Count all possible paths from top left to bottom right of a mXn matrix

The problem is to count all the possible paths from top left to bottom right of a mXn matrix with the constraints that ***from each cell you can either move only to right or down***

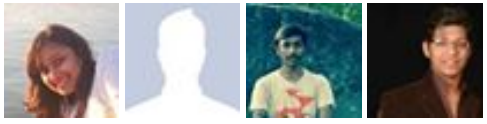
We have discussed a solution to print all possible paths, counting all paths is easier. Let NumberOfPaths(m, n) be the count of paths to reach row number m and column number n in the matrix, NumberOfPaths(m, n) can be recursively written as following.

```
#include <iostream>
using namespace std;

// Returns count of possible paths to reach cell at row number m and c
// number n from the topmost leftmost cell (cell at 1, 1)
int  numberOfPaths(int m, int n)
{
   // If either given row number is first or given column number is fi
   if (m == 1 || n == 1)
        return 1;

   // If diagonal movements are allowed then the last addition
   // is required.
   return  numberOfPaths(m-1, n) + numberOfPaths(m, n-1);
           // + numberOfPaths(m-1,n-1);
}

int main()
{
    cout << numberOfPaths(3, 3);
    return 0;
}
```

Output:

```
6
```

The time complexity of above recursive solution is exponential. There are many overlapping subproblems. We can draw a recursion tree for numberOfPaths(3, 3) and see many overlapping subproblems. The recursion tree would be similar to Recursion tree for Longest Common Subsequence problem.
So this problem has both properties (see this and this) of a dynamic programming problem. Like other typical Dynamic Programming(DP) problems, recomputations of same subproblems can be avoided by constructing a temporary array count[][] in bottom up manner using the above recursive formula.

```
#include <iostream>
using namespace std;

// Returns count of possible paths to reach cell at row number m and c
// number n from the topmost leftmost cell (cell at 1, 1)
int numberOfPaths(int m, int n)
{
    // Create a 2D table to store results of subproblems
    int count[m][n];

    // Count of paths to reach any cell in first column is 1
    for (int i = 0; i < m; i++)
        count[i][0] = 1;

    // Count of paths to reach any cell in first column is 1
    for (int j = 0; j < n; j++)
        count[0][j] = 1;

    // Calculate count of paths for other cells in bottom-up manner us
    // the recursive solution
    for (int i = 1; i < m; i++)
    {
        for (int j = 1; j < n; j++)

            // By uncommenting the last part the code calculatest he t
            // possible paths if the diagonal Movements are allowed
            count[i][j] = count[i-1][j] + count[i][j-1]; //+ count[i-1

    }
    return count[m-1][n-1];
}
```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```
// Driver program to test above functions
int main()
{
    cout << numberOfPaths(3, 3);
    return 0;
}
```

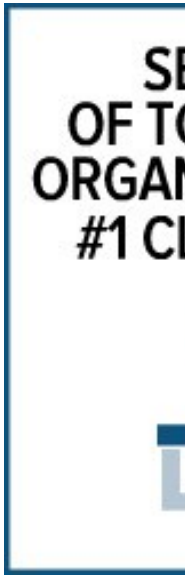
Output:

```
6
```

Time complexity of the above dynamic programming solution is O(mn).

Note the count can also be calculated using the formula (m-1 + n-1)!/(m-1)!(n-1)! as mentioned in the comments of this article.

This article is contributed by **Hariprasad NG**. Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above



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- Find if there is a subarray with 0 sum
- Divide and Conquer | Set 5 (Strassen's Matrix Multiplication)
- Count all possible groups of size 2 or 3 that have sum as multiple of 3

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Vinod Prabhu • a month ago

assuming that I have a 2 rows and 3 columns matrix.

then the number of paths according to this program is 3. but if ai draw paths it

[0,0]->[0,1]->[0,2] ->[1,2]

[0,0]->[0,1]->[1,1] ->[1,2]

[0,0]->[1,0]->[1,1] ->[1,2]

[0,0]->[1,0]->[1,1] ->[0,1]->[0,2]->[1,2]

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newCoder3006 Code without using while loop. We can do it...
Find subarray with given sum · 1 hour ago

Sanjay Agarwal You can also use the this method:...
Count trailing zeroes in factorial of a number · 1 hour ago

#define ROW 5
#define COLUMN 5

using namespace std;

int get_count_paths(int* matrix,int m,int n)
{
return *(matrix+((m-1)*COLUMN)+n-1);
}

void set_count_paths(int* matrix)
{

for(int i =0 ;i < ROW;i++)
for(int j=0;j< COLUMN;j++)
{
if(i==0 ||j ==0)
*(matrix+(i*COLUMN)+j)=1;
else
*(matrix+(i*COLUMN)+j)= *(matrix+((i-1)*COLUMN)+j)+ *(matrix+(i*COLUMN
}
}

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proton • a month ago

// Count of paths to reach any cell in first column is 1
for (int i = 0; i < m; i++)
count[i][0] = 1;
We're starting from top-left with one column at a time...How then to reach any

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Gnanodharan Madhavan • a month ago

Simple recursion to print all the paths.

```
import java.awt.Point;

import java.util.List;

import java.util.ArrayList;

public class printpathofmxmatrix{

private static int TARGET = 100;

private void printPathABofMatrix(int arr[][],int m, int n, List<point> list){

if(m>=arr.length || n>=arr[0].length)

return;

Point point = new Point(m,n);

list.add(point);
```

see more

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Alok Kumar • a month ago

The time complexity of O(m*n) is OK, but we can improve the space complex

```
#include<stdio.h>
#include<stdlib.h>
int ans(int m,int n)
{
if(m<=0||n<=0) return 0;
if(m>n) return ans(n,m);
int dp1[m];
int loop1,loop2;
```

2;loop2">=0;loop2--)
{
dp1[loop2]=dp1[loop2]+dp1[loop2+1];
}
}
return dp1[0];
}
int main()
{
printf("%d\n",ans(5,8));
return 0;
}

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Ritesh Mahato • 3 months ago

@GeeksForGeeks : In second example, the comment should be 'row' and not
// Count of paths to reach any cell in first 'column' is 1
for (int j = 0; j < n; j++)
count[0][j] = 1;

3 • Reply • Share ›

Sekhar • 3 months ago

static int printAllPaths(int[][] a, int rowCount, int colCount, int currX, int currY) {

if (currX == rowCount - 1) {
return 1;
}

if (currY == colCount - 1) {
return 1;
}

```
return printAllPaths(a, rowCount, colCount, currX + 1, currY)
+ printAllPaths(a, rowCount, colCount, currX, currY + 1);
}
```

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trojansmith1990 • 3 months ago

Hi,
Have written here
http://ideone.com/qrYpmc

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Ram • 4 months ago

There are several variations of this problem which are exhaustively covered at

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Subrahmanyan Sankar ➦ Ram • 3 months ago

Thank you for sharing this

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Hari ➦ Ram • 4 months ago

Nice blog thanks for sharing...

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Rohit Mitra • 4 months ago

It can be written as (m + n -2) C (n - 1)

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LK ➦ Rohit Mitra • 4 months ago

Could you please explain?

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to reach the final cell you have to take (m-1) steps to the right a
(m-1)+(n-1)=m+n-2. Out of these (m+n-2) steps any (n-1) step
of ways is (m+n-2)C(n-1) or (m+n-2)C(m-1).

6 ^ | v • Reply • Share ›

h@kumar • 4 months ago

A Short and sweet soln->
(2n-2) C (n-1)

3 ^ | v • Reply • Share ›

sudhakar ➦ h@kumar • 4 months ago

this won't work for large matrix like 1000 x 1000

^ | v • Reply • Share ›

gourav pathak ➦ h@kumar • 4 months ago

Even that would take an O(mn) if m was large

1 ^ | v • Reply • Share ›

Vinod ➦ gourav pathak • a month ago

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[0,0]->[0,1]->[1,1] ->[1,2]

[0,0]->[1,0]->[1,1] ->[1,2]

[0,0]->[1,0]->[1,1] ->[0,1]->[0,2]->[1,2]

1 ^ | v • Reply • Share ›