

Random variable is a variable whose value is unknown and whose value is dependent on outcomes of a random experiment.

Random Variable

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graph TD; A[Random Variable] --> B[Discrete Random variable]; A --> C[Continuous Random variable];
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Discrete Random variable:-

A numerical r.v. takes a countable number of values.

- ❖ No of phone calls received in a day
- ❖ No of Children in the family. Etc

Continuous Random variable:-

A numerical r.v takes all possible value within a certain range.

- ❖ Percentage of marks obtained by Student.
- ❖ Time required to finish the test
- ❖ Height of person .

Probability Distribution

A probability distribution is the mathematical function that gives the probabilities of occurrence of different possible outcomes for an experiment

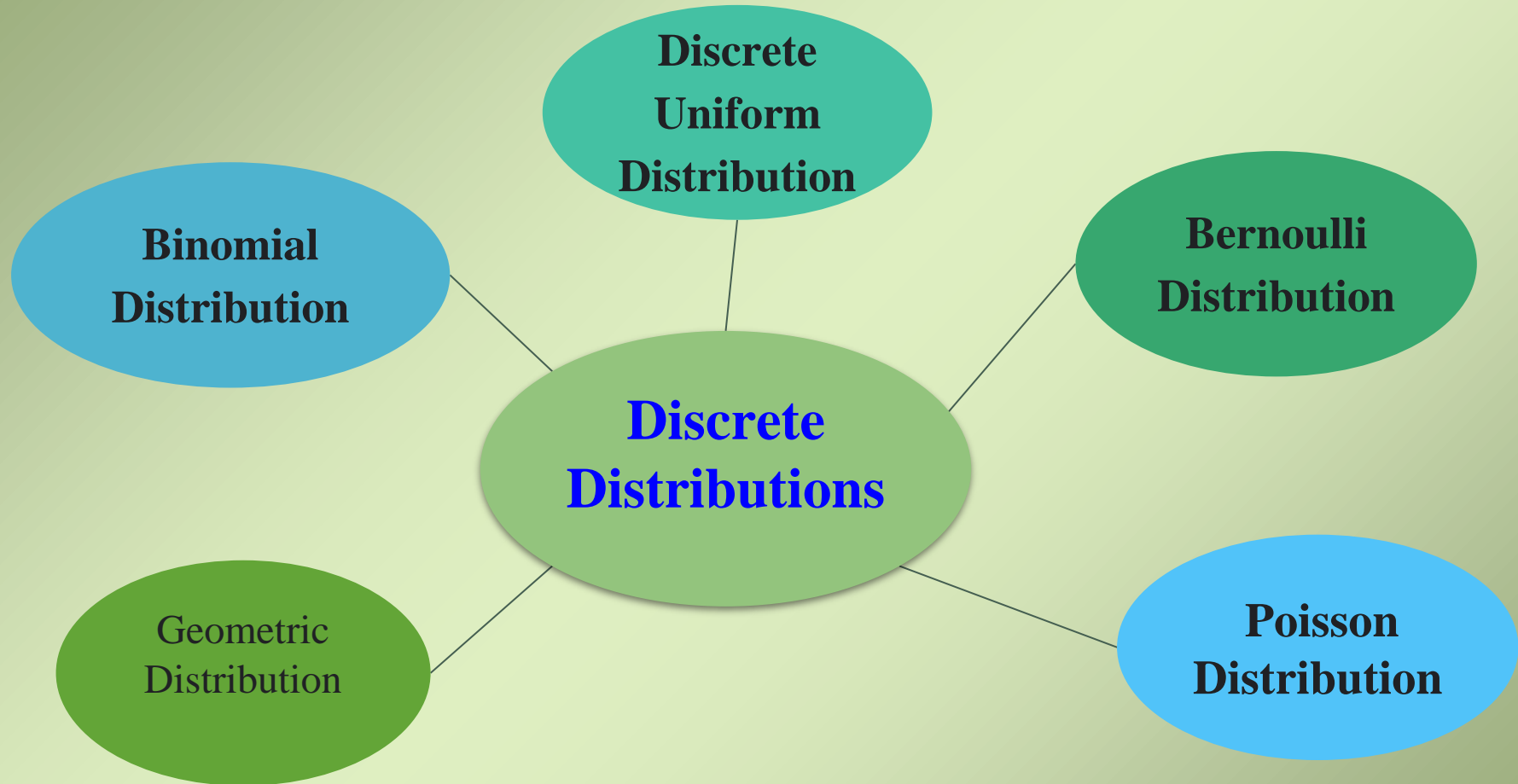
If a random variable X takes value x_1, x_2, \dots, x_n with the Prob. p_1, p_2, \dots, p_n then the Prob distribution is,

X	x_1	x_2	x_n
$P(X)$	p_1	p_2	p_n

X_i	0	1	2
$P(X_i=x_i)$?	0.3	0.5



Discrete Probability Distributions

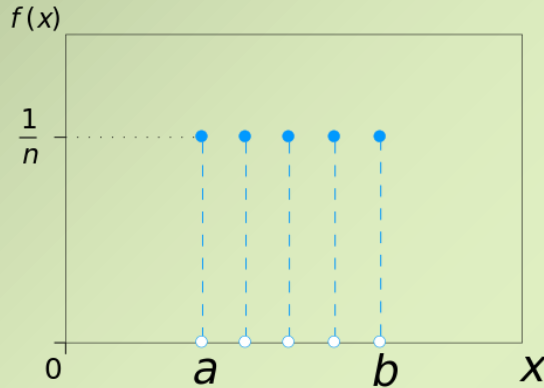


Discrete Uniform Distribution

A discrete uniform distribution is a distribution where finite no. of values are equally observed.

