CARNEGIE MELLON UNIVERSITY, AFRICA

PROJECT REPORT

APPLIED STOCHASTIC PROCESSES

(18-751)

OPTIMIZATION OF AVERAGE WAITING TIME IN BUS TERMINALS USING QUEUING THEORY: KIMIRONKO BUS TERMINAL AS A CASE STUDY

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ABSTRACT

Long waiting time of passengers in bus terminals can have a negative impact both on the passengers and the revenue generated from public bus transportation. Many passengers face physical pain, mental distress, and loss of economic productivity while waiting for the buses at bus stops. The revenue from public transit can be affected when people are discouraged from using bus transit, due to long waiting time in bus terminals. This paper involves the analysis of the queuing systems in Kigali bus terminals, using Kimironko bus terminal as a case study. The waiting time for three queues were studied and the mean waiting time was optimized by using queuing theory and performing bus reallocation. Random search was used as the computational method for finding the optimal bus allocation. The results obtained were promising, as the mean average waiting time reduced was by 51.4%. This result, if applied to the studied bus terminal, can lead to cost-effective benefits for the operators of the public transportation system in Kigali, while preserving the well-being of passengers. Such benefits includes a possible increase in the revenue generated from public bus transportation.

1. INTRODUCTION

1.1 Basic introduction and Background of Study

Queues at bus stations are one of the key concerns in today's life, as taking an example of Kigali city. Research indicates that the current public transport can serve up to 65% of the potential demand but only 37% of the demand is adequately served due to different factors [1]. Waiting in a queue at the bus station and bus stop is one of the biggest problems encountered by the passengers in Kigali. Increasing population has led to the increase in complaints at the bus stations [1]. Therefore, applying queuing theory to the bus stations to reduce the waiting time is a necessary step towards improving the quality of care and services to the passengers.

A queue is a system that contains two main attributes, which are the customers (or jobs) and the service point (also known as servers). For instance, when there is more than one person who wants to use "Tap & Go" (the system that processes payment in bus stations) in a bus station, a queue will be formed. A queue can be described by a number of characteristics, including: the length of the queue, the rate of arrival of jobs to the queue, the service time, among others.

Queuing theory is the mathematical method that can be applied to processes that involve queues or waiting in line. Models that use queuing theory (queuing models) are employed in knowing how to make the best use of limited resources. In this project queuing theory is one of the major tools that will be used in solving the problem that we are considering (as stated in section 1.3) and answering our research questions.

1.2 Aim

The goal of this project is to estimate and optimize the average waiting time of bus passengers in Kigali, using Kimironko Gare as a case study.

1.3 Problem statement

The population of Kigali has increased by 3.34% annually since 2015 [2]. This increase in population is reflected in the queues of customers waiting in line at different service points [1]. One of the service points where the queues experienced are significantly long is at bus terminals, where passengers have to wait in line for an undefined amount of time.

1.4 Justification of Research Problem

Waiting time can have economic ramifications on the public transportation of a country. In the first quarter of 2021, the GDP from transportation in Rwanda decreased by 8.8% of its value in the fourth quarter of 2020 [3]. Passengers are very likely to stop using bus transit when the quality of service, which includes the waiting time, declines [4]. According to a 2014 mobility attitude survey, reduced travel time is the greatest factor that would make people use public transit more [5]. Thus, one way to increase revenue in the transportation sector of a country is to reduce the travel time of passengers [6].

Also, if as mentioned above, optimizing waiting would make more people use public transportation, then there is the possibility that there would be less traffic congestion on the road, as the number of private cars should decrease, and this could translate to a decrease in the occurrences of car accidents, as well as a reduction in environmental pollution caused by vehicles [7].

In addition, passengers' well-being is indirectly affected by waiting time in bus stations. Research which compares actual waiting time of passengers at bus terminals to perceived waiting time has shown that perceived wait time is usually greater than the actual wait time [8], and in general, passengers feel that the time spent on queues in bus stations is more troublesome than the time spent in transit (upon entering the bus) [8]. Hence increased waiting time has a potential impact on the health of passengers [9].

