Learning and the Global Supply Chain

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very primary and incomplete

Abstract

In this paper I combine Costinot et al.(2012) with idea flow theory to investigate the dynamics of global supply chains. The nature of sequential production leads to positive assortative matching: it assigns more skilled workers, who make less mistakes in all stages, to higher stages in the supply chain, which yield higher wage. A worker could split his time to work for income, or search for random meeting with other workers. After meeting, the worker's skill level would be the maximum of his and his opponent's. I characterize the steady state equilibrium where individuals optimally allocate time, skill distribution is invariant, and markets clear.There might be multiple equilibria, both are self-sustaining. Then I use the model to study the dynamic effects of trade liberalization between North and South, and find that: 1) trade and knowledge liberalization can lead to one global equilibrium; 2) exogenous life expectancy could maintain divergence: people with short life expectancy do not invest much in search, and they are specialized in low skill jobs; 3) under communication frictions (the extreme case is that one can only meet others in the same country), divergence might also persist: individuals in skill-abundant country are more willing to search because it's easier to meet a high skill worker. Nevertheless, in case 2) and 3), divergence is alleviated under liberalization compared with autarky.

notes on possible revision 20160524:

1) property right as the source of comparative advantage, instead of life expectancy

2) middle income traps

3) on empirics:

3a) stylized facts of the relation between skill endowment and GSC placement in intro

3b) measure the relationship between skill distribution(eg, college graduate percentage; education institution quality; superior firm rate, innovative/patent percentage, etc) and international specialization status;

3c) how much does it cost to promote a middle income country up to high-income group?

4) extensions: sequential production as the origin of skill-tech complementarity

1 Introduction

background

1)emergence of global supply chain/fragmentation/
vertical specialization/outsourcing/offshoring
2)wage, schooling and inequality under trade liberalization

relation to the literature

1) sequential production, vertical specialization, esp. GSC

Costinot et al.(2012, 2013), Antras and Chor(2013), Baldwin and Venerables(2013), Yi(2003)

earlier papers: Findlay(1978), Dixit and Grossman(1982), Sanyal(1983), Kremer(1983)

2) idea flow theory

Lucas and Moll(2014), Perla and Tonetti(2014), Luttmer(2012), Staley(2011)

based on mean field games: Larsy and Lions (2006, 2007, 2010), Achdou et al.(2014) provide an excellent survey

search-theoretic models for knowledge growth: Javanovic and Rob(1989), Kortum(1997), Eaton and Kortum(2002), Lucas(2009)

applications: Alveraz et al.(2013), Buera and Oberfield(2016), Sampson(2016), Ceicedo et al.(2016),Comin et al.(2012)

3) trade, labor market dynamics and education/human capital investment

Verhoogen(2008), Atkin(2012), Shepherd(2013), Blanchard and Willmann(2016), Acemoglu et al.(2015), Oster and Steinberg(2013), Caselli(2014), Dix-Carneiro(2014), Davidson and Sly(2014), Auer(2015), Greenland and Lopresti(2016)

2 the model

2.1 brief review of Costinot et al.(2012)

there is measure 1 worker, labeled by skill $s \in [\underline{s}, \bar{s}] \subset (0,1)$

skill supply at each level: $L(s) > 0$ on $(\underline{s}, \bar{s}]$

only one final good, subject to sequential production. $\sigma \in [0,1]$

At each stage, the production function is Leontief. Formally, consider two consecutive stages, σ and $\sigma + d\sigma$, with $d\sigma$ infinitesimal. If a firm combines $q(\sigma)$ units of intermediate good σ with $q(\sigma)d\sigma$ units of workers of skill s, its output of intermediate good $\sigma + d\sigma$ is given by

$$q(\sigma + d\sigma) = (1 + \ln s d\sigma) q(\sigma)$$

intermediate good price:$p(\sigma)$

normalize:$p(0) = 0$ and $p(1) = 1$

all markets are competitive

profit maximization requires:

$$p(\sigma + d\sigma) \leq (1 - \ln s d\sigma) p(\sigma) + w(s) d\sigma$$

a free trade equilibrium is the competitive market allocation in which: 1) profit maximization condition holds; 2) all markets clear.

