ...
                                              'FLA_TL');
    [ xT, ...
      xB = FLA_Cont_with_3x1_to_2x1(x0, ...
                                        chi1, ...
                                        x2, ...
                                        'FLA_TOP');
    [ yT, ...
     yB = FLA_Cont_with_3x1_to_2x1(y0, ...
                                        psi1, ...
                                        y2, ...
                                        'FLA_TOP');
 end
 y_out = [yT]
            yB ];
return
```

Click to view .m file Trmvp\_un\_unb\_var1.m. 
← Click for test script test\_Trmvp\_un\_unb\_var1.m. Then copy and paste it into PictureFlame to watch it in action.

**Algorithm:**  $y := \text{Trmvp\_un\_unb\_var2}(U, x, y)$ 

## Partition

$$U \to \left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) ,$$
$$x \to \left(\begin{array}{c|c} x_T \\ \hline x_B \end{array}\right) , y \to \left(\begin{array}{c|c} y_T \\ \hline y_B \end{array}\right)$$

where  $U_{TL}$  is  $0 \times 0$ ,  $x_T$ ,  $y_T$  are  $0 \times 1$  while  $m(U_{TL}) < m(U)$  do

# Repartition

$$\left(\begin{array}{c|c|c}
U_{TL} & U_{TR} \\
\hline
U_{BL} & U_{BR}
\end{array}\right) \rightarrow \left(\begin{array}{c|c|c}
U_{00} & u_{01} & U_{02} \\
\hline
u_{10}^T & v_{11} & u_{12}^T \\
\hline
U_{20} & u_{21} & U_{22}
\end{array}\right),$$

$$\left(\begin{array}{c|c|c}
x_T \\
\hline
x_B
\end{array}\right) \rightarrow \left(\begin{array}{c|c|c}
x_0 \\
\hline
x_1 \\
\hline
x_2
\end{array}\right), \left(\begin{array}{c|c|c}
y_T \\
\hline
y_B
\end{array}\right) \rightarrow \left(\begin{array}{c|c|c}
y_0 \\
\hline
\psi_1 \\
\hline
y_2
\end{array}\right)$$

where  $v_{11}$ ,  $\chi_1$ , and  $\psi_1$  are scalars

$$y_0 := \chi_1 u_{01} + y_0$$
  
$$\psi_1 := \chi_1 v_{11} + \psi_1$$

## Continue with

$$\left(\begin{array}{c|c}
U_{TL} & U_{TR} \\
\hline
U_{BL} & U_{BR}
\end{array}\right) \leftarrow \left(\begin{array}{c|c}
U_{00} & u_{01} & U_{02} \\
\hline
u_{10}^T & v_{11} & u_{12}^T \\
\hline
U_{20} & u_{21} & A_{22}
\end{array}\right),$$

$$\left(\begin{array}{c|c}
x_T \\
\hline
x_B
\end{array}\right) \leftarrow \left(\begin{array}{c|c}
x_0 \\
\hline
x_1 \\
\hline
x_2
\end{array}\right), \left(\begin{array}{c|c}
y_T \\
\hline
y_B
\end{array}\right) \leftarrow \left(\begin{array}{c|c}
y_0 \\
\hline
\psi_1 \\
\hline
y_2
\end{array}\right)$$

```
function [ y_out ] = Trmvp_un_unb_var2( U, x, y )
  [ UTL, UTR, ...
   UBL, UBR ] = FLA_Part_2x2(U, ...
                                0, 0, 'FLA_TL');
  [ xT, ...
   xB = FLA_Part_2x1(x, ...
                         0, 'FLA_TOP');
  [ yT, ...
   yB ] = FLA_Part_2x1( y, ...
0, 'FLA_TOP');
  while (size(UTL, 1) < size(U, 1))
                       U02, ...
    [ U00, u01,
      u10t, upsilon11, u12t, ...
                       U22 ] = FLA_Repart_2x_2_to_3x_3 ( UTL, UTR, ...
      U20, u21,
                                                       UBL, UBR, ...
                                                        1, 1, 'FLA_BR');
    [ x0, ...
      chi1, ...
      x2 = FLA_Repart_2x1_{to_3}x1(xT, ...
                                     xB, \ldots
                                     1, 'FLA_BOTTOM');
    [ y0, ...
      psi1, ...
      y2 = FLA_Repart_2x1_to_3x1(yT, ...
                                     уB, ...
                                     1, 'FLA_BOTTOM');
   y0 = laff_axpy(chi1, u01, y0);
    psi1 = laff_axpy(chi1, upsilon11, psi1);
                                                                  -%
    [ UTL, UTR, ...
      UBL, UBR ] = FLA_Cont_with_3x3_to_2x2 ( U00, u01,
                                                                U02,
                                              u10t\;,\;\;upsilon11\;,\;\;u12t\;,\;\;\ldots
                                              U20, u21,
                                                                U22, ...
                                              'FLA_TL');
    [ xT, ...
      xB = FLA_Cont_with_3x1_to_2x1(x0, ...
                                        chi1, ...
                                        x2, ...
                                        'FLA_TOP');
    [ yT, ...
     yB = FLA_Cont_with_3x1_to_2x1(y0, ...
                                        psi1, ...
                                        y2, ...
                                        'FLA_TOP');
 end
 y_out = [yT]
            yB ];
return
```

Click to view .m file Trmvp\_un\_unb\_var2.m. 
← Click for test script test\_Trmvp\_un\_unb\_var2.m. Then copy and paste it into PictureFlame to watch it in action.

