MapReduce and PageRank

Question 1:

Suppose our input data to a map-reduce operation consists of integer values (the keys are not important). The map function takes an integer i and produces the list of pairs (p,i) such that p is a prime divisor of i. For example, map(12) = [(2,12),(3,12)].

The reduce function is addition. That is, reduce$(p,[i_1,i_2,...,i_k])$ is $(p,i_1+i_2+...+i_k)$.

Compute the output, if the input is the set of integers 15, 21, 24, 30, 49.

Solution:The output of map function is as follows

> map(15) = [(3,15),(5,15)]
>
> map(21) = [(3,21),(7,21)]
>
> map(24) = [(2,24),(3,24)]
>
> map(30) = [(2,30),(3,30),(5,30)]
>
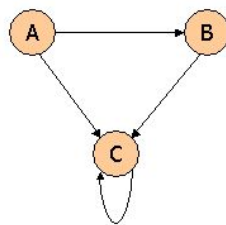> map(49) = [(7,49)]
>
> These are the respective prime divisors of inputs
>
> The output of reduce function is reduce(2,54),reduce(3,90),reduce(5,45),
>
> reduce(7,10).

Question 2:

Consider three Web pages with the following links:

Suppose we compute PageRank with a β of 0.7, and we introduce the additional constraint that the sum of the PageRanks of the three pages must be 3, to handle the problem that otherwise any multiple of a solution will also be a solution. Compute the PageRanks a, b, and c of the three pages A, B, and C, respectively.

solution:-

for ⟹ $a = \beta(c) + (1-\beta)$

$b = \beta(a/2) + (1-\beta)$

$c = \beta(\frac{a}{2} + b + c) + (1-\beta)$

∴ Here $\beta = 0.7$, $a+b+c = 3$

∴ $a = 0.7(0) + (1-0.7) = 0.3$

$b = 0.7(\frac{0.3}{2}) + (1-0.7) = 0.405$
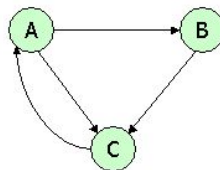
$c = 0.7(\frac{0.3}{2}) + 0.405 + c) + (1-0.7)$

$= 0.405 + 0.7(0.405) + (0.7)(c)$

$= c = 1.7(0.405) + 0.7(c) \Longrightarrow (1-0.7)c = 1.7(0.405)$

$0.3c = 0.6885$

$= c = \dfrac{0.6885}{0.3} = 2.295$

Question 3:

Suppose we compute PageRank with β=0.85. Write the equations for the PageRanks *a*, *b*, and *c* of the three pages A, B, and C, respectively.

Question 5:

Solution :-

formulae :-

$$a = \beta * c + (1-\beta)\tfrac{1}{3}$$

$$b = \beta * \tfrac{a}{2} + (1-\beta)\tfrac{1}{3}$$

$$c = \beta * \left(\tfrac{a}{2} + b\right) + (1-\beta)\tfrac{1}{3}$$

Here,

$$\beta = 0.85$$

$$a = 0.85 * c + (1-0.85)\tfrac{1}{3}$$
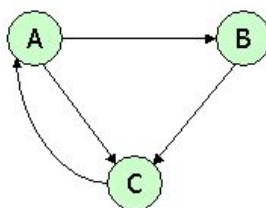
$$= 0.85c + 0.05$$

$$b = 0.85 * 0.5 * a + 0.05$$

$$= 0.425a + 0.05$$

$$c = 0.85 * [0.5 * a + b] + 0.05$$

$$= 0.425a + 0.85b + 0.05$$

Question 4:

Assuming no "taxation," compute the PageRanks *a*, *b*, and *c* of the three pages A, B, and C, using iteration, starting with the "0th" iteration where all three pages have rank *a* = *b* = *c* = 1. Compute as far as the 5th iteration, and also determine what the PageRanks are in the limit.

4Q:-

formulae

$$a = c, \quad b = a/2, \quad c = \frac{a}{2} + b$$

at 0th iteration :-

$$a = 1, \quad b = 1, \quad c = 1$$

1st iteration

$$a - c = 1, \quad b = \frac{1}{2}, \quad c = \frac{1}{2} + 1 = \frac{3}{2}$$

2nd iteration :-

$$a = c = \frac{3}{2}; \quad b = \frac{a}{2} - \frac{1}{2}; \quad c = \frac{1}{2} \pm \frac{1}{2} = 1$$

3rd iteration :-

$$a = c = 1; \quad b = \frac{a}{2} = \frac{3/2}{2} = \frac{3}{4}; \quad c = \frac{3}{4} + \frac{1}{2} = \frac{5}{4}$$

4th iteration

$$a - c = \frac{5}{4}; \quad b = \frac{a}{2} = \frac{1}{2}; \quad c = \frac{5}{4}$$

5th iteration.

$$a = \frac{5}{4}, \quad b = \frac{5}{8}, \quad c = \frac{9}{8}$$