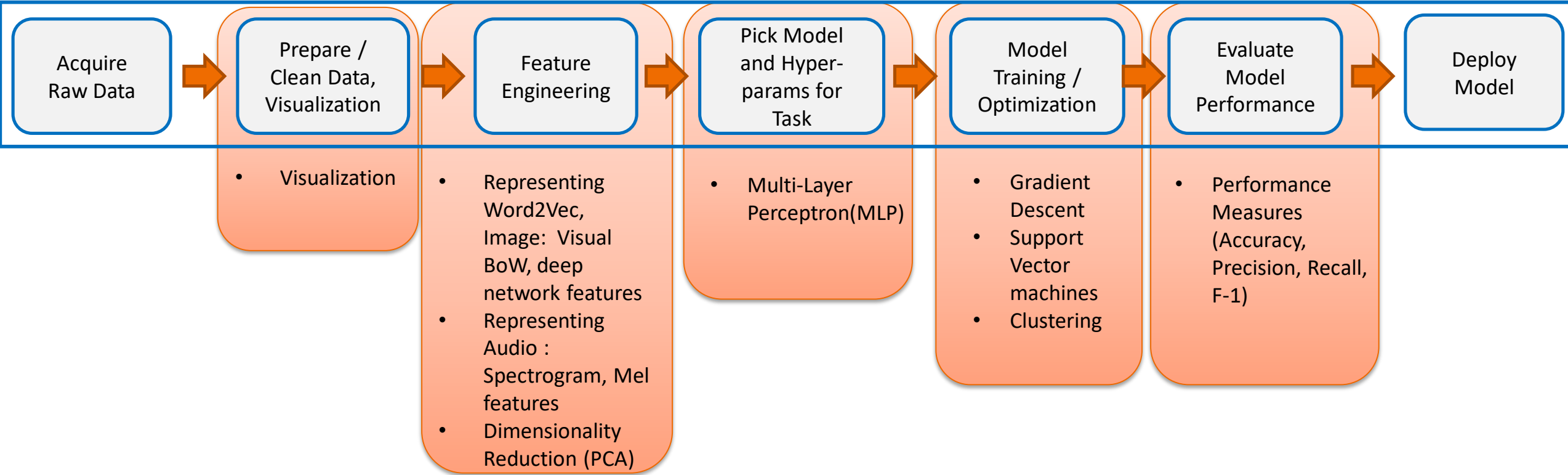


# Focus of this lecture



# Today's Plan

## Session 1:

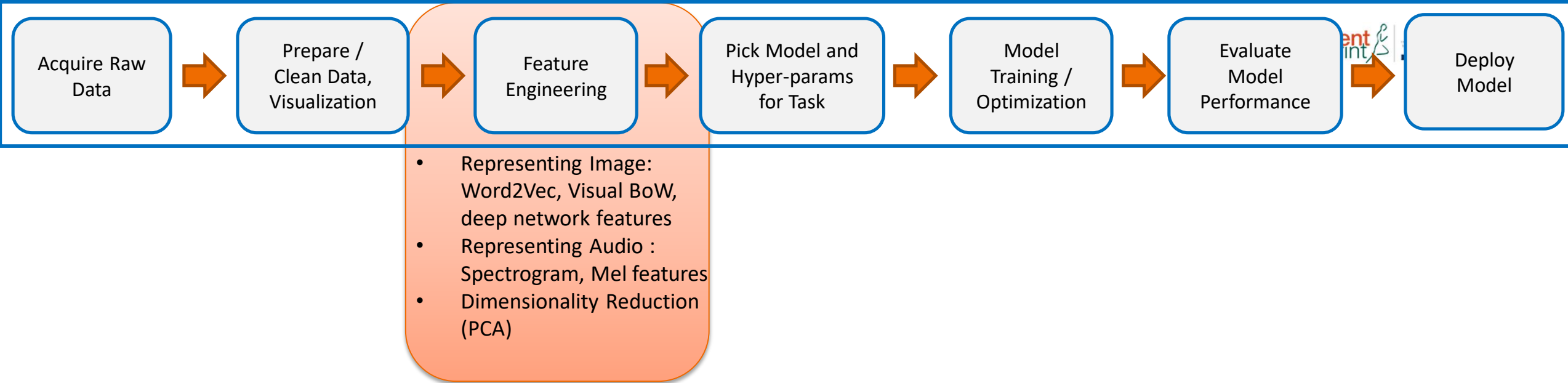
1. Representation
  - Word2Vec
  - Image: Visual BoW, deep network features
  - Audio : Spectrogram, Mel features
2. Dimensionality Reduction (PCA)
3. Visualization

## Session 2:

5. Performance Metrics
6. Gradient Descent
7. MLP
8. SVM with Kernels
9. Clustering

## Session 3:

- Paper Reading



## Feature Engineering

# Word2vec - Intuition

- Marco saw a furry little wampimuk hiding in the tree

- **What is Marco?**
- **What is wampimuk?**



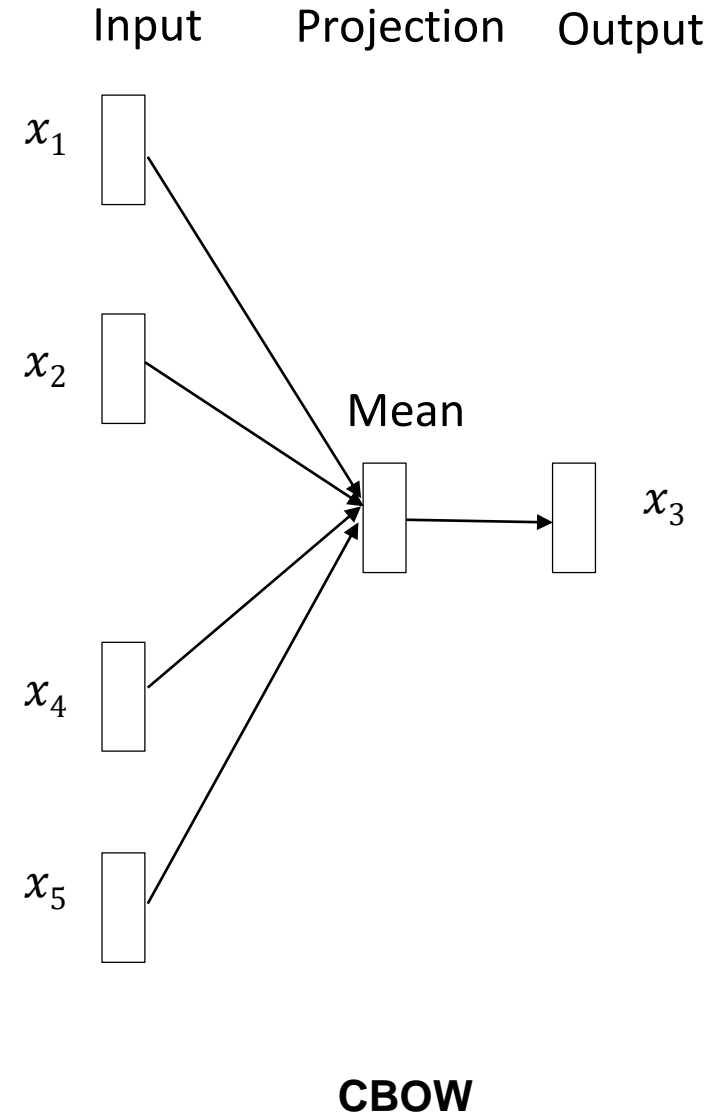
A person



Animal

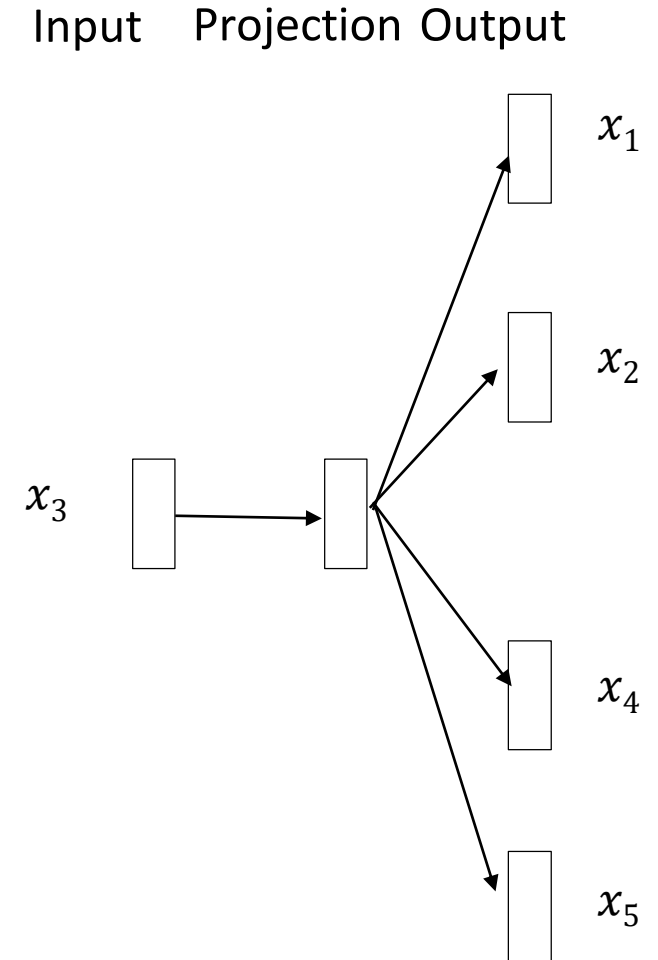
# Variants

- Continuous Bag of Words (CBOW):
  - use a window of words to predict the missing word



# Variants

- **Skip-gram (SG):**
  - use a word to predict the surrounding ones in the specified window.



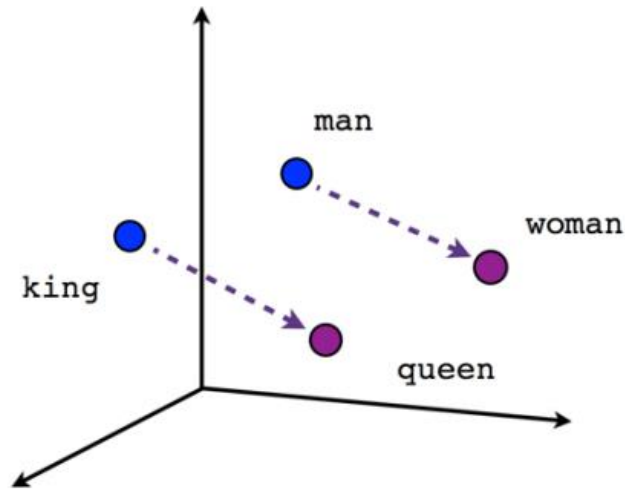
# Word2vec – skip gram examples

## Source Text

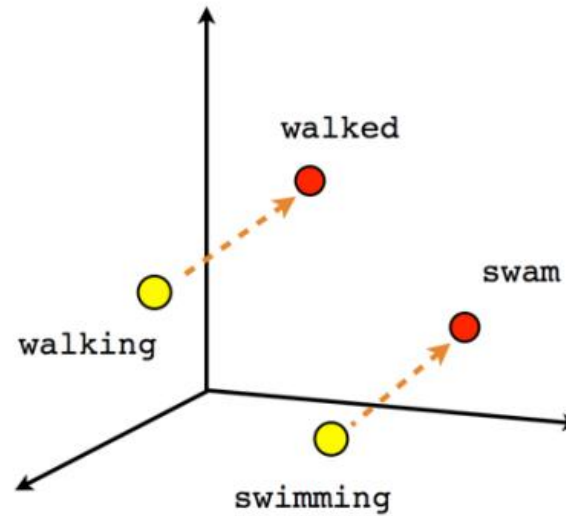
## Training Samples

The quick brown fox jumps over the lazy dog. →	(the, quick) (the, brown)
The quick brown fox jumps over the lazy dog. →	(quick, the) (quick, brown) (quick, fox)
The quick brown fox jumps over the lazy dog. →	(brown, the) (brown, quick) (brown, fox) (brown, jumps)
The quick brown fox jumps over the lazy dog. →	(fox, quick) (fox, brown) (fox, jumps) (fox, over)

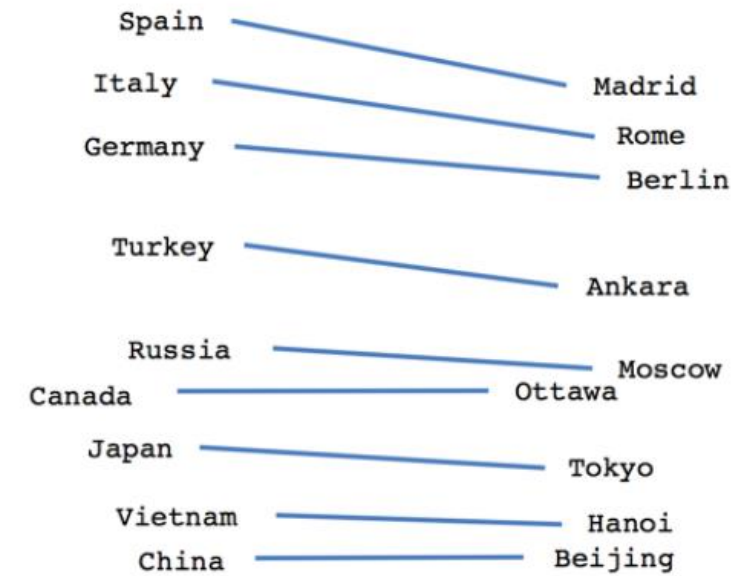
# Examples



Male-Female



Verb tense



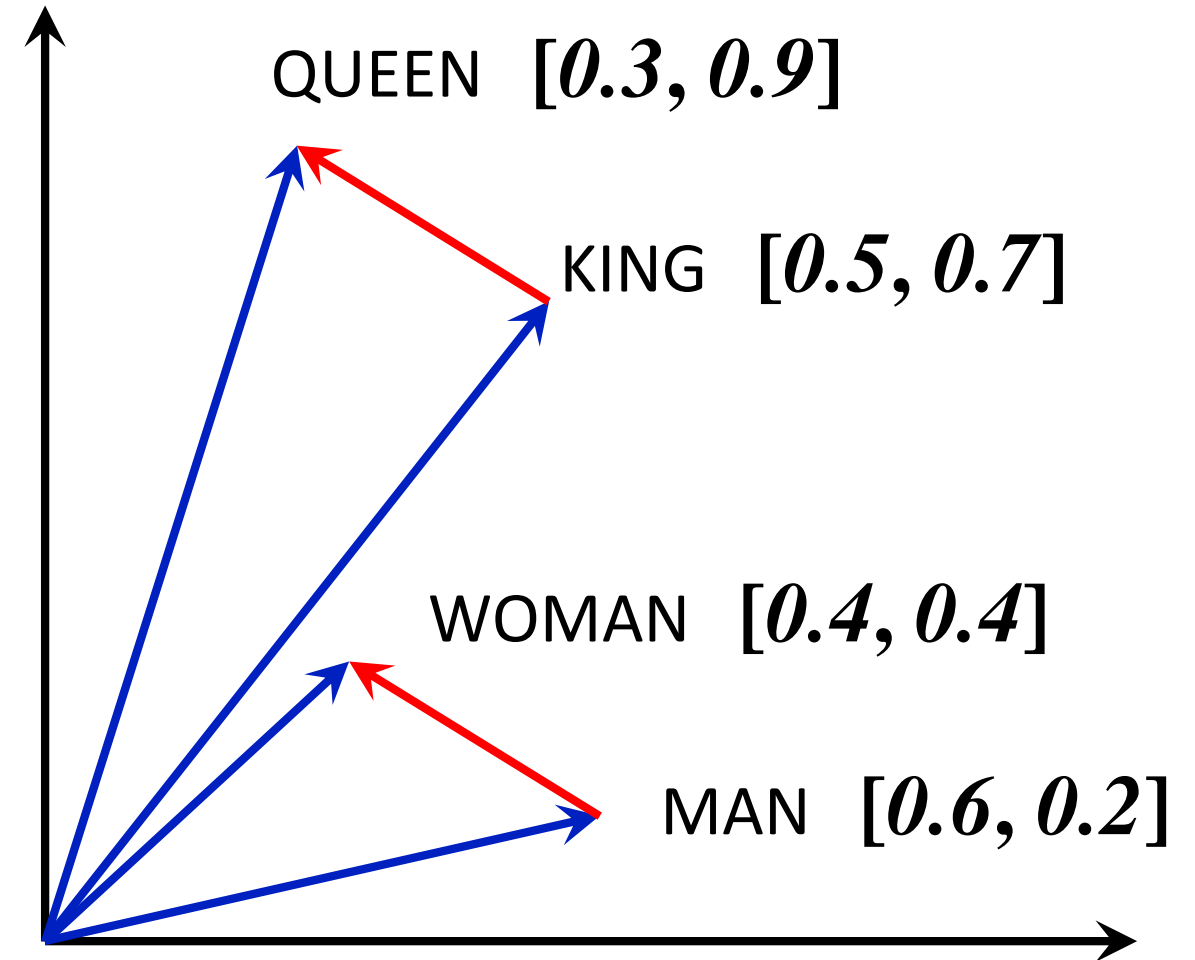
Country-Capital



# Visualising “Word-Arithmetic”

$$\begin{aligned} \text{Vec}(\text{Queen}) \\ = \text{Vec}(\text{King}) - \text{Vec}(\text{Man}) + \text{Vec}(\text{Woman}) \end{aligned}$$

- These fancy arithmetic were imagined (and shown) earlier also; but became popular with Word2Vec.



# More Examples

FRANCE	JESUS	XBOX	REDDISH	SCRATCHED	MEGABITS
AUSTRIA	GOD	AMIGA	GREENISH	NAILED	OCTETS
BELGIUM	SATI	PLAYSTATION	BLUISH	SMASHED	MB/S
GERMANY	CHRIST	MSX	PINKISH	PUNCHED	BIT/S
ITALY	SATAN	IPOD	PURPLISH	POPPED	BAUD
GREECE	KALI	SEGA	BROWNISH	CRIMPED	CARATS
SWEDEN	INDRA	PSNUMBER	GREYISH	SCRAPED	KBIT/S
NORWAY	VISHNU	HD	GRAYISH	SCREWED	MEGAHERTZ
EUROPE	ANANDA	DREAMCAST	WHITISH	SECTIONED	MEGAPIXELS
HUNGARY	PARVATI	GEFORCE	SILVERY	SLASHED	GBIT/S
SWITZERLAND	GRACE	CAPCOM	YELLOWISH	RIPPED	AMPERES

What words have embeddings closest to a given word? From Collobert et al. (2011) (<https://arxiv.org/pdf/1103.0398v1.pdf>)

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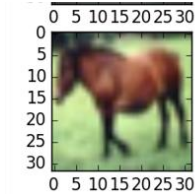
# AI and Problem of Perception

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# Possible Features: Handcrafting



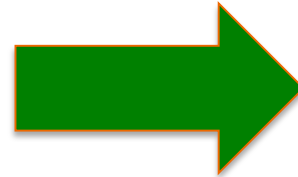
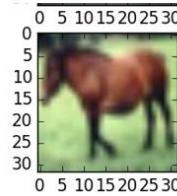
MIN RED
MAX RED
MEAN RED
MIN GREEN
MAX GREEN
MEAN GREEN
MIN BLUE
MAX BLUE
MEAN BLUE

**9 X 1  
FEATURE VECTOR  
PER IMAGE**

## Concerns:

- **Too naïve to capture the visual content?**
- **Too small to represent information?**

# Possible Features: Raw Data Itself



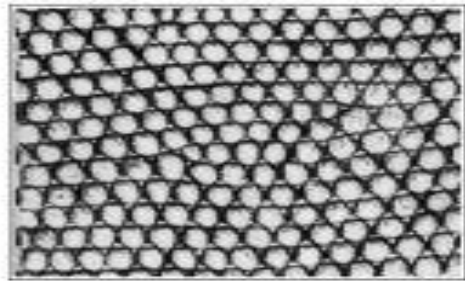
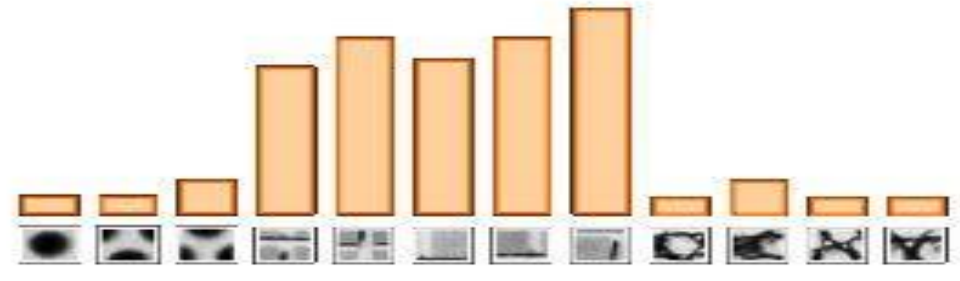
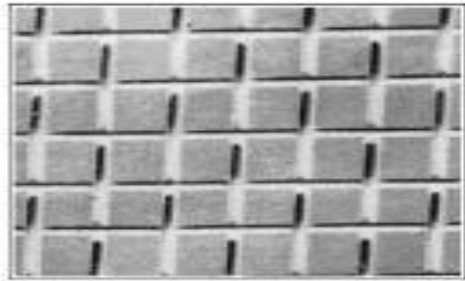
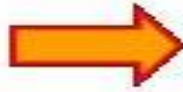
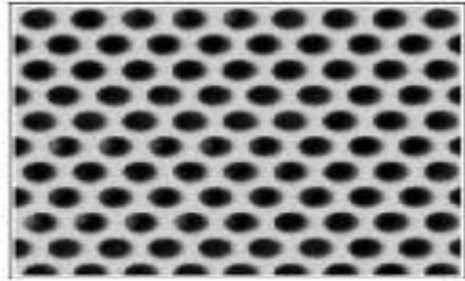
FEATURE VECTOR  
 $32 \times 32 \times 3 = 3072$   
 DIMENSION  
 PER IMAGE ( $d = 3072$ )

3072 X 1  
 vector

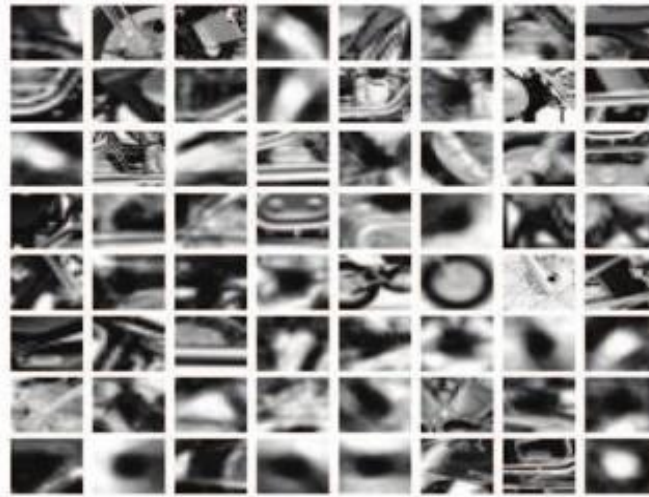
CONCERNS:

- Too big ?
- May be redundancy ?
- Too rigid?

# Visual BoW: Basic Idea



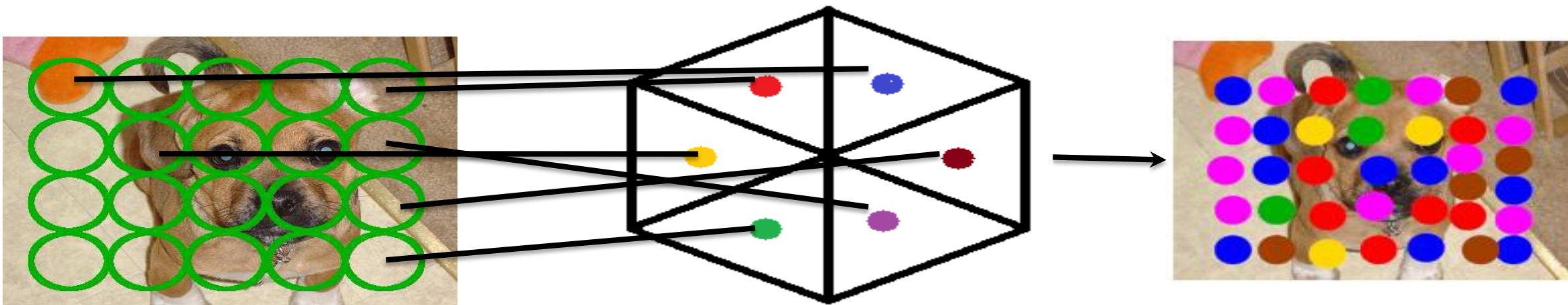
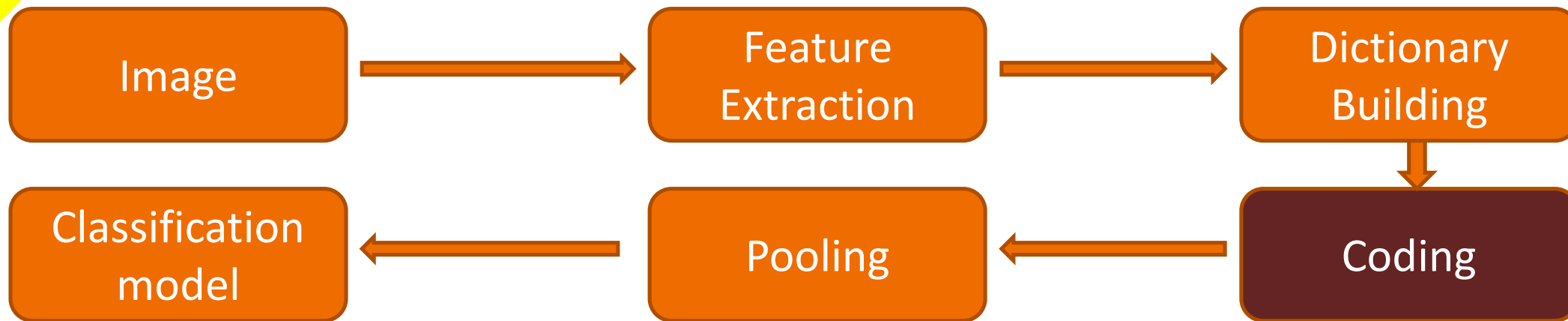
# Bag of Visual Words