LEMMA 1 (Costinot et al., 2012) in a free trade equilibrium, there exists a strictly increasing matching function $M : [\underline{s}, \bar{s}] \to [0,1]$ such that (i) workers with skill s are employed in stage σ if and only if $M(s) = \sigma$, (ii) $M(\underline{s}) = 0$, and (iii) $M(\bar{s}) = 1$.

denote world output at stage σ as $Q(\sigma)$, then they derive the following equations using lemma 1:

$$\frac{Q(M(s + ds))}{Q(M(s))} = 1 + \ln s M'(s) ds$$

$$M'(s) Q(M(s)) = L(s)$$

the first equation simply states that the percentage change in world output between two consecutive stages is determined by the skill of the worker assigned to this stage. the second equation, in turn, equates the demand for workers over this set of stages with the supply of workers assigned to them

then they pin down the equilibrium matching and wage function as follows:

LEMMA 2 (Costinot et al., 2012) in a free trade equilibrium the matching function and wage schedule are given by the solution of two ordinary differential equations

$$\frac{d \ln M'(s)}{ds} = -\ln s e^{\ln M'(s)} + \frac{d \ln L(s)}{ds}$$

$$\frac{d^2 \ln w(s)}{ds^2} = -\frac{1 + sM'(s) \ln s}{s} \frac{d \ln w(s)}{ds} - \left(\frac{d \ln w(s)}{ds}\right)^2 + \frac{M'(s)}{s}$$

with boundary conditions such that:

$$\int_{\underline{s}}^{\bar{s}} \left[\frac{d \ln L(s)}{ds} - \frac{d \ln M'(s)}{ds}\right] \frac{ds}{\ln s} = 1$$

$$w'(\underline{s}),\ w'(\bar{s}) = 1$$

2.2 modified version

now, I modify the model with two considerations: 1)people learns; 2)people dies

2.2.1 work or search for better skills

distinguish labor endowment and actual labor supply(allow for search/learning)

actual labor supply=labor endowment*(1 - time for search and learning)

denote a type s worker's search effort as $a(s)$, then

for $s = \underline{s}$

Pr$(s = \underline{s}) = (1 - a(\underline{s}))\delta$

for $s \in (\underline{s}, \bar{s}]$

$l(s) = (1 - a(s))L(s)$

where δ and $L(s)$ are defined as follows

2.2.2 newly born individuals

every time measure $(1 - \beta)$ individuals, irrelevant to skill levels, are replaced by newly born ones with lowest skill $\underline{s}$

skill distribution: with measure δ on point $\underline{s}$, and density function $L(s) > 0$ on $(\underline{s}, \bar{s}]$

$$\delta + \int_{\underline{s}}^{\bar{s}} L(s')ds' = 1$$

where

$$\int_{\underline{s}}^{\bar{s}} L(s')ds' = \lim_{\Delta s \to 0^+} \int_{\underline{s}+\Delta s}^{\bar{s}} L(s')ds'$$

this means that there are always a mass of individual with lowest skill.

the distribution of skill is neither continuous nor discrete.

(another way to handle this is suppose new worker is drawn from an exogenous distribution, but it's not intuitive as the above assumption)

the problem is to find the maximum stage that $\underline{s}$-type worker handles.

we proceed in three steps:

1) given $l(s)$, fix $\underline{\sigma}$, construct matching function $m_{\underline{\sigma}} : (\underline{\mathsf{s}}, \bar{s}] \to (\underline{\sigma}, 1]$. then for any $Q(\underline{\sigma})$, $Q(1)$ can be found and wrote as a function of $Q(\underline{\sigma})$ and $m_{\underline{\sigma}}$.

2) then for any $\underline{\sigma}$, we can divide the production chain into two parts: $[0, \underline{\sigma}]$, which employs $\underline{\mathsf{s}}$-workers and $(\underline{\sigma}, 1]$, where $(\underline{\mathsf{s}}, \bar{s}]$-labor work. applying Costinot et al.,2013(see appendix A), the discrete version of supply chain:

$$\underline{\sigma} = \frac{1}{\ln \underline{\mathsf{s}}} \ln(1 + \frac{\Pr(s = \underline{\mathsf{s}}) \ln \underline{\mathsf{s}}}{Q(0)})$$

and

$$Q(\underline{\sigma}) = e^{\underline{\sigma} \ln \underline{\mathsf{s}}} Q(0)$$

which relate $Q(\underline{\sigma})$ to $\underline{\sigma}$

3) find $\underline{\sigma}$ to maximize Q(1). then applying second fundamental theorem of welfare economics, the Pareto optimal allocation is also competitive equilibrium.

once $\underline{\sigma}$ is found, we can recover matching function $m_{\underline{\sigma}}$, which pins down wage function $\{w(\underline{\mathsf{s}}), w(\cdot)_{(\underline{\mathsf{s}}, \bar{s}]}\}$.

