Modeling, Manipulating, and Visualizing Continuous Volumetric Data: A Novel Spline-based Approach

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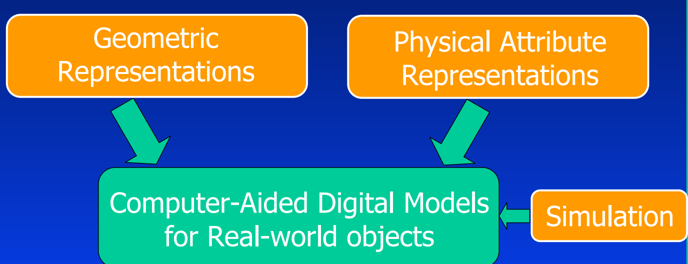
Talk Outline

- Introduction and Motivation
- Spline-based Heterogeneous Volume Modeling
  – Geometric Modeling and Physical Attribute Modeling
  – Feature-Sensitive Volume Reconstruction
- Direct Manipulation of Dynamic Volumetric Models
- Scalar-Field Guided Shape Deformation (if time permits)
- Conclusion

Introduction

- Geometric Design and Reconstruction (x, y, z)
- Physical Attribute Modeling and Reconstruction (d)



Challenges in 3D Modeling and Visualization

- There are no integrated approach and unified paradigm to represent both geometry and attributes for both modeling and visualization purposes.
- Lack of effective, interactive sculpting toolkits for the natural and intuitive manipulation of geometric objects and their associated attributes.
- Reconstruction geometry and its associated attributes simultaneously from an existing discrete model is under-explored.
- More difficult for kinematic & dynamic analysis of physical objects

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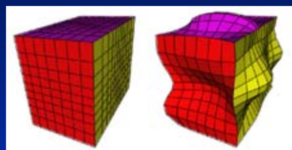
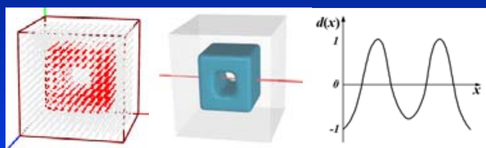
Spline based Volume Modeling

- Our Approach
  – A modeled object is defined as a spline-based analytic formulation.
  – Arbitrary Topology and complicated geometry
  – Collision detection and topological changes
  – Arbitrary resolution and compact storage
  – Nice blending properties
  – Anti-aliasing
  – Ease of manipulation and deformation

B-spline Modeling (Parametric vs. Implicit)





B-spline based Volumetric Implicit Functions

- Scalar trivariate B-spline

$$s(u,v,w)=\sum_{i=0}^{l-1}\sum_{j=0}^{m-1}\sum_{k=0}^{n-1} p_{ijk}B_{i,r}(u)C_{j,s}(v)D_{k,t}(w)$$

$$\mathbf{d}=(\mathbf{B}\otimes\mathbf{C}\otimes\mathbf{D})\mathbf{p}$$

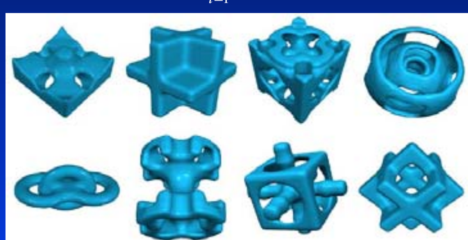
$$\mathbf{d}=[\cdots,s_{ijk},\cdots]^T\ (i\in[0,U],\ j\in[0,V],\ k\in[0,W])$$
$$\mathbf{p}=[\cdots,p_{ijk},\cdots]^T\ (i\in[0,l-1],\ j\in[0,m-1],\ k\in[0,n-1])$$

[Published at PG 2001, SMI 2002, VolVis 2002]

Hybrid, Hierarchical Scalar B-splines

- Spline-based volumetric implicit functions

$$F(x,y,z)=\sum_{i=1}^{N} s_i(\mathbf{T}_i(x,y,z))$$

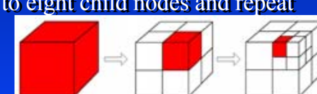


Hierarchical Data Fitting for Rectilinear Volumes

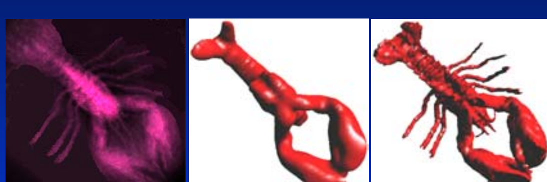
- Create an octree for the entire working space and subdivide the root node to eight child nodes.
- Fit a single scalar, trivariate B-spline to the region of each child node using the least-square technique.
- Evaluate the mean square error (*MSE*) at node *i*,

