Closest Pair of Points | O(nlogn) Implementation

We are given an array of n points in the plane, and the problem is to find out the closest pair of points in the array. This problem arises in a number of applications. For example, in air-traffic control, you may want to monitor planes that come too close together, since this may indicate a possible collision. Recall the following formula for distance between two points p and q.

$$\|pq\| = \sqrt{(p_x - q_x)^2 + (p_y - q_y)^2}.$$

We have discussed a divide and conquer solution for this problem. The time complexity of the implementation provided in the previous post is O(n (Logn)^2). In this post, we discuss an implementation with time complexity as O(nLogn).

Following is a recap of the algorithm discussed in the previous post.

1) We sort all points according to x coordinates.

2) Divide all points in two halves.

3) Recursively find the smallest distances in both subarrays.

4) Take the minimum of two smallest distances. Let the minimum be d.

5) Create an array strip[] that stores all points which are at most d distance away from the middle line dividing the two sets.

6) Find the smallest distance in strip[].

7) Return the minimum of d and the smallest distance calculated in above step 6.

The great thing about the above approach is, if the array strip[] is sorted according to y

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coordinate, then we can find the smallest distance in strip[] in O(n) time. In the implementation discussed in previous post, strip[] was explicitly sorted in every recursive call that made the time complexity O(n (Logn)^2), assuming that the sorting step takes O(nLogn) time.
In this post, we discuss an implementation where the time complexity is O(nLogn). The idea is to presort all points according to y coordinates. Let the sorted array be Py[]. When we make recursive calls, we need to divide points of Py[] also according to the vertical line. We can do that by simply processing every point and comparing its x coordinate with x coordinate of middle line.

Following is C++ implementation of O(nLogn) approach.

```
// A divide and conquer program in C++ to find the smallest distance f
// given set of points.

#include <iostream>
#include <float.h>
#include <stdlib.h>
#include <math.h>
using namespace std;

// A structure to represent a Point in 2D plane
struct Point
{
    int x, y;
};


/* Following two functions are needed for library function qsort().
   Refer: http://www.cplusplus.com/reference/clibrary/cstdlib/qsort/ *

// Needed to sort array of points according to X coordinate
int compareX(const void* a, const void* b)
{
    Point *p1 = (Point *)a,   *p2 = (Point *)b;
    return (p1->x - p2->x);
}
// Needed to sort array of points according to Y coordinate
int compareY(const void* a, const void* b)
{
    Point *p1 = (Point *)a,   *p2 = (Point *)b;
    return (p1->y - p2->y);
}

// A utility function to find the distance between two points
float dist(Point p1, Point p2)
```


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```
{
    return sqrt( (p1.x - p2.x)*(p1.x - p2.x) +
                 (p1.y - p2.y)*(p1.y - p2.y)
               );
}

// A Brute Force method to return the smallest distance between two po
// in P[] of size n
float bruteForce(Point P[], int n)
{
    float min = FLT_MAX;
    for (int i = 0; i < n; ++i)
        for (int j = i+1; j < n; ++j)
            if (dist(P[i], P[j]) < min)
                min = dist(P[i], P[j]);
    return min;
}

// A utility function to find minimum of two float values
float min(float x, float y)
{
    return (x < y)? x : y;
}


// A utility function to find the distance beween the closest points o
// strip of given size. All points in strip[] are sorted accordint to
// y coordinate. They all have an upper bound on minimum distance as d
// Note that this method seems to be a O(n^2) method, but it's a O(n)
// method as the inner loop runs at most 6 times
float stripClosest(Point strip[], int size, float d)
{
    float min = d;  // Initialize the minimum distance as d