**Algorithm:**  $y := \text{Trmvp_ln_unb_var1}(L, x, y)$ 

## Partition

$$L \to \left(\begin{array}{c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right),$$
$$x \to \left(\begin{array}{c|c} x_T \\ \hline x_B \end{array}\right), y \to \left(\begin{array}{c|c} y_T \\ \hline y_B \end{array}\right)$$

where  $L_{TL}$  is  $0 \times 0$ ,  $x_T$ ,  $y_T$  are  $0 \times 1$ while  $m(L_{TL}) < m(L)$  do

## Repartition

$$\left(\begin{array}{c|c|c}
L_{TL} & L_{TR} \\
\hline
L_{BL} & L_{BR}
\end{array}\right) \rightarrow \left(\begin{array}{c|c|c}
L_{00} & l_{01} & L_{02} \\
\hline
l_{10}^T & \lambda_{11} & l_{12}^T \\
\hline
L_{20} & l_{21} & L_{22}
\end{array}\right),$$

$$\left(\begin{array}{c|c|c}
x_T \\
\hline
x_B
\end{array}\right) \rightarrow \left(\begin{array}{c|c}
x_0 \\
\hline
\chi_1 \\
\hline
x_2
\end{array}\right), \left(\begin{array}{c|c|c}
y_T \\
\hline
y_B
\end{array}\right) \rightarrow \left(\begin{array}{c|c}
y_0 \\
\hline
\psi_1 \\
\hline
y_2
\end{array}\right)$$

where  $\lambda_{11}$ ,  $\chi_1$ , and  $\psi_1$  are scalars

$$\psi_1 := l_{10}^T x_0 + \lambda_{11} \chi_1 + \psi_1$$

## Continue with

$$\begin{pmatrix}
L_{TL} & L_{TR} \\
L_{BL} & L_{BR}
\end{pmatrix}
\leftarrow
\begin{pmatrix}
L_{00} & 01 & L_{02} \\
l_{10}^T & \lambda_{11} & l_{12}^T \\
L_{20} & l_{21} & A_{22}
\end{pmatrix},$$

$$\begin{pmatrix}
x_T \\
x_B
\end{pmatrix}
\leftarrow
\begin{pmatrix}
\frac{x_0}{\chi_1} \\
x_2
\end{pmatrix},
\begin{pmatrix}
\frac{y_T}{y_B}
\end{pmatrix}
\leftarrow
\begin{pmatrix}
\frac{y_0}{\psi_1} \\
y_2
\end{pmatrix}$$

```
function [ y_out ] = Trmvp_ln_unb_var1( L, x, y )
  [ LTL, LTR, ...
   LBL, LBR ] = FLA_Part_2x_2(L, ...
                               0, 0, 'FLA_TL');
  [ xT, ...
   xB = FLA_Part_2x1(x, \dots
                         0, 'FLA_TOP');
   yB = FLA_Part_2x1(y, ... 0, 'FLA_TOP');
  while (size(LTL, 1) < size(L, 1))
    [ L00, l01,
                      L02, ...
      110t, lambda11, 112t, ...
                      L22 ] = FLA_Repart_2x_2_{to_3}x_3 (LTL, LTR, ...
                                                      LBL, LBR, ...
                                                      1, 1, 'FLA_BR');
    [ x0, ...
      chi1, ...
      x2 = FLA_Repart_2x1_{to_3}x1(xT, ...
                                     1, 'FLA_BOTTOM');
    [ y0, ...
      psi1, ...
      y2 ] = FLA_Repart_2x1_to_3x1( yT, ...
                                     1, 'FLA_BOTTOM');
    psi1 = laff_dots(l10t, x0, psi1);
    psi1 = laff_dots(lambda11, chi1, psi1);
                                                                  -%
    [ LTL, LTR, ...
     LBL, LBR ] = FLA_Cont_with_3x3_to_2x2(L00, 101,
                                                              L02, ...
                                              110t\ ,\ lambda11\ ,\ l12t\ ,\ \ldots
                                              L20, l21,
                                                              L22, ...
                                              'FLA_TL');
    [ xT, ...
     xB = FLA_Cont_with_3x1_to_2x1(x0, ...
                                        chi1, ...
                                       x2, ...
                                        'FLA_TOP');
    [ yT, ...
     yB = FLA_Cont_with_3x1_to_2x1(y0, ...
                                        psi1, ...
                                       y2, ...
                                        'FLA_TOP');
 end
 y_out = [yT]
            yB ];
return
```

Click to view .m file Trmvp\_ln\_unb\_var1.m. Click for test script test\_Trmvp\_ln\_unb\_var1.m. Then copy\_1 and paste it into PictureFlame to watch it in action.

**Algorithm:**  $y := \text{Trmvp\_ln\_unb\_var2}(L, x, y)$ 

## Partition

$$L \to \left(\begin{array}{c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right),$$
$$x \to \left(\begin{array}{c|c} x_T \\ \hline x_B \end{array}\right), y \to \left(\begin{array}{c|c} y_T \\ \hline y_B \end{array}\right)$$

where  $L_{TL}$  is  $0 \times 0$ ,  $x_T$ ,  $y_T$  are  $0 \times 1$  while  $m(L_{TL}) < m(L)$  do

# Repartition

$$\begin{pmatrix}
L_{TL} & L_{TR} \\
L_{BL} & L_{BR}
\end{pmatrix} \rightarrow
\begin{pmatrix}
L_{00} & l_{01} & L_{02} \\
l_{10}^T & \lambda_{11} & l_{12}^T \\
L_{20} & l_{21} & L_{22}
\end{pmatrix},$$

$$\begin{pmatrix}
x_T \\
x_B
\end{pmatrix} \rightarrow
\begin{pmatrix}
\frac{x_0}{\chi_1} \\
x_2
\end{pmatrix}, \begin{pmatrix}
\frac{y_T}{y_B}
\end{pmatrix} \rightarrow
\begin{pmatrix}
\frac{y_0}{\psi_1} \\
y_2
\end{pmatrix}$$

where  $\lambda_{11}$ ,  $\chi_1$ , and  $\psi_1$  are scalars

$$\psi_1 := \lambda_{11} \chi_1 + \psi_1$$
  
$$y_2 := \chi_1 l_{21} + y_2$$

## Continue with

$$\left(\begin{array}{c|c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c|c|c} L_{00} & 01 & L_{02} \\ \hline l_{10}^T & \lambda_{11} & l_{12}^T \\ \hline L_{20} & l_{21} & A_{22} \end{array}\right), \\
\left(\begin{array}{c|c|c} x_T \\ \hline x_B \end{array}\right) \leftarrow \left(\begin{array}{c|c|c} x_0 \\ \hline \chi_1 \\ \hline x_2 \end{array}\right), \left(\begin{array}{c|c|c} y_T \\ \hline y_B \end{array}\right) \leftarrow \left(\begin{array}{c|c} y_0 \\ \hline \psi_1 \\ \hline y_2 \end{array}\right)$$

```
function [ y_out ] = Trmvp_ln_unb_var2( L, x, y )
  [ LTL, LTR, ...
   LBL, LBR ] = FLA_Part_2x_2(L, ...
                               0, 0, 'FLA_TL');
  [ xT, ...
   xB = FLA_Part_2x1(x, \dots
                         0, 'FLA_TOP');
   yB = FLA_Part_2x1(y, ... 0, 'FLA_TOP');
  while (size(LTL, 1) < size(L, 1))
    [ L00, l01,
                      L02, ...
