Turing Machine 02

Thumrongsak Kosiyatrakul
tkosiyat@cs.pitt.edu

Turing Machine Example

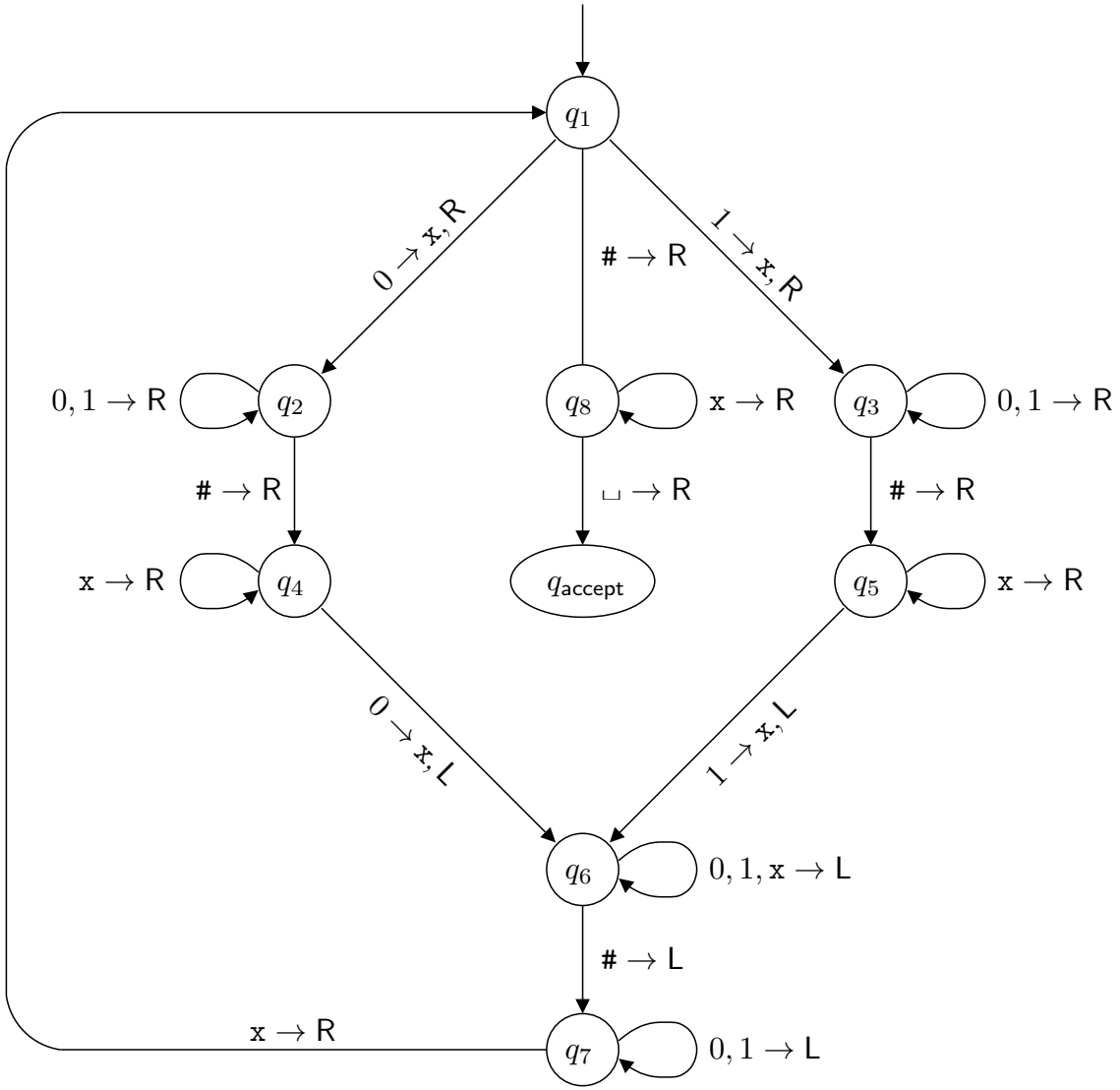
- Design a Turing machine that recognizes the language B where

$$B = \{w\#w \mid w \in \Sigma^*\}$$

- On input string w:
  1. Zig-zag across the tape to corresponding positions on either side of the # symbol to check whether those positions contain the same symbol. If they do not, or if no # is found, $reject$. Cross off symbols as they are checked to keep track of which symbols correspond.
  2. When all symbols to the left of the # have been crossed off, check for any remaining symbols to the right of the #. If any symbols remain, $reject$; otherwise, $accept$.