**Learned Visual  
Vocabulary**

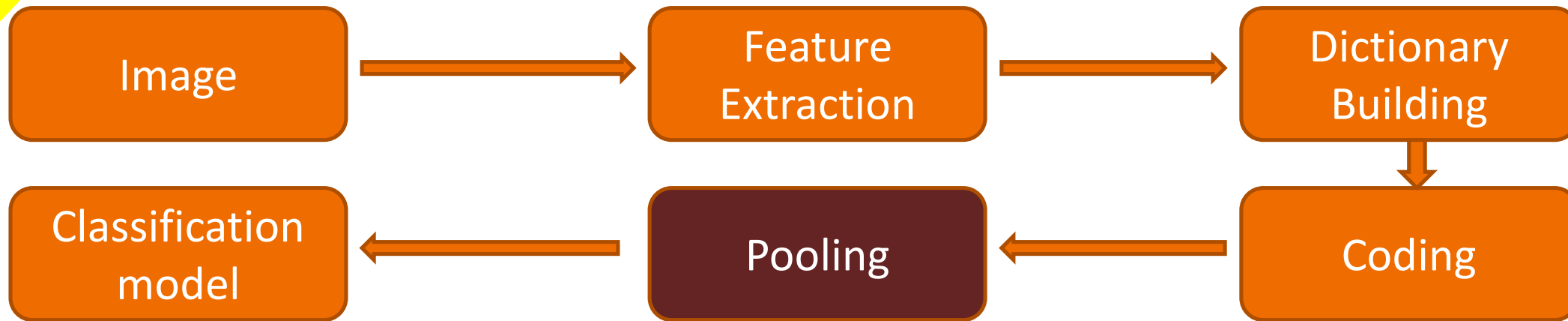




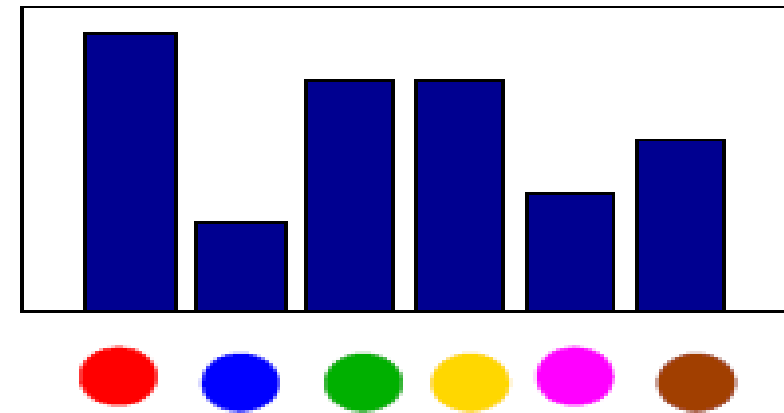


Dictionary/Codebook

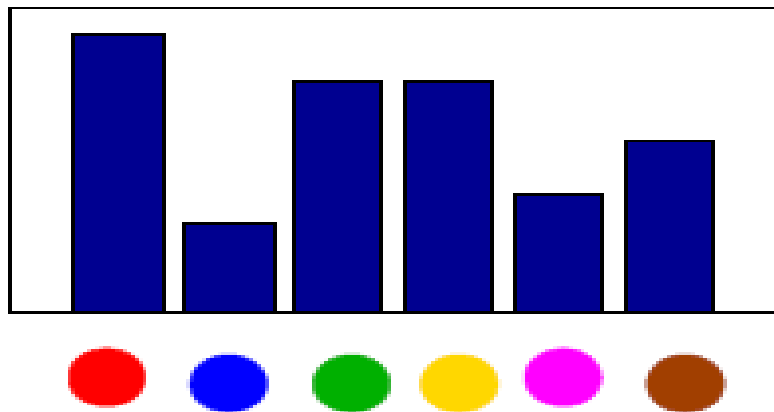
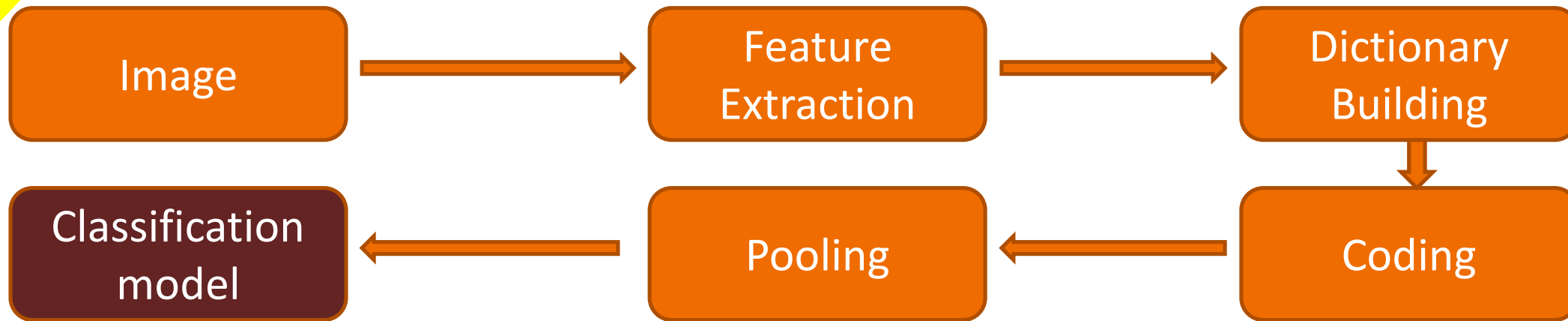




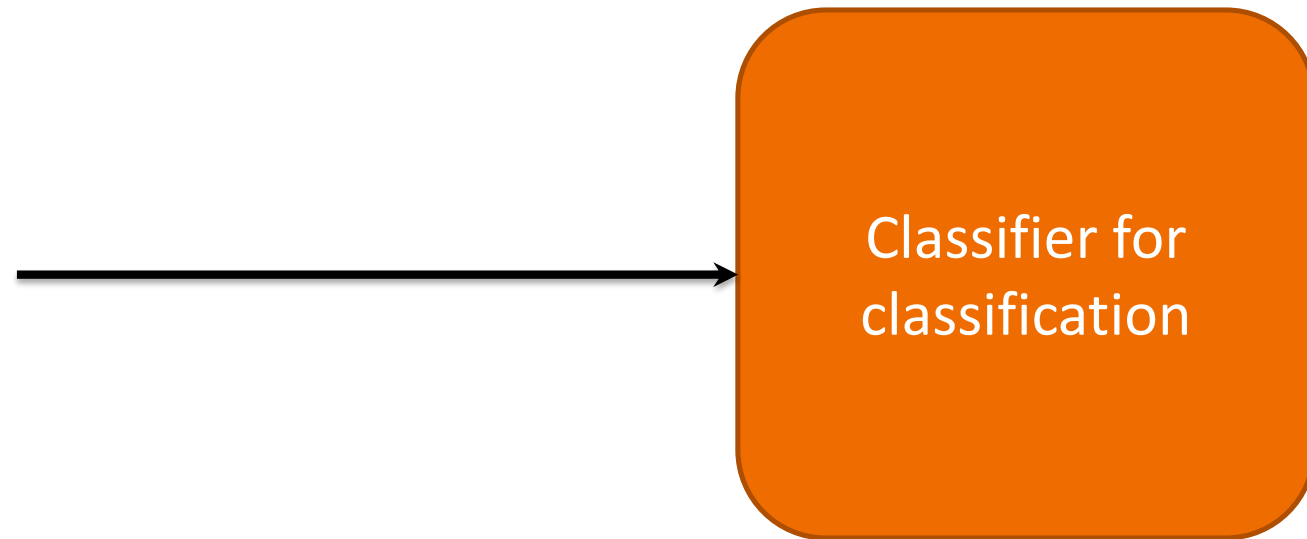
Pooling Function/  
Histogramming



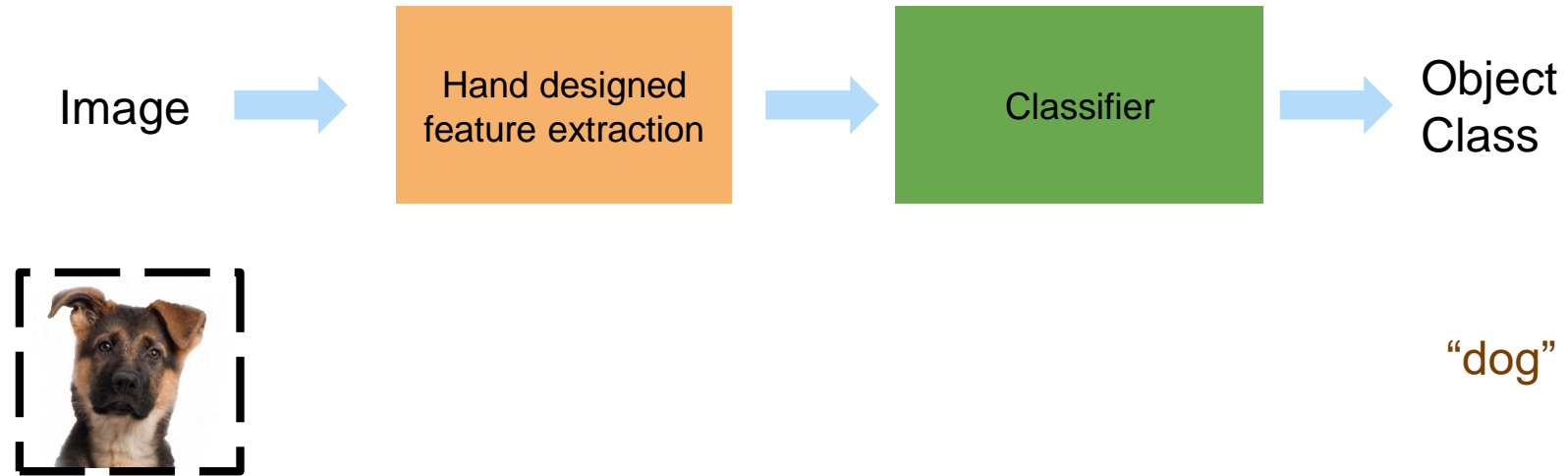
Histogram of Visual Words /  
Image Representation



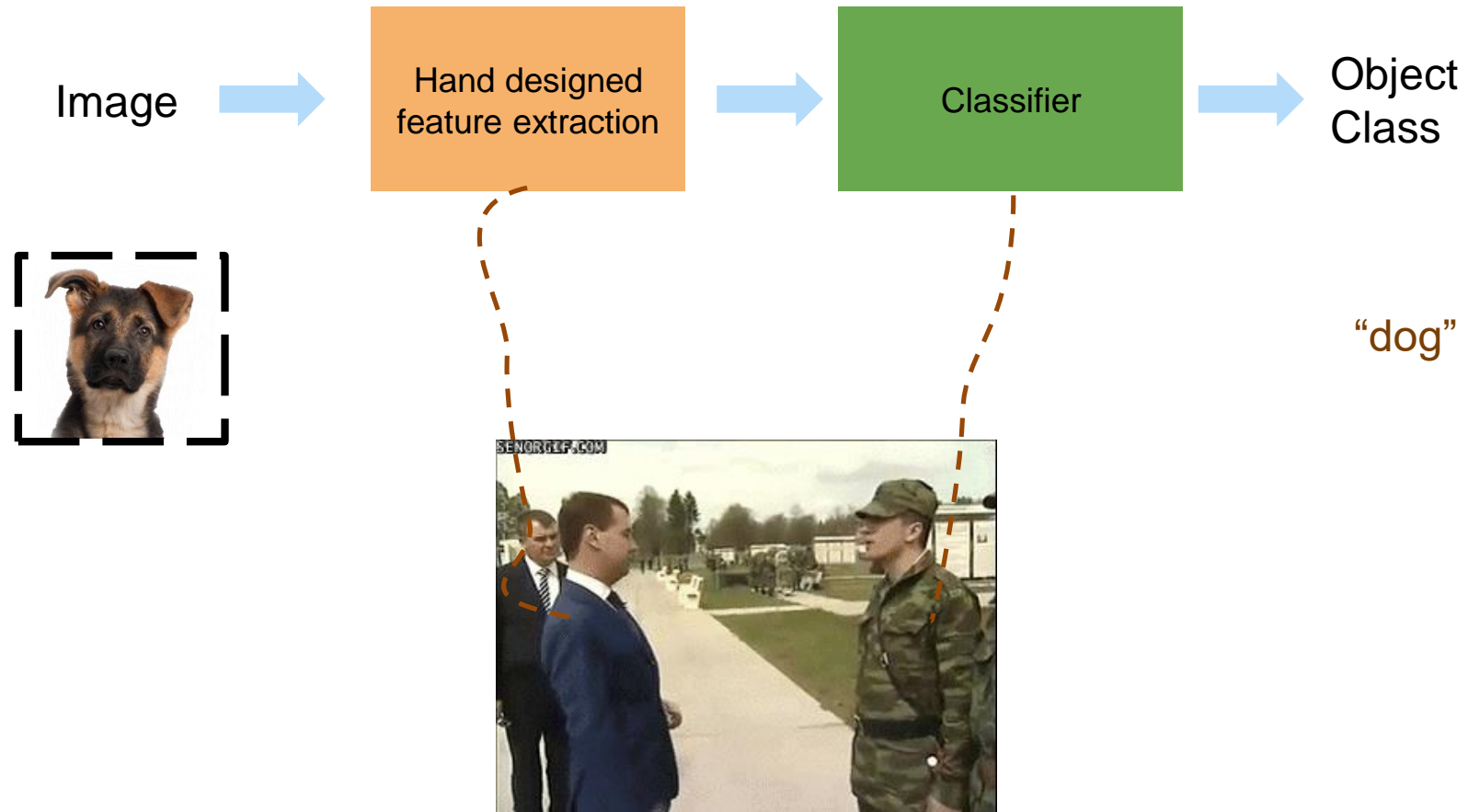
Histogram of Visual Words /  
Image Representation



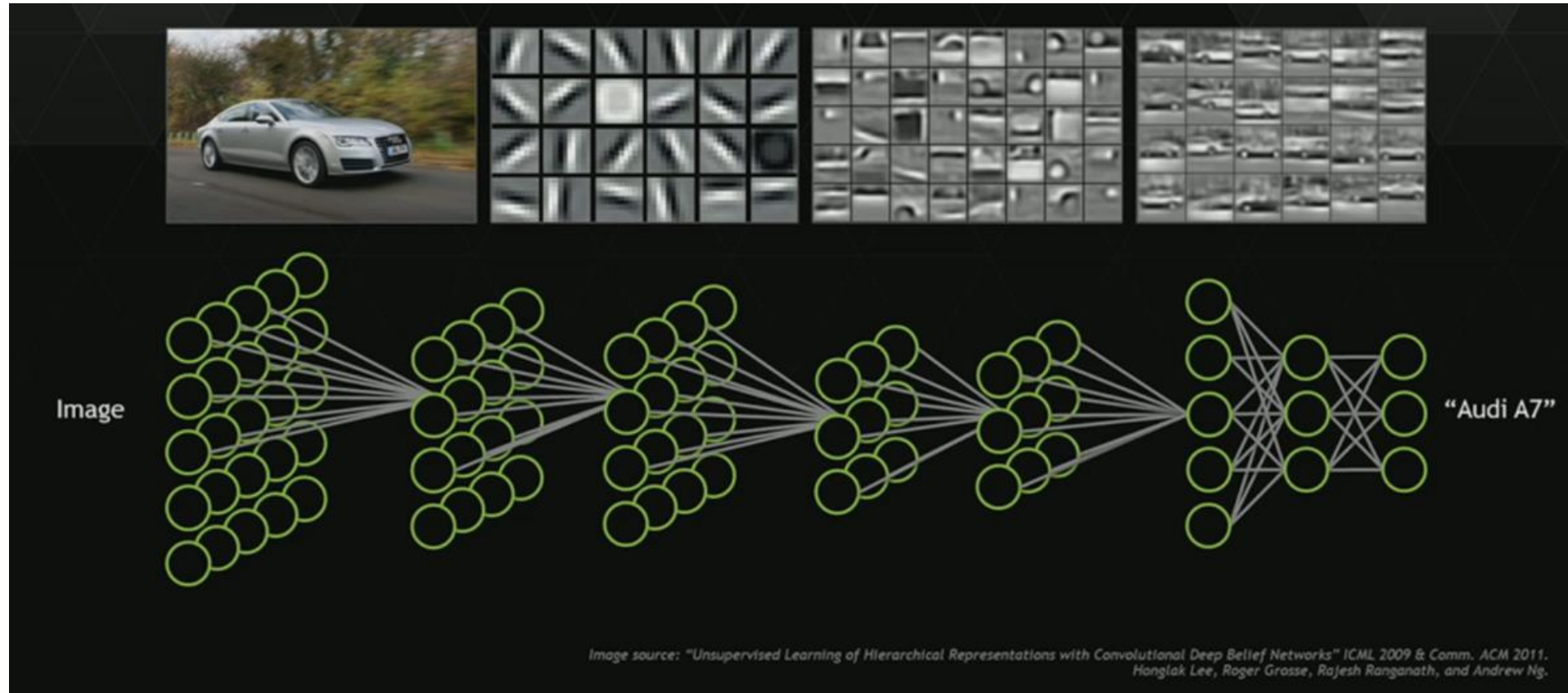
# Object-recognition: conventional approach



# Object-recognition: conventional approach

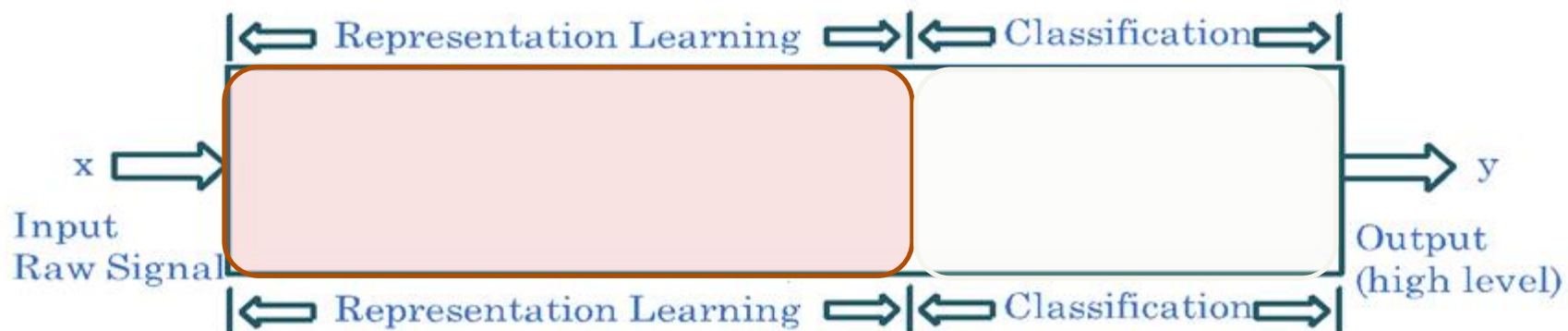
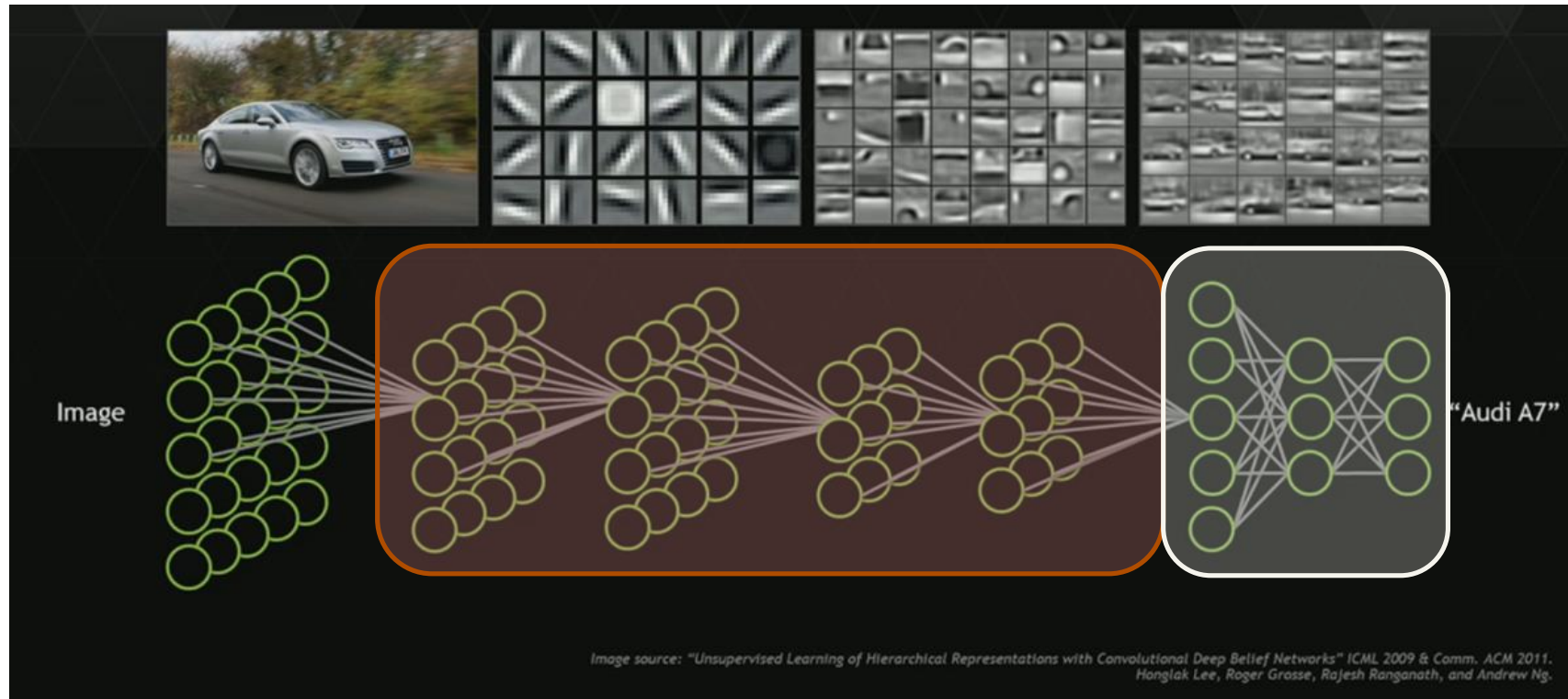


# Object Recognition: Deep Neural Networks



**Data-driven, End-to-End learning, Task-specific feature hierarchy**

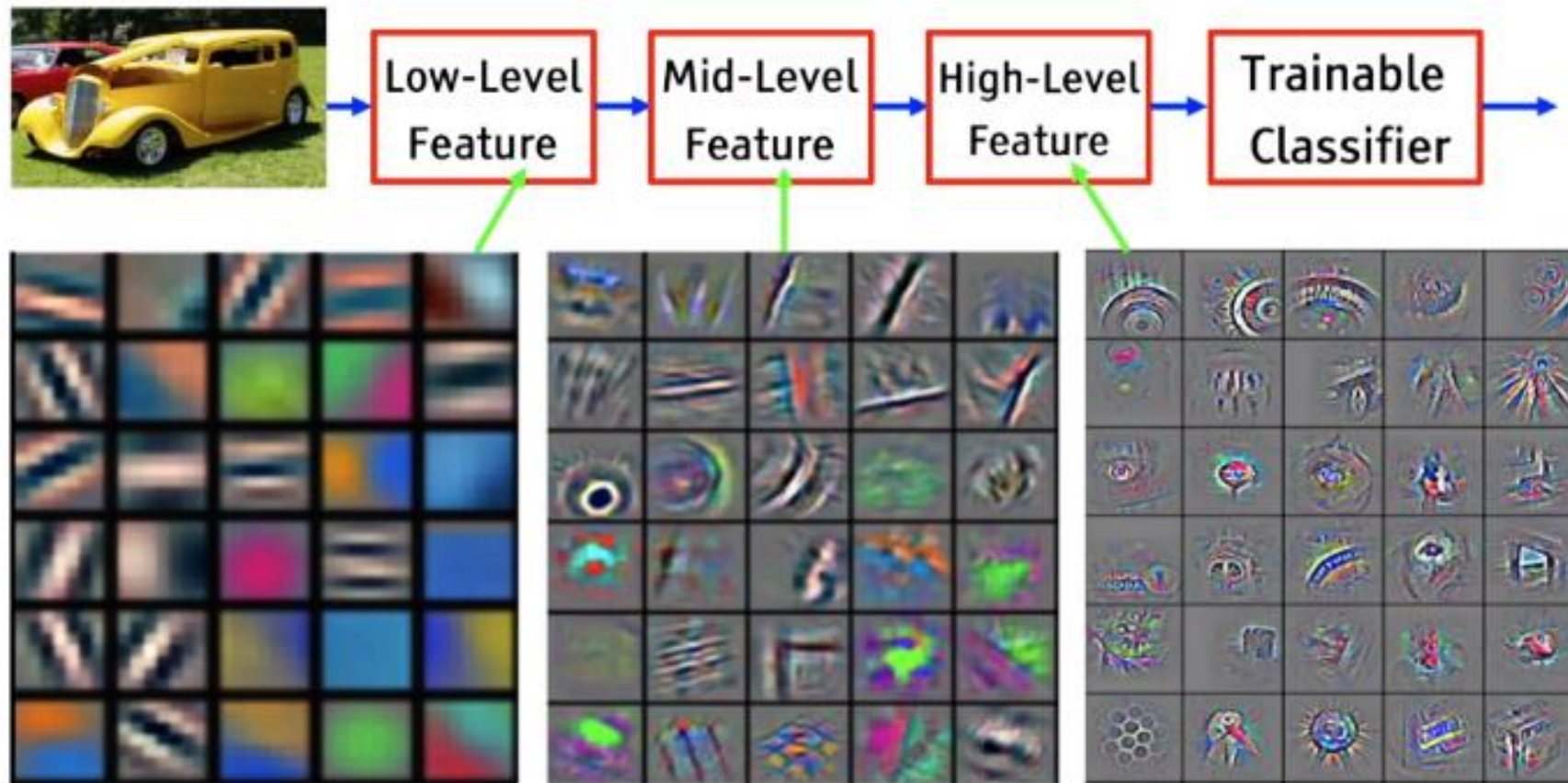
# Summary

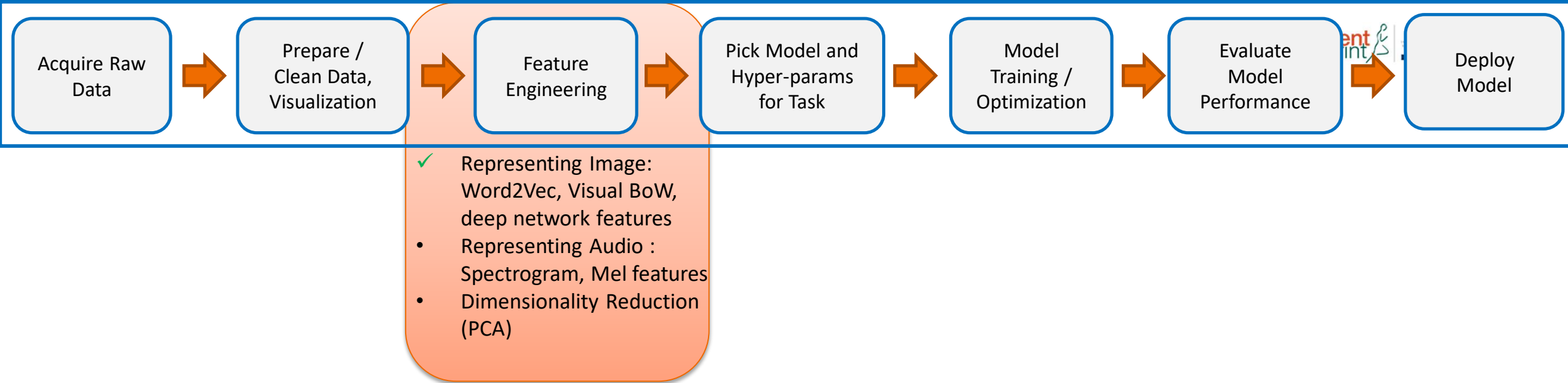




# Deep Learnt Features (2013-XXX)

- It's **deep** if it has **more than one stage** of non-linear feature transformation.