2.3 search, meet and learn

now, equipped with the modifies version of production chain, let's incorporate the dynamic search and learning procedure into the economy.

2.3.1 income and consumption

no investment and capital

individual with skill level s's consumption at time t:

$$c(s,t) = (1 - a(s,t))w(s,t)$$

where $w(s,t)$ is the wage, determined by the above wage function.

utility is CRRA:

$$u(s,t) = u(c(s,t)) = \frac{c(s,t)^{1-\gamma} - 1}{1 - \gamma}$$

2.3.2 search for skills

skill is generated by knowledge, which is inherited by individuals. Knowledge is neither entirely public good nor private good.

1) knowledge is not public good: everyone hold his/her knowledge

2) not private good: individuals could search, discover and learn others' knowledge

the more you search, the higher possibly you meet someone

we simply assume the Poisson rate of meeting as the function of search intensities:

$$\alpha(a(s,t)) = a(s,t)$$

Hamilton-Jacobi-Bellman equation for individual optimization

for $s = \underline{s}$

$$\begin{aligned} V(\underline{s}, t) = \max_{a(\underline{s},t)}\{ \quad & u(\underline{s}, t) + \beta[(1 - a(\underline{s}, t))V(\underline{s}, t) \\ & + a(\underline{s}, t)Pr(s = \underline{s}, t)V(\underline{s}, t) \\ & + a(\underline{s}, t)\int_{\underline{s}}^{\bar{s}} V(s', t)L(s', t)ds'\} \end{aligned}$$

for $s \in (\underline{s}, \bar{s}]$

$$\begin{aligned} V(s, t) = \max_{a(s,t)}\{ \quad & u(s, t) + \beta[(1 - a(s, t))V(s, t) \\ & + a(s, t)(Pr(s = \underline{s},\ t) + \int_{\underline{s}}^{s} L(s', t)ds')V(s, t) \\ & + a(s, t)\int_{s}^{\bar{s}} V(s', t)L(s', t)ds']\} \end{aligned}$$

which means that the present value of a typical skill s at time t equals the sum of 1) the utility flow; 2) the continuation value of next time. the later one could decompose into a) search but without meeting anyone; b) search and meet someone are less or equal to the individual; c) search and meet a person with higher skill.

Kolmogorov Forward equation for distribution evolution

for $s = \underline{s}$

$$Pr(s = \underline{s},\ t + dt) = \beta Pr(s = \underline{s},\ t \ and\ no\ better\ skill\ learned) + 1 - \beta$$

$$= \beta Pr(s = \underline{s}, t)[(1 - a(s, t)) + a(\underline{s}, t)Pr(s = \underline{s}, t)] + 1 - \beta$$

for $s \in (\underline{s}, \bar{s}]$

$$\begin{aligned} \frac{\partial L(s,t)}{\partial t} = \quad & -a(s, t)L(s, t)\int_{s}^{\bar{s}} L(s', t)ds' - (1 - \beta)L(s, t) \\ & + L(s, t)\int_{\underline{s}}^{s} a(s', t)L(s', t)ds' \end{aligned}$$

which states that the change of density of s-type labor equals: 1)outflow into other groups, including a)search and meet a higher skill person, b)die out; 2)inflow into s-group, including individuals with lower skill and meet some one with skill-s.

2.4 steady state equilibrium

DEFINITION (Steady-state Equilibrium) the steady-state equilibrium of dynamic production chain is a tuple

$$(\{\delta_{\underline{s}}, L(\cdot)_{(\underline{s},\bar{s}]}\}, \{\underline{\sigma}, m_{\underline{\sigma}}(\cdot)_{(\underline{s},\bar{s}]}\}, \{w(\underline{s}), w(\cdot)_{(\underline{s},\bar{s}]}\}, \{a(\underline{s}), a(\cdot)_{(\underline{s},\bar{s}]}\})$$

of skill distribution, matching function, wage function and learning policy function on $[\underline{s}, \bar{s}]$, that satisfies:

