Mathematical Models for Estimating Communication Volume in Swarm-Based Multi-Agent Systems

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Abstract

This paper presents an exhaustive mathematical framework for modeling and predicting communication volume within swarm-based multi-agent systems. Through a synthesis of contemporary research, it systematically develops sophisticated analytical models incorporating network topology, messaging protocols, latency constraints, iterative swarm behaviors, and realistic communication environments. Detailed mathematical derivations, illustrative examples, and extensive discussions reinforce the theoretical robustness and practical applicability of the proposed models.

1 Introduction

Swarm-based multi-agent systems represent a significant advancement in decentralized computing, inspired by natural phenomena observed in social insects, birds, and fish. These systems are increasingly utilized in applications ranging from robotics and telecommunications to environmental monitoring and logistics management. Accurate mathematical modeling of communication processes within these systems is crucial to optimizing their performance, scalability, and adaptability.

Swarm-based multi-agent systems represent a transformative advancement in decentralized computing paradigms, inspired by collective behaviors observed in natural organisms such as ant colonies, bee swarms, bird flocks, and fish schools. These systems are predicated upon the principle of emergent intelligence, where relatively simple agents interact locally with their neighbors, following straightforward rules, to produce highly sophisticated and adaptive global behaviors. The decentralized and self-organizing nature of swarm intelligence makes it particularly suitable for complex problem-solving in highly dynamic, uncertain, and distributed environments.

Over the past few decades, swarm intelligence has transitioned from theoretical curiosity to practical necessity across numerous technological and industrial domains. Robotics, telecommunications, logistics, environmental monitoring, surveillance, agriculture, and disaster response have all increasingly leveraged swarm-based approaches due to their inherent robustness, scalability, and adaptability. For example, robotic swarms are extensively deployed in scenarios such as search-and-rescue operations, where autonomous units collaboratively navigate hazardous environments to locate survivors efficiently. Similarly, swarm methodologies have revolutionized logistics management, enabling autonomous drone fleets to deliver goods while dynamically adapting to traffic conditions and operational disruptions.

However, despite significant practical successes, the effective deployment and scalability of swarm systems remain fundamentally constrained by challenges related to inter-agent communication. Communication overhead, latency, reliability, and network topology profoundly influence the overall performance, responsiveness, and scalability of these systems. Efficient communication management is thus paramount, as excessive or poorly managed message exchanges can rapidly degrade system efficiency, consuming computational resources, increasing latency, and compromising reliability.

Addressing these challenges necessitates a rigorous, mathematically grounded approach to quantify, model, and predict communication behaviors and demands within swarm-based systems. Comprehensive mathematical modeling enables researchers and system designers to anticipate network demands, optimize resources proactively, and identify critical bottlenecks. Furthermore, it provides valuable insights into the inherent trade-offs between network complexity, communication overhead, and swarm performance.

To this end, this paper presents a rigorous and exhaustive mathematical framework explicitly designed to estimate and analyze communication volume within swarm-based multi-agent systems. By synthesizing and extending contemporary academic research, we develop advanced mathematical models that rigorously incorporate critical parameters such as network topology, latency constraints, message frequency, layered communication architectures, and dynamic adaptive mechanisms. Each model and formulation is accompanied by detailed derivations, thorough theoretical analyses, and illustrative numerical examples designed to ensure clarity, completeness, and practical applicability.

The contributions of this paper are thus multi-faceted. Firstly, we establish a robust theoretical foundation that bridges gaps between theoretical swarm intelligence studies and their practical computational implementations. Secondly, we provide actionable mathematical tools that enable precise prediction, optimization, and management of communication demands in real-world applications. Finally, by identifying inherent limitations and challenges within current modeling approaches, we outline compelling future research avenues, thereby guiding and stimulating further innovation within the field.

This expanded introduction frames the motivation, objectives, and anticipated outcomes of the paper, establishing a comprehensive basis for subsequent detailed discussions.

2 Fundamental Concepts and Terminology

To facilitate precise analysis and rigorous modeling of swarm-based multi-agent systems, it is crucial to clarify essential terminology and introduce formal mathematical definitions. This section systematically defines foundational concepts and provides illustrative examples from both biological and engineering perspectives.

2.1 Agents and Swarms

An **agent** within a multi-agent system is defined as an autonomous entity capable of decision-making, perception of its environment, and interaction with other agents. Formally, an agent can be mathematically represented as a tuple:

$$A_i = \{S_i, P_i, C_i\}, \tag{1}$$

where:

- S_i represents the agent's internal state variables,
- P_i denotes the set of perception functions through which the agent senses its local environment,
- C_i defines the set of communication actions or interactions the agent can perform with other agents.

A **swarm** is defined as a population of interacting agents, where interactions occur locally based on limited information. Formally, a swarm $\mathcal{S}$ of N agents can be expressed as:

$$\mathcal{S} = \{A_1, A_2, \ldots, A_N\}. \tag{2}$$

Biological Example: An ant colony comprises agents (ants) individually following simple local rules—such as pheromone-based trail-following—to achieve complex tasks collectively, including food foraging and nest construction.

Engineering Example: A swarm of autonomous drones used in agriculture cooperatively surveys crop fields, where each drone autonomously monitors plant health, sharing observations locally to optimize resource allocation.

2.2 Communication Round and Topology

A **communication round** is a discrete interval during which agents within the swarm exchange messages. Each round can be mathematically denoted as a discrete time interval:

$$t \in \{0, 1, 2, \ldots, T\}, \tag{3}$$

where T represents the total number of rounds.

The communication structure among agents is defined by the swarm's **topology**, described by a graph-theoretic framework:

$$\mathcal{G} = (\mathcal{V}, \mathcal{E}), \tag{4}$$

where:

- $\mathcal{V}$ denotes the set of vertices (agents), $\mathcal{V} = \{A_1, A_2, \ldots, A_N\}$,
- $\mathcal{E}$ represents the set of edges, signifying permissible communication links between agents.

Topology significantly influences swarm efficiency, affecting message propagation speed and the robustness of communications. Common topologies include fully connected, ring, grid, random, and hierarchical structures.

Biological Example: In honeybee swarms, communication topology is dynamic, adapting in response to environmental cues, whereby bees perform localized dances conveying positional and resource-quality information to nearby bees.

Engineering Example: Wireless sensor networks deployed in environmental monitoring use structured grid topologies, ensuring systematic coverage and efficient local communication pathways.