      110t, lambda11, 112t, ...
                      L22 ] = FLA_Repart_2x_2_{to_3}x_3 (LTL, LTR, ...
                                                      LBL, LBR, ...
                                                      1, 1, 'FLA_BR');
    [ x0, ...
      chi1, ...
     x2 = FLA_Repart_2x1_to_3x1(xT, ...
                                    1, 'FLA_BOTTOM');
    [ y0, ...
     psi1, ...
     y2 ] = FLA_Repart_2x1_to_3x1( yT, ...
                                    уB, ...
                                    1, 'FLA_BOTTOM');
    psi1 = laff_axpy( chi1, lambda11, psi1 );
   y2 = laff_axpy(chi1, l21, y2);
                                                                 -%
    [ LTL, LTR, ...
     LBL, LBR ] = FLA_Cont_with_3x3_to_2x2(L00, l01,
                                                              L02, ...
                                              110t\ ,\ lambda11\ ,\ l12t\ ,\ \ldots
                                              L20, l21,
                                                              L22, ...
                                              'FLA_TL');
    [ xT, ...
     xB = FLA_Cont_with_3x1_to_2x1(x0, ...
                                        chi1, ...
                                       x2, ...
                                        'FLA_TOP');
    [ yT, ...
     yB = FLA_Cont_with_3x1_to_2x1(y0, ...
                                        psi1, ...
                                       y2, ...
                                        'FLA_TOP');
 end
 y_out = [yT]
           yB ];
return
```

Click to view .m file Trmvp\_ln\_unb\_var2.m. Click for test script test\_Trmvp\_ln\_unb\_var2.m. Then copy\_and paste it into PictureFlame to watch it in action.

**Algorithm:**  $x := \text{Trmv\_un\_unb\_var1}(U, x)$ 

#### Partition

$$U \to \left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right), x \to \left(\begin{array}{c|c} x_T \\ \hline x_B \end{array}\right)$$

where  $U_{TL}$  is  $0 \times 0$ ,  $x_T$  is  $0 \times 1$ while  $m(U_{TL}) < m(U)$  do

# Repartition

$$\left(\begin{array}{c|c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \to \left(\begin{array}{c|c|c} U_{00} & u_{01} & U_{02} \\ \hline u_{10}^T & v_{11} & u_{12}^T \\ \hline U_{20} & u_{21} & U_{22} \end{array}\right), \left(\begin{array}{c} x_T \\ \hline x_B \end{array}\right) \to \left(\begin{array}{c} x_0 \\ \hline \chi_1 \\ \hline x_2 \end{array}\right)$$

where  $v_{11}$  and  $\chi_1$  are scalars

$$\chi_1 := v_{11}\chi_1 \chi_1 := u_{12}^T x_2 + \chi_1$$

## Continue with

$$\left(\begin{array}{c|c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c|c|c} U_{00} & u_{01} & U_{02} \\ \hline u_{10}^T & v_{11} & u_{12}^T \\ \hline U_{20} & u_{21} & A_{22} \end{array}\right), \left(\begin{array}{c} x_T \\ \hline x_B \end{array}\right) \leftarrow \left(\begin{array}{c} x_0 \\ \hline \chi_1 \\ \hline x_2 \end{array}\right)$$

```
function [ x_out ] = Trmv_un_unb_var1( U, x )
  [ UTL, UTR, ...
   UBL, UBR ] = FLA_Part_2x2(U, ...
                                0, 0, 'FLA_TL');
  [ xT, ...
   xB = FLA_Part_2x1(x, ...
                          0, 'FLA_TOP');
  while ( size(UTL, 1) < size(U, 1))
    [ U00, u01,
                        U02, ...
      u10t, upsilon11, u12t, ...
                        U22 ] = FLA_Repart_2x_2_{to_3}x_3 ( UTL, UTR, ...
      U20, u21,
                                                         UBL, UBR, ...
                                                         1, 1, 'FLA_BR');
    [ x0, ...
      chi1, ...
      x2] = FLA_Repart_2x1_to_3x1( xT, ...
                                      1, 'FLA_BOTTOM');
    chi1 = laff_dot( upsilon11, chi1 );
    chi1 = laff_dots(u12t, x2, chi1);
                                                                  ---%
    [ UTL, UTR, ...
      UBL, UBR ] = FLA_Cont_with_3x3_to_2x2(U00, u01,
                                                                 U02, ...
                                               u10t, upsilon11, u12t, ...
                                               U20, u21,
                                                                 U22, ...
                                               'FLA_TL');
    [ xT, ...
      xB \ ] \ = \ FLA\_Cont\_with\_3x1\_to\_2x1\left( \ x0 \, , \ \ldots \right.
                                         chi1, ...
                                         x2, ...
                                         'FLA_TOP');
  end
  x_out = [xT]
            xB];
return
```

Click to view .m file Trmv\_un\_unb\_var1.m. Click for test script test\_Trmv\_un\_unb\_var1.m. Then copy and paste it into PictureFlame to watch it in action.

**Algorithm:**  $x := \text{Trmv\_un\_unb\_var2}(U, x)$ 

#### Partition

$$U \to \left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right), x \to \left(\begin{array}{c|c} x_T \\ \hline x_B \end{array}\right)$$

where  $U_{TL}$  is  $0 \times 0$ ,  $x_T$  is  $0 \times 1$ while  $m(U_{TL}) < m(U)$  do

# Repartition

$$\left(\begin{array}{c|c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \to \left(\begin{array}{c|c|c} U_{00} & u_{01} & U_{02} \\ \hline u_{10}^T & v_{11} & u_{12}^T \\ \hline U_{20} & u_{21} & U_{22} \end{array}\right), \left(\begin{array}{c} x_T \\ \hline x_B \end{array}\right) \to \left(\begin{array}{c} x_0 \\ \hline \chi_1 \\ \hline x_2 \end{array}\right)$$

where  $v_{11}$  and  $\chi_1$  are scalars

$$x_0 := \chi_1 u_{01} + x_0$$
$$\chi_1 := \chi_1 v_{11}$$

## Continue with

$$\left(\begin{array}{c|c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c|c|c} U_{00} & u_{01} & U_{02} \\ \hline u_{10}^T & v_{11} & u_{12}^T \\ \hline U_{20} & u_{21} & A_{22} \end{array}\right), \left(\begin{array}{c} x_T \\ \hline x_B \end{array}\right) \leftarrow \left(\begin{array}{c} x_0 \\ \hline \chi_1 \\ \hline x_2 \end{array}\right)$$

```
function [ x_out ] = Trmv_un_unb_var2( U, x )
  [ UTL, UTR, ...