    // Pick all points one by one and try the next points till the dif
    // between y coordinates is smaller than d.
    // This is a proven fact that this loop runs at most 6 times
    for (int i = 0; i < size; ++i)
        for (int j = i+1; j < size && (strip[j].y - strip[i].y) < min;
            if (dist(strip[i],strip[j]) < min)
                min = dist(strip[i], strip[j]);

    return min;
}

// A recursive function to find the smallest distance. The array Px co
// all points sorted according to x coordinates and Py contains all po
```

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```
// sorted according to y coordinates
float closestUtil(Point Px[], Point Py[], int n)
{
    // If there are 2 or 3 points, then use brute force
    if (n <= 3)
        return bruteForce(Px, n);

    // Find the middle point
    int mid = n/2;
    Point midPoint = Px[mid];


    // Divide points in y sorted array around the vertical line.
    // Assumption: All x coordinates are distinct.
    Point Pyl[mid+1];   // y sorted points on left of vertical line
    Point Pyr[n-mid-1];  // y sorted points on right of vertical line
    int li = 0, ri = 0;  // indexes of left and right subarrays
    for (int i = 0; i < n; i++)
    {
      if (Py[i].x <= midPoint.x)
         Pyl[li++] = Py[i];
      else
         Pyr[ri++] = Py[i];
    }

    // Consider the vertical line passing through the middle point
    // calculate the smallest distance dl on left of middle point and
    // dr on right side
    float dl = closestUtil(Px, Pyl, mid);
    float dr = closestUtil(Px + mid, Pyr, n-mid);

    // Find the smaller of two distances
    float d = min(dl, dr);

    // Build an array strip[] that contains points close (closer than
    // to the line passing through the middle point
    Point strip[n];
    int j = 0;
    for (int i = 0; i < n; i++)
        if (abs(Py[i].x - midPoint.x) < d)
            strip[j] = Py[i], j++;

    // Find the closest points in strip.  Return the minimum of d and
    // distance is strip[]
    return min(d, stripClosest(strip, j, d) );
}
```

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```
// The main functin that finds the smallest distance
// This method mainly uses closestUtil()
float closest(Point P[], int n)
{
    Point Px[n];
    Point Py[n];
    for (int i = 0; i < n; i++)
    {
        Px[i] = P[i];
        Py[i] = P[i];
    }

    qsort(Px, n, sizeof(Point), compareX);
    qsort(Py, n, sizeof(Point), compareY);

    // Use recursive function closestUtil() to find the smallest dista
    return closestUtil(Px, Py, n);
}

// Driver program to test above functions
int main()
{
    Point P[] = {{2, 3}, {12, 30}, {40, 50}, {5, 1}, {12, 10}, {3, 4}}
    int n = sizeof(P) / sizeof(P[0]);
    cout << "The smallest distance is " << closest(P, n);
    return 0;
}
```

Output:

```
The smallest distance is 1.41421
```

Time Complexity: Let Time complexity of above algorithm be T(n). Let us assume that we use a O(nLogn) sorting algorithm. The above algorithm divides all points in two sets and recursively calls for two sets. After dividing, it finds the strip in O(n) time. Also, it takes O(n) time to divide the Py array around the mid vertical line. Finally finds the closest points in strip in O(n) time. So T(n) can expressed as follows
T(n) = 2T(n/2) + O(n) + O(n) + O(n)
T(n) = 2T(n/2) + O(n)
T(n) = T(nLogn)

References:
http://www.cs.umd.edu/class/fall2013/cmsc451/Lects/lect10.pdf

▶ Points and Lines
▶ Points Of
▶ C++ Array
AdChoices
▶ C++ Array
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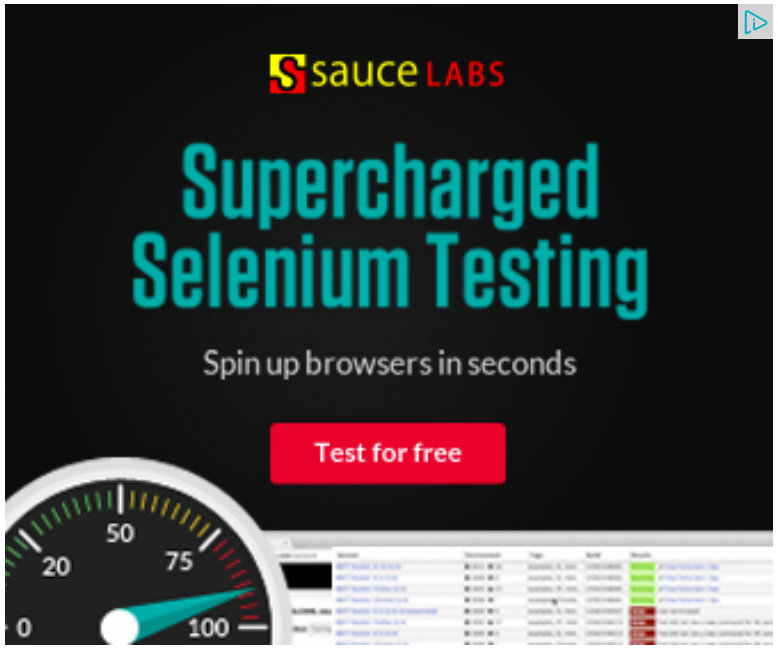
http://www.youtube.com/watch?v=vS4Zn1a9KUc
http://www.youtube.com/watch?v=T3T7T8Ym20M
http://en.wikipedia.org/wiki/Closest_pair_of_points_problem

Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above



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arjomanD • 15 hours ago

Too much Implementation !!
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ColacX • 19 days ago

Point Px[n];

is this valid C++ code? doesn't compile for me.
where is the memory located, on the stack or heap?

Sekhar • a month ago

Does this algorithmn take care of mid && px+mid points for min distance calc
this take care of 3rd and 4th pair in calculation for min distance

Serif • 2 months ago

A simple n log n trailing edge algorithm:

<script src="http://ideone.com/e.js/SzAd4N" type="text/javascript"></script>

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rohan • 5 months ago

GeeksforGeeks

you are using one variable extra in both pyl n pyr as below

Point Pyl[mid+1]; // y sorted points on left of vertical line

if mid==3 for 6 elements then we need array of 3 elements (mid elements inst

Point Pyr[n-mid-1]; // y sorted points on right of vertical line//same reason

kindly update it as following as it creates a lot of confussion

Point Pyl[mid]; // y sorted points on left of vertical line

Point Pyr[n-mid]; // y sorted points on right of vertical line

Guest ➚ rohan • 23 days ago

I don't think so. I am implementing this code right now and when I follov
out_of_range error. When I follow G4G's code, I do not.

Ofer • 5 months ago

An O(n) algorithm using some assumptions:

If you know the maximum distance you're looking for, i.e points with distance g
you can properly discretize your input into integer coordinates, and assuming t
O(logn), and assuming the radius r is constant, then you can design an algorit

Here's an outline of the algorithm:

- For each point, you add the point and its neighbor points within the radius to a
point in the hashmap in that spot, then we know the two points are neighbors,

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distance to a min heap.

- We take the minimum of the min heap as the minimum distance.

Since the heap has only O(logn) points, and so O(log^2(n)) pairs, adding all pa
O(log^2(n)*log(logn)) throughout the algorithm.

As we went over the points once, the runtime is amortized O(n). If we want it (
array instead of the hashmap.

viki • 5 months ago

This line will cause Pyl to go out of bound-

Pyl[i] = Py[i];

5 

GeeksforGeeks Mod viki • 5 months ago

viki, thanks for pointing this out. We have updated the code.

1 

Guest • 5 months ago

I've made this program in C

```
#include<stdio.h>

#include<conio.h>

void abs(int*);//Finds absolute Value

int diff(int,int);//Finds difference

void enter_ar(int *);//To enter Array
```

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```
struct inf{

          int a,b,c;

          }z[50],f;
```

see more

Guest • 5 months ago

I've tried to make this program in C, its output is correct

```
#include<stdio.h>
#include<conio.h>


void abs(int*);//Finds absolute Value
int diff(int,int);//Finds difference
void enter_ar(int *);//To enter Array

struct inf{
   int a,b,c;
   }z[50],f;

struct inf sm(struct inf*,int);//To find smallest in "struct inf" type

void main()
{
```

see more

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1 ^ | v •

Guest • 5 months ago

I tried to make this program in C, its output is correct

```
#include<stdio.h>

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int diff(int,int);//Finds difference

void enter_ar(int *);//To enter Array

struct inf{

          int a,b,c;

          }z[50],f;
```

see more

^ | v •

Vivek VV • 6 months ago

Can someone post the link to part one of this post.
Thanks

^ | v •

rahul ➚ Vivek VV • 6 months ago

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