1) static equilibrium of vertical specialization and market clearing

1a)given skill distribution and learning policy, matching function is pinned down

1b)given matching function, wage function is also determined

2) HJB equation for individual maximization

given skill distribution and wage function, value function and the corresponding learning policy can be derived

3) KF equation for invariant distribution

given the previous skill distribution and learning policy, present skill distribution is invariant

3 trade liberalization

3.1 2*2 settings

two skill level: low and high or unskilled and skilled

correspond to $-\ln s_l = \lambda_l$ and $-\ln s_h = \lambda_h$

denote $\lambda_l/\lambda_h = \lambda > 1$

two countries: north and south

north is relatively skill abundant:

$L_{li} + L_{hi} = 1,\ i = \{S,\ N\}$

$L_{lS} \geq L_{lN}$

for now, lets drop country specific subscript

only low skill need to search and learn, denote $a_l = a$

equilibrium is a tuple $(a, L_l, s^*, w_h/w_l)$

3.1.1 Bellman equations for both type

suppose $\gamma = 1$, so $u(c) = \ln c$

the value function of high skill labor is

$$V_h = max\{\ln w_h + \beta V_h\}$$

i.e., $V_h = \ln w_h/(1-\beta)$

for low skill labor is

$$V_l = \max_{a\in[0,1]}\{\ln(1-a) + \ln w_l + \beta[(1-a)V_l + a(L_h V_h + L_l V_l)]$$

suppose the optimal value is V_l^{opt} when optimal policy is a^{opt}. then we must have:

$V_l^{opt} = \ln w_h/(1-\beta) - 1/(L_h\beta(1-a^{opt}))$ from f.o.c. condition w.r.t a^{opt}

and since the maximum function is defined on a compact set, we have

$$V_l^{opt} = \ln(1-a^{opt}) + \ln w_l + \beta[(1-a^{opt})V_l^{opt} + a^{opt}(L_h V_h + L_l V_l^{opt})]$$

this gives the equation between a, L_l, and w_h/w_l:

$$\ln \frac{w_h}{w_l} = \frac{1-\beta+a\beta L_h}{L_h\beta(1-a)} + \ln(1-a)$$

3.1.2 Kolmogorov Forward equations at equilibrium

outflow=inflow

for low skill group:

$$aL_hL_l + (1-\beta)L_l = 1-\beta$$

for high skill group:

$$(1-\beta)L_h = aL_hL_l$$

in fact they are the same since there is only two groups

so

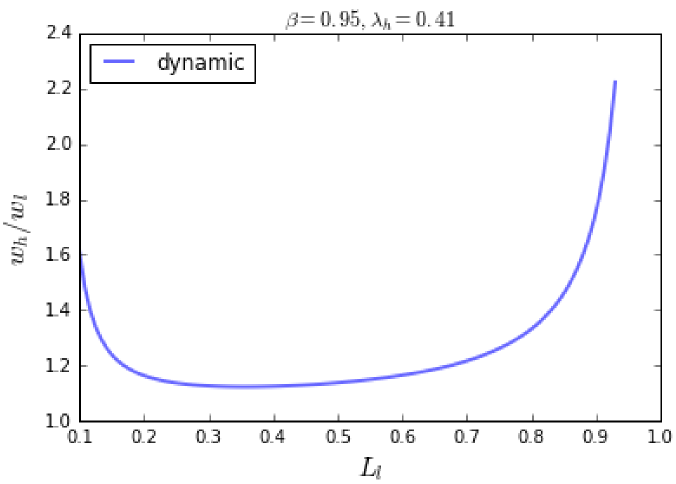
$$a = \frac{1-\beta}{L_l}$$

combining HJB and KF, we derive:

$$\ln \frac{w_h}{w_l} = \frac{1-\beta}{(1-L_l)\beta(L_l+\beta-1)} + \ln(L_l+\beta-1) - \ln L_l$$

and we call this as the "dynamic condition"

Figure: Dynamic Condition



U-shaped intuition:

for L_l low enough: the economy is maintaining a high proportion of skilled workers with a constant flow of newbies. So the search intensity must be high enough to transform these newbies into skilled labors quickly. To provide the incentive to search, the wage gap must be big.

for L_l high enough: there are so many unskilled workers that the possibility to meet an skilled one is relatively small. So people are not willing to search. To compensate the difficulty of search and learning, the prize for success must be high to balance the cost.

