1 Eigenfaces

Eigenfaces refers to an appearance-based approach to face recognition that seeks to capture the variation in a collection of face images and use this information to encode and compare images of individual faces in a holistic (as opposed to a parts-based or feature-based) manner. Specifically, the eigenfaces are the principal components of a distribution of faces, or equivalently, the eigenvectors of the covariance matrix of the set of face images, where an image with N pixels is considered a point (or vector) in N-dimensional space. The idea of using principal components to represent human faces was developed by Sirovich and Kirby (Sirovich and Kirby 1987) and used by Turk and Pentland (Turk and Pentland 1991) for face detection and recognition. The Eigenface approach is considered by many to be the first working facial recognition technology, and it served as the basis for one of the top commercial face recognition technology products. Since its initial development and publication, there have been many extensions to the original method and many new developments in automatic face recognition systems. Eigenfaces is still often considered as a baseline comparison method to demonstrate the minimum expected performance of such a system.

The motivation of Eigenfaces is twofold:

- Extract the relevant facial information, which may or may not be directly related to human intuition of face features such as the eyes, nose, and lips. One way to do so is to capture the statistical variation between face images.

- Represent face images efficiently. To reduce the computation and space complexity, each face image can be represented using a small number of parameters.

The eigenfaces may be considered as a set of features which characterize the global variation among face images. Then each face image is approximated using a subset of the eigenfaces, those associated with the largest eigenvalues. These features account for the most variance in the training set.

Computing the Eigenfaces (Algorithm)

Before generating eigenfaces, face images are normalized to line up the eyes and mouths, then all resampled at the same pixel resolution. Eigenfaces are then extracted out of the image data by means of principal component analysis (PCA) in the following manner:

1. Compute the mean feature vector $\mu = \frac{1}{p} \sum_{k=1}^{p}$ where x_k is a pattern($k = 1 \, to \, p$), p = number of patterns, x is a feature matrix.

2. Find the covariance matrix
   $C = \frac{1}{p} \sum_{k=1}^{p} \{x_k - \mu\}\{x_k - \mu\}^T$ where T represents matrix transposition.

3. Compute Eigen values λ_i and Eigen vectors v_i of covariance matrix $Cv_i = \lambda_i v_i$ ($i = 1, 2, 3..., q$), q = number of features.

4. Estimating high-valued Eigen vectors

   - Arrange all the Eigen values (λ_i) in descending order
   - Choose the threshold value, ϑ

- Number of high valued λ_i can be chosen so as to satisfy the relationship
$$\left(\sum\nolimits_{i=1}^{s} \lambda_i\right)\left(\sum\nolimits_{i=1}^{q} \lambda_i\right)^{-1} \leq \vartheta$$
- Select Eigen vectors corresponding to selected high valued λ_i

5. Extract low dimensional feature vectors (principal components) from raw feature matrix. $P = V^T x$ where V is the matrix of principal components and x is the feature matrix.
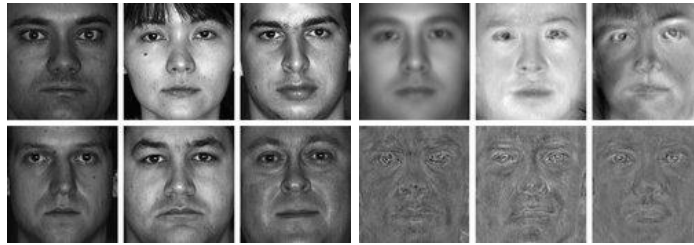

Figure 1

Figure 1 shows some examples from the CMU PIE dataset *(Sim et al. 2003)*, the average face and eigenfaces derived from the dataset.

References:

Blog on Eigenfaces:
https://medium.com/@MarynaL/eigenfaces-3675c94a7d

Blog on Face Recognition using eigenfaces technique:
https://medium.com/@devalshah1619/face-recognition-using-eigenfaces-technique-f221d505d4f7