2.3 Latency

Latency is defined as the time delay encountered during communication between agents. It includes processing delays, propagation delays, and queuing delays. Formally, the latency between two agents A_i and A_j at time t is represented by:

$$L_{ij}(t) = t^j_{\text{received}} - t^i_{\text{sent}}, \tag{5}$$

where:

- t^i_{sent} denotes the timestamp when the message was sent by agent A_i,
- t^j_{received} denotes the timestamp when the message was received by agent A_j.

Aggregate latency constraints significantly influence communication patterns, imposing realistic constraints on how agents interact within given timeframes.

Biological Example: Flocks of birds display minimal latency during rapid directional adjustments, as instantaneous reaction times are crucial to collision avoidance.

Engineering Example: Autonomous vehicle platooning demands extremely low latency communication to maintain safe and efficient spacing between vehicles, necessitating rigorous mathematical modeling and optimization of latency constraints.

2.4 Message Volume

The **message volume**, a core aspect of communication modeling, is formally defined as the total number of discrete messages exchanged among agents within a swarm during specified communication rounds. Given a swarm with N agents, each agent interacting with an average of k other agents per round, the fundamental expression for message volume (M) per communication round is given by:

$$M = N \times k. \tag{6}$$

The cumulative message volume across multiple rounds (T) is then:

$$V_{\text{total}} = N \times k \times T. \tag{7}$$

This simplified model serves as the baseline, which will be expanded further in subsequent sections by incorporating constraints and real-world complexities.

2.5 Information Density

Information density (ρ) within swarm communications quantifies the average amount of actionable or useful information per message exchanged. Formally:

$$\rho = \frac{I_{\text{useful}}}{M_{\text{total}}}, \tag{8}$$

where I_{useful} denotes the sum of useful information content, and M_{total} represents the total message volume. High-density information exchange typically reduces the necessity of frequent messaging, enhancing efficiency.

In subsequent sections, these fundamental definitions will serve as a rigorous basis for complex mathematical modeling, enabling detailed exploration of how various parameters—such as topology, latency, and message frequency—affect communication volume within swarm-based systems.

3 Basic Communication Volume Estimation

Accurate estimation of message volume is foundational to understanding and optimizing swarm-based multi-agent systems. This section rigorously derives the fundamental formulas for estimating communication volume and provides clear, step-by-step justifications along with illustrative examples of incremental complexity.

3.1 Derivation of Basic Messaging Volume Formula

Consider a swarm system comprising N distinct agents. Suppose each agent communicates, on average, with k neighboring agents per communication round. The fundamental assumption here is uniform and symmetrical communication—that is, each agent's messages are considered equal in length and frequency.

The total number of messages (M) exchanged per communication round can be intuitively derived as follows:

1. Each of the N agents initiates communication with exactly k other agents.
2. Under these conditions, the total number of communications initiated by all agents is simply the product of these two quantities.

Therefore, the basic messaging volume per communication round can be mathematically expressed as:

$$M = N \times k. \tag{9}$$

Assumptions in this derivation include:

- Symmetric and uniform communication.
- No consideration yet of redundant or duplicate messages.
- Constant topology and fixed average communication per agent.

3.2 Total Message Volume Over Multiple Rounds

Expanding this formula to estimate cumulative messaging volume across multiple discrete communication rounds is straightforward. If the swarm system operates over a defined number of communication rounds, denoted by T, then the total message volume (V_{total}) can be generalized as:

$$V_{\text{total}} = N \times k \times T. \tag{10}$$

Here, it is implicitly assumed that the topology and average messaging rate remain consistent across all rounds.

3.3 Illustrative Examples

To elucidate the implications and applicability of these formulas, consider the following incremental examples:

Example 1: Basic Scenario

- Number of agents: $N = 10$
- Average neighbors per agent: $k = 2$
- Communication rounds: $T = 1$

Then the message volume per round is simply:

$$M = 10 \times 2 = 20 \text{ messages per round.} \tag{11}$$

Example 2: Multiple Communication Rounds

Expanding the scenario above over multiple rounds:

- Number of rounds: $T = 5$

Thus, total messaging volume across all rounds is:

$$V_{\text{total}} = 10 \times 2 \times 5 = 100 \text{ messages.} \tag{12}$$

Example 3: Increased Communication Density

To further illustrate complexity, consider a larger swarm with denser communication:

- Number of agents: $N = 100$
- Average neighbors per agent: $k = 10$
- Number of rounds: $T = 20$

Applying our formula, we have:

$$V_{\text{total}} = 100 \times 10 \times 20 = 20,000 \text{ messages.} \tag{13}$$

These incremental examples highlight clearly how message volumes scale linearly with the number of agents, the number of neighboring connections per agent, and the duration of the communication.

3.4 Justification of the Formula

This simple linear scaling (Eq. 3 and Eq. 4) is highly valuable as an initial approximation due to its simplicity and interpretability. The formula is justified under several practical and theoretical conditions:

- It accurately captures the initial scaling behavior in typical swarm scenarios.
- It provides a clear baseline against which more sophisticated, constrained models can be compared.
- It assumes that agents interact uniformly, which is often the simplest assumption to start from before introducing real-world complexity.

While more complex factors such as latency, topology variability, message redundancy, and dynamic adaptations are not yet considered, this basic model remains fundamentally useful for preliminary design, benchmarking, and theoretical explorations.

In subsequent sections, we build upon these initial formulas to integrate practical constraints and realistic considerations, providing progressively more sophisticated and accurate estimations. The foundational model estimates messaging volume per communication round as:

$$M = N \times k \tag{14}$$

The total messaging volume over multiple communication rounds is generalized as:

$$V_{total} = N \times k \times T \tag{15}$$

4 Advanced Modeling with Topological and Latency Constraints

While the basic communication model provides valuable initial insights, it does not fully capture the complexities introduced by real-world conditions, such as varying network topologies and communication latencies. To address these critical factors, this section presents advanced mathematical modeling approaches that explicitly incorporate both network topological constraints and latency considerations.

4.1 Incorporation of Network Topology

Network topology significantly impacts the efficiency and effectiveness of inter-agent communications within a swarm. Formally, a swarm's topology can be modeled using graph theory, represented by a graph:

$$\mathcal{G} = (\mathcal{V}, \mathcal{E}), \tag{16}$$

where:

- $\mathcal{V}$ is the set of vertices (agents), such that $|\mathcal{V}| = N$.
- $\mathcal{E}$ represents edges, denoting direct communication links between agents.

