In this thesis we classify singular vectors in scalar parabolic Verma modules for those pairs $(\mathfrak{sl}(n, \mathbb{C}), \mathfrak{p})$ of complex Lie algebras where the homogeneous space $\mathrm{SL}(n, \mathbb{C})/P$ is the Grassmannian of k-planes in $\mathbb{C}^n$. We calculate cohomology of nilpotent radicals with values in certain unitarizable highest weight modules. According to [BH09] these modules have BGG resolutions with weights determined by this cohomology. Such resolutions induce complexes of invariant differential operators on sections of associated bundles over Hermitian symmetric spaces. We describe formal completions of unitarizable highest weight modules that one can use to modify method from [CD01] that constructs sequences of differential operators over any 1-graded (aka almost Hermitian) geometry. We suggest uniform description of octonionic planes that could serve as a basis for better understanding of the exceptional Hermitian symmetric space for group E_6.