Bayesian Inference

AI+X
INSTITUTE FOR ARTIFICIAL INTELLIGENCE

Naive Bayes Models

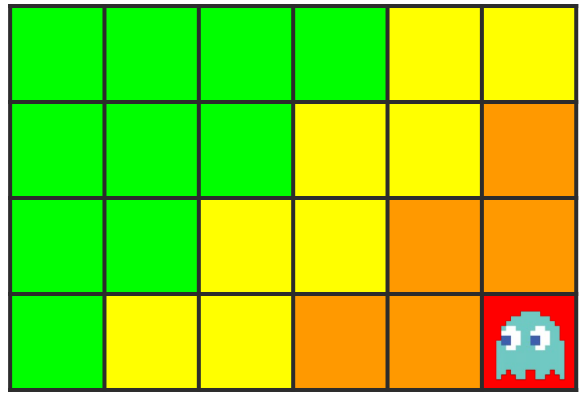
- Sometimes, a single cause directly influences a number of effects, all of which are conditionally independent, given the cause
- The full joint distribution can be written as

$$P(Cause, Effect_1, \dots, Effect_n) = P(Cause) \prod_i P(Effect_i | Cause)$$

Ghostbusters

- A ghost is in the grid somewhere
- Sensor readings tell how close a square is to the ghost (Manhattan distance)
  - On the ghost: red
  - 1 or 2 away: orange
  - 3 or 4 away: yellow
  - 5+ away: green
- Sensors are noisy, but we know P(Color | Distance)

P(red | 3)	P(orange | 3)	P(yellow | 3)	P(green | 3)
0.05	0.15	0.5	0.3



Ghostbusters

- Prior distribution over ghost location: P(G)
  - Let's say this uniform: P(G=(i,j)) = 1/N, with N being the number of grid cells

Figure 2. PacMan board with a hidden ghost.

Ghostbusters

- Sensor reading model: P(R | G)
  - We know what our sensors do (e.g. P(R=green | G=(0,0))

Figure 3. PacMan board with a hidden ghost.

Ghostbusters

- We don’t know where the ghost is located, so we will update the probability of each position being the ghost location

$$P(G = (i,j) \mid R_{4,6} = \text{green}) = \frac{P\left(R_{4,6} = \text{green} \,\middle|\, G = (i,j)\right) P(G = (i,j))}{P(R_{4,6} = \text{green})}$$

Ghostbusters

- Complete sensor model

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
G	0.05	0.05	0.15	0.30	0.30	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95	1.00	1.00	1.00
Y	0.05	0.15	0.25	0.50	0.55	0.30	0.25	0.25	0.25	0.20	0.15	0.15	0.10	0.05	0.00	0.00	0.00
O	0.15	0.55	0.50	0.15	0.10	0.10	0.10	0.05	0.05	0.05	0.05	0.00	0.00	0.00	0.00	0.00	0.00
R	0.75	0.25	0.10	0.05	0.05	0.05	0.05	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Figure 4. Complete sensor model

Knowledge Check 1

What is the probability of the reading $R_{3,5}$ at row 3, column 5 returning green given that the ghost is located at row 2, column 4?

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
G	0.05	0.05	0.15	0.30	0.30	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95	1.00	1.00	1.00
Y	0.05	0.15	0.25	0.50	0.55	0.30	0.25	0.25	0.25	0.20	0.15	0.15	0.10	0.05	0.00	0.00	0.00
O	0.15	0.55	0.50	0.15	0.10	0.10	0.10	0.05	0.05	0.05	0.05	0.00	0.00	0.00	0.00	0.00	0.00
R	0.75	0.25	0.10	0.05	0.05	0.05	0.05	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Figure 5. Complete sensor model