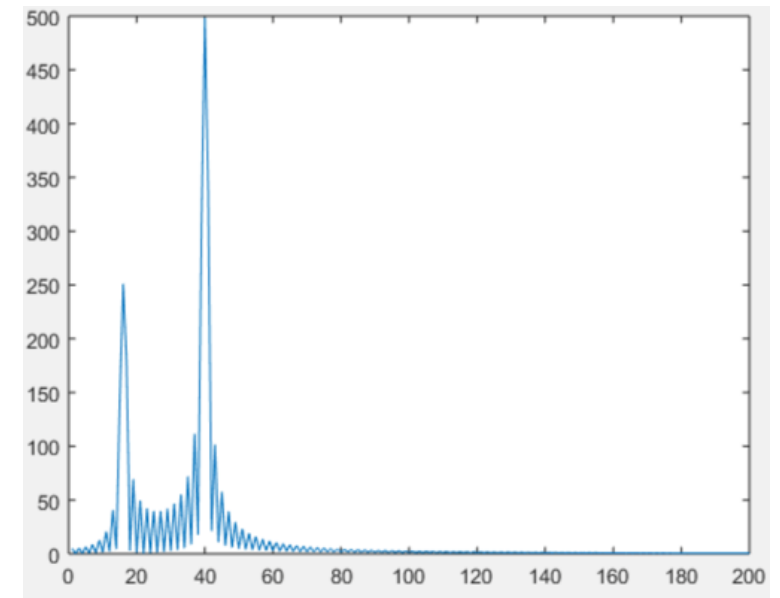
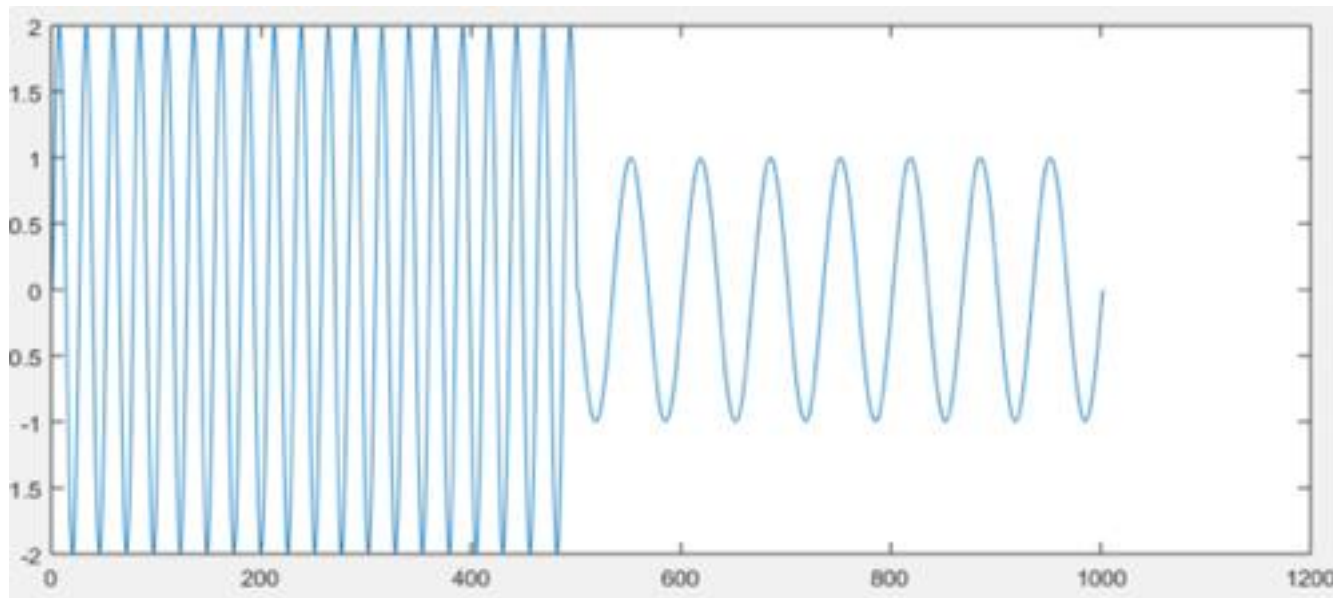
## Speech

(Brief Explanation)

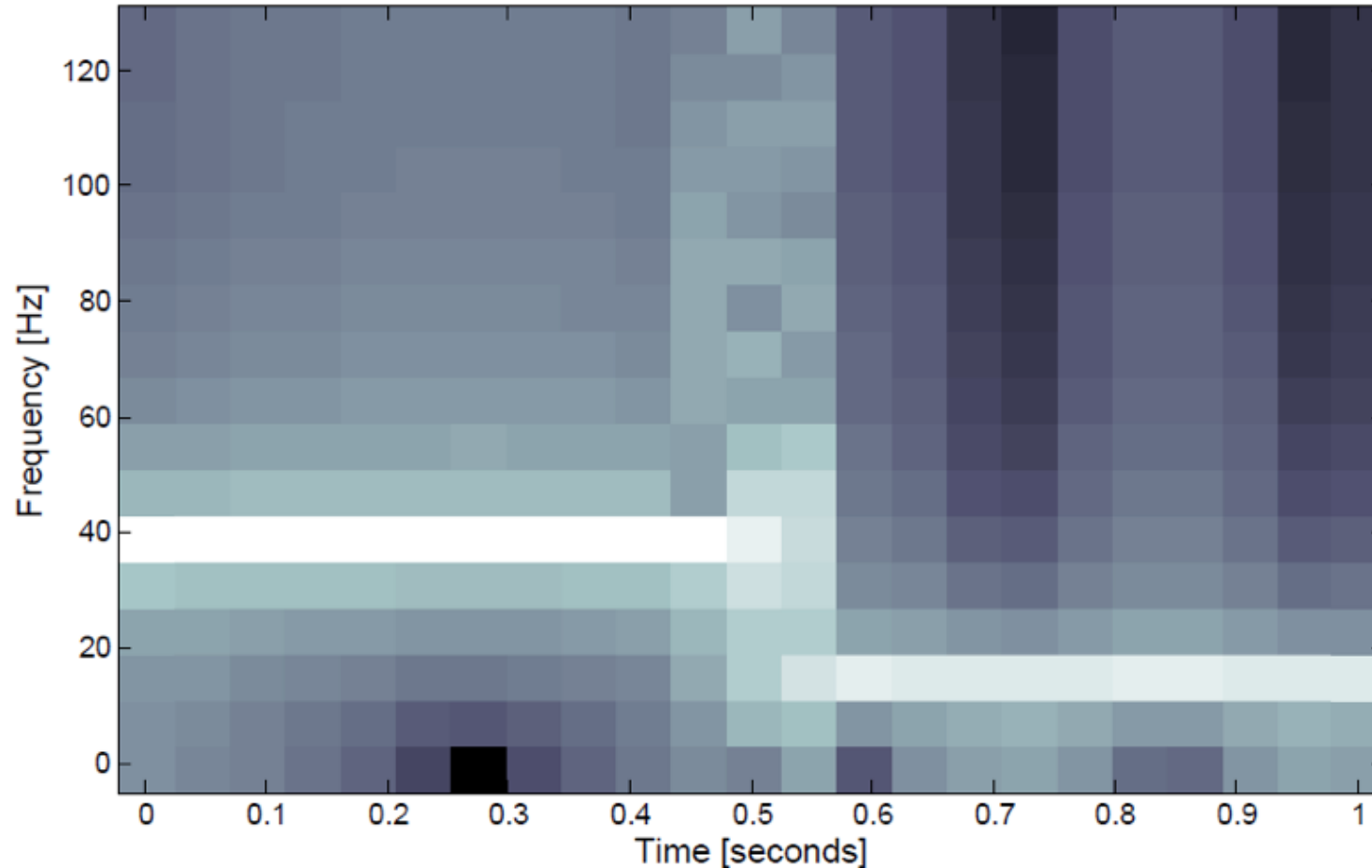


# Example Sound Signal

$$g(t) = \begin{cases} 2 * \sin(2\pi \cdot 39t), & 0 \leq t \leq 1/2 \\ \sin(2\pi \cdot 15t), & 1/2 < t \leq 1 \end{cases}$$



# Spectrogram



Spectrogram of a piecewise monochromatic signal.

Lighter color  $\Rightarrow$  greater DFT magnitude

# Representations

## A: MFCC (Signal processing based; Classical)

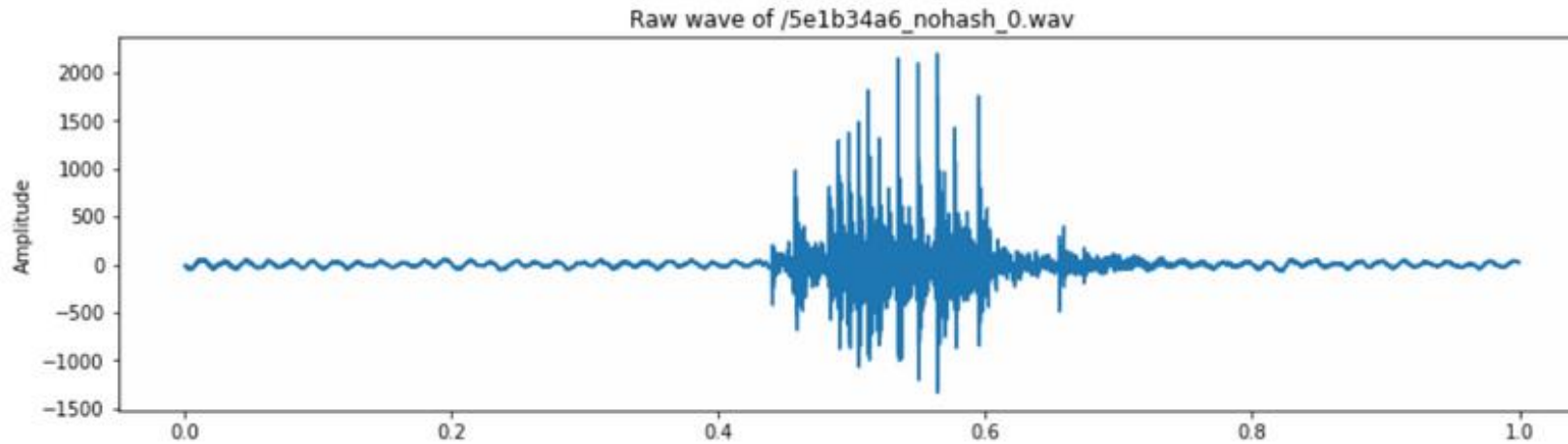
- Mel Frequency Cepstral Coefficients

## B: CNN Based (Modern)

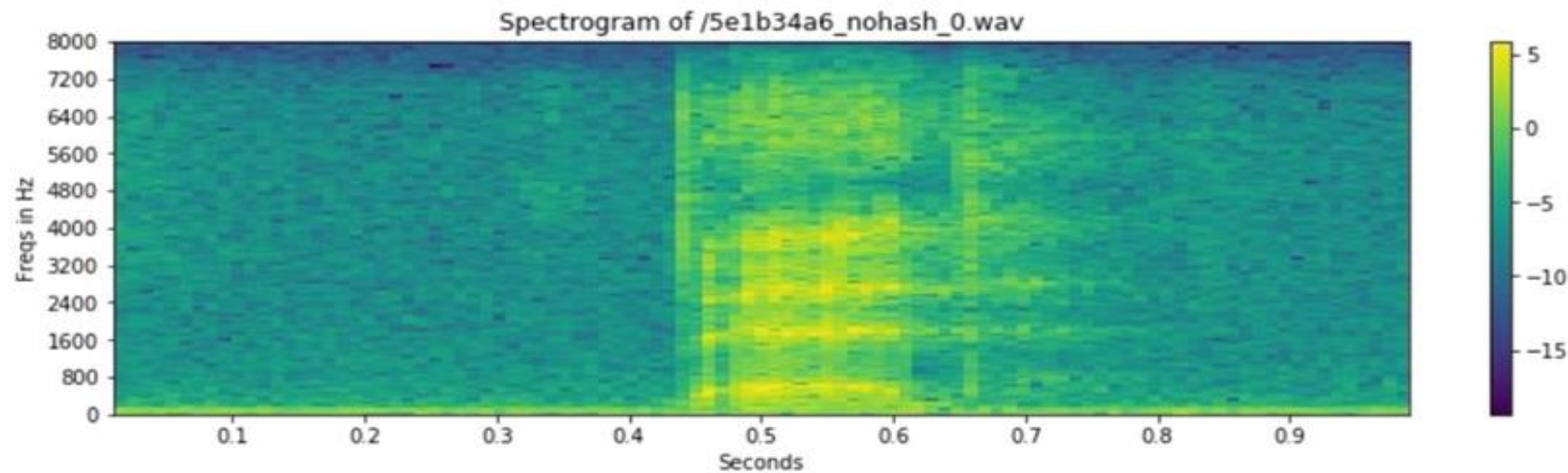
- VGG Features on the Mel Spectrogram

<http://practicalcryptography.com/miscellaneous/machine-learning/guide-mel-frequency-cepstral-coefficients-mfccs/>

# Classical Feature (MFCC)

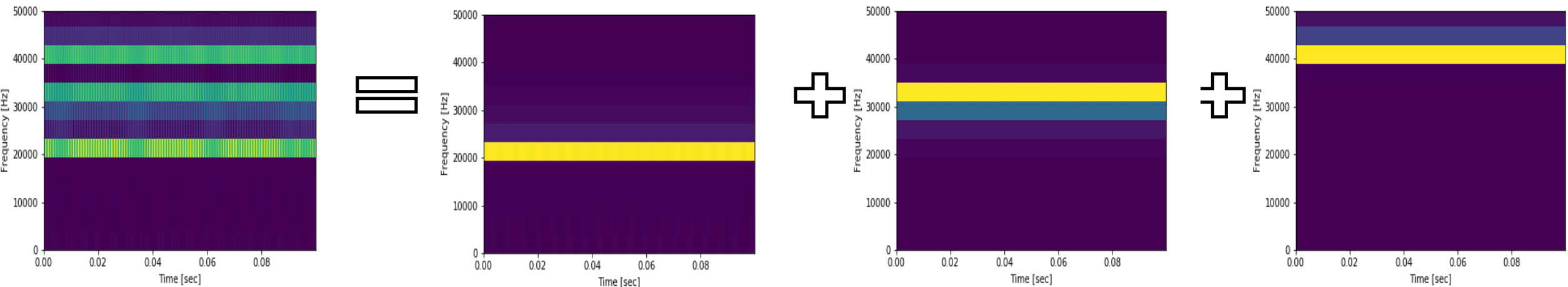
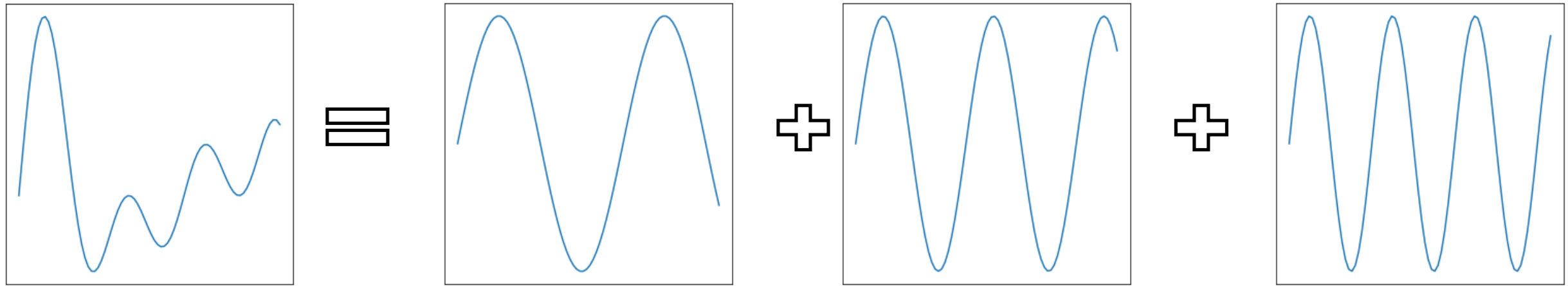


Amplitude  
Vs  
Time



Frequency  
Vs  
Time

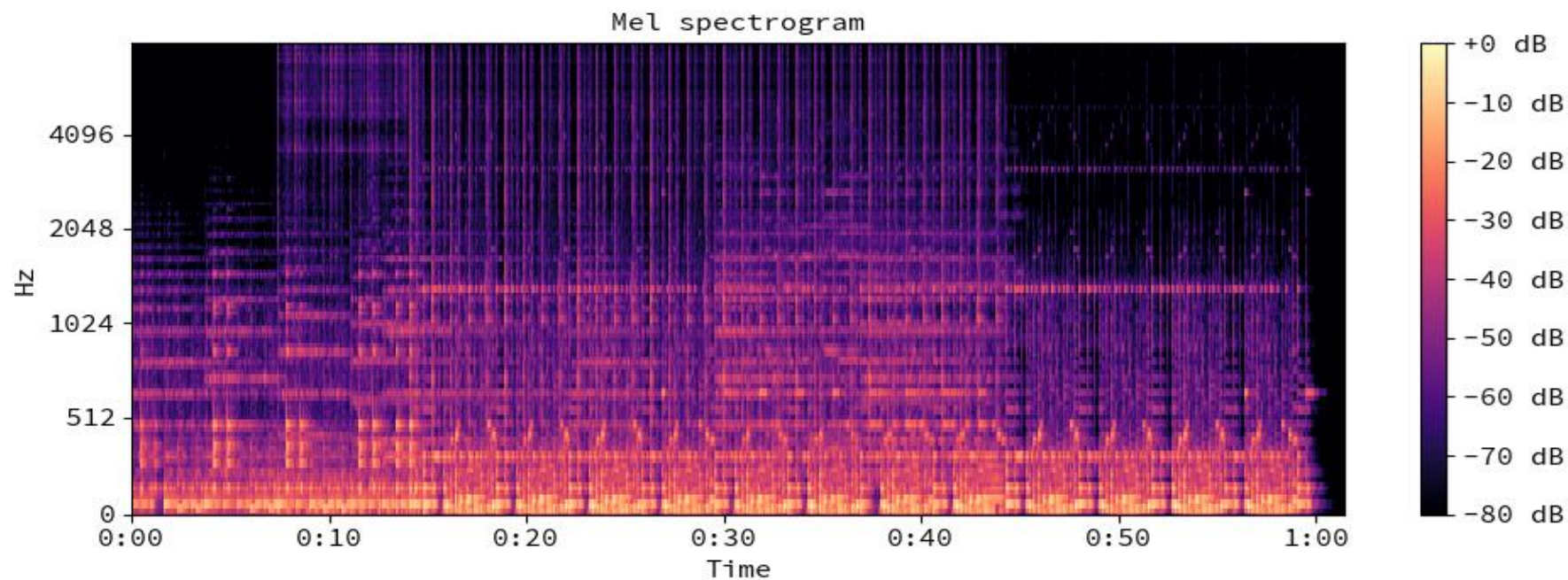
# Any wave is a combination of many sine waves



# Performance on VoxCeleb

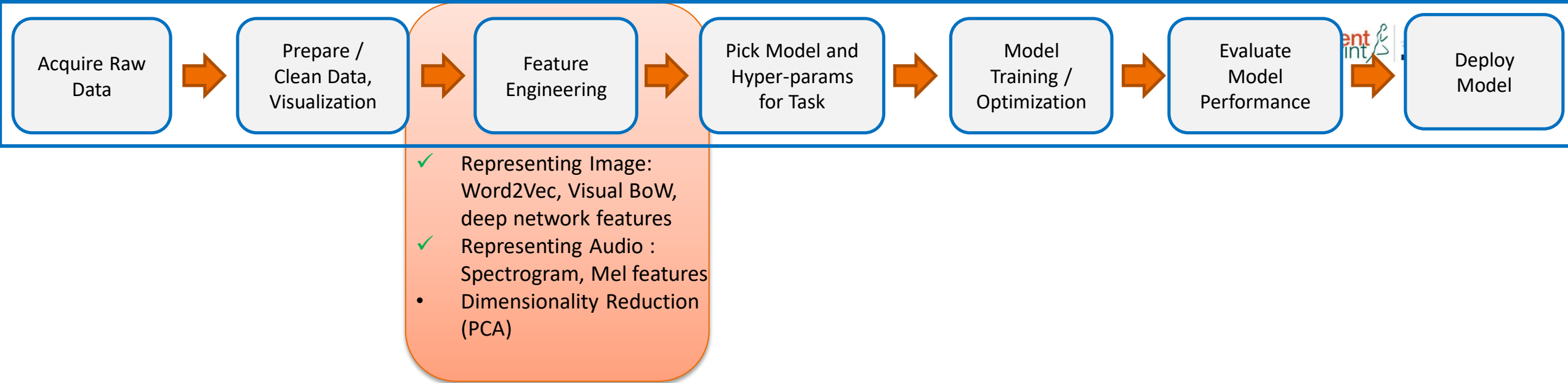
<b>Accuracy</b>	<b>Top-1 (%)</b>	<b>Top-5 (%)</b>
<b>I-vectors + SVM</b>	49.0	56.6
<b>I-vectors + PLDA + SVM</b>	60.8	75.6
<b>CNN-fc-3s no var. norm.</b>	63.5	80.3
<b>CNN-fc-3s</b>	72.4	87.4
<b>CNN</b>	<b>80.5</b>	<b>92.1</b>

# Features from Mel Spectrogram



**MFCC**  
 (Hand coded Classic Features)

<http://practicalcryptography.com/miscellaneous/machine-learning/guide-mel-frequency-cepstral-coefficients-mfccs/>

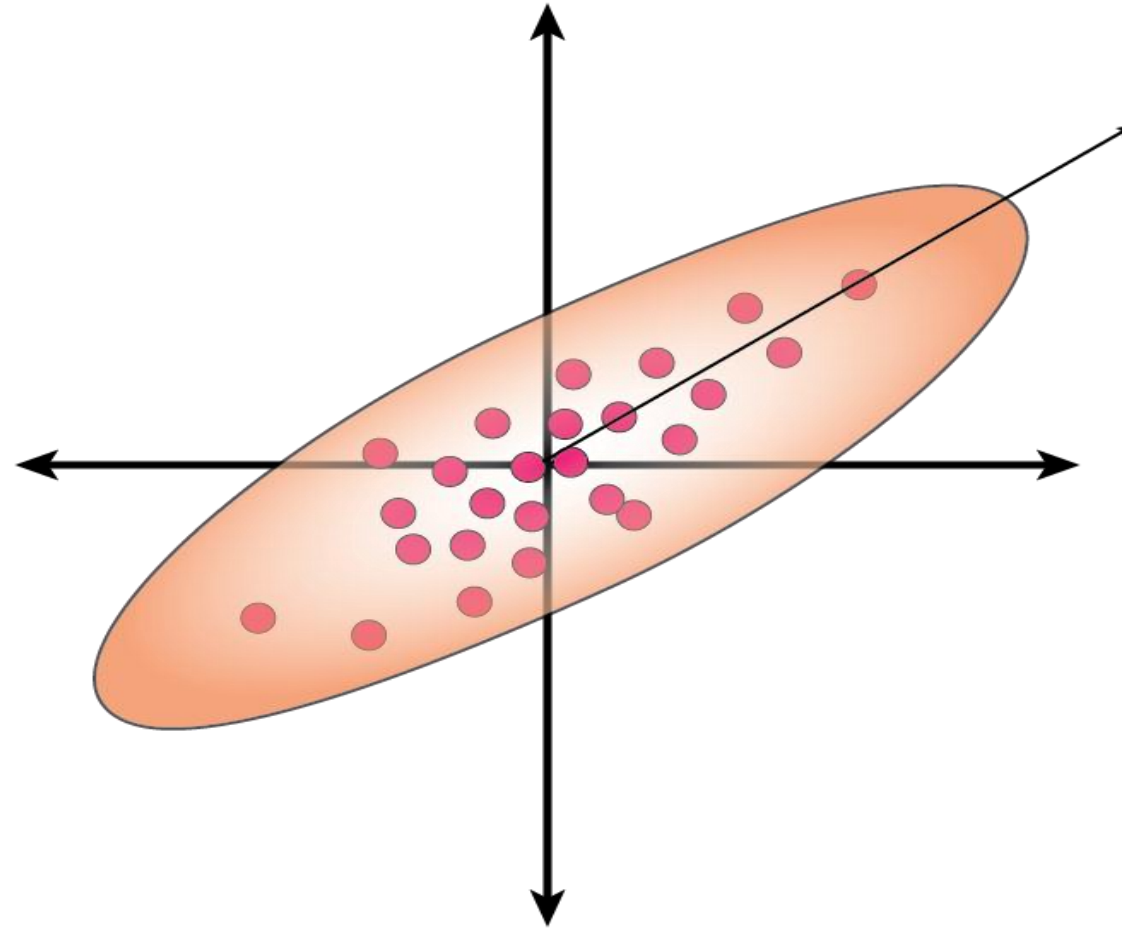


## Principal Component Analysis

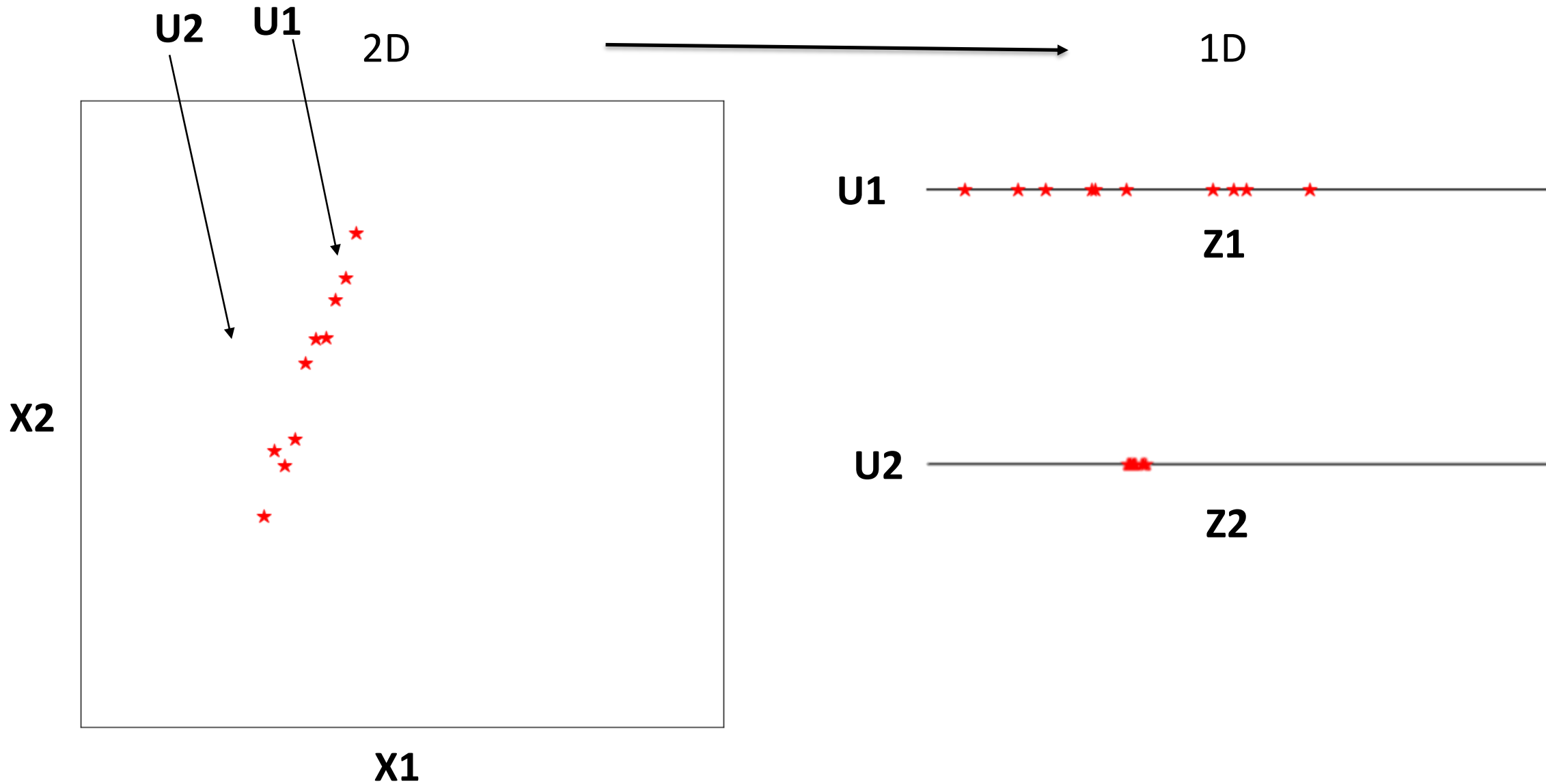
### Simplifying Representations



# PCA



# Dimensionality and Representation



# PCA

$$\hat{\mu} = \frac{1}{N} \sum_{i=1}^N x_i$$

$$\hat{\Sigma} = \frac{1}{N} \sum_{i=1}^N (x_i - \hat{\mu})(x_i - \hat{\mu})^T$$

$$\begin{bmatrix} V_a & C_{a,b} & C_{a,c} & C_{a,d} & C_{a,e} \\ C_{a,b} & V_b & C_{b,c} & C_{b,d} & C_{b,e} \\ C_{a,c} & C_{b,c} & V_c & C_{c,d} & C_{c,e} \\ C_{a,d} & C_{b,d} & C_{c,d} & V_d & C_{d,e} \\ C_{a,e} & C_{b,e} & C_{c,e} & C_{d,e} & V_e \end{bmatrix}$$

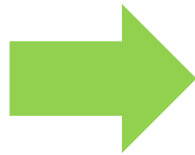
# Review: Eigenvector and Eigenvalue

$$Ax = \lambda x$$

**A: Square Matrix**

**$\lambda$ : Eigenvalue or characteristic value**

**x: Eigenvector or characteristic vector**



- *A is  $d \times d$*
- *x is  $d \times 1$*
- *Lambda is scalar*
- *Max of  $d$  nonzero lambda*
- *Min (N,d)*

$$\begin{bmatrix} 3 & 4 & -2 \\ 1 & 4 & -1 \\ 2 & 6 & -1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ 6 \end{bmatrix}$$

# PCA based Feature Extraction

$r \times 1$

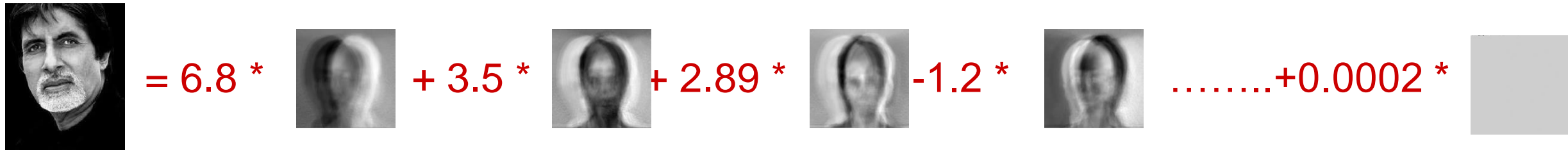
$r \times d$

$d \times 1$

$$\begin{array}{c}
 \begin{array}{c} z_1 \\ z_2 \\ z_3 \\ \vdots \\ \vdots \\ z_r \end{array} \\
 \mathbf{Z}
 \end{array}
 =
 \begin{array}{c}
 \begin{array}{cccccccc}
 u_{11} & u_{12} & u_{13} & \cdot & \cdot & \cdot & \cdot & \cdot & u_{1d} \\
 u_{21} & u_{22} & u_{23} & \cdot & \cdot & \cdot & \cdot & \cdot & u_{2d} \\
 u_{31} & u_{32} & u_{33} & \cdot & \cdot & \cdot & \cdot & \cdot & u_{3d} \\
 \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
 u_{r1} & u_{r2} & u_{r3} & \cdot & \cdot & \cdot & \cdot & \cdot & u_{rd}
 \end{array} \\
 \mathbf{U}
 \end{array}
 \begin{array}{c}
 \begin{array}{c} x_1 \\ x_2 \\ x_3 \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ x_d \end{array} \\
 \mathbf{X}
 \end{array}$$

Each row in  $\mathbf{U}$  is an Eigen vector of co-variance Matrix

# Representing with EigenFaces



$$= 6.8 * \text{[eigenface 1]} + 3.5 * \text{[eigenface 2]} + 2.89 * \text{[eigenface 3]} - 1.2 * \text{[eigenface 4]} \dots\dots\dots + 0.0002 * \text{[eigenface n]}$$

Any face in the database can be represented as a linear combination of the eigen faces.