1.5 Research Questions

Some of the questions we hope to answer by the end of this project are:

1. What is the average waiting time of passengers in Kimironko bus terminal ?
2. Will increasing the number of buses or seats for a particular bus route decrease the waiting time of passengers in the bus route?
3. What effect will such an increase have on the waiting time of other queues?
4. Will reallocating buses decrease the mean waiting time of passengers?

1.6 Objectives of the project

To effectively achieve the aim of this project, we will:

1. Collect data from a bus terminal in Kigali
2. Use queuing theory to estimate the waiting time of each queue
3. Develop a python class that produces the best allocation of buses for all queues

1.7 Literature review

Optimization of waiting time can be, and has been, applied to many fields. In the medical field, optimization of waiting time is a “big deal” as increased waiting time can affect the lives of humans [1]. In hospitals, optimization of wait time has been achieved through several means. I.M. Ibrahim et al., explored ways to decrease holding up time of the patients in one of the crisis clinics in Malaysia where the patients used to hold up for a long period of time. They carried out simulation optimization in Arena, and used the OpQuest in Arena Software Simulation to find the optimal combination of resources that would minimize waiting time [10].

In the transportation sector, which is of major concern to us in this study, optimization of waiting time has been achieved through various means. Queueing theory have been used in Banks to evaluate their performances in Nigeria two GTBank and Ecobank in Lagos. The queue characteristics have been analyzed using Multi-Server Queuing Model to determine waiting time and to find the optimal service level. The results showed that customers in Ecobank spent more time in the queue and in the system than at GTBank [20]. Husam Asfoor in his study of '' Analysis of Waiting Time Reduction in a Private Hospital in the Middle East" was able to examine the patient waiting time using queueing theory by evaluating and analyzing the average wait time at the hospital facilities. After recommending the changes in the hospital , some improvements were seen[21].

Other methods asides from queuing theory (or in addition to queuing theory) have also being applied to queuing systems. In the paper called "Modeling queueing systems" Angela Zoi Leontas came up with the theory of queueing systems where they used markovian property with measuring the effectiveness of the queueing systems with exponential interarrival and service time which were eventually used to optimize the performance of the given system [22]. C. Zheng et al. studied the spacing of bus stations as a way of reducing the overall travel time of passengers – time spent outside the bus and time spent inside the bus. They applied the game theory of Nash bargaining problems as the technique for minimizing the overall travel time of passengers [6]. Other researchers have explored optimization of the number of buses and bus routes as ways of minimizing waiting time of passengers. Bus schedules have also been created as a way of meeting the increasing number of passengers waiting in bus terminals, while also maximizing the use of resources (buses) [11], and reducing passengers' waiting time. E. Hassannayebi et al., optimized the waiting time by suggesting a model for train scheduling to reduce the number of people on terminals[12]. The mathematical optimization model for on-line train predicts the numbers of passengers in each given period and reschedule the timetable accordingly. They used mixed-integer linear (MILP) and non-linear programming (MINLP) models. From the output they gained in experiment of those techniques, the efficiency of the linear optimization model was superior when compared with the non-linear formulation. In the paper entitled "Application of Queuing Theory to Optimize Waiting-Time in Hospital Operations", D. Yaduvanshi et al. studied the out-patient department from different dimensions by incorporating SWOT analysis which was used in breaking down the queuing problem and knowing where the hospital operations could be improved. Afterwards, queuing theory was applied to the areas that needed improvements knowing the area of improvement to make the working and processes better [23].

In order to optimize passengers' waiting time, a model of the queuing system is required. It is this model that would provide us with an estimate of the expected waiting of passengers, among other things. The models used in describing queuing systems are known as queuing models. In operations management, the study of waiting in line is known as the queuing theory.

