Вариант №11

Вид классов: `classification`

Random state: `5`

Class sep: `1.5`

Для всех:

`n_features = 2`

`n_redundant = 0`

`n_informative = 2`

`n_clusters_per_class = 1`

`n_classes = 4`

`n_samples = 100`

```
In [1]:  from sklearn.datasets import make_classification
```

Загрузка выборки согласно варианту №11

```
In [2]:  X, y = make_classification(n_samples=100,
                                     n_features=2,
                                     n_redundant=0,
                                     n_informative=2,
                                     n_clusters_per_class=1,
                                     n_classes=4,
                                     random_state=5,
                                     class_sep=1.5)
```
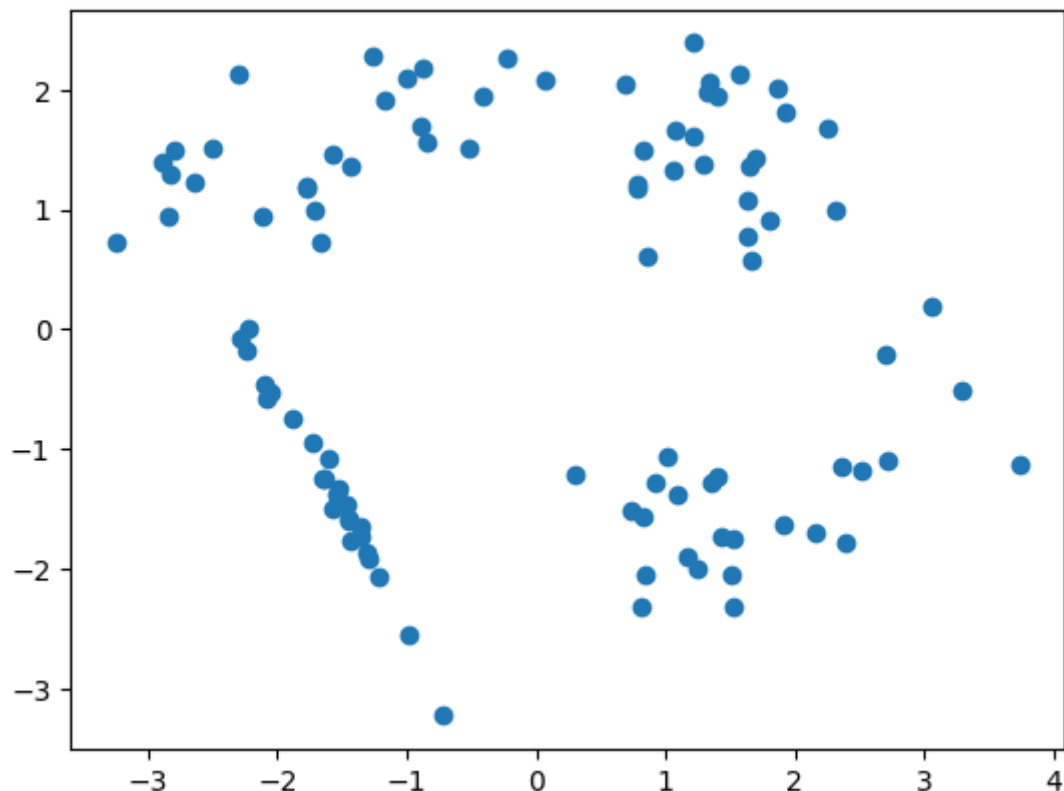
Отображение выборки на графике

```
In [3]: import matplotlib.pyplot as plt
```

```
In [4]: plt.scatter(X[:, 0], X[:, 1])
```

Out[4]: <matplotlib.collections.PathCollection at 0x23816fe7dc0>

Иерархическая кластеризация выборки

```
In [5]: from scipy.cluster.hierarchy import linkage, dendrogram
```

Расстояние ближайшего соседа (single)

```
In [6]: mergings_single = linkage(X, method='single')
        mergings_single
```

```
Out[6]: array([[4.30000000e+01, 7.40000000e+01, 1.04010734e-02, 2.00000000e+00],
               [4.00000000e+01, 5.00000000e+01, 1.12542017e-02, 2.00000000e+00],
               [4.60000000e+01, 4.90000000e+01, 1.83059535e-02, 2.00000000e+00],
               [4.20000000e+01, 8.90000000e+01, 2.85527765e-02, 2.00000000e+00],
               [5.30000000e+01, 9.70000000e+01, 5.04282799e-02, 2.00000000e+00],
               [4.40000000e+01, 7.70000000e+01, 5.33827361e-02, 2.00000000e+00],
               [3.00000000e+00, 6.90000000e+01, 6.17673195e-02, 2.00000000e+00],
               [3.10000000e+01, 9.90000000e+01, 6.72403955e-02, 2.00000000e+00],
               [1.20000000e+01, 2.40000000e+01, 7.32365299e-02, 2.00000000e+00],
               [1.10000000e+01, 6.80000000e+01, 8.39490552e-02, 2.00000000e+00],
               [6.10000000e+01, 9.00000000e+01, 8.49616595e-02, 2.00000000e+00],
               [2.00000000e+00, 1.08000000e+02, 9.09643390e-02, 3.00000000e+00],
               [1.30000000e+01, 1.06000000e+02, 9.13774467e-02, 3.00000000e+00],
               [0.00000000e+00, 1.10000000e+02, 9.20819789e-02, 3.00000000e+00],
               [1.80000000e+01, 5.50000000e+01, 9.83732271e-02, 2.00000000e+00],
               [4.00000000e+00, 6.60000000e+01, 9.88681987e-02, 2.00000000e+00],
               [1.01000000e+02, 1.13000000e+02, 1.10925457e-01, 5.00000000e+00],
               [6.50000000e+01, 1.14000000e+02, 1.11640492e-01, 3.00000000e+00],
               [4.80000000e+01, 8.30000000e+01, 1.11845503e-01, 2.00000000e+00],
               [3.30000000e+01, 1.04000000e+02, 1.13206885e-01, 3.00000000e+00],
               [3.80000000e+01, 5.20000000e+01, 1.15293650e-01, 2.00000000e+00],
               [3.20000000e+01, 1.19000000e+02, 1.16955665e-01, 4.00000000e+00],
               [1.16000000e+02, 1.21000000e+02, 1.28520035e-01, 9.00000000e+00],
               [1.05000000e+02, 1.22000000e+02, 1.32507899e-01, 1.10000000e+01],
               [1.00000000e+02, 1.23000000e+02, 1.33193406e-01, 1.30000000e+01],
               [2.30000000e+01, 9.80000000e+01, 1.37848277e-01, 2.00000000e+00],
               [9.00000000e+00, 7.30000000e+01, 1.43682550e-01, 2.00000000e+00],
               [4.10000000e+01, 1.18000000e+02, 1.48754298e-01, 3.00000000e+00],
               [3.90000000e+01, 9.40000000e+01, 1.53854892e-01, 2.00000000e+00],
               [1.70000000e+01, 9.30000000e+01, 1.56712084e-01, 2.00000000e+00],
               [3.00000000e+01, 8.00000000e+01, 1.61672046e-01, 2.00000000e+00],
               [9.60000000e+01, 1.24000000e+02, 1.66986790e-01, 1.40000000e+01],
               [3.70000000e+01, 8.40000000e+01, 1.71866607e-01, 2.00000000e+00],
               [8.10000000e+01, 1.31000000e+02, 1.72043909e-01, 1.50000000e+01],
               [2.60000000e+01, 1.33000000e+02, 1.81684528e-01, 1.60000000e+01],
               [6.30000000e+01, 1.27000000e+02, 1.92726906e-01, 4.00000000e+00],
               [8.00000000e+00, 1.02000000e+02, 1.93598822e-01, 3.00000000e+00],
               [2.10000000e+01, 8.50000000e+01, 2.02968978e-01, 2.00000000e+00],
               [2.50000000e+01, 4.50000000e+01, 2.03402121e-01, 2.00000000e+00],
               [6.70000000e+01, 1.30000000e+02, 2.14842182e-01, 3.00000000e+00],
               [5.00000000e+00, 1.00000000e+01, 2.18215653e-01, 2.00000000e+00],
               [5.40000000e+01, 1.38000000e+02, 2.28966419e-01, 3.00000000e+00],
               [6.00000000e+00, 7.10000000e+01, 2.33805372e-01, 2.00000000e+00],
               [6.40000000e+01, 1.11000000e+02, 2.39553421e-01, 4.00000000e+00],
               [1.60000000e+01, 6.00000000e+01, 2.42386418e-01, 2.00000000e+00],
               [4.70000000e+01, 1.29000000e+02, 2.48902858e-01, 3.00000000e+00],
               [9.20000000e+01, 1.41000000e+02, 2.49069862e-01, 4.00000000e+00],
               [7.90000000e+01, 1.44000000e+02, 2.49075408e-01, 3.00000000e+00],
               [1.28000000e+02, 1.42000000e+02, 2.49434643e-01, 4.00000000e+00],
               [5.90000000e+01, 1.37000000e+02, 2.53228457e-01, 3.00000000e+00],
               [3.60000000e+01, 1.34000000e+02, 2.55324875e-01, 1.70000000e+01],
               [1.12000000e+02, 1.50000000e+02, 2.60323662e-01, 2.00000000e+01],
               [7.50000000e+01, 1.25000000e+02, 2.60686027e-01, 3.00000000e+00],
               [7.20000000e+01, 1.36000000e+02, 2.62098265e-01, 4.00000000e+00],
               [1.50000000e+01, 1.52000000e+02, 2.63296702e-01, 4.00000000e+00],
               [7.00000000e+00, 2.90000000e+01, 2.72955732e-01, 2.00000000e+00],
               [1.09000000e+02, 1.46000000e+02, 2.73368293e-01, 6.00000000e+00],
               [1.07000000e+02, 1.49000000e+02, 2.74571669e-01, 5.00000000e+00],
               [8.20000000e+01, 1.03000000e+02, 2.80325225e-01, 3.00000000e+00],
               [1.48000000e+02, 1.58000000e+02, 2.81428379e-01, 7.00000000e+00],
```

```
        [1.20000000e+02, 1.57000000e+02, 2.92426396e-01, 7.00000000e+00],
        [5.60000000e+01, 1.35000000e+02, 2.93681764e-01, 5.00000000e+00],
        [1.15000000e+02, 1.54000000e+02, 3.00201809e-01, 6.00000000e+00],
        [1.17000000e+02, 1.51000000e+02, 3.07197502e-01, 2.30000000e+01],
        [1.40000000e+02, 1.43000000e+02, 3.13498443e-01, 6.00000000e+00],
        [1.00000000e+00, 1.45000000e+02, 3.28112403e-01, 4.00000000e+00],
        [7.80000000e+01, 1.26000000e+02, 3.29062308e-01, 3.00000000e+00],
        [7.60000000e+01, 1.61000000e+02, 3.41039132e-01, 6.00000000e+00],
        [1.32000000e+02, 1.53000000e+02, 3.43084630e-01, 6.00000000e+00],
        [1.65000000e+02, 1.66000000e+02, 3.47195581e-01, 7.00000000e+00],
        [8.60000000e+01, 9.50000000e+01, 3.49253096e-01, 2.00000000e+00],
        [9.10000000e+01, 1.64000000e+02, 3.53291975e-01, 7.00000000e+00],
        [1.56000000e+02, 1.59000000e+02, 3.53433739e-01, 1.30000000e+01],
        [5.70000000e+01, 1.71000000e+02, 3.57539555e-01, 8.00000000e+00],
        [5.10000000e+01, 1.70000000e+02, 3.64489121e-01, 3.00000000e+00],
        [1.55000000e+02, 1.62000000e+02, 3.67584994e-01, 8.00000000e+00],
        [1.72000000e+02, 1.73000000e+02, 3.89014271e-01, 2.10000000e+01],
        [8.70000000e+01, 1.68000000e+02, 4.08622118e-01, 7.00000000e+00],
        [1.47000000e+02, 1.75000000e+02, 4.10733988e-01, 1.10000000e+01],
        [1.69000000e+02, 1.74000000e+02, 4.45682122e-01, 1.00000000e+01],
        [1.60000000e+02, 1.78000000e+02, 4.54349088e-01, 1.80000000e+01],
        [2.20000000e+01, 1.67000000e+02, 4.67201729e-01, 7.00000000e+00],
        [6.20000000e+01, 1.76000000e+02, 5.12178550e-01, 2.20000000e+01],
        [3.40000000e+01, 1.80000000e+02, 5.20635523e-01, 1.90000000e+01],
        [1.40000000e+01, 1.82000000e+02, 5.33975873e-01, 2.30000000e+01],
        [2.00000000e+01, 7.00000000e+01, 5.37671619e-01, 2.00000000e+00],
        [1.90000000e+01, 1.63000000e+02, 5.37865192e-01, 2.40000000e+01],
        [2.80000000e+01, 1.84000000e+02, 5.76206296e-01, 2.40000000e+01],
        [1.77000000e+02, 1.79000000e+02, 5.95228790e-01, 1.70000000e+01],
        [1.39000000e+02, 1.83000000e+02, 5.95738421e-01, 2.20000000e+01],
        [1.81000000e+02, 1.88000000e+02, 6.06595322e-01, 2.40000000e+01],
        [1.87000000e+02, 1.90000000e+02, 6.17090098e-01, 4.80000000e+01],
        [5.80000000e+01, 1.91000000e+02, 6.60097462e-01, 4.90000000e+01],
        [2.70000000e+01, 1.85000000e+02, 6.60910510e-01, 3.00000000e+00],
        [8.80000000e+01, 1.86000000e+02, 7.26987998e-01, 2.50000000e+01],
        [3.50000000e+01, 1.93000000e+02, 7.72337717e-01, 4.00000000e+00],
        [1.89000000e+02, 1.95000000e+02, 8.25273048e-01, 2.60000000e+01],
        [1.92000000e+02, 1.94000000e+02, 9.17929103e-01, 7.40000000e+01],
        [1.96000000e+02, 1.97000000e+02, 1.09136522e+00, 1.00000000e+02]])
```

