Extending Instance-based and Linear Models

AI+X
INSTITUTE FOR ARTIFICIAL INTELLIGENCE

Extending instance-based learning and linear models

- Instance-based learning
  - Pruning and reducing the number of exemplars
  - Weighted attributes
  - Generalized exemplars and distance functions
- Extending linear models
  - Support vector machines, kernel ridge regression, kernel perceptrons
  - Multilayer perceptrons and radial basis function networks
  - Gradient descent
- Numeric prediction with local linear models
  - Model Trees
  - Learning rule sets with model trees
  - Locally weighted linear regression

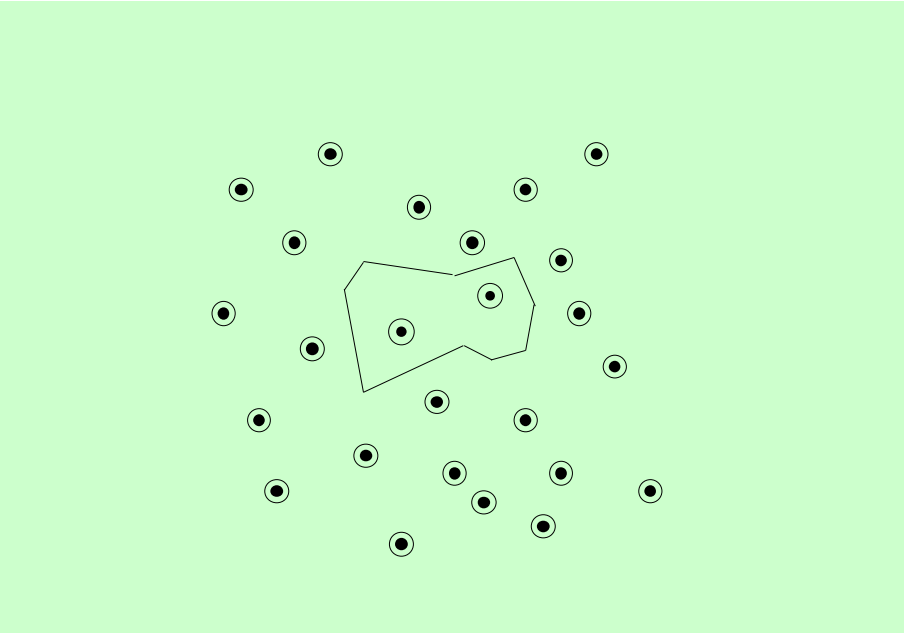
Instance Based Learning

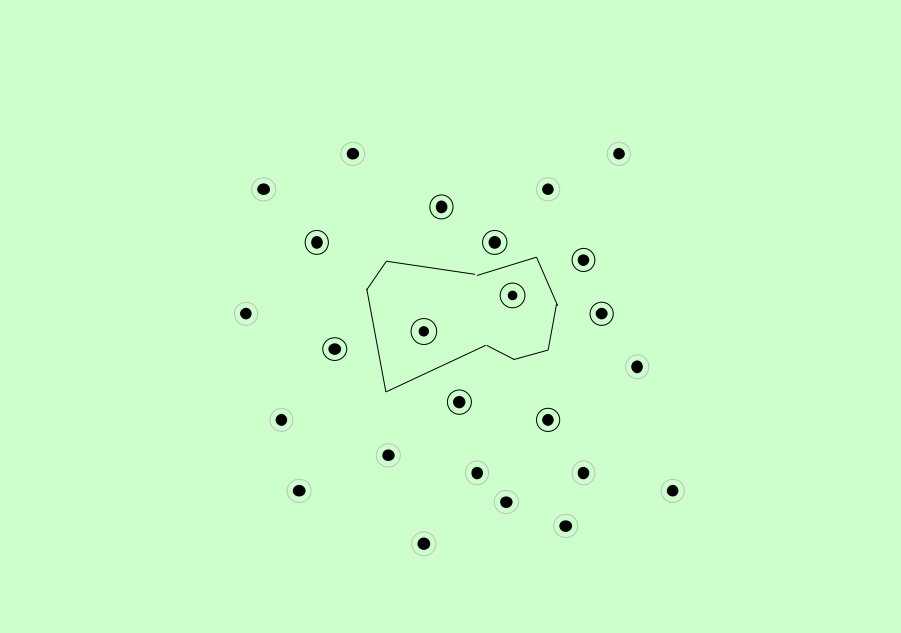
Instance-based learning

Practical problems of 1-nearest-neighbour scheme:

- Slow (but: fast tree-based approaches exist)
  - Remedy: remove irrelevant data
- Noise (but: *k* -NN copes quite well with noise)
  - Remedy: remove noisy instances
- All attributes deemed equally important
  - Remedy: weight attributes (or simply select)
- Doesn’t perform explicit generalization
  - Remedy: rule-based NN approach

Learning prototypes





- Only those instances involved in a decision need to be stored
- Noisy instances should be filtered out
- Idea: only use *prototypical* examples

Speed up classification, combat noise

- David Aha's IB2: save memory, speed up classification
  - Work incrementally
  - Only incorporate misclassified instances
  - Problem: noisy data gets incorporated
- David Aha's IB3: deal with noise
  - Discard instances that do not perform well
  - Compute confidence intervals for
    1. Each instance's success rate
    2. Default accuracy of the instance's class
  - Accept/reject instances according to performance
    1. Accept if lower limit of 1 exceeds upper limit of 2
    2. Reject if upper limit of 1 is below lower limit of 2

Weight attributes

- David Aha's IB4: weight each attribute (weights can be class-specific)
- Weighted Euclidean distance:

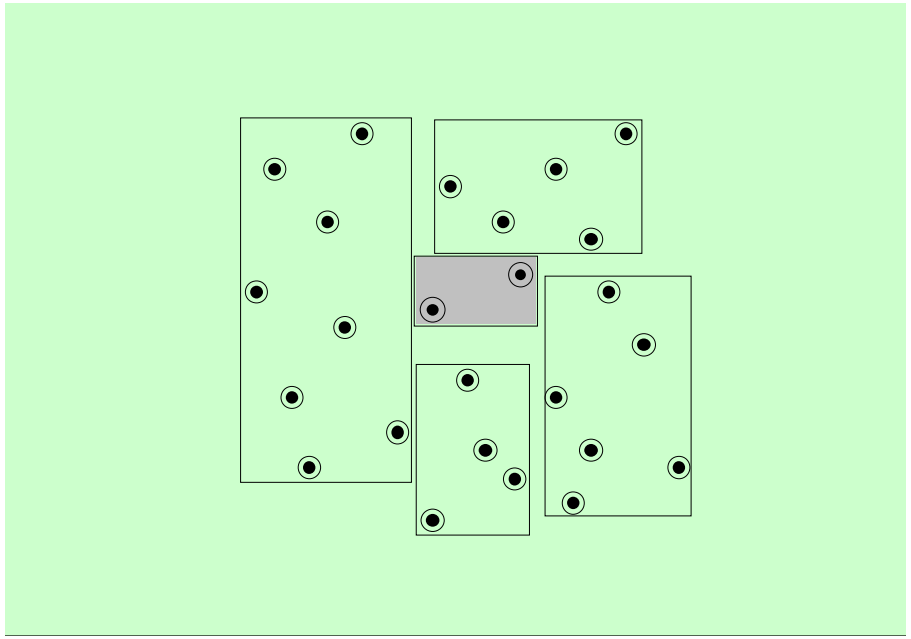
$$\sqrt{(w_1^2(x_1 - y_1)^2 + \ldots + w_n^2(x_n - y_n)^2)}$$

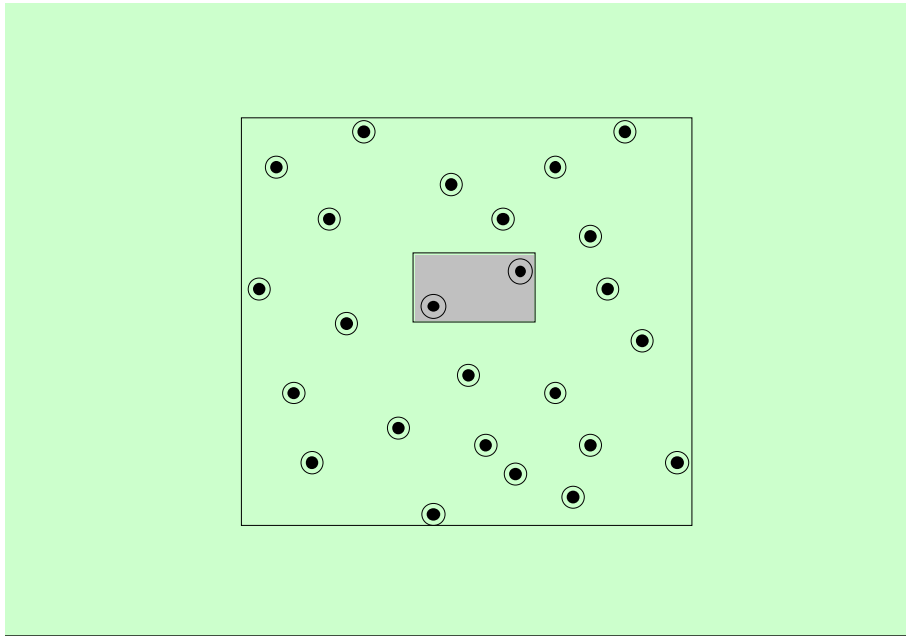
- Update weights based on nearest neighbor
  - Class correct: increase weight
  - Class incorrect: decrease weight
  - Amount of change for *i* th attribute depends on $|x_i - y_i|$

Generalized exemplars

- Generalize instances into *hyperrectangles*
  - Online: incrementally modify rectangles
  - Offline version: seek small set of rectangles that cover the instances
- Important design decisions:
  - Allow overlapping rectangles?
    - Requires conflict resolution
  - Allow nested rectangles?
  - Dealing with uncovered instances?