$$\varepsilon_i=\frac{1}{N_i}\sum_{j=0}^{N_i}(d_j-f_i(x_j))^2$$

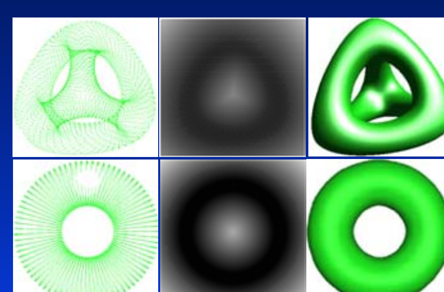
- If ε_i is less than the user-specified error bound , then mark the node as a leaf node.
- Otherwise, subdivide the node *i* to eight child nodes and repeat the above.



Fitting Examples



Fitting Examples (Point Clouds)



Simplex Spline based Heterogeneous Modeling

- An integrated approach for representing, modeling, and rendering of multi-dimensional, physical attributes across any volumetric objects of complicated geometry and arbitrary topology.
- Our model makes use of a more general and flexible tetrahedral domain and offers a compact continuous representation at the same time. It directly facilitates multiresolution modeling.
- The C^{n-1} continuity and C^0 continuity can both be modeled with ease. Such flexibility also allows us to model continuous or discontinuous distribution in the attribute field.
- Using time-varying knots instead of fixed knots offers more freedom and improves accuracy for approximation. The knots are explicitly and automatically determined by optimizing certain objective function.

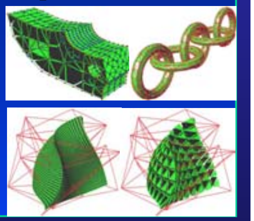
[Submitted to ACM Solid Modeling 04]

Trivariate Simplex Spline Volumes

- Trivariate simplex spline is based on a tetrahedral domain, which can be of complicated geometry and arbitrary topology.
- For a general trivariate simplex spline, each domain tetrahedron I has its own set of control points.
- Mathematical formulation:

$$\mathbf{s}(\mathbf{u}) = \sum_{I\in\Omega}\sum_{|\beta|=n} \mathbf{c}^I_\beta N^I_\beta(\mathbf{u})$$

$$N^I_\beta(\mathbf{u}) = |\,d(\mathbf{p}_i,\mathbf{q}_j,\mathbf{r}_k,\mathbf{s}_l)\,|\, M(\mathbf{u}\,|\,V^I_\beta)$$

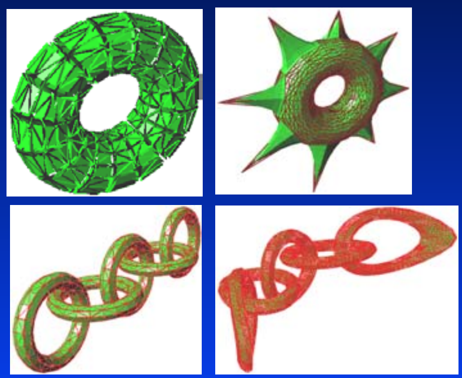


Coupling Solid Geometry and Physical Attributes

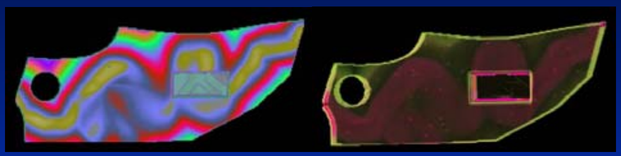
- A control coefficient (and possibly other vector-based quantities with n components) is associated with a corresponding control point and evaluated with the geometry simultaneously over the same tetrahedral domain.

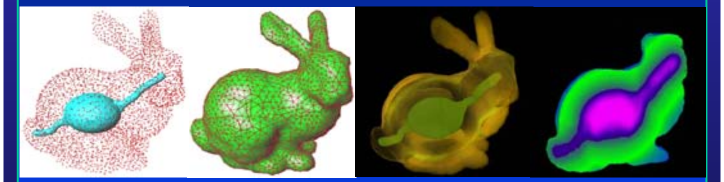
$$\begin{bmatrix}\mathbf{g}\\ \mathbf{s}\end{bmatrix}(\mathbf{x}) = \sum \begin{bmatrix}\mathbf{g}^I_\beta\\ \mathbf{p}^I_\beta\end{bmatrix} N^I_\beta(\mathbf{x})$$

