삼각함수 공식 정리

1. Trigonometric Functions of Special Angles (특수각일때의 함수값)

Angle	$\frac{\pi}{6}=30°$	$\frac{\pi}{4}=45°$	$\frac{\pi}{3}=60°$
sin	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
cos	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$
tan	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$

2. Functions in terms of Angles in the First Quadrant (일사분면각으로 표현한 함수)

n 이 정수일때,

	$-\alpha$	$\frac{\pi}{2}\pm\alpha$	$\pi\pm\alpha$	$\frac{3\pi}{2}\pm\alpha$	$2n\pi\pm\alpha$
sin	$-\sin\alpha$	$\cos\alpha$	$\mp\sin\alpha$	$-\cos\alpha$	$\pm\sin\alpha$
cos	$\cos\alpha$	$\mp\sin\alpha$	$-\cos\alpha$	$\pm\sin\alpha$	$+\cos\alpha$
tan	$-\tan\alpha$	$\mp\cot\alpha$	$\pm\tan\alpha$	$\mp\tan\alpha$	$\pm\cot\alpha$

3. Fundamental Identities (기본 항등식)

(1) Reciprocal relations (상호관계)

$$\sin\alpha=\frac{1}{\csc\alpha},\qquad \cos\alpha=\frac{1}{\sec\alpha},\qquad \tan\alpha=\frac{\sin\alpha}{\cos\alpha}=\frac{1}{\cot\alpha}$$

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(2) Pythagorean theorem (피타고라스 정리)

$$\sin^2 z + \cos^2 z = 1$$

$$\sec^2 z - \tan^2 z = 1$$

$$\csc^2 z - \cot^2 z = 1$$

(3) Product relations (곱의 관계)

$$\sin\alpha = \tan\alpha\cos\alpha, \qquad \sin\alpha = \tan\alpha\cos\alpha$$

$$\tan\alpha = \sin\alpha\sec\alpha, \qquad \cot\alpha = \cos\alpha\csc\alpha$$

$$\sec\alpha = \csc\alpha\tan\alpha, \qquad \csc\alpha = \sec\alpha\cot\alpha$$

(4) Quotient relations (지수관계)

$$\sin\alpha = \frac{\tan\alpha}{\sec\alpha}, \qquad \cos\alpha = \frac{\cot\alpha}{\csc\alpha}, \qquad \tan\alpha = \frac{\sin\alpha}{\cos\alpha}$$

4. One Circular Function in terms of Another (다른 삼각함수로 표현한 삼각함수)

	$\sin x$	$\cos x$	$\tan x$
$\sin x =$	$\sin x$	$\sqrt{1-\cos^2 x}$	$\frac{\tan x}{\sqrt{1+\tan^2 x}}$
$\cos x =$	$\sqrt{1-\sin^2 x}$	$\cos x$	$\frac{1}{\sqrt{1+\tan^2 x}}$
$\tan x =$	$\frac{\sin x}{\sqrt{1-\sin^2 x}}$	$\frac{\sqrt{1-\cos^2 x}}{\cos x}$	$\tan x$

5. Circulation Functions in terms of Exponentials (지수함수로형태로 나타낸 삼각함수)

$$\cos z = \frac{e^{iz} + e^{-iz}}{2}, \qquad e^{iz} = \cos z + i\sin z$$

$$\sin z = \frac{e^{iz} - e^{-iz}}{2i}, \qquad e^{-iz} = \cos z - i \sin z$$

$$\tan z = \frac{\sin z}{\cos z} = \frac{e^{iz} - e^{-iz}}{i(e^{iz} + e^{-iz})}$$

6. Angle Sum and Difference Relationships (각도 합과 차의 관계)

$$\sin(\alpha \pm \beta) = \sin\alpha \cos\beta \pm \cos\alpha \sin\beta$$

$$\cos(\alpha \pm \beta) = \cos\alpha \cos\beta \mp \sin\alpha \sin\beta$$

$$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha \tan\beta}$$

7. Multiple-angle Formulae (배각공식)

$$\sin 2\alpha = 2\sin\alpha\cos\alpha = \frac{2\tan\alpha}{1+\tan^2\alpha}$$

$$\cos 2\alpha = 2\cos^2\alpha - 1 = 1 - 2\sin^2\alpha = \cos^2\alpha - \sin^2\alpha = \frac{1-\tan^2\alpha}{1+\tan^2\alpha}$$

$$\tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha}$$

$$\sin 3\alpha = -4\sin^3\alpha + 3\sin\alpha$$

$$\cos 3\alpha = 4\cos^3\alpha - 3\cos\alpha$$

$$\tan 3\alpha = \frac{-\tan^3\alpha + 3\tan\alpha}{-3\tan^2\alpha + 1}$$

8. Half-Angle Formulae (반각공식)

$$\cos\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{2}}$$ ($\frac{\alpha}{2}$ 이 일사분면이나 사사분면에 있으면 양수,아니면 음수)

$$\sin\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{2}}$$ ($\frac{\alpha}{2}$ 이 일사분면이나 이사분면에 있으면 양수,아니면 음수)

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$$\tan\frac{\alpha}{2}=\frac{1-\cos\alpha}{\sin\alpha}=\frac{\sin\alpha}{1+\cos\alpha}=\pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}$$

($\frac{\alpha}{2}$ 이 일사분면이나 삼사분면에 있으면 양수,아니면 음수)

9. Powers of Circular Functions (삼각함수의 지수)

$$\sin^2\alpha=\frac{1}{2}(1-\cos 2\alpha),\qquad \cos^2\alpha=\frac{1}{2}(1+\cos 2\alpha)$$

$$\sin^3\alpha=\frac{1}{4}(-\sin 3\alpha+3\sin\alpha),\qquad \cos^3\alpha=\frac{1}{4}(\cos 3\alpha+3\cos\alpha)$$

$$\tan^2\alpha=\frac{1-\cos 2\alpha}{1+\cos 2\alpha},$$

10. Products of Sine and Cosine (곱의 공식)

$$\cos\alpha\cos\beta=\frac{1}{2}\cos(\alpha-\beta)+\frac{1}{2}\cos(\alpha+\beta)$$

$$\sin\alpha\sin\beta=\frac{1}{2}\cos(\alpha-\beta)-\frac{1}{2}\cos(\alpha+\beta)$$

$$\sin\alpha\cos\beta=\frac{1}{2}\sin(\alpha-\beta)+\frac{1}{2}\sin(\alpha+\beta)$$

11. Sums of Circular Functions (합의 공식)

$$\sin\alpha\pm\sin\beta=2\sin\frac{\alpha\pm\beta}{2}\cos\frac{\alpha\mp\beta}{2}$$

$$\cos\alpha+\cos\beta=2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\cos\alpha-\cos\beta=-2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$$

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