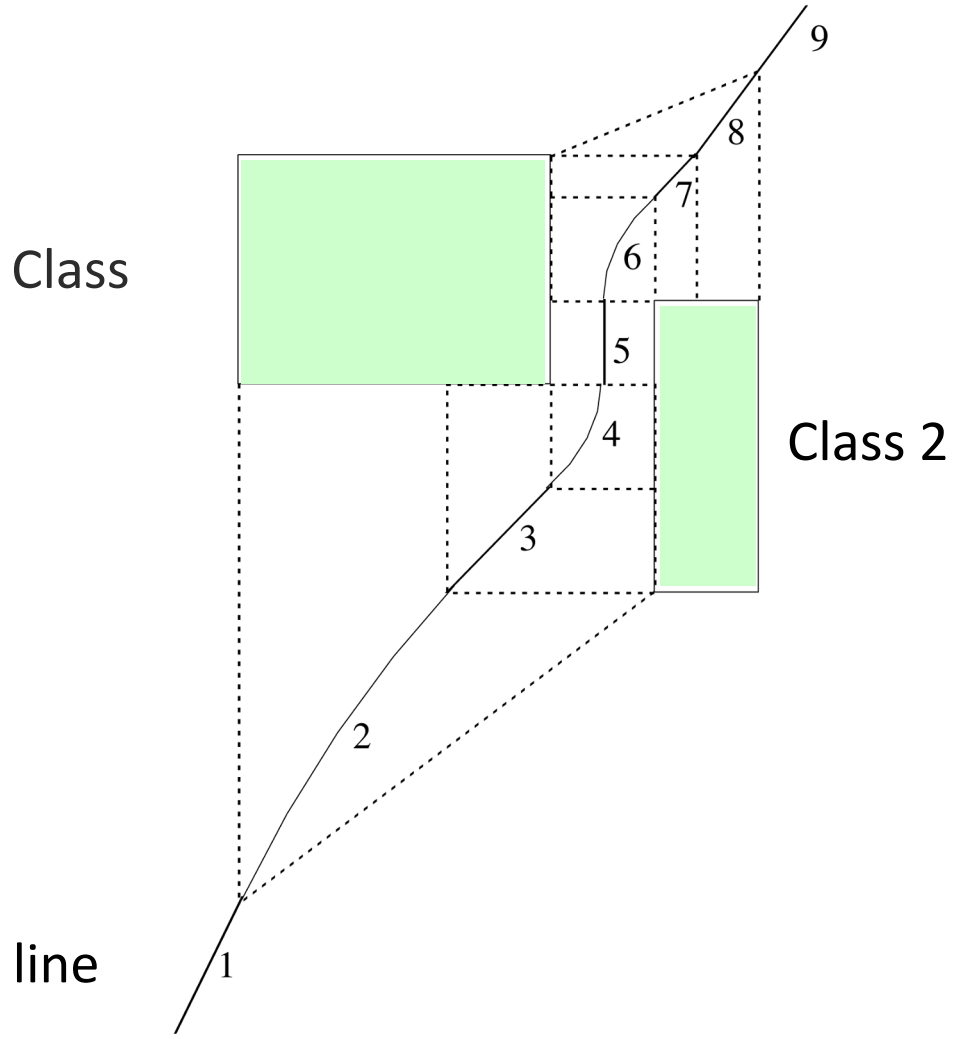
Rectangular generalizations





- Nearest-neighbor rule is used outside rectangles
- Rectangles are rules! (But they can be more conservative than “normal” rules.)
- Nested rectangles are rules with exceptions

Separating generalized exemplars



Generalized distance functions

- Problem with Euclidean distance, etc.: only natural for purely numeric datasets
- Transformation-based approach to designing distance functions can be applied more generally
- Given: some transformation operations on attributes
- *K* similarity* = probability of transforming instance A into B by chance
  - Average over all transformation paths
  - Weight paths according their probability *(need way of measuring this)*
- Uniform way of dealing with different attribute types
- Easily generalized to give distance between *sets* of instances

Discussion and Bibliographic Notes

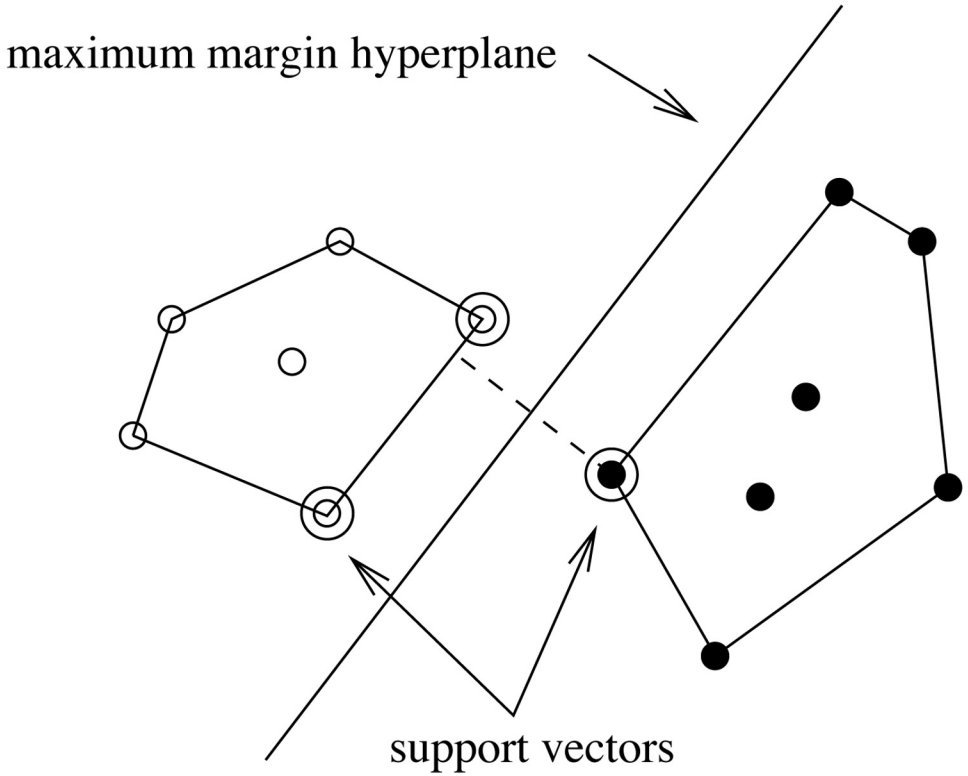
- Nearest-neighbor methods gained popularity in machine learning through the work of Aha (1992)
- Salzberg (1991) suggested that generalization with nested exemplars can achieve high classification accuracy
- Wettschereck and Dietterich (1994) argued that these results were fortuitous and did not hold in other domains
- Martin (1995) explored the idea that overgeneralization that occurs when hyperrectangles nest or overlap is problematic
- The generalized distance function based on transformations is described by Cleary and Trigg (1995)

Extending Linear Models

Support vector machines

- *Support vector machines* are algorithms for learning linear classifiers
- Resilient to overfitting because they learn a particular linear decision boundary:
  - The *maximum margin hyperplane*
- Fast in the nonlinear case
  - Use a mathematical trick to avoid creating “pseudo-attributes”
  - The nonlinear space is created implicitly

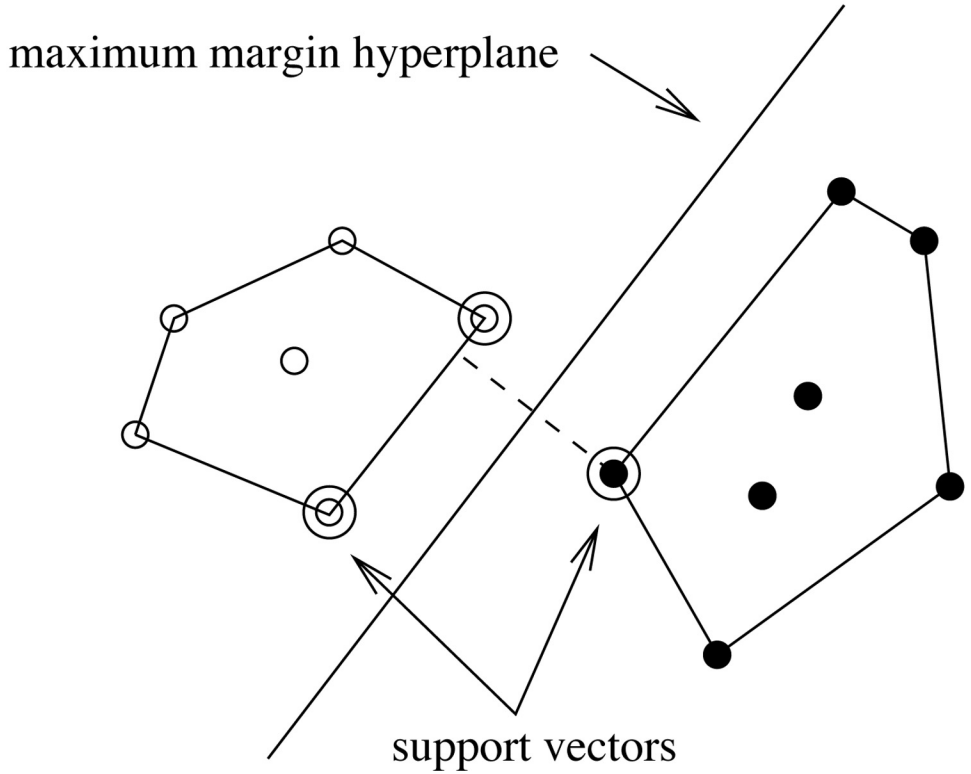
The maximum margin hyperplane



The instances closest to the maximum margin hyperplane are called *support vectors*

Support vectors

- The support vectors define the maximum margin hyperplane
- All other instances can be deleted without changing its position and orientation



- The hyperplane

$$x = w_0 + w_1 a_1 + w_2 a_2$$

can be written as

$$x = b + \sum_{i \text{ is a supp. vector}} \alpha_i y_i \vec{a}(i) \cdot \vec{a}$$

Finding support vectors

$$x = b + \sum_{i \text{ is a supp. vector}} \alpha_i y_i \vec{a}(i) \cdot \vec{a}$$

- Support vector: training instance for which $\alpha_i > 0$
- Determining α_i and b ?
- —A *constrained quadratic optimization* problem
  - Off-the-shelf tools for solving these problems
  - However, special-purpose algorithms are faster
  - Example: Platt's *sequential minimal optimization* (SMO) algorithm
- Note: the method discussed so far assumes separable data!