Turing Machine M_1

- A Turing machine that decides $B = \{w\#w \mid w \in \Sigma^*\}$.

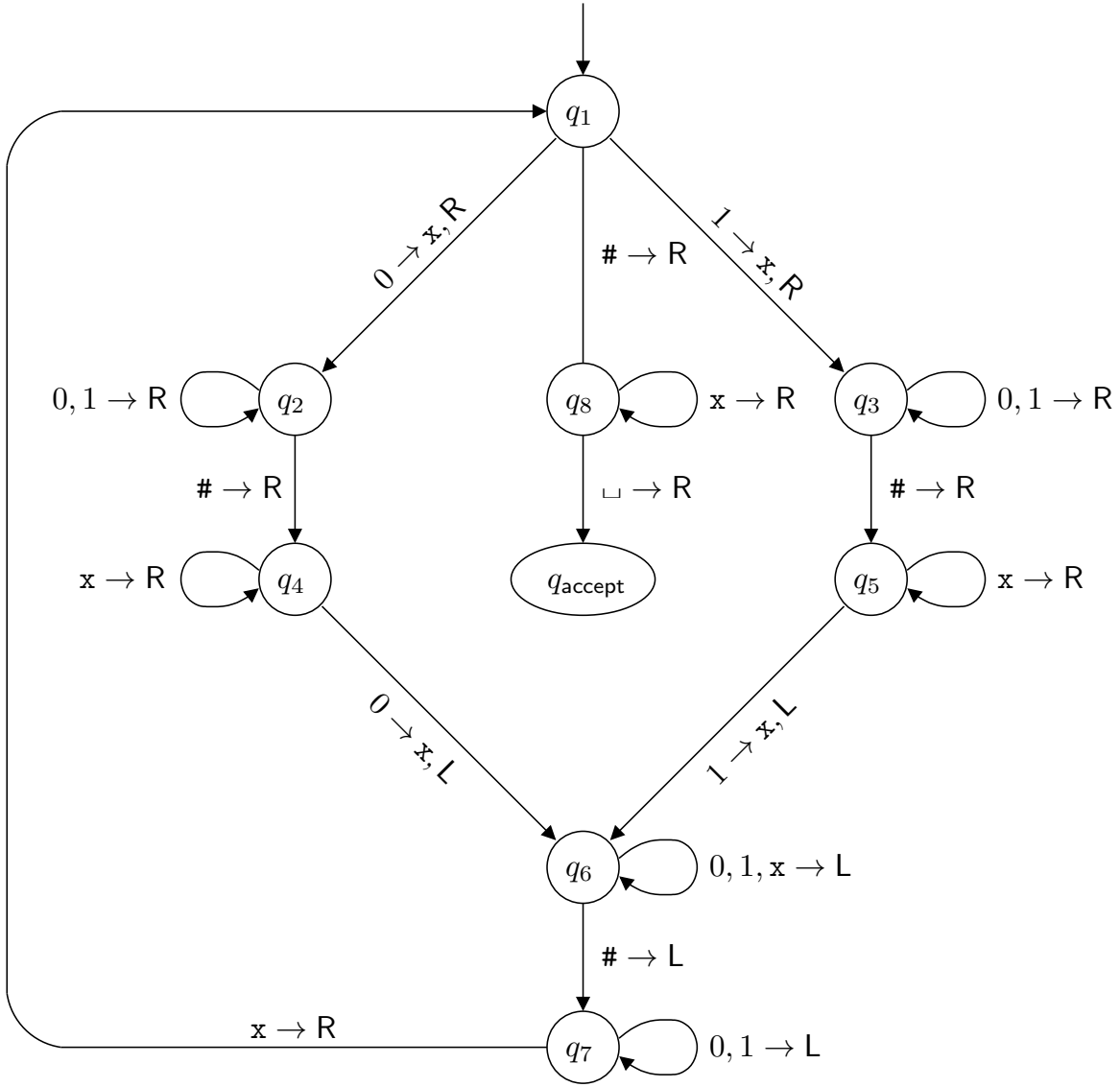


Turing Machine M_1

- $\Sigma = \{0, 1, \#\}$
- $\Gamma = \{0, 1, \#, \mathtt{x}, \sqcup\}$
- $1 \to \mathtt{x}, \mathrm{R}$
  - Read 1, write x, and move the tape head to the right direction
- $\# \to \mathrm{R}$
  - A shorthand notation for $\# \to \#, \mathrm{R}$
  - Read #, write #, and move the tape head to the right direction
- $0, 1 \to \mathrm{L}$
  - A shorthand notation representing two transitions:
    - $0 \to \mathrm{L}$ which is $0 \to 0, \mathrm{L}$
    - $1 \to \mathrm{L}$ which is $1 \to 1, \mathrm{L}$
- q_{reject} is omitted
  - All missing exiting arrows go to q_{reject}
  - Assume that it moves the tape head to the right direction before entering q_{reject} without writing a symbol (writing the same symbol it read)

