



Bayesian Inference

Naive Bayes Models

- Sometimes, a single cause directly influences a number of effects, all of which are conditionally independent, given the cause
- The full joint distribution can be written as

$$P(Cause, Effect_1, \dots, Effect_n) = P(Cause) \prod_i P(Effect_i | Cause)$$

Ghostbusters

- A ghost is in the grid somewhere
- Sensor readings tell how close a square is to the ghost (Manhattan distance)
 - On the ghost: red
 - 1 or 2 away: orange
 - 3 or 4 away: yellow
 - 5+ away: green
- Sensors are noisy, but we know $P(\text{Color} \mid \text{Distance})$

$P(\text{red} \mid 3)$	$P(\text{orange} \mid 3)$	$P(\text{yellow} \mid 3)$	$P(\text{green} \mid 3)$
0.05	0.15	0.5	0.3



Ghostbusters

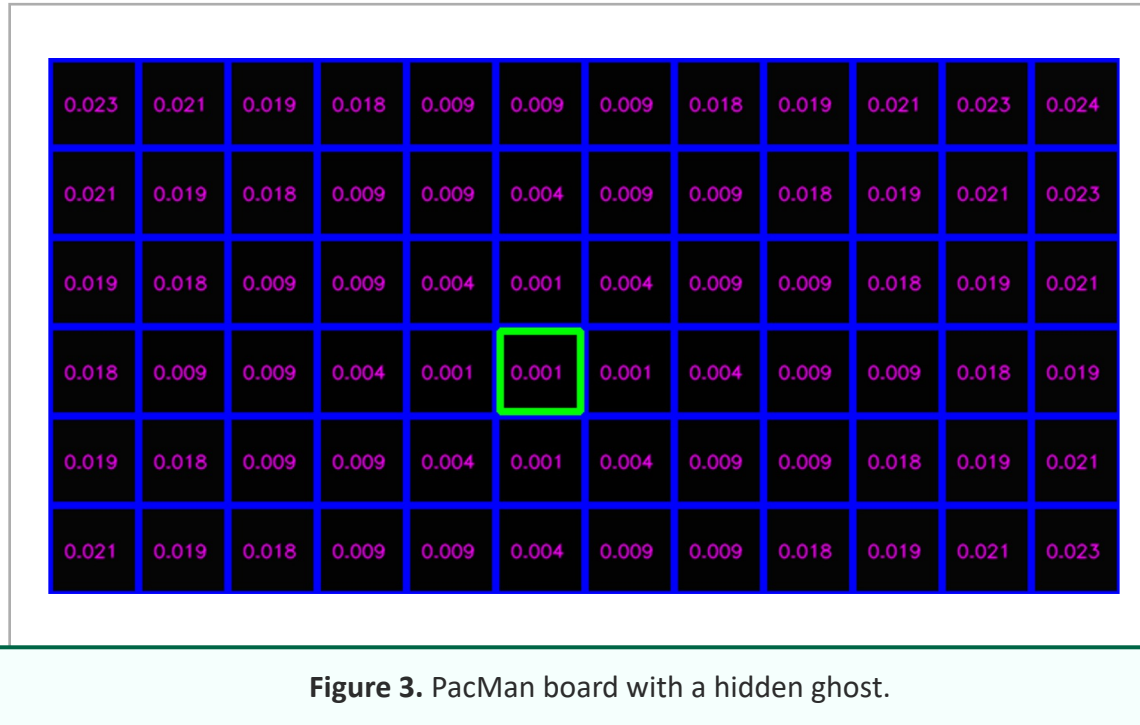
- Prior distribution over ghost location: $P(G)$
 - Let's say this uniform: $P(G=(i,j)) = 1/N$, with N being the number of grid cells



Figure 2. PacMan board with a hidden ghost.