3.1.3 organization of sequential production

now, consider the production side. discrete version, so use Costinot et al.(2013)

denote the maximum stage unskilled workers can handle as s^*

matching function

$$s^* = 0 - \frac{1}{\lambda_l} \ln(1 - \frac{\lambda_l l_l}{Q_0})$$

$$1 = s^* - \frac{1}{\lambda_h} \ln(1 - \frac{\lambda_h L_h}{Q^*})$$

$$Q^* = e^{-\lambda_l s^*} Q_0$$

where $l_l = (1-a)L_l = L_l + \beta - 1$

then we get the equation for s^*, denote $e^{-\lambda_h s^*} = t$

$$(L_h + \lambda l_l)t^\lambda - \lambda l_l e^{-\lambda_h} t^{\lambda-1} - L_h = 0$$

assume $\lambda = 2$, we have closed form solution:

$$s^* = -\frac{1}{\lambda_h} \ln \left(\frac{l_l e^{-\lambda_h} + \sqrt{l_l^2 e^{-2\lambda_h} + L_h(L_h + 2l_l)}}{L_h + 2l_l} \right)$$

which state s^*(here it is just a point, and represents the matching function) is a function of a and L_h

wage function

next, from wage function, we have:

$$w_h = w_l + (\lambda_l - \lambda_h)p_l$$

$$p_l = (e^{\lambda_l s^*} - 1)\frac{w_l}{\lambda_l}$$

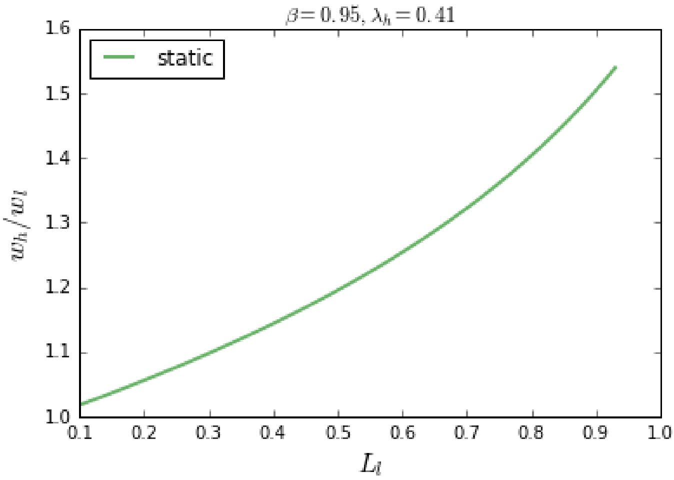
$$1 = e^{\lambda_h(1-s^*)}p_l + (e^{\lambda_h(1-s^*)} - 1)\frac{w_h}{\lambda_h}$$

in fact, we only need the first two equations to derive the wage ratio between high and low skill labors:

$$
\begin{aligned}
\frac{w_h}{w_l} &= 1+\frac{\lambda-1}{\lambda}(e^{\lambda\lambda_h s^*}-1) \\
&= \frac{1}{2}\left[\left(\frac{l_l e^{-\lambda_h}+\sqrt{l_l^2 e^{-2\lambda_h}+L_h(L_h+2l_l)}}{L_h}\right)^{-2}+1\right] \\
&= \frac{1}{2}\left[\left(\frac{(L_l+\beta-1)e^{-\lambda_h}+\sqrt{(L_l+\beta-1)^2 e^{-2\lambda_h}+(1-L_l)(2\beta-L_l-1)}}{2\beta-L_l-1}\right)^{-2}+1\right]
\end{aligned}
$$

which state relative wage as a function of low skill proportion, and we label this as the "static condition"

Figure: Static Condition



strictly increasing intuition(according to Costinot et al.,2012, 2013):

1) scarcity of skilled labors are working on higher stages, and performing less stages, while the former dominate the later

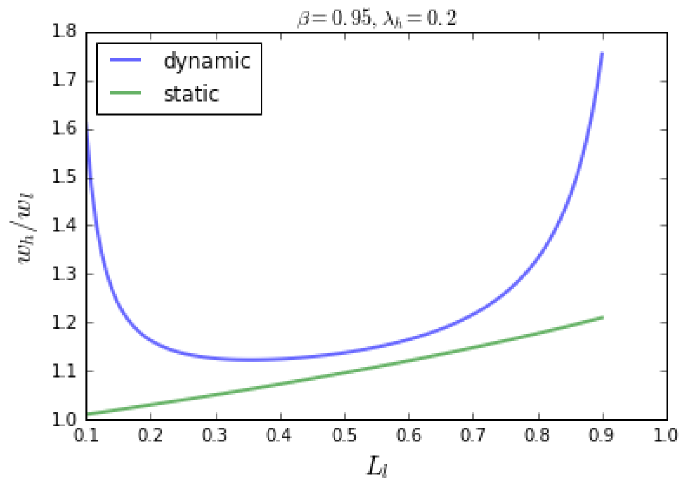
2) high stages has high intermediate good share

both raise relative wage.

