Introduction to Computational Science
University of Amsterdam
September 2023

Assignment 1: SIR model (ODE)

Due Monday October 2nd 2023 @ 23:59

Introduction

The SIR model is an epidemiological model that computes the spread of an infectious disease through a population of people. It is used to compute the fraction of susceptible (S), infected (I), and recovered (R) individuals at any given time through the spread of an infectious disease. The model can be written as a set of first order differential equations:

$$\frac{dS}{dt} = -\beta SI \tag{1}$$

$$\frac{dI}{dt} = \beta SI - \gamma I \tag{2}$$

$$\frac{dR}{dt} = \gamma I \tag{3}$$

Here β is the parameter for the rate of infection and γ is the parameter for the rate of recovery. A first assumption can be made keeping the population in our model "closed" meaning $S + I + R = 1$.

Problem 1: Numerical integration of the SIR model

In the basic model we can assume the population is fixed $S + I + R = 1$. The spread of infection goes in one direction from susceptible to infected to recovered. Once an individual is recovered he or she can no longer become susceptible.

1. Numerically integrate the above ODEs to show two scenarios, one in which there is an epidemic and one in which there is no epidemic. What is the parameter that determines if the infectious disease will become an epidemic? Provide phase space diagrams for both scenarios and use these phase space diagrams to analyse the system.

2. Given below is the historical data from a case of an influenza outbreak situation. The outbreak was in a boys school with a total of 763 boys. We can assume that one infected boy started the epidemic (day 0). Try to fit an SIR model to the data and estimate the parameters of the model.

Day	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Number	1	3	8	28	75	221	291	255	235	190	125	70	28	12	5

3. Think about and design potential vaccination strategies (at least two) for the school case. These plans can be as simple or as elaborate as you like. Form a hypothesis and design an experiment to evaluate and compare these potential vaccination plans.

Problem 2: Demography

Now we want to start to add more terms to our model. Lets start to consider demography, i.e. Birth and death rates.

1. Add birth and natural death rate terms into model. (Hint: you can keep $N_{population}$ constant in this case). How does the dynamics change? Run the model to capture an endemic state. Do you see oscillatory behavior in the fraction of infected population? What are requirements to observe this behavior? What is the frequency and amplitude of these oscillations (Hint: use Fourier analysis)?

2. Add an infection induced mortality term to the model (in this case assume $N_{population} \neq$ constant), which will involve a probability of dying while infected. Explain what changes in the model behaviour you observe as this probability approaches 1.

Problem 3: Variants of the SIR model

In this problem we will investigate variants of the SIR model. you are free to pick any variant of the SIR model. Some example variants you may consider are SIER, MSIR, SEIS, MSEIR, MSEIRS etc.

1. Numerically solve this model and analyse the behaviour of the model under different parameter conditions. You can do this using phase space plots, or other plots.

2. Add seasonal effects into the model (Hint: add a time varying sinusoidal rate of infection) show how the behaviour of the model changes with different levels of seasonal forcing.