

Divide and Conquer | Set 5 (Strassen's Matrix Multiplication)

Given two square matrices A and B of size $n \times n$ each, find their multiplication matrix.

Naive Method

Following is a simple way to multiply two matrices.

```
void multiply(int A[][N], int B[][N], int C[][N])
{
    for (int i = 0; i < N; i++)
    {
        for (int j = 0; j < N; j++)
        {
            C[i][j] = 0;
            for (int k = 0; k < N; k++)
            {
                C[i][j] += A[i][k]*B[k][j];
            }
        }
    }
}
```

Time Complexity of above method is $O(N^3)$.

Divide and Conquer

Following is simple Divide and Conquer method to multiply two square matrices.

- 1) Divide matrices A and B in 4 sub-matrices of size $N/2 \times N/2$ as shown in the below diagram.
- 2) Calculate following values recursively. $ae + bg$, $af + bh$, $ce + dg$ and $cf + dh$.

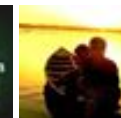
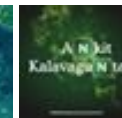
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$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \times \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} ae + bg & af + bh \\ ce + dg & cf + dh \end{bmatrix}$$

A B C

A, B and C are square matrices of size $N \times N$
 a, b, c and d are submatrices of A, of size $N/2 \times N/2$
 e, f, g and h are submatrices of B, of size $N/2 \times N/2$

In the above method, we do 8 multiplications for matrices of size $N/2 \times N/2$ and 4 additions. Addition of two matrices takes $O(N^2)$ time. So the time complexity can be written as

$$T(N) = 8T(N/2) + O(N^2)$$

From **Master's Theorem**, time complexity of above method is $O(N^3)$ which is unfortunately same as the above naive method.

Simple Divide and Conquer also leads to $O(N^3)$, can there be a better way?

In the above divide and conquer method, the main component for high time complexity is 8 recursive calls. The idea of **Strassen's method** is to reduce the number of recursive calls to 7. Strassen's method is similar to above simple divide and conquer method in the sense that this method also divide matrices to sub-matrices of size $N/2 \times N/2$ as shown in the above diagram, but in Strassen's method, the four sub-matrices of result are calculated using following formulae.



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$$\begin{aligned} p1 &= a(f - h) \\ p3 &= (c + d)e \\ p5 &= (a + d)(e + h) \\ p7 &= (a - c)(e + f) \end{aligned}$$

$$\begin{aligned} p2 &= (a + b)h \\ p4 &= d(g - e) \\ p6 &= (b - d)(g + h) \end{aligned}$$

The A x B can be calculated using above seven multiplications.
Following are values of four sub-matrices of result C

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \times \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} p5 + p4 - p2 + p6 & p1 + p2 \\ p3 + p4 & p1 + p5 - p3 - p7 \end{bmatrix}$$

A B C

A, B and C are square matrices of size N x N
a, b, c and d are submatrices of A, of size N/2 x N/2
e, f, g and h are submatrices of B, of size N/2 x N/2
p1, p2, p3, p4, p5, p6 and p7 are submatrices of size N/2 x N/2

Time Complexity of Strassen's Method

Addition and Subtraction of two matrices takes $O(N^2)$ time. So time complexity can be written as

$$T(N) = 7T(N/2) + O(N^2)$$

From Master's Theorem, time complexity of above method is $O(N^{\log_2 7})$ which is approximately $O(N^{2.8074})$

Generally Strassen's Method is not preferred for practical applications for following reasons.

- 1) The constants used in Strassen's method are high and for a typical application Naive method works better.
- 2) For Sparse matrices, there are better methods especially designed for them.
- 3) The submatrices in recursion take extra space.
- 4) Because of the limited precision of computer arithmetic on noninteger values, larger errors

stack!

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accumulate in Strassen's algorithm than in Naive Method (Source: CLRS Book)

References:

Introduction to Algorithms 3rd Edition by Clifford Stein, Thomas H. Cormen, Charles E. Leiserson, Ronald L. Rivest

<https://www.youtube.com/watch?v=LOLebQ8nKHA>

<https://www.youtube.com/watch?v=QXY4RskLQcI>

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16



5



1

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