In [7]:
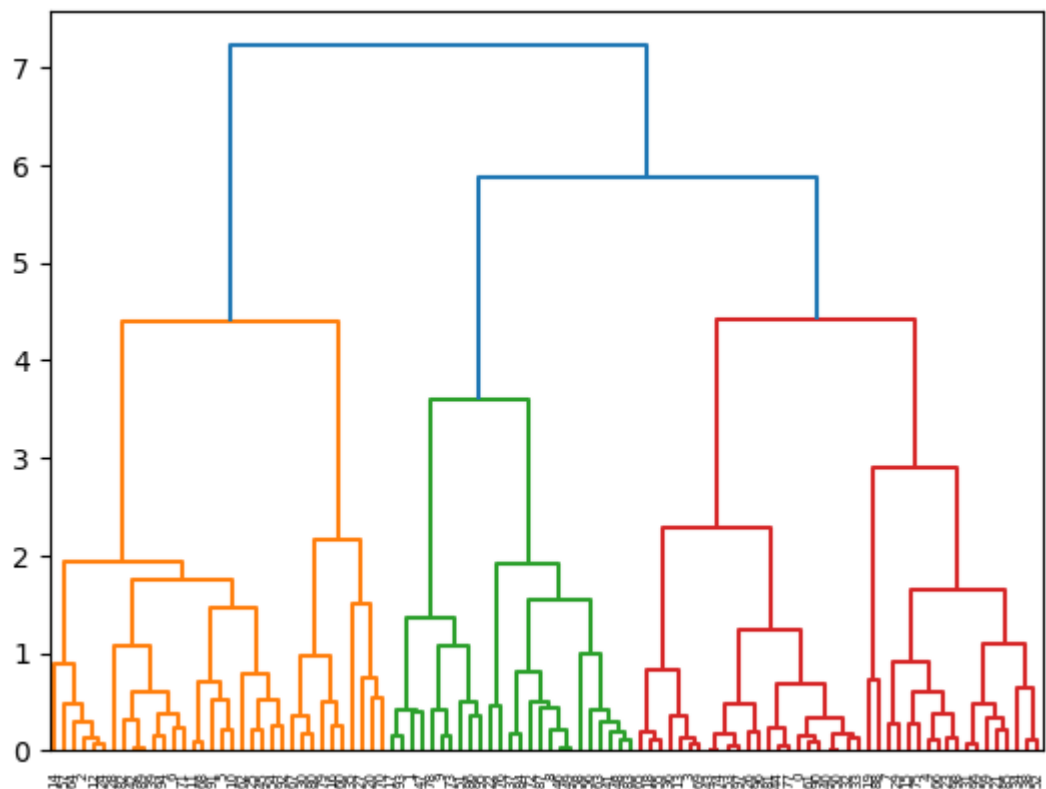```python
dendrogram(mergings_single)
plt.show()
```
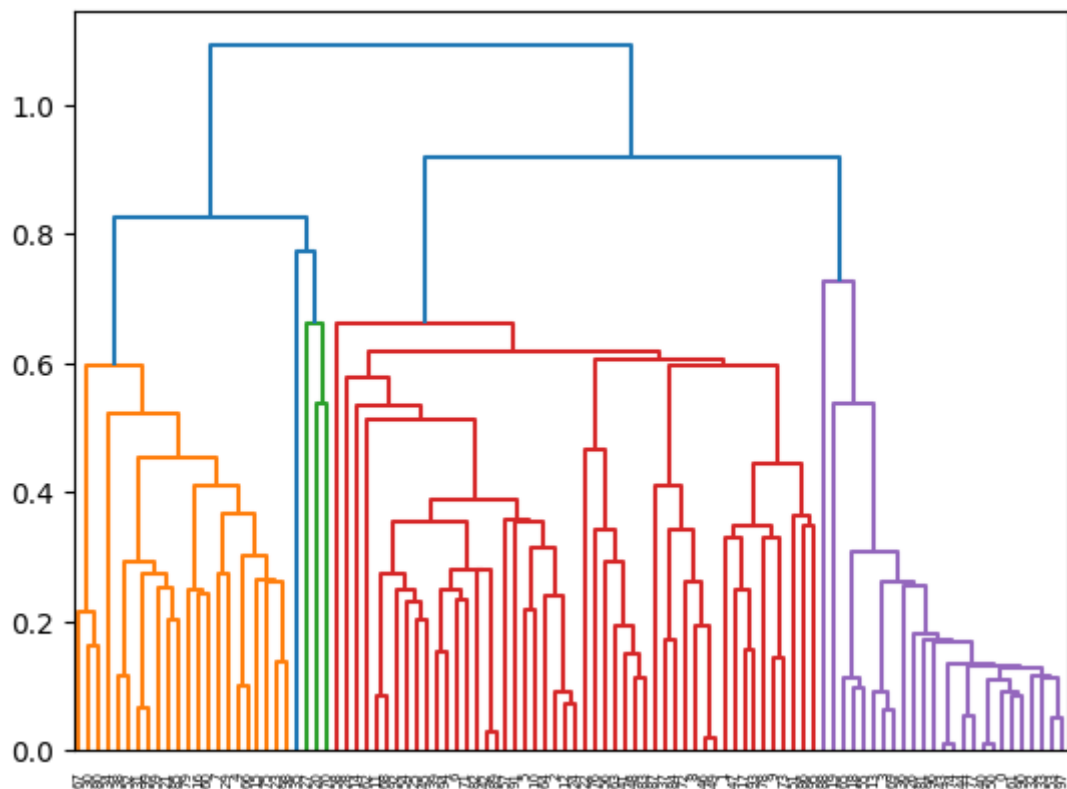
## Расстояние дальнего соседа (complete)