# PCA/Eigen Face Algorithm: Detail

1. Compute the mean feature vector

$$\mu = \frac{1}{P} \sum_{k=1}^P x_k, \text{ where, } x_k \text{ is a pattern } (k = 1 \text{ to } p), p = \text{number of patterns, } x \text{ is the feature matrix}$$

2. Find the covariance matrix

$$C = \frac{1}{P} \sum_{k=1}^P \{x_k - \mu\} \{x_k - \mu\}^T \text{ where, } T \text{ represents matrix transposition}$$

3. Compute Eigen values  $\lambda_i$  and Eigen vectors  $v_i$  of covariance matrix

$$Cv_i = \lambda_i v_i \quad (i = 1, 2, 3, \dots, q), q = \text{number of features}$$

4. Estimating high-valued Eigen vectors

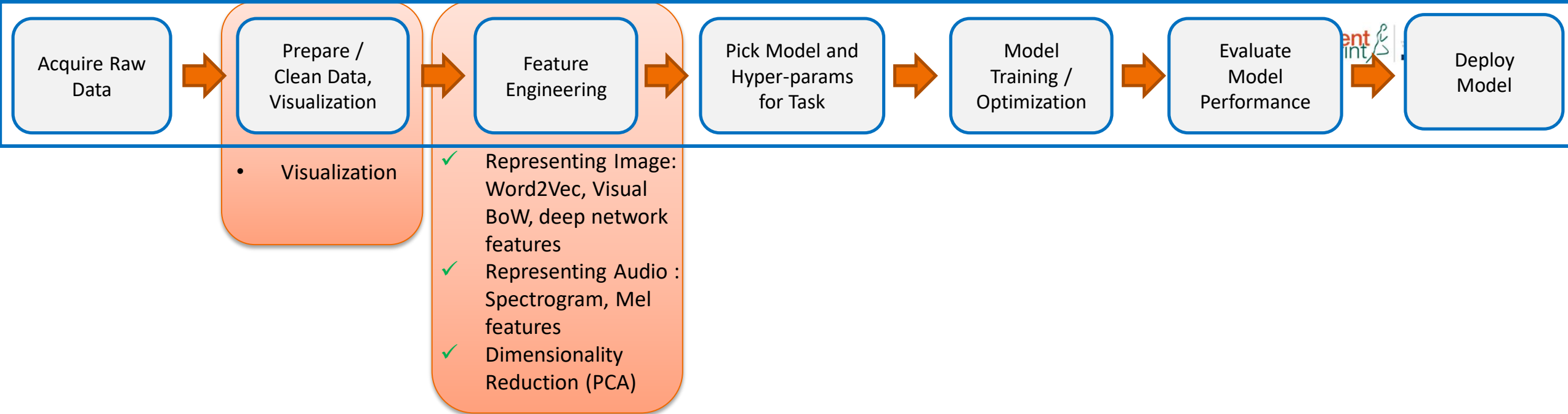
- (i) Arrange all the Eigen values ( $\lambda_i$ ) in descending order
- (ii) Choose a threshold value,  $\theta$
- (iii) Number of high-valued  $\lambda_i$  can be chosen so as to satisfy the relationship

$$\left( \sum_{i=1}^s \lambda_i \right) \left( \sum_{i=1}^q \lambda_i \right)^{-1} \geq \theta, \text{ where, } s = \text{number of high valued } \lambda_i \text{ chosen}$$

- (iv) Select Eigen vectors corresponding to selected high valued  $\lambda_i$

5. Extract low dimensional feature vectors (principal components) from raw feature matrix.

$$P = V^T x, \text{ where, } V \text{ is the matrix of principal components and } x \text{ is the feature matrix}$$

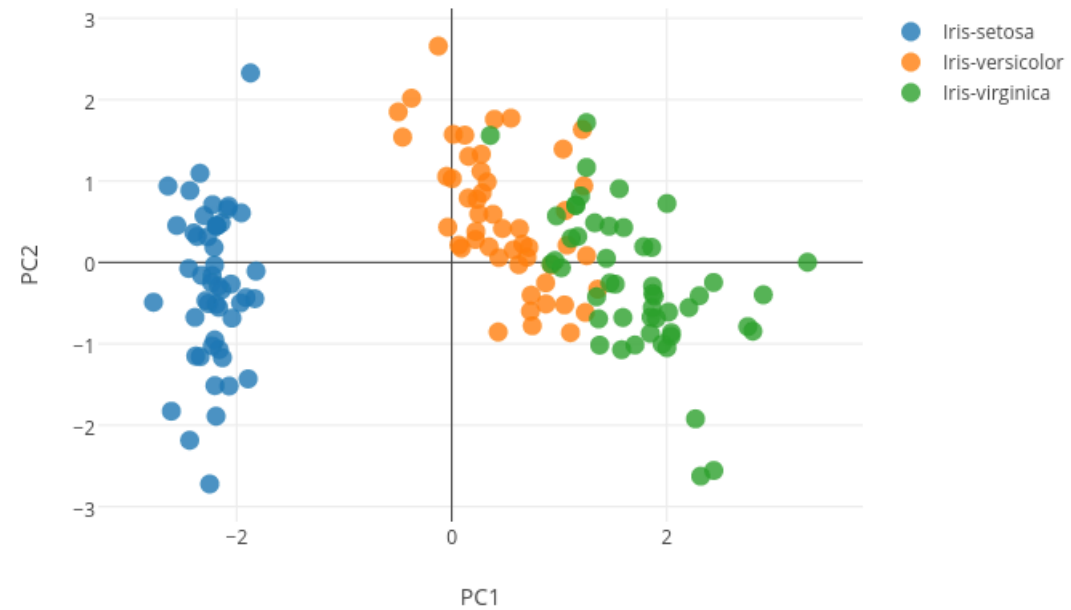


## Data Visualization

When Data is High-Dimensional



# PCA



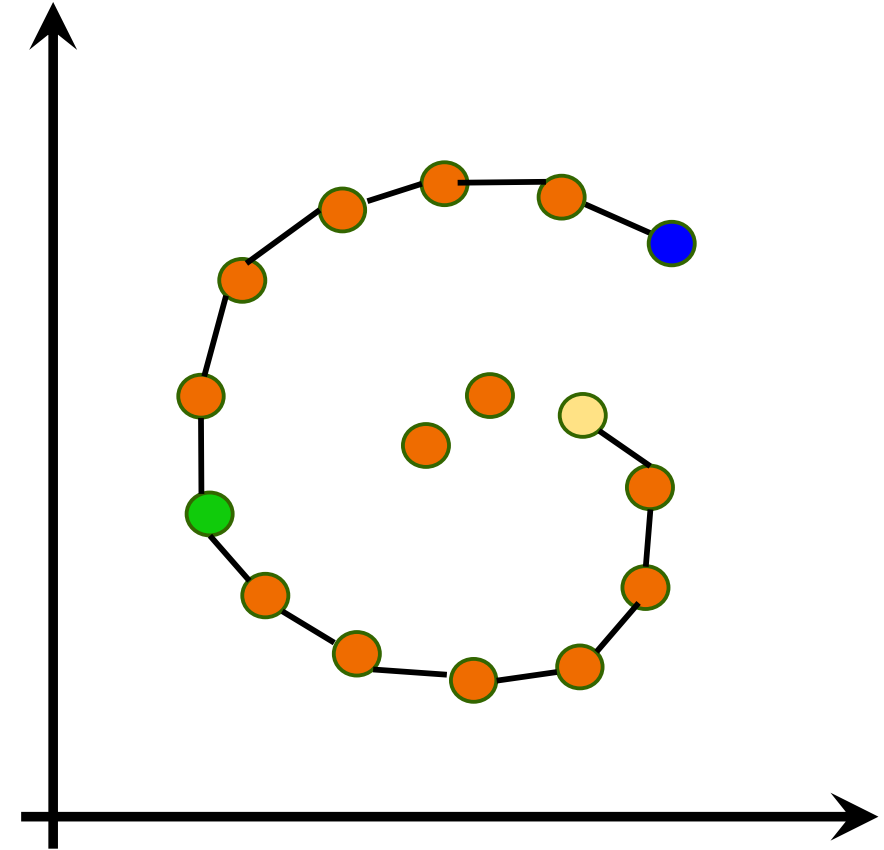
PLOT ON 2 PRINCIPLE COMPONENTS

# MDS (Multidimensional scaling)

- Minimize an objective function that measures the discrepancy between similarities in the data and similarities in the map.
- Distance between samples in “high” dimension and “low” dimension is same (or D-d) is minimized.

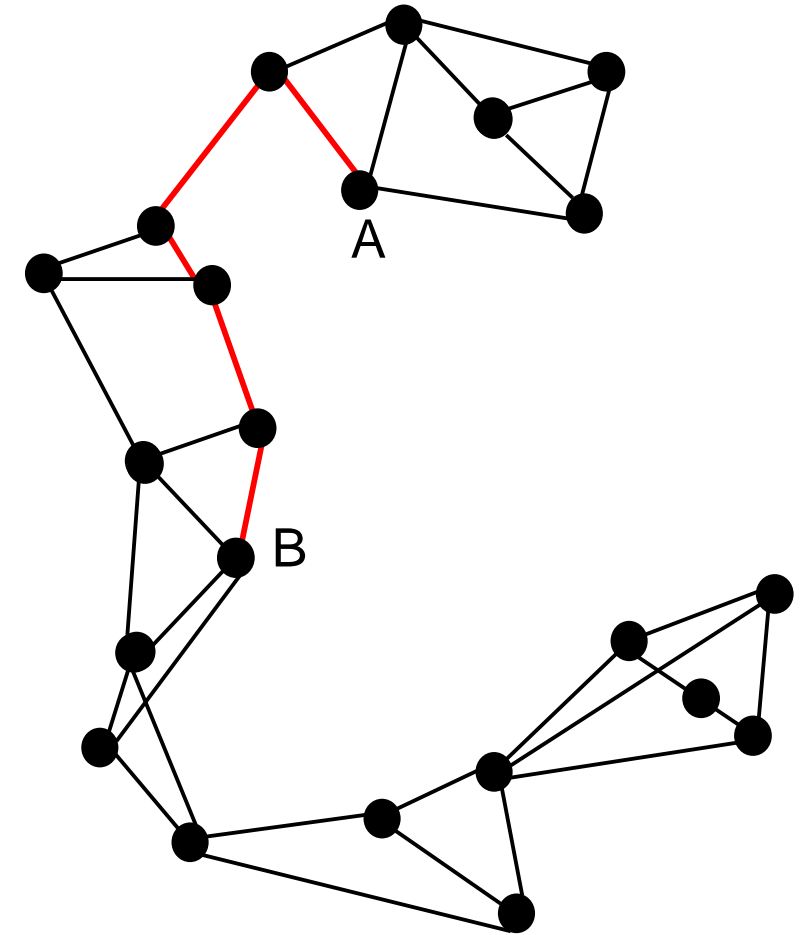
# ISOMAP (Isometric Mapping)

- $d(\text{blue}, \text{yellow}) > d(\text{blue}, \text{green})$
- Is Euclidean metric the right distance metric?
- How to robustly measure distances along the manifold?



# ISOMAP

- How does ISOMAP measure the MD?
- Connect each data point to its K nearest neighbors in the high-dimensional space.
- **Link weights:** True Euclidean distances.
- $MD(A, B) = ShortestPath(A, B)$  in this **neighborhood graph**.
- Compute the low-dimensional embedding as in Metric MDS.



# LLE: Locally Linear Embedding

- **Idea:** Preserve the structure of local neighbourhood

$$\mathbf{x}_i \approx \sum_j w_{ij} \mathbf{x}_j$$

- **Approach:**
  - Represent each point as a weighted combination of its Neighbours in HD. Remember the  $w_{ij}$ s.
  - Find a LD representation that minimize the representation error:

$$Cost = \sum_i \left\| \mathbf{y}_i - \sum_{j \in N(i)} w_{ij} \mathbf{y}_j \right\|^2$$

- The weights  $w_{ij}$  refer to the amount of contribution the point  $x_i$  has while reconstructing the point  $x_i$ . The cost function is minimized under two constraints: (a) Each data point  $x_i$  is reconstructed only from its neighbors, thus enforcing  $w_{ij}$  to be zero if point  $x_j$  is not a neighbor of the point  $x_i$  and (b) The sum of every row of the weight matrix equals 1.
- Also  $\mathbf{y}$ s should have unit variance across each dimension.

# SNE and t-SNE

- **Idea is simple:** Instead of distance think about probabilities.  $P_{ij}$  as the probability of  $j$  in the neighborhood of  $i$ .
- For each point, we have now a probability vector (of size  $N$ ).
  - SNE uses Gaussian. T-SNE uses another t-distribution (with 1 degree of freedom).
- We want these prob vectors to be the same in low dimensional.
- Optimize using gradient descent.

# Computing the LD Embedding

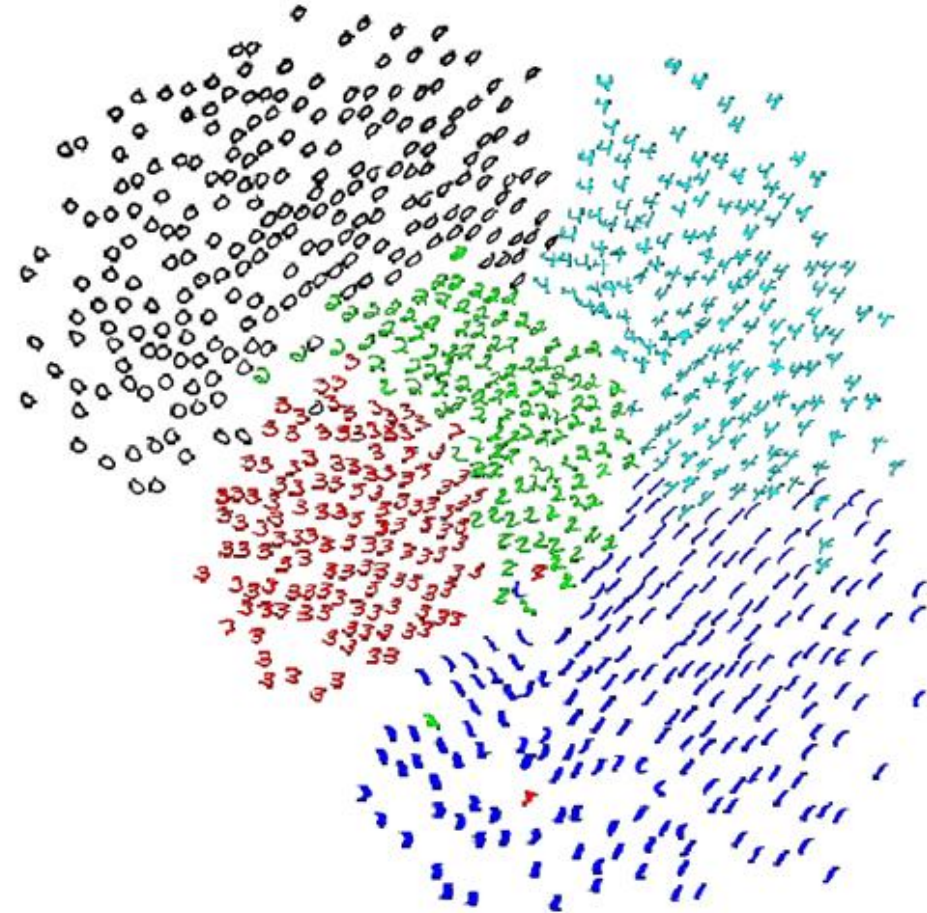
$$Cost = \sum_i KL(P_j \parallel Q_i) = \sum_i \sum_j p_{j|i} \log \frac{p_{j|i}}{q_{j|i}}$$

- For points where  $p_{ij}$  is large and  $q_{ij}$  is small we lose a lot.
  - Nearby points in high-D really want to be nearby in low-D

## PCA on MNIST (0-9)

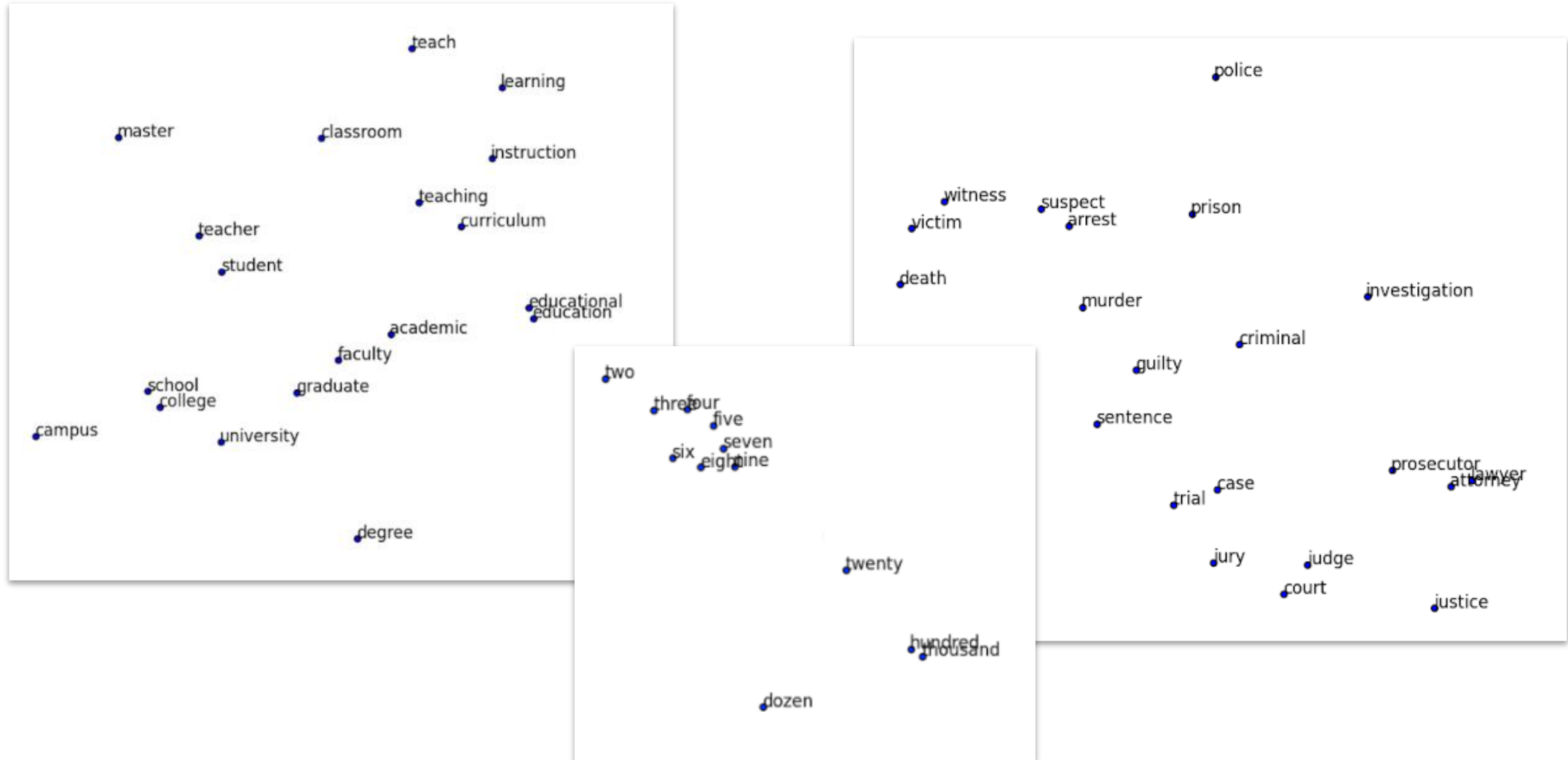


## SNE on MNIST (0-5)

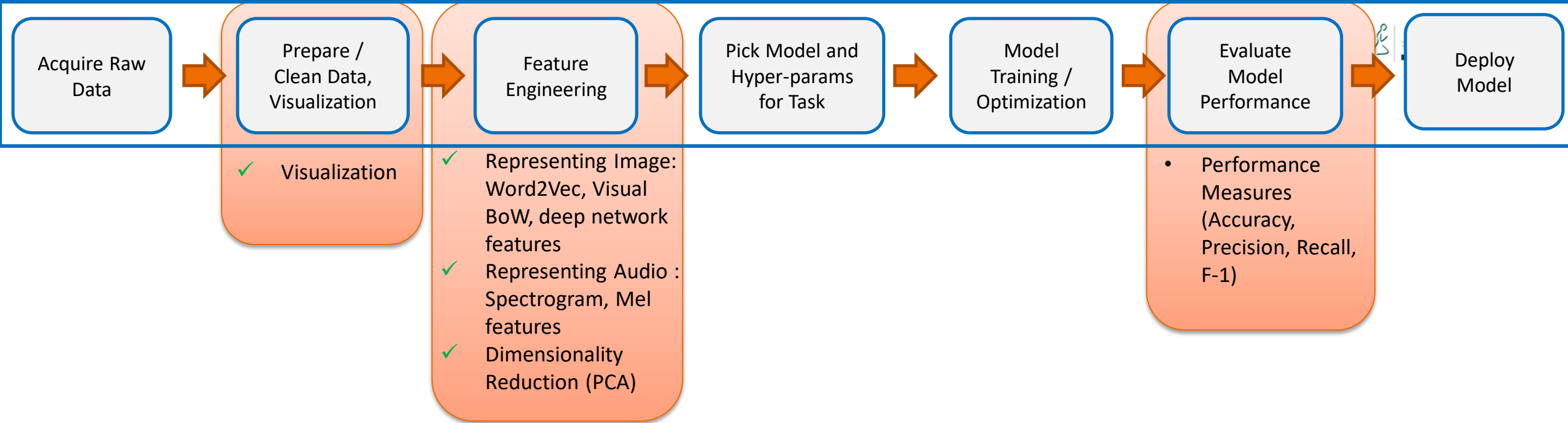




# Word2vec



<http://nlp.yvespeirsman.be/blog/visualizing-word-embeddings-with-tsne/>



## Performance Metrics

# Key accuracy measures and terminologies

- Classification Error =  $\frac{\text{errors}}{\text{total}}$
- =  $\frac{FP + FN}{TP + TN + FP + FN}$

	Predicted: NO	Predicted: YES	
n=165 Actual: NO	TN = 50	FP = 10	60
Actual: YES	FN = 5	TP = 100	105
	55	110	

- Accuracy = 1 - Error =  $\frac{\text{correct}}{\text{total}}$
- =  $\frac{TP + TN}{TP + TN + FP + FN}$

# Revisiting scenarios where metrics are appropriate

- When you do cancer screening what do you care?
  - High TP and Low FN
- When you classify between “apple” and “orange”
  - High Accuracy
- Automatic Firing on detecting a violation.
  - Very low FP

# Precision and Recall

$$Precision = \frac{TP}{(TP + FP)}$$

$$Recall = \frac{TP}{(TP + FN)}$$

# Precision and Recall – examples

- A system which needs to launch a missile at a terrorist hideout located in a dense urban area.
  - Precision not 100% ➡ civilian casualties
- A system which needs to identify cancer-risk patients
  - Recall not 100% ➡ some patients will die of cancer

# F-measure: Combines Precision and Recall

- What to do when one classifier has better Precision but worse Recall, while other classifier behaves exactly opposite?
  - F-measure (Information Retrieval)

$$F_1 = \frac{2}{\frac{1}{Recall} + \frac{1}{Precision}}$$

# F-measure

- What to do when one classifier has better Precision but worse Recall, while other classifier behaves exactly opposite?
  - F-measure (Information Retrieval)

$$F_1 = \frac{2}{\frac{1}{Recall} + \frac{1}{Precision}}$$

$$Precision = \frac{TP}{(TP + FP)}$$

$$Recall = \frac{TP}{(TP + FN)}$$

- F1 measure punishes extreme values more !
- Definition of Recall and Precision have same numerator, different denominators. A sensible way to combine them is harmonic mean.