The adjacency matrix (A) associated with graph $\mathcal{G}$ mathematically encapsulates the topology, defined by:

$$A_{ij} = \begin{cases} 1, & \text{if a direct communication link exists between agents } i \text{ and } j, \\ 0, & \text{otherwise.} \end{cases} \tag{17}$$

Thus, the actual average number of neighbors per agent (k_{actual}) can be rigorously computed as:

$$k_{\text{actual}} = \frac{1}{N} \sum_{i=1}^{N} \sum_{j=1}^{N} A_{ij}. \tag{18}$$

This adjustment reflects the realistic scenario where communication opportunities are explicitly limited by physical, environmental, or logistical constraints.

4.2 Mathematical Modeling of Latency Constraints

Latency, defined as the delay experienced in message transmission, critically influences communication feasibility and effectiveness. Following Wilson et al. (2024), latency constraints can be formally incorporated into the communication model as follows.

Define the latency between two agents A_i and A_j as L_{ij}. Given a maximum allowable latency threshold τ, communication between these agents is considered feasible only if:

$$L_{ij} \leq \tau. \tag{19}$$

To integrate latency constraints, we introduce the Heaviside step function $\Theta(x)$, mathematically defined by:

$$\Theta(x) = \begin{cases} 1, & \text{if } x \geq 0, \\ 0, & \text{if } x < 0. \end{cases} \tag{20}$$

Consequently, the latency-adjusted communication degree of an agent A_i becomes:

$$k_i(\tau) = \sum_{j=1}^{N} A_{ij}\, \Theta(\tau - L_{ij}), \tag{21}$$

and the swarm-wide average effective communication degree, accounting for latency, is then:

$$k(\tau) = \frac{1}{N} \sum_{i=1}^{N} \sum_{j=1}^{N} A_{ij}\, \Theta(\tau - L_{ij}). \tag{22}$$

4.3 Integrated Model with Both Topology and Latency Constraints

The previous sections have separately addressed topology and latency constraints. Combining these constraints, the refined model of total messaging volume per communication round is expressed as:

$$M(\tau) = N \times k(\tau), \tag{23}$$

where $k(\tau)$ explicitly considers both the topology (via adjacency matrix A) and the latency constraints (via the Heaviside step function).

The cumulative messaging volume across T rounds with these combined constraints is thus generalized as:

$$V_{\text{total}}(\tau) = N \times k(\tau) \times T. \tag{24}$$

4.4 Illustrative Examples with Integrated Constraints

To clearly demonstrate the impact of topology and latency constraints, consider the following illustrative scenarios:

Example 1: Simple Grid Topology with Latency Constraints

Consider a simple grid topology consisting of $N = 25$ agents arranged in a 5×5 grid. Each agent ideally communicates with 4 neighbors (up, down, left, right). Suppose latency between adjacent agents is $L_{ij} = 50$ ms, and the latency threshold is $\tau = 100$ ms. Communication between diagonal agents is infeasible due to higher latency.

In this scenario:

$$k(\tau) = 4 \quad \text{(since all adjacent neighbors satisfy } L_{ij} \leq \tau\text{)}, \tag{25}$$

thus, messaging volume per round is:

$$M(\tau) = 25 \times 4 = 100 \text{ messages per round.} \tag{26}$$

Example 2: Random Topology with Varied Latencies

Consider a random topology of $N = 50$ agents, where the average ideal neighbors per agent is $k = 8$. Due to environmental variability, latencies range randomly between 20 ms and 150 ms, with a latency threshold set at $\tau = 80$ ms.

To determine $k(\tau)$, assume empirical measurement indicates that approximately 70% of the links satisfy $L_{ij} \leq \tau$. Thus:

$$k(\tau) = 8 \times 0.70 = 5.6, \tag{27}$$

and messaging volume per round becomes:

$$M(\tau) = 50 \times 5.6 = 280 \text{ messages per round.} \tag{28}$$

This example clearly demonstrates how latency constraints significantly reduce effective messaging volume relative to the ideal scenario.

4.5 Justification of the Advanced Model

This advanced mathematical formulation incorporating topology and latency constraints significantly enhances the realism and practical applicability of swarm communication models. The inclusion of topological constraints ensures accurate representation of agent interactions, while latency considerations explicitly address critical timing constraints commonly encountered in practical deployments.

Furthermore, these sophisticated mathematical models provide crucial insight into designing effective swarm communication strategies, offering clear guidance on optimal topology structures and latency management techniques. These insights are invaluable for network designers seeking to optimize swarm performance under realistic operational conditions.

In subsequent sections, this rigorous model serves as a foundational basis for even more complex scenarios involving layered communication structures and dynamic adaptations.

5 Layered Communication Architectures

Real-world swarm-based systems often employ multi-layered communication structures, where interactions among agents are not merely uniform but structured into distinct phases, each characterized by different message types, purposes, and associated complexities. Such architectures significantly enhance the flexibility, responsiveness, and robustness of swarm-based systems. In this section, we mathematically formalize and rigorously analyze multi-phase communication architectures, explicitly modeling the concepts of reputation, state management, and agent expertise.

5.1 Formalization of Layered Communication

We define a layered communication model consisting of three primary interaction phases:

1. **Generation Phase (g)**: Agents propose new information or tasks.
2. **Analysis Phase (a)**: Agents evaluate proposed information using expertise and state information.
3. **Evaluation Phase (e)**: Agents make decisions influenced by analysis and reputation metrics.

Mathematically, the average number of communications per agent per round within this layered structure is expressed as:

$$k_{\text{layered}} = g + a + e. \tag{29}$$

Consequently, the total message volume over T rounds becomes:

$$V_{\text{layered}} = N \times (g + a + e) \times T. \tag{30}$$

5.2 Modeling Reputation in Communication

In realistic swarm-based systems, agents possess varying levels of trustworthiness or **reputation**, which significantly influences communication dynamics. Reputation affects whether an agent's communication is accepted and propagated through the swarm.

Formally, the reputation of an agent A_i at time t is denoted by $R_i(t)$, where:

$$0 \leq R_i(t) \leq 1, \tag{31}$$

with $R_i(t) = 1$ indicating fully trusted agents and $R_i(t) = 0$ indicating completely untrusted agents.

During the evaluation phase, reputation-weighted acceptance probability (P_{accept}) of agent A_j receiving information from agent A_i is modeled as:

$$P_{\text{accept}}(i, j, t) = \frac{R_i(t)}{\sum_{l \in \mathcal{N}_j} R_l(t)}, \tag{32}$$

where $\mathcal{N}_j$ is the set of neighbors communicating with agent A_j.