A $P(R_{3,5} = \text{green} \mid G = (2,4)) = 15\%$

B $P(R_{3,5} = \text{green} \mid G = (2,4)) = 25\%$

C $P(R_{3,5} = \text{green} \mid G = (2,4)) = 30\%$

D $P(R_{3,5} = \text{green} \mid G = (2,4)) = 5\%$

Ghostbusters

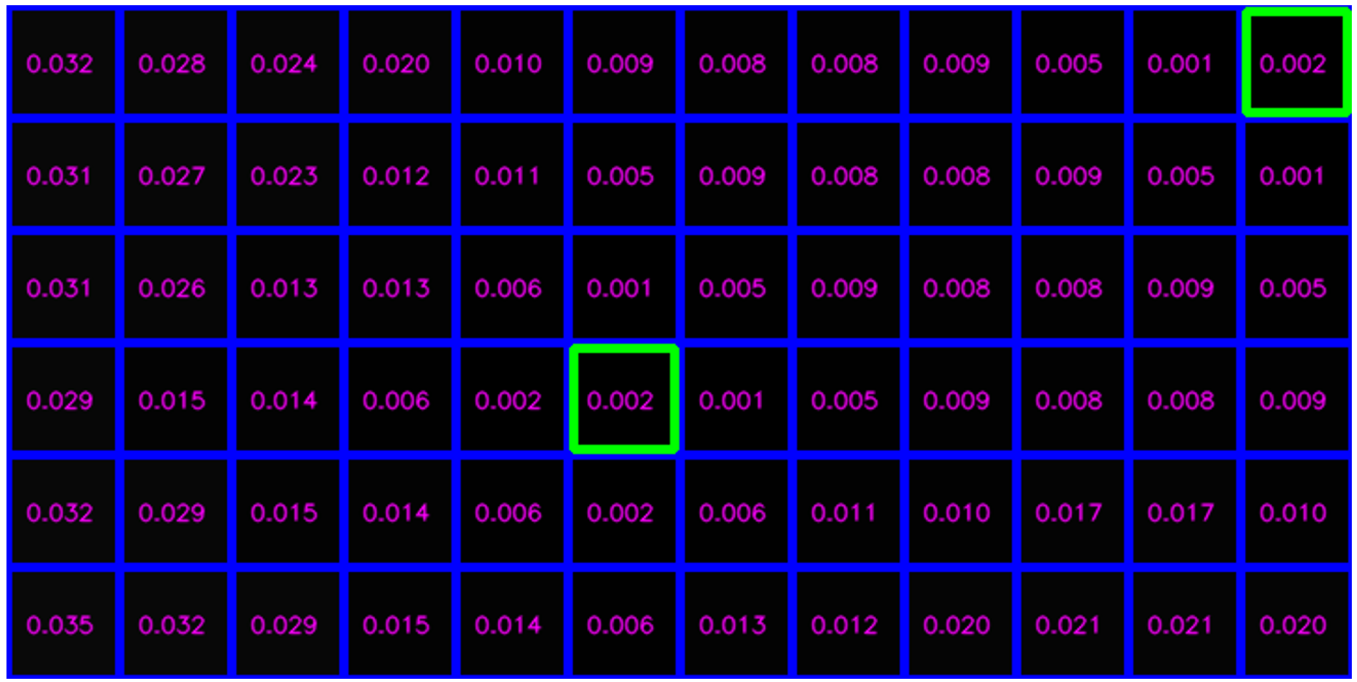


Figure 6. PacMan board with a hidden ghost

Ghostbusters

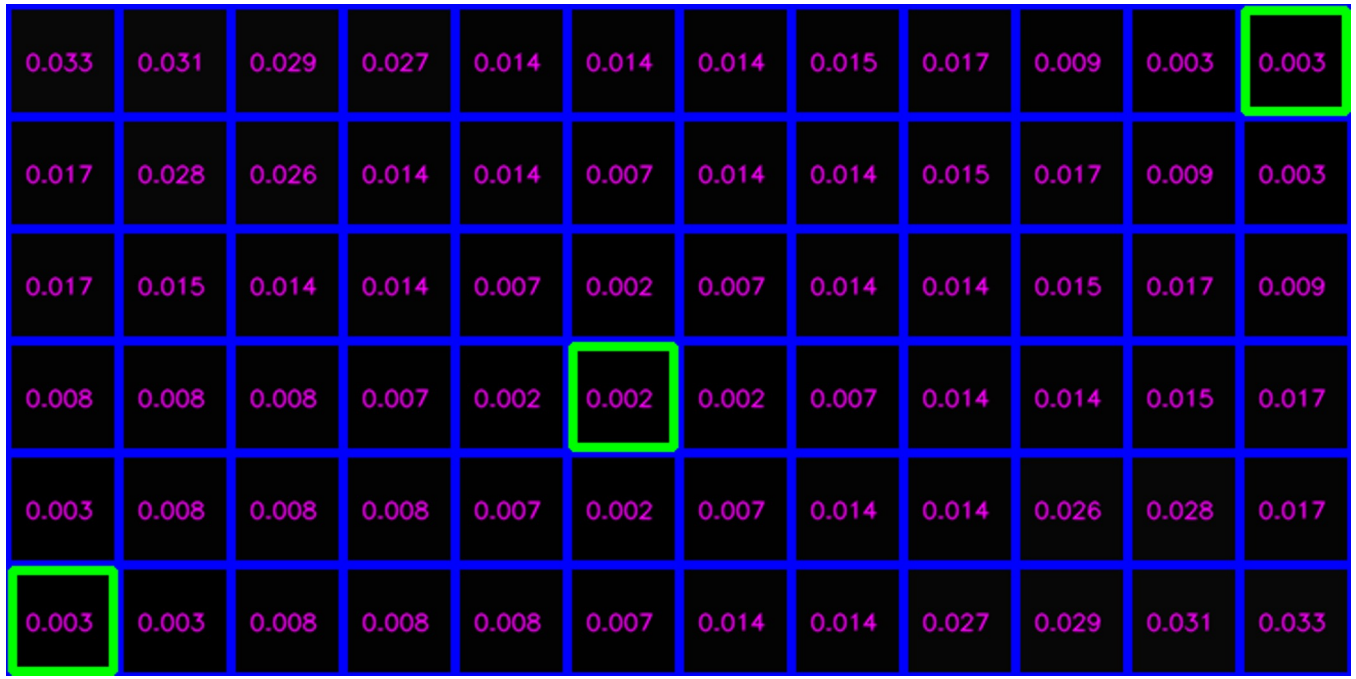


Figure 7. PacMan board with a hidden ghost

