機統作業 HW7 資訊系 邱冠維 H34086149

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#7.12

$f(x_1, x_2) = f(x_1) f(x_2) = e^{-(x_1+x_2)}$, $x_1, x_2 > 0$

Inverse function: $x_1 = y_1 y_2$, $x_2 = y_1 - y_1 y_2$ (for $y_1 > 0$, $0 < y_2 < 1$)

$$J = \begin{vmatrix} \frac{\partial x_1}{\partial y_1} & \frac{\partial x_1}{\partial y_2} \\ \frac{\partial x_2}{\partial y_1} & \frac{\partial x_2}{\partial y_2} \end{vmatrix} = \begin{vmatrix} y_2 & y_1 \\ 1-y_2 & -y_1 \end{vmatrix} = -y_1$$

$g(y_1, y_2) = f(y_1 y_2, y_1 - y_1 y_2)\,|J| = y_1 e^{-y_1}$, $(y_1 > 0,\ 0 < y_2 < 1)$

$\Rightarrow g(y_1) = \int_0^1 y_1 e^{-y_1}\, dy_2 = y_1 e^{-y_1}$, $y_1 > 0$

$g(y_2) = \int_0^\infty y_1 e^{-y_1}\, dy_1 = \Gamma(2) = 1$, $0 < y_2 < 1$

$\because g(y_1, y_2) = g(y_1) g(y_2)$

$\therefore Y_1, Y_2$ are independent

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#7.14

$y = x^2 \to x = \pm\sqrt{y}$, $-1 < \pm\sqrt{y} < 1$, $0 < y < 1$

case 1. $x = \sqrt{y}$ $\qquad J = \frac{dx}{dy} = \pm\frac{1}{2\sqrt{y}}$

$g(y) = f(\sqrt{y}) \cdot \left|\frac{1}{2\sqrt{y}}\right| = \frac{1+\sqrt{y}}{4\sqrt{y}}$

case 2. $x = -\sqrt{y}$

$g(y) = f(-\sqrt{y}) \cdot \left|\frac{-1}{2\sqrt{y}}\right| = \frac{1-\sqrt{y}}{4\sqrt{y}}$

$\Rightarrow g(y) = \frac{1+\sqrt{y}}{4\sqrt{y}} + \frac{1-\sqrt{y}}{4\sqrt{y}} = \frac{1}{2\sqrt{y}}$ (for $0 < y < 1$) #

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#7.18

$$M_x(t) = E(e^{tx}) = p\sum_{x=1}^{\infty} e^{tx}q^{x-1} = \frac{p}{q}\sum_{x=1}^{\infty}(e^t q)^x = \frac{pe^t}{1-qe^t}$$

$$e^t \times q < 1 \rightarrow e^t < \frac{1}{q} \quad t < \ln q$$

$$M'_x(0) = \left.\frac{(1-qe^t)pe^t + pqe^{2t}}{(1-qe^t)^2}\right|_{t=0} = \frac{(1-q)p+pq}{(1-q)^2} = \frac{1}{p}$$

$$M''_x(0) = \left.\frac{(1-qe^t)^2pe^t + 2pqe^{2t}(1-qe^t)}{(1-qe^t)^4}\right|_{t=0} = \frac{2-p}{p^2}$$

$$\mu = M'_x(0) = \frac{1}{p} \ \#$$

$$\sigma^2 = M''_x(0) - \mu^2 = \frac{q}{p^2} \ \#$$

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7.22

$$M_x = (1-2t)^{-\frac{v}{2}}$$

$$M_x'(0) = v(1-2t)^{\frac{-v}{2}-1}\Big|_{t=0} = v$$

$$M_x''(0) = (v^2+2v)(1-2t)^{\frac{-v}{2}-2}\Big|_{t=0} = v^2+2v$$

$$\sigma^2 = M_x''(0) - (M_x'(0))^2 = 2v \ \#$$

Matlab 1_b

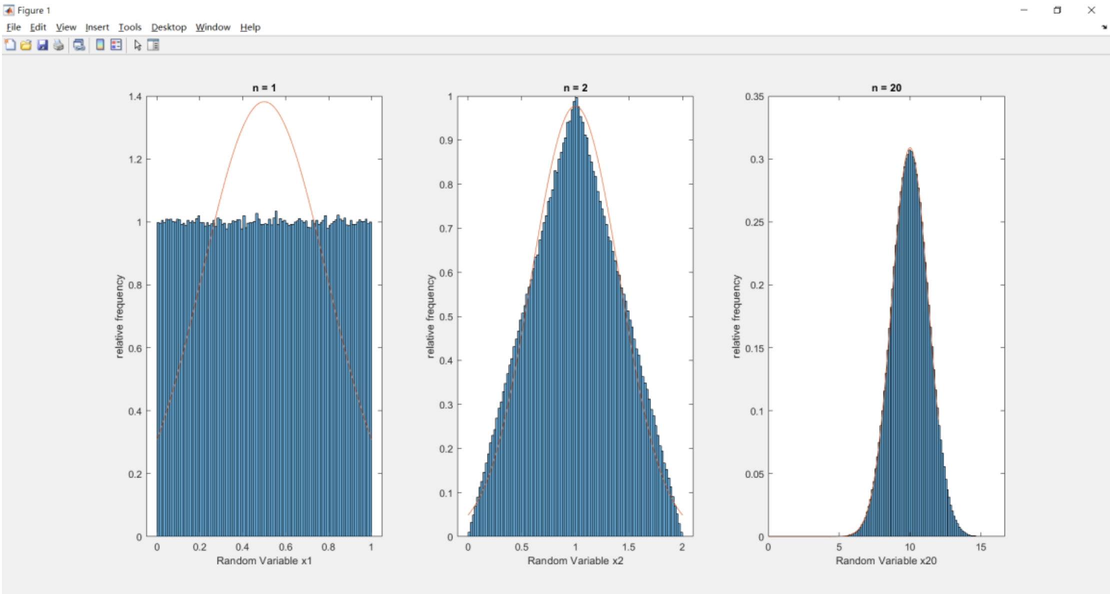

Figure 1
File Edit View Insert Tools Desktop Window Help
n = 1
relative frequency
Random Variable x1
n = 2
relative frequency
Random Variable x2
n = 20
relative frequency
Random Variable x20

Matlab 1_b

當 n=1 時，Irwin-Hall distribution 會是長方形的圖形，也就等於是平均分佈的圖形。用此圖形去近似常態分佈會有非常大的誤差。

當 n=2 時，Irwin-Hall distribution 會是三角形的圖形，對於接近平均值的資料會接近似於常態分佈，然而對於遠離平均的資料則不適合。

當 n=20 時，Irwin-Hall distribution 大約已經是常態分佈的圖形了，對於幾乎所有資料都很接近常態分佈。