5.3 State Management Modeling

Agents maintain internal states (S_i) representing memory, context-awareness, or environmental knowledge. State management influences message volume, as agents selectively communicate based on current states.

The state update function for an agent A_i at time t receiving information from a neighbor agent A_j is modeled as:

$$S_i(t+1) = f\big(S_i(t), I_{ji}(t), R_j(t)\big), \tag{33}$$

where:

- $I_{ji}(t)$ is the information received from agent A_j at time t.
- $f(\cdot)$ is a state-update function influenced by information quality and sender reputation.

Higher reputation and more relevant information positively impact state updates, reinforcing productive communication links.

5.4 Expertise and Information Analysis

Expertise refers to an agent's capacity to accurately assess and process received information. The expertise level (E_i) of agent A_i is formally defined as:

$$0 \leq E_i \leq 1, \tag{34}$$

with $E_i = 1$ signifying maximum expertise (perfect analysis capability).

During the analysis phase, the probability of agent A_i correctly evaluating information I received at time t is modeled as:

$$P_{\text{correct}}(I, i) = E_i \cdot Q(I), \tag{35}$$

where $Q(I)$ denotes inherent quality or accuracy of the information.

5.5 Detailed Illustrative Example

To concretely illustrate these concepts, consider a swarm-based scenario in environmental monitoring:

Scenario Setup:

- Swarm size: $N = 100$ agents.
- Each round includes 3 phases: Generation ($g = 2$ messages), Analysis ($a = 2$ messages), Evaluation ($e = 1$ message).
- Agents have varying reputations and expertise levels.
- Communication occurs over $T = 50$ rounds.

Step 1: Initial Computation of Basic Volume
Initial total volume without constraints is straightforwardly calculated as:

$$V_{\text{layered}} = 100 \times (2 + 2 + 1) \times 50 = 25,000 \text{ messages}. \tag{36}$$

Step 2: Reputation-based Adjustments
Assume average reputation $\bar{R} = 0.75$. Reputation-based acceptance probability reduces message propagation efficiency proportionally:

Adjusted effective volume per round considering reputation (k_R):

$$k_R = (g + a + e) \cdot \bar{R} = 5 \times 0.75 = 3.75. \tag{37}$$

Thus, revised volume over $T = 50$ rounds is:

$$V_{\text{reputation}} = N \times k_R \times T = 100 \times 3.75 \times 50 = 18,750 \text{ messages.} \tag{38}$$

Step 3: Expertise and State Management Adjustments

Suppose average agent expertise $\bar{E} = 0.8$, and average information quality $Q(I) = 0.9$. Probability of correct information evaluation is:

$$P_{\text{correct}} = \bar{E} \times Q(I) = 0.8 \times 0.9 = 0.72. \tag{39}$$

Given agents update states based on correctly evaluated information, the effective communication adjusted further for expertise and state update accuracy ($k_{E,S}$):

$$k_{E,S} = k_R \times P_{\text{correct}} = 3.75 \times 0.72 = 2.7. \tag{40}$$

Revised final volume over 50 rounds incorporating all constraints becomes:

$$V_{\text{final}} = N \times k_{E,S} \times T = 100 \times 2.7 \times 50 = 13,500 \text{ messages.} \tag{41}$$

5.6 Significance of the Layered Model

This layered communication architecture, enriched by reputation, state management, and expertise modeling, provides significantly deeper insights into realistic communication dynamics. It demonstrates how agents selectively manage communication, emphasizing the importance of trusted and competent interactions. This structured approach not only reduces unnecessary communication overhead but also enhances the overall effectiveness and reliability of swarm-based systems.

This comprehensive layered model thus serves as a robust foundation for further advancements in swarm communication systems, guiding practical deployment strategies and inspiring future theoretical explorations.

6 Dynamic Modeling Using swarmDMD

The static models discussed thus far provide valuable insights into the steady-state communication behavior of swarm-based multi-agent systems. However, real-world swarm scenarios are inherently dynamic, characterized by evolving communication patterns, changing network conditions, and adaptive agent behaviors. To capture these dynamics rigorously, this section introduces and explores an advanced mathematical modeling approach known as Dynamic Mode Decomposition (DMD), specifically tailored to swarm systems, termed **swarmDMD**.

6.1 Theoretical Foundation of swarmDMD

swarmDMD, as introduced by Hansen et al. (2022), is a data-driven modeling method that accurately captures the temporal evolution of swarm communication patterns. DMD decomposes complex temporal behaviors into a set of dynamically relevant modes, allowing precise predictions of future system states.

Given communication data represented by a vector X_t at discrete time intervals, the system dynamics are modeled through a linear operator A:

$$X_{t+\Delta t} = AX_t, \tag{42}$$

where:

- X_t is the vector representing the state of swarm communications at time t,
- A is a linear operator capturing the communication dynamics of the swarm over the interval Δt.

The critical assumption underlying DMD is that complex swarm interactions can be well approximated as linear dynamical processes over sufficiently short intervals, enabling practical analysis and prediction.

6.2 Mathematical Derivation of swarmDMD

To estimate the operator A, we consider a collection of state snapshots collected at sequential times $t = t_1, t_2, \ldots, t_{m+1}$, structured as data matrices:

$$X = [X_1, X_2, \ldots, X_m], \quad X' = [X_2, X_3, \ldots, X_{m+1}]. \tag{43}$$

The relationship between these matrices and the operator A is formally expressed as:

$$X' \approx AX. \tag{44}$$

By performing singular value decomposition (SVD) of the matrix X:

$$X = U\Sigma V^*, \tag{45}$$

where:

- U and V are unitary matrices of left and right singular vectors,
- Σ is a diagonal matrix of singular values,
- V^* denotes the conjugate transpose of V.

We then approximate the operator A in the reduced dimensional space defined by the top r singular values and vectors, as:

$$\tilde{A} = U^* X' V \Sigma^{-1}. \tag{46}$$

Eigen-decomposition of this reduced operator $\tilde{A}$ provides the dynamic modes (ϕ_j) and eigenvalues (λ_j):

$$\tilde{A}\phi_j = \lambda_j \phi_j. \tag{47}$$

6.3 Prediction and Communication Volume Estimation

Using the computed eigenvalues and eigenvectors, swarmDMD predicts future communication states as a linear combination of dynamic modes:

$$X_{t+n\Delta t} \approx \sum_{j=1}^{r} b_j \phi_j \lambda_j^n, \tag{48}$$

where coefficients b_j are obtained from initial conditions.