Nonlinear SVMs

- We can create a nonlinear classifier by creating new “pseudo” attributes from the original attributes in the data
  - “Pseudo” attributes represent attribute combinations
  - E.g.: all polynomials of degree 2 that can be formed from the original attributes
- We can learn a linear SVM from this extended data
- The linear SVM in the extended space is a non-linear classifier in the original attribute space
- Overfitting often not a significant problem with this approach because the maximum margin hyperplane is stable
  - There are often comparatively few support vectors relative to the size of the training set
- Computation time still an issue
  - Each time the dot product is computed, all the “pseudo attributes” must be included

A mathematical trick

- Avoid computing the “pseudo attributes”
- Compute the dot product before doing the nonlinear mapping
- Example: $$x = b + \sum_{i \text{ is a supp. vector}} \alpha_i y_i (\vec{a}(i) \cdot \vec{a})^n$$
- Corresponds to a map into the instance space spanned by all products of *n* attributes

Other kernel functions

- Mapping is called a “kernel function”
- Polynomial kernel

$$x = b + \sum_{i \text{ is a supp. vector}} \alpha_i y_i (\vec{a}(i) \cdot \vec{a})^n$$

- We can use others:

$$x = b + \sum_{i \text{ is a supp. vector}} \alpha_i y_i K(\vec{a}(i) \cdot \vec{a})$$

- Only requirement:

$$K(\vec{x}_i, \vec{x}_j) = \phi(\vec{x}_i) \cdot \phi(\vec{x}_j)$$

← *K*() can be written as a dot product in a feature space create by the implicit feature mapping *Φ*()

- Examples:

$$K(\vec{x}_i, \vec{x}_j) = (\vec{x}_i \cdot \vec{x}_j + 1)^d,$$

$$K(\vec{x}_i, \vec{x}_j) = \exp\left(\frac{-(\vec{x}_i - \vec{x}_j)^2}{2\sigma^2}\right)$$

$$K(\vec{x}_i, \vec{x}_j) = \tanh(\beta \vec{x}_i \cdot \vec{x}_j + b) \quad *$$

Noise

- Have assumed that the data is separable (in original or transformed space)
- Can apply SVMs to noisy data by introducing a "noise" parameter *C*
  - Also known as *regularization* parameter
- *C* bounds the influence of any one training instance on the decision boundary
  - Based on the following constraint: $0 \leq \alpha_i \leq C$
- A "soft" margin is maximized based on this constraint
- Still a quadratic optimization problem
- Have to determine *C* by experimentation

Sparse data

- SVM algorithms speed up dramatically if the data is *sparse* (i.e., many values are 0)
- Why? Because they compute lots and lots of dot products
- Sparse data **->** can compute dot products very efficiently
  - Iterate only over non-zero values
- SVMs can process sparse datasets with 10,000s of attributes

Support vector regression

- Maximum margin hyperplane only applies to classification
- However, idea of support vectors and kernel functions can be used for regression
- Basic method is the same as in linear regression: want to minimize error
- Difference A: ignore errors smaller than e and use absolute error instead of squared error
- Difference B: simultaneously aim to maximize flatness of function
- User-specified parameter e defines “tube”

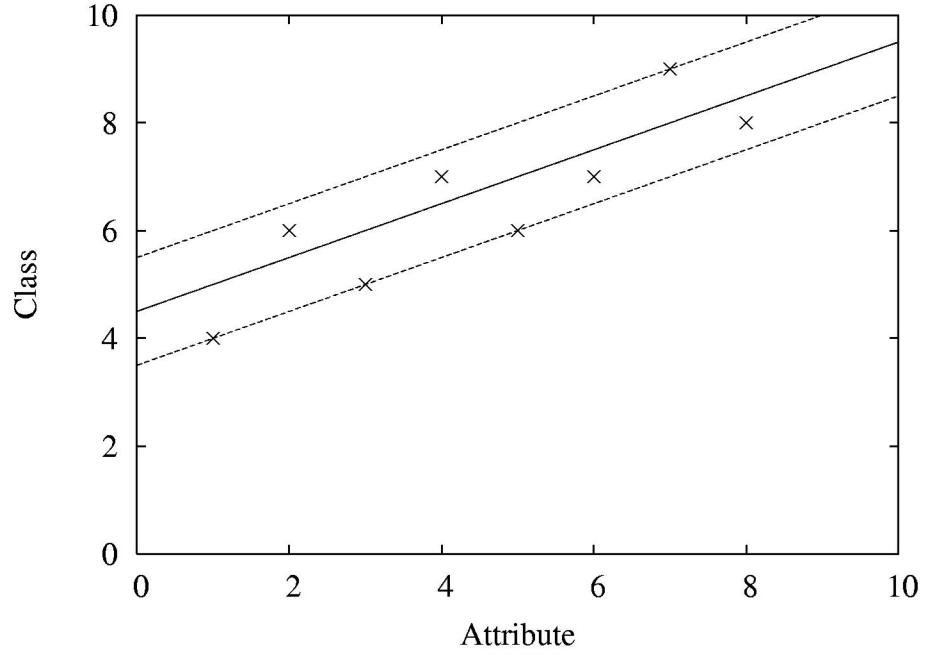
More on SVM regression

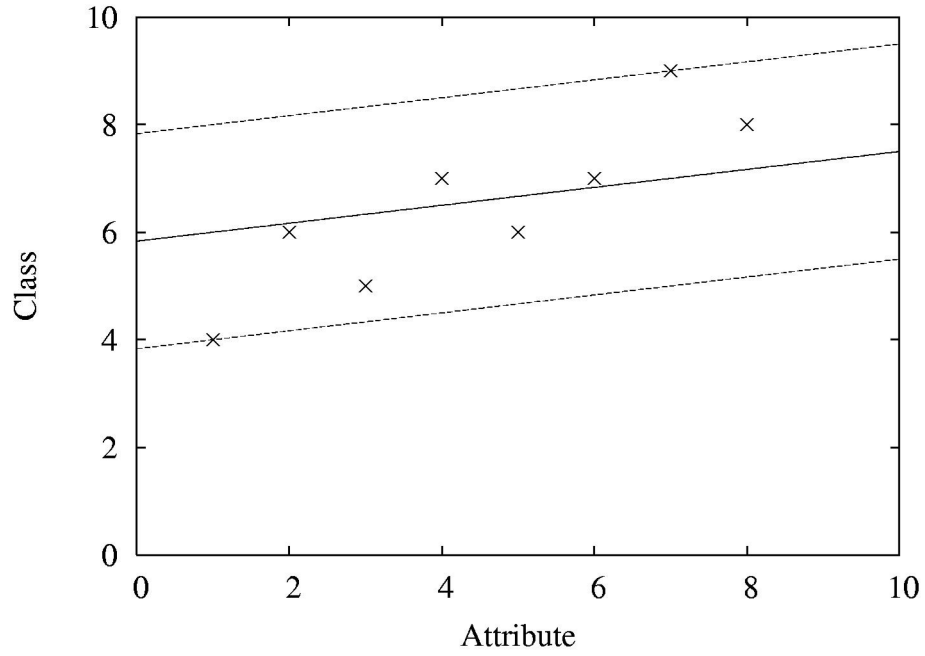
- If there are tubes that enclose all the training points, the flattest of them is used
  - E.g.: mean is used if 2e > range of target values
- Model can be written as:

$$x = b + \sum_{i \text{ is a supp. vector}} \alpha_i \vec{a}(i) \cdot \vec{a}$$

- Support vectors: points on or outside tube
- Dot product can be replaced by kernel function
- In contrast to the classification case, the coefficients a_i may be negative (in the classification case, we have the class values)
- No tube that encloses all training points?
  - Requires trade-off between error and flatness
  - Controlled by upper limit *C* on absolute value of coefficients a_i

$\varepsilon = 2$





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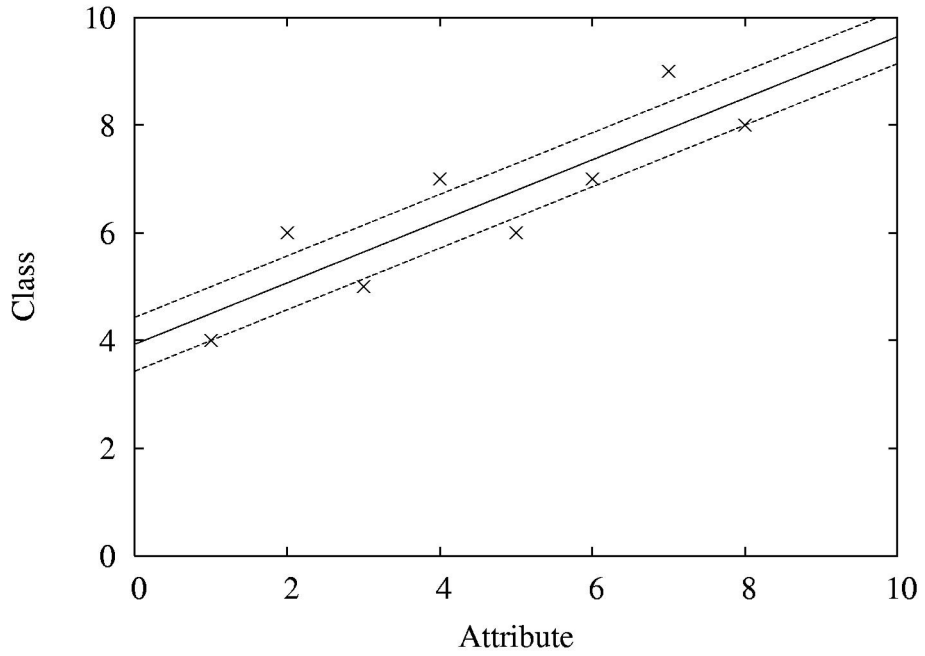


