

## Lower bound for comparison based sorting algorithms

The problem of sorting can be viewed as following.

**Input:** A sequence of  $n$  numbers  $\langle a_1, a_2, \dots, a_n \rangle$ .

**Output:** A permutation (reordering)  $\langle a'_1, a'_2, \dots, a'_n \rangle$  of the input sequence such that  $a'_1 \leq a'_2 \leq \dots \leq a'_n$ .

A sorting algorithm is comparison based if it uses comparison operators to find the order between two numbers. Comparison sorts can be viewed abstractly in terms of decision trees. A decision tree is a **full binary tree** that represents the comparisons between elements that are performed by a particular sorting algorithm operating on an input of a given size. The execution of the sorting algorithm corresponds to tracing a path from the root of the decision tree to a leaf. At each internal node, a comparison  $a_i \square a_j$  is made. The left subtree then dictates subsequent comparisons for  $a_i \square a_j$ , and the right subtree dictates subsequent comparisons for  $a_i > a_j$ . When we come to a leaf, the sorting algorithm has established the ordering. So we can say following about the decision tree.

- 1) Each of the  $n!$  permutations on  $n$  elements must appear as one of the leaves of the decision tree for the sorting algorithm to sort properly.
- 2) Let  $x$  be the maximum number of comparisons in a sorting algorithm. The maximum height of the decision tree would be  $x$ . A tree with maximum height  $x$  has at most  $2^x$  leaves.

After combining the above two facts, we get following relation.

$$n! \leq 2^x$$

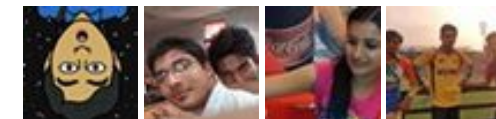
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Taking Log on both sides.

$$\log_2 n! \leq x$$

Since  $\log_2 n! = \Theta(n \log n)$ , we can say  
 $x = \Omega(n \log_2 n)$

Therefore, any comparison based sorting algorithm must make at least  $\Omega(n \log_2 n)$  comparisons to sort the input array, and Heapsort and merge sort are asymptotically optimal comparison sorts.

#### References:

Introduction to Algorithms, by Thomas H. Cormen, Charles E. Leiserson, Ronald L. Rivest and Clifford Stein

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**Kanhaiya Kumawat** • 4 months ago

there is typo in the last line: "Therefore, any comparison based sorting algorithm sort the input array, and Heapsort and merge sort are asymptotically optimal with its  $n \log n$  rather than  $\log n$ .

^ | v ·



**GeeksforGeeks** Mod → Kanhaiya Kumawat • 3 months ago

Thanks for pointing this out. We have corrected the typo.

^ | v ·



**wgpshashank** • 3 years ago

More Info ..

<http://www.it-c.dk/courses/ITM/F2003/Sorting.pdf>

^ | v ·

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**lovocas** · 3 years ago

" A decision tree is a full binary tree"

I am confused by this statement. :-<

A full binary tree is a tree has  $2^{(k+1)}-1$  nodes ?k is the heright,the root&#039s

I think desision tree is just a binary tree, whose nodes has either 2 or zero nod

^ | v ·



**GeeksforGeeks** → lovocas · 3 years ago

@lovocas: The statement looks correct. Please see the following [Wiki](#)

A full binary tree (sometimes proper binary tree or 2-tree or strictly bina  
other than the leaves has two children.

To avoid confusion, we have updated the post and added the Wiki link

^ | v ·



**Yang** → GeeksforGeeks · 7 months ago

What if I write a "really bad" algorithm and it asks the same que  
answer every time. Then, the Decision tree will still be full binar

^ | v ·



**lovocas** → GeeksforGeeks · 3 years ago

oh, thanks very much , got it!

^ | v ·

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