Modeling of Solid Geometry of Volumes



Attribute Editing Using Control Coefficients





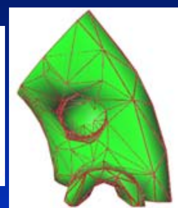
Feature-Sensitive Volume Reconstruction

- Problem Statement:
  - Given a set $\mathbf{P} = \{\mathbf{p}_i\}_{i=1}^m$ of points $\mathbf{p}_i = \{x_i, y_i, z_i, d_i\} \in R^4$, find a trivariate simplex spline volume $\mathbf{s}: R^3 \to R^4$, that approximates P.
- The fitting algorithm
  1. Create a tetrahedral domain for the entire volume;
  2. Minimize the square error by treating control vectors as free variables.

     $$\min E = \sum_{i=1}^m (\mathbf{p}_i - \mathbf{s}(x_i, y_i, z_i))^2.$$
  3. For each node of the tetrahedralization, if the fitting error in its 1-ring neighboring tetrahedra is too large, minimize the square error by treating the knots associated with the node as free variables.
  4. For each tetrahedron, if its fitting error is too large, subdivide it into four tetrahedra and repeat previous two steps.

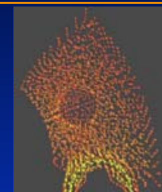
Feature-Sensitive Volume Reconstruction

1. Finding a good initial tetrahedralization
   – Geometric features
   – Attribute field features

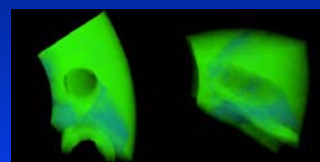



2. Adjustment of Free knots according to features
3. Local adaptive refinement according to features

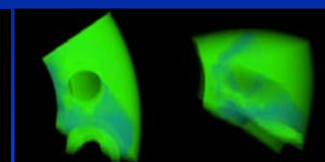
Fitting Examples



The original dataset

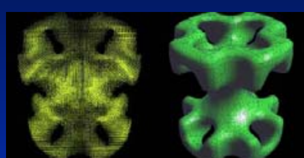
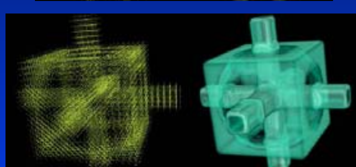


Fitting with control vectors only



Fitting with both control vectors and knots

Fitting Examples

Visualization of Simplex Spline Volumes

- Direct Volume Rendering
  – X-Ray volume rendering

$$\int_0^L M(\mathbf{x}_c + t\mathbf{d}_c|\mathbf{V})dt = \frac{n}{n+1}\sum_{j=0}^{3}\lambda(\mathbf{x}_c)\int_0^L M(\mathbf{x}_c + t\mathbf{d}_c|\mathbf{V}\setminus\{\mathbf{x}_{k_j}\})dt.$$

When $n = 0$,

$$\int_0^L M(\mathbf{x}_c + t\mathbf{d}_c|\mathbf{V})dt = \frac{L}{\mathrm{Vol}(\mathbf{V})}.$$

  – General volume rendering
- Multiresolution Marching Tetrahedra Algorithm

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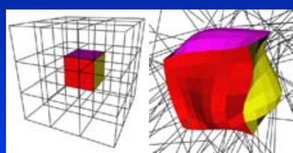
Interacting with Dynamic Volumetric Models

- Integrate volumetric spline functions and physics-based modeling into one single framework: *Haptics-based Dynamic Volumetric Modeling*
- Physics-based techniques provide a natural, force-based interface to facilitate direct manipulation and intuitive shape design
- Haptic interface permits users to interactively sculpt virtual materials and feel the physical presence with force feedback
- Various sculpting tools are available

[To appear in the journal: IEEE TVCG 2003]

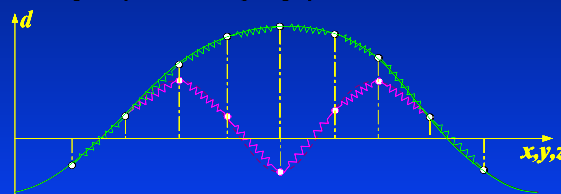
Integrating Elasticity with Geometry

- Integrating elasticity with geometry is straightforward.
- Allocate mass-points on sampling points, springs on edges of cells.
- In order to prevent shearing, angular springs may be used to preserve the regular geometry.