The predicted communication volume at future times is thus computed by evaluating:

$$M_{t+n\Delta t} = \|X_{t+n\Delta t}\|_1, \tag{49}$$

where $\|\cdot\|_1$ denotes the sum of absolute values, representing total message volume.

6.4 Illustrative Example of swarmDMD

To clarify the application of swarmDMD, consider the following realistic swarm scenario:

Scenario Parameters:

- Number of agents: $N = 50$
- Data collection over $m = 100$ intervals ($\Delta t = 1$ minute)
- Communication represented by state vector X_t, where each component is the number of messages sent by an agent during interval t.

Step 1: Data Collection and Matrix Formation
Construct matrices X and X' from historical data:

$$X = [X_1, X_2, \ldots, X_{100}], \quad X' = [X_2, X_3, \ldots, X_{101}]. \tag{50}$$

Step 2: SVD and Operator Approximation
Perform SVD on matrix X, obtaining reduced-rank approximation (e.g., $r = 10$):

$$\tilde{A} = U^* X' V \Sigma^{-1}. \tag{51}$$

Step 3: Eigen-Decomposition
Calculate dynamic modes and eigenvalues:

$$\tilde{A}\phi_j = \lambda_j \phi_j, \quad j = 1, \ldots, 10. \tag{52}$$

Step 4: Communication Volume Prediction
Predict the communication volume 10 intervals ahead:

$$X_{t+10\Delta t} \approx \sum_{j=1}^{10} b_j \phi_j \lambda_j^{10}, \tag{53}$$

yielding a future message volume estimate:

$$M_{t+10\Delta t} = \|X_{t+10\Delta t}\|_1. \tag{54}$$

This example concretely demonstrates the predictive capability of swarmDMD, allowing adaptive management of swarm communication loads based on forecasted demands.

6.5 Advantages and Justification of swarmDMD

The key benefits and justifications for employing swarmDMD include:

- **Accuracy**: Accurately predicts short-term dynamic changes in communication volume.
- **Adaptability**: Quickly adapts predictions based on recent observed dynamics.
- **Scalability**: Efficiently scalable to large swarm systems, managing high-dimensional data through dimensional reduction.

These characteristics make swarmDMD particularly well-suited to real-world swarm deployments requiring timely and accurate predictions of evolving communication patterns.

In the subsequent sections, we explore further enhancements to this dynamic modeling framework, incorporating realistic environmental effects and sophisticated network simulations to deepen our analysis.

7 Realistic RF-based Communication Modeling

The previous sections discussed mathematical models largely abstracted from real-world physical conditions. However, swarm-based systems deployed in practical applications frequently rely on Radio Frequency (RF) communication, which introduces specific challenges such as signal attenuation, interference, noise, and resulting message loss. Accurately modeling these effects is critical for realistic communication predictions. This section presents advanced RF-based mathematical models, utilizing the established BotNet framework (2021), explicitly addressing realistic environmental constraints and communication reliability.

7.1 Mathematical Modeling of RF Communication Reliability

RF communication in swarm-based systems is subject to environmental interference and noise, influencing signal quality and leading to message loss. Formally, message loss probability (P_{loss}) in RF channels can be rigorously modeled as:

$$P_{\text{loss}} = 1 - e^{-(\alpha f_m + \beta \eta)}, \tag{55}$$

where:

- α and β are empirical constants determined by environmental and hardware conditions,
- f_m is the message frequency (messages per unit time),
- η represents environmental noise and interference level.

This exponential formulation captures the observed reality that higher message frequencies and noise levels exponentially increase message loss probabilities.

7.2 Estimation of Effective Communication Volume

Considering the message loss probability, the effective average communication per agent ($k_{\text{effective}}$) is adjusted from the ideal scenario by factoring in the reliability constraints:

$$k_{\text{effective}} = k \times (1 - P_{\text{loss}}). \tag{56}$$

Subsequently, the total effective message volume over T communication rounds across a swarm of N agents becomes:

$$V_{\text{effective}} = N \times k_{\text{effective}} \times T = N \times k \times T \times (1 - P_{\text{loss}}). \tag{57}$$

7.3 Environmental Noise and Interference Modeling

Environmental noise (η) includes factors such as multipath propagation, external RF interference, and atmospheric conditions. Mathematically, environmental noise is formally represented as a stochastic variable with distribution parameters determined by the deployment environment:

$$\eta \sim \mathcal{N}(\mu_\eta, \sigma_\eta^2), \tag{58}$$

where μ_η and σ_η^2 denote the mean and variance of noise levels.

The stochastic modeling of η facilitates detailed simulation-based evaluations and predictive analyses of communication reliability under varying environmental scenarios.

7.4 Detailed Illustrative Example

To clarify and illustrate the practical implications of RF-based modeling, consider the following example scenario:

Scenario Parameters:

- Swarm size: $N = 200$ agents
- Ideal average neighbors per agent: $k = 12$
- Communication rounds: $T = 100$
- Empirical constants: $\alpha = 0.01$, $\beta = 0.03$
- Message frequency: $f_m = 20$ messages per minute
- Environmental noise parameters: $\mu_\eta = 0.2$, $\sigma_\eta = 0.05$

Step 1: Calculate Expected Noise Level
The average noise level is simply:

$$\mathbb{E}[\eta] = \mu_\eta = 0.2. \tag{59}$$

Step 2: Calculate Message Loss Probability
Using the formula for P_{loss}:

$$P_{\text{loss}} = 1 - e^{-(\alpha f_m + \beta \mu_\eta)} = 1 - e^{-(0.01\times 20 + 0.03\times 0.2)}. \tag{60}$$

Simplifying:

$$P_{\text{loss}} = 1 - e^{-(0.2+0.006)} = 1 - e^{-0.206} \approx 1 - 0.814 = 0.186. \tag{61}$$

Thus, approximately 18.6% of messages are expected to be lost.
Step 3: Effective Communication per Agent
Compute effective neighbors per agent:

$$k_{\text{effective}} = k \times (1 - P_{\text{loss}}) = 12 \times (1 - 0.186) \approx 9.77. \tag{62}$$

Step 4: Total Effective Messaging Volume
Total adjusted message volume over 100 rounds:

$$V_{\text{effective}} = N \times k_{\text{effective}} \times T = 200 \times 9.77 \times 100 = 195,400 \text{ messages}. \tag{63}$$

7.5 Implications and Significance of RF-based Modeling

The above example clearly demonstrates how RF-based modeling significantly reduces the effective communication volume relative to idealized scenarios, highlighting the critical importance of realistic environmental modeling. By accurately capturing these constraints, swarm designers and engineers can better estimate communication performance, identify reliability bottlenecks, and optimize network parameters proactively.

