

The first LMS Bath-WIMCS Analysis Day took place in Swansea on Friday 20th March:

<http://math.swansea.ac.uk/staff/vm/LMS-WIMCS-Bath-2015/>

Our intention was that each meeting in this series should involve a selection of topics from those proposed in the application, to encourage the widest possible attendance. In fact all three topics (calculus of variations and nonlinear PDEs; asymptotics, homogenisation and applications; spectral theory and related topics) were represented in this meeting, although a common theme emerged around homogenisation which appeared in different guises in four of the five talks.

Federica Dragoni (Cardiff) spoke on ‘Stochastic homogenisation for Hamilton-Jacobi equations’. She reviewed two 1999 papers (of Souganidis and of Rezakhanlou and Tarver) which deal with the equation

$$u_t^\varepsilon + H(x/\varepsilon, Du, \omega) = 0, \quad x \in \mathbb{R}^N,$$

in which ω is a random variable and the Hamiltonian H satisfies a special Lipschitz condition with respect to its first argument. It turns out that if H is stationary ergodic then the homogenised problem is deterministic, in the sense that the viscosity solutions converge to a function which does not depend on ω .

In the second part of her talk, Dr Dragoni spoke about her own recent work with Mannucci and Marchi, dealing with stochastic homogenisation for a Hamiltonian of the form $H(x/\varepsilon, \sigma(x)Du, \omega)$ in which σ is a matrix of Carnot type. Carnot groups are anisotropic at every scale, so the scale x/ε has to be adapted to the new group structure and additional difficulties arise from the fact that the method of approximating initial conditions by affine functions now fails.

Alexander Kiselev (Lviv, Ukraine) spoke on ‘Mathematical modelling of scattering in the presence of metamaterial objects’. His talk reviewed the models in the literature to date and presented recent joint work with Cherednichenko in one and two dimensions, in particular on the rigorous construction of scattering matrix in the low-energy limit. Their approach allows not only a superlens of an arbitrary geometry, but also illuminates the origin and particulars of the process of cloaking. They showed that the problem of cloaking can be explained in terms of ‘resonances’ for the compact scatterer, whose spectral properties give rise to cloaking.

Valery Smyshlyayev (UCL) spoke on ‘Two-scale homogenisation of a general class of high-contrast PDE systems’. The first part of this talk was a review suitable for a general audience of mathematical analysts, while the second part presented joint work with Cooper (formerly Leverhulme-WIMCS Fellow in Cardiff, now in Montpellier, soon to move to Bath) and Kamotski (UCL). For a general class of high-contrast PDE systems, they show that under a decomposition assumption a two-scale version of the strong resolvent convergence holds, with a well-defined two-scale limit operator. This implies two-scale convergence of semigroups with applications to a wide class of micro-resonant time-dependent problems.

Matthew Lettington (Cardiff) gave our only talk in analytic number theory, on ‘Higher dimensional interlacing Fibonacci sequences, continued fractions and Chebyshev polynomials.’ He presented joint work with Hindmarsh and Pryce studying higher-dimensional interlacing Fibonacci sequences and their multi-dimensional continued fractions, generated both from Chebyshev type functions and m -dimensional recurrence relations. For each m , there exist both rational and integer versions of these sequences and the underlying p -adic structure of the rational sequence allows recovery of enables the integer sequence. In particular, for the positive index sequences, one can clear fractions by knowing the number of prime divisors of $2m+1$; in the negative index case the ‘excess’ prime factors can be removed using Weisman’s congruence. When $2m+1$ is a prime these two processes come into alignment. From either the rational or the integer sequences, Lettington showed how to construct a continued fraction vector in \mathbb{Q}^m , which converges to an irrational algebraic point in \mathbb{R}^m . The sequence terms can be expressed as simple recurrences, trigonometric sums, binomial polynomials and as sums over ratios of powers of the diagonals of the regular unit m -gon. These sequences exhibit a ‘rainbow’ quality, corresponding to the Fleck numbers at negative indices and the m -dimensional Fibonacci numbers at positive indices. The families of orthogonal generating polynomials defining the recurrence relations employed turn out to be divisible by the minimal polynomials of certain algebraic numbers, and the three-term recurrences and differential equations for these polynomials can be found.

Lucia Scardia (Bath) spoke on ‘Homogenisation of dislocation dynamics’. Plastic deformation of a metal is caused by the movement of curve-like defects (called dislocations) in its crystal lattice. It is not known how to use this microscale information to make theoretical predictions at the macroscopic scale. Nevertheless, macroscopic plasticity is heavily dependent on dynamic properties of the dislocation curves. Mora, Peletier and Scardia recently upscaled a time-dependent system of discrete, interacting dislocations by combining Gamma-convergence methods with the theory of rate-independent systems. In the continuum limit they obtain an evolution law for the dislocation density. The talk presented this result and possible extensions to more realistic complex systems.

Attendance	Male/Female
Invited speakers	3/2
Research students (host inst)	4/2
Research students (other insts)	5/0
Other participants (host inst)	6/1
Other participants (other insts)	2/0
Total	20/5