Tutorial 2

Course: Operations Research-IT-205

1. Consider a standard form linear programming problem (i.e., Maximize $Z = \sum c_i x_i$ s.t. $Ax = b$) Suppose that it has the following canonical tableau:

						b
c	0	-4	0	0	4	-8
	0	1	0	1	-1	5
A	1	2	0	0	-2	6
	0	3	1	0	-3	7

(a) Find the basic feasible solution corresponding to the above canonical tableau, and the corresponding value of the objective function.
(b) Find all the reduced cost coefficient values associated with the above canonical tableau.
(c) Does the given linear programming problem have feasible solutions with arbitrarily positive objective function values?
(d) Suppose column 2 enters the basis. Find the canonical tableau for the new basis.

2. Consider the following problem.

$$\text{Maximize } Z = -x_1 - 2x_2 - x_3$$

$$\text{subject to } x_1 + x_2 + 2x_3 \leq 12$$

$$x_1 + x_2 - x_3 \leq 1$$

$$x_1, x_2, x_3 \geq 0$$

(a) Construct the dual problem.
(b) Use duality theory to show that the optimal solution for the primal problem has $Z \leq 0$.

3. Consider the following problem.

$$\text{Maximize } Z = x_1 + 2x_2$$

$$\text{subject to } -x_1 + x_2 \leq -2$$

$$4x_1 + x_2 \leq 4$$

$$x_1, x_2 \geq 0$$

(a) Demonstrate graphically that this problem has no feasible solutions.
(b) Construct the dual problem.
(c) Demonstrate graphically that the dual problem has an unbounded objective function.

4. Define the functions $f : \mathbb{R}^2 \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}^2$ by $f(x_1, x_2) = (x_1^2/6 + x_2^2/4)$, $g(t) = [3t+5, 2t-6]^T$. Let $F : \mathbb{R} \to \mathbb{R}$ be given by $F(t) = f(g(t))$. Evaluate $\frac{dF}{dt}(t)$ using the chain rule.

5. Suppose that $f(x) = o(g(x))$. Show that for any given $\epsilon > 0$, there exists $\delta > 0$ such that if $||x|| < \delta$, then $||f(x)|| < \epsilon|g(x)|$.

6. Use above exercise to show that if functions $f : \mathbb{R}^n \to \mathbb{R}$ and $g : \mathbb{R}^n \to \mathbb{R}$ satisfy $f(x) = -g(x) + o(g(x))$ and $g(x) > 0$ for all $x \neq 0$, then for all $x \neq 0$ sufficiently small, we have $f(x) < 0$.

7. Write down the Taylor series expansion of the following functions about the given points x_0. Neglect terms of order three or higher.
(a) $f(x) = x_1 e^{-x_2} + x_2 + 1$, $x_0 = [1, 0]^T$
(b) $f(x) = x_1^4 + 2x_1^2 x_2^2 + x_2^4$, $x_0 = [1, 1]^T$

8. Consider the problem

$$\text{Minimize } f(x)$$

$$\text{subject to } x \in \Omega,$$

where $f \in C^2$. For each of the following specifications for Ω, x^* and f, determine if the given point x^* is
(i) definately a local minimizer,
(ii) definately not a local minimizer, or
(iii) possibly a local minimizer.
Justify your answer.
(a) $f : \mathbb{R}^2 \to \mathbb{R}, \Omega = \{x = [x_1, x_2]^T | x_1 \geq 1\}, x^* = [1, 2]^T$ and $\nabla f(x^*) = [1, 1]^T$
(b) $f : \mathbb{R}^2 \to \mathbb{R}, \Omega = \{x = [x_1, x_2]^T | x_1 \geq 1, x_2 \geq 2\}, x^* = [1, 2]^T$ and $\nabla f(x^*) = [1, 0]^T$.
(c) $f : \mathbb{R}^2 \to \mathbb{R}, \Omega = \{x = [x_1, x_2]^T | x_1 \geq 0, x_2 \geq 0\}, x^* = [1, 2]^T$ and $\nabla f(x^*) = [0, 0]^T$, and Hessian $F(x^*) = I$ (Identity matrix)
(d) $f : \mathbb{R}^2 \to \mathbb{R}, \Omega = \{x = [x_1, x_2]^T | x_1 \geq 1, x_2 \geq 2\}, x^* = [1, 2]^T$ and $\nabla f(x^*) = [1, 0]^T$, and Hessian

$$F(x^*) = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

9. Consider the function $f : \mathbb{R}^2 \to \mathbb{R}$ given below:

$$F(x) = x^T \begin{bmatrix} 1 & 2 \\ 4 & 7 \end{bmatrix} x + x^T \begin{bmatrix} 3 \\ 5 \end{bmatrix} + 6$$

(a) Find the gradient and Hessian of f at the point $[1, 1]^T$
(b) Find the directional derivative of f at $[1, 1]^T$ with respect to a unit vector in the direction of maximal rate of increase.
(c) Find a point that satisfies the FONC (interior case) for f. Does this point satisfy the SONC (for a minimizer)?