   UBL, UBR ] = FLA_Part_2x2(U, ...
                                0, 0, 'FLA_TL');
  [ xT, ...
   xB = FLA_Part_2x1(x, ...
                          0, 'FLA_TOP');
  while ( size(UTL, 1) < size(U, 1))
    [ U00, u01,
                        U02, ...
      u10t, upsilon11, u12t, ...
                        U22 ] = FLA_Repart_2x_2_{to_3}x_3 ( UTL, UTR, ...
      U20, u21,
                                                         UBL, UBR, ...
                                                         1, 1, 'FLA_BR');
    [ x0, ...
      chi1, ...
      x2] = FLA_Repart_2x1_to_3x1( xT, ...
                                      1, 'FLA_BOTTOM');
    x0 = laff_axpy(chi1, u01, x0);
    chi1 = laff_dot( upsilon11, chi1 );
                                                                  ---%
    [ UTL, UTR, ...
      UBL, UBR ] = FLA_Cont_with_3x3_to_2x2(U00, u01,
                                                                 U02, ...
                                               u10t, upsilon11, u12t, ...
                                               U20, u21,
                                                                 U22, ...
                                               'FLA_TL');
    [ xT, ...
      xB \ ] \ = \ FLA\_Cont\_with\_3x1\_to\_2x1\left( \ x0 \, , \ \ldots \right.
                                         chi1, ...
                                         x2, ...
                                         'FLA_TOP');
  end
  x_out = [xT]
            xB];
return
```

Click to view .m file Trmv\_un\_unb\_var2.m. 
Click for test script test\_Trmv\_un\_unb\_var2.m. Then copy and paste it into PictureFlame to watch it in action.

**Algorithm:**  $[x] := \text{Trmv_ln_unb_var}1(L, x)$ 

Partition

$$L \to \left(\begin{array}{c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right), x \to \left(\begin{array}{c|c} x_T \\ \hline x_B \end{array}\right)$$

where  $L_{BR}$  is  $0 \times 0$ ,  $x_B$  has 0 rows while  $m(L_{BR}) < m(L)$  do

Repartition

$$\left(\begin{array}{c|c|c}
L_{TL} & L_{TR} \\
\hline
L_{BL} & L_{BR}
\end{array}\right) \rightarrow \left(\begin{array}{c|c|c}
L_{00} & l_{01} & L_{02} \\
\hline
l_{10}^T & \lambda_{11} & l_{12}^T \\
\hline
L_{20} & l_{21} & L_{22}
\end{array}\right), \left(\begin{array}{c}
x_T \\
\hline
x_B
\end{array}\right) \rightarrow \left(\begin{array}{c}
x_0 \\
\hline
\chi_1 \\
\hline
x_2
\end{array}\right)$$

where  $\lambda_{11}$  is  $1 \times 1$ ,  $\chi_1$  has 1 row

$$\chi_1 := \lambda_{11} \chi_1 
\chi_1 := l_{10}^T x_0 + \chi_1$$

Continue with

$$\begin{pmatrix} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{pmatrix} \leftarrow \begin{pmatrix} L_{00} & l_{01} & L_{02} \\ \hline l_{10}^T & \lambda_{11} & l_{12}^T \\ L_{20} & l_{21} & L_{22} \end{pmatrix}, \begin{pmatrix} x_T \\ \hline x_B \end{pmatrix} \leftarrow \begin{pmatrix} x_0 \\ \hline \chi_1 \\ \hline x_2 \end{pmatrix}$$

```
function [ x_out ] = Trmv_ln_unb_var1( L, x )
  [ LTL, LTR, ...
   LBL, LBR ] = FLA_Part_2x2(L, ...
                                0, 0, 'FLA_BR');
 [ xT, ...
   xB = FLA_Part_2x1(x, ...
                          0, 'FLA_BOTTOM');
 while ( size(LBR, 1) < size(L, 1))
    [ L00, l01,
                      L02, ...
      110t, lambda11, l12t, ...
      L20, 121,
                      L22 ] = FLA_Repart_2x_2_{to_3}x_3 (LTL, LTR, ...
                                                       LBL, LBR, ...
                                                       1, 1, 'FLA_TL');
    [ x0, ...
      {\tt chil} \; , \quad \dots
      x2 ] = FLA_Repart_2x1_to_3x1( xT, ...
                                     хB, ...
                                     1, 'FLA_TOP');
    chi1 = laff_dot( lambda11, chi1);
    chi1 = laff_dot(l10t, x0, chi1);
                                                                  --%
    [ LTL, LTR, ...
                                                               L02, ...
     LBL, LBR ] = FLA\_Cont\_with\_3x3\_to\_2x2 (L00, l01,
                                               110t\ ,\ lambda11\ ,\ l12t\ ,\ \ldots
                                               L20, 121,
                                                               L22, ...
                                               'FLA_BR');
   [ xT, ...
     xB = FLA_Cont_with_3x1_to_2x1(x0, ...
                                        chi1, ...
                                        x2, ...
                                        'FLA_BOTTOM');
 end
 x_out = [xT]
            xB];
return
```

**Algorithm:**  $[x] := \text{Trmv_lnb_var2}(L, x)$ 

Partition

$$L \to \left(\begin{array}{c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right), x \to \left(\begin{array}{c|c} x_T \\ \hline x_B \end{array}\right)$$

where  $L_{BR}$  is  $0 \times 0$ ,  $x_B$  has 0 rows while  $m(L_{BR}) < m(L)$  do

Repartition

$$\left(\begin{array}{c|c|c}
L_{TL} & L_{TR} \\
\hline
L_{BL} & L_{BR}
\end{array}\right) \to \left(\begin{array}{c|c|c}
L_{00} & l_{01} & L_{02} \\
\hline
l_{10}^T & \lambda_{11} & l_{12}^T \\
\hline
L_{20} & l_{21} & L_{22}
\end{array}\right), \left(\begin{array}{c}
x_T \\
\hline
x_B
\end{array}\right) \to \left(\begin{array}{c}
x_0 \\
\hline
\chi_1 \\
\hline
x_2
\end{array}\right)$$

where  $\lambda_{11}$  is  $1 \times 1$ ,  $\chi_1$  has 1 row

 $x_2 := \chi_1 l_{21} + x_2$  $\chi_1 := \chi_1 \lambda_{11}$ 

Continue with

$$\left(\begin{array}{c|c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c|c|c} L_{00} & l_{01} & L_{02} \\ \hline l_{10}^T & \lambda_{11} & l_{12}^T \\ \hline L_{20} & l_{21} & L_{22} \end{array}\right), \left(\begin{array}{c} x_T \\ \hline x_B \end{array}\right) \leftarrow \left(\begin{array}{c} x_0 \\ \hline \chi_1 \\ \hline x_2 \end{array}\right)$$

```
function [ x_out ] = Trmv_ln_unb_var2( L, x )
  [ LTL, LTR, ...
