The eigen vectors are $\frac{1}{3}$ and $\begin{matrix} 1 \\ -1/3 \end{matrix}$

The eigen values are $2+3c = \lambda \rightarrow \lambda = 2+3\cdot 3 = 11$ and

$\lambda = 2+3\cdot(-1/3) = 1$

Q3

~~Answ~~

Given the eigen vector of some matrix be

$M = [1, 3, 4, 5, 7]$

To get the unit eigen vector of given matrix, we need to divide each component by square root of sum of squares in the same direction.

Sum of squares $= 1^2+3^2+4^2+5^2+7^2 = 100$ and its square root is 10.

Unit eigen vector $= [1/10, 3/10, 4/10, 5/10, 7/10]$.

Q4 The given three points in a 2-D space are $(1,1), (2,2)$ and $(3,4)$.

We should construct a matrix whose rows correspond to points and columns correspond to dimensions of the space.

Then, the matrix will be

$M = \begin{matrix} 1 & 1 \\ 2 & 2 \\ 3 & 4 \end{matrix}$ $\quad M^T M = \begin{matrix} 1 & 2 & 3 \\ 1 & 2 & 4 \end{matrix} \cdot \begin{matrix} 1 & 1 \\ 2 & 2 \\ 3 & 4 \end{matrix} = \begin{matrix} 14 & 17 \\ 17 & 21 \end{matrix}$

Assignment - 8

(1) Let c1 be [2/7, 3/7, 6/7], c2 be [6/7, 2/7, -3/7] and c3 be [x, y, z].

The dot product of any two columns must be zero.

$c1.c2 = (2/7 * 6/7) + (3/7 * 2/7) + (6/7 * -3/7) = 0$

$c2.c3 = (6/7 * x) + (2/7 * y) + (-3/7 * z) = 0 \rightarrow 6x + 2y - 3z = 0$ - Eq1

$c3.c1 = (x * 2/7) + (y * 3/7) + (z * 6/7) = 0 \rightarrow 2x + 3y + 6z = 0 \rightarrow$ Eq 2

$2 * eq1 + Eq2 \rightarrow 12x + 4y - 6z + 2x + 3y + 6z = 0 \rightarrow 14x + 7y = 0 \rightarrow y = -2x.$

$3 * eq2 - eq1 \rightarrow 6x + 9y + 18z - 6x - 2y + 3z = 0 \rightarrow 7y + 21z = 0 \rightarrow y = -3z$

$x : y : z = -2 : 1 : -3.$

(2)

Let the given matrix be $A = \begin{matrix} 2 & 3 \\ 3 & 10 \end{matrix}$ and the eigen vector be of the form $\begin{matrix} 1 \\ e \end{matrix}$,

$Ax = \lambda x \rightarrow \begin{matrix} 2 & 3 \\ 3 & 10 \end{matrix} \cdot \begin{matrix} 1 \\ e \end{matrix} = \lambda * \begin{matrix} 1 \\ e \end{matrix} \rightarrow 2 + 3e = \lambda$ and

$3 + 10e \rightarrow 3 + 10e = (2 + 3e)e$

$3e^2 - 8e + 3 = 0 \rightarrow e = 3, -1/3$

Question - 6

Probability with which we choose row = $\frac{\text{sum of squares of elements in the rows}}{\text{sum of squares of elements in the matrix.}}$

sum of squares of elements in the matrix $= 12 * 13 * 25/6 = 3900/6 = 650$

$P(R1) = \frac{1^2+2^2+3^2}{650} = 14/650 = 0.02$ $P(R3) = \frac{7^2+8^2+9^2}{650} = \frac{194}{650} = 0.298$

$P(R2) = \frac{4^2+5^2+6^2}{650} = 77/650 = 0.12$ $P(R4) = \frac{10^2+11^2+12^2}{650} = \frac{365}{650} = 0.56.$

Question - 5

Moore-Penrose pseudoinverse means the matrix having diagonal elements replaced by 1 and divided by corresponding elements of given matrix and the other elements will be zero.

Moore-Penrose pseudoinverse of given matrix

$$\begin{matrix} 1 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 0 \end{matrix}.$$