02.03.2024 (суббота)

7.049. $\left(81^{\frac{1}{4}-\frac{1}{2}\log_9 4}+25^{\log_{125}8}\right)\cdot 49^{\log_7 2}$. $=\left(\frac{3}{4}+4\right)\cdot 4=$

$=\frac{3}{\cancel{4}}\cdot\cancel{4}+4\cdot 4=3+16=19$

$$81^{\frac{1}{4} - \frac{1}{2}\log_9 4} = 81^{\frac{1}{4}} \cdot 81^{-\frac{1}{2}\log_9 4} =$$

$$= \sqrt[4]{81} \cdot 9^{\cancel{2}\cdot\left(-\frac{1}{\cancel{2}}\right)\log_9 4} = 3 \cdot 9^{\log_9 \frac{1}{4}} =$$

$$= 3 \cdot \frac{1}{4} = \frac{3}{4}$$

$$\sqrt[m]{a^n} = a^{\frac{n}{m}}$$

$$\sqrt[2]{a^1} = a^{\frac{1}{2}}$$

$$4^{-1} = \frac{1}{4}$$

$\log_{5^3} 2^3 = \log_5 2$

$25^{\log_{125} 8}$

5^2

$5^{2\log_5 2} = 5^{\log_5 4} = 4$

$a^{\log_a b} = b$

$49^{\log_7 2} = 7^{2\log_7 2} = 7^{\log_7 4} = 4$

Свойство	Пример
$\log_a b^m = m \log_a b$	$\log_2 3^5 = 5 \log_2 3$
$\log_{a^n} b = \frac{1}{n} \log_a b$	$\log_{2^3} 5 = \frac{1}{3} \log_2 5$
$\log_{a^n} b^m = \frac{m}{n} \log_a b$	$\log_{2^3} 5^7 = \frac{7}{3} \log_2 5$
$\log_a b + \log_a c = \log_a(b*c)$	$\log_2 3 + \log_2 5 = \log_2(3*5)$
$\log_a b - \log_a c = \log_a \frac{b}{c}$	$\log_2 3 - \log_2 5 = \log_2 \frac{3}{5}$
$\log_a a = 1$	$\log_2 2 = 1$
$\log_a 1 = 0$	$\log_2 1 = 0$
$\log_a b = \frac{\log_c b}{\log_c a}$	$\log_2 5 = \frac{\log_3 5}{\log_3 2}$
$\log_a b = \frac{1}{\log_b a}$	$\log_2 3 = \frac{1}{\log_3 2}$
$a^{\log_c b} = b^{\log_c a}$	$2^{\log_5 3} = 3^{\log_5 2}$

7.050. $\dfrac{81^{\frac{1}{\log_5 9}}+3^{\frac{3}{\log_{\sqrt{6}} 3}}}{409}\left((\sqrt{7})^{\frac{2}{\log_{25} 7}}-125^{\log_{25} 6}\right).$

$$\log_a b = \frac{1}{\log_b a}$$

$$81^{\frac{1}{\log_5 9}} = 81^{\log_9 5} = 9^{\log_9 25} = 25$$

$$3^{\frac{3}{\log_{\sqrt{6}} 3}} = 3^{3\cdot\frac{1}{\log_{\sqrt{6}} 3}} = 3^{3\cdot\log_3\sqrt{6}} = 3^{\log_3\sqrt{6}^3} = \sqrt{6}^3$$

$$\sqrt{6}^3 = \sqrt{6}\cdot\sqrt{6}\cdot\sqrt{6} = 6\sqrt{6} = 6^1\cdot 6^{\frac{1}{2}} = 6^{\frac{3}{2}}$$

$$\sqrt{7}^{\frac{2}{\log_{25} 7}} = \sqrt{7}^{2\cdot\frac{1}{\log_{25} 7}} = \sqrt{7}^{2\cdot\log_7 25} = 7^{\log_7 25} = 25$$