Ghostbusters

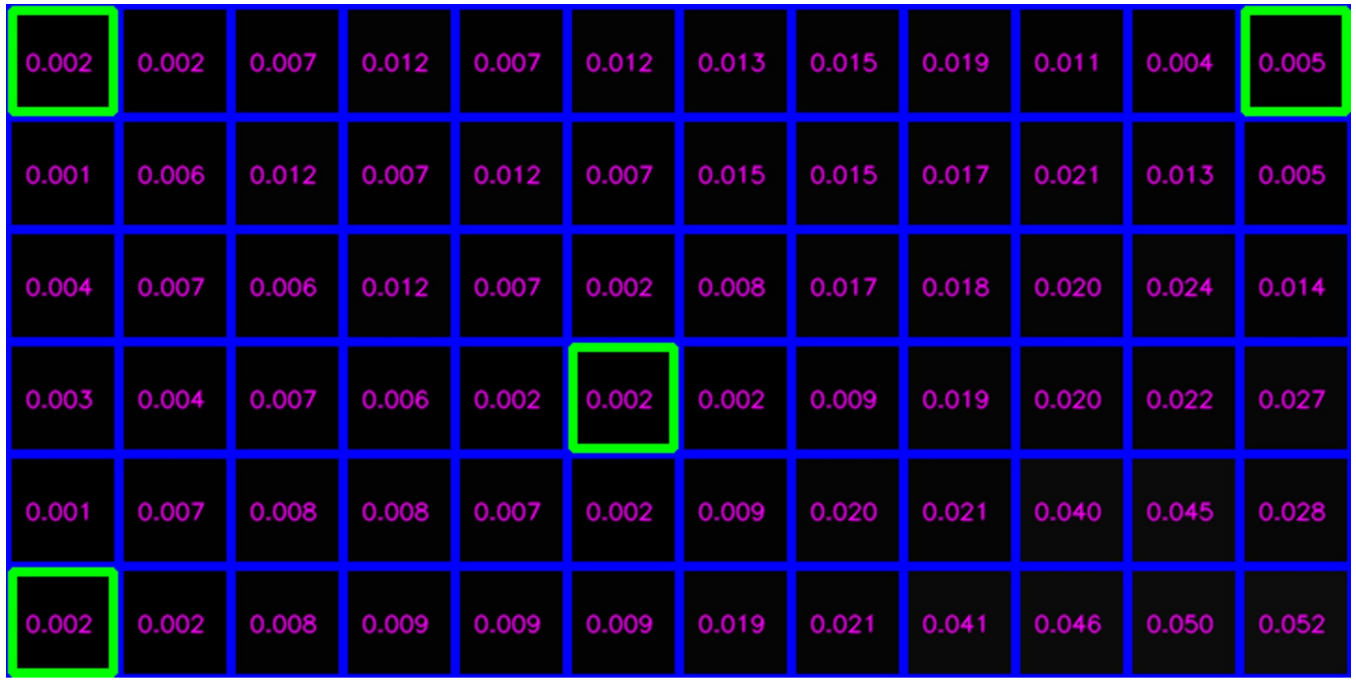


Figure 8. PacMan board with a hidden ghost

Ghostbusters

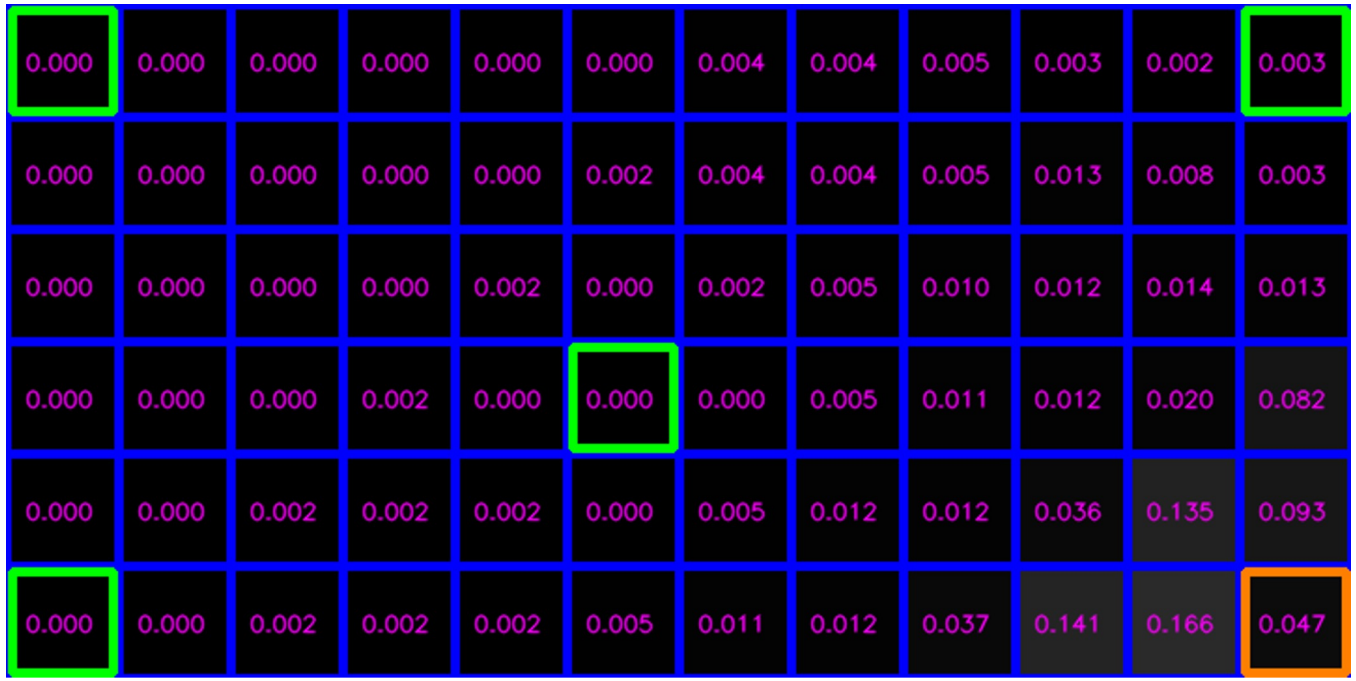


Figure 9. PacMan board with a hidden ghost

