Assignment #3

Dr. Chung-Hsien Kuo

Duration: 2 weeks

Please write report to show and explain your solution.

The content should have *student name* and *student ID*. The report file must be named as **report_<ID>.pdf**, e.g., report_D10907813.pdf

This assignment is individual assignment. Any teamwork or cheating behavior are prohibited and will be marked 0 if being figured out.

Question 1: Perceptron (15pts)

Given a perceptron with input $X = (x_1, x_2)^T$, weight vector $(\theta, w_1, w_2)^T = (-2, 1, 1)^T$ and activation function is the step function: $f_{\text{step}}(x) = \begin{cases} -1, & x < 0 \\ +1, & x \geq 0 \end{cases}$.

a) The decision boundary is a surface/line that separates the data point into region, which are the classes in which they belong. Plot decision boundary that is realized by this perceptron in the partition of $\mathbb{R}^2$ and mark the area where the perceptron outputs 1. (5pts)

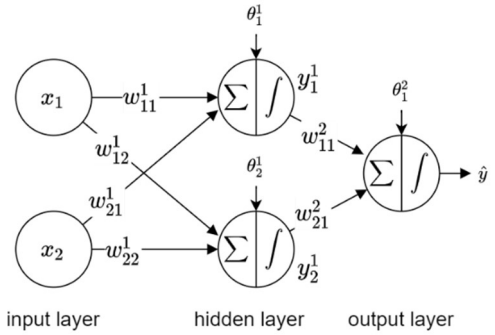
   Note: In this question, you can do program, or write on paper and then capture and put it the report or use the following online tool: https://www.geogebra.org/graphing?lang=en to plot your solution.

b) Suppose $X_1 = (2, -5)^T$, what is the corresponding output of this perceptron? (5pts)

c) Same as question b), what is the corresponding output of this perceptron with the tanh activation function $\tanh = \frac{1}{1+e^{-2x}} - 1$ instead? (5pts)

Question 2: Multi-Layer Perceptron - MLP (60pts)

Given a MLP as following. All neurons use the sigmoid function. The network has two input variables x_1 and x_2 and one output variable y.



a) Calculate the output of all neurons $(y_1^1, y_2^1, \hat{y})$ and the error made by the MLP when applying input $X = (x_1, x_2)^T = (10, -2)^T$, weights $w^1 =$

$\begin{pmatrix} w_{11}^1 & w_{12}^1 \\ w_{21}^1 & w_{22}^1 \end{pmatrix} = \begin{pmatrix} 0.5 & 0.9 \\ 0.4 & 1.0 \end{pmatrix}$ and $w^2 = \begin{pmatrix} w_{11}^2 \\ w_{21}^2 \end{pmatrix} = \begin{pmatrix} -1.2 \\ 1.1 \end{pmatrix}$, bias $\theta^1 = \begin{pmatrix} \theta_1^1 \\ \theta_2^1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and $\theta^2 = (\theta_1^2) = (1)$, and the target value $y = 1$. (30pts)

b) Use back-propagation technique to update weights w^1 and w^2 with the learning rate is equal to 0.1. (30pts)

Question 3: Perceptron Learning (10pts)

A student implemented MLP using following training data and computed a decision boundary as $x_1 + x_2 - 1 = 0$

x_1	x_2	label
1	2	1
2	3	1
4	4.5	1
0	1.5	1
0	0	0
0.5	0	0
0.5	1	0
−1	0	0

When he applied this model on the validation samples, he noticed that the accuracy was low. He debugged the code and found that the separating boundary is not computed accurately.

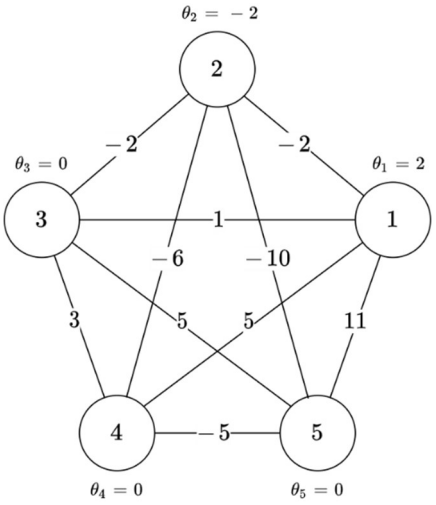
a) Which point(s) are misclassified in the above samples? (5pts)

b) Suggest appropriate corrections to the parameters of the separating boundary? (5pts)

Question 4: Hopfield Networks (15pts)

c) Can the vector $(1, 0, -1, 0, 1)$ be stored in a 5-neuron Hopfield network? If so, what are the weights for a network with that vector stored in it? If so, why not? (5pts)

d) Given the Hopefield network as following.



Write down the corresponding weight matrix, such that weight w_{ij} is in

column i and row j. Write down the threshold vector θ, such that element θ_i is the bias of neuron i. (5pts)

e) Find one or more stable state of the given network. (5pts)