$\varepsilon = 0.5$

Kernel Ridge Regression

- For classic linear regression using squared loss, only simple matrix operations are needed to find the parameters
- This is not the case for support vector regression because a different loss function is used
- Requires use of numeric optimization technique such as sequential minimal optimization
- Can we combine the power of the kernel trick with the simplicity of standard least-squares regression?
  - Yes! This yields *kernel ridge regression*

Comments on kernel ridge regression

- Like in an SVM, the predicted class value for a test instance is expressed as a weighted sum of dot products
- But: all training instances are involved in this sum:

$$\sum_{j=1}^{n} \alpha_j \mathbf{a}_j \cdot \mathbf{a}$$

- Unlike in an SVM, **all** training instances participate – not just support vectors
  - No sparseness in the solution (no support vectors)
- Also, loss in ridge regression does not ignore errors smaller than e
- Moreover, squared error is used instead of absolute error so regression model is more sensitive to outliers

Performing kernel ridge regression

- The penalized loss function that is optimized by kernel ridge regression is

$$\sum_{i=1}^{n}\left(y_i-\sum_{j=1}^{n}\alpha_j\mathbf{a}_j\cdot\mathbf{a}_i\right)^2+\lambda\sum_{i,j=1}^{n}\alpha_i\alpha_j\mathbf{a}_j\cdot\mathbf{a}_i$$

- The user-specified parameter λ determines closeness of fit to the training data
- The coefficients can be found using matrix operations
- Standard regression – invert an m × m matrix ($O(m^3)$), m = #attributes
- Kernel ridge regression – invert an n × n matrix ($O(n^3)$), n = #instances
- Has an advantage if
  - a non-linear fit is desired or
  - there are more attributes than training instances

The kernel perceptron

- We can use the "kernel trick" to make a non-linear classifier using the perceptron learning rule
- Observation: in perceptron learning rule, weight vector is modified by adding or subtracting training instances
- Hence, we can represent the learned weight vector using all instances that have been misclassified:
  - This means we can use $\sum_i \sum_j y(j) a'(j)_i a_i$

    instead of $\sum_i w_i a_i$

    (where *y* is either -1 or +1)
- Now swap summation signs: $\sum_j y(j) \sum_i a'(j)_i a_i$
- Can be expressed as: $\sum_j y(j) \vec{a}'(j)_i \cdot \vec{a}_i$
- Can replace dot product by kernel: $\sum_j y(j) K(\vec{a}'(j)_i \cdot \vec{a}_i)$

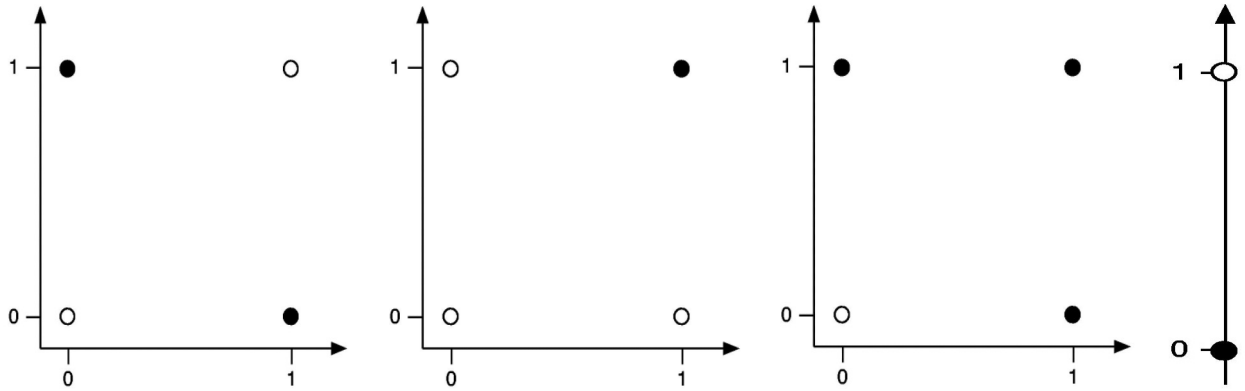
Comments on kernel perceptron

- Finds separating hyperplane in space created by kernel function (if it exists)
  - But: doesn't find maximum-margin hyperplane
- Easy to implement, supports incremental learning
- Perceptron can be made more stable by using all weight vectors encountered during learning, not just the last one (yields the *voted perceptron*)
  - Weight vectors vote on prediction (vote based on number of successful classifications since inception)

Multilayer perceptrons

- Using kernels is only one way to build nonlinear classifier based on perceptrons
- Can create network of perceptrons to approximate arbitrary target concepts
- A *multilayer perceptron* is an example of an artificial neural network build from perceptrons
- Consists of: input layer, hidden layer(s), and output layer
- Structure of MLP is usually found by experimentation
- Parameters can be found using *backpropagation*