# F-measure

- Use when
  - FP and FN are ‘equally costly’
  - You don’t expect results to change when more data is added
  - TN is high (e.g. face detector)

$$\frac{2}{\frac{1}{Recall} + \frac{1}{Precision}}$$

$$Precision = \frac{TP}{(TP + FP)}$$

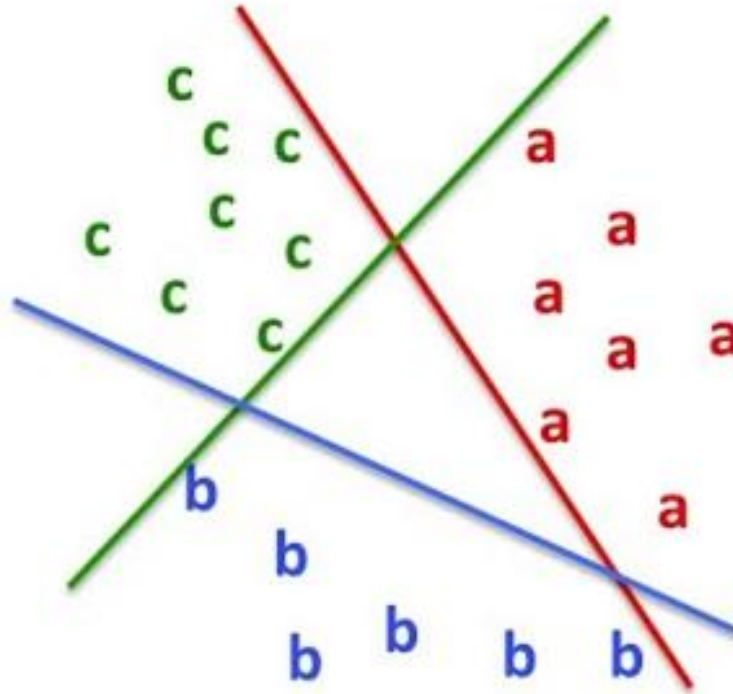
$$Recall = \frac{TP}{(TP + FN)}$$

# Utility and Cost

- Sometimes, there is a cost for each error
  - E.g. Earthquake prediction
    - False positive: Cost of preventive measures
    - False negative: Cost of recovery
- Detection Cost (Event detection) -Can be applied to example above
  - $\text{Cost} = C_{\text{FP}} * \text{FP} + C_{\text{FN}} * \text{FN}$

# How to use 2-class measures for multi-class ?

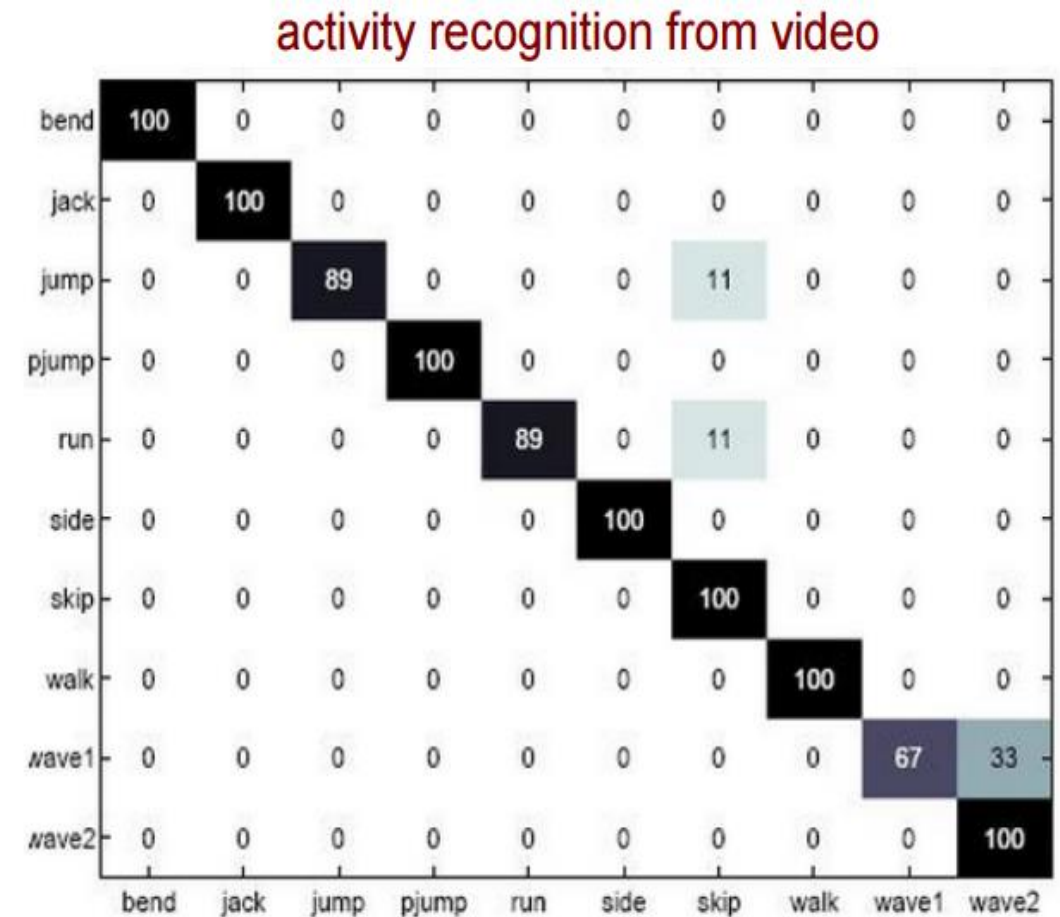
- Convert into 2-class problem(s) !



# Multi-class problems - Confusion matrix

n=165		Predicted:	Predicted:	
		NO	YES	
Actual:				
NO	TN = 50	FP = 10	60	
Actual:				
YES	FN = 5	TP = 100	105	
		55	110	

actual class



predicted class

Courtesy:  
vision.jhu.edu

Avg. accuracy may not be very meaningful  
with imbalanced class label distribution

# + Doctor Strange (2016)

★ 7.5/10  
424,942

★ Rate This

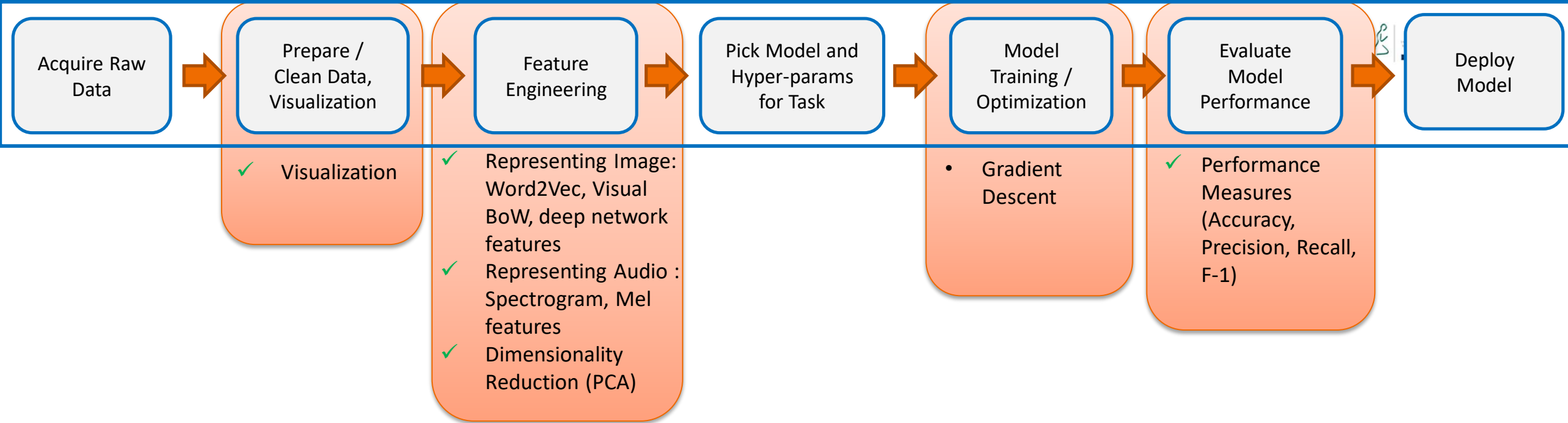
PG-13 | 1h 55min | Action, Adventure, Fantasy | 4 November 2016 (USA)



While on a journey of physical and spiritual healing, a brilliant neurosurgeon is drawn into the world of the mystic arts.

## Two Metrics for Multi-label case

- Jaccard Distance:  $1 - \text{intersection} / \text{union}$
- Hamming Loss:  $\text{mismatches} / \text{total}$



## Gradient Descent

# The Problem

- Find  $\mathbf{w}$ , given examples:  $(x_i, y_i), i = 1, 2, \dots, n$
- Supervised situation: output label  $y_i$  is available for all training  $n$  samples
- **Objective:** predict  $y$  values for a novel input  $x$  that we have not seen before
  - Called generalization in Machine Learning
- How do we find  $w$ ? **Ans: Gradient descent!**



# Loss Function

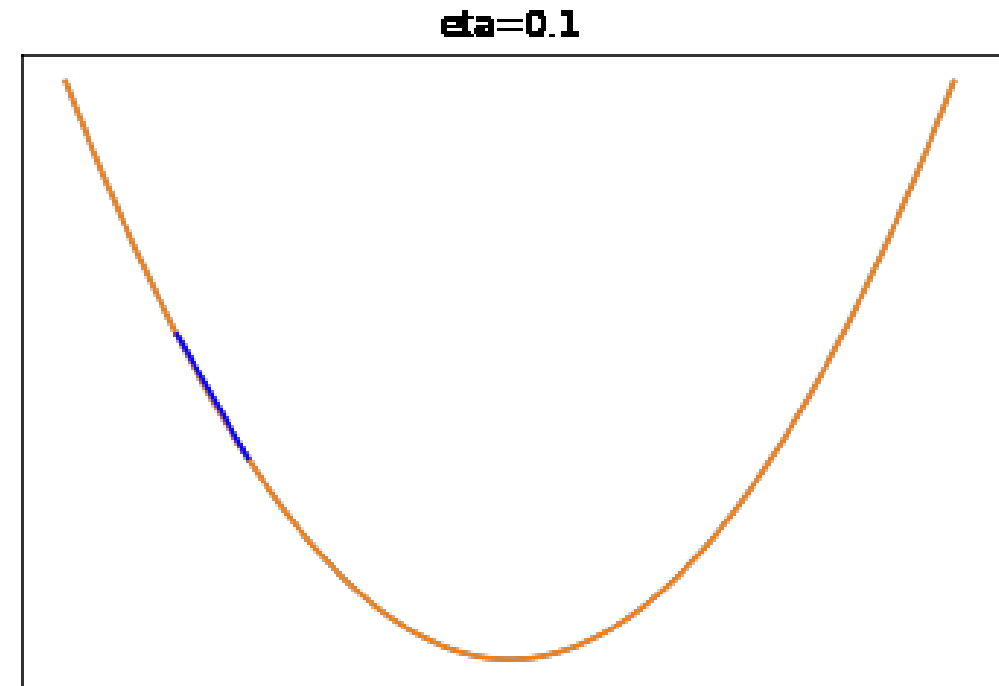
- **Error/Loss (L):** A function of the difference between the actual value or label ( $y_i$ ) and the predicted value ( $f(w, x_i)$ )
- **Total Loss:**

$$J = \sum_{i=1}^n L(y_i, f(w, x_i))$$

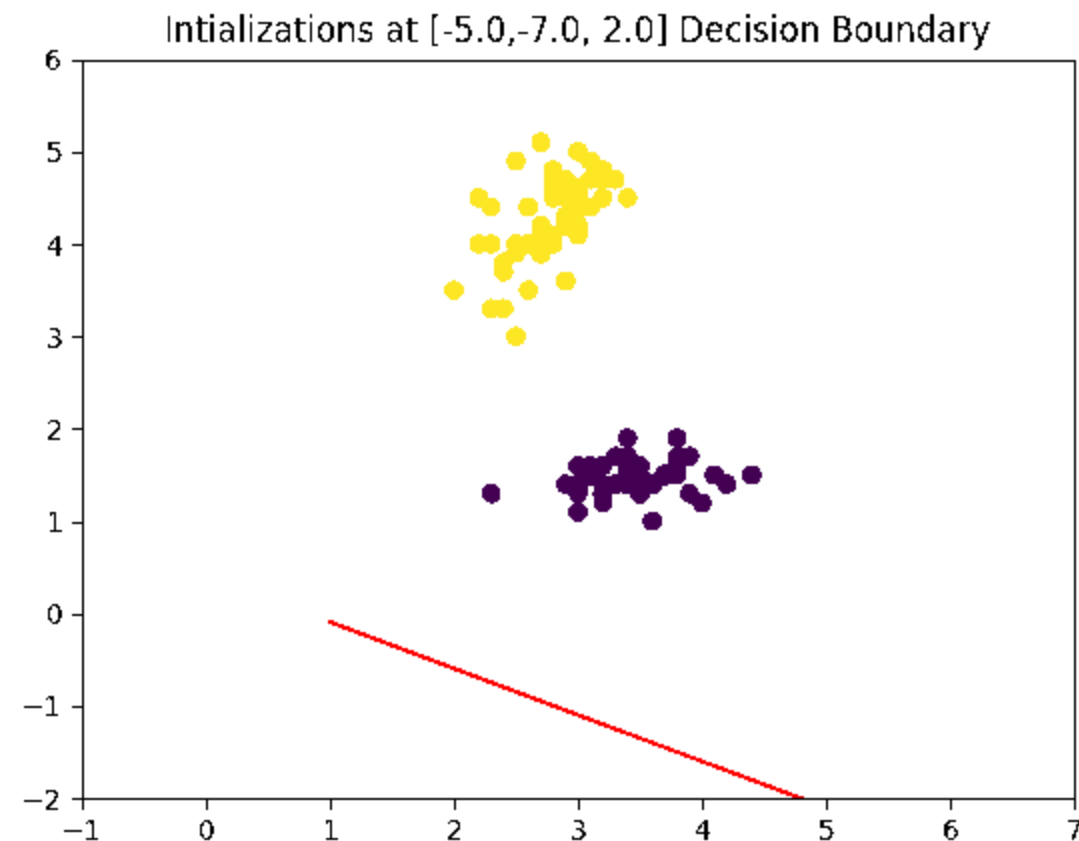
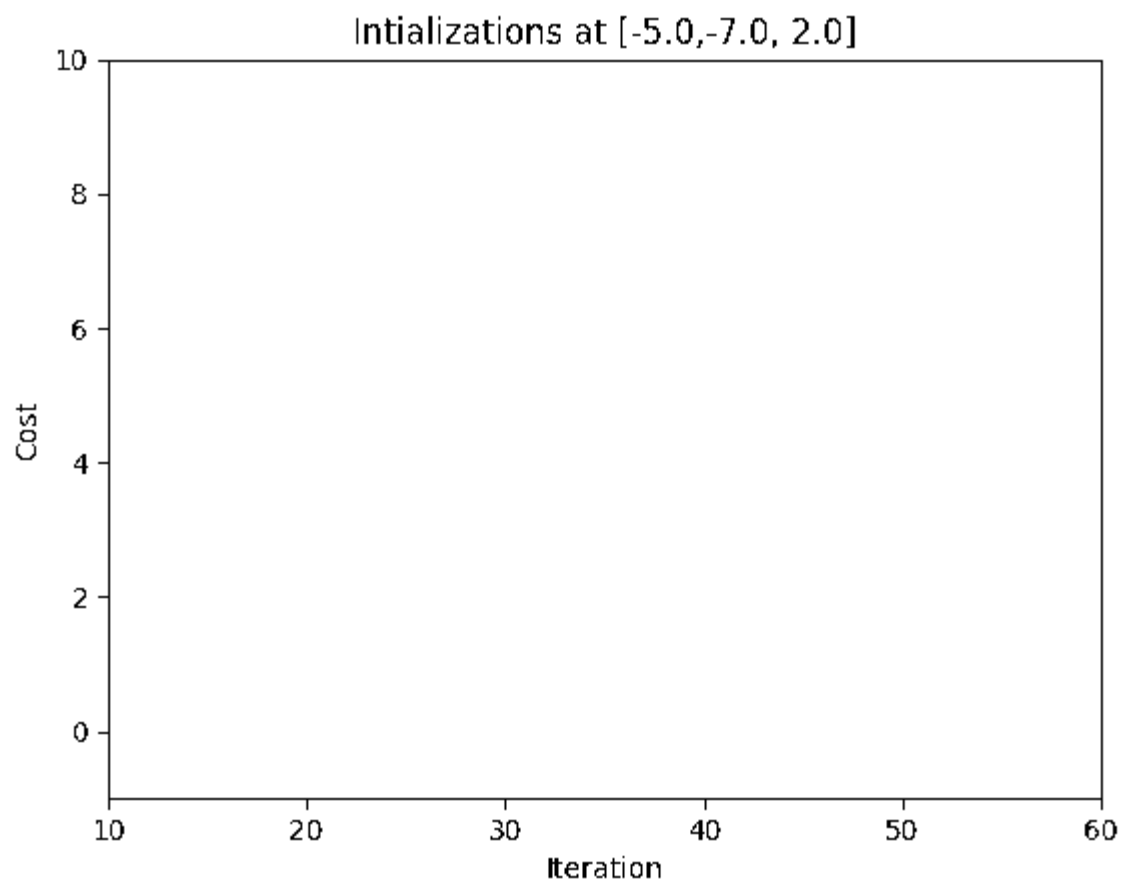
the sum of **loss** over  $n$  labelled training samples:  $(x_i, y_i)$
- **Strategy:** Start with some initial values for  $w$  and bring the predicted values closer to the corresponding labels (or minimize  $J$ ) by adjusting  $w$ .

# Gradient Descent

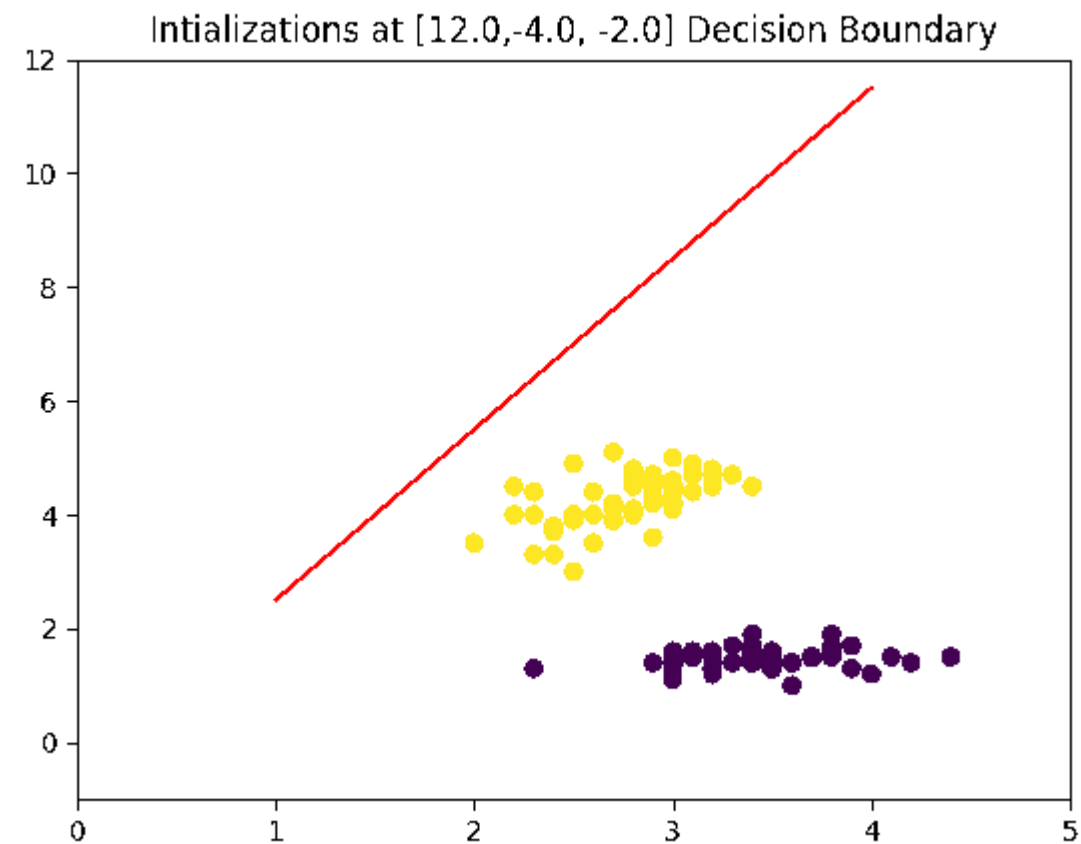
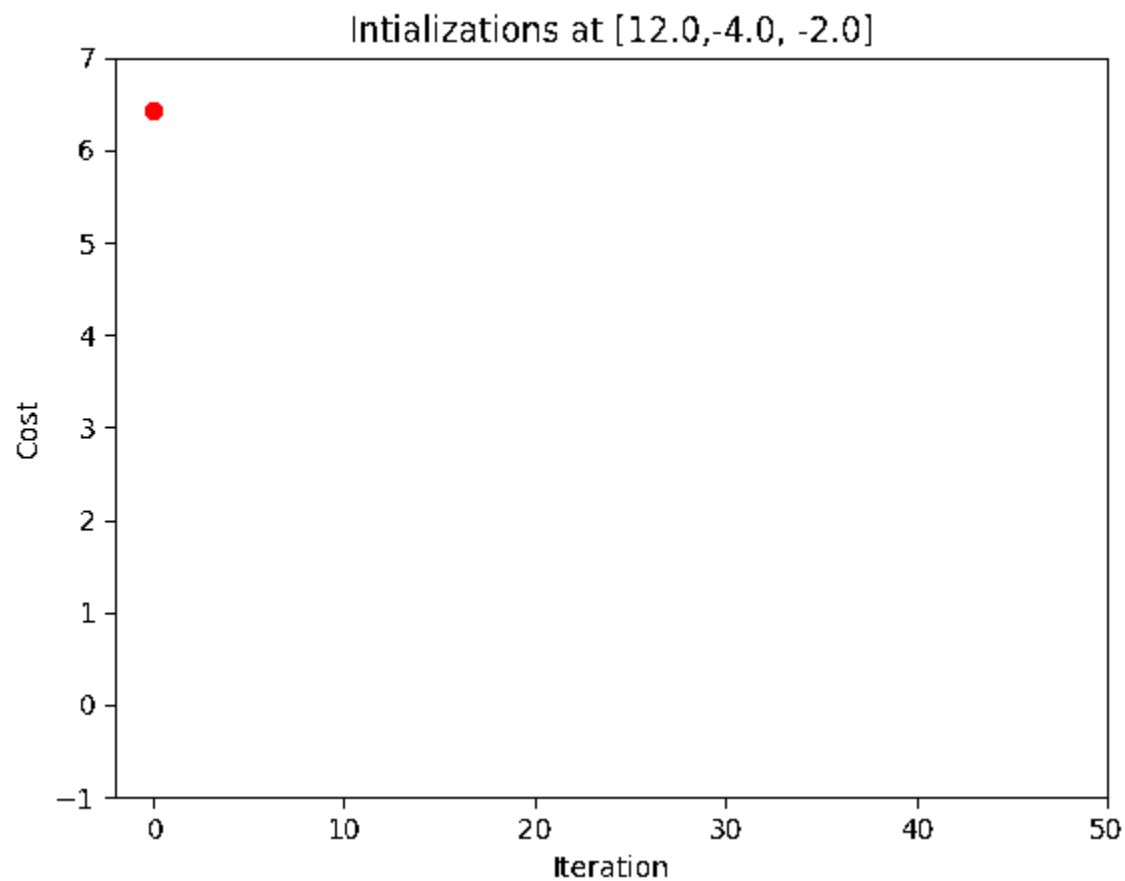
- Minimum of the function lies in the opposite direction of gradient
- Start with a guess
- Take a step against gradient:
- $w' = w - \eta \nabla J(w)$



# Gradient Descent - Initialization

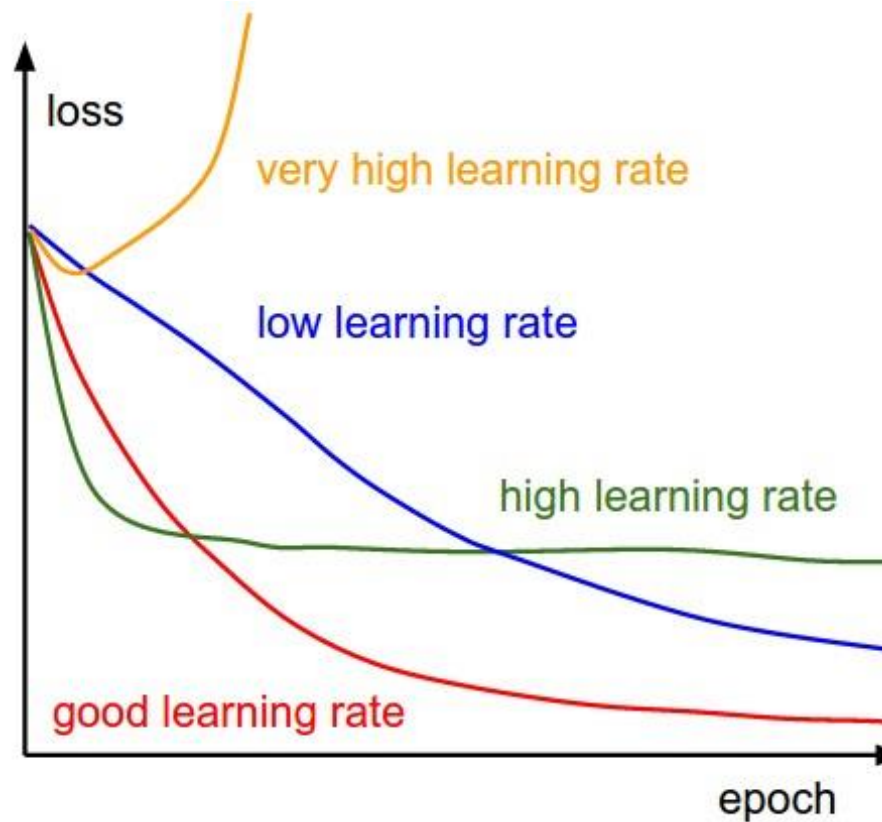


# Gradient Descent - Initialization



Scale varied for better visibility

# Gradient Descent - Learning rate



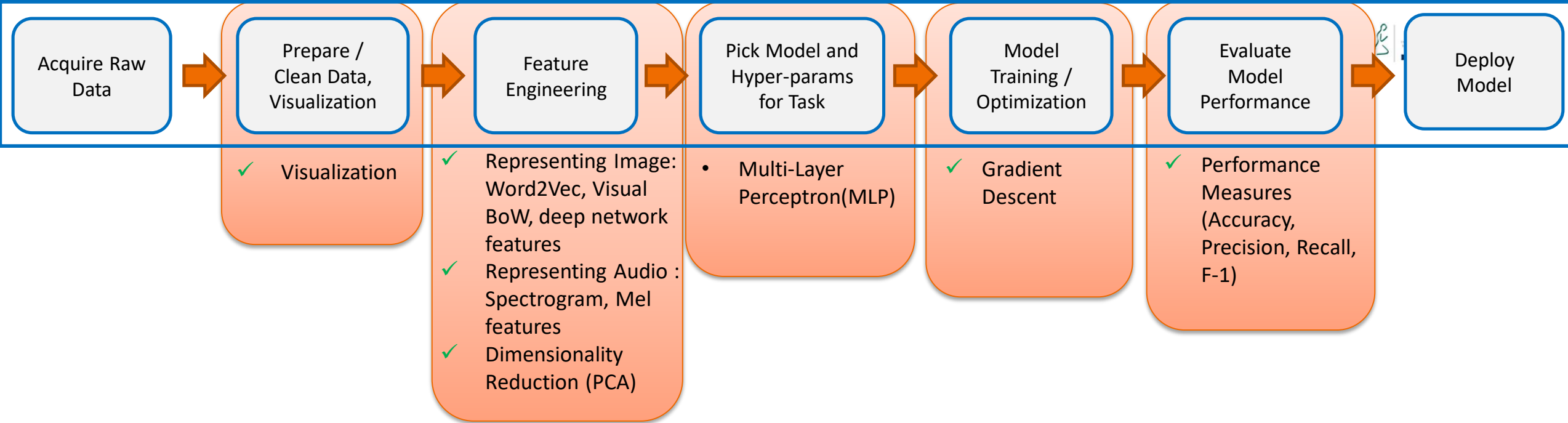
<http://cs231n.github.io/neural-networks-3/#baby>