3.1.4 existence and multiplicity of equilibrium

conditional on parameter values, there might be no equilibrium, one equilibrium or two equilibria

Figure: No Equilibrium

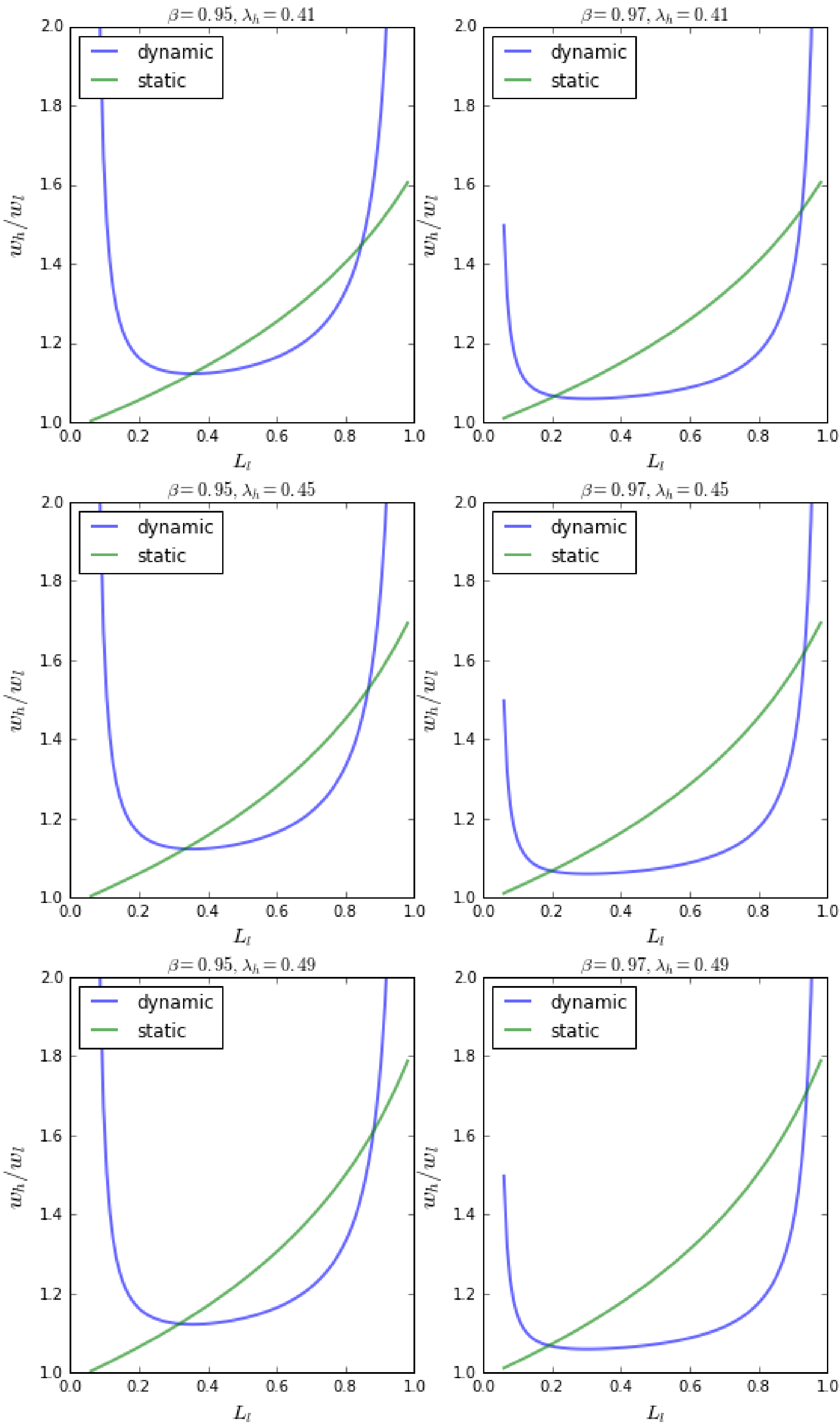


3.1.5 comparative static analysis of parameters

now lets focus on the setting with multiple equilibria, denote the one with higher proportion of skilled labor (i.e., low proportion of unskilled labor) as high equilibrium, another as low equilibrium.

