19.12.2023 (вторник)

Решите неравенство:

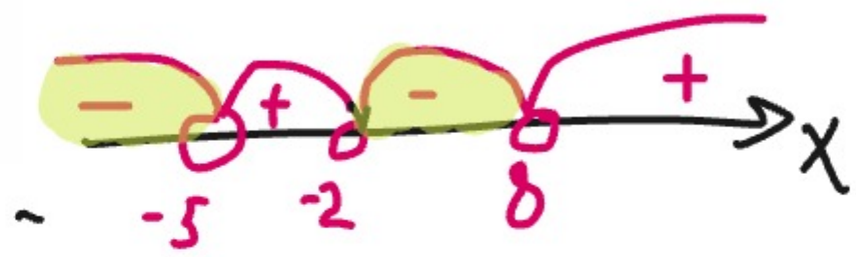
1) $(x+2)(x-8)(x+5)<0$;

2) $(x+5)^2(x-6)(8-x)\geqslant 0$;

3) $\dfrac{x}{x-3}+\dfrac{2}{x}-\dfrac{2}{x^2-3x}\leqslant 0$.

4) $(x^2-36)\sqrt{x+4}\geqslant 0$.

1) $(x+2)(x-8)(x+5)<0$



$(x+2)\cdot(x-8)\cdot(x+5)=0$

$x=-2$

$x=8$

$x=-5$

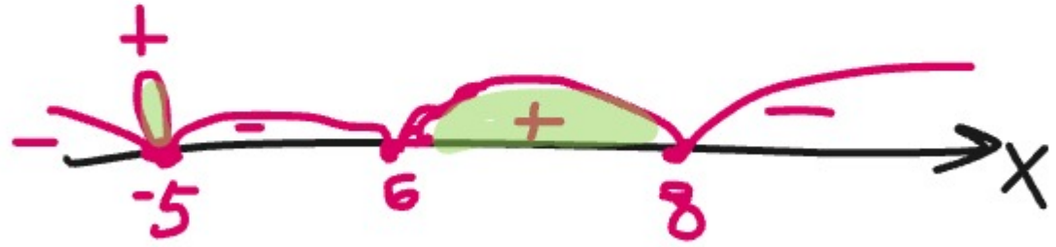
$x\in(-\infty;-5)\cup(-2;8)$

2) Так как:

$(x+5)^2=(x+5)(x+5)$

$\Downarrow$

$(x+5)(x+5)(x-6)(8-x)\geq 0$



$x=-5$

$x=6$

$x=8$

$x\in\{5\}\cup[6;8]$

3) $\frac{x}{x-3} + \frac{2}{x} - \frac{2}{x^2-3x} \leq 0$

$x-3 \neq 0; x \neq 3$

$x \neq 0$

$x^2 - 3x \neq 0, x(x-3) \neq 0; \; x \neq 0; x \neq 3$

$\left.\begin{matrix} x \neq 0 \\ x \neq 3 \end{matrix}\right\}$ ОДЗ

$\frac{x^{\cdot x}}{x-3} + \frac{2^{\cdot (x-3)}}{x} - \frac{2}{x(x-3)} \leq 0$

$\frac{x^2+2x-6-2}{x(x-3)} \leq 0; \quad \frac{x^2+2x-8}{x(x-3)} \leq 0$

$$\frac{x^2+2x-8}{x(x-3)} \leq 0$$

$$x^2+2x-8=0$$

$$\Delta = b^2-4ac = 36 = 6^2$$

$$x = \frac{-b \pm \sqrt{\Delta}}{2a}$$

$$x = \frac{-2+6}{2} = \frac{4}{2} = 2$$

$$x = \frac{-2-6}{2} = \frac{-8}{2} = -4$$

	−	+	−
x	−	+	+
$x-3$	−	−	+
$\frac{x^2+2x-8}{x(x-3)}$	+	−	+

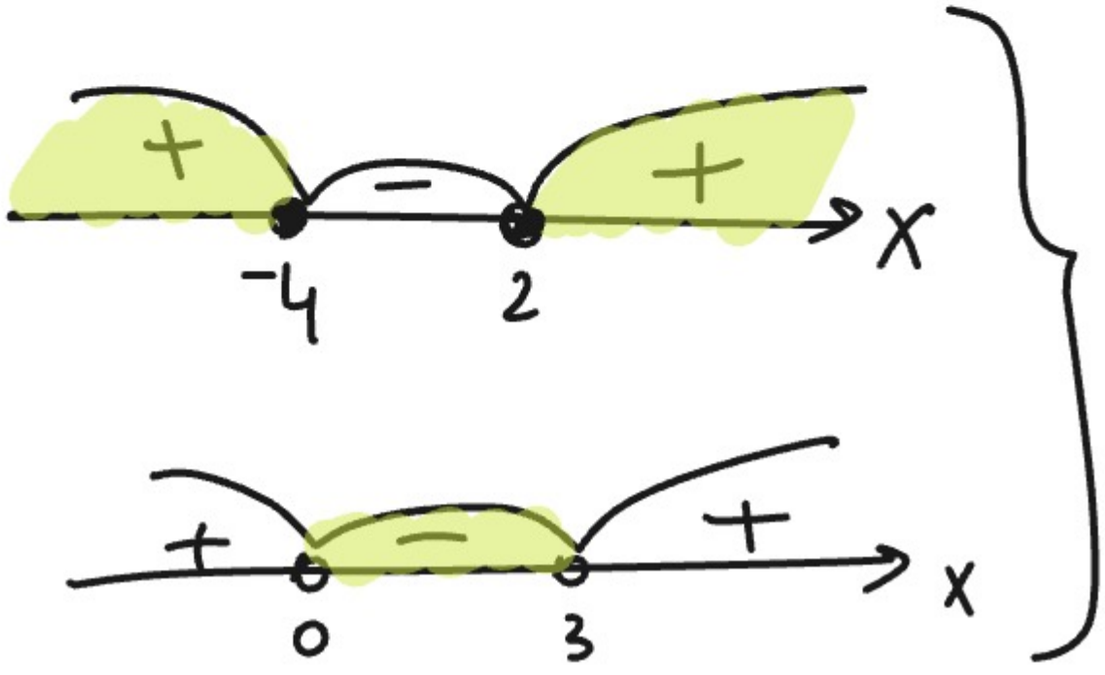
$$\begin{cases} x^2+2x-8 \geq 0 \\ x(x-3) < 0 \end{cases}$$

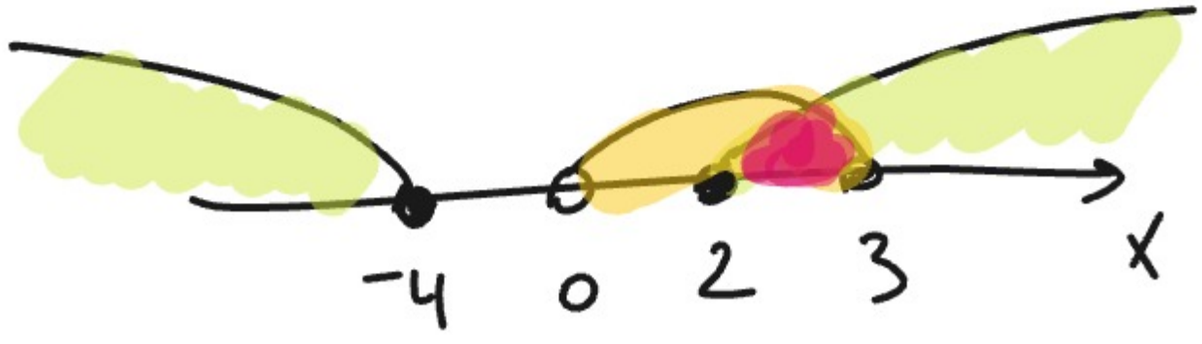
$$\begin{cases} x^2+2x-8 \leq 0 \\ x(x-3) > 0 \end{cases}$$