```python
mergings_complete = linkage(X, method='complete')
mergings_complete
```

In [8]:

```
Out[8]: array([[4.30000000e+01, 7.40000000e+01, 1.04010734e-02, 2.00000000e+00],
               [4.00000000e+01, 5.00000000e+01, 1.12542017e-02, 2.00000000e+00],
               [4.60000000e+01, 4.90000000e+01, 1.83059535e-02, 2.00000000e+00],
               [4.20000000e+01, 8.90000000e+01, 2.85527765e-02, 2.00000000e+00],
               [5.30000000e+01, 9.70000000e+01, 5.04282799e-02, 2.00000000e+00],
               [4.40000000e+01, 7.70000000e+01, 5.33827361e-02, 2.00000000e+00],
               [3.00000000e+00, 6.90000000e+01, 6.17673195e-02, 2.00000000e+00],
               [3.10000000e+01, 9.90000000e+01, 6.72403955e-02, 2.00000000e+00],
               [1.20000000e+01, 2.40000000e+01, 7.32365299e-02, 2.00000000e+00],
               [1.10000000e+01, 6.80000000e+01, 8.39490552e-02, 2.00000000e+00],
               [6.10000000e+01, 9.00000000e+01, 8.49616595e-02, 2.00000000e+00],
               [1.80000000e+01, 5.50000000e+01, 9.83732271e-02, 2.00000000e+00],
               [4.00000000e+00, 6.60000000e+01, 9.88681987e-02, 2.00000000e+00],
               [4.80000000e+01, 8.30000000e+01, 1.11845503e-01, 2.00000000e+00],
               [3.80000000e+01, 5.20000000e+01, 1.15293650e-01, 2.00000000e+00],
               [2.00000000e+00, 1.08000000e+02, 1.23092784e-01, 3.00000000e+00],
               [3.20000000e+01, 3.30000000e+01, 1.24237414e-01, 2.00000000e+00],
               [1.30000000e+01, 1.06000000e+02, 1.31830688e-01, 3.00000000e+00],
               [2.30000000e+01, 9.80000000e+01, 1.37848277e-01, 2.00000000e+00],
               [9.00000000e+00, 7.30000000e+01, 1.43682550e-01, 2.00000000e+00],
               [0.00000000e+00, 1.10000000e+02, 1.51991231e-01, 3.00000000e+00],
               [3.90000000e+01, 9.40000000e+01, 1.53854892e-01, 2.00000000e+00],
               [1.70000000e+01, 9.30000000e+01, 1.56712084e-01, 2.00000000e+00],
               [3.00000000e+01, 8.00000000e+01, 1.61672046e-01, 2.00000000e+00],
               [1.01000000e+02, 1.16000000e+02, 1.65428634e-01, 4.00000000e+00],
               [1.00000000e+02, 1.04000000e+02, 1.65687613e-01, 4.00000000e+00],
               [3.70000000e+01, 8.40000000e+01, 1.71866607e-01, 2.00000000e+00],
               [6.50000000e+01, 1.11000000e+02, 1.81293977e-01, 3.00000000e+00],
               [2.60000000e+01, 9.60000000e+01, 1.81684528e-01, 2.00000000e+00],
               [4.10000000e+01, 1.13000000e+02, 2.00142017e-01, 3.00000000e+00],
               [2.10000000e+01, 8.50000000e+01, 2.02968978e-01, 2.00000000e+00],
               [2.50000000e+01, 4.50000000e+01, 2.03402121e-01, 2.00000000e+00],
               [8.00000000e+00, 1.02000000e+02, 2.11866903e-01, 3.00000000e+00],
               [5.00000000e+00, 1.00000000e+01, 2.18215653e-01, 2.00000000e+00],
               [8.10000000e+01, 1.05000000e+02, 2.25353756e-01, 3.00000000e+00],
               [6.00000000e+00, 7.10000000e+01, 2.33805372e-01, 2.00000000e+00],
               [1.60000000e+01, 6.00000000e+01, 2.42386418e-01, 2.00000000e+00],
               [5.40000000e+01, 9.20000000e+01, 2.49069862e-01, 2.00000000e+00],
               [1.50000000e+01, 7.50000000e+01, 2.63296702e-01, 2.00000000e+00],
               [7.00000000e+00, 2.90000000e+01, 2.72955732e-01, 2.00000000e+00],
               [6.40000000e+01, 1.15000000e+02, 2.87543352e-01, 4.00000000e+00],
               [6.30000000e+01, 1.29000000e+02, 2.94195216e-01, 4.00000000e+00],
               [8.20000000e+01, 1.03000000e+02, 3.08147116e-01, 3.00000000e+00],
               [1.20000000e+02, 1.24000000e+02, 3.28348381e-01, 7.00000000e+00],
               [5.90000000e+01, 1.30000000e+02, 3.37963424e-01, 3.00000000e+00],
               [8.60000000e+01, 9.50000000e+01, 3.49253096e-01, 2.00000000e+00],
               [6.70000000e+01, 1.23000000e+02, 3.57238356e-01, 3.00000000e+00],
               [3.60000000e+01, 1.17000000e+02, 3.60669557e-01, 4.00000000e+00],
               [1.21000000e+02, 1.35000000e+02, 3.70479586e-01, 4.00000000e+00],
               [1.12000000e+02, 1.18000000e+02, 3.77945354e-01, 4.00000000e+00],
               [1.00000000e+00, 4.70000000e+01, 3.86546804e-01, 2.00000000e+00],
               [7.80000000e+01, 1.19000000e+02, 4.10020390e-01, 3.00000000e+00],
               [1.22000000e+02, 1.50000000e+02, 4.10453510e-01, 4.00000000e+00],
               [5.60000000e+01, 1.41000000e+02, 4.17478255e-01, 5.00000000e+00],
               [8.70000000e+01, 1.32000000e+02, 4.27610198e-01, 4.00000000e+00],
               [2.20000000e+01, 7.60000000e+01, 4.67201729e-01, 2.00000000e+00],
               [1.25000000e+02, 1.28000000e+02, 4.73722937e-01, 6.00000000e+00],
               [5.70000000e+01, 1.40000000e+02, 4.79183235e-01, 5.00000000e+00],
               [1.07000000e+02, 1.44000000e+02, 4.81208453e-01, 5.00000000e+00],
               [7.90000000e+01, 1.36000000e+02, 4.91428433e-01, 3.00000000e+00],
```