Ghostbusters

- Sensor reading model: $P(R \mid G)$
 - We know what our sensors do (e.g. $P(R=\text{green} \mid G=(0,0))$)



Ghostbusters

- We don't know where the ghost is located, so we will update the probability of each position being the ghost location

$$P(G = (i, j) \mid R_{4,6} = \text{green}) = \frac{P(R_{4,6} = \text{green} \mid G = (i, j)) P(G = (i, j))}{P(R_{4,6} = \text{green})}$$

Ghostbusters

- Complete sensor model

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
G	0.05	0.05	0.15	0.30	0.30	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95	1.00	1.00	1.00
Y	0.05	0.15	0.25	0.50	0.55	0.30	0.25	0.25	0.25	0.20	0.15	0.15	0.10	0.05	0.00	0.00	0.00
O	0.15	0.55	0.50	0.15	0.10	0.10	0.10	0.05	0.05	0.05	0.05	0.00	0.00	0.00	0.00	0.00	0.00
R	0.75	0.25	0.10	0.05	0.05	0.05	0.05	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Figure 4. Complete sensor model

Knowledge Check 1



What is the probability of the reading $R_{3,5}$ at row 3, column 5 returning green given that the ghost is located at row 2, column 4?

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
G	0.05	0.05	0.15	0.30	0.30	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95	1.00	1.00	1.00
Y	0.05	0.15	0.25	0.50	0.55	0.30	0.25	0.25	0.25	0.20	0.15	0.15	0.10	0.05	0.00	0.00	0.00
O	0.15	0.55	0.50	0.15	0.10	0.10	0.10	0.05	0.05	0.05	0.05	0.00	0.00	0.00	0.00	0.00	0.00
R	0.75	0.25	0.10	0.05	0.05	0.05	0.05	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Figure 5. Complete sensor model

A

$$P(R_{3,5} = \text{green} \mid G = (2,4)) = 15\%$$

B

$$P(R_{3,5} = \text{green} \mid G = (2,4)) = 25\%$$

C

$$P(R_{3,5} = \text{green} \mid G = (2,4)) = 30\%$$

D

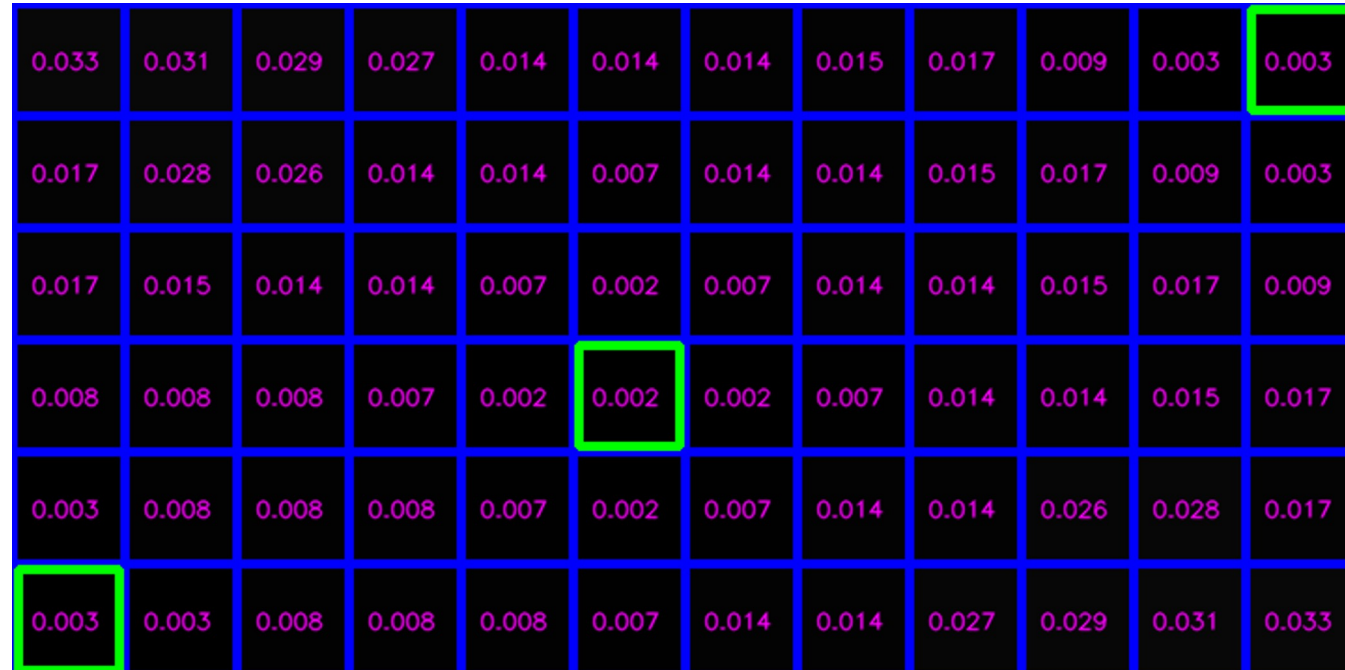
$$P(R_{3,5} = \text{green} \mid G = (2,4)) = 5\%$$