$$\log_{a^n} b^m = \frac{m}{n}\log_a b$$

$$125^{\log_{25} 6} = 5^{3\log_{5^2} 6} = 5^{\log_{5^2} 6^3} = 5^{\frac{3}{2}\log_5 6} =$$

$$= 5^{\log_5 6^{\frac{3}{2}}} = 6^{\frac{3}{2}}$$

$$\frac{25+6^{\frac{3}{2}}}{409}\cdot\left(25-6^{\frac{3}{2}}\right)=\frac{\left(25+6^{\frac{3}{2}}\right)\left(25-6^{\frac{3}{2}}\right)}{409}=$$

$$=\frac{25^2-\left(6^{\frac{3}{2}}\right)^2}{409}=\frac{625-216}{409}=\frac{409}{409}=1$$

7.043. $\dfrac{\left(27^{\frac{1}{\log_2 3}}+5^{\log_{25}49}\right)\left(81^{\frac{1}{\log_4 9}}-8^{\log_4 9}\right)}{3+5^{\frac{1}{\log_{16}25}}\cdot 5^{\log_5 3}}. = \dfrac{(8+7)(16-27)}{3+4\cdot 3} = \dfrac{\cancel{15}\cdot(-11)}{\cancel{15}} = -11$

1) $27^{\frac{1}{\log_2 3}} = 3^{\log_3 2^3} = 8$

2) $5^{\log_{25}49} = 5^{\log_{5^2}7^2} = 5^{\log_5 7} = 7$

3) $81^{\frac{1}{\log_4 9}} = 9^{\log_9 4^2} = 16$

4) $8^{\log_4 9} = 2^{\log_2 3^3} = 27$

$5^{\log_5 4} = 4$; $5^{\log_5 3} = 3$

7.013. $\left(\left(\sqrt[5]{27}\right)^{\frac{x}{4}-\sqrt{\frac{x}{3}}}\right)^{\frac{x}{4}+\sqrt{\frac{x}{3}}}=\sqrt[4]{3^7}.$

$x \geq 0$

$(a^n)^m = a^{n \cdot m}$

$$\sqrt[5]{27}^{\left(\frac{x}{4}-\sqrt{\frac{x}{3}}\right)\left(\frac{x}{4}+\sqrt{\frac{x}{3}}\right)} = \sqrt[4]{3^7}$$

$\sqrt[4]{3^7} = 3^{\frac{7}{4}}$

$\sqrt[m]{a^n} = a^{\frac{n}{m}}$

$$\sqrt[5]{27}^{\frac{x^2}{16}-\frac{x}{3}} = \sqrt[4]{3^7}$$

$$\sqrt[5]{3^3}^{\frac{x^2}{16}-\frac{x}{3}} = \sqrt[4]{3^7}$$

$$3^{\frac{3}{5}\left(\frac{x^2}{16}-\frac{x}{3}\right)} = 3^{\frac{7}{4}}$$

$$\frac{3}{5}\left(\frac{x^2}{16}-\frac{x}{3}\right)=\frac{7}{4}$$

$$\frac{3x^2}{5\cdot 16}-\frac{\cancel{3}x}{5\cdot\cancel{3}}-\frac{7}{4}=0$$

$$\frac{3x^2}{80}-\frac{x^{\backslash 16}}{5}-\frac{7^{\backslash 20}}{4}=0$$

$$\frac{3x^2-16x-140}{80}=0 \quad \Big|\cdot 80$$

$$3x^2-16x-140=0$$

$$D=256+1680=1936=44^2$$

$$x=\frac{16+44}{6}=10$$

$$x=\frac{16-44}{6}=-\frac{28}{6}=-\frac{14}{3}$$

так как x находится под корнем, то он не может быть отрицательным

Ответ: 10

$\log_a b$; $\lg x = \log_{10} x$; $\ln x = \log_e x$ $\quad \log_a b = \log_a c$

$e \simeq 2,72$ $\quad b = c$

7.064. $\lg(\sqrt{6+x}+6)=\dfrac{2}{\log_{\sqrt{x}} 10}$.