   LBL, LBR ] = FLA_Part_2x2(L, ...
                               0, 0, 'FLA_BR');
 [ xT, ...
   xB = FLA_Part_2x1(x, ...
                         0, 'FLA_BOTTOM');
 while ( size(LBR, 1) < size(L, 1))
    [ L00, 101,
                      L02, ...
     110t, lambda11, l12t, ...
     L20, 121,
                     L22 ] = FLA_Repart_2x_2_{to_3}x_3 (LTL, LTR, ...
                                                     LBL, LBR, ...
                                                      1, 1, 'FLA_TL');
    [ x0, ...
      chi1, ...
     x2 ] = FLA_Repart_2x1_to_3x1( xT, ...
                                    хB, ...
                                    1, 'FLA_TOP');
    x2 = laff_axpy(chi1, l21, x2);
    chi1 = laff_dot( chi1, lambda11 );
                                                                --%
    [ LTL, LTR, ...
                                                              L02, ...
     LBL, LBR ] = FLA\_Cont\_with\_3x3\_to\_2x2 (L00, l01,
                                              110t\ ,\ lambda11\ ,\ l12t\ ,\ \ldots
                                              L20, 121,
                                                              L22, ...
                                              'FLA_BR');
   [ xT, ...
     xB = FLA_Cont_with_3x1_to_2x1(x0, ...
                                       chi1, ...
                                       x2, ...
                                        'FLA_BOTTOM');
 end
 x_out = [xT]
           xB];
return
```

Click to view .m file Trmv\_ln\_unb\_var2.m. 
Click for test script test\_Trmv\_ln\_unb\_var2.m. Then copy and paste it into PictureFlame to watch it in action.

**Algorithm:**  $y := \text{Symv\_u\_unb\_var1}(A, x, y)$ 

## Partition

$$A \to \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right),$$

$$x \to \left(\begin{array}{c|c} x_T \\ \hline x_B \end{array}\right), y \to \left(\begin{array}{c|c} y_T \\ \hline y_B \end{array}\right)$$

where  $A_{TL}$  is  $0 \times 0$ ,  $x_T$ ,  $y_T$  are  $0 \times 1$  while  $m(A_{TL}) < m(A)$  do

## Repartition

$$\begin{pmatrix}
A_{TL} & A_{TR} \\
A_{BL} & A_{BR}
\end{pmatrix} \rightarrow
\begin{pmatrix}
A_{00} & a_{01} & A_{02} \\
a_{10}^T & \alpha_{11} & a_{12}^T \\
A_{20} & a_{21} & A_{22}
\end{pmatrix},$$

$$\begin{pmatrix}
x_T \\
x_B
\end{pmatrix} \rightarrow
\begin{pmatrix}
\frac{x_0}{\chi_1} \\
x_2
\end{pmatrix},
\begin{pmatrix}
\frac{y_T}{y_B}
\end{pmatrix} \rightarrow
\begin{pmatrix}
\frac{y_0}{\psi_1} \\
y_2
\end{pmatrix}$$

where  $\alpha_{11}$ ,  $\chi_1$ , and  $\psi_1$  are scalars

$$\psi_1 := a_{01}^T x_0 + \alpha_{11} \chi_1 + a_{12}^T x_2 + \psi_1$$

## Continue with

$$\left(\begin{array}{c|c}
A_{TL} & A_{TR} \\
\hline
A_{BL} & A_{BR}
\end{array}\right) \leftarrow \left(\begin{array}{c|c}
A_{00} & a_{01} & A_{02} \\
\hline
a_{10}^T & \alpha_{11} & a_{12}^T \\
\hline
A_{20} & a_{21} & A_{22}
\end{array}\right),$$

$$\left(\begin{array}{c|c}
x_T \\
\hline
x_B
\end{array}\right) \leftarrow \left(\begin{array}{c|c}
x_0 \\
\hline
\chi_1 \\
\hline
x_2
\end{array}\right), \left(\begin{array}{c|c}
y_T \\
\hline
y_B
\end{array}\right) \leftarrow \left(\begin{array}{c|c}
y_0 \\
\hline
\psi_1 \\
\hline
y_2
\end{array}\right)$$

```
function [ y_out ] = Symv_u_unb_var1( A, x, y )
  [ ATL, ATR, ...
   ABL, ABR = FLA_Part_2x2(A, ...
                               0, 0, 'FLA_TL');
  [ xT, ...
   xB = FLA_Part_2x1(x, ...
                         0, 'FLA_TOP');
  [ yT, ...
   yB = FLA_Part_2x1(y, ... 0, 'FLA_TOP');
 while ( size( ATL, 1 ) < size( A, 1 )
    [ A00, a01,
                     A02, ...
      a10t, alpha11, a12t, ...
                     A22 ] = FLA_Repart_2x_2_{to_3}x_3 (ATL, ATR, ...
                                                    ABL, ABR, ...
                                                     1, 1, 'FLA_BR');
    [ x0, ...
      chi1, ...
      x2 = FLA_Repart_2x1_{to_3}x1(xT, ...
                                    xB, \ldots
                                    1, 'FLA_BOTTOM');
    [ y0, ...
     psi1, ...
     y2 = FLA_Repart_2x1_to_3x1(yT, ...
                                    уB, ...
                                    1, 'FLA_BOTTOM');
    psi1 = laff_dots(a01, x0, psi1);
    psi1 = laff_dots( alpha11, chi1, psi1);
    psi1 = laff_dots(a12t, x2, psi1);
    [ ATL, ATR, ...
                                                            A02, ...
     ABL, ABR ] = FLA_Cont_with_3x3_{to_2}x2 ( A00, a01,
                                             a10t, alpha11, a12t, ...
                                             A20, a21,
                                                             A22, ...
                                              'FLA_TL');
    [ xT, ...
     xB = FLA_Cont_with_3x1_to_2x1(x0, ...
