Tutorial 1

Course: Operations Research-IT-205

1. The WorldLight Company produces two light fixtures (products 1 and 2) that require both metal frame parts and electrical components. Management wants to determine how many units of each product to produce so as to maximize profit. For each unit of product 1, 1 unit of frame parts and 2 units of electrical components are required. For each unit of product 2, 3 units of frame parts and 2 units of electrical components are required. The company has 200 units of frame parts and 300 units of electrical components. Each unit of product 1 gives a profit of Rs 100, and each unit of product 2, up to 60 units, gives a profit of Rs 200. Any excess over 60 units of product 2 brings no profit, so such an excess has been ruled out.
   (a) Formulate a linear programming model for this problem.
   (b) Use the graphical method to solve this model. What is the resulting total profit?

2. This is your lucky day. You have just won a Rs 1,00,000 prize. You are setting aside Rs 40,000 for expenses, but you have decided to invest the other Rs 60,000. Upon hearing this news, two different friends have offered you an opportunity to become a partner in two different entrepreneurial ventures, one planned by each friend. In both cases, this investment would involve expending some of your time next summer as well as putting up cash. Becoming a full partner in the first friends venture would require an investment of Rs 50,000 and 400 hours, and your estimated profit (ignoring the value of your time) would be Rs 40,500. The corresponding figures for the second friends venture are Rs 40,000 and 500 hours, with an estimated profit to you of Rs 40,500. However, both friends are flexible and would allow you to come in at any fraction of a full partnership you would like. If you choose a fraction of a full partnership, all the above figures given for a full partnership (money investment, time investment, and your profit) would be multiplied by this same fraction. Because you were looking for an interesting summer job anyway (maximum of 600 hours), you have decided to participate in one or both friends ventures in whichever combination would maximize your total estimated profit. You now need to solve the problem of finding the best combination.
   (a) Formulate a linear programming model for this problem.
   (b) Use the graphical method to solve this model. What is your total estimated profit?

3. Use the graphical method to solve this problem:

$$\text{Minimize} \quad Z = 3x_1 + 2x_2$$

$$\text{subject to} \quad x_1 + 2x_2 \leq 12$$

$$2x_1 + 3x_2 = 12$$

$$2x_1 + x_2 \geq 8$$

$$x_1 \geq 0, x_2 \geq 0.$$

4. Consider the optimization problem:

$$\text{minimize } Z = c_1|x_1| + \cdots + c_n|x_n|$$
$$\text{subject to } Ax = b$$

where $c_i \neq 0,\ i = 1, \ldots, n$. Convert the above problem into an equivalent standard form.

5. Consider the system of equations:

$$\begin{bmatrix} 2 & -1 & 2 & -1 & 3 \\ 1 & 2 & 3 & 1 & 0 \\ 1 & 0 & -2 & 0 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 14 \\ 5 \\ -10 \end{bmatrix}$$

Check if the system has basic solutions. If yes, find all basic solutions.

6. Consider the linear programming model

$$\text{Maximize } \ Z = x_1 + 2x_2$$
$$\text{subject to } \quad x_1 + 3x_2 \leq 8$$
$$x_1 + x_2 \leq 4$$
$$x_1 \geq 0, x_2 \geq 0$$

(a) Use graphical analysis to identify all the corner-point solutions for this model. Label each as either feasible or infeasible.
(b) Calculate the value of the objective function for each of the CPF solutions. Use this information to identify an optimal solution.
(c) Solve the problem using simplex method.

7. Consider the following problem:

$$\text{Maximize } Z = 2x_1 + 4x_2 + 3x_3$$
$$\text{subject to } \ x_1 + 3x_2 + 2x_3 \leq 30$$
$$x_1 + x_2 + x_3 \leq 24$$
$$3x_1 + 5x_2 + 3x_3 \leq 60$$
$$\text{and } \ x_1, x_2, x_3 \geq 0$$

You are given the information that $x_1 > 0, x_2 = 0, , x_3 > 0$ in the optimal solution.
(a) Describe how you can use this information to adapt the simplex method to solve this problem in the minimum possible number of iterations (when you start from the usual initial BF solution).

8. Consider the following problem.

$$\text{maximize } Z = 5x_1 + x_2 + 3x_3 + 4x_4$$

$$\text{subject to } \; x_1 - 2x_2 + 4x_3 + 3x_4 \leq 20$$

$$-4x_1 + 6x_2 + 5x_3 - 4x_4 \leq 40$$

$$2x_1 - 3x_2 + 3x_3 + 8x_4 \leq 50$$

$$x_i \geq 0, \quad i = 1, ..., 4$$

Work through the simplex method step by step to demonstrate that Z is unbounded.

9. Consider the following problem.

$$\text{Minimize } Z = 2x_1 + x_2 + 3x_3$$

$$\text{subject to } \; 5x_1 + 2x_2 + 7x_3 = 420$$

$$3x_1 + 2x_2 + 5x_3 \geq 280$$

$$x_i \geq 0, \quad i = 1, 2, 3$$

Solve using two-phase simplex method.