Turing Machine M_1

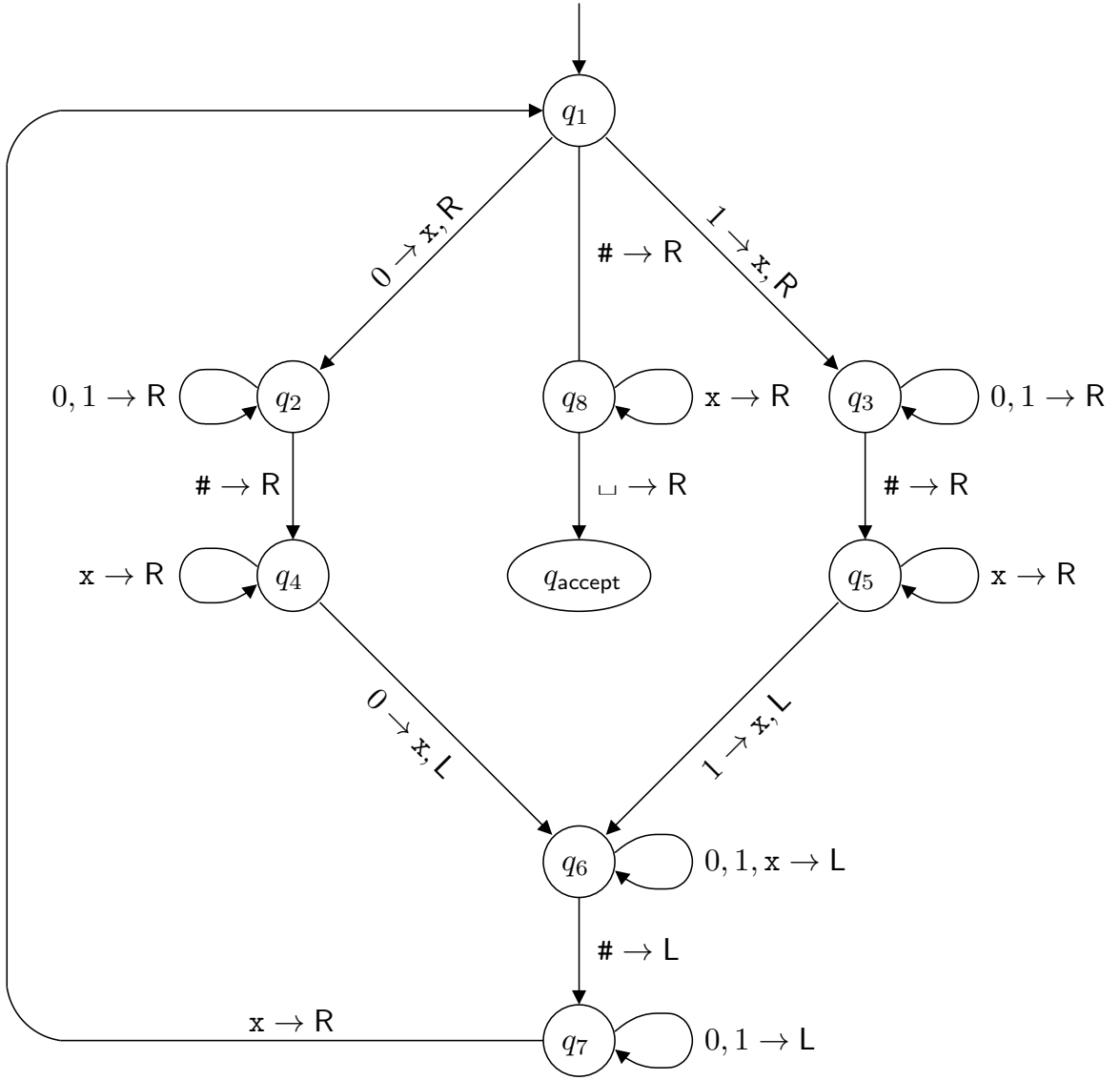
Computing 01#01



$q_1 01\#01$
$\mathtt{x} q_2 1\#01$
$\mathtt{x}1 q_2 \#01$
$\mathtt{x}1\# q_4 01$
$\mathtt{x}1 q_6 \#\mathtt{x}1$
$\mathtt{x} q_7 1\#\mathtt{x}1$
$q_7 \mathtt{x}1\#\mathtt{x}1$
$\mathtt{x} q_1 1\#\mathtt{x}1$
$\mathtt{xx} q_3 \#\mathtt{x}1$
$\mathtt{xx}\# q_5 \mathtt{x}1$
$\mathtt{xx}\#\mathtt{x} q_5 1$
$\mathtt{xx}\# q_6 \mathtt{xx}$
$\mathtt{xx} q_6 \#\mathtt{xx}$
$\mathtt{x} q_7 \mathtt{x}\#\mathtt{xx}$
$\mathtt{xx} q_1 \#\mathtt{xx}$
$\mathtt{xx}\# q_8 \mathtt{xx}$
$\mathtt{xx}\#\mathtt{x} q_8 \mathtt{x}$
$\mathtt{xx}\#\mathtt{xx} q_8 \sqcup$
$\mathtt{xx}\#\mathtt{xx} \sqcup q_{\text{accept}}$

