

# Algebraic Actions

- Commutative Swap
  - Users may swap the positions of two nodes in the same tag quadrant. The resulting tag is equivalent. There is no limit to how many times the user can do this.
- Associative Tag Merge
  - If an inner tag has the same orientation as its enclosing tag, the elements in the inner tag can be extracted and put into the outer tag i.e. an NS tag in an NS tag or an EW tag in an EW tag.
    - When merging an EW tag inside an EW tag, if the inner EW tag contains elements in the W quadrant, then that element will be moved into the W quadrant of the outer tag.
    - The same works with NS tags
- Associative Tag Introduction
  - Elements in the same tag quadrant may be put in another tag of the same orientation e.i. Two elements in the N quadrant of a NS tag may be grouped in an NS tag.
- Distribute
  - When an addition tag and an element (i.e. literal, variable, or another tag) is inside a multiplication tag, the element may be dragged into that addition tag causing that element to be distributed through.
- Splitting fractions
  - When an EW tag is enclosed in a NS tag with an element in the S quadrant, then that NS tag may be split up
  - $(x + 3)/2 = x/2 + 3/2$
- Factor
  - When two or more elements in an addition tag have a common factor, that factor may be pulled out into a multiplication tag with the remaining addition tag
  - All elements in the tag must have that common factor. If a few elements have a common factor, then those elements must be enclosed using another EW tag.
- Combining fractions
  - When two or more NS tags with the same element in the S quadrant are in an EW tag, then they be combined into one NS tag.
  - $x/2 + 3/2 = (x + 3)/2$
- Cancelling
  - If an element appears both the “red” and “blue” sides of a tag, they can be dragged together and cancel out into the identity.
- Identity Laws
  - When adding a new element to a tag, adds in opposite side as well (only 1-0, but want to add x in, makes it  $(x+1)-(x)$ )

- When users introduce an element into an addition tag, that element will be added to the subtraction tag and vice versa.
  - When users introduce an element into a multiplication tag, that element will be added to the division tag and vice versa.
  - Any # of 0s in the addition tag can be freely added and removed.
  - Any # of 1s in the multiplication tag can be freely added and removed.
- Factor Literals
  - Z3
    - $2 \rightarrow 1 + 1$
    - $1 \rightarrow 2 + 2$
    - $0 \rightarrow 2 + 1$
    - $1 \rightarrow 2 * 2$
- Merge Literals
  - Z3
    - $1 + 2 = 0$
    - $1 + 1 = 2$
    - $2 + 2 = 1$
    - $2 * 2 = 1$
- Multiplicative Zero Introduction
  - $0 \rightarrow 0 * \text{anything}$
- Additive Zero Introduction
  - $\text{anything} \rightarrow 0 + \text{anything}$



