Differential Equations	Name
MAA121/MAG131 Sheet 1	Number
Hand in by 11/02/2010	Year
	Mark: **/10**
	date marked: / /2010

Please attach your working, with this sheet at the front.

These handouts, Lecture Notes and other support materials can be downloaded at:
`http://www-maths.swan.ac.uk/staff/vm/VMteaching.html`

The mark for this assignment does not count towards your final mark.

1. Plutonium 239, virtually nonexistent in nature, is one of the radioactive materials, used in the production of nuclear weapons, and is a byproduct of the generation of power in a nuclear reactor. Its half–life is approximately 24 000 years. What is the value of the decay rate k in the model for the radioactive decay (see p.1 of the Lecture Notes) that should be used for this isotope ?

2. In 1947 a large collection of papirus scrolls, including the oldest known manuscript version of portions of the Old Testament, was found in a cave near Dead Sea; they have come to be known as the 'Dead Sea Scrolls'. The scroll containing the book of Isaiah was dated in 1994 using the radiocarbon technique; it was found to contain between 75% and 77% of the initial level of Carbon-14. Between which dates was the scroll written ?

3. A round table hangs on the wall of the castle in Winchester. Many would like to believe that this is the Round Table of King Arthur, who (so legend would have it) was at the height of his powers in about AD 500. If the table dates from this time, what proportion of the original Carbon-14 would remain? In 1976 the table was dated using the radiocarbon technique, and 91.6% of the original quantity of Carbon-14 was found. From when the table date ?

4. Radiocarbon dating is an extremely delicate process. Suppose that the percentage of Carbon-14 is known to lie in the range $0.99p$ to $1.01p$. What is the range of possible dates for the sample ?

Differential Equations
MAA121/MAG131 Sheet 1 (Solutions)

1. The half–life in the radioactive decay model (1) is

$$T_{1/2} = \frac{\log(2)}{k}.$$

If $T_{1/2} = 24000$ then

$$k = \frac{\log(2)}{T_{1/2}} \simeq 2.9 \times 10^{-5}.$$

2. We have

$$N(1994) = pN(T_0) = N(T_0)e^{-k(1994-T_0)},$$

where $0.75 \leq p \leq 0.77$. Taking the logarithm gives

$$\log(p) = -k(1994 - T_0),$$

and so

$$T_0 = 1994 + \frac{\log p}{k}.$$

where $k = 1.216 \times 10^{-4}$, according to (4). This gives (approximately)

$$-372 < T_0 < -155,$$

dating scrolls between $372\,BC$ and $155\,BC$.

3. If the table dates from $500\,AD$ then we would expect

$$N(t) = N(500)e^{-k(t-500)},$$

and so in 2008 we have

$$N(2008) = N(500)e^{-1508\,k}.$$

The proportion of Carbon-14 isotopes remaining should there be

$$e^{-1508\,k} \simeq 83\%.$$

However, we in fact have 91.6% remaining in 1976. Therefore, by formula (5),

$$T_0 = 1976 + \frac{\log 0.916}{k} \approx 1255,$$

so the table probably dates from the rein of the English King Edward I, who took the throne in $1270\,AD$ (once the wood was well seasoned) and had a passion for all things Arthurian.

4. Suppose that a proportion αp of the original Carbon-14 remain. Then

$$\alpha p N(T_0) = N(t) = N(T_0)e^{-k(t-T_0)}.$$

Taking the logarithm on both sides and using $\log(\alpha p) = \log(\alpha) + \log(p)$, we obtain

$$\log(\alpha) + \log(p) = -k(t - T_0).$$

Hence

$$T_0 = t + \frac{\log p}{k} + \frac{\log \alpha}{k}.$$

Denote by S_0 the value of the above expression when $\alpha = 1$, that is

$$S_0 = t + \frac{\log p}{k}.$$

Then for $\alpha = 0.99$ we obtain

$$T_0 \simeq S_0 - 82.65,$$

while for a proportion $1.01p$ we obtain

$$T_0 \simeq S_0 + 81.83.$$

Thus 1% error can give a difference of over 160 years in the estimated date!