Turing Machine M_1

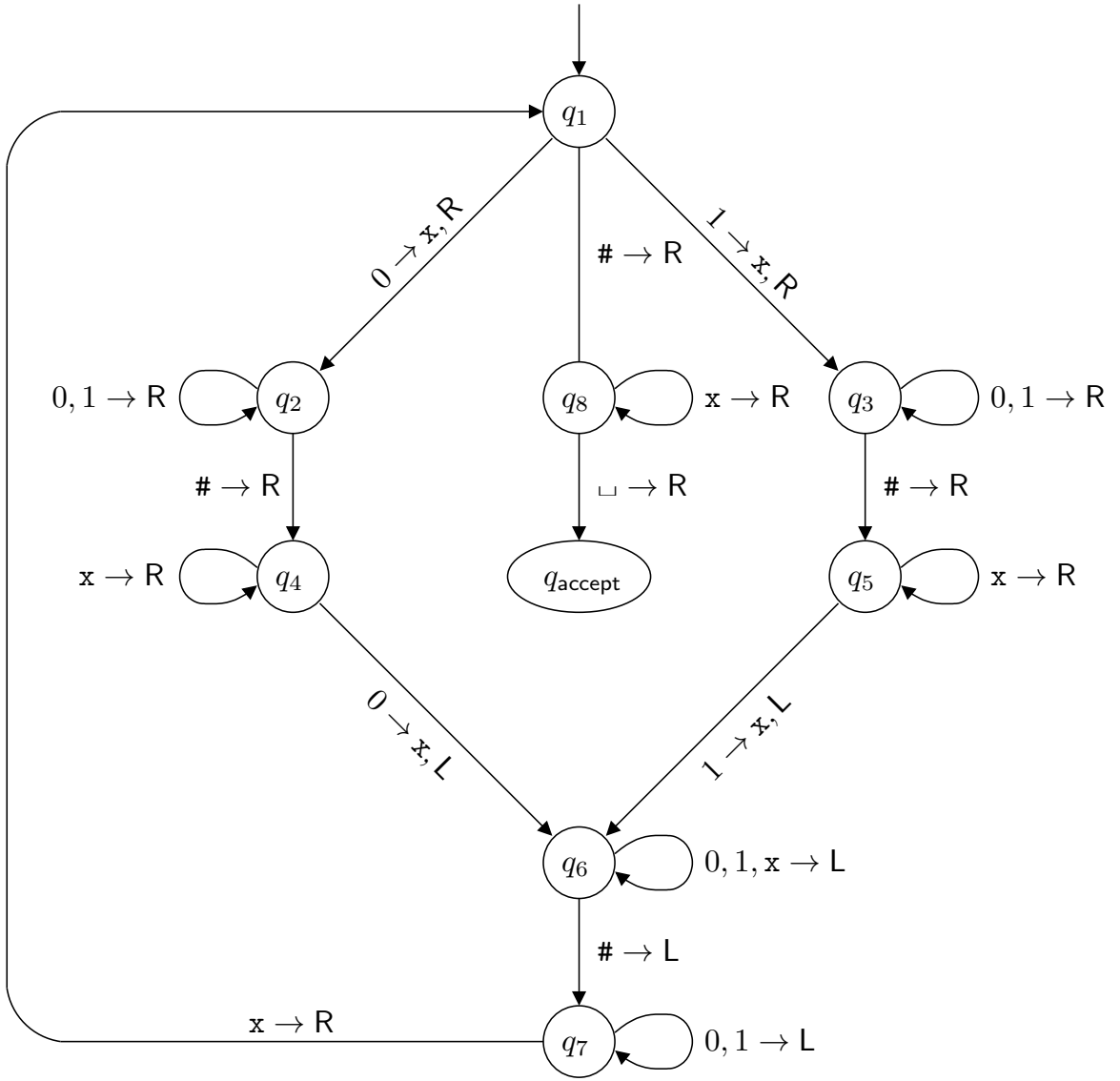
Computing 01#10



$q_1 01\#10$
$\mathtt{x} q_2 1\#10$
$\mathtt{x}1 q_2 \#10$
$\mathtt{x}1\# q_4 10$
$\mathtt{x}1\#1 q_{\text{reject}} 0$

Turing Machine M_1

Computing 01 (no # symbol)



$q_1 01$
$\mathtt{x} q_2 1$
$\mathtt{x} 1 q_2 \sqcup$
$\mathtt{x} 1 \sqcup q_{\text{reject}}$

Turing Machine Example

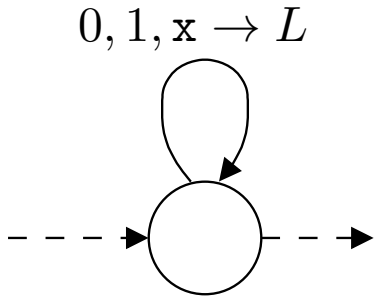
- Design a Turing machine that recognizes the language A where

$$A = \{0^{2^n} \mid n \geq 0\}$$

- On input string w:
  1. Sweep left to right across the tape, crossing off every other 0
  2. If in stage 1 the tape contained a single 0, $accept$
  3. If in stage 1 the tape contained more than a single 0 and the number of 0s was odd, $reject$
  4. Return the head to the left-hand end of the tape
  5. Go to stage 1