$$\begin{cases} x^2+2x-8 \geq 0 \\ x(x-3) < 0 \end{cases}$$

$$\begin{cases} (x-2)(x+4) \geq 0 \\ x(x-3) < 0 \end{cases}$$

$x \neq 0$

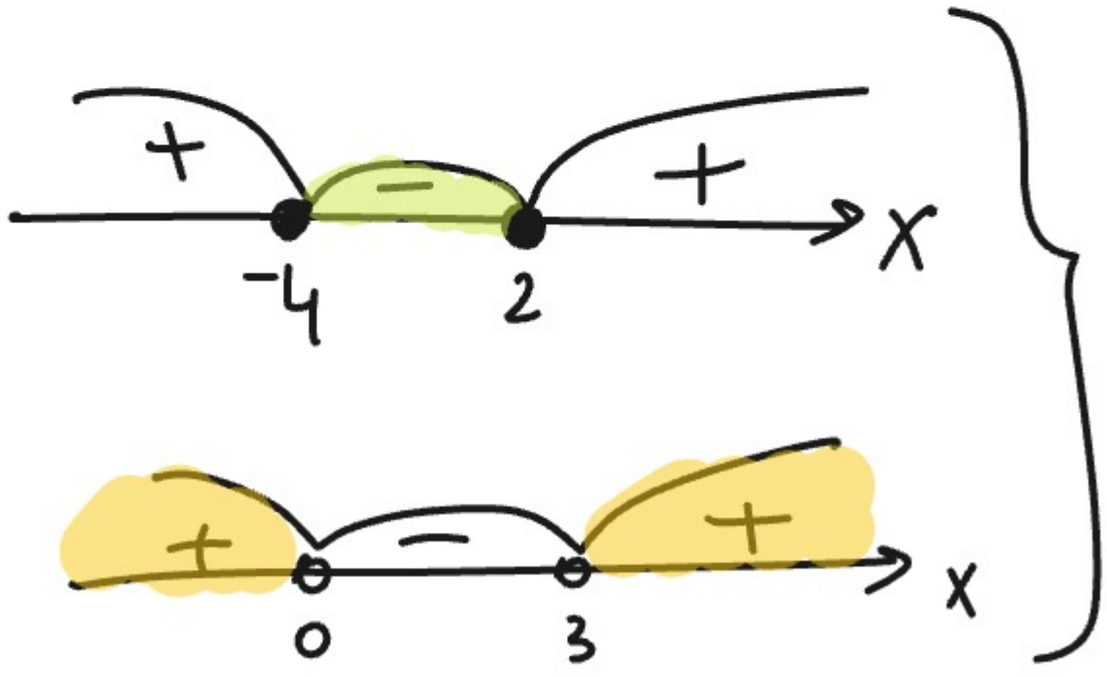


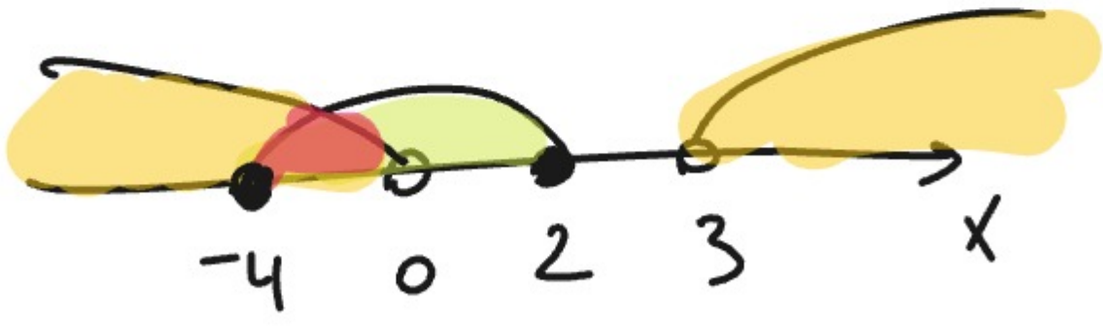


$x \in [2;3)$

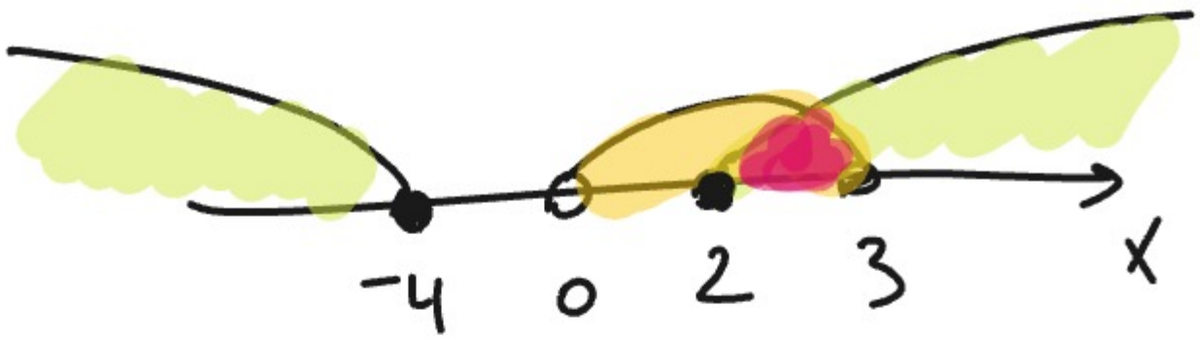
$$\begin{cases} x^2+2x-8 \le 0 \\ x(x-3) > 0 \end{cases}$$

