date . . No.

4.24

(a) $E(X^2Y-2XY)=\sum_{x=0}^{3}\sum_{y=0}^{2}(x^2y-2xy)f(x,y)$

y \ x	0	1	2	3
0	0	$\frac{3}{70}$	$\frac{9}{70}$	$\frac{3}{70}$
1	$\frac{2}{70}$	$\frac{18}{70}$	$\frac{18}{70}$	$\frac{2}{70}$
2	$\frac{3}{70}$	$\frac{9}{70}$	$\frac{3}{70}$	0

$f(x,y)$

$E(x^2y-2xy)=1\cdot1\cdot(1-2)\cdot\frac{18}{70}+1\cdot2\cdot(1-2)\cdot\frac{9}{70}$
$+3\cdot1\cdot(3-2)\cdot\frac{2}{70}=-\frac{3}{7}$ #

(b) $\mu_x=E(x)=\sum x\cdot g(x)=0+\frac{30}{70}+2\cdot\frac{30}{70}+3\cdot\frac{5}{70}=1.5$

$\mu_y=E(y)=\sum y\cdot h(y)=0+\frac{40}{70}+2\cdot\frac{15}{70}=1$

$\mu_x-\mu_y=1.5-1=0.5$ #

4.44

$E(XY)=\sum_x\sum_y xyf(x,y)=1\cdot1\cdot\frac{18}{70}+2\cdot1\cdot\frac{18}{70}+3\cdot1\cdot\frac{2}{70}$
$+1\cdot2\cdot\frac{9}{70}+2\cdot2\cdot\frac{3}{70}=\frac{9}{7}$

$\sigma_{xy}=E[(x-\mu_x)(y-\mu_y)]=(x-\mu_x)(y-\mu_y)f(x,y)$
$=E(xy)-\mu_x\mu_y$

由上一題得知 $\mu_x\mu_y=1.5\cdot1=1.5$

$\Rightarrow \sigma_{xy}=\frac{9}{7}-1.5=-\frac{3}{14}$ #

date No.

#4.60

y \ x	2	4
1	0.15	0.1
3	0.25	0.25
5	0.15	0.1

$f(x,y)$

$E(X) = 2\cdot 0.55 + 4\cdot 0.45 = 2.9$

$E(Y) = 1\cdot 0.25 + 3\cdot 0.5 + 5\cdot 0.25 = 3$

(a) $E(2X-3Y) = 2E(X) - 3E(Y) = -3.2$ #

(b) $E(XY) = E(X)\cdot E(Y) = 8.7$

#4.78 $P(\mu - 2\sigma < X < \mu + 2\sigma)$

$\sigma^2 = E(X^2) - [E(X)]^2$

$E(X) = \int_0^1 x\,30x^2(1-x)^2\,dx = \frac{1}{2}$

$E(X^2) = \int_0^1 x^2\,30x^2(1-x)^2\,dx = \frac{2}{7}$

$\sigma^2 = \frac{2}{7} - \frac{1}{4} = \frac{1}{28} \Rightarrow \sigma = \frac{1}{2\sqrt{7}}$

$P(\mu - 2\sigma < X < \mu + 2\sigma) = \int_{\frac{1}{2}-\frac{1}{\sqrt{7}}}^{\frac{1}{2}+\frac{1}{\sqrt{7}}} 30x^2(1-x)^2\,dx = 0.96998$ #

by chebbyshev's theorem

$P(\mu - 2\sigma < X < \mu + 2\sigma) > 1 - \frac{1}{2^2} = \frac{3}{4} = 0.75$ #

#4.98 $g(x)$:

(a) $\begin{cases} g(0) = \sum_{\lambda=0}^{2} f(0,\lambda) = 0.2 \\ g(1) = \sum_{\lambda=0}^{2} f(1,\lambda) = 0.32 \\ g(2) = \sum_{\lambda=0}^{2} f(2,\lambda) = 0.48 \\ g(x) = 0 \quad \text{if } x \neq 0,1,2 \end{cases}$

$h(y)$:

$\begin{cases} h(0) = \sum_{\lambda=0}^{2} f(\lambda,0) = 0.26 \\ h(1) = \sum_{\lambda=0}^{2} f(\lambda,1) = 0.35 \\ h(2) = \sum_{\lambda=0}^{2} f(\lambda,2) = 0.39 \\ h(y) = 0 \quad \text{if } y \neq 0,1,2 \end{cases}$

(b) $E(X) = 0 \cdot 0.2 + 1 \cdot 0.32 + 2 \cdot 0.48 = 1.28$ #

$E(X^2) = 0^2 \cdot 0.2 + 1^2 \cdot 0.32 + 2^2 \cdot 0.48 = 2.24$

$Var(X) = 2.24 - 1.28^2 = 0.6016$ #

(c) $E(X|Y=2) = 1 \cdot \frac{5}{39} + 2 \cdot \frac{30}{39} = \frac{65}{39}$

$E(X^2|Y=2) = 1^2 \cdot \frac{5}{39} + 2^2 \cdot \frac{30}{39} = \frac{125}{39}$

$Var(X|Y=2) = \frac{125}{39} - \left(\frac{65}{39}\right)^2 = \frac{50}{117}$ #