Moving the Tape Head Back

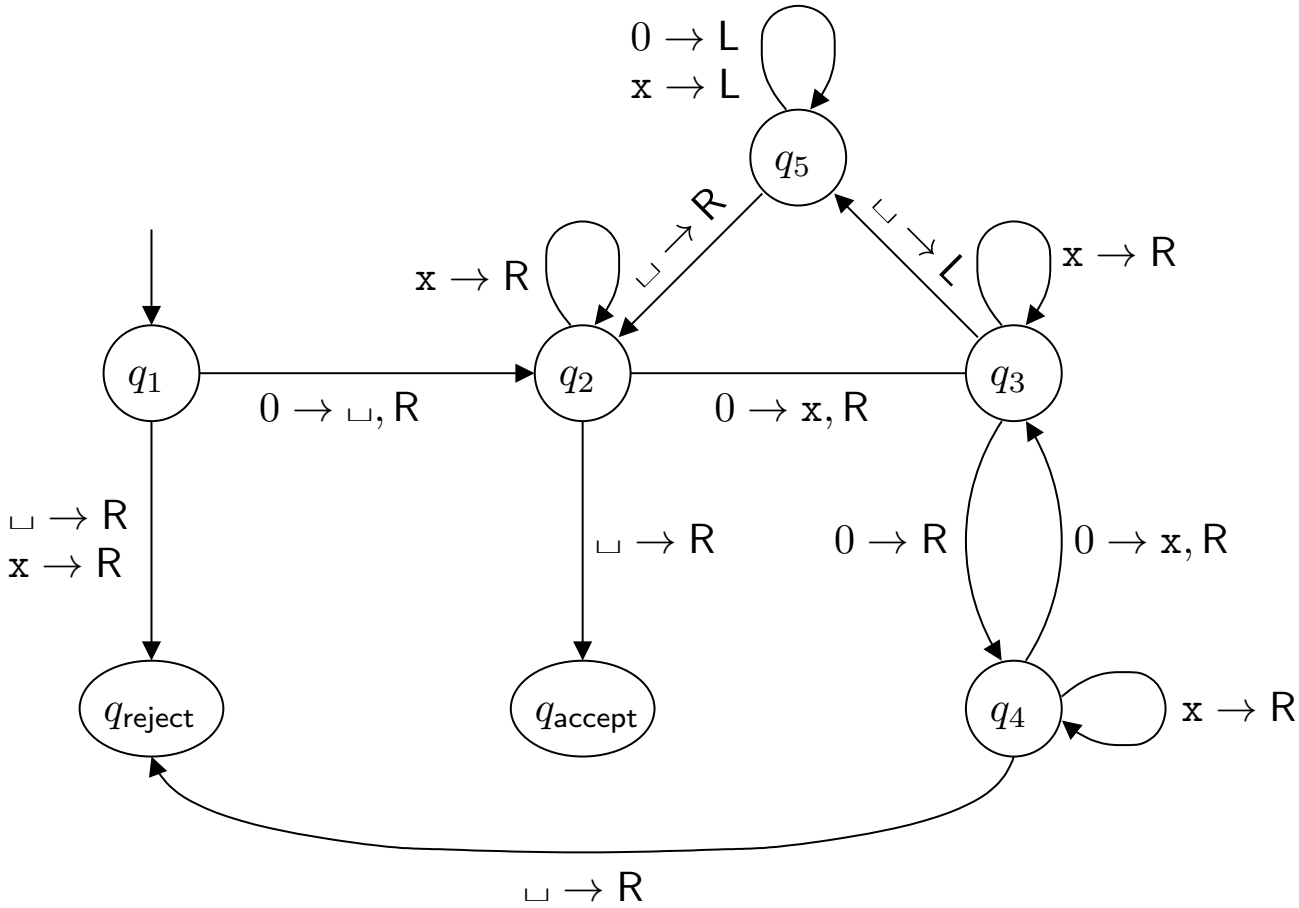
- Suppose we want to move the tape head back to the left most end of the tape
  - Need a state to do that