Queuing models are ubiquitous in the analysis of service, production and many other situations, given the limited resources or providers[13]. Basic concepts of queuing models, and in some cases the performance of a queuing system, has been measured by using a mathematical analysis. For instance, Wang et. al., studied the stationary distribution along with the distribution of waiting time for a stable system of an M/M/1 queuing model, and they found that the queue was only stable when the service rate exceeded arrival rate [14]. In some other project, Bai et. al. 2016, examined the arrival of recipients (customers), along with the possibility that the customer might get tired and leave the queue. The researchers came up with a queueing model that describes the length of time customers are willing to wait for the service. Their model enabled them to know how many customers can be too much that can lead to dissatisfaction of customers, along with knowing how many customers can be inadequate to cause an underutilization of resources[15]. In their paper, "Comparative Study on Different Queuing Models to Reduce Waiting Time in Brahmaso Clinic", T. Aung et al. considered multiple servers M/M/2 to M/M/3 and M/M/4 to M/M/5 queuing models in order to reduce waiting time and also analyze and compare queuing parameters and performance measures of the system. He found that the waiting time in the system has been reduced by increasing a server to queuing models M/M/4 and M/M/2 [16].

1.8 Methodology

In order to complete this project within the given timeframe, we have chosen to study a bus terminal which we assume to be representative of bus terminals in Kigali city. Kimironko Gare is a central bus terminal in Kimironko, Kigali, where buses heading to destinations including: Zindiro, Down Town, Kicukiro, Masizi, Kabuga, Nyabugogo, Kinyinga pick up passengers.

Figure 1: A Queue of passengers waiting for Buses at Kimironko Gare at Night

Dataset

Data for three destinations – Kabuga, Zindiro and Down Town – were collected from the bus station. The 3 chosen destinations have varying conditions. For instance, at night, the queue of passengers heading towards Kabuga is a lot busier than the queue of passengers heading towards Down Town. The data collected includes information on the arrival time and departure time of passengers and buses, as well as the number of arrivals of passengers and buses. The data was collected from 7:15PM to 8:00PM. The data can be found on the links below:

https://github.com/aanuorioke/Optimization-of-Waiting-Time-in-Kimironko-Queuing-Theory

The data that was collected for Zindiro and Kabuga are samples (from the entire queue). This is because the rate at which passengers were arriving to these queues was high and we were limited in the number of data collectors.

The data that was collected was used in the following ways:

1) To understand the distribution of interarrival times for the three queues (i.e. whether or not they follow the theoretical exponential distribution)
2) To calculate the parameters associated with a queuing system
3) To validate our python program that re-allocates buses to queues

Research Approach

Using python, Chi-square goodness of fit test was used to check if the inter-arrival times of the three queues follow an exponential distribution or not. The results are contained in a later section.

The service times of the three queues have minimum and maximum values which are affected by traffic and time of day. The service time was assumed to be uniformly distributed.

In the bus terminal in Kimironko, passengers do not enter the buses in parallel, that is, if 10 buses are available, buses are filled one after the other. Hence, our queuing system is not considered as a multi-server queuing system. In a multi-server queuing system, servers do not have to be filled (used) one after the other. Our system is taken to be a one-server system.

Based on these facts and assumptions, our system was modeled as a G/G/1 queuing system. Where the first G represents an interarrival time that is not exponentially distributed and the second G represents a service-time that is non-exponential.

Approach for Parameter Estimation

Jupyter Notebook and Python were used for our analysis.

As mentioned, queuing theory was applied to the 3 queues under consideration.

The parameters involved in queuing theory include: arrival rate, service rate and utilization factor. These parameters are used to get an estimate of the average waiting time, in accordance with Little's law.