                                       chi1, ...
                                       x2, ...
                                       'FLA_TOP');
    [ yT, ...
     yB = FLA_Cont_with_3x1_to_2x1(y0, ...
                                       psi1, ...
                                       y2, ...
                                       'FLA_TOP');
 end
 y_out = [yT]
           yB ];
return
```

Click to view .m file Symv\_u\_unb\_var1.m. Click for test script test\_Symv\_u\_unb\_var1.m. Then copy and paste it into PictureFlame to watch it in action.

**Algorithm:**  $y := \text{Symv\_u\_unb\_var2}(A, x, y)$ 

Partition

$$A \to \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right),$$
$$x \to \left(\begin{array}{c|c} x_T \\ \hline x_B \end{array}\right), y \to \left(\begin{array}{c|c} y_T \\ \hline y_B \end{array}\right)$$

where  $A_{TL}$  is  $0 \times 0$ ,  $x_T$ ,  $y_T$  are  $0 \times 1$  while  $m(A_{TL}) < m(A)$  do

Repartition

$$\left(\begin{array}{c|c}
A_{TL} & A_{TR} \\
\hline
A_{BL} & A_{BR}
\end{array}\right) \rightarrow \left(\begin{array}{c|c}
A_{00} & a_{01} & A_{02} \\
\hline
a_{10}^T & \alpha_{11} & a_{12}^T \\
\hline
A_{20} & a_{21} & A_{22}
\end{array}\right),$$

$$\left(\begin{array}{c|c}
x_T \\
\hline
x_B
\end{array}\right) \rightarrow \left(\begin{array}{c|c}
x_0 \\
\hline
\chi_1 \\
\hline
x_2
\end{array}\right) , \left(\begin{array}{c|c}
y_T \\
\hline
y_B
\end{array}\right) \rightarrow \left(\begin{array}{c|c}
y_0 \\
\hline
\psi_1 \\
\hline
y_2
\end{array}\right)$$

where  $\alpha_{11}$ ,  $\chi_1$ , and  $\psi_1$  are scalars

$$y_0 := \chi_1 a_{01} + y_0$$
  

$$\psi_1 := \alpha_{11} \chi_1 + \psi_1$$
  

$$y_2 := \chi_1 (a_{12}^T)^T + y_2$$

## Continue with

$$\left(\begin{array}{c|c}
A_{TL} & A_{TR} \\
\hline
A_{BL} & A_{BR}
\end{array}\right) \leftarrow \left(\begin{array}{c|c}
A_{00} & a_{01} & A_{02} \\
\hline
a_{10}^T & \alpha_{11} & a_{12}^T \\
\hline
A_{20} & a_{21} & A_{22}
\end{array}\right),$$

$$\left(\begin{array}{c|c}
x_T \\
\hline
x_B
\end{array}\right) \leftarrow \left(\begin{array}{c|c}
x_0 \\
\hline
x_1 \\
\hline
x_2
\end{array}\right), \left(\begin{array}{c|c}
y_T \\
\hline
y_B
\end{array}\right) \leftarrow \left(\begin{array}{c|c}
y_0 \\
\hline
y_1 \\
\hline
y_2
\end{array}\right)$$

```
function [ y_out ] = Symv_u_unb_var2( A, x, y )
  [ ATL, ATR, ...
   ABL, ABR = FLA_Part_2x2(A, ...
                               0, 0, 'FLA_TL');
  [ xT, ...
   xB = FLA_Part_2x1(x, ...
                         0, 'FLA_TOP');
  [ yT, ...
   yB = FLA_Part_2x1(y, ... 0, 'FLA_TOP');
 while ( size( ATL, 1 ) < size( A, 1 )
    [ A00, a01,
                    A02, ...
      a10t, alpha11, a12t, ...
                    A22 ] = FLA_Repart_2x_2_{to_3}x_3 (ATL, ATR, ...
                                                    ABL, ABR, ...
                                                    1, 1, 'FLA_BR');
    [ x0, ...
      chi1, ...
     x2 = FLA_Repart_2x1_to_3x1(xT, ...
                                    xB, \ldots
                                    1, 'FLA_BOTTOM');
    [ y0, ...
     psi1, ...
     y2 = FLA_Repart_2x1_to_3x1(yT, ...
                                    уB, ...
                                    1, 'FLA_BOTTOM');
   y0 = laff_axpy(chi1, a01, y0);
    psi1 = laff_axpy(chi1, alpha11, psi1);
   y2 = laff_axpy(chi1, a12t, y2);
    [ ATL, ATR, ...
                                                            A02, ...
     ABL, ABR ] = FLA_Cont_with_3x3_{to_2}x2 ( A00, a01,
                                             a10t, alpha11, a12t, ...
                                             A20, a21,
                                                            A22, ...
                                             'FLA_TL');
    [ xT, ...
     xB = FLA_Cont_with_3x1_to_2x1(x0, ...
                                       chi1, ...
                                       x2, ...
                                       'FLA_TOP');
    [ yT, ...
     yB = FLA_Cont_with_3x1_to_2x1(y0, ...
                                       psi1, ...
                                       y2, ...
                                       'FLA_TOP');
 end
 y_out = [yT]
           yB ];
return
```

**Algorithm:**  $y := \text{Symv_L_UNB_VAR1}(A, x, y)$ 

## Partition

$$A \to \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right),$$

$$x \to \left(\begin{array}{c|c} x_T \\ \hline x_B \end{array}\right), y \to \left(\begin{array}{c|c} y_T \\ \hline y_B \end{array}\right)$$

where  $A_{TL}$  is  $0 \times 0$ ,  $x_T$ ,  $y_T$  are  $0 \times 1$  while  $m(A_{TL}) < m(A)$  do

## Repartition

$$\begin{pmatrix}
A_{TL} & A_{TR} \\
A_{BL} & A_{BR}
\end{pmatrix} \rightarrow
\begin{pmatrix}
A_{00} & a_{01} & A_{02} \\
a_{10}^T & \alpha_{11} & a_{12}^T \\
A_{20} & a_{21} & A_{22}
\end{pmatrix},$$

$$\begin{pmatrix}
x_T \\
x_B
\end{pmatrix} \rightarrow
\begin{pmatrix}
\frac{x_0}{\chi_1} \\
x_2
\end{pmatrix},
\begin{pmatrix}
\frac{y_T}{y_B}
\end{pmatrix} \rightarrow
\begin{pmatrix}
\frac{y_0}{\psi_1} \\
\frac{\psi_1}{y_2}
\end{pmatrix}$$

where  $\alpha_{11}$ ,  $\chi_1$ , and  $\psi_1$  are scalars

$$\psi_1 := a_{10}^T x_0 + \alpha_{11} \chi_1 + a_{21}^T x_2 + \psi_1$$

## Continue with

$$\left(\begin{array}{c|c}
A_{TL} & A_{TR} \\
\hline
A_{BL} & A_{BR}
\end{array}\right) \leftarrow \left(\begin{array}{c|c}
A_{00} & a_{01} & A_{02} \\
\hline
a_{10}^T & \alpha_{11} & a_{12}^T \\
\hline
A_{20} & a_{21} & A_{22}
\end{array}\right),$$

$$\left(\begin{array}{c|c}
x_T \\
\hline
x_B
\end{array}\right) \leftarrow \left(\begin{array}{c|c}
x_0 \\
\hline
\chi_1 \\
\hline
x_2
\end{array}\right), \left(\begin{array}{c|c}
y_T \\
\hline
y_B
\end{array}\right) \leftarrow \left(\begin{array}{c|c}
y_0 \\
\hline
\psi_1 \\
\hline
y_2
\end{array}\right)$$

```
function [ y_out ] = Symv_l_unb_var1( A, x, y )
  [ ATL, ATR, ...