# Mini Batch Gradient Descent

## Pseudo Code

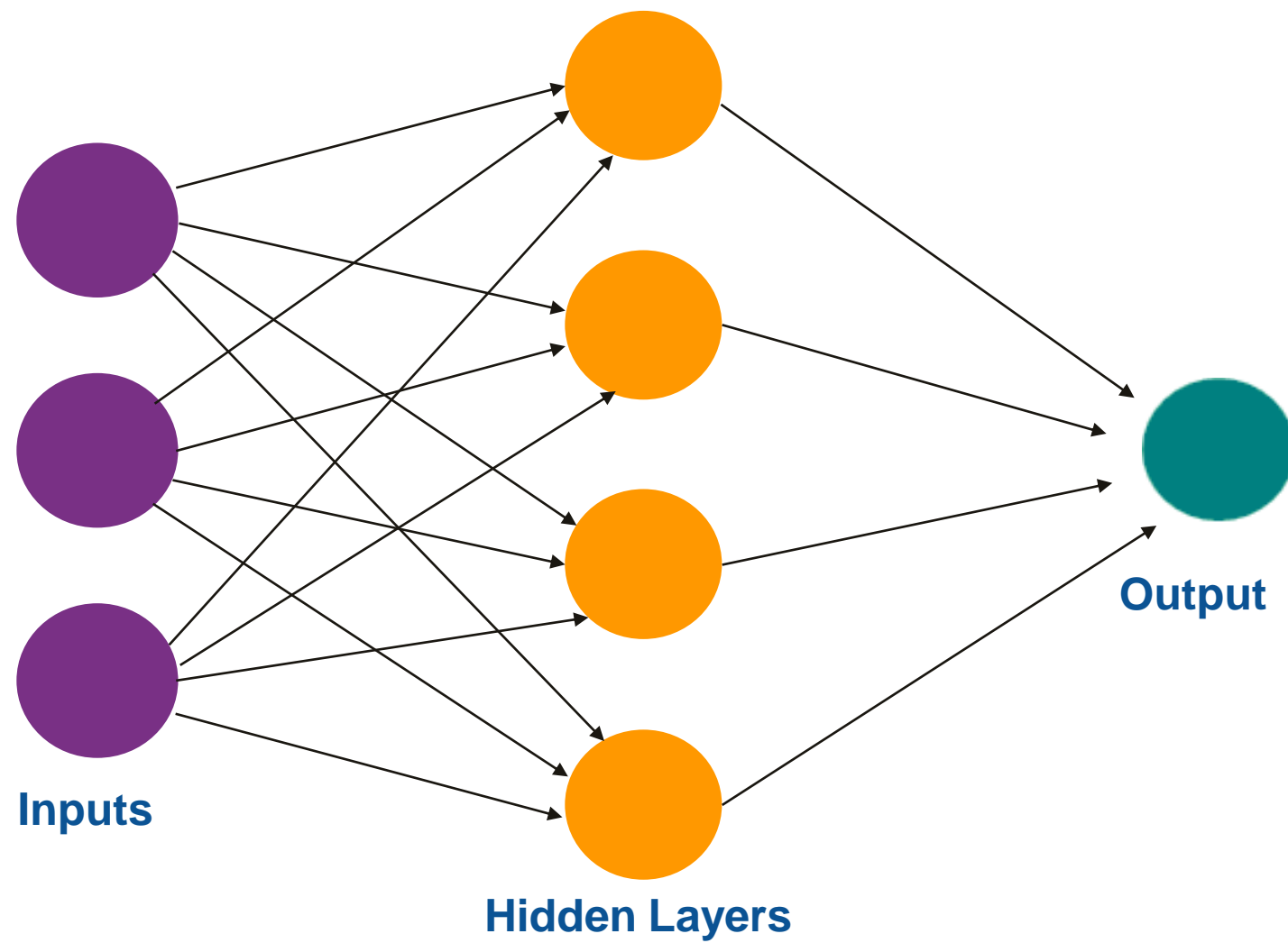
```

model = initialization(...)
train_data = load_training_data()
for i in 1...n: // epochs
    Tb1, Tb2, Tb3, ....., Tbm = split_training_batches(train_data)
    for Tbj in Tb1, Tb2, Tb3, ....., Tbm: // mini batches
        error = 0
        for X, y in Tbj: // samples in mini batch Tbj
            predictions = predict(X, model)
            error += calculate_error(y, predictions)
        gradient = differentiate(model_params, error)
        model = update_model(model, gradient)
    
```



## MLP

# MLP

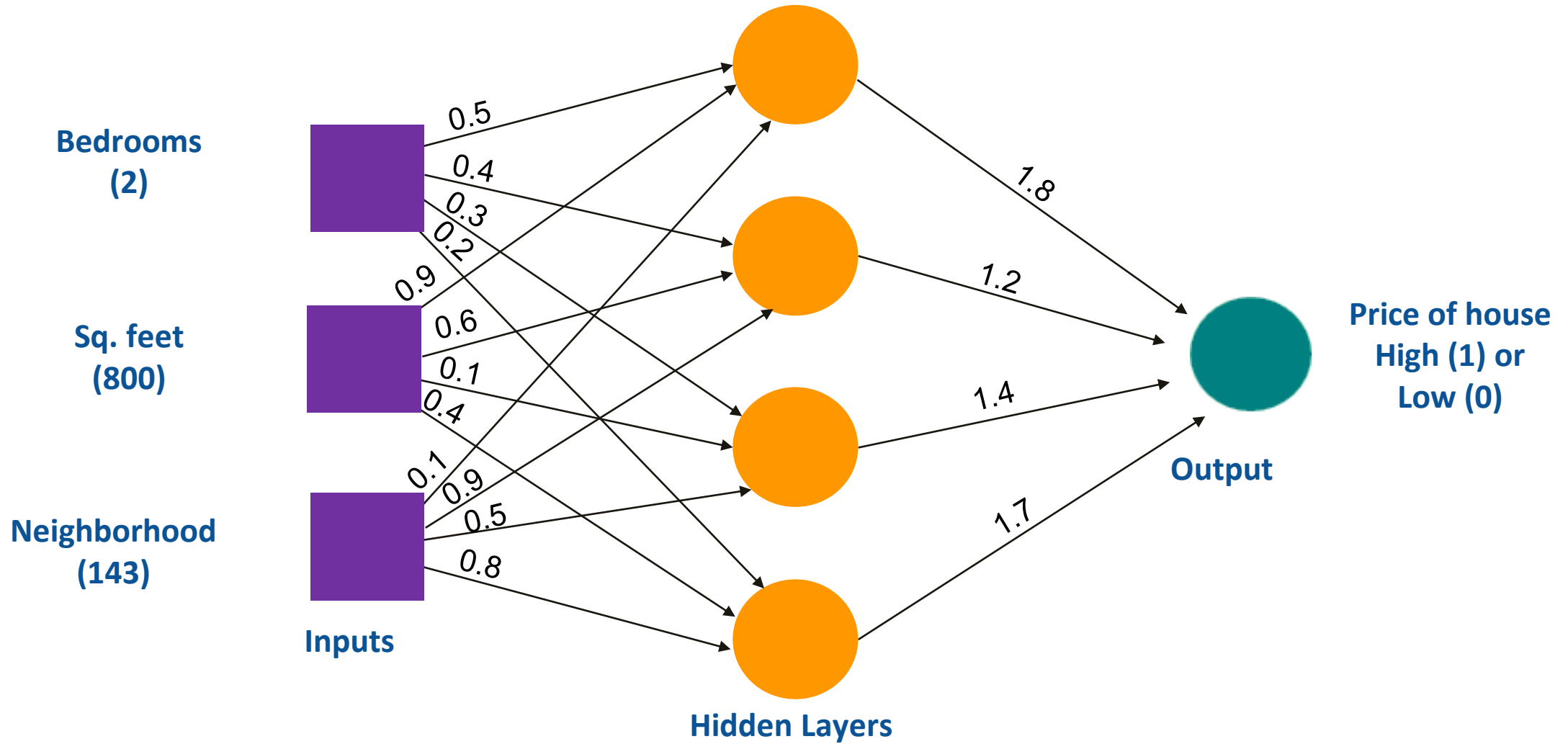




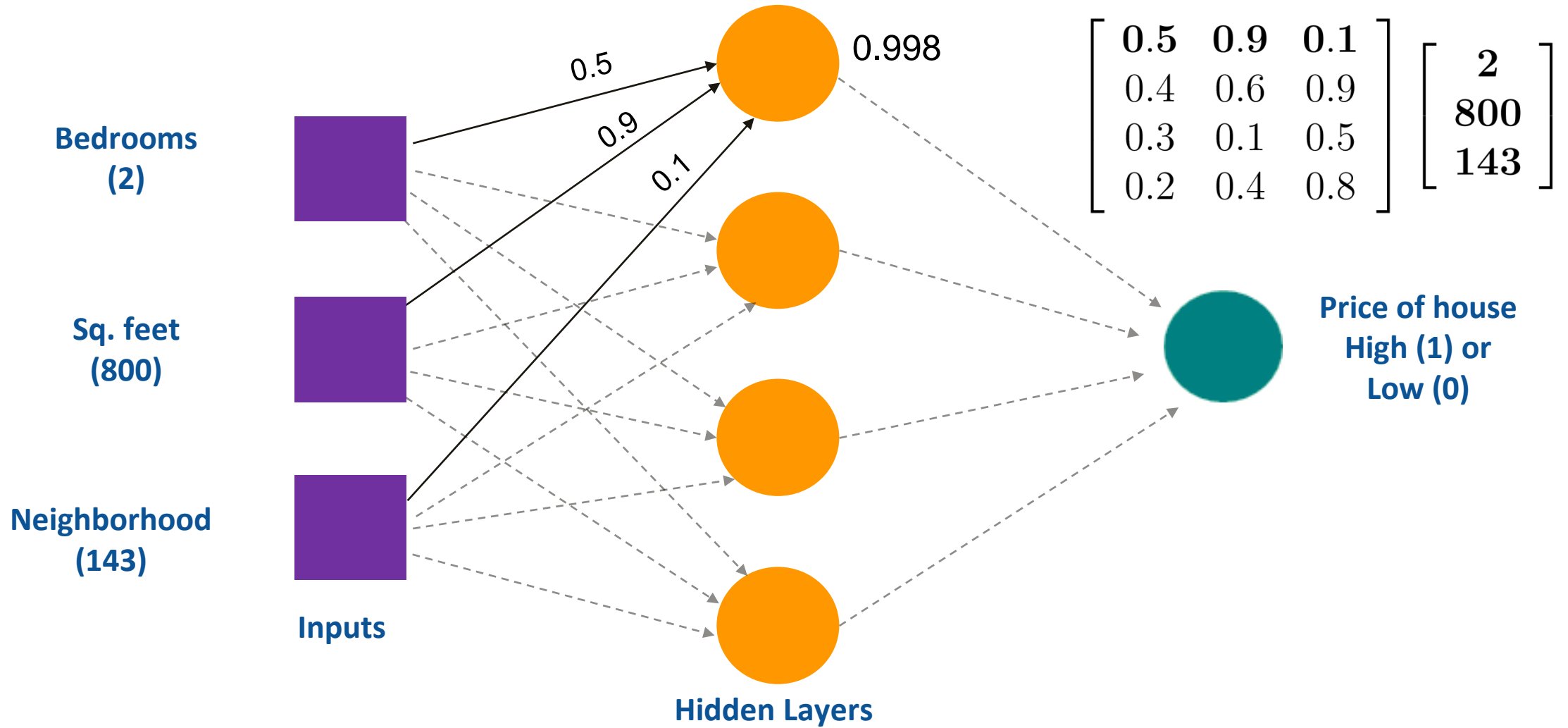
# Eg. House price from attributes

Bedrooms	Sq. Feet	Neighborhood (no. of houses in the locality)	Price high or low? High (1), Low (0)
3	2000	90	1
2	800	143	0
2	850	167	0
1	550	267	0
4	2000	396	1