The following equations for G/G/1, gotten from Gosavi [19], were used to estimate the waiting time of each queue.

$$Arrival\ rate,\ \boldsymbol{\lambda} = \frac{1}{E[interarrival - time]}$$

$$Service\ rate,\ \boldsymbol{\mu} = \frac{1}{E[Service - time]}$$

$$Utilization\ rate,\ \boldsymbol{\rho} = \frac{\boldsymbol{\lambda}}{c\boldsymbol{\mu}}$$

$$Mean\ number\ of\ passengers,\ L_q = \frac{\boldsymbol{\rho}^2(1 + C_s^2)(C_a^2 + \boldsymbol{\rho}^2 C_s^2)}{2(1 - \boldsymbol{\rho})(1 + \boldsymbol{\rho}^2 C_s^2)}$$

$$Squared\ coefficient\ of\ variation\ of\ interarrival\ time,\ C_a^2 = \frac{Variance\ of\ inter - arrival}{(E[Interarrival])^2}$$

$$Squared\ coefficient\ of\ variation\ of\ service\ time,\ C_s^2 = \frac{Variance\ of\ serivice\ time}{(E[service\ time])^2}$$

$$Average\ Waiting\ time\ of\ passengers, W_q^{G/G/1} = \frac{L_q}{\boldsymbol{\lambda}}$$

Where

c is the number of buses (servers) in the system.

Service time was calculated using equation (8).

$$Service\ time = \frac{journey\ time}{number\ of\ seats} \tag{8}$$

Where the journey time is the time it takes the bus to go from Kimironko Gare to the destination in consideration, together with the time it takes to load all buses at once (i.e Departure time of last bus - Arrival time of first bus)

$$Journey\ time,\ t_j = t_d + t_l$$

Where,

t_d is the time it takes to go from Kimironko to the destination in consideration

t_l is the time it takes to load all buses at once

Approach used in reallocating buses

In this project, the goal of bus re-allocation is to reduce the mean waiting time of the three queues. The mean waiting time is given by:

$$Mean\ waiting\ time,\ t_m = \frac{t_{q1} + t_{q2} + t_{q3}}{3}$$

Where,

t_{q1} is the estimated waiting time of queue 1

t_{q2} is the estimated waiting time of queue 2

t_{q3} is the estimated waiting time of queue 3

Re-allocation of buses was done using random search. Firstly, all buses were combined together. Then a random allocation of buses was done, such that the least number of buses a queue can get is 1. Using this new allocation of buses, the waiting time for each queue is estimated with queuing theory, and the mean waiting time, t_m is calculated. This random generation of number of buses assigned to each queue was repeated many times (10,000). We also took into account the fact that all buses do not have an equal number of seats. The allocation of buses that gave the minimum mean waiting time was chosen to be our optimal bus re-allocation.

Advantages and Limitations of Research Approach

The queuing theory is an efficient mathematical model that can be applied to optimize various parameters in a queuing system. It is designed to work only for systems in equilibrium. A queuing system becomes unstable when the utilization factor exceeds 1. In such a case, queuing theory is limited, in that it cannot be used to estimate the waiting time of the system. In an unstable queuing system, the waiting time will tend to grow infinitely as the number of passengers grows infinitely. In order to apply queuing theory in estimating the waiting time of passengers in an unstable system, more buses must be added to bring the system into equilibrium.

Given that the number of ways of re-allocating buses can be numerous, depending on the total number of buses in the queuing system, randomly generating different bus allocations (random search) quickly helped us to see what we can expect if we went through every possible allocation of buses (brute force). Although this is a quicker and accurate way of getting the optimal bus allocation, it is possible that we might have missed a combination that would have given us a lesser waiting time. Also, the mean waiting time obtained will be random since we used random search.

1.9 Results and Discussion

On obtaining our data, we calculated the average waiting time for each queue without using queuing theory. Average Waiting time of passengers was obtained from the Arrival and Departure time of passengers:

$$Waiting\ Time = Departure\ Time - Arrival\ Time \tag{1}$$

The result for average waiting time obtained is shown in table 1.

From Table 1, the mean waiting time is 11.07 mins.

<table>
<tr><th>Destination</th><th>Average Waiting time (mins)</th><th>Mean Waiting Time (mins)</th></tr>
<tr><td>Down Town</td><td>3</td><td rowspan="3">11.07</td></tr>
<tr><td>Zindiro</td><td>9.84</td></tr>
<tr><td>Kabuga</td><td>20.37</td></tr>
</table>

Table 1: Average Waiting Time from Data without using Queuing Theory

From the data, PMFs of inter-arrival times for the three queues were obtained. Figures 2, 3 and 4 show the results.

