

$$\sum_{i,j \in -1,0,1} A_{G,0,0}^{i,j} U_{i,j}^h = 0 \quad (1)$$

Setting $2h_\xi = h_\xi^- + h_\xi^+$ with ξ being x or y .

The values of the nonzero entries are the following:

Four corner nodes ($i, j \in \{-, +\}$):

$$A_{G,0,0}^{i,j} = \frac{k^2 h_x^i h_y^j}{36} + \frac{h_x^i}{6h_y^j} + \frac{h_y^j}{6h_x^i}$$

Four side nodes:

$$\begin{aligned} A_{G,0,0}^{\pm,0} &= \frac{k^2 h_x^\pm h_y}{9} + \frac{2h_y}{3h_x^\pm} - \frac{h_x^\pm}{3} \frac{h_y}{h_y^- h_y^+} \\ A_{G,0,0}^{0,\pm} &= \frac{k^2 h_x h_y^\pm}{9} + \frac{2h_x}{3h_y^\pm} - \frac{h_y^\pm}{3} \frac{h_x}{h_x^- h_x^+} \end{aligned}$$

Center node:

$$A_{G,0,0}^{0,0} = \frac{4k^2 h_x h_y}{9} - \frac{4h_x}{3} \frac{h_y}{h_y^- h_y^+} - \frac{4h_y}{3} \frac{h_x}{h_x^- h_x^+}$$

Setting $\alpha_\xi^\pm h_\xi = h_\xi^\pm$ with ξ being x or y .

Four side nodes:

$$\begin{aligned} A_{G,0,0}^{\pm,0} &= \frac{k^2 h_x^\pm h_y}{9} + \frac{2h_y}{3h_x^\pm} - \frac{h_x^\pm}{3\alpha_y^- \alpha_y^+ h_y} \\ A_{G,0,0}^{0,\pm} &= \frac{k^2 h_x h_y^\pm}{9} + \frac{2h_x}{3h_y^\pm} - \frac{h_y^\pm}{3\alpha_x^- \alpha_x^+ h_x} \end{aligned}$$

Center node:

$$A_{G,0,0}^{0,0} = \frac{4k^2 h_x h_y}{9} - \frac{4h_x}{3\alpha_y^- \alpha_y^+ h_y} - \frac{4h_y}{3\alpha_x^- \alpha_x^+ h_x}$$

$$k^2 = \frac{6 \left(2f_x^+ h_x^- \left((h_x^+)^2 h_y - (h_y^+)^2 h_y^- - h_y^+ (h_y^-)^2 \right) + 2f_x^- h_x^+ \left((h_x^-)^2 h_y - (h_y^-)^2 h_y^- - h_y^+ (h_y^-)^2 \right) + 2f_y^+ h_y^- \right)}{9}$$

Terme indépendant en haut

$$\begin{aligned} &+ 12h_x^- h_y^+ \left((h_x^+)^2 + (h_y^-)^2 \right) \\ &\quad + 12h_x^- h_y^- \left((h_x^+)^2 + (h_y^+)^2 \right) \\ &\quad + 12h_x^+ h_y^+ \left((h_x^-)^2 + (h_y^-)^2 \right) \\ &\quad + 12h_x^+ h_y^- \left((h_x^-)^2 + (h_y^+)^2 \right) \end{aligned}$$

Termes en f en haut

$$12f_x^+ h_x^- \left((h_x^+)^2 h_y - (h_y^+)^2 h_y^- - h_y^+ (h_y^-)^2 \right) + 12f_x^- h_x^+ \left((h_x^-)^2 h_y - (h_y^+)^2 h_y^- - h_y^+ (h_y^-)^2 \right) \\ + 12f_y^+ h_y^- \left(h_x (h_y^+)^2 - (h_x^+)^2 h_x^- - h_x^+ (h_x^-)^2 \right) + 12f_y^- h_y^+ \left(h_x (h_y^-)^2 - h_x^+ (h_x^-)^2 - (h_x^+)^2 h_x^- \right)$$

Terme indépendant en bas (en facteur des 4h)

$$16h_x h_y$$

Termes en f en bas (en facteur des 4h)

$$4f_x^+ h_x^+ h_y \quad + \quad 4f_x^- h_x^- h_y \quad + \quad 4f_y^+ h_x h_y^+ \quad + \quad 4f_y^- h_x h_y^-$$

Termes en g en bas (en facteur des 4h)

$$+ h_x^+ h_y^+ g^{++} + h_x^+ h_y^- g^{+-} + h_x^- h_y^+ g^{-+} + h_x^- h_y^- g^{--}$$