```
       [5.10000000e+01, 1.45000000e+02, 4.98298432e-01, 3.00000000e+00],
       [7.20000000e+01, 1.54000000e+02, 4.99519577e-01, 5.00000000e+00],
       [1.31000000e+02, 1.37000000e+02, 5.12764429e-01, 4.00000000e+00],
       [9.10000000e+01, 1.33000000e+02, 5.19014270e-01, 3.00000000e+00],
       [2.00000000e+01, 7.00000000e+01, 5.37671619e-01, 2.00000000e+00],
       [1.38000000e+02, 1.49000000e+02, 5.92196264e-01, 6.00000000e+00],
       [1.42000000e+02, 1.48000000e+02, 6.06680317e-01, 7.00000000e+00],
       [3.40000000e+01, 1.14000000e+02, 6.34303030e-01, 3.00000000e+00],
       [1.34000000e+02, 1.43000000e+02, 6.73125645e-01, 1.00000000e+01],
       [1.09000000e+02, 1.63000000e+02, 6.93630998e-01, 5.00000000e+00],
       [1.90000000e+01, 8.80000000e+01, 7.26987998e-01, 2.00000000e+00],
       [2.70000000e+01, 1.64000000e+02, 7.38621102e-01, 3.00000000e+00],
       [6.20000000e+01, 1.62000000e+02, 7.78190207e-01, 5.00000000e+00],
       [1.26000000e+02, 1.61000000e+02, 8.00014146e-01, 7.00000000e+00],
       [1.27000000e+02, 1.47000000e+02, 8.18422473e-01, 7.00000000e+00],
       [1.40000000e+01, 1.57000000e+02, 8.86866276e-01, 6.00000000e+00],
       [1.39000000e+02, 1.65000000e+02, 9.08689026e-01, 8.00000000e+00],
       [1.46000000e+02, 1.59000000e+02, 9.62064229e-01, 6.00000000e+00],
       [5.80000000e+01, 1.53000000e+02, 9.98541792e-01, 6.00000000e+00],
       [1.51000000e+02, 1.60000000e+02, 1.06387033e+00, 6.00000000e+00],
       [2.80000000e+01, 1.66000000e+02, 1.08050850e+00, 8.00000000e+00],
       [1.58000000e+02, 1.67000000e+02, 1.09101798e+00, 8.00000000e+00],
       [1.56000000e+02, 1.68000000e+02, 1.22766930e+00, 1.60000000e+01],
       [1.52000000e+02, 1.79000000e+02, 1.35062609e+00, 1.00000000e+01],
       [1.69000000e+02, 1.72000000e+02, 1.45826959e+00, 1.00000000e+01],
       [3.50000000e+01, 1.71000000e+02, 1.49456176e+00, 4.00000000e+00],
       [1.73000000e+02, 1.78000000e+02, 1.54083535e+00, 1.30000000e+01],
       [1.76000000e+02, 1.81000000e+02, 1.64807863e+00, 1.60000000e+01],
       [1.80000000e+02, 1.84000000e+02, 1.75325751e+00, 1.80000000e+01],
       [1.55000000e+02, 1.86000000e+02, 1.91792647e+00, 1.50000000e+01],
       [1.75000000e+02, 1.88000000e+02, 1.93125299e+00, 2.40000000e+01],
       [1.77000000e+02, 1.85000000e+02, 2.15928506e+00, 1.00000000e+01],
       [1.74000000e+02, 1.82000000e+02, 2.29149696e+00, 2.30000000e+01],
       [1.70000000e+02, 1.87000000e+02, 2.90738778e+00, 1.80000000e+01],
       [1.83000000e+02, 1.89000000e+02, 3.58796363e+00, 2.50000000e+01],
       [1.90000000e+02, 1.91000000e+02, 4.39795306e+00, 3.40000000e+01],
       [1.92000000e+02, 1.93000000e+02, 4.41833809e+00, 4.10000000e+01],
       [1.94000000e+02, 1.96000000e+02, 5.87120715e+00, 6.60000000e+01],
       [1.95000000e+02, 1.97000000e+02, 7.22235680e+00, 1.00000000e+02]])
```

In [9]:
```python
dendrogram(mergings_complete)
plt.show()
```
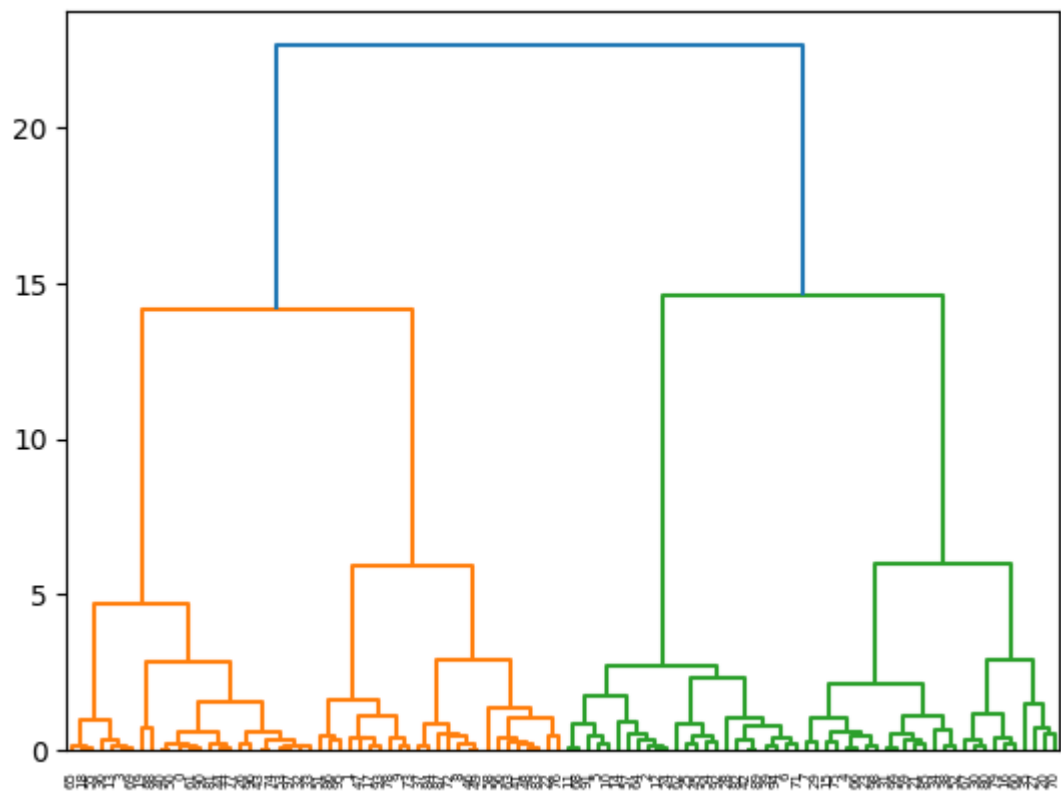
Расстояние Уорда (Ward)

```
mergings_ward = linkage(X, method='ward')
mergings_ward
```

```
Out[10]: array([[4.30000000e+01, 7.40000000e+01, 1.04010734e-02, 2.00000000e+00],
                [4.00000000e+01, 5.00000000e+01, 1.12542017e-02, 2.00000000e+00],
                [4.60000000e+01, 4.90000000e+01, 1.83059535e-02, 2.00000000e+00],
                [4.20000000e+01, 8.90000000e+01, 2.85527765e-02, 2.00000000e+00],
                [5.30000000e+01, 9.70000000e+01, 5.04282799e-02, 2.00000000e+00],
                [4.40000000e+01, 7.70000000e+01, 5.33827361e-02, 2.00000000e+00],
                [3.00000000e+00, 6.90000000e+01, 6.17673195e-02, 2.00000000e+00],
                [3.10000000e+01, 9.90000000e+01, 6.72403955e-02, 2.00000000e+00],
                [1.20000000e+01, 2.40000000e+01, 7.32365299e-02, 2.00000000e+00],
                [1.10000000e+01, 6.80000000e+01, 8.39490552e-02, 2.00000000e+00],
                [6.10000000e+01, 9.00000000e+01, 8.49616595e-02, 2.00000000e+00],
                [1.80000000e+01, 5.50000000e+01, 9.83732271e-02, 2.00000000e+00],
                [4.00000000e+00, 6.60000000e+01, 9.88681987e-02, 2.00000000e+00],
                [4.80000000e+01, 8.30000000e+01, 1.11845503e-01, 2.00000000e+00],
                [3.80000000e+01, 5.20000000e+01, 1.15293650e-01, 2.00000000e+00],
                [2.00000000e+00, 1.08000000e+02, 1.17599744e-01, 3.00000000e+00],
                [3.20000000e+01, 3.30000000e+01, 1.24237414e-01, 2.00000000e+00],
                [1.30000000e+01, 1.06000000e+02, 1.26020017e-01, 3.00000000e+00],
                [0.00000000e+00, 1.10000000e+02, 1.36555687e-01, 3.00000000e+00],
                [2.30000000e+01, 9.80000000e+01, 1.37848277e-01, 2.00000000e+00],
                [9.00000000e+00, 7.30000000e+01, 1.43682550e-01, 2.00000000e+00],
                [3.90000000e+01, 9.40000000e+01, 1.53854892e-01, 2.00000000e+00],
                [1.70000000e+01, 9.30000000e+01, 1.56712084e-01, 2.00000000e+00],
                [3.00000000e+01, 8.00000000e+01, 1.61672046e-01, 2.00000000e+00],
                [6.50000000e+01, 1.11000000e+02, 1.64301470e-01, 3.00000000e+00],
                [1.04000000e+02, 1.16000000e+02, 1.69321882e-01, 4.00000000e+00],
                [3.70000000e+01, 8.40000000e+01, 1.71866607e-01, 2.00000000e+00],
                [2.60000000e+01, 9.60000000e+01, 1.81684528e-01, 2.00000000e+00],
                [4.10000000e+01, 1.13000000e+02, 1.93097488e-01, 3.00000000e+00],
                [2.10000000e+01, 8.50000000e+01, 2.02968978e-01, 2.00000000e+00],
                [2.50000000e+01, 4.50000000e+01, 2.03402121e-01, 2.00000000e+00],
                [5.00000000e+00, 1.00000000e+01, 2.18215653e-01, 2.00000000e+00],
                [1.01000000e+02, 1.18000000e+02, 2.24500678e-01, 5.00000000e+00],
                [8.10000000e+01, 1.05000000e+02, 2.29432000e-01, 3.00000000e+00],
                [6.00000000e+00, 7.10000000e+01, 2.33805372e-01, 2.00000000e+00],
                [8.00000000e+00, 1.02000000e+02, 2.34094759e-01, 3.00000000e+00],
                [1.60000000e+01, 6.00000000e+01, 2.42386418e-01, 2.00000000e+00],
                [5.40000000e+01, 9.20000000e+01, 2.49069862e-01, 2.00000000e+00],
                [1.50000000e+01, 7.50000000e+01, 2.63296702e-01, 2.00000000e+00],
                [7.00000000e+00, 2.90000000e+01, 2.72955732e-01, 2.00000000e+00],
                [6.30000000e+01, 1.28000000e+02, 3.02806343e-01, 4.00000000e+00],
                [6.40000000e+01, 1.15000000e+02, 3.08315715e-01, 4.00000000e+00],
                [5.90000000e+01, 1.29000000e+02, 3.24289711e-01, 3.00000000e+00],
                [1.00000000e+02, 1.25000000e+02, 3.26732183e-01, 6.00000000e+00],
                [6.70000000e+01, 1.23000000e+02, 3.27319906e-01, 3.00000000e+00],
                [8.20000000e+01, 1.03000000e+02, 3.39734452e-01, 3.00000000e+00],
                [8.60000000e+01, 9.50000000e+01, 3.49253096e-01, 2.00000000e+00],
                [3.60000000e+01, 1.17000000e+02, 3.61418771e-01, 4.00000000e+00],
                [4.70000000e+01, 1.22000000e+02, 3.75967965e-01, 3.00000000e+00],
                [1.21000000e+02, 1.34000000e+02, 4.07843267e-01, 4.00000000e+00],
                [1.00000000e+00, 1.48000000e+02, 4.13879796e-01, 4.00000000e+00],
                [7.80000000e+01, 1.20000000e+02, 4.21170104e-01, 3.00000000e+00],
                [5.60000000e+01, 1.40000000e+02, 4.22923299e-01, 5.00000000e+00],
                [7.90000000e+01, 1.36000000e+02, 4.27523782e-01, 3.00000000e+00],
                [5.10000000e+01, 1.46000000e+02, 4.61999122e-01, 3.00000000e+00],
                [2.20000000e+01, 7.60000000e+01, 4.67201729e-01, 2.00000000e+00],
                [1.12000000e+02, 1.19000000e+02, 4.71909881e-01, 4.00000000e+00],
                [1.30000000e+02, 1.37000000e+02, 4.72630797e-01, 4.00000000e+00],
                [7.20000000e+01, 1.35000000e+02, 4.85915566e-01, 4.00000000e+00],
                [9.10000000e+01, 1.31000000e+02, 4.96911794e-01, 3.00000000e+00],
```