$$\begin{cases} (x-2)(x+4) \le 0 \\ x(x-3) > 0 \end{cases}$$

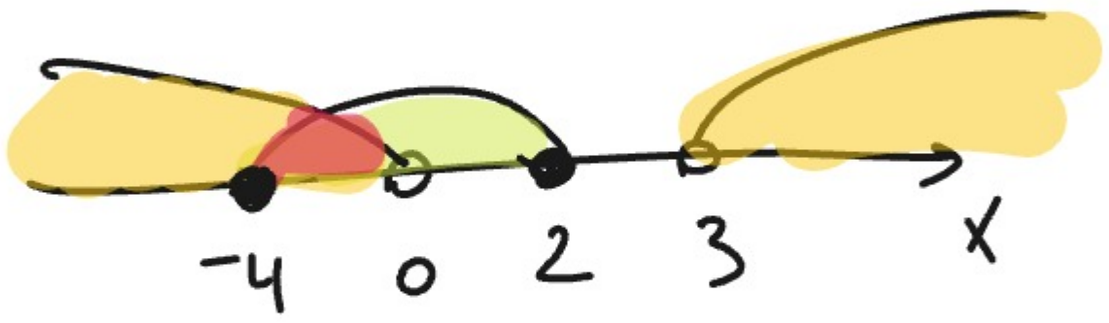




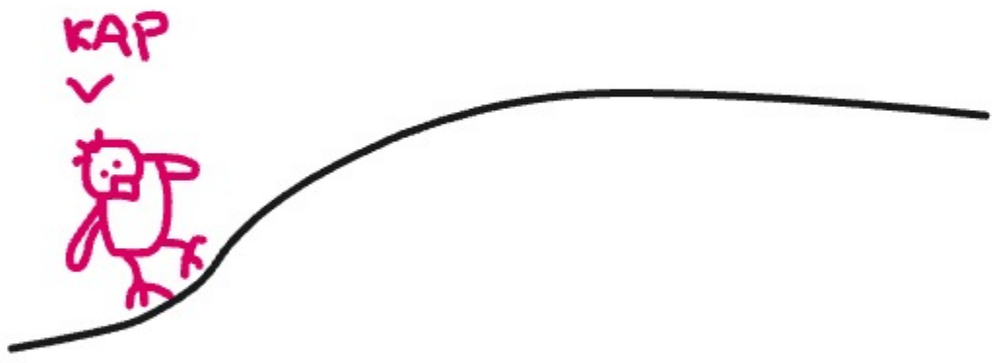
$x \in [-4;0)$



$x \in [2;3)$



$x \in [-4;0)$



$$x \in [-4;0) \cup [2;3)$$

$$\sqrt{2x+3} - \sqrt{4-x} = 2$$

$$\left(\sqrt{2x+3} - \sqrt{4-x}\right)^2 = 4$$

$$2x+3-2\sqrt{2x+3}\cdot\sqrt{4-x}+4-x=4$$

$$x+7-2\sqrt{2x+3}\cdot\sqrt{4-x}=4$$

$$-2\sqrt{2x+3}\cdot\sqrt{4-x}=-3-x \quad |\cdot(-1)$$