Ghostbusters

0.032	0.028	0.024	0.020	0.010	0.009	0.008	0.008	0.009	0.005	0.001	0.002
0.031	0.027	0.023	0.012	0.011	0.005	0.009	0.008	0.008	0.009	0.005	0.001
0.031	0.026	0.013	0.013	0.006	0.001	0.005	0.009	0.008	0.008	0.009	0.005
0.029	0.015	0.014	0.006	0.002	0.002	0.001	0.005	0.009	0.008	0.008	0.009
0.032	0.029	0.015	0.014	0.006	0.002	0.006	0.011	0.010	0.017	0.017	0.010
0.035	0.032	0.029	0.015	0.014	0.006	0.013	0.012	0.020	0.021	0.021	0.020

Figure 6. PacMan board with a hidden ghost

Ghostbusters



0.033	0.031	0.029	0.027	0.014	0.014	0.014	0.015	0.017	0.009	0.003	0.003
0.017	0.028	0.026	0.014	0.014	0.007	0.014	0.014	0.015	0.017	0.009	0.003
0.017	0.015	0.014	0.014	0.007	0.002	0.007	0.014	0.014	0.015	0.017	0.009
0.008	0.008	0.008	0.007	0.002	0.002	0.002	0.007	0.014	0.014	0.015	0.017
0.003	0.008	0.008	0.008	0.007	0.002	0.007	0.014	0.014	0.026	0.028	0.017
0.003	0.003	0.008	0.008	0.008	0.007	0.014	0.014	0.027	0.029	0.031	0.033

Figure 7. PacMan board with a hidden ghost

Ghostbusters

0.002	0.002	0.007	0.012	0.007	0.012	0.013	0.015	0.019	0.011	0.004	0.005
0.001	0.006	0.012	0.007	0.012	0.007	0.015	0.015	0.017	0.021	0.013	0.005
0.004	0.007	0.006	0.012	0.007	0.002	0.008	0.017	0.018	0.020	0.024	0.014
0.003	0.004	0.007	0.006	0.002	0.002	0.002	0.009	0.019	0.020	0.022	0.027
0.001	0.007	0.008	0.008	0.007	0.002	0.009	0.020	0.021	0.040	0.045	0.028
0.002	0.002	0.008	0.009	0.009	0.009	0.019	0.021	0.041	0.046	0.050	0.052

Figure 8. PacMan board with a hidden ghost

Ghostbusters

0.000	0.000	0.000	0.000	0.000	0.000	0.004	0.004	0.005	0.003	0.002	0.003
0.000	0.000	0.000	0.000	0.000	0.002	0.004	0.004	0.005	0.013	0.008	0.003
0.000	0.000	0.000	0.000	0.002	0.000	0.002	0.005	0.010	0.012	0.014	0.013
0.000	0.000	0.000	0.002	0.000	0.000	0.000	0.005	0.011	0.012	0.020	0.082
0.000	0.000	0.002	0.002	0.002	0.000	0.005	0.012	0.012	0.036	0.135	0.093
0.000	0.000	0.002	0.002	0.002	0.005	0.011	0.012	0.037	0.141	0.166	0.047

Figure 9. PacMan board with a hidden ghost

Ghostbusters

0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.003	0.002	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.003	0.003	0.006	0.004
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.003	0.005	0.030	0.037
0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.003	0.005	0.054	0.223	0.139
0.000	0.000	0.000	0.000	0.000	0.001	0.003	0.005	0.055	0.233	0.074	0.078

Figure 10. PacMan board with a hidden ghost

Ghostbusters

0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.002	0.001	0.001
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.008	0.013	0.011	0.011
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.008	0.090	0.337	0.063	0.063
0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.008	0.092	0.105	0.123	0.118	0.118

Figure 11. PacMan board with a hidden ghost

Ghostbusters

0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.002	0.009	0.003	0.003
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.002	0.063	0.708	0.044	0.044
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.012	0.029	0.086	0.033	0.033

Figure 12. PacMan board with a hidden ghost

Knowledge Check 2



Can we stop sensing the environment and assume the ghost is located at row 5 column 11?