```
            [8.70000000e+01, 1.58000000e+02, 5.02297257e-01, 5.00000000e+00],
            [5.70000000e+01, 1.41000000e+02, 5.16836078e-01, 5.00000000e+00],
            [2.00000000e+01, 7.00000000e+01, 5.37671619e-01, 2.00000000e+00],
            [1.07000000e+02, 1.42000000e+02, 5.63112143e-01, 5.00000000e+00],
            [1.32000000e+02, 1.33000000e+02, 5.93905502e-01, 8.00000000e+00],
            [1.38000000e+02, 1.56000000e+02, 6.24276480e-01, 6.00000000e+00],
            [1.27000000e+02, 1.43000000e+02, 6.25457818e-01, 8.00000000e+00],
            [3.40000000e+01, 1.14000000e+02, 6.66711016e-01, 3.00000000e+00],
            [1.90000000e+01, 8.80000000e+01, 7.26987998e-01, 2.00000000e+00],
            [2.70000000e+01, 1.62000000e+02, 7.47359106e-01, 3.00000000e+00],
            [1.45000000e+02, 1.49000000e+02, 7.71396209e-01, 7.00000000e+00],
            [6.20000000e+01, 1.57000000e+02, 8.25851718e-01, 5.00000000e+00],
            [1.26000000e+02, 1.60000000e+02, 8.58521999e-01, 7.00000000e+00],
            [1.09000000e+02, 1.59000000e+02, 8.62686127e-01, 5.00000000e+00],
            [1.40000000e+01, 1.61000000e+02, 8.86018458e-01, 6.00000000e+00],
            [1.24000000e+02, 1.47000000e+02, 9.96357425e-01, 7.00000000e+00],
            [1.52000000e+02, 1.55000000e+02, 1.06393225e+00, 7.00000000e+00],
            [1.39000000e+02, 1.65000000e+02, 1.07129978e+00, 8.00000000e+00],
            [2.80000000e+01, 1.70000000e+02, 1.07499258e+00, 8.00000000e+00],
            [1.63000000e+02, 1.67000000e+02, 1.09223870e+00, 8.00000000e+00],
            [1.50000000e+02, 1.51000000e+02, 1.13694339e+00, 7.00000000e+00],
            [1.44000000e+02, 1.53000000e+02, 1.17389985e+00, 6.00000000e+00],
            [5.80000000e+01, 1.76000000e+02, 1.39179991e+00, 8.00000000e+00],
            [3.50000000e+01, 1.69000000e+02, 1.47348244e+00, 4.00000000e+00],
            [1.64000000e+02, 1.66000000e+02, 1.54847783e+00, 1.60000000e+01],
            [1.54000000e+02, 1.80000000e+02, 1.59674719e+00, 1.00000000e+01],
            [1.73000000e+02, 1.74000000e+02, 1.76398138e+00, 1.10000000e+01],
            [1.77000000e+02, 1.79000000e+02, 2.14874689e+00, 1.60000000e+01],
            [1.71000000e+02, 1.78000000e+02, 2.30905774e+00, 1.30000000e+01],
            [1.86000000e+02, 1.88000000e+02, 2.72473310e+00, 2.40000000e+01],
            [1.68000000e+02, 1.84000000e+02, 2.82461218e+00, 1.80000000e+01],
            [1.72000000e+02, 1.82000000e+02, 2.88749778e+00, 1.50000000e+01],
            [1.81000000e+02, 1.83000000e+02, 2.89735933e+00, 1.00000000e+01],
            [1.75000000e+02, 1.90000000e+02, 4.72830196e+00, 2.50000000e+01],
            [1.85000000e+02, 1.91000000e+02, 5.93627196e+00, 2.50000000e+01],
            [1.87000000e+02, 1.92000000e+02, 5.98223284e+00, 2.60000000e+01],
            [1.93000000e+02, 1.94000000e+02, 1.41555302e+01, 5.00000000e+01],
            [1.89000000e+02, 1.95000000e+02, 1.45798509e+01, 5.00000000e+01],
            [1.96000000e+02, 1.97000000e+02, 2.26041572e+01, 1.00000000e+02]])
```

In [11]:
```python
dendrogram(mergings_ward)
plt.show()
```
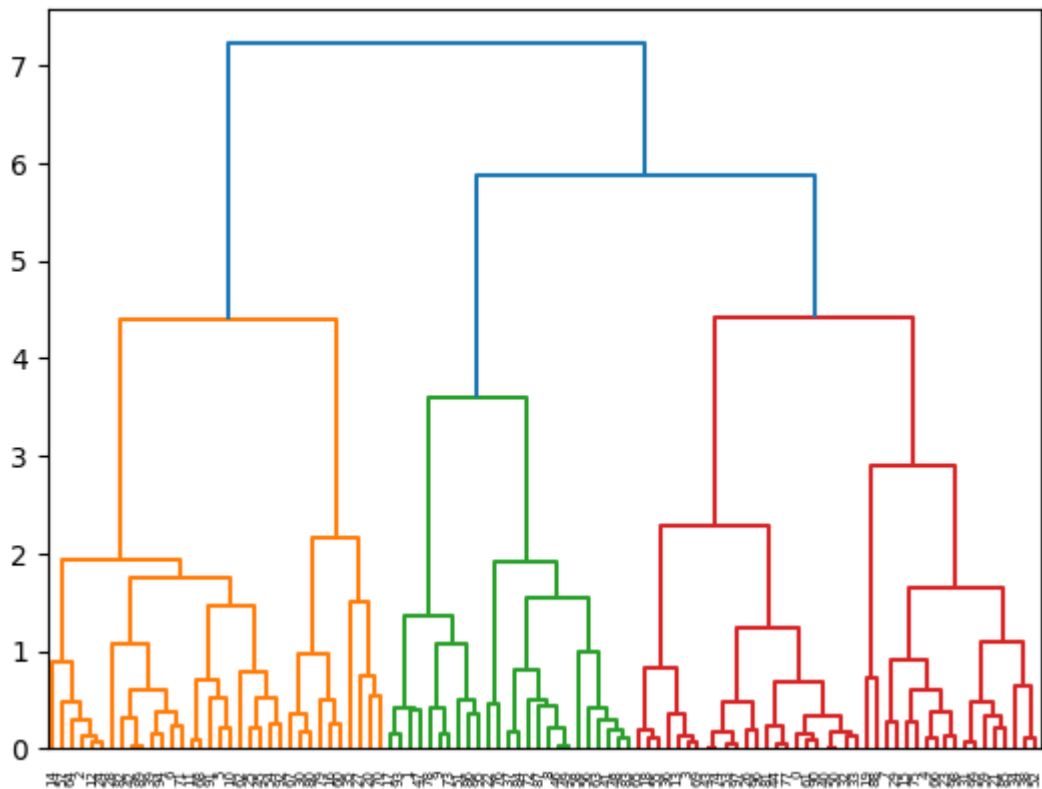
Выбор лучшего разбиения

```
In [12]: mergings_complete = linkage(X, method='complete')
         mergings_complete
```