Ghostbusters

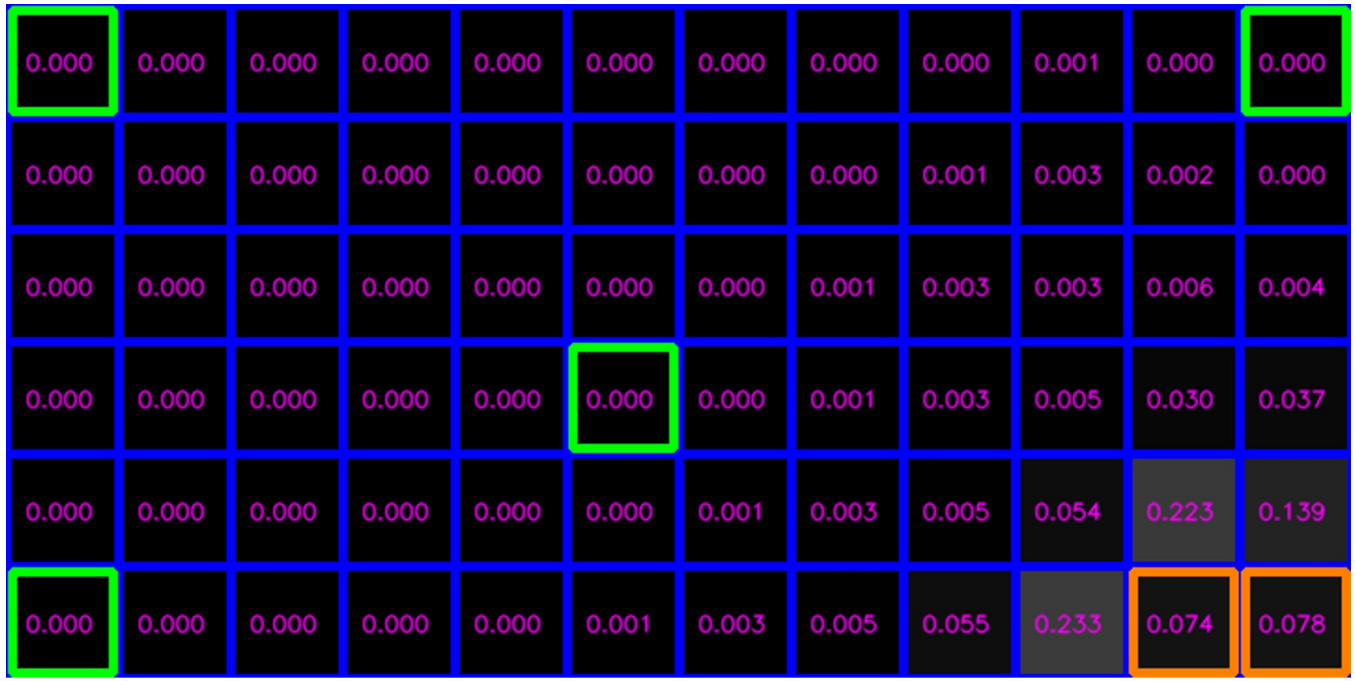


Figure 10. PacMan board with a hidden ghost

Ghostbusters

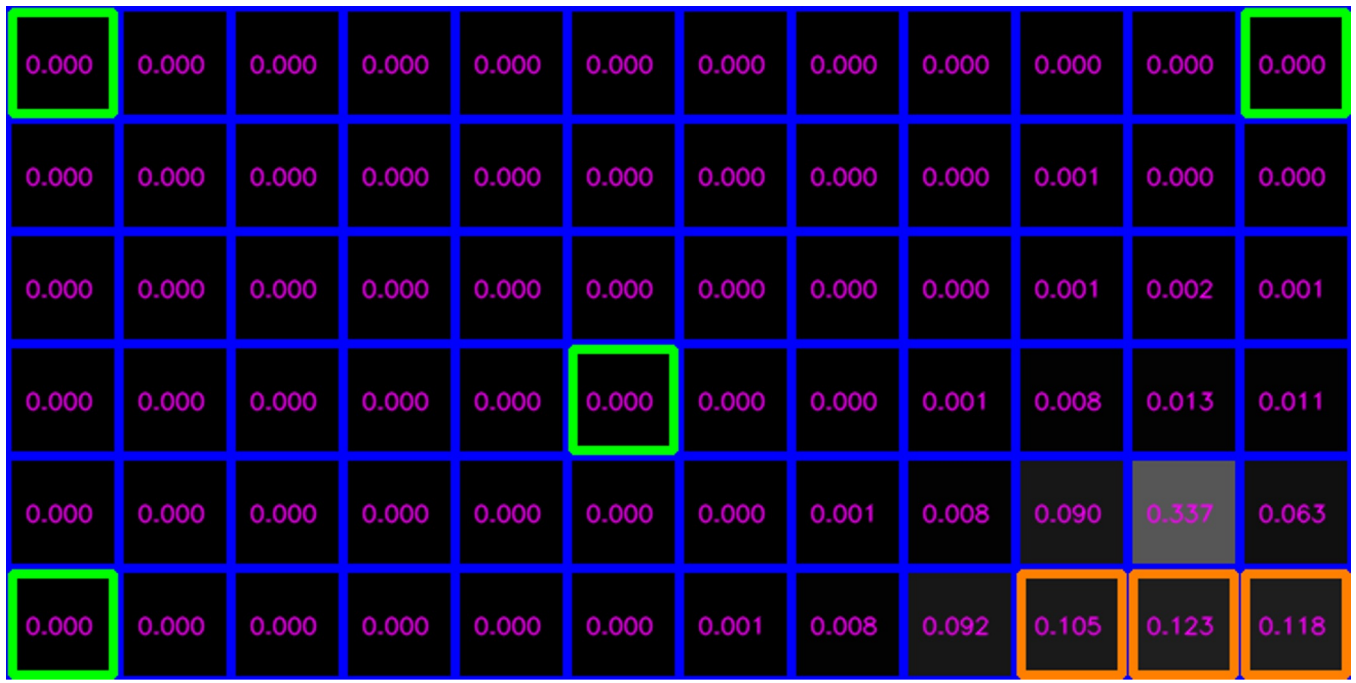


Figure 11. PacMan board with a hidden ghost