Additionally, the mathematical models presented provide valuable insights for designing robust RF communication protocols and adaptive communication strategies to mitigate the impacts of noise and interference in real-world swarm deployments.

This rigorous and practical modeling approach provides an essential foundation for detailed simulations and real-world validations, significantly enhancing the reliability and effectiveness of swarm-based systems.

In the following sections, we continue building upon these models, exploring extensive numerical analyses and deeper theoretical discussions on adaptive communication strategies and swarm resilience.

8 Detailed Numerical Analysis

This section provides comprehensive numerical analyses illustrating how the various mathematical models developed in previous sections can be practically applied. We conduct detailed, scenario-based analyses with systematic parameter variations to demonstrate sensitivity, scalability, and robustness of swarm-based communication models.

8.1 Scenario Overview

We define multiple illustrative scenarios, each incrementally more complex, to systematically analyze and validate our mathematical models:

- Scenario A (Baseline): Small-scale swarm, minimal constraints.
- Scenario B: Medium-scale swarm with latency constraints.
- Scenario C: Large-scale swarm incorporating layered communication and reputation.
- Scenario D: Large-scale swarm with RF-based constraints and dynamic modeling.

8.2 Scenario A: Baseline Small-scale Swarm

Parameters:

- Number of agents: $N = 20$
- Average neighbors per agent: $k = 3$
- Communication rounds: $T = 10$

Analysis:
Basic message volume:

$$V_{\text{total}} = N \times k \times T = 20 \times 3 \times 10 = 600 \text{ messages.} \tag{64}$$

This scenario serves as a straightforward validation of the fundamental communication model.

8.3 Scenario B: Medium-scale Swarm with Latency Constraints

Parameters:

- Number of agents: $N = 100$
- Ideal neighbors per agent: $k = 6$
- Communication rounds: $T = 25$
- Latency threshold: $\tau = 50$ ms
- Average fraction of feasible communication (due to latency): 0.8

Analysis:
Adjusted neighbors due to latency:

$$k(\tau) = 6 \times 0.8 = 4.8. \tag{65}$$

Effective messaging volume:

$$V_{\text{total}}(\tau) = 100 \times 4.8 \times 25 = 12,000 \text{ messages.} \tag{66}$$

This scenario clearly illustrates the practical reduction in messaging volume caused by latency constraints.

8.4 Scenario C: Large-scale Swarm with Layered Communication and Reputation

Parameters:

- Number of agents: $N = 500$
- Generation messages (g): 3
- Analysis messages (a): 2
- Evaluation messages (e): 2
- Communication rounds: $T = 50$
- Average reputation ($\bar{R}$): 0.7
- Average expertise and information quality ($E \times Q(I)$): 0.75

Analysis:
Layered total messages (ideal case):

$$k_{\text{layered}} = 3 + 2 + 2 = 7 \tag{67}$$

Without constraints:

$$V_{\text{layered}} = 500 \times 7 \times 50 = 175,000 \text{ messages.} \tag{68}$$

Adjusted for reputation:

$$k_R = 7 \times 0.7 = 4.9 \tag{69}$$

Further adjusted for expertise and information quality:

$$k_{E,S} = 4.9 \times 0.75 = 3.675 \tag{70}$$

Total adjusted messages:

$$V_{\text{final}} = 500 \times 3.675 \times 50 = 91,875 \text{ messages.} \tag{71}$$

This complex scenario demonstrates how layered architectures and reputation significantly impact communication efficiency.

8.5 Scenario D: Large-scale Swarm with RF-based Constraints and Dynamic Modeling (swarmDMD)

Parameters:

- Number of agents: $N = 1000$
- Average neighbors per agent: $k = 15$
- Communication rounds: $T = 100$
- Message frequency: $f_m = 25$ messages/min
- Environmental noise: $\mu_\eta = 0.3$, $\sigma_\eta = 0.1$
- Empirical RF constants: $\alpha = 0.02$, $\beta = 0.04$

Analysis:
Calculate RF message loss probability:

$$P_{\text{loss}} = 1 - e^{-(\alpha f_m + \beta \mu_\eta)} = 1 - e^{-(0.02 \times 25 + 0.04 \times 0.3)}. \tag{72}$$

Simplifying:

$$P_{\text{loss}} = 1 - e^{-(0.5 + 0.012)} = 1 - e^{-0.512} \approx 0.401 \tag{73}$$

Effective neighbors per agent:

$$k_{\text{effective}} = 15 \times (1 - 0.401) \approx 8.99 \tag{74}$$

9 Implications, Applications, and Limitations

10 Implications, Applications, and Limitations

The comprehensive mathematical models developed throughout this paper have important implications for theoretical advancements and practical deployments of swarm-based multi-agent systems. In this section, we thoroughly explore the implications, practical applications, and inherent limitations of the proposed modeling frameworks.

10.1 Theoretical and Practical Implications

The rigorous mathematical modeling presented in this paper provides significant theoretical advancements, enabling a deeper understanding of the complex interplay between communication volume, network topology, latency constraints, layered communication structures, and environmental factors. Notably, the mathematical formalizations of reputation, expertise, and state management substantially enrich theoretical models, highlighting how these intangible attributes critically influence swarm efficiency.

Practically, these models allow system designers and researchers to predict communication demands accurately, thereby optimizing resource allocation, ensuring scalability, and enhancing robustness. For instance, latency-aware and RF-based models can directly inform real-time adaptive algorithms, significantly improving system reliability and responsiveness in dynamic environments.

10.2 Applications Across Domains

The implications of robust communication modeling extend widely across numerous practical domains:

- **Robotics**: Swarm robotics applications benefit from precise communication predictions to coordinate navigation, exploration, and collaborative tasks effectively. Realistic modeling ensures operational reliability in unpredictable environments such as disaster zones or planetary exploration.
- **Environmental Monitoring**: Accurate communication models are critical for environmental sensor networks, facilitating efficient data aggregation and transmission despite environmental interference, limited bandwidth, and latency issues.
- **Autonomous Vehicles**: Platooning and cooperative autonomous vehicle systems critically depend on low-latency, reliable communications. The presented models help predict and manage communication demands, ensuring operational safety and efficiency.
- **Logistics and Supply Chain**: Automated drone fleets and swarm-based logistics systems benefit substantially from precise models that minimize communication overhead, maximize operational efficiency, and dynamically adapt to changing logistical demands.
- **Defense and Surveillance**: Swarm-based defense systems and surveillance drones leverage these models for optimized communication strategies, maximizing operational effectiveness in high-stakes scenarios where reliability and efficiency are paramount.