```
Out[12]: array([[4.30000000e+01, 7.40000000e+01, 1.04010734e-02, 2.00000000e+00],
                [4.00000000e+01, 5.00000000e+01, 1.12542017e-02, 2.00000000e+00],
                [4.60000000e+01, 4.90000000e+01, 1.83059535e-02, 2.00000000e+00],
                [4.20000000e+01, 8.90000000e+01, 2.85527765e-02, 2.00000000e+00],
                [5.30000000e+01, 9.70000000e+01, 5.04282799e-02, 2.00000000e+00],
                [4.40000000e+01, 7.70000000e+01, 5.33827361e-02, 2.00000000e+00],
                [3.00000000e+00, 6.90000000e+01, 6.17673195e-02, 2.00000000e+00],
                [3.10000000e+01, 9.90000000e+01, 6.72403955e-02, 2.00000000e+00],
                [1.20000000e+01, 2.40000000e+01, 7.32365299e-02, 2.00000000e+00],
                [1.10000000e+01, 6.80000000e+01, 8.39490552e-02, 2.00000000e+00],
                [6.10000000e+01, 9.00000000e+01, 8.49616595e-02, 2.00000000e+00],
                [1.80000000e+01, 5.50000000e+01, 9.83732271e-02, 2.00000000e+00],
                [4.00000000e+00, 6.60000000e+01, 9.88681987e-02, 2.00000000e+00],
                [4.80000000e+01, 8.30000000e+01, 1.11845503e-01, 2.00000000e+00],
                [3.80000000e+01, 5.20000000e+01, 1.15293650e-01, 2.00000000e+00],
                [2.00000000e+00, 1.08000000e+02, 1.23092784e-01, 3.00000000e+00],
                [3.20000000e+01, 3.30000000e+01, 1.24237414e-01, 2.00000000e+00],
                [1.30000000e+01, 1.06000000e+02, 1.31830688e-01, 3.00000000e+00],
                [2.30000000e+01, 9.80000000e+01, 1.37848277e-01, 2.00000000e+00],
                [9.00000000e+00, 7.30000000e+01, 1.43682550e-01, 2.00000000e+00],
                [0.00000000e+00, 1.10000000e+02, 1.51991231e-01, 3.00000000e+00],
                [3.90000000e+01, 9.40000000e+01, 1.53854892e-01, 2.00000000e+00],
                [1.70000000e+01, 9.30000000e+01, 1.56712084e-01, 2.00000000e+00],
                [3.00000000e+01, 8.00000000e+01, 1.61672046e-01, 2.00000000e+00],
                [1.01000000e+02, 1.16000000e+02, 1.65428634e-01, 4.00000000e+00],
                [1.00000000e+02, 1.04000000e+02, 1.65687613e-01, 4.00000000e+00],
                [3.70000000e+01, 8.40000000e+01, 1.71866607e-01, 2.00000000e+00],
                [6.50000000e+01, 1.11000000e+02, 1.81293977e-01, 3.00000000e+00],
                [2.60000000e+01, 9.60000000e+01, 1.81684528e-01, 2.00000000e+00],
                [4.10000000e+01, 1.13000000e+02, 2.00142017e-01, 3.00000000e+00],
                [2.10000000e+01, 8.50000000e+01, 2.02968978e-01, 2.00000000e+00],
                [2.50000000e+01, 4.50000000e+01, 2.03402121e-01, 2.00000000e+00],
                [8.00000000e+00, 1.02000000e+02, 2.11866903e-01, 3.00000000e+00],
                [5.00000000e+00, 1.00000000e+01, 2.18215653e-01, 2.00000000e+00],
                [8.10000000e+01, 1.05000000e+02, 2.25353756e-01, 3.00000000e+00],
                [6.00000000e+00, 7.10000000e+01, 2.33805372e-01, 2.00000000e+00],
                [1.60000000e+01, 6.00000000e+01, 2.42386418e-01, 2.00000000e+00],
                [5.40000000e+01, 9.20000000e+01, 2.49069862e-01, 2.00000000e+00],
                [1.50000000e+01, 7.50000000e+01, 2.63296702e-01, 2.00000000e+00],
                [7.00000000e+00, 2.90000000e+01, 2.72955732e-01, 2.00000000e+00],
                [6.40000000e+01, 1.15000000e+02, 2.87543352e-01, 4.00000000e+00],
                [6.30000000e+01, 1.29000000e+02, 2.94195216e-01, 4.00000000e+00],
                [8.20000000e+01, 1.03000000e+02, 3.08147116e-01, 3.00000000e+00],
                [1.20000000e+02, 1.24000000e+02, 3.28348381e-01, 7.00000000e+00],
                [5.90000000e+01, 1.30000000e+02, 3.37963424e-01, 3.00000000e+00],
                [8.60000000e+01, 9.50000000e+01, 3.49253096e-01, 2.00000000e+00],
                [6.70000000e+01, 1.23000000e+02, 3.57238356e-01, 3.00000000e+00],
                [3.60000000e+01, 1.17000000e+02, 3.60669557e-01, 4.00000000e+00],
                [1.21000000e+02, 1.35000000e+02, 3.70479586e-01, 4.00000000e+00],
                [1.12000000e+02, 1.18000000e+02, 3.77945354e-01, 4.00000000e+00],
                [1.00000000e+00, 4.70000000e+01, 3.86546804e-01, 2.00000000e+00],
                [7.80000000e+01, 1.19000000e+02, 4.10020390e-01, 3.00000000e+00],
                [1.22000000e+02, 1.50000000e+02, 4.10453510e-01, 4.00000000e+00],
                [5.60000000e+01, 1.41000000e+02, 4.17478255e-01, 5.00000000e+00],
                [8.70000000e+01, 1.32000000e+02, 4.27610198e-01, 4.00000000e+00],
                [2.20000000e+01, 7.60000000e+01, 4.67201729e-01, 2.00000000e+00],
                [1.25000000e+02, 1.28000000e+02, 4.73722937e-01, 6.00000000e+00],
                [5.70000000e+01, 1.40000000e+02, 4.79183235e-01, 5.00000000e+00],
                [1.07000000e+02, 1.44000000e+02, 4.81208453e-01, 5.00000000e+00],
                [7.90000000e+01, 1.36000000e+02, 4.91428433e-01, 3.00000000e+00],
```

```
       [5.10000000e+01, 1.45000000e+02, 4.98298432e-01, 3.00000000e+00],
       [7.20000000e+01, 1.54000000e+02, 4.99519577e-01, 5.00000000e+00],
       [1.31000000e+02, 1.37000000e+02, 5.12764429e-01, 4.00000000e+00],
       [9.10000000e+01, 1.33000000e+02, 5.19014270e-01, 3.00000000e+00],
       [2.00000000e+01, 7.00000000e+01, 5.37671619e-01, 2.00000000e+00],
       [1.38000000e+02, 1.49000000e+02, 5.92196264e-01, 6.00000000e+00],
       [1.42000000e+02, 1.48000000e+02, 6.06680317e-01, 7.00000000e+00],
       [3.40000000e+01, 1.14000000e+02, 6.34303030e-01, 3.00000000e+00],
       [1.34000000e+02, 1.43000000e+02, 6.73125645e-01, 1.00000000e+01],
       [1.09000000e+02, 1.63000000e+02, 6.93630998e-01, 5.00000000e+00],
       [1.90000000e+01, 8.80000000e+01, 7.26987998e-01, 2.00000000e+00],
       [2.70000000e+01, 1.64000000e+02, 7.38621102e-01, 3.00000000e+00],
       [6.20000000e+01, 1.62000000e+02, 7.78190207e-01, 5.00000000e+00],
       [1.26000000e+02, 1.61000000e+02, 8.00014146e-01, 7.00000000e+00],
       [1.27000000e+02, 1.47000000e+02, 8.18422473e-01, 7.00000000e+00],
       [1.40000000e+01, 1.57000000e+02, 8.86866276e-01, 6.00000000e+00],
       [1.39000000e+02, 1.65000000e+02, 9.08689026e-01, 8.00000000e+00],
       [1.46000000e+02, 1.59000000e+02, 9.62064229e-01, 6.00000000e+00],
       [5.80000000e+01, 1.53000000e+02, 9.98541792e-01, 6.00000000e+00],
       [1.51000000e+02, 1.60000000e+02, 1.06387033e+00, 6.00000000e+00],
       [2.80000000e+01, 1.66000000e+02, 1.08050850e+00, 8.00000000e+00],
       [1.58000000e+02, 1.67000000e+02, 1.09101798e+00, 8.00000000e+00],
       [1.56000000e+02, 1.68000000e+02, 1.22766930e+00, 1.60000000e+01],
       [1.52000000e+02, 1.79000000e+02, 1.35062609e+00, 1.00000000e+01],
       [1.69000000e+02, 1.72000000e+02, 1.45826959e+00, 1.00000000e+01],
       [3.50000000e+01, 1.71000000e+02, 1.49456176e+00, 4.00000000e+00],
       [1.73000000e+02, 1.78000000e+02, 1.54083535e+00, 1.30000000e+01],
       [1.76000000e+02, 1.81000000e+02, 1.64807863e+00, 1.60000000e+01],
       [1.80000000e+02, 1.84000000e+02, 1.75325751e+00, 1.80000000e+01],
       [1.55000000e+02, 1.86000000e+02, 1.91792647e+00, 1.50000000e+01],
       [1.75000000e+02, 1.88000000e+02, 1.93125299e+00, 2.40000000e+01],
       [1.77000000e+02, 1.85000000e+02, 2.15928506e+00, 1.00000000e+01],
       [1.74000000e+02, 1.82000000e+02, 2.29149696e+00, 2.30000000e+01],
       [1.70000000e+02, 1.87000000e+02, 2.90738778e+00, 1.80000000e+01],
       [1.83000000e+02, 1.89000000e+02, 3.58796363e+00, 2.50000000e+01],
       [1.90000000e+02, 1.91000000e+02, 4.39795306e+00, 3.40000000e+01],
       [1.92000000e+02, 1.93000000e+02, 4.41833809e+00, 4.10000000e+01],
       [1.94000000e+02, 1.96000000e+02, 5.87120715e+00, 6.60000000e+01],
       [1.95000000e+02, 1.97000000e+02, 7.22235680e+00, 1.00000000e+02]])
```

In [13]:
```python
dendrogram(mergings_complete)
plt.show()
```