as shown in the following figure:

Figure : Equilibrium Under Different Parameter Sets



the effect of λ_h an increase in λ_h will raise the static condition curve up, while keep the dynamic condition curve constant. this will raise the equilibrium relative wage at both equilibria;and for high equilibrium, unskilled proportion, and critical stage will decrease, and learning intensity increases correspondingly; the opposite is true for low equilibrium.

the effect of β an increase in β will drag down the entire dynamic curve while drag up the left side of static curve. for low equilibrium, relative wage, unskilled proportion and critical stage will raise, while learning intensity will decrease. The opposite is true for high equilibrium.

3.2 full liberalization

3.2.1 convergence to global equilibrium

consider now two country at different equilibrium, i.e., $L_{lS} > L_l^e > L_{lN}$, north high, south low

what about the global economy when they are entirely integrated

here, "entirely" means not only trade flow is without frictions, but also knowledge.

the global equilibrium can be either of the equilibrium

what determines the realization?

1) country size

2) people's belief and expectation

3) government policy and global cooperation

4) etc, etc

but now, we focus two source to maintain divergence:

3.2.2 different β

consider now individuals has country specific survival rate, i.e., $\beta_S < \beta_N$

other things equal

DEFINITION (Steady-state Free Trade Equilibrium With Different Survival Rate) equilibrium is tuple $(a_s, L_s, a_n, L_n, w_h/w_l, s^*)$

1) market clear

1a) given $(a_s, L_{ls}, a_n, L_{ln})$, global skill distribution $l_l = (1-a_s)L_{ls} + (1-a_n)L_{ln}$, $L_h = L_{hs} + L_{hn}$ is settled and determines global critical stage s^*

1b) then, s^* determines global wage ratio w_h/w_l

2) Bellman equation for individuals in n and s

2a) north: given w_h/w_l and L_{ln}, determine a_n

2b) south: given w_h/w_l and L_{ls}, determine a_s

3) KF equation for individuals in n and s

3a) north: given a_n, find an invariant L_{ln} which balances inflow and outflow into each skill group

3b) south: given a_s, find an invariant L_{ls}

we are especially interested in the situation where north are at high equilibrium and south at low:

compared with autarky, full liberalization with country specific β will

i) balance the skill premium and critical stage between North and South, not surprisingly, and

ii) force BOTH countries to decrease the unskilled labor proportion and increase search intensity

Table I: Global Equilibrium

β		0.95			0.97	
λ_h	0.41	0.45	0.49	0.41	0.45	0.49
High Equilibrium						
w_h/w_l	1.1227	1.1231	1.1240	1.0658	1.0674	1.0689
s^*	0.2676	0.2445	0.2261	0.1508	0.1404	0.1317
L_l	0.3551	0.3343	0.3180	0.2074	0.1983	0.1908
a	0.1408	0.1496	0.1572	0.1447	0.1513	0.1572
Low Equilibrium						
w_h/w_l	1.4480	1.5269	1.6109	1.5359	1.6202	1.7110
s^*	0.7802	0.7997	0.8146	0.8883	0.8963	0.9026
L_l	0.8468	0.8665	0.8818	0.9252	0.9336	0.9403
a	0.0590	0.0577	0.0567	0.0324	0.0321	0.0319
$\beta_{s,n} = 0.95, 0.97$	n: high	s: low		n: high	s: high	
w_h/w_l	1.3086	1.3226	1.3367	1.0598	1.0600	1.0598
s^*	0.5863	0.5531	0.5254	0.1377	0.1259	0.1152
L_{lN}	0.0703	0.0692	0.0681	0.3024	0.3024	0.3024
a_N	0.4267	0.4337	0.4405	0.0992	0.0992	0.0992
L_{lS}	0.7865	0.7949	0.8028	0.3555	0.3553	0.3553
a_S	0.0636	0.0629	0.0623	0.1406	0.1407	0.1407
With Frictions						
w_h/w_l	1.1231	1.1499	1.1729	1.1710	1.1913	1.2114
s^*	0.2685	0.2913	0.3031	0.3587	0.3600	0.3598
L_{lN}	0.3333	0.2189	0.1891	0.0910	0.0860	0.0821
a_N	0.1500	0.2284	0.2643	0.3297	0.3487	0.3652
L_{lS}	0.3787	0.5544	0.6216	0.7938	0.8146	0.8311
a_S	0.1320	0.0902	0.0804	0.0378	0.0368	0.0361

