

Meshing of Triply-Periodic Smooth Surfaces in CGAL

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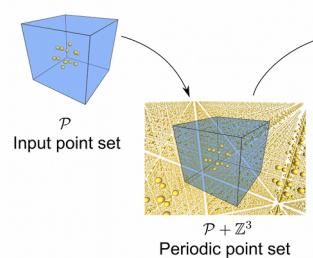
Delaunay triangulations in \mathbb{T}^3 [CT09]

point set
 \downarrow
 $DT(\mathcal{P} + \mathbb{Z}^3)$
infinite periodic point set
Delaunay triangulation

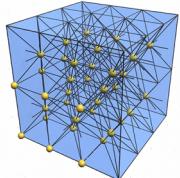
$$\mathbb{T}^3 = \mathbb{R}^3 / \mathbb{Z}^3 \quad \text{flat torus}$$

$$\pi : \mathbb{R}^3 \rightarrow \mathbb{T}^3 \quad \text{quotient map}$$

Definition: $DT_{\mathbb{T}}(\mathcal{P}) := \pi(DT(\mathcal{P} + \mathbb{Z}^3))$
Delaunay triangulation of \mathcal{P} in \mathbb{T}^3 ,
if it is a **simplicial complex**.



There is a set \mathcal{Q} of 36 points
such that $DT_{\mathbb{T}}(\mathcal{Q} \cup \mathcal{P})$
is a **simplicial complex**
for any point set \mathcal{P} .



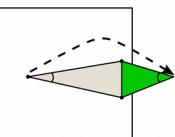
Algorithm

1. Initialize with point set \mathcal{Q} of 36 points
2. Insert points one by one
into the Delaunay triangulation

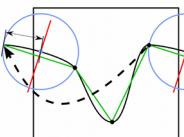
CGAL Periodic Triangulations [CT10]

- generic, robust, fully dynamic
- Voronoi functionality
- randomized worst-case optimal $O(n^2)$
- efficient in practice

Periodic refinement criteria



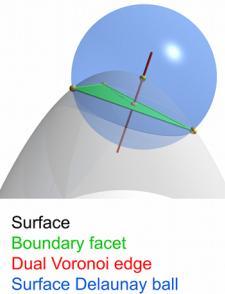
Aspect criterion
translate the vertex to
compute the angle



Uniform size criterion
translate the vertex to
compute the **sphere radius**

Curvature criterion
translate the **facet center**
to compute the distance
to the surface

Mesher surfaces [BO05]



Refinement criteria

Aspect criterion

- lower bound on minimum angle of **boundary facet**

Uniform size criterion

- upper bound on radius of **surface Delaunay ball**

Curvature criterion

- upper bound on distance between center of a **boundary facet** and center of its **surface Delaunay ball**

Given a C^2 -continuous surface S without boundary,
the algorithm generates a triangulated surface with

- the same topological type as S
- close to S for the Hausdorff distance
- good approximation for normals, areas, and curvature



Algorithm

bad facets:
boundary facets that do not
meet the refinement criteria

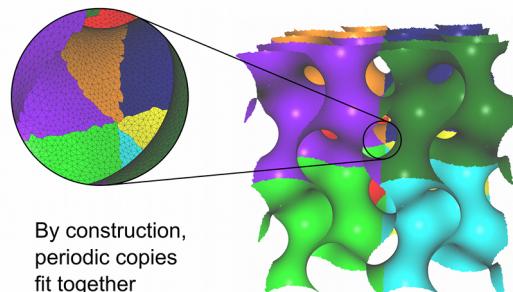
1. Initialization step
Find points on surface and construct their Delaunay triangulation.
2. Refinement step
For each **bad facet**, insert the center of its surface Delaunay ball
into the Delaunay triangulation.

Triangulation

CGAL Surface Mesher [RY10]

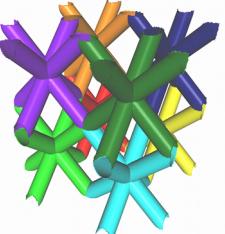
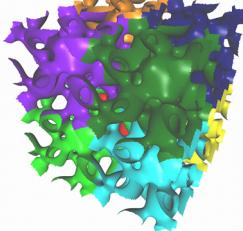
Criteria

Result

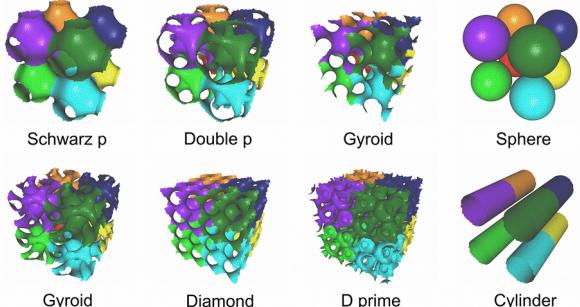


Examples

Angular bound 30°, Radius bound 0.01, Distance bound 0.01:



Angular bound 30°, Radius bound 0.03, Distance bound 0.03:



Current work: Volume meshes

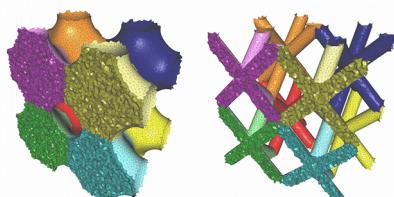
Facet criteria

Angular bound: 30°
Radius bound: 0.03
Distance bound: 0.03

Cell criteria

Radius-edge bound: 2
Radius bound: 0.03

(Joint work with Mikhail Bogdanov)



References

- [BO05] Jean-Daniel Boissonnat and Steve Oudot. Provably good surface sampling and approximation. In *Graphical Models*, 67:405–451, 2005.
- [CT09] Manuel Caroli and Monique Teillaud. Computing 3D Periodic Triangulations. In *Proceedings of the 17th European Symposium on Algorithms*, LNCS 5757, pages 37–48, 2009.
- [CT10] Manuel Caroli and Monique Teillaud. 3D Periodic Triangulations. In *CGAL 3.6 User and Reference Manual*, 2010.
- [RY10] Laurent Rineau and Mariette Yvinec. 3D Surface Mesh Generation. In *CGAL 3.6 User and Reference Manual*, 2010.

This work was partially sponsored by the ANR (Agence Nationale de la Recherche) under the "Triangles" project of the Programme Blanc (No. ANR-07-BLAN-0319) <http://www-sop.inria.fr/geometrie/collaborations/triangles>