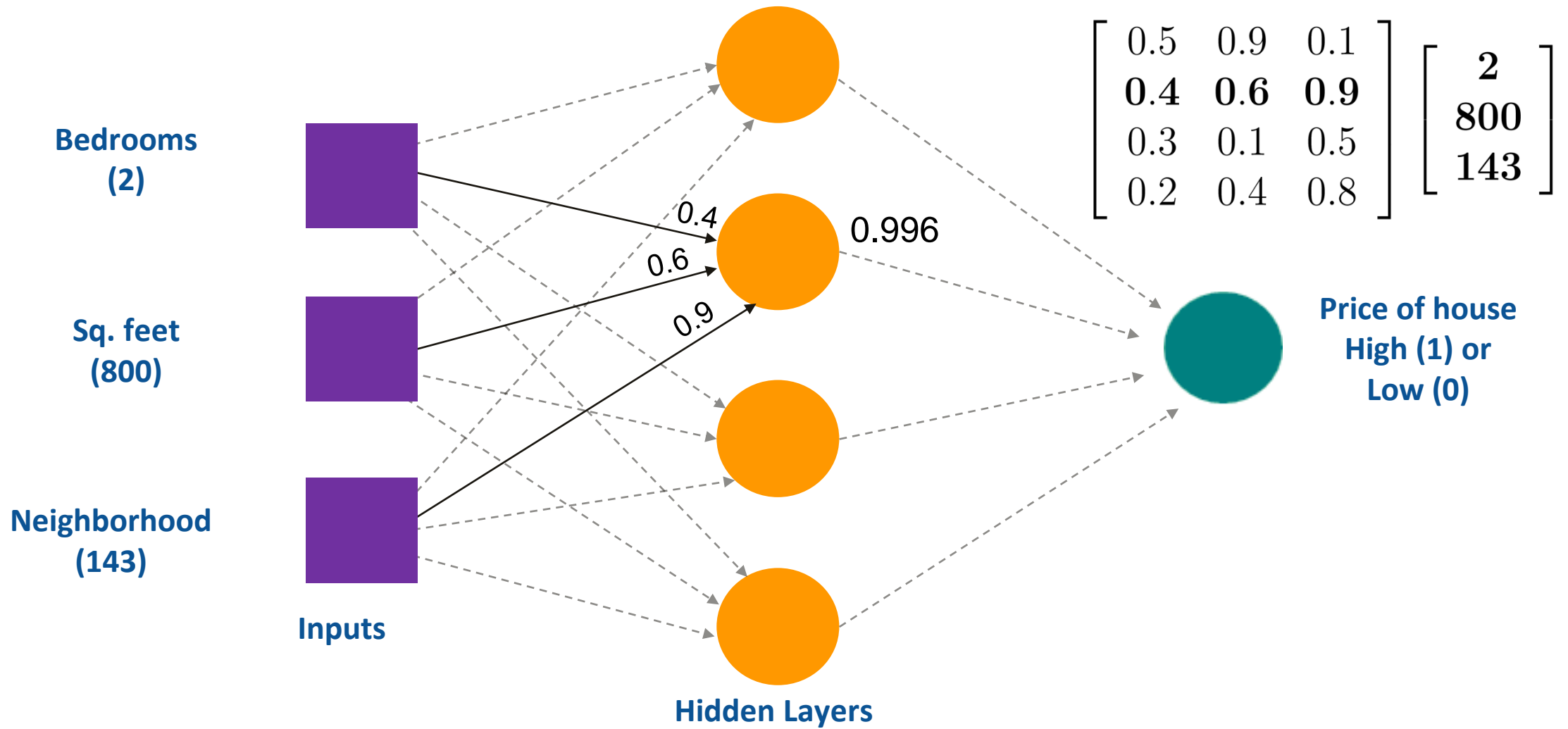
# Initialize weights



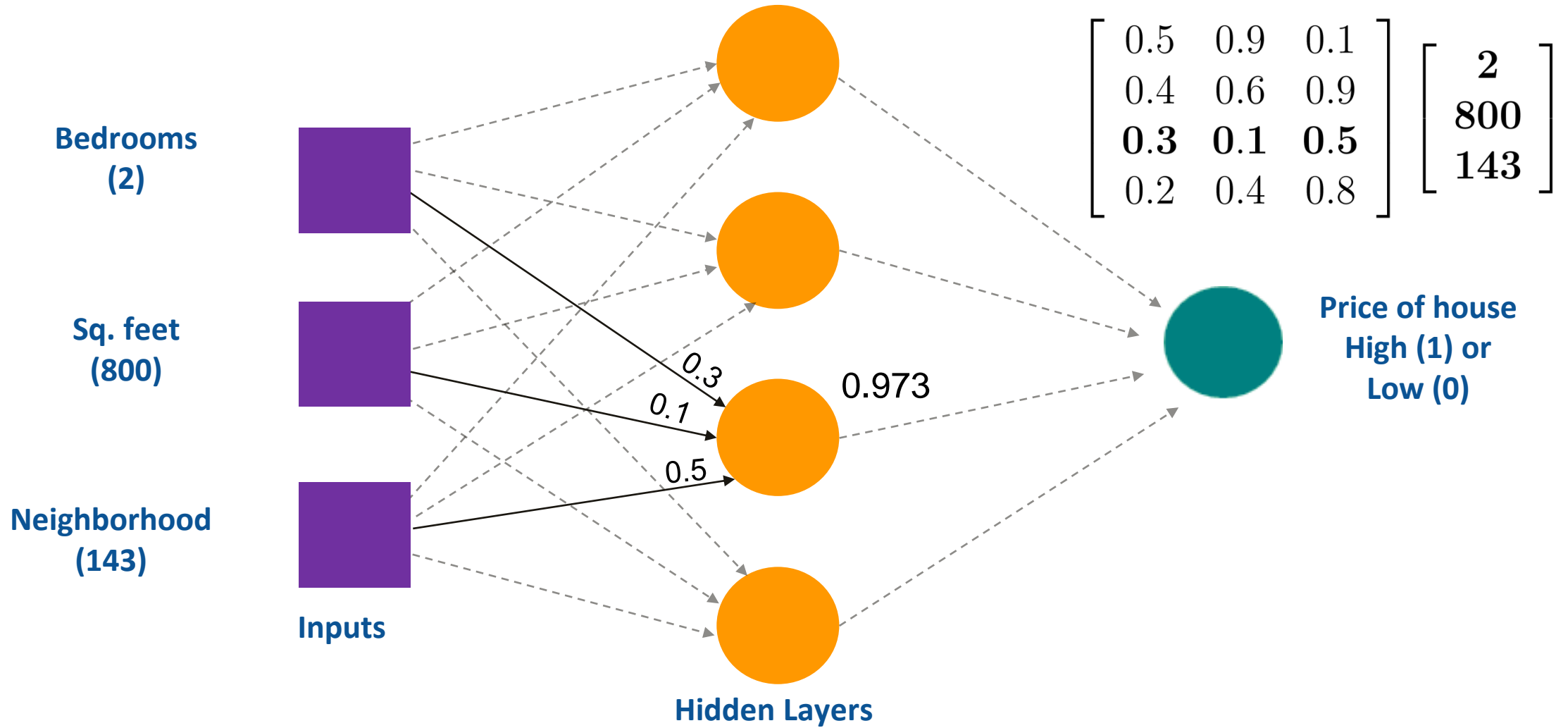
# Weights at the first neuron



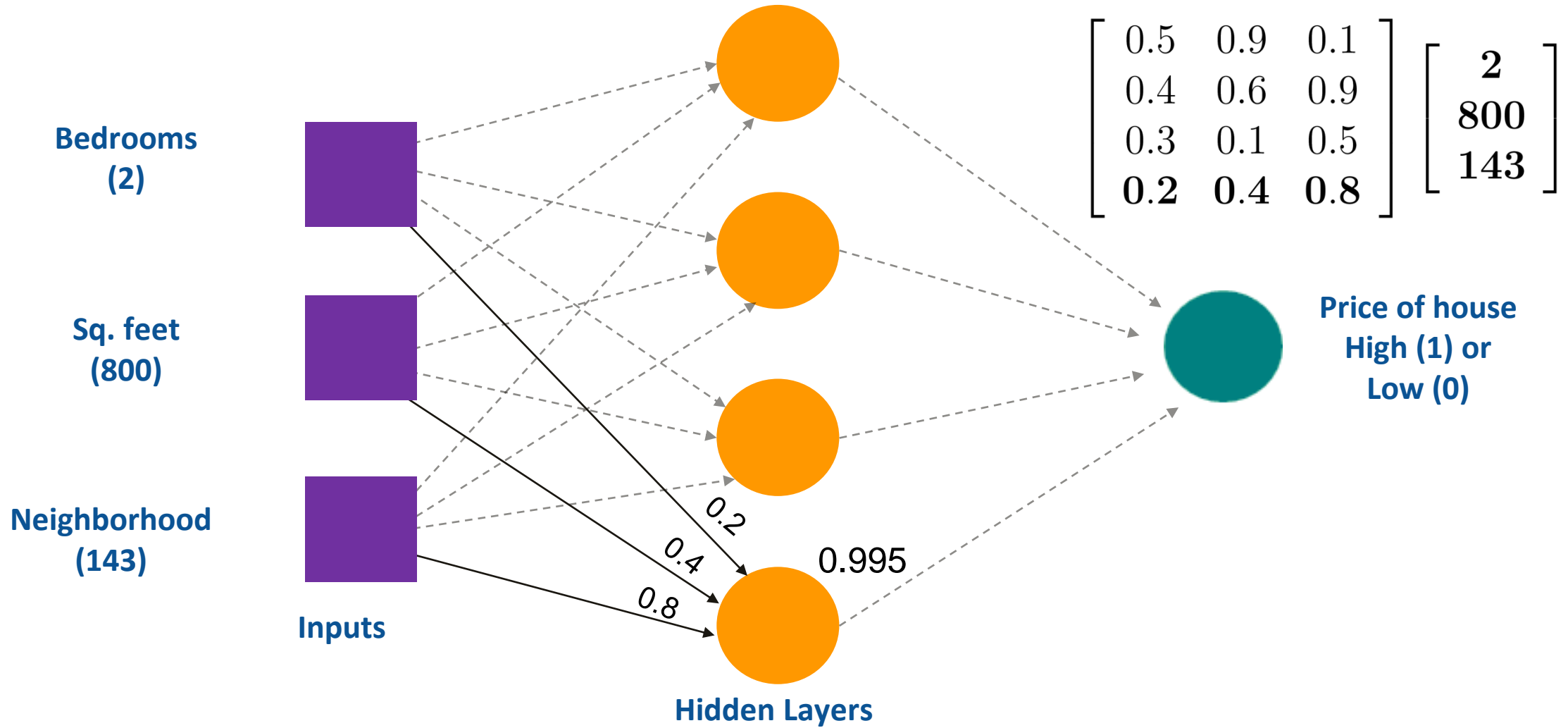
# Weights at the second neuron



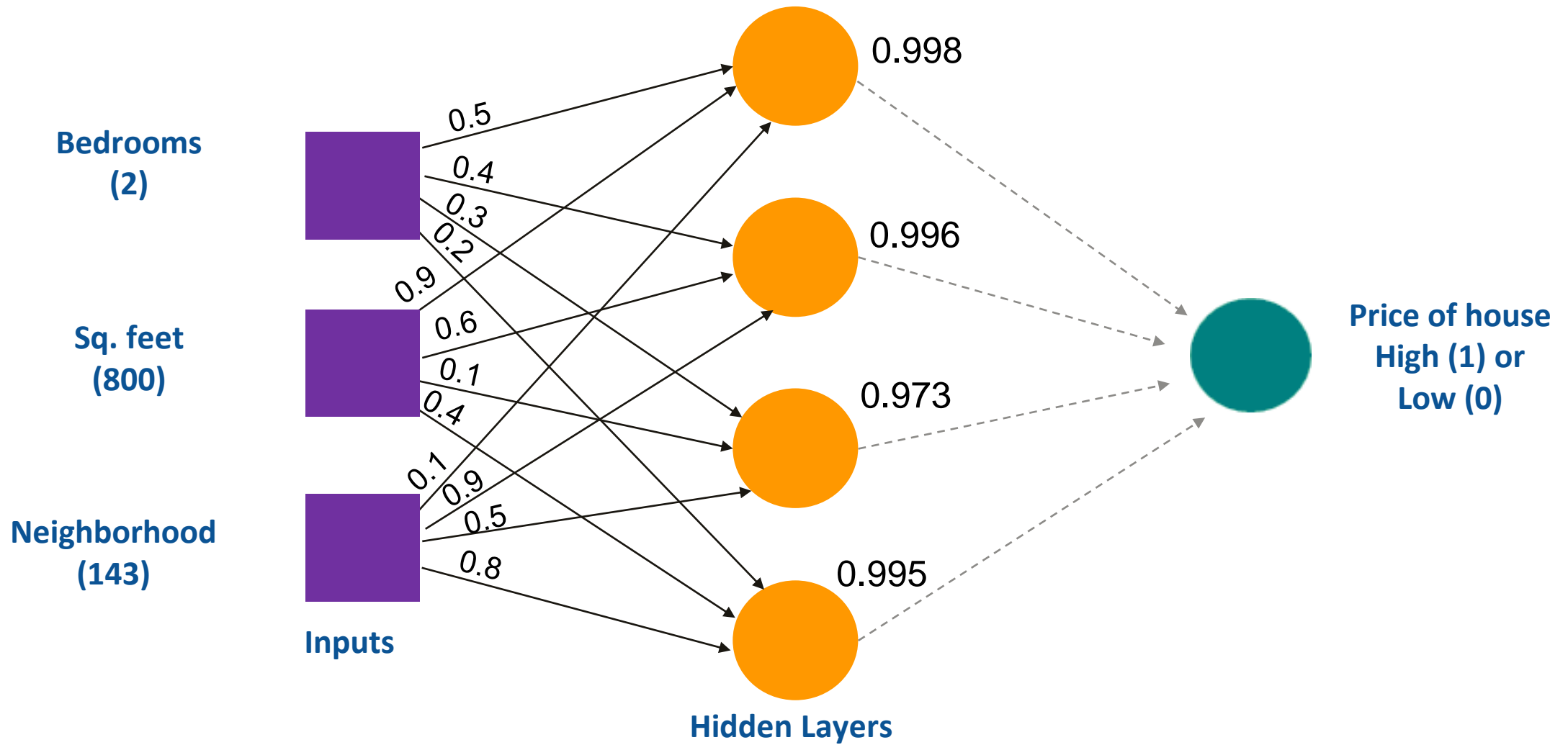
# Weights at the third neuron



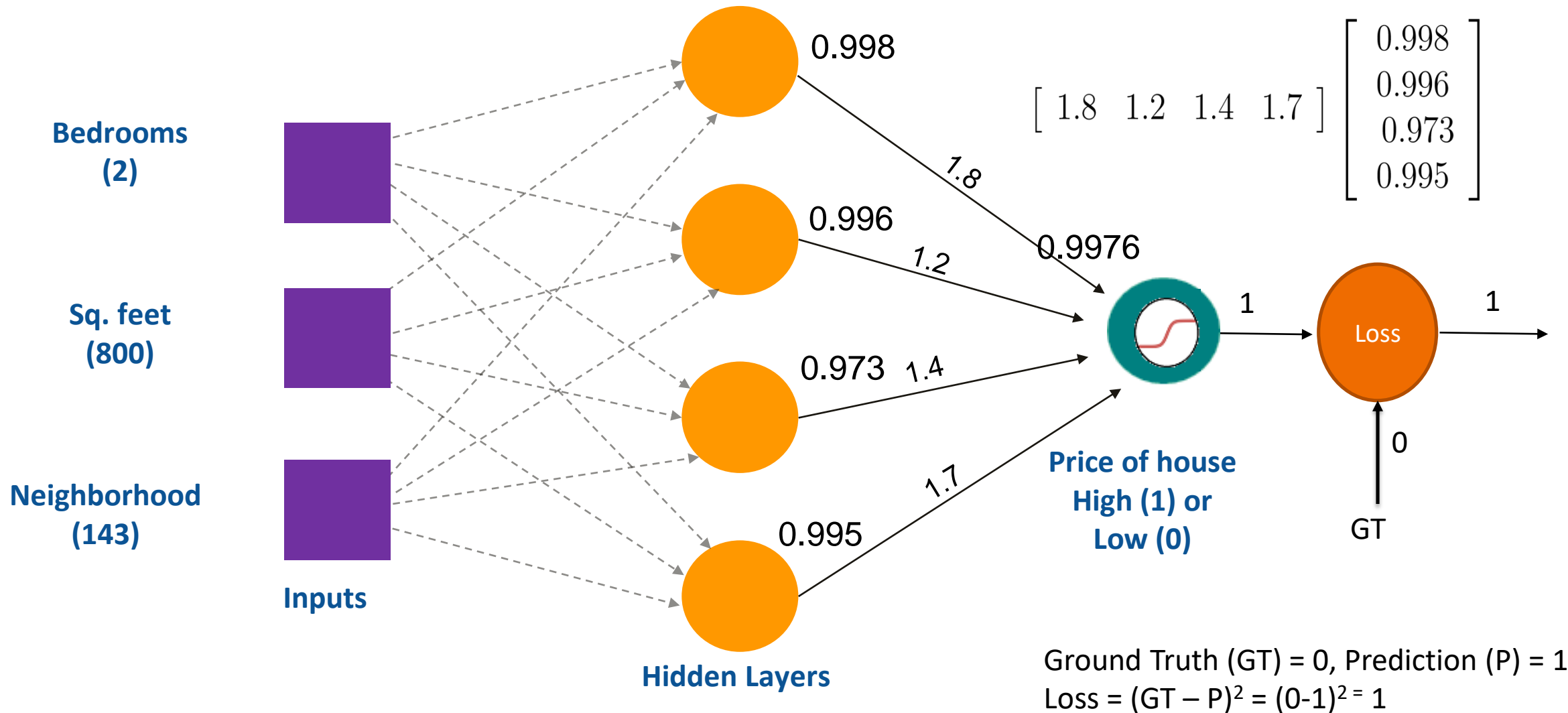
# Weights at the fourth neuron



# Inputs to the next layer

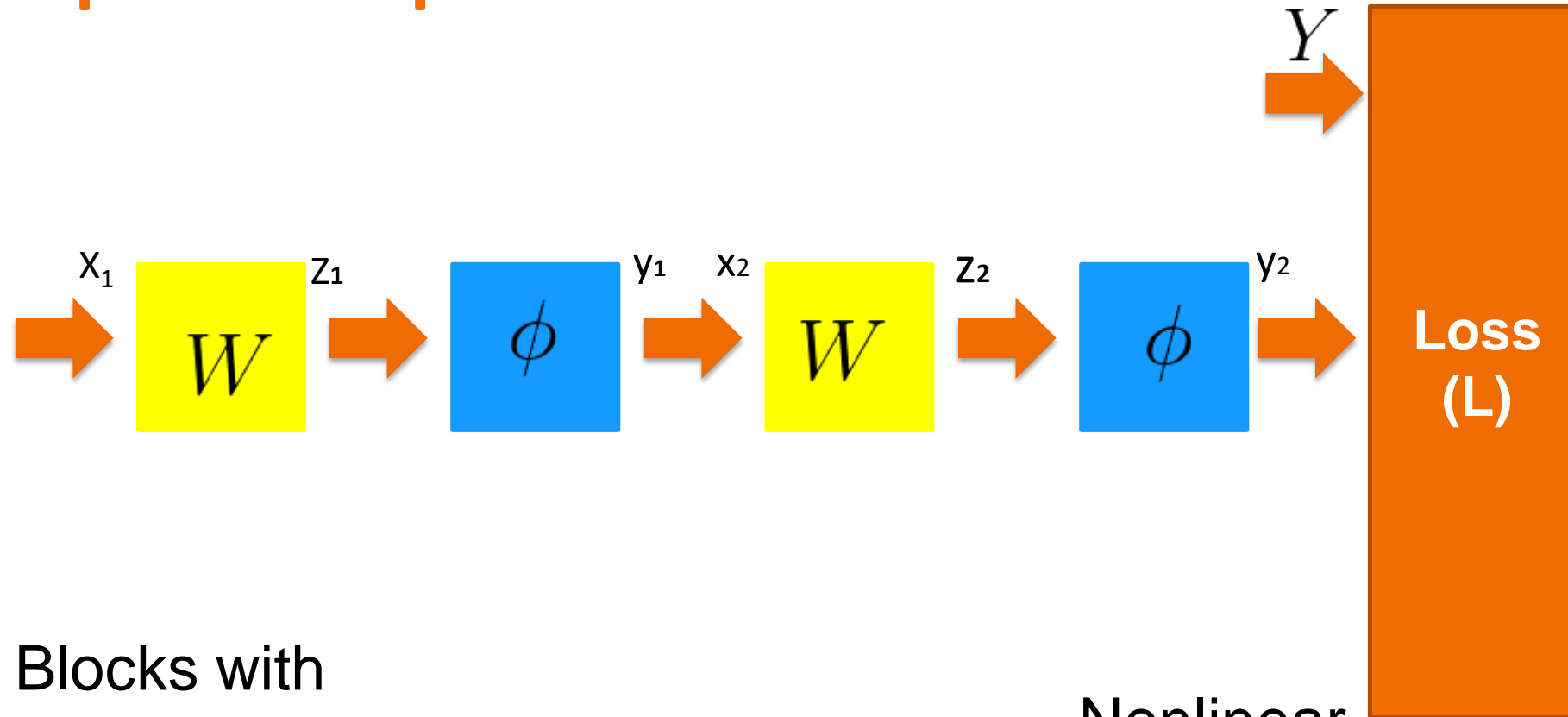


# Computations in next layer





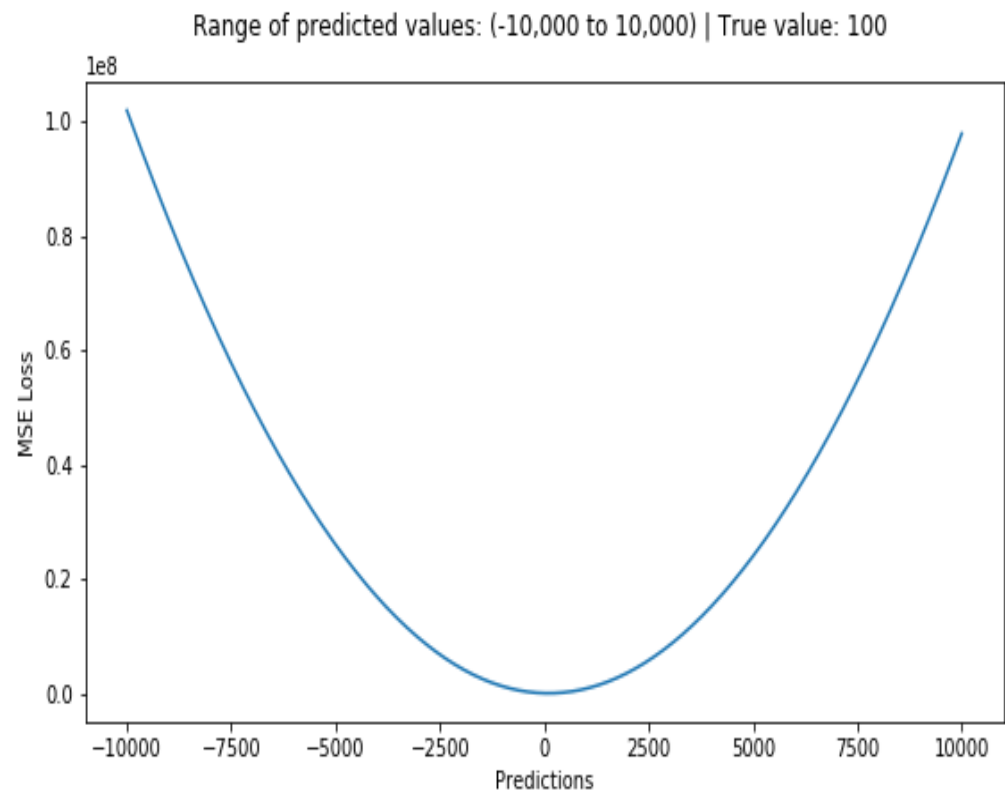
# A simpler view point



Blocks with  
 Learnable  
 parameters  
 Matrix  
 Multiplication

Nonlinear  
 functions  
 (often non  
 learnable)

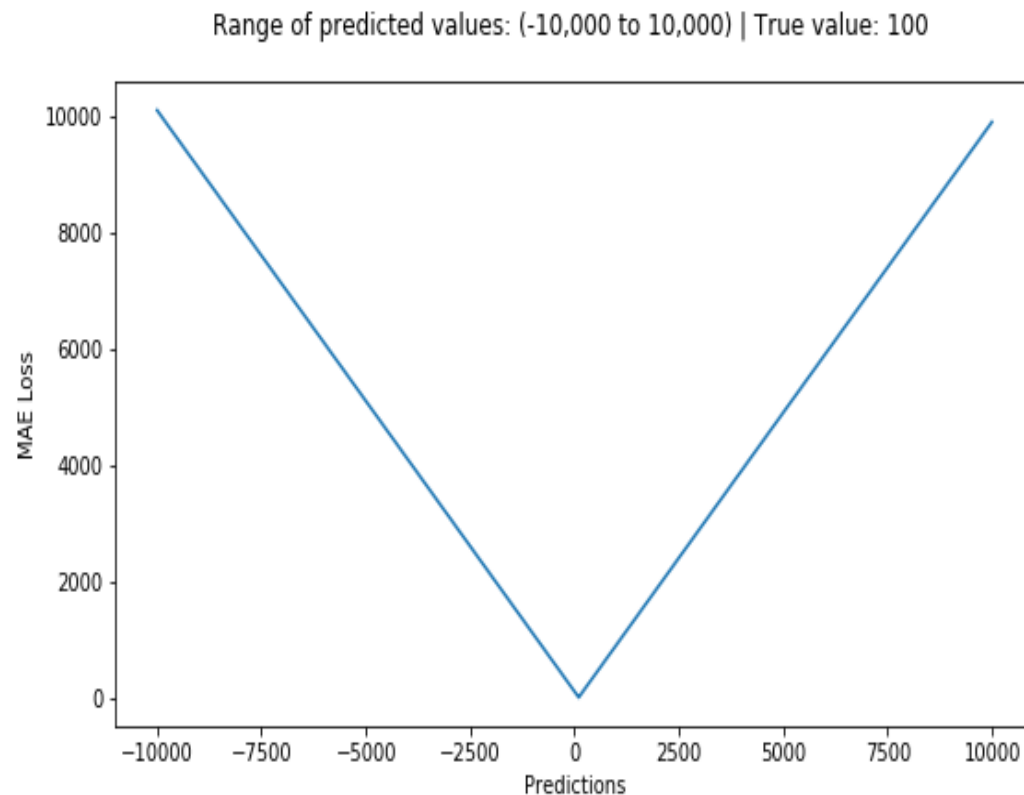
# Loss Functions



**Mean Squared Loss**

$$L(y, y') = (y - y')^2$$

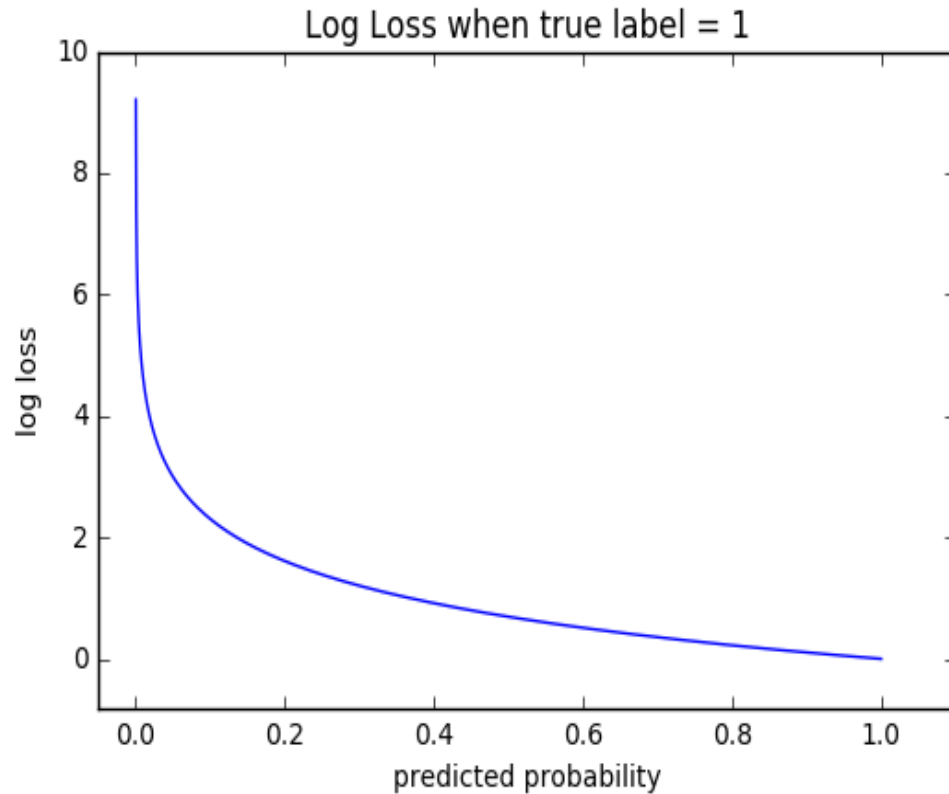
$y$  - Actual value  
 $y'$  - Predicted value



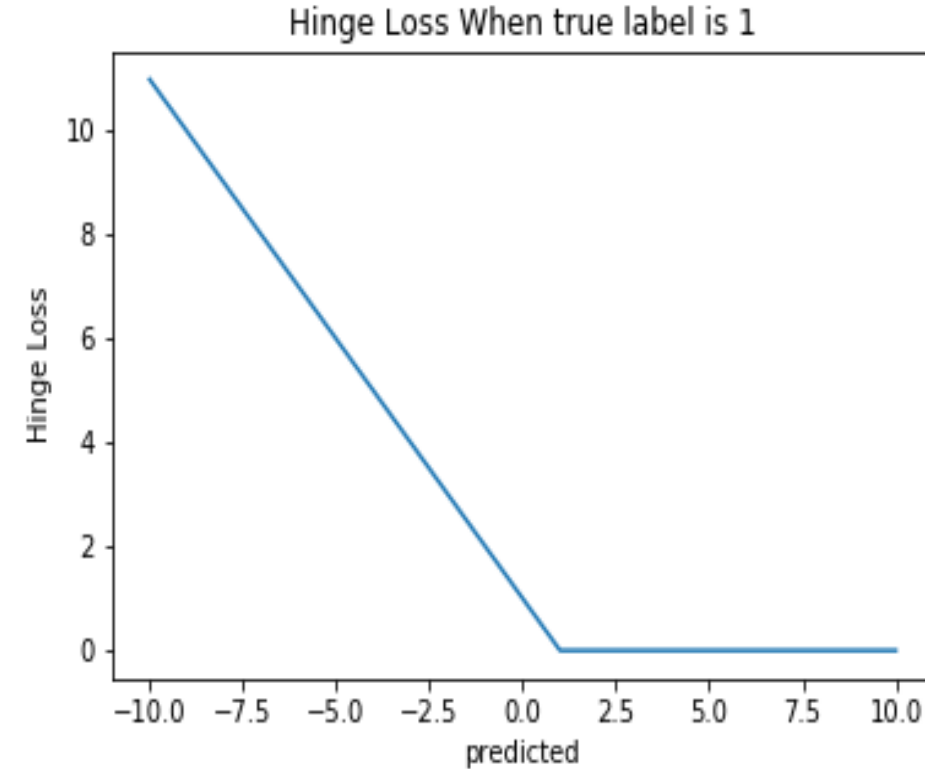
**Mean Absolute Loss**

$$L(y, y') = |y - y'|$$

# Loss Functions



**Cross Entropy Loss**



**Hinge Loss**

$$L(y, y') = -(y \log(y') + (1 - y) \log(1 - y'))$$

$y$  - Actual value  
 $y'$  - Predicted value

$$L(y, y') = \max(0, 1 - y * y')$$

# Softmax

- Normalizes the output.
- K is total number of classes

$$z_n = \frac{e^{x_n}}{\sum_{i=1}^K e^{x_i}}$$

```
Out[12]: array([ 6.,  0.,  5.,  3.,  8.])
```

```
In [8]: exp = (np.e)**(x)
exp
```

executed in 6ms, finished 01:47:23 2018-08-21

```
Out[8]: array([ 4.03428793e+02,  1.00000000e+00,  1.48413159e+02,
                2.00855369e+01,  2.98095799e+03])
```

```
In [9]: sigma_e = np.sum(exp)
sigma_e
```

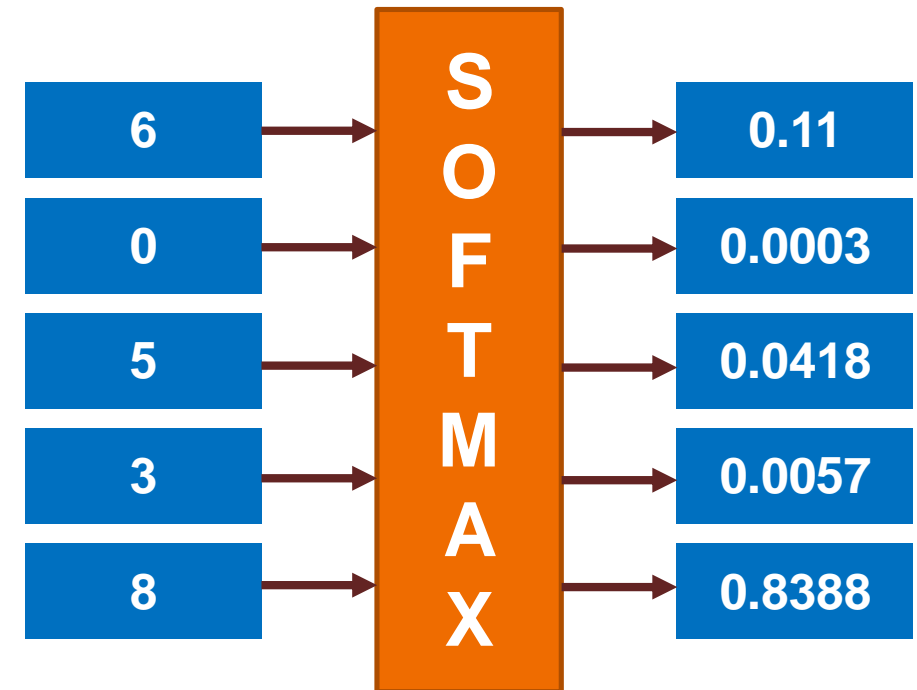
executed in 9ms, finished 01:47:25 2018-08-21

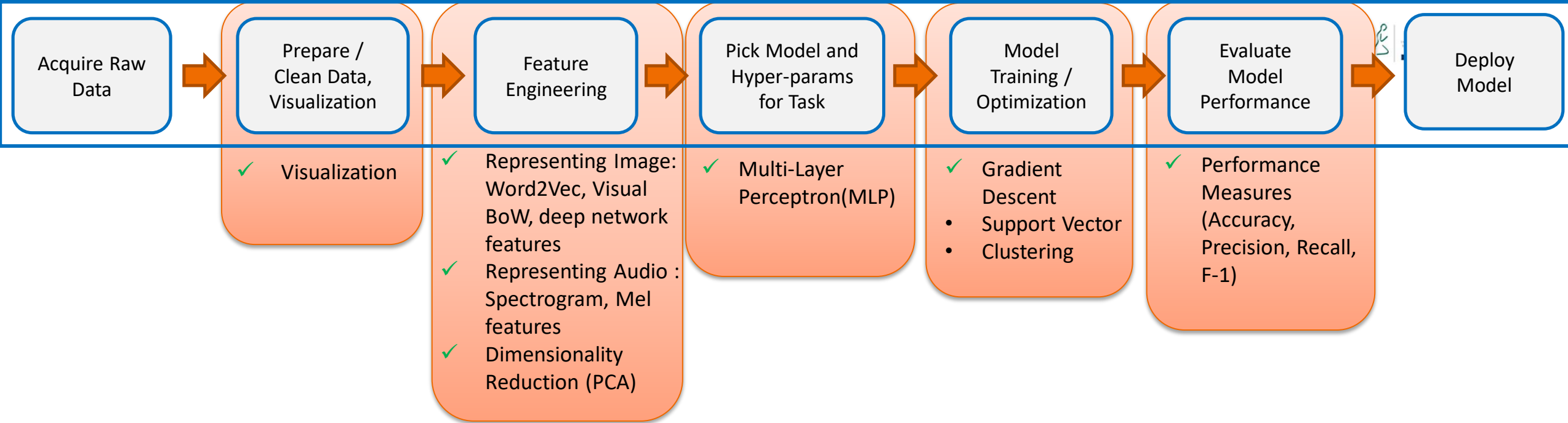
```
Out[9]: 3553.8854765602264
```

```
In [11]: z = exp/sigma_e
z
```

executed in 8ms, finished 01:47:34 2018-08-21

```
Out[11]: array([ 1.13517669e-01,  2.81382168e-04,  4.17608165e-02,
                 5.65171192e-03,  8.38788421e-01])
```





## SVMs and Kernels

### Kernel as Similarity Function

# Linear Classifier

- Decision boundary: Hyperplane

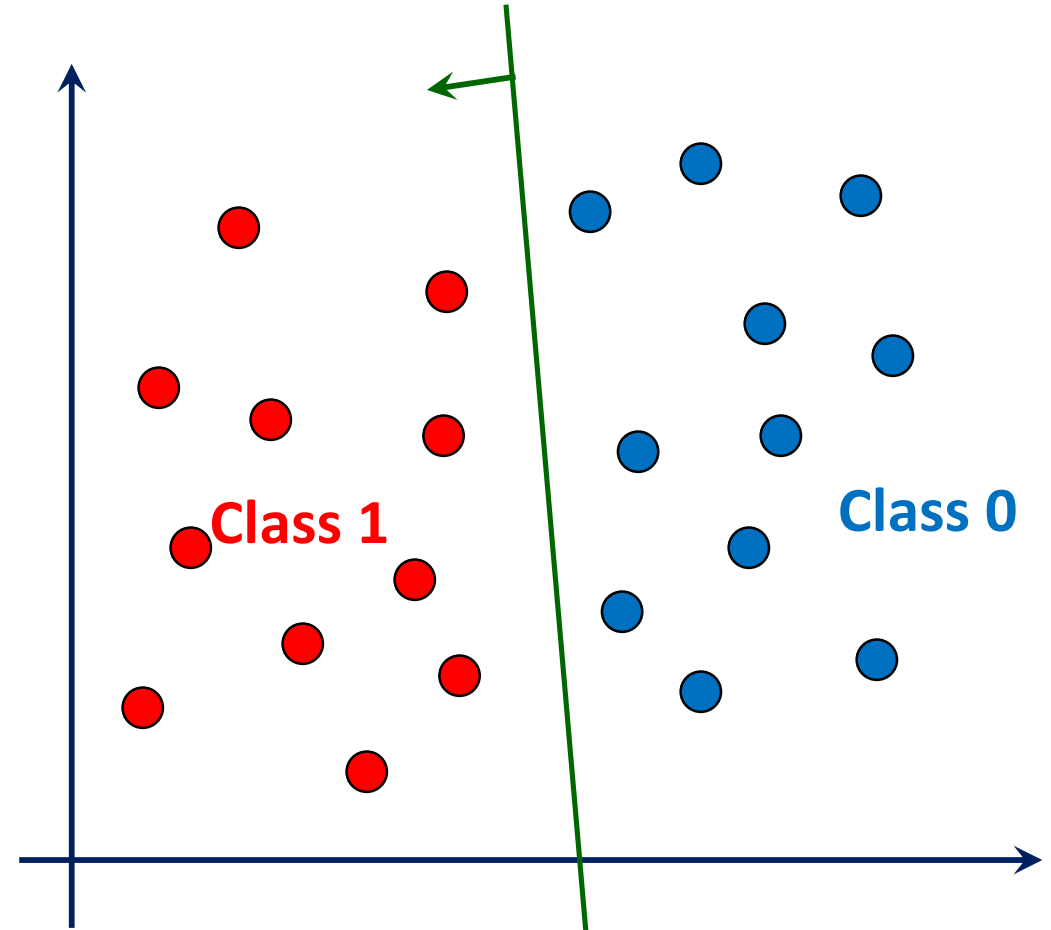
$$w^T x = 0$$

- Class 1 lies on the positive side

$$w^T x > 0$$

- Class 0 lies on the negative side

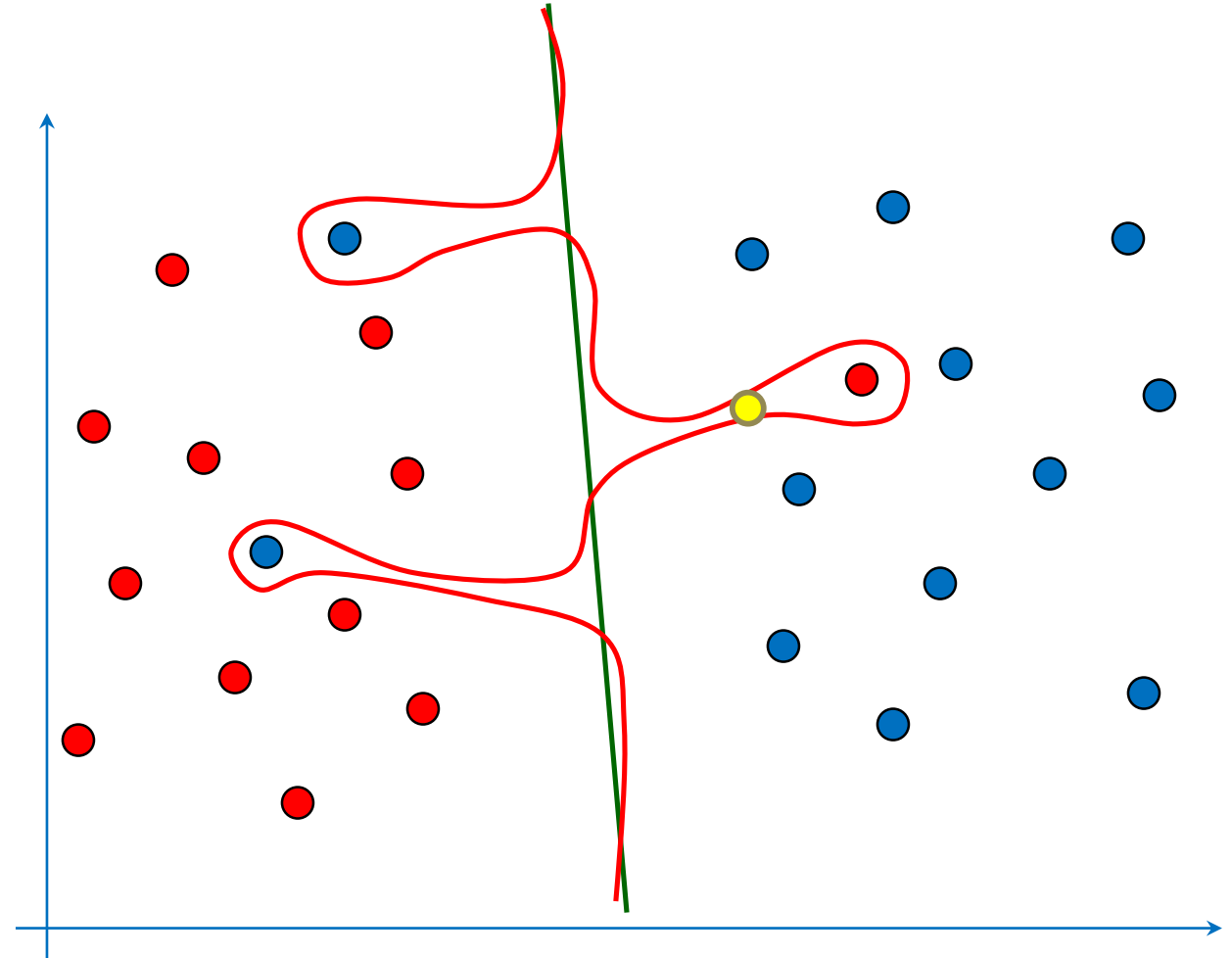
$$w^T x < 0$$



# Why Linear? Generalization vs. Complexity

- Is it good to use a complex curve to reduce training error?