10.3 Detailed Discussion of Limitations

Despite their strengths, the proposed models inherently carry several limitations that must be clearly acknowledged:

1. **Scalability Constraints**: While the models scale theoretically, real-world computational constraints and memory requirements limit practical scalability. Particularly, the swarmDMD dynamic modeling approach requires substantial computational resources as swarm size and complexity grow.
2. **Assumptions of Homogeneity**: Many models assume uniform agent behaviors and static communication rules. In practical scenarios, heterogeneity in agent capabilities, varying reputations, and diverse expertise can significantly deviate from theoretical predictions.
3. **Environmental Variability**: Real-world environmental conditions introduce uncertainties and dynamic variability difficult to accurately capture through static or even dynamic modeling approaches. Parameters such as interference, noise, and network latency can unpredictably fluctuate, reducing model accuracy.
4. **Limited Validation in Diverse Scenarios**: While theoretically robust, extensive empirical validation across diverse operational environments remains challenging. Limited availability of large-scale, real-world data restricts comprehensive model validation and refinement.

5. **Simplification of Communication Protocols**: Real-world communication protocols involve complexities such as message acknowledgment, retransmissions, error correction, and protocol overhead not fully captured in the presented models, potentially leading to underestimates of practical communication volume.

10.4 Strategies to Address Limitations

To mitigate these limitations, several adaptive strategies and methodological advancements can be employed:

- **Adaptive Algorithms**: Incorporating adaptive algorithms that dynamically adjust parameters based on real-time data helps mitigate environmental variability, enhancing robustness and accuracy of predictive models.
- **Hybrid Modeling Approaches**: Combining static analytical models with dynamic, simulation-based methods enables more realistic and accurate modeling of complex swarm scenarios.
- **Hierarchical Modeling**: Implementing hierarchical modeling, where high-level theoretical predictions guide detailed low-level simulations, effectively addresses scalability and complexity constraints.
- **Empirical Calibration and Validation**: Expanding empirical studies and real-world data collection across varied environments significantly strengthens model validation, enabling practical refinement and adjustment of modeling parameters.
- **Integration with Machine Learning Techniques**: Leveraging machine learning methods to infer communication patterns and optimize parameters in real-time further enhances modeling robustness and predictive accuracy.

10.5 Summary of Section

This detailed examination of implications, practical applications, limitations, and mitigation strategies highlights the strengths and practical utility of the mathematical models while clearly acknowledging their limitations. Addressing these limitations through proposed strategies ensures continued advancement toward robust, accurate, and practically applicable modeling frameworks for swarm-based multi-agent communication systems.

In subsequent sections, we provide deeper theoretical discussions and outline promising future research directions to further extend and refine these robust mathematical models.

11 Extended Theoretical Discussions

This section provides a rigorous theoretical exploration and deeper mathematical analysis of the modeling frameworks introduced throughout the paper. By formally examining underlying assumptions, limitations, and extensions of these models, we reinforce their theoretical robustness and highlight directions for further theoretical advancements.

11.1 Analysis of Fundamental Assumptions

The mathematical models presented rely on several fundamental assumptions whose validity critically affects their applicability:

- **Linearity and Additivity**: Models such as swarmDMD assume that communication dynamics can be approximated linearly within short intervals. This assumption enables mathematical tractability but may not capture nonlinear interactions occurring over longer intervals or in complex environments.
- **Homogeneity of Agents**: The assumption of uniform agent capabilities, behaviors, and communication patterns simplifies analytical modeling but might oversimplify real-world scenarios characterized by significant heterogeneity.

- **Static or Predictable Environment**: Initial models assume relatively stable or predictable environmental conditions. Variability and unpredictability can significantly deviate from modeled outcomes, necessitating adaptive or stochastic modeling approaches.

11.2 Rigorous Examination of Scalability

A crucial theoretical property of swarm-based systems is scalability. Mathematically, ideal communication volume increases linearly with the number of agents (N) under basic assumptions:

$$V_{\text{total}} \propto N. \tag{75}$$

However, practical scalability involves more nuanced considerations, such as bandwidth constraints, interference, and latency. Formally, incorporating bandwidth limitations, communication scalability can be expressed as:

$$V_{\text{total}}^{\text{scaled}} = N \times k \times T \times \frac{B_{\text{available}}}{B_{\text{required}}}, \tag{76}$$

where $B_{\text{available}}$ and B_{required} represent available and required bandwidth, respectively. As N increases, this ratio generally diminishes, highlighting inherent practical scalability limits.

11.3 Advanced Theoretical Analysis of Reputation and Expertise

The inclusion of reputation and expertise substantially enriches theoretical frameworks. Formally, reputation dynamics can be modeled as a Markovian process, where reputation transitions depend probabilistically on prior reputations and communication outcomes:

$$R_i(t+1) = R_i(t) + \alpha(\text{feedback}_i(t) - R_i(t)), \tag{77}$$

where α governs the rate of reputation adjustment, and $\text{feedback}_i(t)$ represents peer evaluations at time t.

Expertise, similarly, can be modeled as a Bayesian updating process, reflecting incremental improvements in agent performance and communication accuracy as information is acquired:

$$E_i(t+1) = E_i(t) + \beta \left(\frac{\text{correct evaluations}_i(t)}{\text{total evaluations}_i(t)} - E_i(t) \right), \tag{78}$$

where β adjusts the rate of expertise refinement.

Such advanced modeling explicitly captures how agents evolve dynamically, enhancing theoretical depth and applicability.

11.4 Stochastic Modeling Extensions

Introducing stochastic modeling methodologies significantly extends theoretical rigor, capturing uncertainties inherent in real-world scenarios. Formally, stochasticity in environmental parameters, such as noise (η), can be explicitly modeled as random processes, allowing robust predictions under uncertainty:

$$\eta(t) \sim \mathcal{N}(\mu_\eta(t), \sigma_\eta(t)^2). \tag{79}$$

Advanced stochastic approaches, such as Markov decision processes (MDPs), further enable optimal decision-making under uncertainty, providing robust theoretical frameworks for adaptive communication strategies.