Examples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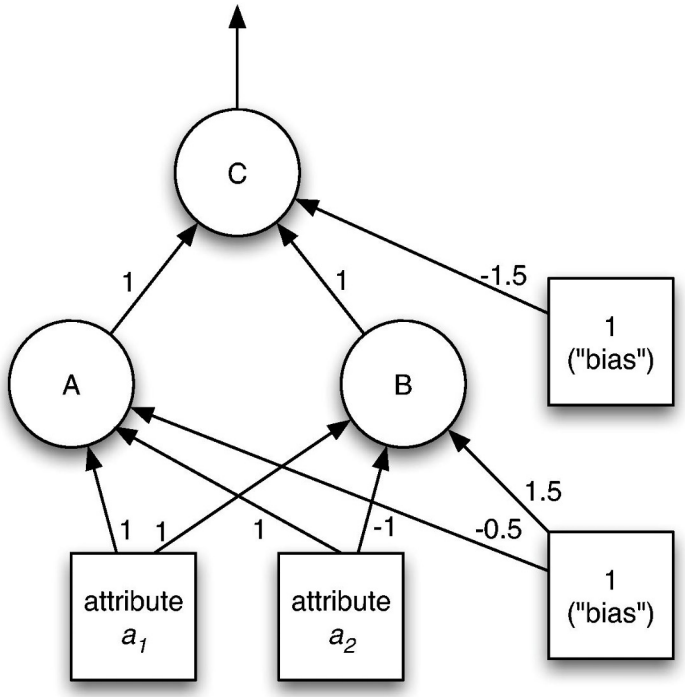
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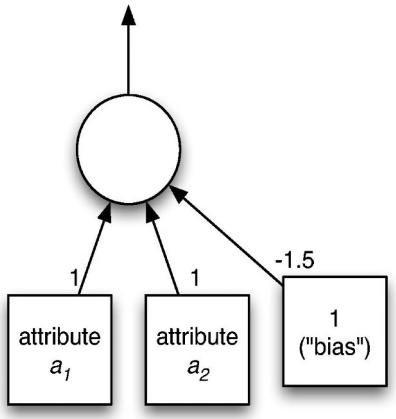
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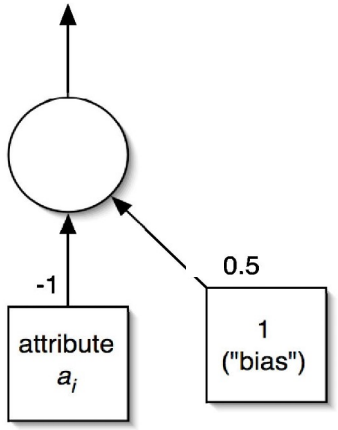
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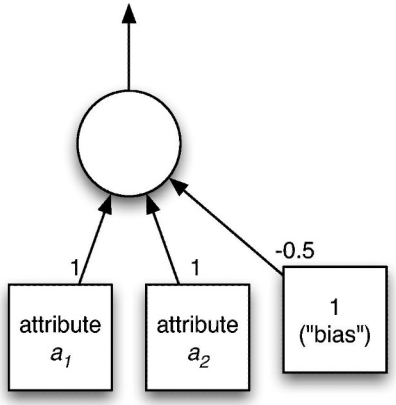
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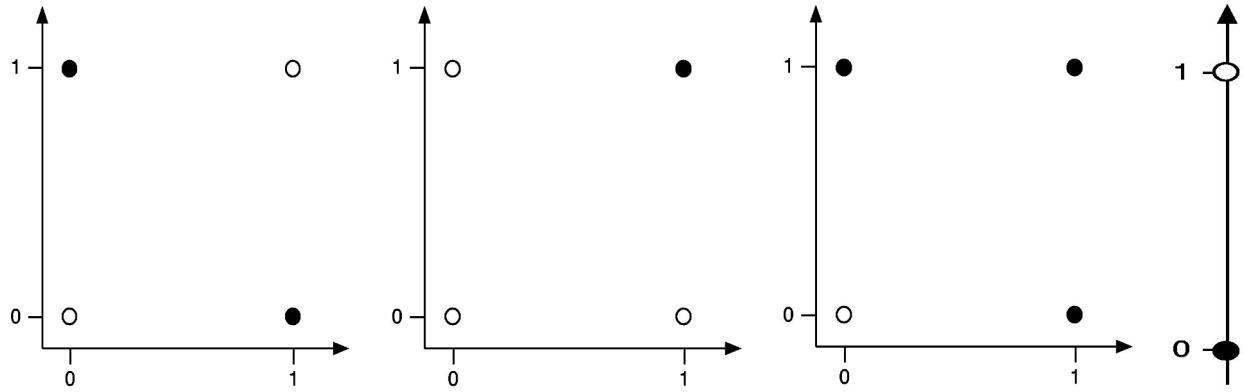
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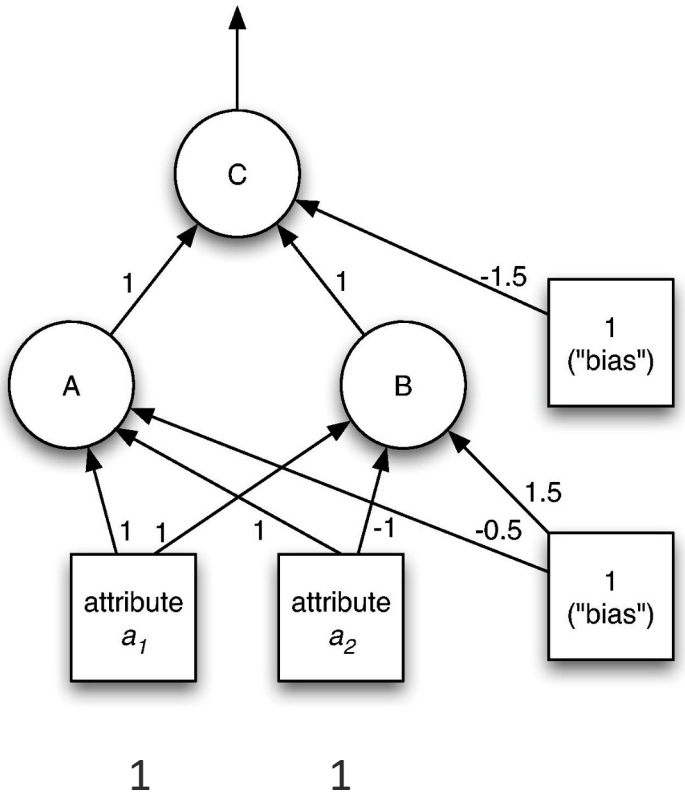
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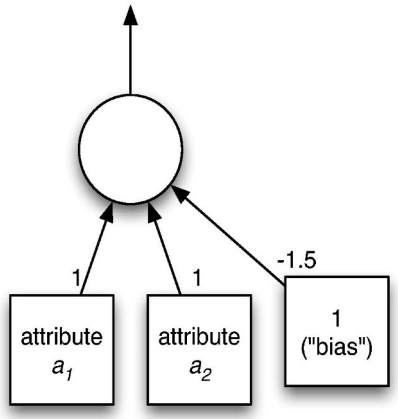
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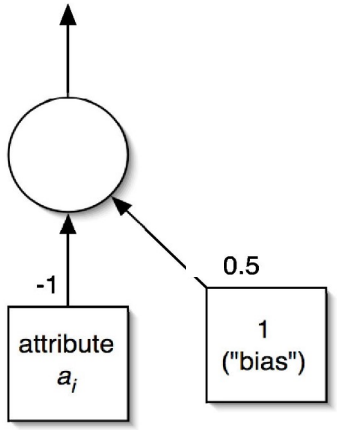
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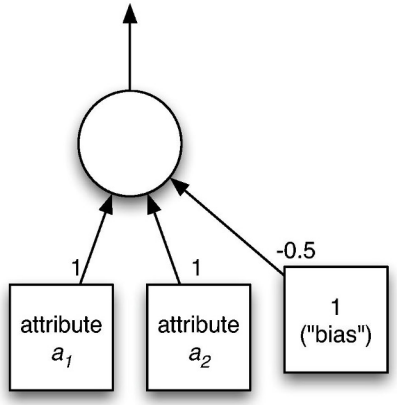
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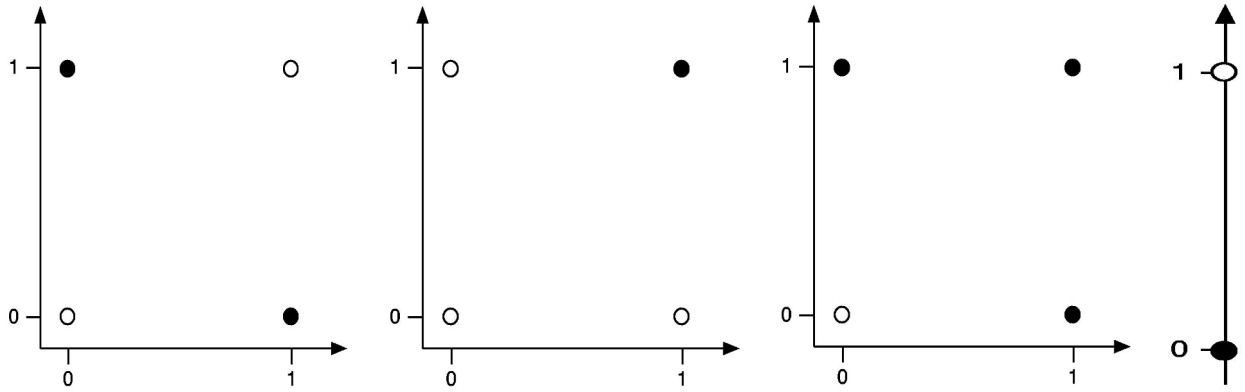
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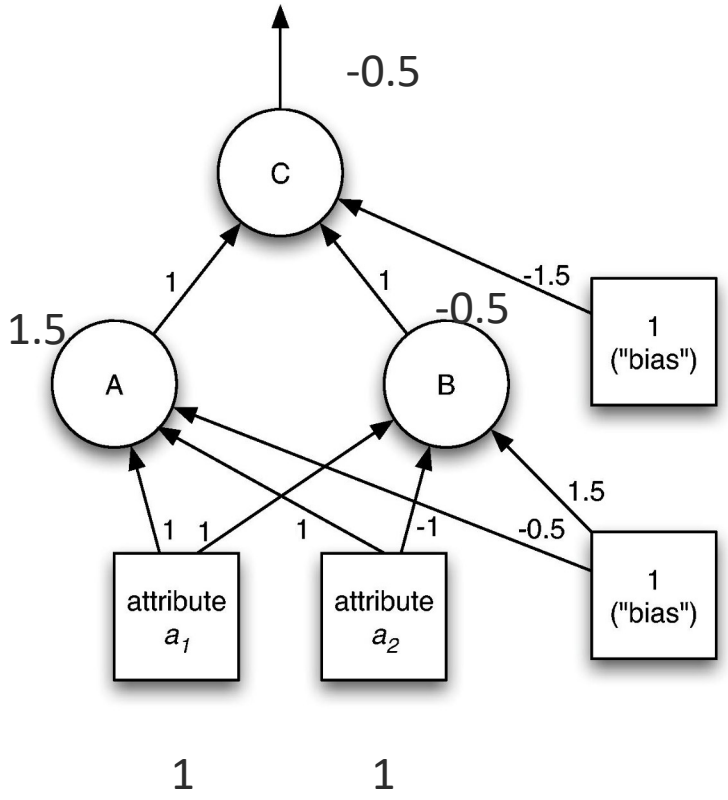
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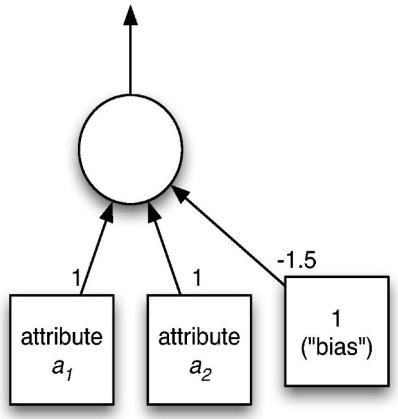
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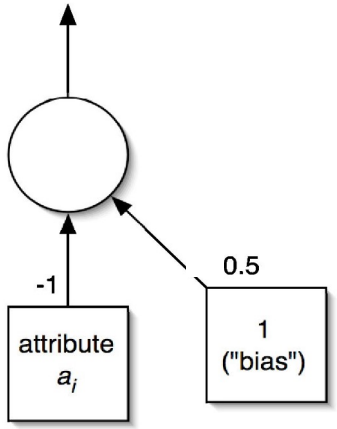
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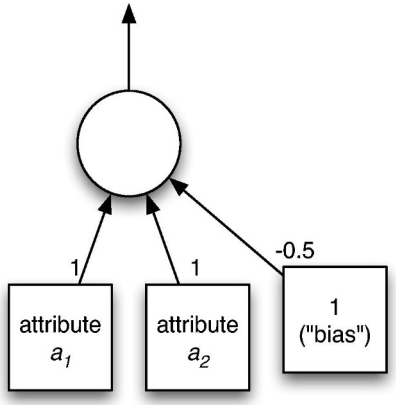
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Backpropagation

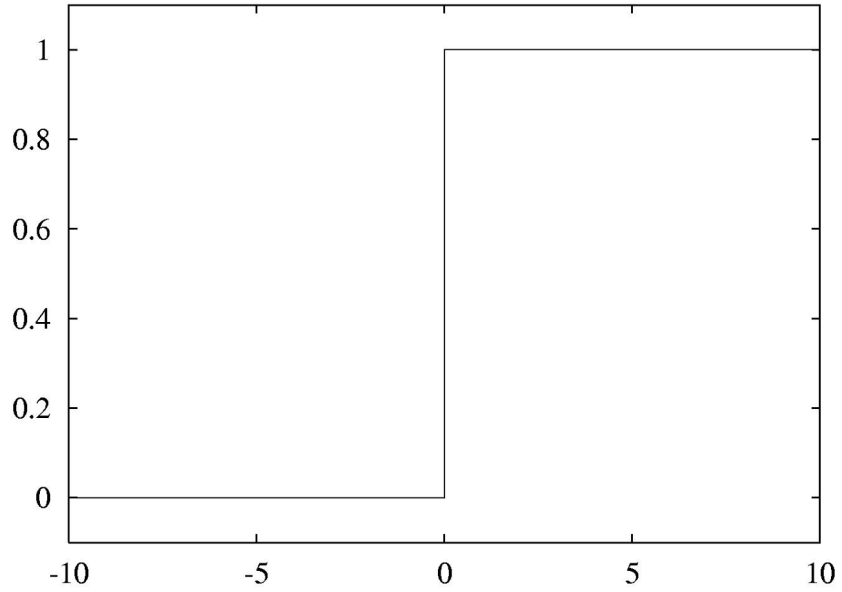
- How to learn the weights given a network structure?
- Cannot simply use perceptron learning rule because we have hidden layer(s)
- Function we are trying to minimize: error
- Can use a general function minimization technique called *gradient descent*
  - *Activation function* needs to provide gradient information: can use *sigmoid function* instead of threshold function