   ABL, ABR ] = FLA_Part_2x_2(A, ...)
                               0, 0, 'FLA_TL');
  [ xT, ...
   xB = FLA_Part_2x1(x, ...
                         0, 'FLA_TOP');
  [ yT, ...
   yB = FLA_Part_2x1(y, ... 0, 'FLA_TOP');
 while ( size( ATL, 1 ) < size( A, 1 )
    [ A00, a01,
                     A02, ...
      a10t, alpha11, a12t, ...
                     A22 ] = FLA_Repart_2x_2_{to_3}x_3 (ATL, ATR, ...
                                                    ABL, ABR, ...
                                                     1, 1, 'FLA_BR');
    [ x0, ...
      chi1, ...
     x2 = FLA_Repart_2x1_to_3x1(xT, ...
                                    xB, \ldots
                                    1, 'FLA_BOTTOM');
    [ y0, ...
     psi1, ...
     y2 = FLA_Repart_2x1_to_3x1(yT, ...
                                    уB, ...
                                    1, 'FLA_BOTTOM');
    psi1 = laff_dots(al0t, x0, psi1);
    psi1 = laff_dots( alpha11, chi1, psi1);
    psi1 = laff_dots(a21, x2, psi1);
    [ ATL, ATR, ...
                                                             A02, ...
     ABL, ABR ] = FLA_Cont_with_3x3_{to_2}x2 ( A00, a01,
                                              a10t, alpha11, a12t, ...
                                             A20, a21,
                                                             A22, ...
                                              'FLA_TL');
    [ xT, ...
     xB = FLA_Cont_with_3x1_to_2x1(x0, ...
                                       chi1, ...
                                       x2, ...
                                       'FLA_TOP');
    [ yT, ...
     yB = FLA_Cont_with_3x1_to_2x1(y0, ...
                                       psi1, ...
                                       y2, ...
                                        'FLA_TOP');
 end
 y_out = [yT]
           yB ];
return
```

Click to view .m file Symv\_l\_unb\_var1.m. Click for test script test\_Symv\_l\_unb\_var1.m. Then copy and paste it into PictureFlame to watch it in action.

**Algorithm:**  $y := \text{Symv_L_unb_var2}(A, x, y)$ 

**Partition** 

$$A \to \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right),$$

$$x \to \left(\begin{array}{c|c} x_T \\ \hline x_B \end{array}\right), y \to \left(\begin{array}{c|c} y_T \\ \hline y_B \end{array}\right)$$

where  $A_{TL}$  is  $0 \times 0$ ,  $x_T$ ,  $y_T$  are  $0 \times 1$  while  $m(A_{TL}) < m(A)$  do

Repartition

$$\begin{pmatrix}
A_{TL} & A_{TR} \\
A_{BL} & A_{BR}
\end{pmatrix} \rightarrow
\begin{pmatrix}
A_{00} & a_{01} & A_{02} \\
a_{10}^T & \alpha_{11} & a_{12}^T \\
A_{20} & a_{21} & A_{22}
\end{pmatrix},$$

$$\begin{pmatrix}
x_T \\
x_B
\end{pmatrix} \rightarrow
\begin{pmatrix}
\frac{x_0}{\chi_1} \\
\frac{\chi_1}{x_2}
\end{pmatrix},
\begin{pmatrix}
\frac{y_T}{y_B}
\end{pmatrix} \rightarrow
\begin{pmatrix}
\frac{y_0}{\psi_1} \\
\frac{\psi_1}{y_2}
\end{pmatrix}$$

where  $\alpha_{11}$ ,  $\chi_1$ , and  $\psi_1$  are scalars

$$y_0 := \chi_1(a_{10}^T)^T + y_0$$
  

$$\psi_1 := \alpha_{11}\chi_1 + \psi_1$$
  

$$y_2 := \chi_1 a_{21} + y_2$$

## Continue with

$$\left(\begin{array}{c|c}
A_{TL} & A_{TR} \\
\hline
A_{BL} & A_{BR}
\end{array}\right) \leftarrow \left(\begin{array}{c|c}
A_{00} & a_{01} & A_{02} \\
\hline
a_{10}^T & \alpha_{11} & a_{12}^T \\
\hline
A_{20} & a_{21} & A_{22}
\end{array}\right),$$

$$\left(\begin{array}{c|c}
x_T \\
\hline
x_B
\end{array}\right) \leftarrow \left(\begin{array}{c|c}
x_0 \\
\hline
x_1 \\
\hline
x_2
\end{array}\right), \left(\begin{array}{c|c}
y_T \\
\hline
y_B
\end{array}\right) \leftarrow \left(\begin{array}{c|c}
y_0 \\
\hline
y_1 \\
\hline
y_2
\end{array}\right)$$

```
function [ y_out ] = Symv_l_unb_var2( A, x, y )
  [ ATL, ATR, ...
   ABL, ABR = FLA_Part_2x2(A, ...
                               0, 0, 'FLA_TL');
  [ xT, ...
   xB = FLA_Part_2x1(x, ...