Ghostbusters

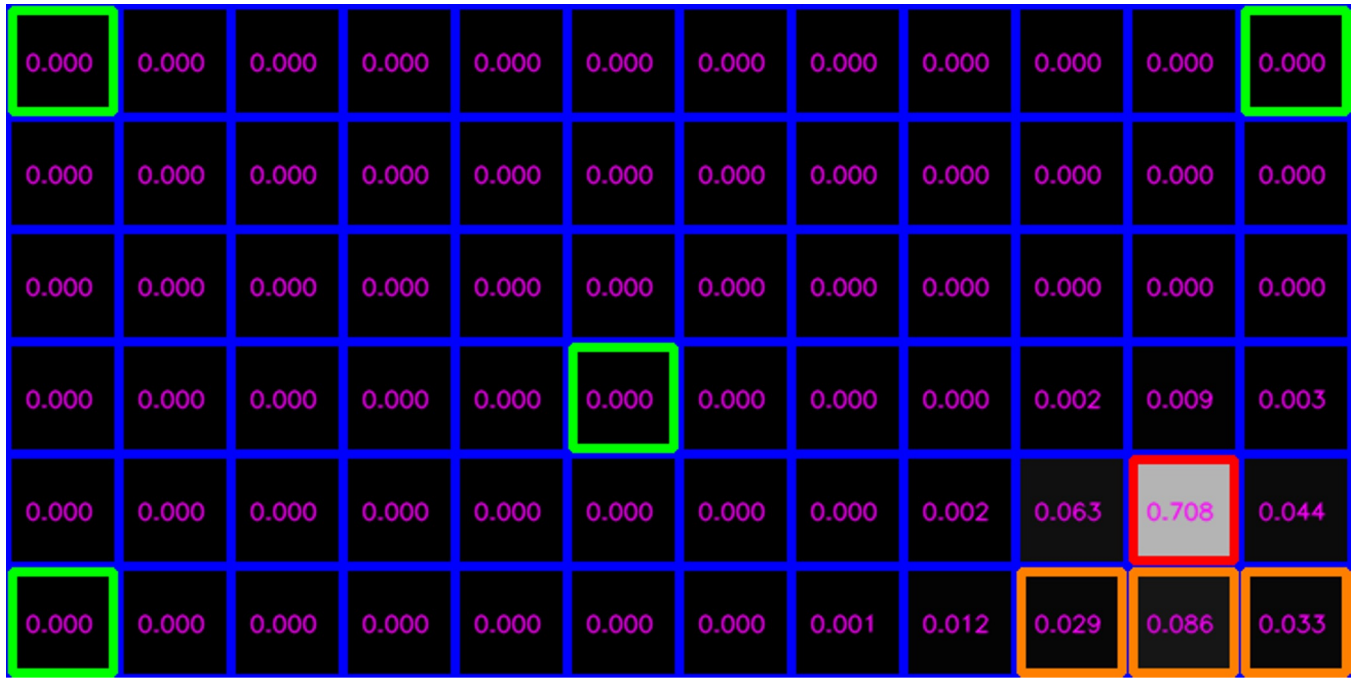


Figure 12. PacMan board with a hidden ghost

Knowledge Check 2

Can we stop sensing the environment and assume the ghost is located at row 5 column 11?