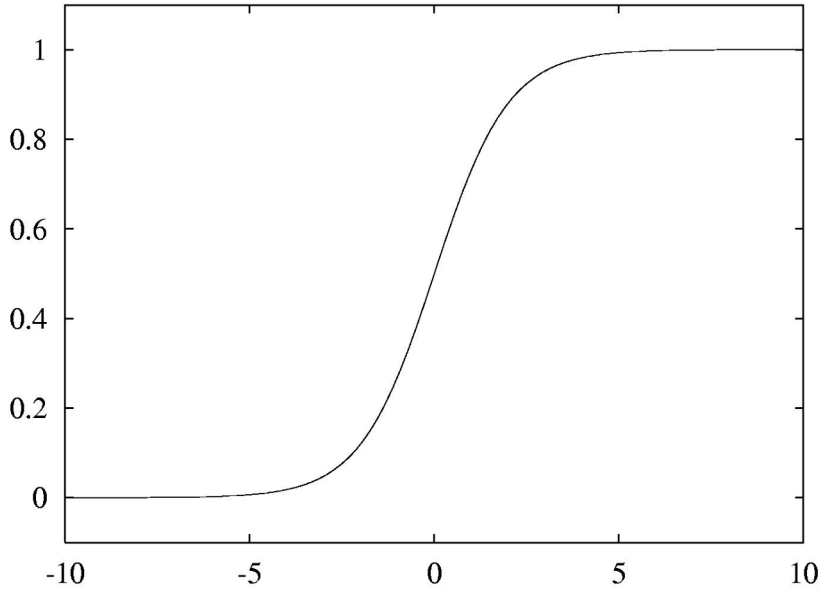
$$f(x) = \frac{1}{1 + \exp(-x)}$$

  - Loss function also needs to provide gradient information: cannot use zero-one loss, but can use squared error

$$E = \frac{1}{2}(y - f(x))^2$$

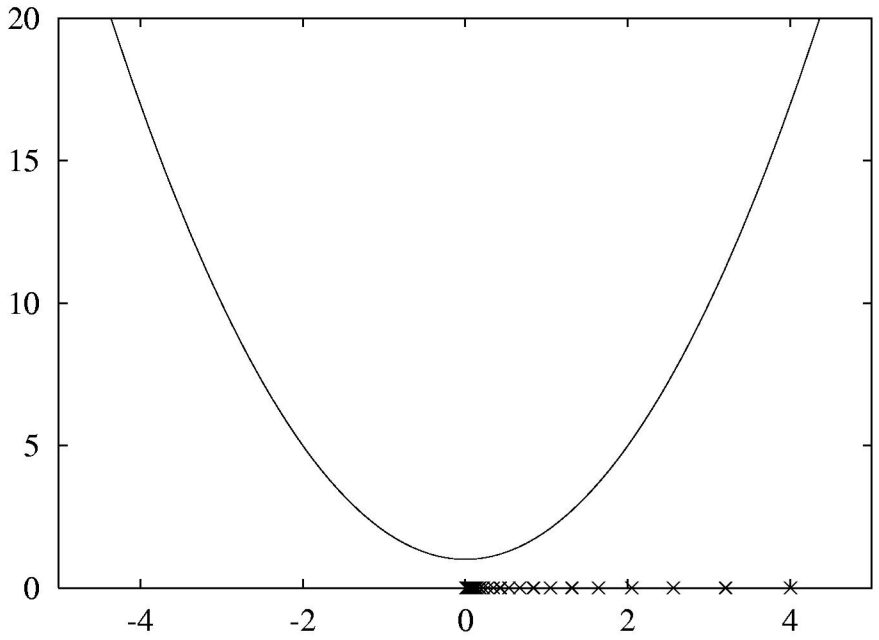
Threshold vs. sigmoid activation function





Gradient descent example

- Function: x^2+1
- Derivative: $2x$
- Learning rate: 0.1
- Start value: 4



- *Can only find a local minimum!*

Minimizing the error I

Need to find partial derivative of error function with respect to each parameter (i.e., weight):

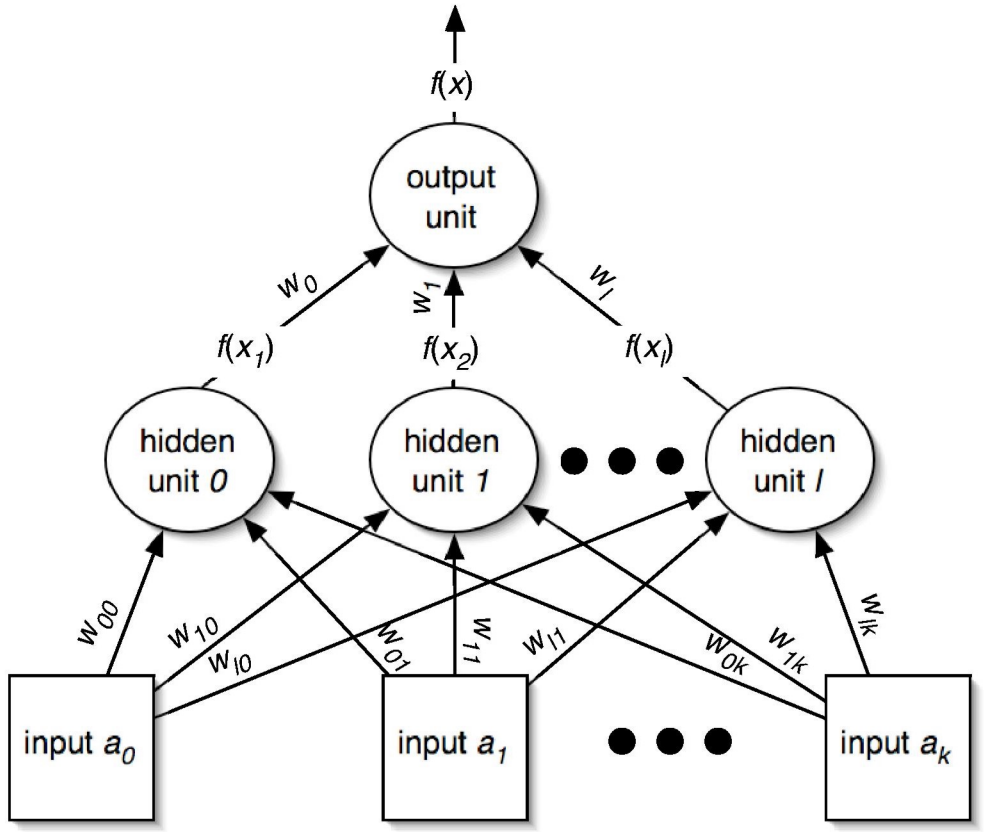
$$\frac{dE}{dw_i} = (f(x) - y)\frac{df(x)}{dw_i}$$

$$\frac{df(x)}{dx} = f(x)(1 - f(x))$$

$$x = \sum_i w_i f(x_i)$$

$$\frac{df(x)}{dw_i} = f'(x) f(x_i)$$

$$\frac{dE}{dw_i} = (f(x) - y) f'(x) f(x_i)$$



Minimizing the error II

What about the weights for the connections from the input to the hidden layer? More application of the chain rule…

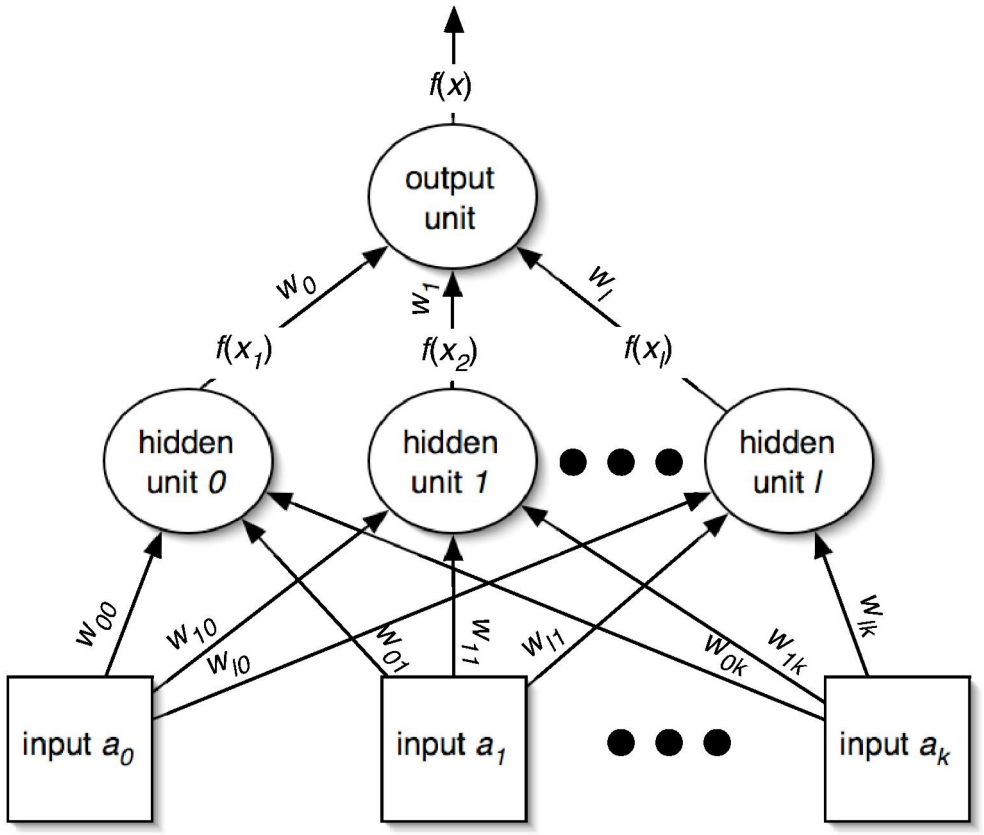
$$\frac{dE}{dw_{ij}} = \frac{dE}{dx}\frac{dx}{dw_{ij}} = (f(x) - y)f'(x)\frac{dx}{dw_{ij}}$$

$$x = \sum_i w_i f(x_i)$$

$$\frac{dx}{dw_{ij}} = w_i \frac{df(x_i)}{dw_{ij}}$$

$$\frac{df(x_i)}{dw_{ij}} = f'(x_i)\frac{dx}{dw_{ij}} = f'(x_i)a_i$$

$$\frac{dE}{dw_{ij}} = (f(x) - y)f'(x)w_i f'(x_i)a_i$$



Remarks

- The same process works for multiple hidden layers and multiple output units (e.g., for multiple classes)
- Can update weights after all training instances have been processed or incrementally:
  - *batch learning* vs. *stochastic backpropagation*
  - Weights are initialized to small random values
- How to avoid overfitting?
  - *Early stopping*: use validation set to check when to stop
  - *Weight decay*: add penalty term to error function
- How to speed up learning?
  - *Momentum*: re-use proportion of old weight change
  - Use optimization method that employs 2nd derivative