$$2\sqrt{2x+3}\cdot\sqrt{4-x}=3+x \quad |\uparrow^2$$

- 2.

$$4\cdot(2x+3)\cdot(4-x)=9+6x+x^2$$

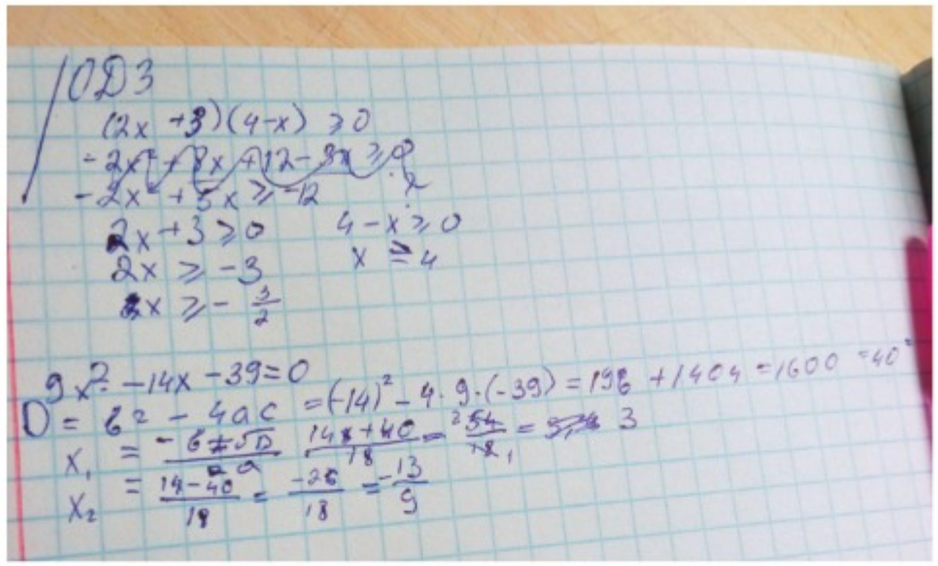
$$(8x+12)\cdot(4-x)=9+6x+x^2$$

$$32x-8x^2+48-12x-9-6x-1x^2=0$$

$$14x-9x^2+39=0$$

$$-9x^2+14x+39=0 \quad /(-1)$$

$$9x^2-14x-39=0$$



$$x = -\frac{13}{9} \approx -1{,}44;\ x=3$$

ПРОВЕРКА

$$\sqrt{2x+3}-\sqrt{4-x}=2$$

$$\sqrt{2\cdot3+3}-\sqrt{4-3}=2$$

$$3-1=2$$

$$2=2 \text{ (верно)}$$