	1	2	3	4	5	6	7	8	9	10	11	12
1	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
2	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
4	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.002	0.009	0.003
5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.002	0.063	0.708	0.044
6	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.012	0.029	0.086	0.033

Figure 13. PacMan board with a hidden ghost

A

Yes, as $P(G = (5,11)) > 50\%$

B

No, because our sensor is too noisy

C

Only if 70.8% is high enough for our application purpose

Ghostbusters

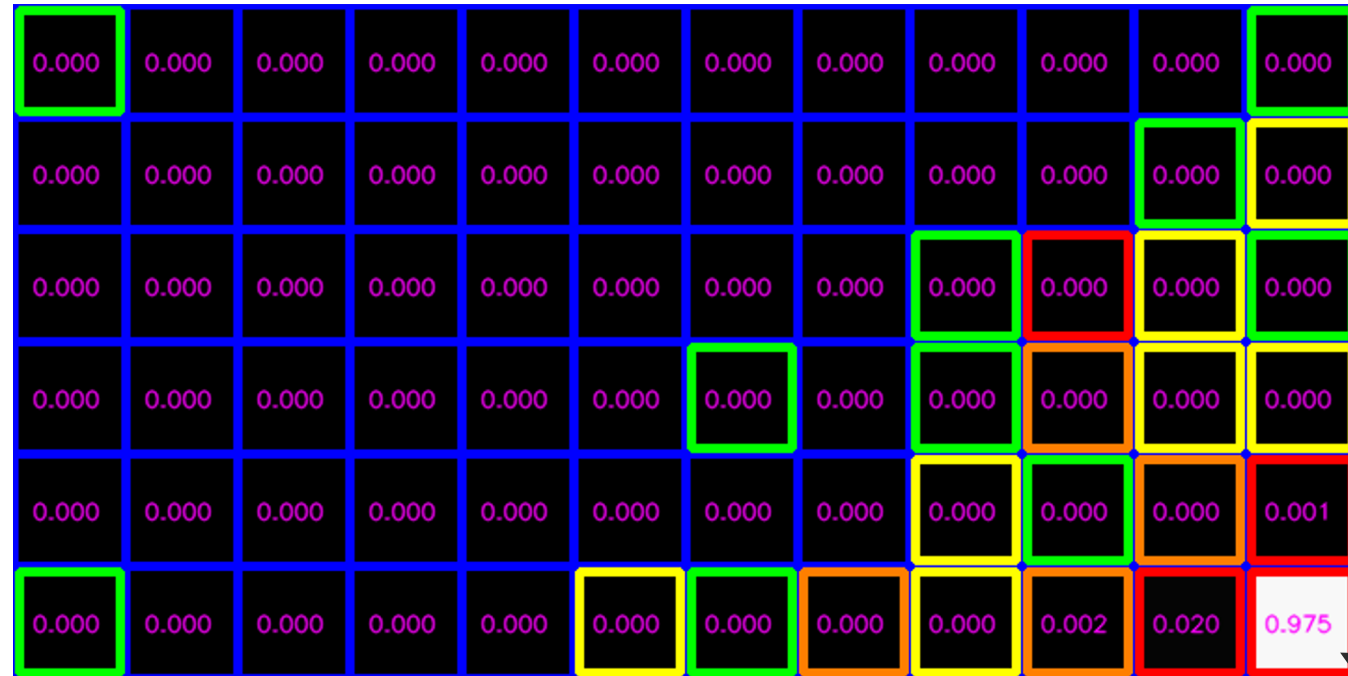


Figure 14. PacMan board with a hidden ghost. The noisier the sensor, the more measurements are needed to reduce the uncertainty.



You have reached the end
of the lecture.



Image/Figure References

Figure 2-3. PacMan board with a hidden ghost

Figure 4-5. Complete sensor model.

Figure 6-13. PacMan board with a hidden ghost

Figure 14. PacMan board with a hidden ghost. The noisier the sensor, the more measurements are needed to reduce the uncertainty.

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