Assignment - Study of the sensitivity of gradient descent and Newton methods

Objective:
The objective of this assignment is to study the sensitivity and behavior of the Gradient Descent and Newton methods in estimating perspective transformations. Both methods were applied with three different learning rates to observe their convergence and the quality of the fit.

Methodology:
Gradient Descent and Newton Methods were employed to estimate perspective transformations.
Three different learning rates were experimented with for each method to analyze the convergence and fit quality.
The quality of the fit was studied visually and quantitatively in terms of final residual values.

Observations:

Newton's Method:
Learning Rate: 1
Residual Error: 0.4319
Converged rapidly, achieving a lower residual error.
Transformation Matrix (T) and Inverse Transformation (T_inv) showcased substantial deviation from the initial parameters.

Learning Rate: 0.1
Residual Error: 0.4319
Similar convergence and residual error as with a learning rate of 1, indicating good performance to the learning rate in this range.

Learning Rate: 0.01
Residual Error: 0.4319
Achieved similar convergence and residual error as the higher learning rates, showing consistency in performance.

Gradient Descent Method:
Learning Rate: $1\times10-8$
Residual Error: 0.9501
Exhibited similar performance as the other learning rates, indicating lesser sensitivity to learning rate variations in this range.

Learning Rate: $1\times10-9$
Residual Error: 0.9501
Maintained a consistent residual error and convergence pattern with a slight change in the learning rate.

Learning Rate: $1\times10-10$
Residual Error: 0.9501
Exhibited similar performance as the other learning rates, indicating lesser sensitivity to learning rate variations in this range.

Analysis:

Convergence Speed:
Newton's Method converged more quickly and effectively compared to the Gradient Descent Method across different learning rates.
The rapid convergence of Newton's method can be attributed to its utilization of second-order information (Hessian), enabling more informed updates.

Residual Error:
Newton's Method achieved lower residual errors consistently across different learning rates, indicating a more accurate fit.
The Gradient Descent Method, while consistent across learning rates, retained a higher residual error, implying a less accurate fit.

Sensitivity to Learning Rate:
Newton's method exhibited robustness and consistency across varying learning rates, maintaining similar residual errors and convergence patterns.
The Gradient Descent Method also displayed consistent performance across different learning rates but was less effective in reducing the residual error.

Conclusion:
I analyzed the Newton and Gradient Descent methods for their sensitivity and effectiveness in estimating perspective transformations. Newton's method showcased superior performance in terms of rapid convergence and lower residual errors, demonstrating its effectiveness in achieving accurate fits. Both methods displayed consistent performance across different learning rates, with Newton's method being less sensitive to learning rate variations and more effective in achieving precise estimations. The comparative analysis provided valuable insights into the characteristics and applicability of these techniques in computer vision tasks.