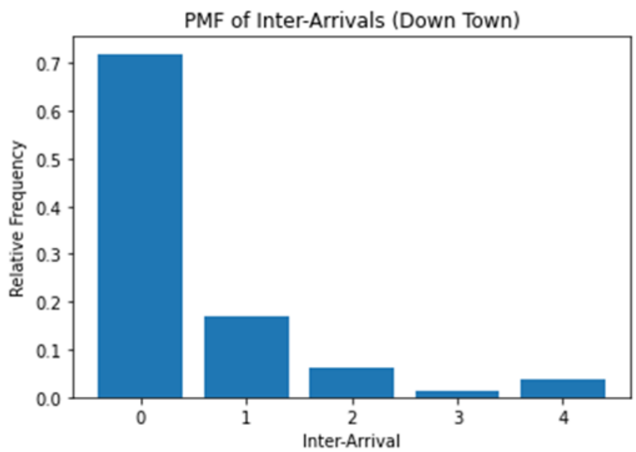

Figure 2: PMF of Inter-Arrivals for Down Town

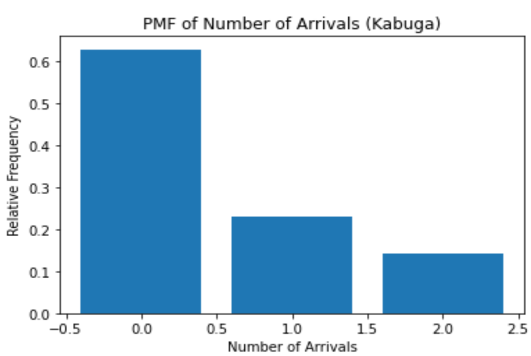

Figure 3: PMF of Inter-Arrivals for Kabuga

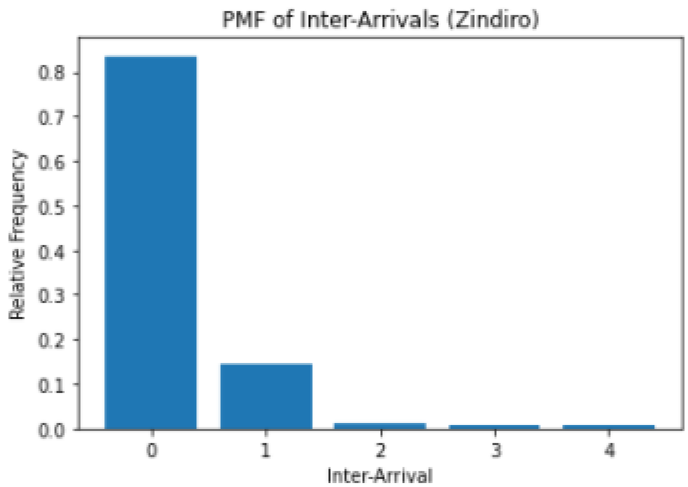

Figure 4: PMF of Inter-Arrivals for Zindiro

As seen from the PMF plots in Figure 2 - 4, it is not easy to say whether the distributions follow an exponential distribution or not. Hence we used the Chi-square goodness of fit test to get a better idea of the distribution of inter-arrival times.

The results for the Chi-square goodness of fit test, which helped us in deciding whether to use an M/M/1 queuing system or a G/G/1 queuing system, is shown in Table 2:

Destination	**p-value**
Down Town	$1.5 * 10^{-9}$
Zindiro	$3.34 * 10^{-49}$
Kabuga	$1.33 * 10^{-9}$

Table 2: Result from Chi-square goodness of fit test

Since the p-values for all three destinations was less than a chosen significance level of 0.05, we rejected the null hypothesis that the interarrival distribution is exponential. Hence we used the G/G/1 queuing system.

The arrival rate, service rate, utilization rate and average waiting time of each queue were estimated using queuing theory. The results obtained are shown in Table 3.