  - This will result in an infinite loop
- Recall that if the tape head is at the left-most square and the transition is L, it will stay at the same place
  - There is no signal telling a TM that its tape head is currently at the left-most square
- The trick is to mark the left-most symbol (depending on Γ)
  - Turn 0 or 1 to $\sqcup$
  - Turn 0 to $\dot{0}$ or $\mathtt{x}$
  - Turn 1 to $\dot{1}$ or $\sqcup$

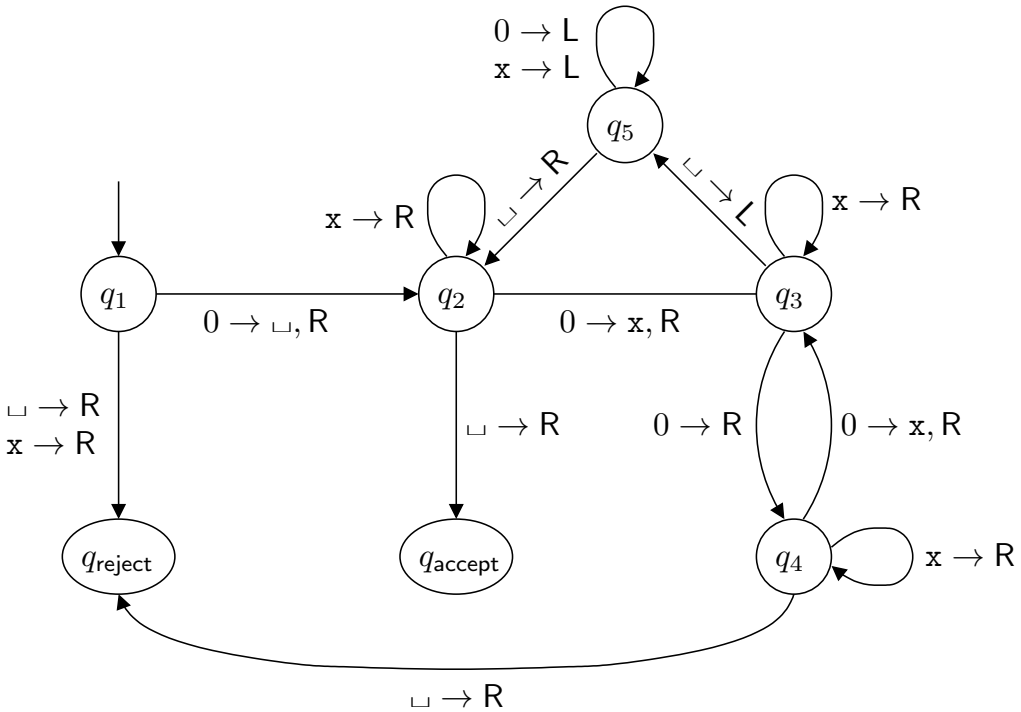
Turing Machine M_2

- A Turing machine that decides $A = \{0^{2^n} \mid n \geq 0\}$.