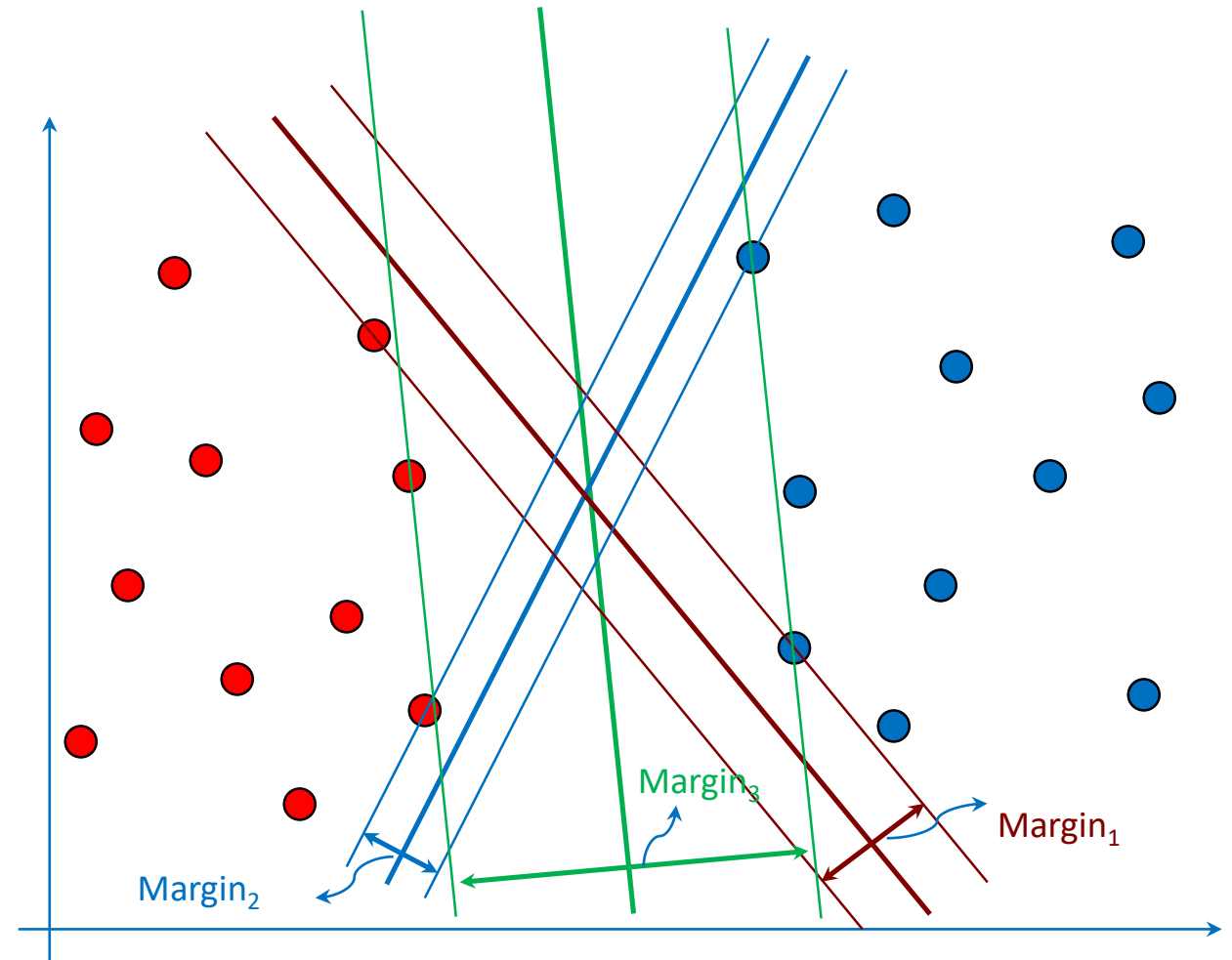
Are both solutions equally good?



# Margin: The No-mans Band

- **Margin:** Width of a band around decision boundary without any training samples
- Margin varies with the position and orientation of the separating hyperplane

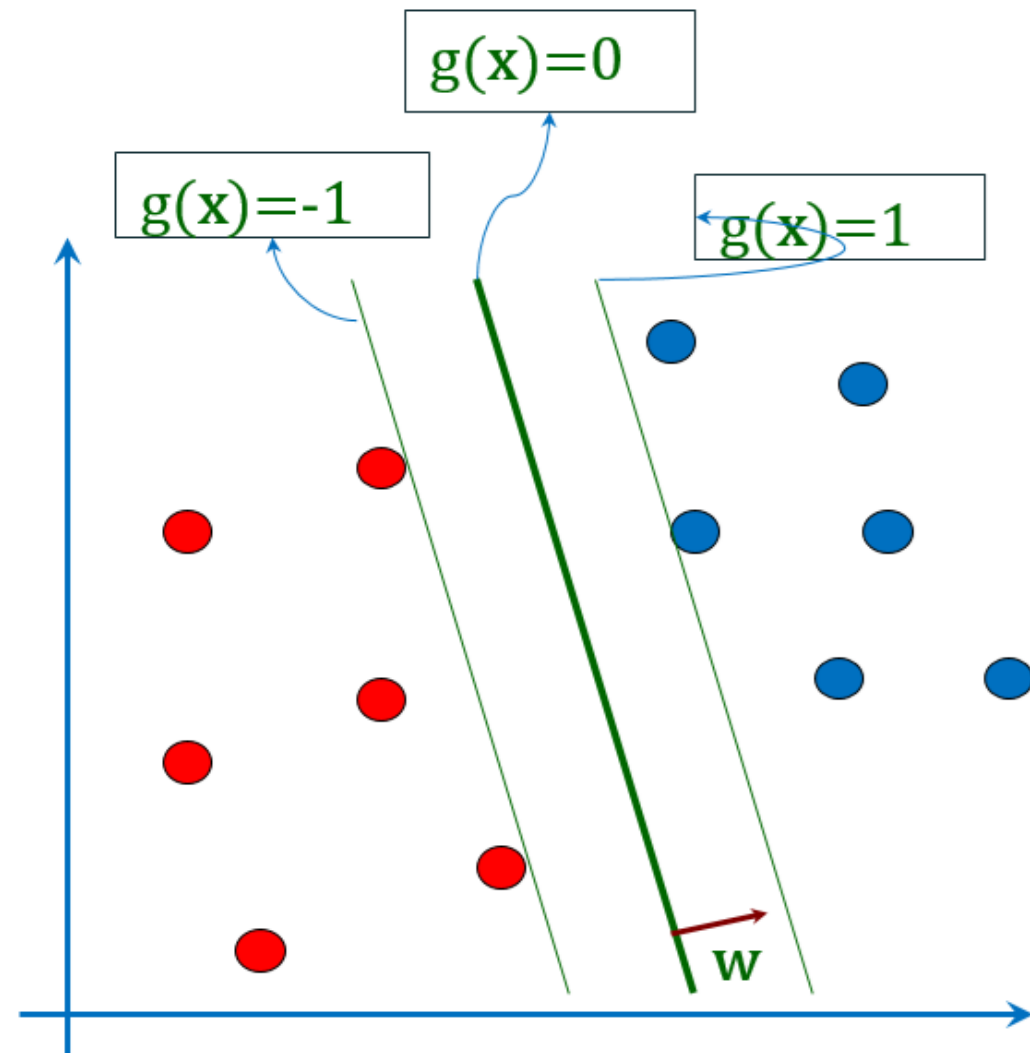
Is a Larger Margin better? Why?





# SVM: Formulation

- Let  $g(X) = W^T X + b$
- We want to maximize margin:
  - $W^T X_i + b \leq -1$  for  $y_i = -1$
  - $W^T X_i + b \geq 1$  for  $y_i = 1$
  - Or  $y_i(W^T X_i + b) \geq 1$  for  $i$ .



# SVM Optimization: Convex Optimization

$$\min \frac{1}{2} W^T W$$

$$y_i(W^T x_i + b) - 1 \geq 0 \forall i$$

$$y_i \in \{1, -1\}$$

---


$$J_d(\alpha) = \sum_{i=1}^N \alpha_i - \frac{1}{2} \alpha_i \alpha_j y_i y_j \boxed{x_i^T x_j} \quad \sum_{i=1}^N \alpha_i y_i = 0$$

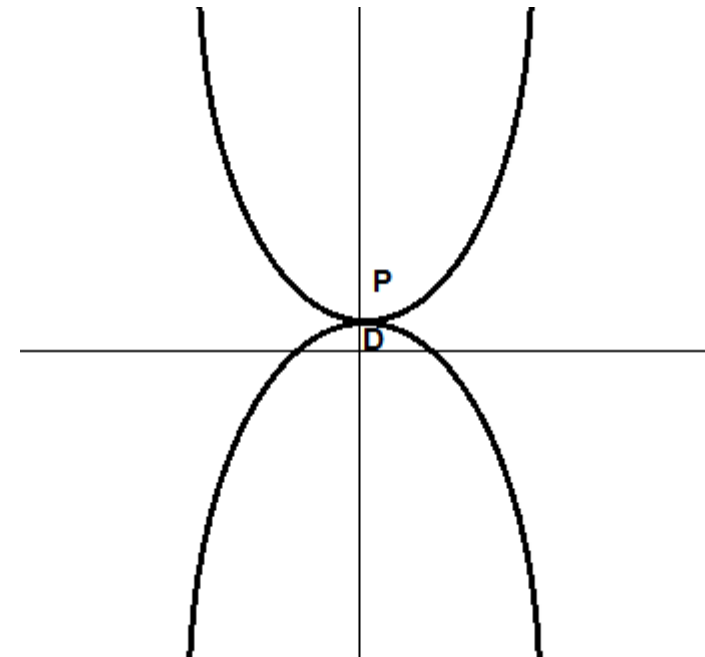
$$W = \sum_{i=1}^N \boxed{\alpha_i y_i x_i}$$

Support Vectors

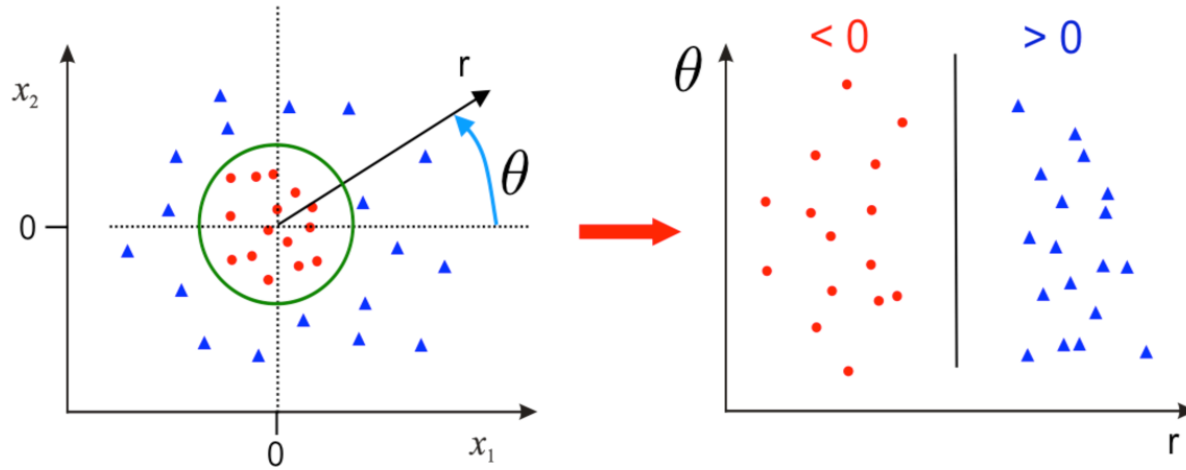
Convex Opt.  
Only dot products

$$W^T X = \sum_{i=1}^N \alpha_i y_i \boxed{x_i^T x}$$

Only dot products



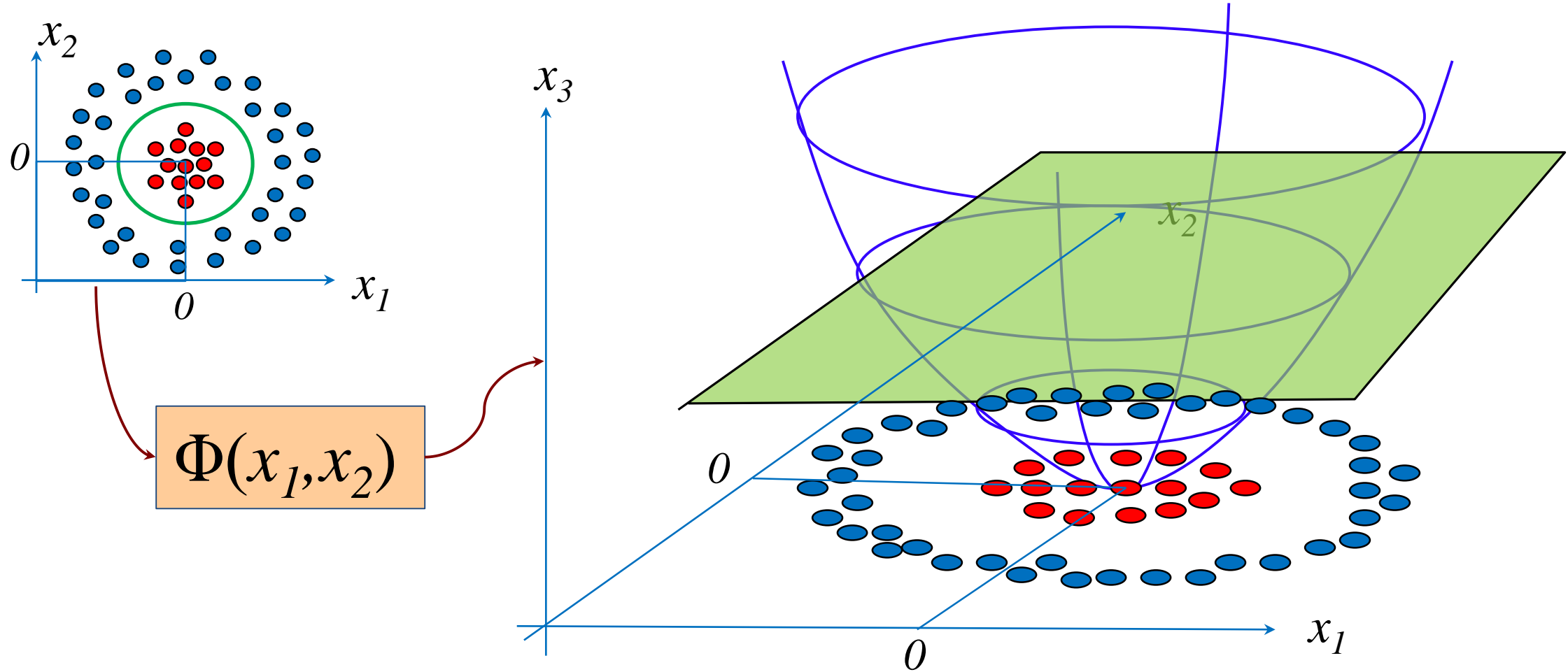
# Nonlinearity with Feature Maps



$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \rightarrow \begin{pmatrix} r \\ \theta \end{pmatrix}$$

- With a “smart” feature map, a linearly non-separable problem can be converted to a separable problem!!

# Non-linear Mapping



$\Phi$  is a non-linear mapping into a possibly high-dimensional space

# Kernel Strategy

$$w = \sum_{i=1}^N \alpha_i y_i x_i$$

What we need is only  $w^T x = \sum_{i=1}^N \alpha_i y_i x_i^T x$

**We can do the same in a new feature space:**

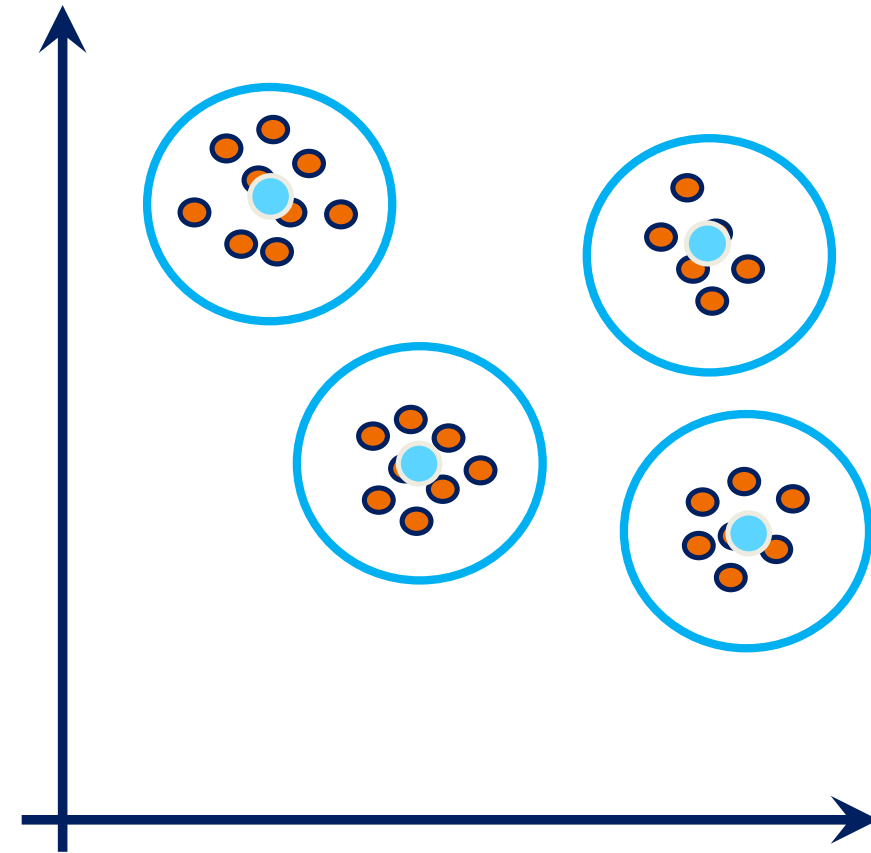
$$w^T x = \sum_{i=1}^N \alpha_i y_i \phi(x_i)^T \phi(x) \qquad w^T x = \sum_{i=1}^N \alpha_i y_i K(x_i, x)$$

# Clustering

Identifying Similar Patterns

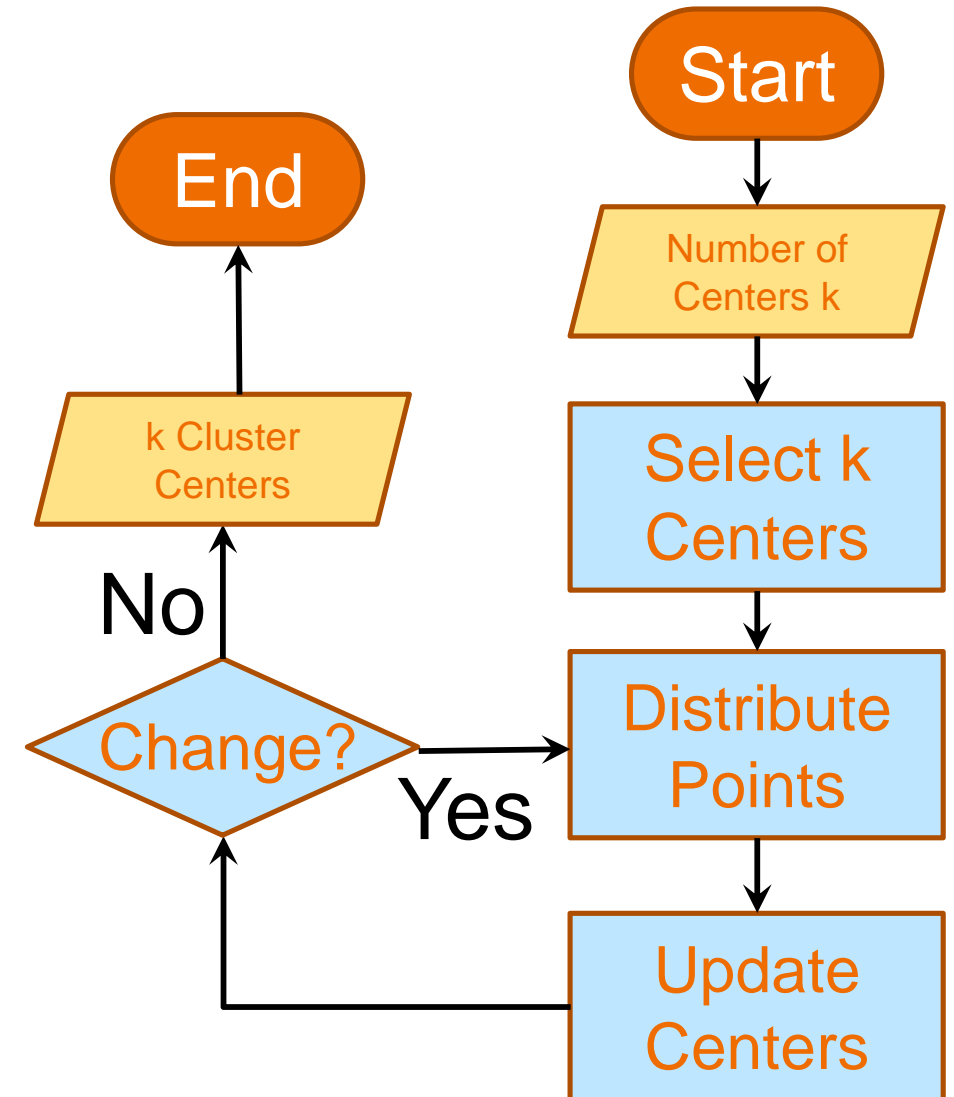
# K-Means

- You are given  $N$  points
- How do we find  $k$  clusters?
  - What if we know the cluster centers?
- How do we find the cluster centers?
  - What if we know the  $k$  clusters?



# K-Means

1. Input:  $k$  (number of clusters)
2. Randomly select  $k$  centers
3. Distribute Points
4. Update Centers
5. Repeat 3,4 till convergence
6. Output: Cluster centers

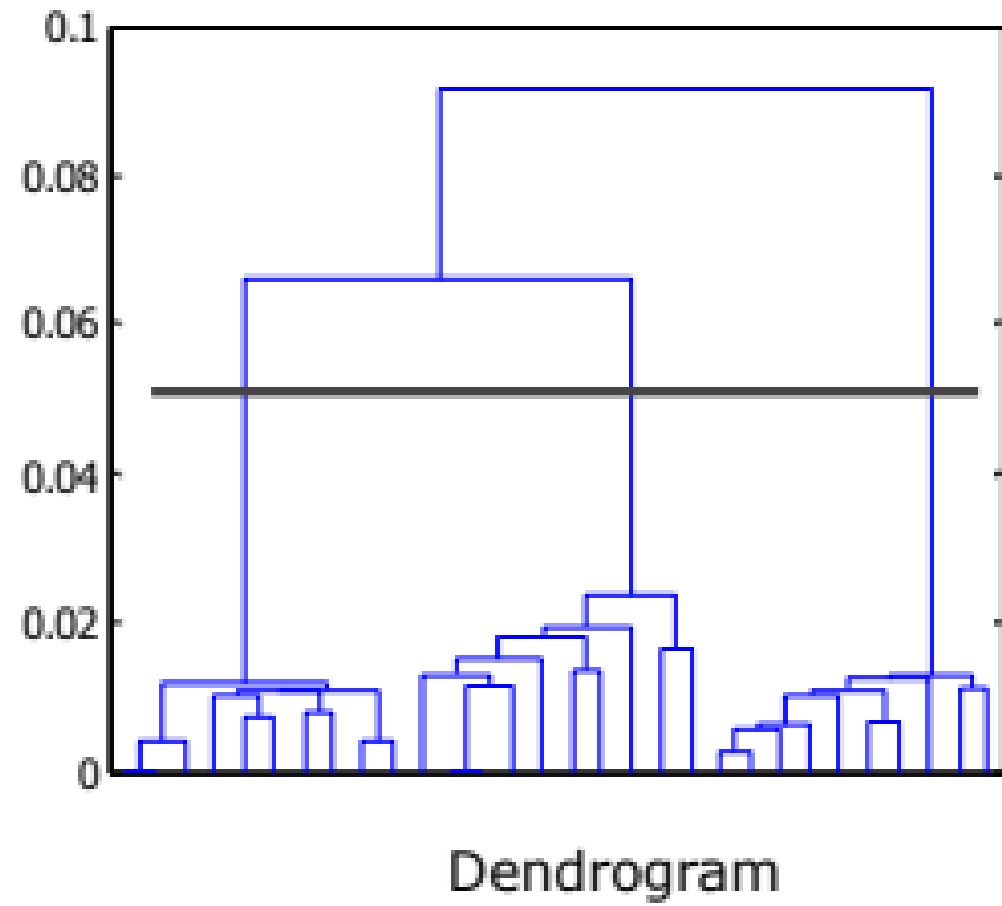
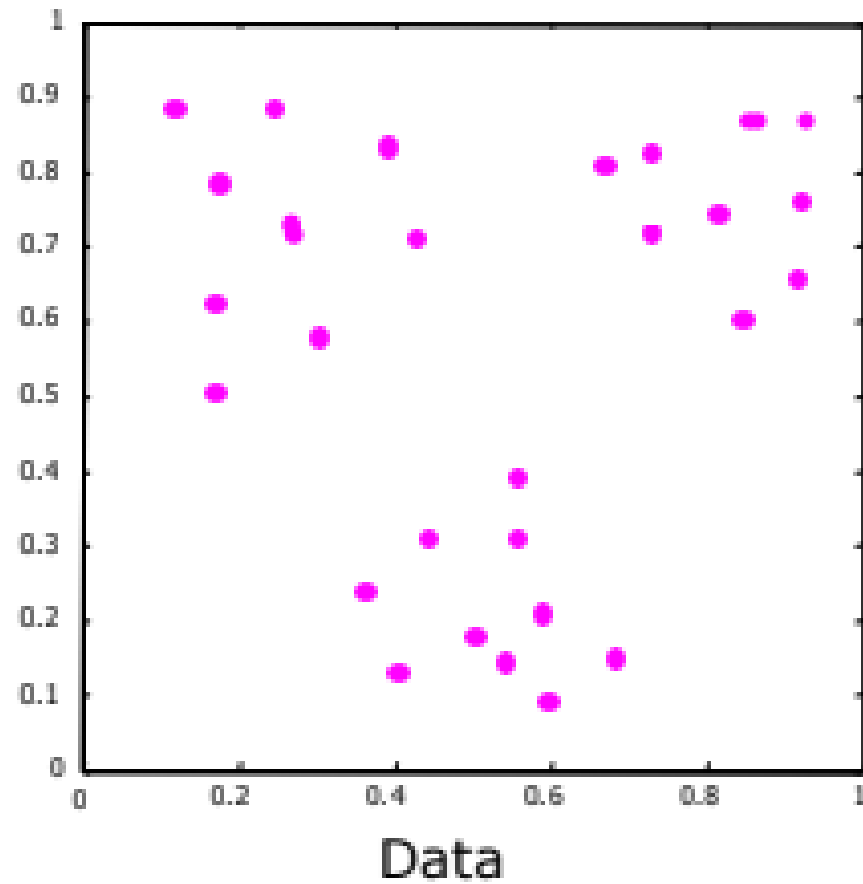




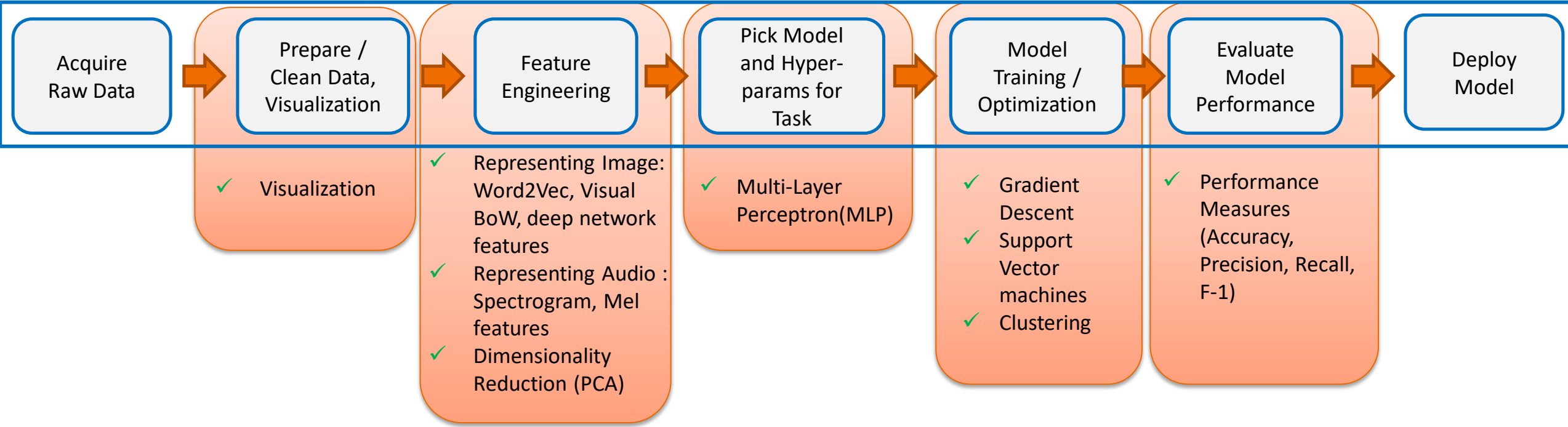
# Single-Link Algorithm

- Form a hierarchy for the data points (dendrogram), which can be used to partition the data
- The “closest” data points are joined to form a cluster at each step

# Single-Link Algorithm



# Summary



**Thanks!!**

**Questions?**