Destination	Number of buses	Number of seats	Arrival Rate (passengers/mins)	Service Rate	Utilization Rate	Average Waiting Time using queuing theory (mins)
Town	10	220	2.07	2.93	0.7	2.22
Zindiro	4	332	4.97	5.53	0.90	8.34
Kabuga	4	88	1.7	1.6	1.062	N/A

Table 3: Estimated Average Waiting Time

As seen in Table 3, Kabuga does not have an average waiting time when queuing theory was applied to it. This is because with a utilization rate of 1.062, Kabuga's utilization rate is greater than 1. This indicates that passengers are arriving faster than they can be served by the system, so the queue and waiting time will grow infinitely. This infinite growth in waiting time can be seen in Table 2, where Kabuga's average waiting time is 20.37 minutes. Thus the queue for Kabuga is unstable. Due to this instability, queuing theory cannot be used to estimate Kabuga's average waiting time. To bring Kabuga to stability, more buses or seats must be added to its queue.

Also, comparing the waiting times in table 2 and 3 for Down Town and Zindiro, we can see there is a deviation between the actual waiting time (Table 2) and the estimated waiting time (Table 3). For Down Town, there is a deviation of 0.78 mins. For Zindiro, the deviation is 1.5 mins.

Re-allocation of buses was done for the three queues, and the mean waiting time was estimated. The performance of each re-allocation that was done was measured by comparing the mean waiting time after re-allocation with the mean waiting time before re-allocation (Table 1). The percentage change in mean waiting time was then used to judge whether the re-allocation was effective or not.

Re-allocation of buses was done for the three queues, and the mean waiting time was estimated. The result is as shown in table 4.

Destination	Optimal Number of buses	Optimal Number of seats	Individual Waiting Time (mins)	Mean Waiting Time (mins)

Town	6	193	3.94	5.38
Zindiro	10	342	6.51	
Kabuga	2	105	5.67	

Table 4: Mean Waiting Time after Reallocating Buses

From the results in Table 4, the mean waiting time for the three queues reduced by 51.4% in comparison to the actual mean waiting time (Table 1). Given that we were not expecting as much decrease as this, we consider this to be a significant result.

Although the waiting time of Down Town increased by 31%, the average waiting time of Zindiro and Kabuga decreased by 33% and 72% respectively. The increase in waiting time Town is as expected. The decrease in waiting time for Kabuga was expected, though we did not expect a 72% decrease in Kabuga's waiting time.

It can be seen that bringing Kabuga into stability, by increasing the number of seats assigned to its queue, greatly reduces its average waiting time. This would be due to the fact that the system can now handle the number of arrivals.

Also, reallocation of buses was done for two queues. The result obtained when the buses for Town and Zindiro were re-allocated is as shown in Table 5. The result obtained for bus reallocation between Town and Kabuga is shown in Table 6.

Destination	Optimal Number of buses	Optimal Number of Seats	Individual Waiting Time (mins)	Mean Waiting Time (mins)
Town	6	193	3.94	4.37
Zindiro	8	359	4.79	

Table 5: Re-allocation of buses for Town and Zindiro

Destination	Optimal	Optimal	Individual	Mean Waiting Time

<table>
<tr><th></th><th>Number of buses</th><th>Number of Seats</th><th>Waiting Time (mins)</th><th>(mins)</th></tr>
<tr><td>Town</td><td>9</td><td>198</td><td>3.46</td><td rowspan="2">3.66</td></tr>
<tr><td>Kabuga</td><td>5</td><td>110</td><td>3.86</td></tr>
</table>

Table 6: Reallocation of buses for Town and Kabuga

From Table 5, the mean waiting time for Town and Zindiro decreased from 6.42 mins (Table 1) to 4.37 mins. This represents a 32% decrease in mean waiting time. From Table 6, the mean waiting time for Town and Kabuga decreased from 11.68 mins (Table 1) to 3.66 mins. This represents a 69% decrease in mean waiting time.

