Swansea Summer School in Nonlinear PDEs

Minicourses abstracts

Martino Bardi

Università degli Studi di Padova

Fully nonlinear degenerate elliptic equations: qualitative properties of viscosity solutions

Abstract. The course is an introduction to the theory of viscosity solutions to fully nonlinear degenerate elliptic equations. After presenting some model problems and motivating the definitions, I will illustrate the basic properties of viscosity solutions, in particular stability and the comparison principles.
In the second part I'll discuss two qualitative properties of subsolutions: the Strong Maximum Principle, i.e., if a subsolution in an open connected set attains an interior maximum then it is constant, and the Liouville property, i.e., if a subsolution in the whole space is bounded form above then it is constant. They are standard results for classical solutions of linear elliptic PDEs, and many extensions are known. I will show how the viscosity methods allow to turn around the difficulties of non-smooth solutions, fully nonlinear equations, and their possible degeneracies

Serena Dipierro

University of Western Australia

Nonlocal equations and applications

Abstract. We present some equation of nonlocal type arising in material sciences (atom dislocations in crystals) and biology (optimal foraging). Topics will include:

- Averaging procedures and the Malmheden Theorem
- All functions are locally s-harmonic
- Harmonic extensions
- Atom dislocations in crystals

Louis Jeanjean
Université de Franche-Comté

On nonlinear quantum graphs

Abstract. The nonlinear Schrodinger equation is a ubiquitous model in physics, with many applications in fields as diverse as Bose-Einstein condensation or non-linear optics. In many physical situations, the underlying space (i.e. the domain on which the solutions of the equation are defined) is essentially one-dimensional and can be modeled as a metric graph, also called a quantum graph, i.e. a locally one-dimensional domain obtained by gluing together one–dimensional, possibly unbounded intervals, the edges, through the identification of some of their endpoints, the vertices. The mathematical study of this type of model is very recent and is growing incredibly. As when the non-linear Schrodinger equation is placed on the entire space the study of stationary solutions is the subject of special attention. One can then focus either on solutions with a given frequency or on solutions having a prescribed mass, namely a prescribed L2 norm. This second type of solution is particularly interesting from a physical point of view, as this quantity is preserved over time.
In this course, we will concentrate on this second type of solution. First, we will look at the mass sub-critical case, whose study naturally leads us to consider minimization problems. Secondly, we will study the mass-supercritical case, where a minimization approach is no longer possible. In particular, we will present the various tools, such as variational methods, blow-up techniques or Morse index estimates, that are needed to deal with this case.

Enrico Valdinoci
University of Western Australia

Nonlocal minimal surfaces and phase transitions

Abstract. We present some results about nonlocal perimeters and their applications to models of capillarity and long-range phase transitions. Topics will include:

- Landau theory of phase transitions and long-range interactions
- Gamma-convergence and convergence of level sets
- Inequalities from differential and integral geometry
- Regularity of (non)local minimal surfaces
- A conjecture of De Giorgi

Jean Van Schaftingen

UCLouvain

Sobolev mappings into manifolds

Abstract. Sobolev mappings arise naturally in many partial differential equations and calculus of variations problems arising in physical models, geometry and computer graphics. Although they can be naturally defined as Sobolev functions satisfying almost everywhere a nonlinear manifold constraint, not all the properties of classical Sobolev spaces pass to the corresponding spaces of mappings. In particular the approximation by smooth maps and the surjectivity of the traces on fractional Sobolev spaces fail when the integrability exponent is small. The obstacles to these properties are intimately connected to obstruction theory and the finiteness or triviality of some homotopy groups of the target manifold. In these lectures, I will describe these pathologies and the nonlinear constructions leading to positive results, and relate them to the lifting and homotopy problems.

Guest lectures abstracts

John Ball

Heriot-Watt University and Maxwell Institute for Mathematical Sciences

Understanding material microstructure

Abstract. Under temperature changes or loading, alloys can form beautiful patterns of microstructure that largely determine their macroscopic behaviour. These patterns result from phase transformations involving a change of shape of the underlying crystal lattice, together with the requirement that such changes in different parts of the crystal fit together geometrically. Similar considerations apply to plastic slip. The lecture will explain both successes in explaining such microstructure mathematically, and how resolving deep open questions of the calculus of variations could lead to a better understanding.

Nicolas Dirr

Cardiff University

Fluctuations of SPDES and interacting particle systems

Abstract. Stochastic Partial Differential Equations and interacting particle systems are both meant to model the same physical reality, i.e. a deterministic evolution subject to stochastic fluctuations. Here we focus on the Simple Symmetric Exclusion Process and its deterministic limit under diffusive scaling, the heat equation. If we take into account fluctuations, then the large deviations rate functionals and fluctuation fields of the particles coincide formally with those of a stochastic heat equation with nonlinear divergence form noise. This equation has no solution for white noise, i.e. for randomness on all scales. The particles however, have a natural cut off on small scales: The particle size. This indicates

that the SPDE is a useful approximation of the particle system at intermediate scales much larger than the particle size but smaller than the macroscopic scale. On these scales, regularized noise is a natural choice and for such noise a solution theory has been developed by B. Fehrman and B. Gess. We will sketch this concept of solution and explain the Large Deviations and Central Limit Theorem for them. Then we will present numerical evidence to show that the SPDE nonlinear noise term is a good approximation for the particle systems on intermediate scales. This is joint work with Benjamin Fehrman and Benjamin Gess.

Juan Davila

University of Bath

Overhanging solitary waves with vorticity

Abstract. We find solutions for an overdetermined elliptic problem, resulting in solitary waves that are non-graphical, within a fluid with constant vorticity. Although numerical evidence supports their existence, constructing these nearly singular solutions remains challenging using complex variables or bifurcation theory. Our method resembles the desingularisation process used for constant mean curvature surfaces. This work is a collaboration with Manuel del Pino, Monica Musso, and Miles Wheeler from the University of Bath.

Xue-Mei Li

Imperial College London

Non-linear wave equation with rough random Gaussian noise

Abstract. We consider a stochastic wave equations on the 2 dinensional torus with non-linear potential, with distributional valued noise. I will discuss an attempt with X. Ren, attempting to make sense of a solution theory by a renormalization procedure.

Florian Theil

University of Warwick

Convergence of particle systems to kinetic equations

Abstract. We consider the asymptotic behaviour of solutions of particle systems in the limit where the number of particles tends to infinity. In the case of short-range interactions and small densities it is expected that the solutions approximate solutions of the Boltzmann equation. Since the convergence is only known for short time, we consider regularisations of the particle evolution where the expected number of collisions per particle is bounded. Here we can establish convergence for large times and study finite-size corrections.