Radial basis function networks

- *RBF network*: another type of *feedforward network,* with two layers (plus the input layer)
- Hidden units represent points in instance space and activation depends on distance to these points
  - To this end, distance is converted into a similarity score using a Gaussian activation function
  - Width of Gaussian may be different for each hidden unit
  - Points of equal activation of units in hidden layer form hypersphere (or hyperellipsoid) as opposed to hyperplane
- Output layer is the same as in a multi-layer perceptron

Learning RBF networks

- Parameters to be learned: centers and widths of the RBFs + weights in output layer
- Can learn the two sets of parameters independently and still get fairly accurate models
  - E.g.: clusters from *k*-means can be used to form basis functions
  - Linear model for output layer can be based on fixed RBFs found using clustering, which makes learning very efficient
- However, for best accuracy it is best to train the entire network in a fully supervised manner
  - Can use the same methods that are used for training multilayer perceptrons
- Disadvantage of standard RBF networks: no built-in attribute weighting based on relevance
  - But: can introduce attribute weights into the distance function
- RBF networks are related to RBF SVMs, which have a basis function centered on each support vector

Stochastic gradient descent

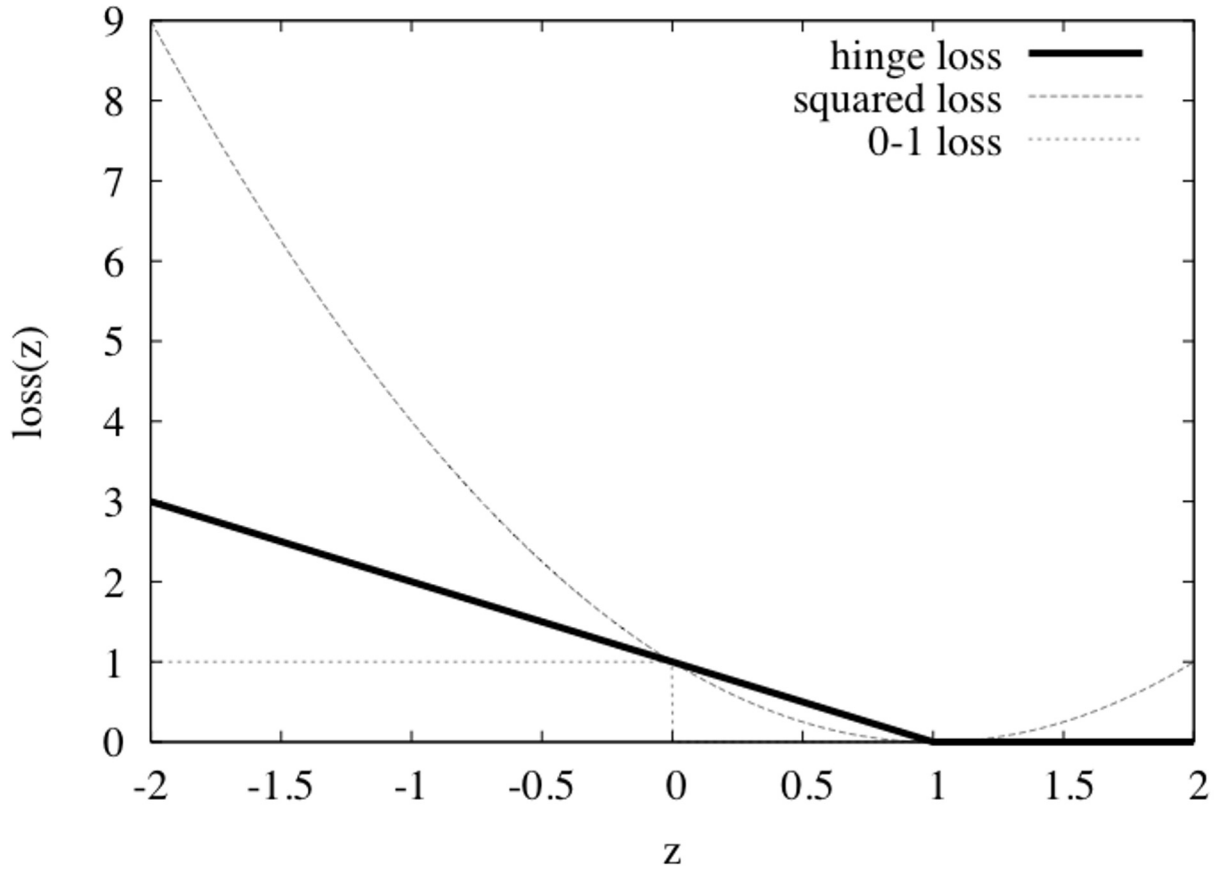
- We have have seen gradient descent + stochastic gradient descent for learning weights in a neural network
- Gradient descent is a general-purpose optimization technique
  - Can be applied whenever the objective function is *differentiable*
  - Actually, can be used even when the objective function is not completely differentiable!
  - This based on the concept of *subgradients*, which we will not get into here
- One application: learning linear models – e.g. linear SVMs or logistic regression
- Very fast, simple method for learning from large datasets

Stochastic gradient descent cont.

- Learning linear models using gradient descent is easier than optimizing non-linear neural networks
- Objective function has a single global minimum rather than several local minima
- Stochastic gradient descent is fast, uses little memory and is suitable for incremental online learning
- Let us look at how to apply stochastic gradient descent to learn a linear support vector machine

Loss functions

- For SVMs, the error function (to be minimized) is called the *hinge loss*



$z = yf(x)$, where the class y is either -1 or $+1$

Optimizing the hinge loss

- In the linearly separable case, the hinge loss is 0 for a function that successfully separates the data
  - The *maximum margin* hyperplane is given by the smallest weight vector that achieves 0 hinge loss
  - Corresponding optimization problem that needs to be solved:

user-specified regularization parameter

$$\lambda \|w\|^2 + \sum \max(1 - \underbrace{y\mathbf{w}\mathbf{x}}_{z}, 0)$$

- But: hinge loss is not differentiable at $z = 1$; cannot compute gradient for all values of z
  - Can use *subgradient* – something that resembles a gradient
  - Can use 0 at $z = 1$
  - In fact, loss is 0 for $z \leq 1$, so we can focus on $z \leq 1$ and proceed as usual with stochastic gradient descent
- Also yields a solution if the data is not separable

Discussion and Bibliographic Notes

- SVMs stem from statistical learning theory (Vapnik 1999)
- A good starting point for exploration is a tutorial by Burges (1998)
- Soft-margin SVMs were discussed by Cortes and Vapnik (1995)
- Tutorial on support vector regression: Smola and Schölkopf (2004)
- Schölkopf et al. (1999) present support vector regression with just one parameter instead of two (C and ε)
- Fletcher (1987) covers constrained quadratic optimization
- The SMO algorithm for training SVMs is due to Platt (1998)
- Ridge regression was introduced by Hoerl and Kennard (1970)
- Hastie et al. (2009) give a good description of kernel ridge regression
- Kernel ridge regression is equivalent to Gaussian process regression, a Bayesian approach that also provides estimates of uncertainty

Discussion and Bibliographic Notes

- The kernel perceptron is due to Freund and Schapire (1999)
- Cristianini and Shawe-Taylor (2000) provide an introduction to support vector machines and kernel-based methods
- Shawe-Taylor and Cristianini (2004) and Schölkopf and Smola (2002) cover kernel-based learning in detail
- Bishop (1995) provides an excellent introduction to both multilayer perceptrons and RBF networks
- Kivinen et al. (2002), Zhang (2004) and Shalev-Shwartz et al. (2007) explore gradient methods for SVMs
- Kivinen et al. and Shalev-Shwartz et al. provide heuristics for setting the learning rate for gradient descent

Numeric Prediction with Local Linear Models

Numeric prediction (aka regression)

- Counterparts exist for all classification schemes previously discussed
  - Decision trees, rule learners, SVMs, etc.
- (Almost) all classification schemes can be applied to regression problems using discretization:
  - Discretize the class into intervals
  - Predict weighted average of interval representatives (e.g., midpoints)
  - Weight according to class probabilities
- We will cover a couple of approaches to regression that are based on building local linear models
  - Model trees (+ a rule learning algorithm based on them) and locally weighted linear regression

Regression trees

- Like decision trees, but:
  - Splitting criterion: minimize intra-subset variation
  - Termination criterion: std. dev. becomes small
  - Pruning criterion: based on numeric error measure
  - Prediction: Leaf predicts average class value of instances
- Yields piecewise constant functions
- Easy to interpret
- More sophisticated version: *model trees*

Model trees

- Build a regression tree
- Each leaf **->** linear regression function
- Smoothing: factor in ancestor's predictions
  - Smoothing formula: $$p' = \frac{np + kq}{n + k}$$
  - Same effect can be achieved by incorporating ancestor models into the leaves
- Need linear regression function at each *node*
- At each node, use only a subset of attributes to build linear regression model
  - Those occurring in subtree
  - (+maybe those occurring in path to the root)
- Fast: tree usually uses only a small subset of the attributes

Building the tree

- Splitting: standard deviation reduction $$SDR = sd(T) - \sum_i \left|\frac{T_i}{T}\right| \times sd(T_i)$$

- Termination of splitting process:
  - Standard deviation < 5% of its value on full training set
  - Too few instances remain (e.g., < 4)

Pruning:

- Heuristic estimate of absolute error of linear regression models:

$$\frac{n+v}{n-v} \times \text{average\_absolute\_error}$$

- Greedily remove terms from LR models to minimize estimated error
- Proceed bottom up: compare error of LR model at internal node to error of subtree (this happens before smoothing is applied)
- Heavy pruning: single model may replace whole subtree

Nominal attributes

- Convert nominal attributes to binary ones
  - Sort attribute values by their average class values
  - If attribute has *k* values,
    generate *k* – 1 binary attributes
    - *i* th attribute is 0 if original nominal value is part of the first *i* nominal values in the sorted list, and 1 otherwise
- Treat binary attributes as numeric in linear regression models and when selecting splits
- Can prove: best SDR split on one of the new binary attributes is the best (binary) SDR split on original nominal attribute
- In practice this process is not applied at every node of the tree but globally at the root node of the tree
  - Splits are no longer optimal but runtime and potential for overfitting are reduced this way

Missing values

- Modify splitting criterion: $$SDR = \frac{m}{|T|} \times \left[sd(T) - \sum_i \left| \frac{T_i}{T} \right| \times sd(T_i) \right]$$

- To determine which subset an instance goes into, use *surrogate splitting*
  - Split on the attribute whose correlation with attribute whose value is missing is greatest
  - Problem: complex and time-consuming
  - Simple solution: always use the class as surrogate attribute
  - Class can only be used at training time
- Test set: replace missing value with average

Surrogate splitting based on class

- Choose split point based on instances with known values
- Split point divides instances into 2 subsets
  - *L* (smaller class average)
  - *R* (larger)
- *m* is the average of the two averages
- For an instance with a missing value:
  - Choose *L* if class value < *m*
  - Otherwise *R*
- Once full tree is built, replace missing values with averages of corresponding leaf nodes
  - Linear regression models can then be built on the completed ("imputed") dataset

Pseudo-code for M5'

- Let us consider the pseudo code for the model tree inducer M5’
- Four methods:
  - Main method: *MakeModelTree*
  - Method for splitting: *split*
  - Method for pruning: *prune*
  - Method that computes error: *subtreeError*
- We will briefly look at each method in turn
- We will assume that the linear regression method performs attribute subset selection based on error (discussed previously)
- Nominal attributes are replaced globally at the root node

MakeModelTree

```
MakeModelTree (instances)
{
  SD = sd(instances)
  for each k-valued nominal attribute
    convert into k-1 synthetic binary attributes
  root = newNode
  root.instances = instances
  split(root)
  prune(root)
  printTree(root)
}
```

split

```
split(node)
{
  if sizeof(node.instances) < 4 or
     sd(node.instances) < 0.05*SD
    node.type = LEAF
  else
    node.type = INTERIOR
    for each attribute
      for all possible split positions of attribute
        calculate the attribute's SDR
    node.attribute = attribute with maximum SDR
    split(node.left)
    split(node.right)
}
```

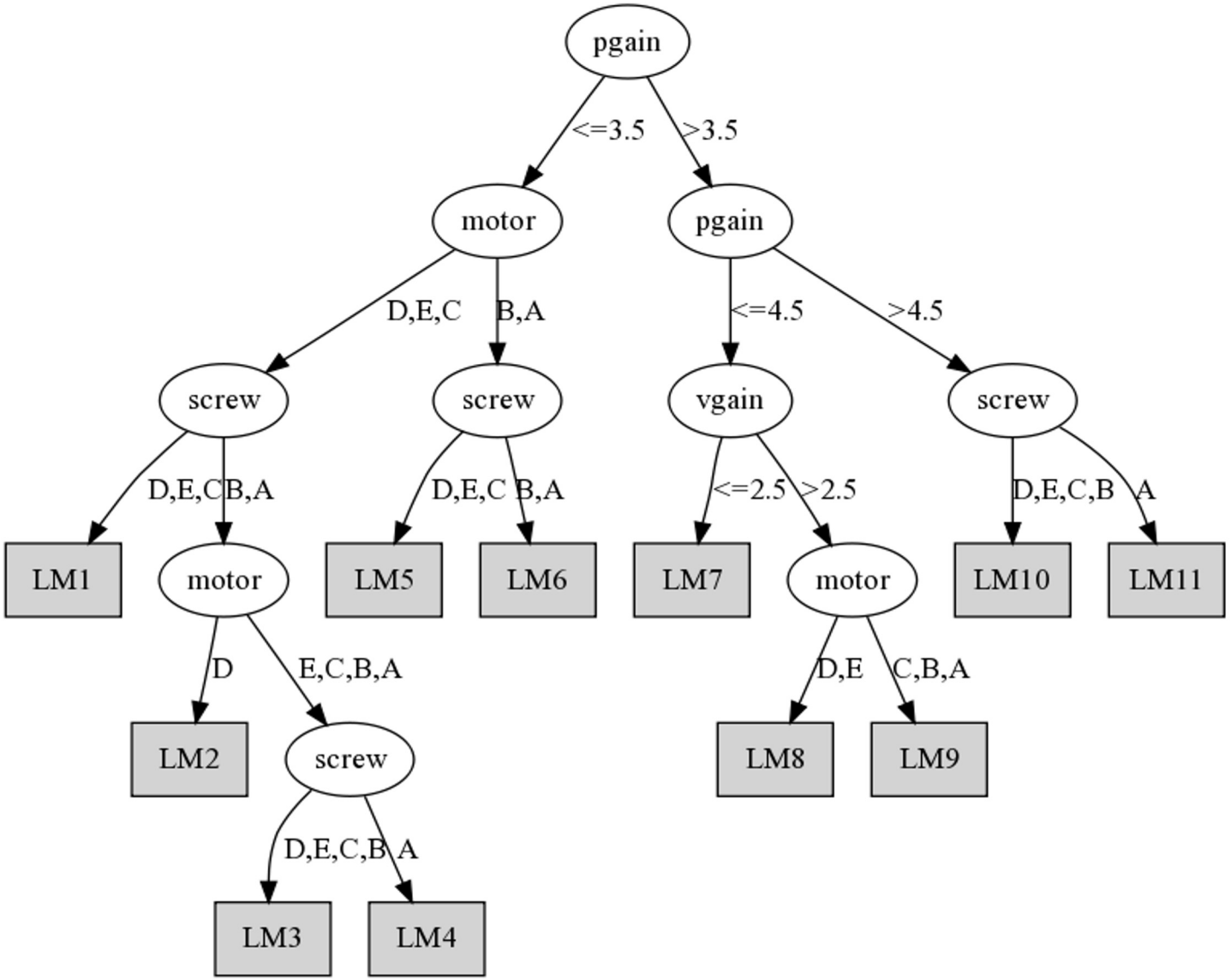
prune

```
prune(node)
{
  if node = INTERIOR then
    prune(node.leftChild)
    prune(node.rightChild)
    node.model = linearRegression(node)
    if subtreeError(node) > error(node) then
      node.type = LEAF
}
```

subtreeError

```
subtreeError(node)
{
  l = node.left; r = node.right
  if node = INTERIOR then
    return (sizeof(l.instances)*subtreeError(l)
            + sizeof(r.instances)*subtreeError(r))
              /sizeof(node.instances)
  else return error(node)
}
```

Model tree for servo data



Rules from model trees

- PART algorithm generates classification rules by building partial decision trees
- Can use the same method to build rule sets for regression
  - Use model trees instead of decision trees
  - Use variance instead of entropy to choose node to expand when building a partial tree
- Rules that are generated will have linear models on right-hand side
- Caveat: using smoothed trees may not be appropriate due to the separate-and-conquer strategy used in rule learning
  - Empirical evidence shows that smoothing does not help
- Full trees can be used instead of partial trees at the expense of runtime

Locally weighted regression

- Locally weighted regression is a numeric prediction method that combines
  - instance-based learning
  - linear regression
- It is a “lazy” learning method:
  - Computes new regression function for each test instance at prediction time
  - Works incrementally
- Weights training instances
  - according to distance to test instance
  - builds linear regression model from weighted data
  - requires weighted version of linear regression (straightforward)
- Advantage: nonlinear approximation
- Slow if implemented using brute-force search; however, fast data structures can be used for the nearest-neighbor search

Design decisions

- Weighting functions:
  - Inverse Euclidean distance
  - Gaussian kernel applied to Euclidean distance
  - Triangular kernel used the same way
  - etc.
- Empirically, performance does not appear to depend much on the weighting method that is used
- Ideally, weighting function has *bounded support* so that most training instances receive weight 0 and can be ignored
- *Smoothing parameter* is used to scale the distance function for computation of the weights
  - Multiply distance by inverse of this parameter
  - Possible choice: distance to the *k*th nearest training instance (renders choice of smoothing parameter data dependent)

Discussion and Bibliographic Notes

- Regression trees were introduced in the "*classification and regression trees*", or CART system (Breiman et al., 1984)
- The method of handling nominal attributes and the surrogate device for dealing with missing values were included in CART
- M5 model trees were first described by Quinlan (1992)
- The M5' version is given by Wang and Witten (1997)
- Using model trees (although not partial trees) for generating rule sets has been explored by Hall et al. (1999)
- There are many variations of locally weighted learning.
  - Statisticians have considered using locally quadratic models
  - They have applied locally weighted logistic regression to classification
  - Frank et al. (2003) evaluated the use of locally weighted learning in conjunction with Naïve Bayes
  - Atkeson et al. (1997) provide a survey on locally weighted learning

You have reached the end of the lecture.

Reference:

I. H. Witten, E. Frank,M. A. Hall and C. J. Pal(2016).*Data Mining: Practical Machine Learning Tools and Techniques.* Morgan Kaufmann

Assets 01 Structural Assets

Label

Label
To emphasize and draw attention to a main point or content. To label content

Main Point
Use to present main points in a list.
can be used in cyan, slate and lime.

01

Numbered Point
Use to present main steps/process/points that require numbering, or lettering.

Arrow
highlight different elements of an image

A Clickable Quiz

To highlight important quotes.

- Angela Forero

Propose a question using this asset

Important information using this asset

Term: definition