Figure 13. PacMan board with a hidden ghost

A. Yes, as P(G = (5,11)) > 50%

B. No, because our sensor is too noisy

C. Only if 70.8% is high enough for our application purpose

Ghostbusters

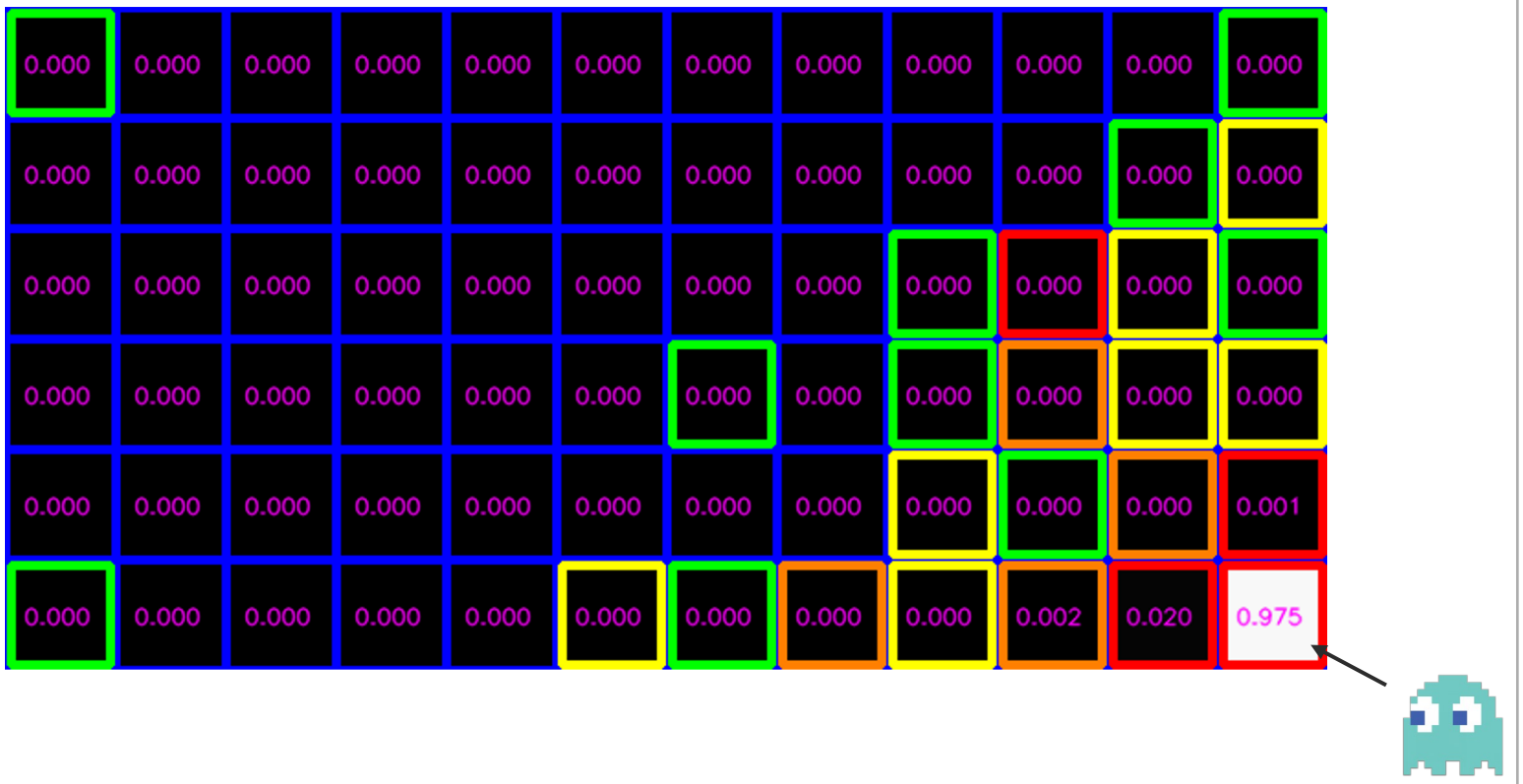


Figure 14. PacMan board with a hidden ghost. The noisier the sensor, the more measurements are needed to reduce the uncertainty.

You have reached the end of the lecture.

Image/Figure References

Figure 2-3. PacMan board with a hidden ghost
Figure 4-5. Complete sensor model.
Figure 6-13. PacMan board with a hidden ghost
Figure 14. PacMan board with a hidden ghost. The noisier the sensor, the more measurements are needed to reduce the uncertainty.
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