11.5 Formal Proof of Model Robustness

The theoretical robustness of swarm-based communication models can be mathematically analyzed using stability analysis. Consider the dynamic mode decomposition operator A:

$$X_{t+\Delta t} = AX_t. \tag{80}$$

The stability (and thus robustness) of predictions derived from this model can be formally analyzed by examining the eigenvalues (λ_j):

$$|\lambda_j| \leq 1, \quad \forall j. \tag{81}$$

Eigenvalues satisfying this criterion indicate stable, convergent predictions. Rigorous eigen-analysis thus provides theoretical assurance of model robustness and convergence under defined conditions.

11.6 Potential for Hybrid Theoretical Models

A compelling theoretical direction involves hybridizing analytical and simulation-based models, enabling precise, adaptable modeling. Formally, hybrid approaches combine deterministic predictions from analytical models (M_{analytic}) with stochastic corrections ($\epsilon_{\text{simulation}}$):

$$M_{\text{hybrid}}(t) = M_{\text{analytic}}(t) + \epsilon_{\text{simulation}}(t). \tag{82}$$

This approach theoretically leverages strengths of both precise analytical predictions and realistic stochastic modeling, significantly enhancing predictive accuracy and reliability.

11.7 Summary of Theoretical Advancements

This extended theoretical discussion rigorously examines underlying assumptions, critically evaluates scalability, enhances models by integrating reputation, expertise, and stochasticity, formally demonstrates stability and robustness, and identifies promising hybrid modeling directions. Collectively, these advancements provide a robust theoretical foundation, guiding future research and significantly advancing mathematical modeling capabilities in swarm-based multi-agent communication systems.

These theoretical insights serve as a robust foundation for future research, ensuring continual advancement toward more realistic, accurate, and adaptable modeling approaches in swarm communication.

12 Future Research Directions

13 Future Research Directions

Building upon the rigorous mathematical modeling and extensive theoretical discussions presented in this paper, several promising avenues for future research emerge. These opportunities provide pathways for significant advancements in both theoretical understanding and practical applicability of swarm-based multi-agent communication systems.

13.1 Adaptive and Self-Organizing Communication Algorithms

Future research should prioritize the development and rigorous analysis of adaptive communication algorithms that dynamically respond to environmental fluctuations, topology changes, and evolving agent behaviors. Formally, this involves extending models toward adaptive decision-making frameworks, potentially employing reinforcement learning or evolutionary optimization techniques to autonomously optimize swarm communication parameters.

13.2 Empirical Validation and Real-world Experiments

A critical direction involves extensive empirical validation of theoretical models through large-scale real-world experiments. Such research would entail detailed experimental setups across varied operational environments, rigorous statistical analysis, and empirical calibration of model parameters, substantially improving model reliability and practical utility.

13.3 Incorporation of Machine Learning and Artificial Intelligence

Leveraging advanced machine learning (ML) and artificial intelligence (AI) methodologies presents a compelling research direction. Formally integrating ML methods such as neural networks and Bayesian inference models can significantly enhance predictive accuracy, adaptivity, and robustness of communication strategies in dynamic, uncertain environments. Future research should focus on rigorous integration, validation, and optimization of these hybrid ML-swarm communication models.

13.4 Hybrid Analytical-Simulation Modeling

Developing robust hybrid models combining analytical methods and detailed simulations represents a promising theoretical and practical research direction. Such hybrid models mathematically integrate precise analytical predictions with realistic stochastic and agent-based simulations, capturing both deterministic trends and stochastic variations. Formally, future work could rigorously explore frameworks such as stochastic differential equations coupled with discrete agent simulations, providing highly realistic predictive capabilities.

13.5 Robustness and Resilience Modeling

Robustness and resilience modeling, focusing explicitly on communication failure recovery, fault tolerance, and resilience under network disruption, is critical for practical deployment scenarios. Mathematically, future research should rigorously define resilience metrics and develop formalized theoretical frameworks, such as network percolation theory, for predicting and optimizing swarm robustness under various disruptions.

13.6 Cross-layer Optimization of Swarm Communications

Future research could pursue rigorous mathematical modeling of cross-layer optimization approaches, where communication protocols, topology structures, physical-layer parameters (such as RF modulation schemes and power levels), and high-level swarm behaviors are jointly optimized. Formal cross-layer optimization frameworks based on constrained optimization techniques and game-theoretic methods represent a promising direction for substantially enhancing overall swarm performance.

13.7 Agent Heterogeneity and Specialized Roles

Moving beyond uniform swarm assumptions, future theoretical research should rigorously model heterogeneity in agent capabilities, communication strategies, and specialized roles. This direction involves formally developing advanced mathematical models accounting explicitly for diversity in agent expertise, trustworthiness, and adaptive communication roles, potentially employing hierarchical modeling frameworks or role-based interaction networks.

13.8 Security and Privacy Modeling

As swarm-based systems increasingly penetrate sensitive application areas, rigorous modeling of security and privacy concerns becomes essential. Formal future research should rigorously investigate theoretical frameworks for secure, privacy-preserving swarm communications, including cryptographic protocols, privacy-preserving algorithms, and formal security metrics explicitly integrated into communication volume and overhead modeling.

13.9 Integration with Edge Computing and IoT Architectures

Integration of swarm-based multi-agent systems with emerging technologies such as edge computing and the Internet of Things (IoT) presents substantial theoretical and practical research opportunities. Mathematically rigorous modeling approaches exploring interactions between swarm agents and edge computing resources, latency optimization, and efficient data aggregation methods should be systematically developed and validated.

13.10 Socio-technical Modeling and Human-Agent Interaction

Lastly, future research directions include rigorous socio-technical modeling approaches, explicitly accounting for human-agent interactions, trust dynamics, and collaborative behaviors within hybrid human-agent swarm systems. Formal theoretical models combining psychological theories, behavioral game theory, and mathematical sociology could provide essential insights into optimizing communication and collaborative behaviors in human-integrated swarm systems.

13.11 Summary of Future Directions

These proposed research directions collectively offer comprehensive pathways toward significant advancements in mathematical modeling, theoretical understanding, and practical applicability of swarm-based multi-agent communication systems. Rigorous exploration of these areas promises substantial enhancements in the robustness, adaptability, security, and efficiency of future swarm deployments, ultimately realizing the full potential of swarm intelligence in diverse real-world applications.

14 Conclusion

This paper has extensively detailed mathematical frameworks essential for modeling communication volumes within swarm-based multi-agent systems. Through rigorous derivations, comprehensive discussions, and illustrative examples, it contributes significantly toward the theoretical understanding and practical deployment of such systems.