```
In [14]: import numpy as np

         def update_cluster_centers(X, c):
             centers = np.zeros((4, 2))
             for i in range(1, 5):
                 ix = np.where(c == i)
                 centers[i - 1, :] = np.mean(X[ix, :], axis=1)
             return centers
```
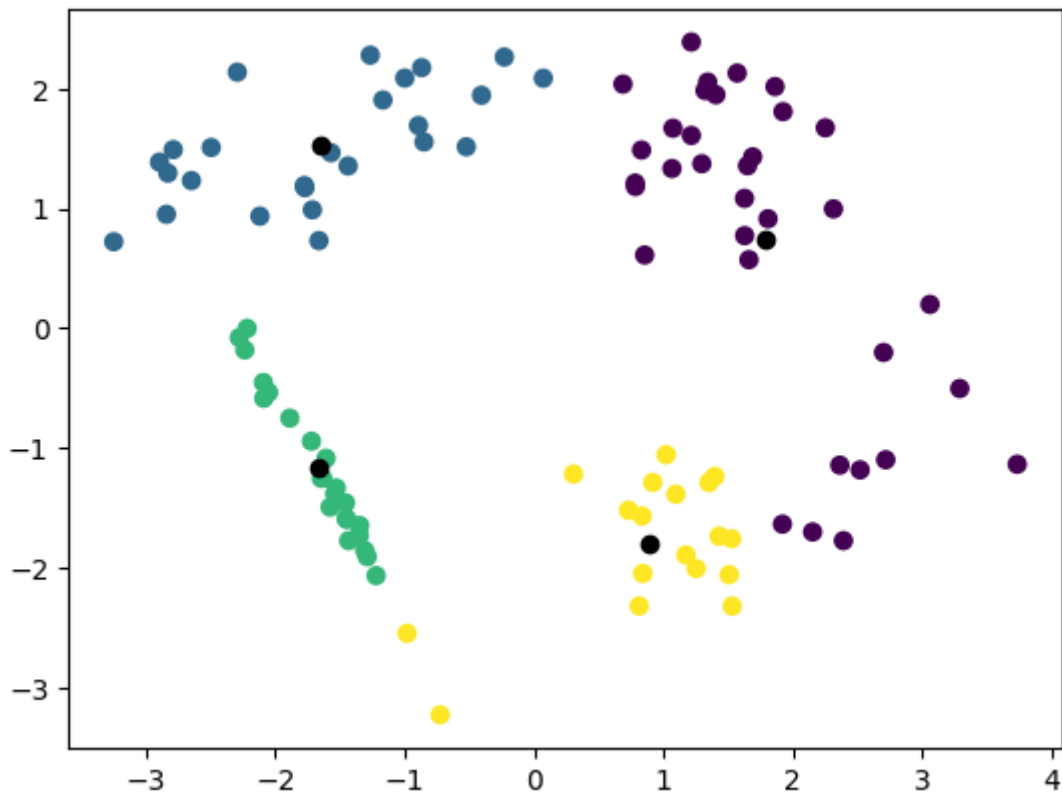
```
In [15]: from scipy.cluster.hierarchy import fcluster
```

```
In [16]: T = fcluster(mergings_complete, 4, criterion='maxclust')
         clusters = update_cluster_centers(X, T)
         clusters
```

```
Out[16]: array([[ 1.78328185,  0.75033971],
                [-1.64954738,  1.52368342],
                [-1.67057742, -1.16988193],
                [ 0.8884734 , -1.80177278]])
```

```
In [17]: plt.scatter(X[:, 0], X[:, 1], c=T)
         plt.scatter(clusters[:, 0], clusters[:, 1], c='black')
```

```
Out[17]: <matplotlib.collections.PathCollection at 0x23864b223e0>
```

Вычисление характеристик

In [18]:
```python
from sklearn.metrics.pairwise import euclidean_distances
```

In [19]:
```python
# Сумма квадратов расстояний до центроида (inertia)
sum_sq_dist = np.zeros(4)
for i in range(1, 5):
    ix = np.where(T == i)
    sum_sq_dist[i - 1] = np.sum(euclidean_distances(*X[ix, :], [clusters[i - 1]]
sum_sq_dist = np.sum(sum_sq_dist) / 4
sum_sq_dist
```

Out[19]: 31.087034614354266

In [20]:
```python
# Сумма средних внутрикластерных расстояний
sum_avg_intercluster_dist = np.zeros(4)
for i in range(1, 5):
    ix = np.where(T == i)
    sum_avg_intercluster_dist[i - 1] = np.sum(euclidean_distances(*X[ix, :], [cl
sum_avg_intercluster_dist = np.sum(sum_avg_intercluster_dist) / 4
sum_avg_intercluster_dist
```

Out[20]: 1.1148321806219426

In [21]:
```python
# Сумма межкластерных расстояний
sum_intercluster_dist = np.sum(euclidean_distances(clusters, clusters))
sum_intercluster_dist
```

Out[21]: 39.37587131636027

Кластеризация выборки методом k-средних

In [22]:
```python
from sklearn.cluster import KMeans
```

In [23]:
```python
models = []
predicted_values = []

for k in range(1, 11):
    kmeans = KMeans(n_clusters=k)
    kmeans.fit(X)
    models.append(kmeans)
    predicted_values.append(kmeans.predict(X))
```

In [24]:
```python
# Средней суммы квадратов расстояний до центроида
sum_sq_dist_avg = []
for it, kmean in enumerate(models):
    sum_sq_dist_avg.append(kmean.inertia_ / (it + 1))
sum_sq_dist_avg
```

Out[24]:
```
[545.7162442721157,
 144.93385251944756,
 54.50937795574206,
 20.56516779612909,
 12.702615089960599,
 7.573488639064396,
 5.197171687175967,
 4.034885804323963,
 3.1414549568038117,
 2.50427153253386]
```

In [25]:
```python
plt.plot(range(1, 11), sum_sq_dist_avg, '-o')
```

Out[25]:
```
[<matplotlib.lines.Line2D at 0x23817a7ef80>]
```
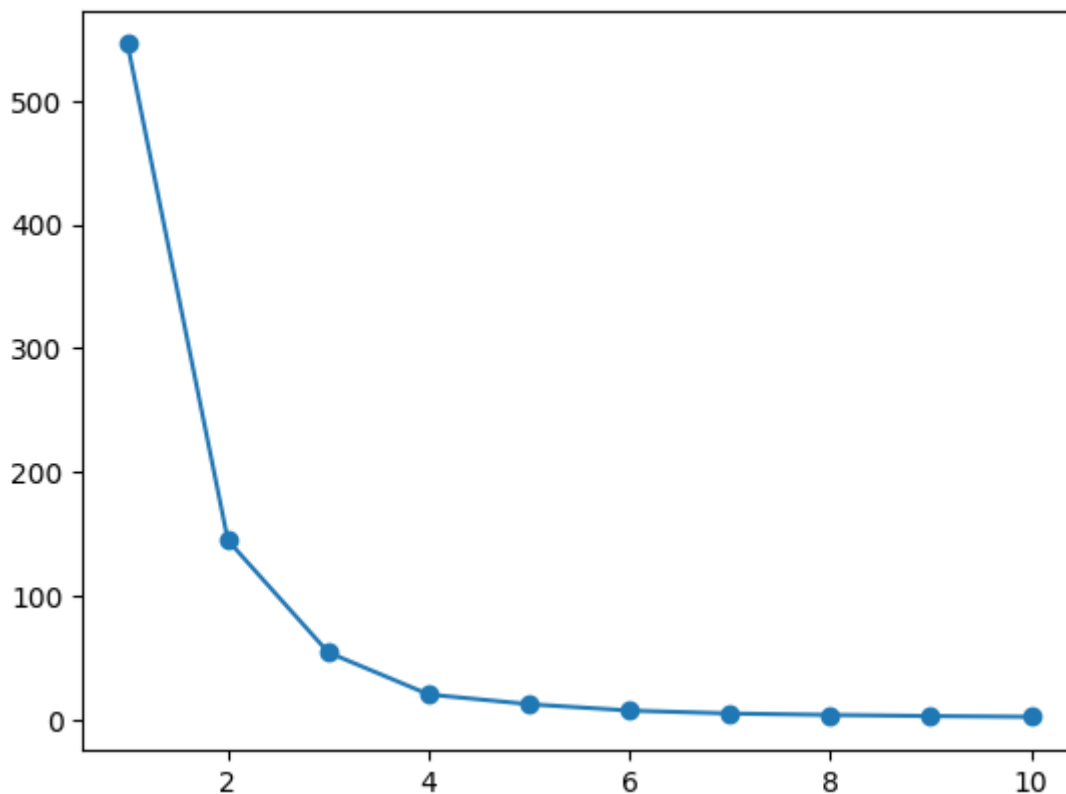
```python
# Средней суммы средних внутрикластерных расстояний
new_centers = [kmean.cluster_centers_ for kmean in models]

