Schrodinger's Smoke

Suman Swaroop
sswaroop@cse.iitb.ac.in

C Vishwesh
vishwesh@cse.iitb.ac.in

April 13, 2018

1 Problem Statement

We describe a new approach for the purely Eulerian simulation of in-compressible fluids. In it, the fluid state is represented by a C^2 -valued wave function evolving under the Schrodinger equation subject to incompressibility constraints. The resulting algorithm is simple, unconditionally stable, and efficient.Grid based approaches battle loss of vorticity with various devices, while purely Lagrangian approaches have their own host of problems ranging from inadequate control of sample density (particles) and the need for sophisticated multiple solvers. This method does not require any Lagrangian techniques for advection or to counteract the loss of vorticity and does a robust simulation of intricate phenomena such as vortical wakes and interacting vortex filaments, even on modestly sized grids at a quality level comparable to purely Lagrangian methods.

2 Key Idea

This approach is completely grid-based and Eulerian. The key idea here is that instead of describing the fluid evolution in terms of velocity or vorticity field, ISF evolves a two component wave function($\psi = (\psi_1, \psi_2)$) which encodes the fluid state on a 3D domain. The classical density ρ and fluid velocity v can be obtained as follows

$$\rho = |\psi|^2 \text{ and } \rho v_\alpha = \hbar \langle \frac{\partial \psi}{\partial x_\alpha}, i\psi \rangle$$

The time evolution of the wave function is governed by the Schrodinger equation given by,

$$i\hbar\dot{\psi} = -\frac{\hbar^2}{2}\Delta\psi + p\psi$$

subject to the constraints $\langle \Delta\psi, i\psi \rangle_{\Re} = 0$ and $|\psi|^2 = 1$ which correspond to the incompressibility constraints $\text{div(v)} = 0$ and $\rho = 1$ in the classical sense. Here p is a scalar potential and is referred to as pressure in analogy to the Euler equation.

3 Implementation Approach

- Set up a 3D lattice with each voxel storing wave function $\psi_v \in C^2$, the real valued pressure $q_v \in \Re$, and real valued divergence $\xi_v \in \Re$. The discrete velocity 1-form is defined on directed edges as $vw \in E, \eta_v w = \hbar\langle \psi_v, \psi_w \rangle$ and stored in staggered grid fashion at the vertices
- Initialization of ψ : ψ_1 is initialized with the desired initial state, and ψ_2 is set to ϵ (a small value to guard against zeroes during normalization). We can add a vortex filament by constructing a complex function ψ_1 which has its zero set as the single filament curve γ. ψ for multiple filaments are just the componentwise product of single filament ψs. We can give initial velocities to different regions of the smoke by using the velocity constraint projection algorithm given, which enforces a plane wave in the given region. This can also be used to simulate obstacles.
- Overall time integration(time evolution of ψ) uses operator splitting. For each timestep we first perform a linear Schrodinger evolution requiring integration of Laplace term followed by normalization and pressure projection in order. Gravity and buoyancy can be included later and solved easily by integration and operator splitting.
- Once we simulate ψ we can obtain velocity vector field from it and can render the smoke using a technique used to render Eulerian fluids.
- The ISF simulation will be done in c++ using the fftw library for fourier transforms. The rendering of the smoke will be done in RenderMan or Mitsuba.

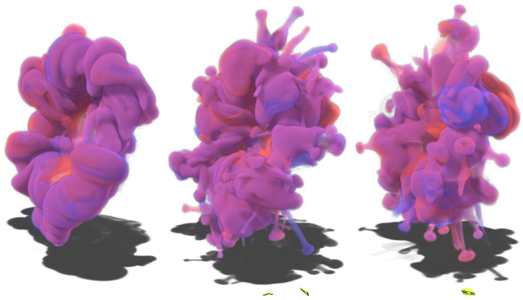

Figure 1: a) Eulerian b) Lagrangian c) Paper's method

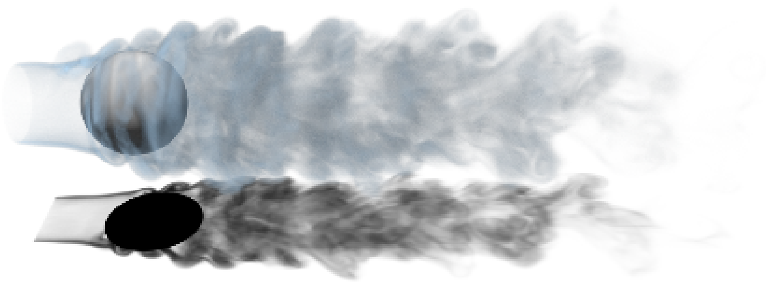

Figure 8: Frame 600 of a spherical obstacle of radius 0.4m *in a* 1m s^{-1} *flow.*

Figure 2: Stationary obstacle in a background flow

4 Result Summary

It can be seen in Fig. 1, Schrodinger's smoke simulation is comparable to Lagrangian method.The ISF simulation captures the vortical structures and their dynamics even at modest resolutions with fidelity rivaling purely Lagrangian methods.

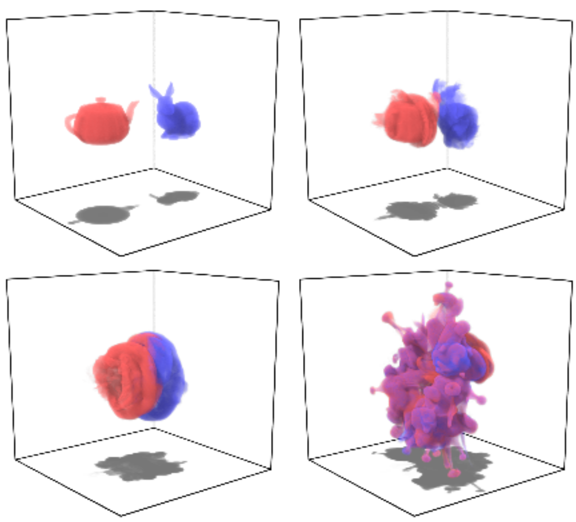

Figure 3: Bunny and teapot initialized with constant velocities. Shows inertial motion dynamics.

5 Challenges and Limitations

5.1 Challenges

- Since the rendering technique was not mentioned explicitly, we are facing the challenge of the rendering the smoke efficiently and accurately.
- The paper's foundation is based on higher level mathematics and physics and hence we couldn't understand many of the proofs and derivations clearly.

5.2 Limitations

- The simple splitting method we employ for time integration exhibits loss of kinetic energy. Even though its impact is not as problematic
- We do not know how to express arbitrary forces at the level of ψ.
- There is an upper bound on the velocity that can be represented without aliasing