In discrete uniform distribution the probability remains same for each value.

Here, pmf = $P(X=x) = 1/n$, where $x = 1, 2, 3, \dots, n$.



Situation where its applicable

1. The birthday of a person may occur on either Sunday, Monday, ..., Saturday with the same probabilities $1/7$.



Bernoulli Distribution

A discrete random variable X in Bernoulli distribution takes value 1 with probability p and the value 0 with probability $q = 1 - p$.

Here we perform single experiment where we ask yes-no questions. For the outcome we get two values: yes is success with probability p and no is failure with probability q . In other words single success/failure experiment is called bernoulli trial.

Ex.

If $P(\text{yes}) = P(X=1) = 0.2$ then $P(\text{no}) = P(X=0) = 0.8$

$$\text{Pmf} = \begin{cases} q = 1 - p & \text{if } k = 0 \\ p & \text{if } k = 1 \end{cases} \quad \text{where} \quad 0 \leq p \leq 1$$

Situation where its applicable

1. Tossing of a coin.
2. Result of exam either pass or fail
3. Probability of having a boy child.



Factorial:-

The Product of first n natural numbers is called a factorial.

Factorial is denoted by the symbol “ ! “

$$n! = n*(n-1) *(n-2)*.....3*2*1.$$

Example:-

$$\begin{aligned} 5! &= 5*4*3*2*1 \\ &= 120 \end{aligned}$$

$$\begin{aligned} 3! &= 3*2*1 \\ &= 6. \end{aligned}$$

Combination:-

A group of same or all of given objects without considering their order is called combination and is given by,

$$nC_x = \frac{n!}{(n-x)! * x!}$$

$$\begin{aligned} nC_x &= \frac{n!}{(n-x)! * x!} \\ 5C_3 &= \frac{5!}{(5-3)! * 3!} \\ 5C_3 &= \frac{5 * 4 * 3 * 2 * 1}{2! * 3!} \\ 5C_3 &= \frac{5 * 4 * 3 * 2 * 1}{(2 * 1) * (3 * 2 * 1)} \\ 5C_3 &= \frac{60}{6} \\ 5C_3 &= 10 \end{aligned}$$

Binomial Distribution:-

Binomial distribution has two parameters 'n' and 'p'. The Binomial distribution is simply counting a things, ex. No of defective products.

Here we perform n independent experiment where we ask yes-no questions. For the outcomes we get two values: yes is success with probability p and no is failure with probability q. Here we're interested in calculating no. of successes probability.

$$\text{Pmf} = f(k, n, p) = \Pr(k; n, p) = \Pr(X = k) = \binom{n}{k} p^k (1 - p)^{n-k}$$

For $k = 0, 1, 2, \dots, n$ where,

$$\binom{n}{k} = \frac{n!}{k!(n-k)!} = \text{Different way of distributing } k \text{ successes in a sequence of } n \text{ trials.}$$

and P = probability of Success , q = probability of Failure, n = No of trials.

$P(X=k)$ means probability of getting exactly k successes in n independent Bernoulli trials.

k successes occur with probability p^k

$n - k$ failures occur with probability $(1 - p)^{n-k}$

Note: $n=1$, the Binomial distribution is a Bernoulli distribution.

Binomial Distribution:-

Situation Where its Applicable

1. Suppose it is known that 2% of all credit card transactions in a certain region are fraudulent. If there are 100 transactions per day in a certain region. What is the probability that 2 fraudulent transactions occur in a given day.
2. The likelihood that a patient with a heart attack dies of the attack is 0.04 (i.e., 4 of 100 die of the attack). Suppose we have 5 patients who suffer a heart attack, what is the probability 1 will die from the attack?

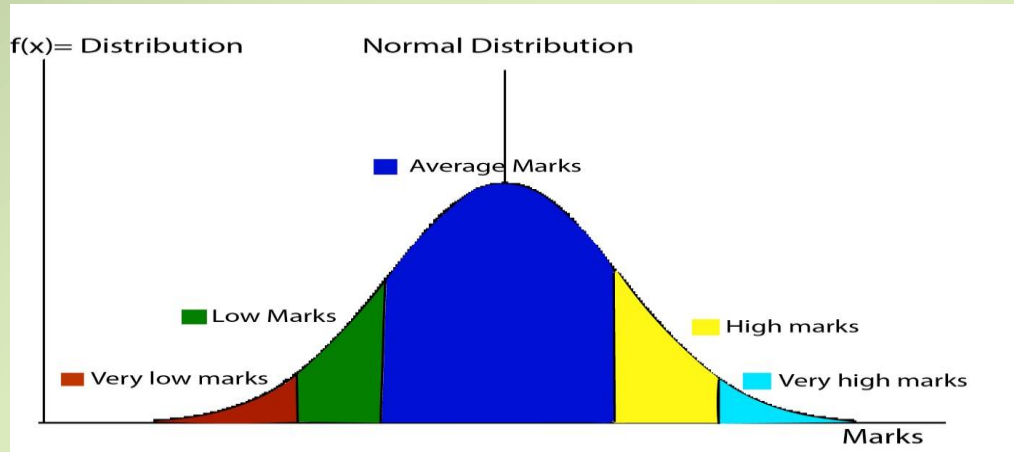


Continuous Probability Distributions

Normal Distribution:-

Normal Distribution is also called as Gaussian distribution. It is type of Continuous probability distribution.

It has two parameters μ is a mean and σ is a standard deviation.



Properties of Normal Distribution:-

1. The Normal Distribution is bell Shaped(Symmetric).
2. The mean, median and mode are equal for Normal Dist.
3. Most of the Discrete distribution tends to Normal Dist as N tends to infinity.

Examples:-

1. Weight of the newborn Babies.
2. Marks of the students.

