

CSCI 516 - Fundamental Concepts in Computing and Machine Organization

Homework Assignment 3

Due on 11/01/2022 (Tuesday), 11:55PM

Requirement

- You need to do your assignment independently.
- You need to submit ".pdf" file which contains your solutions to D2L

Questions

1. Describe with an example how addition, subtraction, multiplication, and division is done in CPU using the binary number system.

2. Convert each of the following decimal values to IEEE-754 single and IEEE-754 double precision representation. Write your converted result in hexadecimal format. Clearly show all the steps.

- 3.75
- -12.5

3. Convert each of the following IEEE-754 floating point representation to decimal values. Clearly show all the steps.

- 0x40200000
- 0xC1080000

1. Addition:

$$\begin{array}{r} 0011010 + 001100 \\ = 00100110 \end{array}$$

Calculations:

Carry

$$\begin{array}{r} \textcircled{11} \\ 0011010 \\ + 0001100 \\ \hline 0100110 \end{array}$$

Subtraction:

$$\begin{array}{r} 0011010 \\ - 0001100 \\ \hline = 0001110 \end{array}$$

Calculations:

Borrow

$$\begin{array}{r} 001\cancel{1}010 \\ - 0001100 \\ \hline 0001110 \end{array}$$

Multiplication:

$$\begin{array}{r} 0011010 \times 0001100 \\ = 0100111000 \end{array}$$

Calculations:

$$\begin{array}{r} 0000000 \\ 0000000\times \\ 0011010\times\times \\ 0011010\times\times\times \\ \hline 0100111000 \end{array}$$

Division:

$$\begin{array}{r} 000110 \div 101010 \\ = 111 \end{array}$$

Calculations:

$$\begin{array}{r} 111 \\ \hline 000110 \sqrt{101010} \\ - 110 \\ \hline + 001 \\ - 110 \\ \hline 110 \\ - 110 \\ \hline 0 \end{array}$$

2.

Calculations :

3.75 to Hexadecimal

$$= 0x40700000$$

Since the number is positive, the sign bit is 0.

After normalizing the number
 $3.75 = 1.111 \times 2^1$

$$\begin{aligned} \text{Exponent} &= 1 + 127 (\text{bias}) \\ &= 10000000 \end{aligned}$$

Mantissa =

11100000000000000000000000000000

Therefore the binary number is

0100 0000 0111 0000 0000 0000 0000 0000

Which gives us a hexadecimal of

$$0x40700000$$

-12.5 to Hexadecimal

$$= 0xc1480000$$

After normalizing the number

$$-12.5 = 1.10000101001 \times 2^3$$

$$\text{Exponent} = 3 + 127 (\text{bias}) = 10000010$$

Calculations:

Since the number is negative the sign bit is 1.

Mantissa = 10010000000000000000000000000000

Therefore the binary number is

$$110000010100100000000000$$

Which gives us a hexadecimal

of

$$0xc1480000$$

3.

$0x40200000$ to decimal
= 2.5

Calculations:
Convert hexadecimal
to binary

4 | 0 | 2 | 0 | 0 | 0 | 0 | 0
0100 | 0000 | 0010 | 0000 | 0000 | 0000 | 0000 | 0000

IEEE 754 format is

Sign	Exponent	Mantissa
1 bit	8 bits	23 bits

Convert exponent to decimal

$$\Rightarrow 10000000 = 128$$

It is value implicit since $1 \leq \text{Exponent} \leq 254$

Implicit form is $(-1)^s (1.M) \times 2^{E-127}$
 $s = \text{sign bit}$

$0x C1080000$ to decimal Calculations:
 $= -8.5$ Convert hexadecimal
 to binary

C	1	0	8	0	0	0	0
1100	0001	0000	1000	0000	0000	0000	0000

IEEE 754 format is

Sign	Exponent	Mantissa
1 bit	8 bits	23 bits

Convert exponent to decimal

$$\Rightarrow 10000010 = 130$$

It is value implicit since $1 \leq \text{Exponent} \leq 254$

Implicit form is $(-1)^s (1.M) \times 2^{E-127}$
 $s = \text{sign bit}$

$$(-1)^1 \times (1.001) \times 2^{130-127}$$

$$-1 \times [1 + 0 \times (\frac{1}{2}) \times 0 + (\frac{1}{2^2}) \times 1 + 0 \times \frac{1}{2^3} + 1 \times \frac{1}{2^4}] \times 2^3$$

$$- [1 + \frac{1}{16}] \times 2^3$$

$$-\frac{17}{18} \times 8 = -8.5$$