Based on the results obtained in this project, we believe that the average waiting time of bus passengers in bus terminals in Kigali (at night) can be reduced by re-allocating buses. This can in turn lead to benefits for the public transportation system of Rwanda. The public transportation sector of the country could increase their revenue from bus transit, as more passengers are likely going to be willing to use bus transit with a reduction in waiting time.

In terms of cost, the re-allocation that was achieved is a cost-effective way for the government to minimize average waiting time of bus passengers. Another way, which may or may not be cost effective is to increase the number of buses or seats of a queue that got more buses or seats after re-allocation was done. Comparing the results for Kabuga in Table 3 and 4, we can see that prior to re-allocation, Kabuga had 4 buses which had 88 seats in total (22 seats each). But after reallocation, Kabuga got 2 buses with 105 seats. This implies that it got one bus that has 83 seats and another bus that has 22 seats. So if bus operators want to maintain the waiting time of Town, they would need to purchase one big bus and a small bus for Kabuga.

Future Work

Due to the constraint in time that we experienced, we were unable to get data for a whole day (24 hours). Given that the conditions of bus terminals vary with time and periods of the day, our solution would only work for the period (night time) in which we collected our data. This implies that our solution cannot be generalized for all times of the day. Hence, in future work, it is important to consider other periods of the day during data collection. To achieve even better results, the data can be collected over a week or month.

In addition, we were limited in the number of data collectors we had. Hence it is possible to have missed some passengers that arrived, given that the rate of arrivals for Kabuga and town were high. It is therefore important to employ more accurate means of collecting data in the future.

Lastly, since we used random search in finding the optimal mean average time and the number of simulations we ran is not equal to the total number of possible bus allocations, our results can be random. Hence, in future work, it is recommended to use brute force or other methods that would yield consistent results.

1.10 Conclusion

In this project, we used queuing theory to optimize the mean waiting time of three bus passenger queues in a bus terminal in Kigali - Kimironko Gare. Data was collected manually over a period of 45 mins. A G/G/1 queuing system was used in estimating the waiting time of queues. Optimization of mean waiting time was done by reallocating buses. A significant reduction (51.4%) was achieved in mean waiting time of passengers. The results obtained suggest that the mean waiting time of passengers in bus terminals in Kigali can be reduced.

DIVISION OF WORK

Aanuoluwapo Orioke

- Literature review
- Data Collection and analysis for Town
- Code development and algorithm development for bus reallocation

Vital Hakizimana

- Literature review
- Data Collection for Town and analysis for Zindiro and Kabuga
- Development of algorithm used in the reallocation of buses

Moise Uwimana

- Literature review
- Data collection and analysis for Zindiro
- Development of algorithm used in the reallocation of buses

Gabriel Ntwari

- Literature review
- Data collection and analysis for Kabuga
- Development of algorithm used in the reallocation of buses

References

1. D. Niyonsenga, "Assessing public transport supply for Kigali, Rwanda," M.S. thesis, Dept. of Urban and Regional Planning & Geo-Info. Mgmt. Univ. Twente, Enschede, The Netherlands, 2012. [Online]. Available: https://www.google.com/url?sa=t&rct=j&q=&esrc=s&source=web&cd=&cad=rja&uact=8&ved=2ahUKEwi-i_yiyIjzAhXUEcAKHdF3CxwQFnoECAQQAQ&url=http%3A%2F%2Fessay.utwente.nl%2F84857%2F1%2Fniyonsenga.pdf&usg=AOvVaw1wsslKix2C9emfBYSvwiRp
2. World Population Review, "Kigali Population 2021 (Demographics, Maps, Graphs)", *Worldpopulationreview.com*, 2021. [Online]. Available: https://worldpopulationreview.com/world-cities/kigali-population. [Accessed: 09- Oct- 2021].
3. Trading economics, "Rwanda GDP From Transport | 2021 Data | 2022 Forecast |", *Tradingeconomics.com*, 2021. [Online]. Available: https://tradingeconomics.com/rwanda/gdp-from-transport. [Accessed: 09- Oct- 2021].