sum_avg_intercluster_dist_avg = []
for k, kmean in enumerate(models):
    intercluster_sum = np.zeros(4)
    for i in range(4):
        ix = np.where(predicted_values[k] == i)
        if len(ix[0]) == 0:
            intercluster_sum[i - 1] = 0
        else:
            intercluster_sum[i - 1] = np.sum(euclidean_distances(*X[ix, :], [kme
    sum_avg_intercluster_dist_avg.append(np.sum(intercluster_sum) / (k + 1))
sum_avg_intercluster_dist_avg
```

Out[26]: [5.457162442721157,
 13.134942139552361,
 13.21168207036577,
 14.25587147571919,
 10.089408399483421,
 11.172896629070635,
 7.346492245325464,
 7.423014177427671,
 10.74275476303583,
 4.536319517477247]

In [27]: `plt.plot(range(1, 11), sum_avg_intercluster_dist_avg, '-o')`

Out[27]: [<matplotlib.lines.Line2D at 0x2381ac67dc0>]
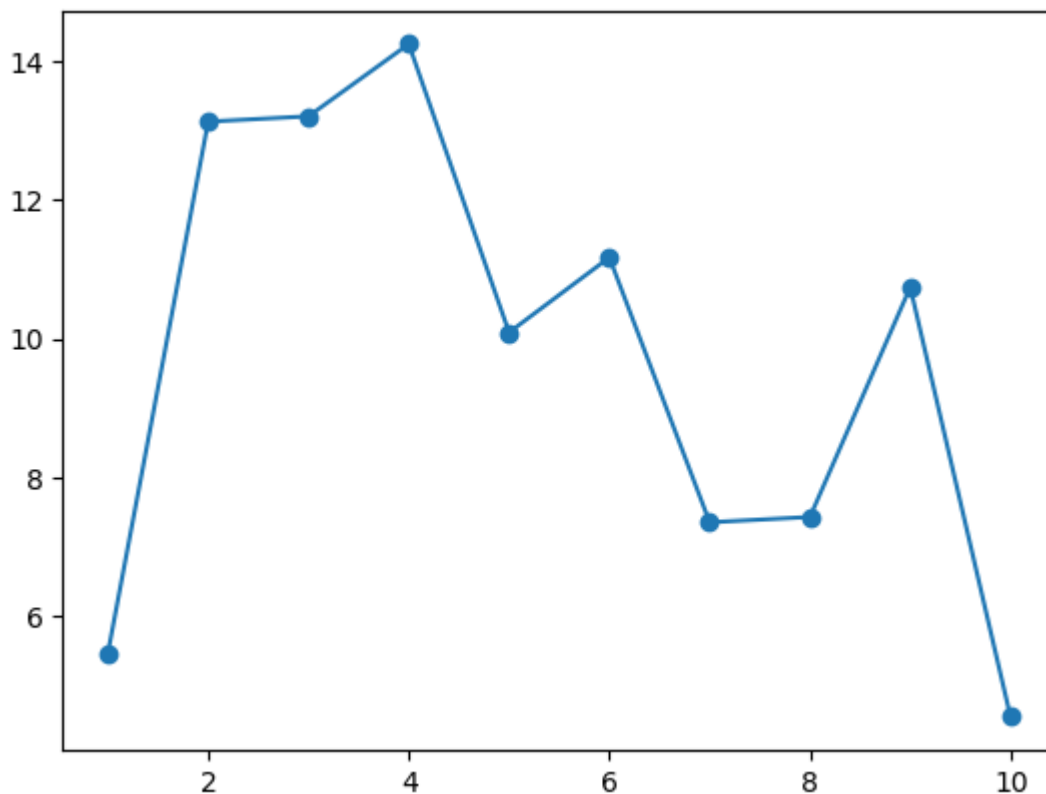
```
In [28]: # Средней суммы межкластерных расстояний от количества кластеров
         sum_intercluster_dist_avg = []

         for k, kmean in enumerate(models):
             value = np.sum(euclidean_distances(kmean.cluster_centers_, kmean.cluster_cer
             sum_intercluster_dist_avg.append(value / (k + 1))
         sum_intercluster_dist_avg
```

```
Out[28]: [0.0,
          3.1996932328025496,
          6.801441493599005,
          10.388727979540153,
          13.252244338636618,
          17.165246982909057,
          20.174583394404824,
          22.952887415646607,
          26.05386642784076,
          28.95120558692425]
```

```
In [29]: plt.plot(range(1, 11), sum_intercluster_dist_avg, '-o')
```

```
Out[29]: [<matplotlib.lines.Line2D at 0x2381acdbc70>]
```
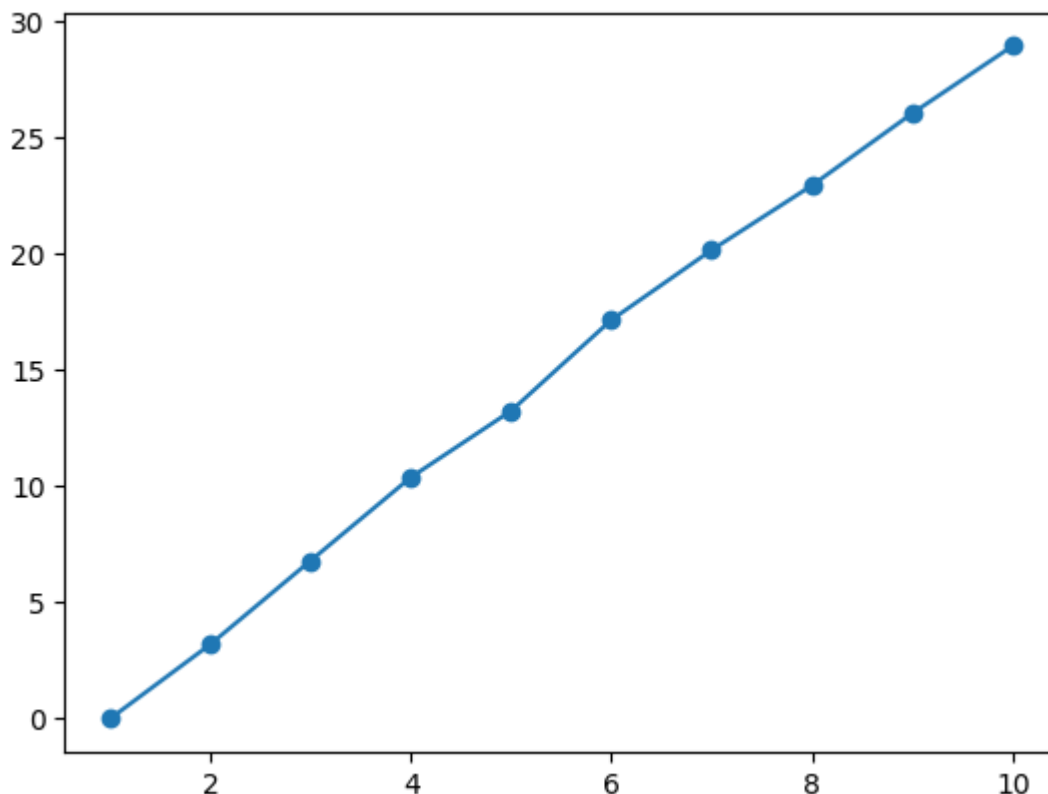
Составление сравнительной таблицы

```
In [30]:  import pandas as pd
```

```
In [31]:  columns = pd.MultiIndex.from_product([['Иерархический метод', 'Метод k-средних']
                                                ['Сумма квадратов расстояний до центроида'
          df = pd.DataFrame(columns=columns)
          df
```

Out[31]:

	Иерархический метод				Метод k-
	Сумма квадратов расстояний до центроида	Сумма средних внутрикластерных расстояний	Сумма межкластерных расстояний	Сумма квадратов расстояний до центроида	Сумма средних внутрикластерных расстояний	межкла рас

```
In [32]:  df['Иерархический метод', 'Сумма квадратов расстояний до центроида'] = [sum_sq_d
          df['Иерархический метод', 'Сумма средних внутрикластерных расстояний'] = [sum_av
          df['Иерархический метод', 'Сумма межкластерных расстояний'] = [sum_intercluster_

          df['Метод k-средних', 'Сумма квадратов расстояний до центроида'] = sum_sq_dist_a
          df['Метод k-средних', 'Сумма средних внутрикластерных расстояний'] = sum_avg_int
          df['Метод k-средних', 'Сумма межкластерных расстояний'] = sum_intercluster_dist_

          df
```

Out[32]:

	Иерархический метод			Метод К	
	Сумма квадратов расстояний до центроида	**Сумма средних внутрикластерных расстояний**	**Сумма межкластерных расстояний**	**Сумма квадратов расстояний до центроида**	**Сумма средних внутрикластерных расстояний**	**межкл ра**
0	31.087035	1.114832	39.375871	545.716244	5.457162
1	31.087035	1.114832	39.375871	144.933853	13.134942
2	31.087035	1.114832	39.375871	54.509378	13.211682
3	31.087035	1.114832	39.375871	20.565168	14.255871
4	31.087035	1.114832	39.375871	12.702615	10.089408
5	31.087035	1.114832	39.375871	7.573489	11.172897
6	31.087035	1.114832	39.375871	5.197172	7.346492
7	31.087035	1.114832	39.375871	4.034886	7.423014
8	31.087035	1.114832	39.375871	3.141455	10.742755
9	31.087035	1.114832	39.375871	2.504272	4.536320

In [33]:
```python
df.to_excel('result.xlsx')
```