$x > 0$

$\lg(\sqrt{6+x}+6) = 2\cdot\dfrac{1}{\log_{\sqrt{x}} 10}$

$\lg(\sqrt{6+x}+6) = 2\log_{10}\sqrt{x}$

$\lg(\sqrt{6+x}+6) = \lg\sqrt{x}^2$

$\sqrt{6+x}+6 = \sqrt{x}^2$

$\sqrt{6+x}+6 = x$

$\sqrt{6+x} = x-6 \mid \uparrow 2$

$6+x = (x-6)^2$

$6+x = x^2-12x+36$

$x^2-12x+36-6-x=0$

$x^2-13x+30=0$

$D = 169-120 = 49 = 7^2$

$x = \dfrac{13+7}{2} = 10$

$x = \dfrac{13-7}{2} = 3$

7.066. $\lg(5-x)-\frac{1}{3}\lg(35-x^3)=0.$

$$\lg(5-x)=\frac{1}{3}\lg(35-x^3)$$

$$\lg(5-x)=\lg(35-x^3)^{\frac{1}{3}}$$

$$5-x=(35-x^3)^{\frac{1}{3}} \quad \Big| \uparrow 3$$

$$(5-x)^3=35-x^3 \qquad (a-b)^3=a^3-3a^2b+3ab^2-b^3$$

$$125-75x+15x^2-\cancel{x^3}-35+\cancel{x^3}=0$$

$$15x^2-75x+90=0 \quad \Big| :15$$

$$x^2-5x+6=0$$

$$\Delta=25-24=1$$

$$x=3;\ x=2$$

7.071. $\underbrace{\lg 5+\lg(x+10)}_{\log_a b+\log_a c=\log_a(b\cdot c)}=\underbrace{1}_{\log_a a=1}-\lg(2x-1)+\lg(21x-20)$.

$$\lg(5\cdot(x+10))=\underbrace{\lg 10-\lg(2x-1)}_{\log_a b-\log_a c=\log_a\frac{b}{c}}+\lg(21x-20)$$

$$\lg(5\cdot(x+10))=\underbrace{\lg\left(\frac{10}{2x-1}\right)+\lg(21x-20)}_{\log_a b+\log_a c=\log_a(b\cdot c)}$$

$$\lg(5\cdot(x+10))=\lg\left(\frac{10}{2x-1}\cdot(21x-20)\right)$$

$$5\cdot(x+10)=\frac{10}{2x-1}\cdot(21x-20)$$

$$\frac{5x+50}{1}=\frac{10(21x-20)}{2x-1}$$

$$\frac{5x+50}{1} = \frac{10(21x-20)}{2x-1}$$

$$(5x+50)(2x-1) = 10(21x-20)\cdot 1$$

$$10x^2 - 5x + 100x - 50 = 210x - 200$$

$$10x^2 + 95x - 50 - 210x + 200 = 0$$

$$10x^2 - 115x + 150 = 0 \quad | :5$$

$$2x^2 - 23x + 30 = 0$$

$$D = 529 - 4 \cdot 2 \cdot 30 = 289 = 17^2$$

$$x = \frac{23+17}{4} = 10$$

$$x = \frac{23-17}{4} = 1{,}5$$

7.073. $\log_5\sqrt{x-9}-\log_5 10+\log_5\sqrt{2x-1}=0.$

$x \geq 0$

$\log_5\sqrt{x-9}+\log_5\sqrt{2x-1}=\log_5 10$

$\log_5\left(\left(\sqrt{x-9}\right)\left(\sqrt{2x-1}\right)\right)=\log_5 10$

$\left(\sqrt{x-9}\right)\left(\sqrt{2x-1}\right)=10 \quad |\uparrow^2$

$(x-9)(2x-1)=100$

$2x^2-x-18x+9-100=0$

$2x^2-19x-91=0$

$D=361-4\cdot 2\cdot(-91)=1089=33^2$

$x=\frac{19+33}{4}=\frac{52}{4}=13$

$x=\frac{19-33}{4}=\frac{-14}{4}=-\frac{7}{2}=-3{,}5$