Tutorial 3

Course: Operations Research-IT-205

1. Let $f : \mathbb{R} \to \mathbb{R}$ be given by $f(x) = (x-a)^4$, where $a \in \mathbb{R}$ is a constant. Apply Newton's method to the problem of minimizing of f.
   a. Write down equation for Newton's method.
   b. Let $y_k = |x_k - a|$. Show that the sequence $\{y_k\}$ satisfies $y_{k+1} = \frac{2}{3}y_k$.
   c. Show that x_k converges to x_0 for any initial guess x_0.

2. Let $f : \mathbb{R} \to \mathbb{R}$ be given by $f(x) = x^{\frac{4}{3}}$. Minimize f.
   a. Write down the algorithm for Newton's method applied to this problem.
   b. Show that as long as starting point is not zero, the algorithm in (a) does not converge to 0.

3. Consider the function $f : \mathbb{R}^2 \to \mathbb{R}$ given by

$$f(x) = \frac{3}{2}(x_1^2 + x_2^2) + 2x_1x_2 - (x_1 + x_2) + 1$$

   Apply Steepest descent method to find first approximation to local Minimizer. Start with $x_0 = (1, 1)$.

4. Maximize $x_2^2 + x_3^2$ subject to $x_1^2 + x_2^2 + x_3^2 - 1 = 0$

5. Maximize x_1x_2 subject to $x_1^2 + 4x_2^2 = 1$

6. Minimize $2x_1 + 3x_2 - 4$ subject to $x_1x_2 = 6$