4. Y. Freemark, “Talking transit: speeding up the bus,” *Metropolitan Planning Council,* Jun 23, 2016. [Online], Available: https://www.metroplanning.org/news/7328/Talking-Transit-Speeding-up-the-bus.
5. RSG, “Who’s on board: 2014 mobility attitude survey,”
6. C. Zheng, S. Zheng, G. Ma, “The Bus Station Spacing Optimization Based on Game Theory,” in *Advances in Mechanical Engineering*, Jan. 2015. doi:10.1155/2014/453979
7. I. Hartmann Tolić, K. Nyarko, A. Ceder, “Optimization of Public Transport Services to Minimize Passengers’ Waiting Times and Maximize Vehicles’ Occupancy Ratios,” in *Electronics*. vol. 9. 360, 2020.
8. R. G. Mishalani, M. M. McCord, “Passenger Wait time perceptions at bus stops: empirical results and impacts on evaluation real-time bus arrival information,” in *J public Trans*, vol. 9, no. 2, 2006, pp 89 – 106.
9. W. Leong, K. Goh, S. Hess, P. Murphy, “Improving bus service reliability: The Singapore experience,” in *Res. Transp. Econ.,* vol 59, 2016, pp 40 – 49.
10. I. M. Ibrahim, C.-Y. Liong, S. A. Bakar and e. al., "Minimizing patient waiting time in emergency department of public hospital using simulation optimization approach," *AIP Conference Proceedings,* vol. 1830, no. 1, 2017. [online: https://doi.org/10.1063/1.4980949]
11. A. Ceder, “Methods for creating bus timetables,” in *Transp. Res. Part A: General,* 1987, vol. 21, no. 1, pp 59 – 83.
12. E. Hassannayebi, S. H. Zegordi, M. Yaghini, M. R. Amin-Naseri, “Timetable optimization models and methods forminimizing passenger waiting time at public transit terminals,” in *Transportation Planning andTechnology*, 2017 DOI: 10.1080/0381060.2017.1283156
13. H. Mehri, T. Djemel, H. Kammoun, “Solving of waiting lines models in the airport using queuing theory model and linear programming the practice case,” in *A.I.M.H.B,* 2006. Hal-00263072v2..
14. L. Wang, C. Xu, “Queue’s waiting time,” [Online] Available: https://dornsife.usc.edu/assets/sites/406/docs/505b/Project_Queue-5.pdf

15. J. Bai, K. C. So, C. Tang. "A queuing model for managing small projects under uncertainties," in *Eur. J. Oper. Res.* vol 253, no 3, 2016, pp 777-790.
16. T. Aung, L. Naing, "Comparative Study on Different Queuing Models to Reduce Waiting Time inL Brahmaso Clinic," *International Journal of Scientific and Research Publications,* vol. 9, no. 6, June 2018, pp. 4-5, 2019.
17. G. Xin, W. Wang, "Model passengers' travel time for conventional bus stop," in *J. Appl. Math.*, 2014, Article ID 986546. [Online]. Available: http://dx.doi.org/10.1155/2014/986546.
18. S. Shanmugasundaram, P. Umarani, "Queuing Theory Applied in Our Day to Day Life," in *IJSER*, April 2015, vol. 6, no. 4.
19. A. Gosavi, "Tutorial for Use of Basic Queueing Formulas," Notes for Department of Engineering Management and Systems Engineering, Missouri S & T
20. A. FARAYIBI, "Investigating the Application of Queue Theory in Nigerian Banking systems," Munich Personal RePEc Archive, pp. 6-8, 2016.
21. Asfoor, Husam, "An Analysis of Waiting Time Reduction in a Private Hospital in the Middle East" (2014). Applied Research Projects. 41. https://doi.org/10.21007/chp.hiim.0036.
22. Leontas, Angela Zoi, "Modeling queueing systems" (2006). Theses Digitization Project. 3101. https://scholarworks.lib.csusb.edu/etd-project/3101
23. A. S. P. V. M. Deepak Yaduvanshi, "Application of Queuing Theory to Optimize Waiting-Time in Hospital Operations," *oscm,* vol. 12, no. 3, p. 1, 2019