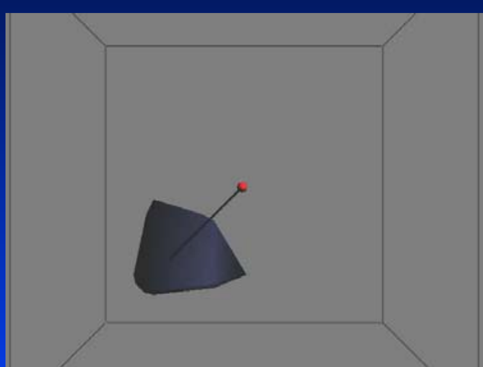


Integrating Elasticity with Attributes

- Density springs
  - "*Density Springs*" do not change the geometric positions of the mass points. Instead, they permit the density change of mass points.
  - When manipulating the attribute field, the density values are changed by the mass-spring system.



Dynamic Volumetric Model (Video)



Simulation

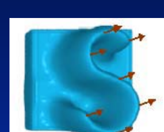
- Lagrangian dynamics:

$$\mathbf{M}\ddot{\mathbf{d}} + \mathbf{D}\dot{\mathbf{d}} + \mathbf{K}\mathbf{d} = \mathbf{f}_{\mathbf{d}}$$

- **M** is a mass matrix, **D** is a damping matrix, **K** is a stiffness matrix.
- The applied force at every mass-point is the summation of all possible external forces.

Force Mapping & Force Tools

Deformation with point-based force tools



Deformation with curve-based and area-based force tools

Chisel and Squirt operation

Joining with a force tool

Force tools allow users to sculpt solid interior without breaking the outer material

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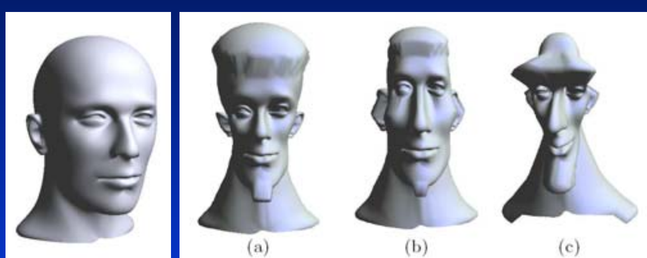
SFD Algorithm

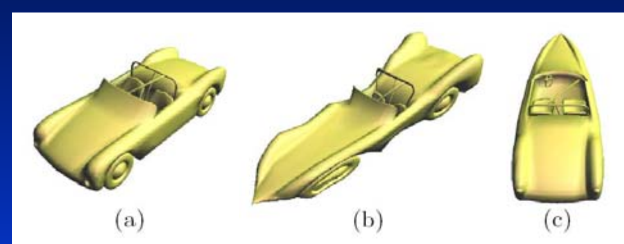
- Embed an entire model or a part of the model into a scalar field and calculate the scalar values at all the vertices of that embedded part.
- Constrain the vertices on the level sets where they originally reside by enforcing vertex-flow constraints during the deformation process.

[Published at SM 2003 and To appear in the journal: The Visual Computer 2003]

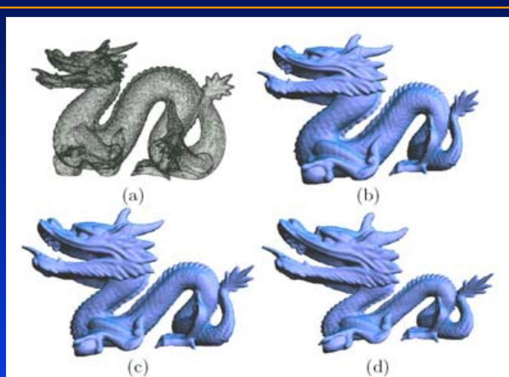
SFD of the Mannequin Polygonal Model



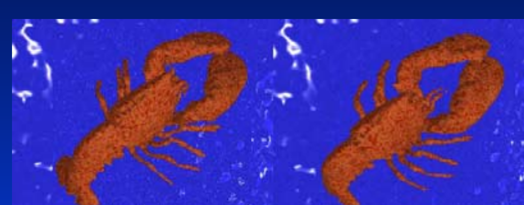
SFD of the Car Polygonal Model



SFD Applied to Point Set Objects



SFD Applied to Volumetric Objects



Conclusion

- A novel spline-based approach for multiresolution modeling, manipulating, and visualizing volumetric data.
- Employ the trivariate B-spline to model rectilinear volume, and use the trivariate simplex spline to model unstructured grid volume.
- The data fitting algorithms reconstruct a compact continuous representation and offer a data reduction capability.
- Our spline-based paradigm has the unique advantages.
- Enhancing the volume graphics with dynamics and haptics, and offer a natural interaction.
- Scalar-field guided shape deformation supports large scale deformation for arbitrary geometric and volumetric objects.

Acknowledgements

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Thank you