Turing Machine M_2

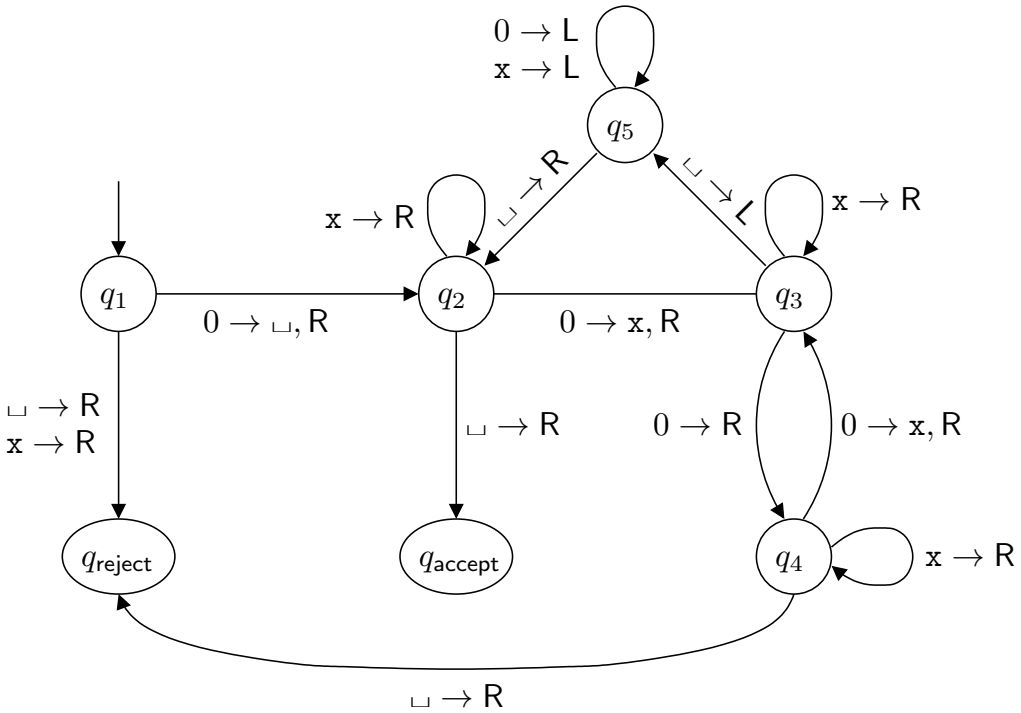
Computing 0000



$q_1 0000$
$\sqcup q_2 000$
$\sqcup \mathtt{x} q_3 00$
$\sqcup \mathtt{x} 0 q_4 0$
$\sqcup \mathtt{x} 0 \mathtt{x} q_3 \sqcup$
$\sqcup \mathtt{x} 0 q_5 \mathtt{x}$
$\sqcup \mathtt{x} q_5 0 \mathtt{x}$
$\sqcup q_5 \mathtt{x} 0 \mathtt{x}$
$q_5 \sqcup \mathtt{x} 0 \mathtt{x}$
$\sqcup q_2 \mathtt{x} 0 \mathtt{x}$
$\sqcup \mathtt{x} q_2 0 \mathtt{x}$
$\sqcup \mathtt{xx} q_3 \mathtt{x}$
$\sqcup \mathtt{xxx} q_3 \sqcup$
$\sqcup \mathtt{xx} q_5 \mathtt{x}$
$\sqcup \mathtt{x} q_5 \mathtt{xx}$
$\sqcup q_5 \mathtt{xxx}$
$q_5 \sqcup \mathtt{xxx}$
$\sqcup q_2 \mathtt{xxx}$
$\sqcup \mathtt{x} q_2 \mathtt{xx}$
$\sqcup \mathtt{xx} q_2 \mathtt{x}$
$\sqcup \mathtt{xxx} q_2 \sqcup$
$\sqcup \mathtt{xxx} \sqcup q_{\text{accept}}$

Turing Machine M_2

Computing 000



q_1000
$\sqcup q_200$
$\sqcup \mathtt{x} q_30$
$\sqcup \mathtt{x} 0 q_4 \sqcup$
$\sqcup \mathtt{x} 0 \sqcup q_{\text{reject}}$

Designing a TM

- Designing a TM in a form of a state diagram is hard
  - Need to keep in mind out the content of the tape
  - Need to worry about moving tape head
- But for a simple problem, it is not that bad
- Let's create a TM that shift all symbols on the tape to the right by one square by inserting the blank symbol at the left-most square and move the tape head back to the left-most square
  - Let $\Sigma = \{0, 1\}$ and $\Gamma = \{0, 1, \sqcup\}$
  - Here are some input/output (on the tape)

Input	Output
01101	$\sqcup 01101$
0	$\sqcup 0$
111	$\sqcup 111$
ε	ε

  - Note that this TM always accepts all strings over Σ

Designing a TM

- Solution