                         0, 'FLA_TOP');
  [ yT, ...
   yB = FLA_Part_2x1(y, ... 0, 'FLA_TOP');
 while ( size( ATL, 1 ) < size( A, 1 )
    [ A00, a01,
                    A02, ...
      a10t, alpha11, a12t, ...
                    A22 ] = FLA_Repart_2x_2_{to_3}x_3 (ATL, ATR, ...
                                                    ABL, ABR, ...
                                                    1, 1, 'FLA_BR');
    [ x0, ...
      chi1, ...
     x2 = FLA_Repart_2x1_to_3x1(xT, ...
                                    xB, \ldots
                                    1, 'FLA_BOTTOM');
    [ y0, ...
     psi1, ...
     y2 = FLA_Repart_2x1_to_3x1(yT, ...
                                    уB, ...
                                    1, 'FLA_BOTTOM');
   y0 = laff_axpy(chi1, a10t, y0);
    psi1 = laff_axpy(chi1, alpha11, psi1);
   y2 = laff_axpy(chi1, a21, y2);
    [ ATL, ATR, ...
                                                            A02, ...
     ABL, ABR ] = FLA_Cont_with_3x3_{to_2}x2 ( A00, a01,
                                             a10t, alpha11, a12t, ...
                                             A20, a21,
                                                            A22, ...
                                             'FLA_TL');
    [ xT, ...
     xB = FLA_Cont_with_3x1_to_2x1(x0, ...
                                       chi1, ...
                                       x2, ...
                                       'FLA_TOP');
    [ yT, ...
     yB = FLA_Cont_with_3x1_to_2x1(y0, ...
                                       psi1, ...
                                       y2, ...
                                       'FLA_TOP');
 end
 y_out = [yT]
           yB ];
return
```

**Algorithm:**  $y := \text{Symv\_u\_unb\_var3}(A, x, y)$ 

**Partition** 

$$A \to \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right),$$
$$x \to \left(\begin{array}{c|c} x_T \\ \hline x_B \end{array}\right), y \to \left(\begin{array}{c|c} y_T \\ \hline y_B \end{array}\right)$$

where  $A_{TL}$  is  $0 \times 0$ ,  $x_T$ ,  $y_T$  are  $0 \times 1$  while  $m(A_{TL}) < m(A)$  do

Repartition

$$\left(\begin{array}{c|c}
A_{TL} & A_{TR} \\
\hline
A_{BL} & A_{BR}
\end{array}\right) \rightarrow \left(\begin{array}{c|c}
A_{00} & a_{01} & A_{02} \\
\hline
a_{10}^T & \alpha_{11} & a_{12}^T \\
\hline
A_{20} & a_{21} & A_{22}
\end{array}\right),$$

$$\left(\begin{array}{c|c}
x_T \\
\hline
x_B
\end{array}\right) \rightarrow \left(\begin{array}{c|c}
x_0 \\
\hline
x_1 \\
\hline
x_2
\end{array}\right), \left(\begin{array}{c|c}
y_T \\
\hline
y_B
\end{array}\right) \rightarrow \left(\begin{array}{c|c}
y_0 \\
\hline
\psi_1 \\
\hline
y_2
\end{array}\right)$$

where  $\alpha_{11}$ ,  $\chi_1$ , and  $\psi_1$  are scalars

$$y_0 := \chi_1 a_{01} + y_0$$
  

$$\psi_1 := \alpha_{11} \chi_1 + \psi_1$$
  

$$\psi_1 := a_{01}^T x_0 + \psi_1$$

## Continue with

$$\left(\begin{array}{c|c}
A_{TL} & A_{TR} \\
\hline
A_{BL} & A_{BR}
\end{array}\right) \leftarrow \left(\begin{array}{c|c}
A_{00} & a_{01} & A_{02} \\
\hline
a_{10}^T & \alpha_{11} & a_{12}^T \\
\hline
A_{20} & a_{21} & A_{22}
\end{array}\right),$$

$$\left(\begin{array}{c|c}
x_T \\
\hline
x_B
\end{array}\right) \leftarrow \left(\begin{array}{c|c}
x_0 \\
\hline
x_1 \\
\hline
x_2
\end{array}\right), \left(\begin{array}{c|c}
y_T \\
\hline
y_B
\end{array}\right) \leftarrow \left(\begin{array}{c|c}
y_0 \\
\hline
y_1 \\
\hline
y_2
\end{array}\right)$$

```
function [ y_out ] = Symv_u_unb_var3( A, x, y )
  [ ATL, ATR, ...
   ABL, ABR = FLA_Part_2x2(A, ...
                               0, 0, 'FLA_TL');
  [ xT, ...
   xB = FLA_Part_2x1(x, ...
                         0, 'FLA_TOP');
  [ yT, ...
   yB = FLA_Part_2x1(y, ... 0, 'FLA_TOP');
 while ( size( ATL, 1 ) < size( A, 1 )
    [ A00, a01,
                    A02, ...
      a10t, alpha11, a12t, ...
                    A22 ] = FLA_Repart_2x_2_{to_3}x_3 (ATL, ATR, ...
                                                    ABL, ABR, ...
                                                    1, 1, 'FLA_BR');
    [ x0, ...
      chi1, ...
     x2 = FLA_Repart_2x1_to_3x1(xT, ...
                                    xB, \ldots
                                    1, 'FLA_BOTTOM');
    [ y0, ...
     psi1, ...
     y2 = FLA_Repart_2x1_to_3x1(yT, ...
                                    уB, ...
                                    1, 'FLA_BOTTOM');
   y0 = laff_axpy(chi1, a01, y0);
    psi1 = laff_axpy(chi1, alpha11, psi1);
    psi1 = laff_dots(a01, x0, psi1);
    [ ATL, ATR, ...
                                                            A02, ...
     ABL, ABR ] = FLA_Cont_with_3x3_{to_2}x2 ( A00, a01,
                                             a10t, alpha11, a12t, ...
                                             A20, a21,
                                                            A22, ...
                                             'FLA_TL' );
    [ xT, ...
     xB = FLA_Cont_with_3x1_to_2x1(x0, ...
                                       chi1, ...
                                       x2, ...
                                       'FLA_TOP');
    [ yT, ...
     yB = FLA_Cont_with_3x1_to_2x1(y0, ...
                                       psi1, ...
                                       y2, ...
                                       'FLA_TOP');
 end
 y_out = [yT]
           yB ];
return
```

Click to view .m file Symv\_u\_unb\_var3.m. Click for test script test\_Symv\_u\_unb\_var3.m. Then copy and paste it into PictureFlame to watch it in action.