3.3 communication frictions

now, individuals are only allowed to search in there own country

other things equal

(i.e., the production function the same, Labor measure 1, and the organization of production is globalized without any trade frictions)

if both country are at the same steady state equilibrium it does not matter

but, what if they are starting from different equilibrium(high vs low) when trade opens

DEFINITION (Steady-state Free Trade Equilibrium With Communication Frictions) equilibrium is a tuple $(a_s, L_{ls}, a_n, L_{ln}, w_h/w_l, s^*)$ with

1)market clear,

2)Bellman,

3)KF.

compared with autarky, trade liberalization with extreme communication frictions will

1) also balance the skill premium and critical stage between North and South, not surprisingly, and

2) force BOTH countries to decrease unskilled labor proportions, and increase search intensity

4 migration (tbw)

what if high skill labor is able to move from south to north

5 empirics

6 conclusion

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appendix

1. sketch of Costinot et al.(2013)

in fact, Costinot et al.(2012) is the generalized and continuous version of 2013. the former is published at AER P&P, the later at REStud.

countries indexed by $c \in \mathcal{C} \equiv \{1, \ldots, C\}$

labor supply and wage in country c: L_c and w_c

stage $s \in \mathcal{S} \equiv (0, S]$. at each stage, producing one unit of intermediate good requires one unit of the intermediate good produced in the previous stage and one unit of labor

Poisson mistake rate $\lambda_c > 0$. assume λ_c is strictly decreasing in c.

then we have

$$q(s+ds) = (1 - \lambda_c ds) q(s)$$

profit maximization requires that for all $c \in \mathcal{C}$,

$$\begin{aligned} p(s+ds) \leq \quad & (1 + \lambda_c ds) p(s) + w_c ds \\ p(s+ds) = \quad & (1 + \lambda_c ds) p(s) + w_c ds \ if \ Q_c(s') > 0 \ for \ all \ s' \in (s, s+ds] \end{aligned}$$

where $Q_c(s')$ denotes total output at stage s' in country c.

good and labor market clearing require that

$$\sum_{c=1}^{C} Q_c(s_2) - \sum_{c=1}^{C} Q_c(s_1) = -\int_{s_1}^{s_2} \sum_{c=1}^{C} \lambda_c Q_c(s) ds, \ for \ all \ s_1 \leq s_2$$

$$\int_0^S Q_c(s) ds = L_c, \ for \ all \ c \in \mathcal{C}$$

Definition 1(Costinot et al, 2013)

A free trade equilibrium corresponds to output levels $Q_c(\cdot) : \mathcal{S} \to \mathbb{R}^+$ for all $c \in \mathcal{C}$, wage $w_C \in \mathbb{R}^+$ for all $c \in \mathcal{C}$, and intermediate good prices $p(\cdot) : \mathcal{S} \to \mathbb{R}^+$ such that the above three conditions hold

Proposition 1(Costinot et al, 2013)

In any free trade equilibrium, there exists a sequence of stages $S_0 \equiv 0 < S_1 < \ldots < S_C = S$ such that for all $s \in \mathcal{S}$ and $c \in \mathcal{C}$, $Q_c(s) > 0$if and only if $s \in (S_{c-1}, S_c]$.

denote $Q_c \equiv Q_c(S_c)$, then

Lemma 1(Costinot et al, 2013)

In any free trade equilibrium, the pattern of vertical specialization and export levels satisfy the following system of first-order non-linear difference equations:

$$S_c = S_{c-1} - \left(\frac{1}{\lambda_c}\right) \ln\left(1 - \frac{\lambda_c L_c}{Q_{c-1}}\right), \ for \ all \ c \in \mathcal{C}$$

$$Q_c = e^{-\lambda_c (S_c - S_{c-1})} Q_{c-1}, \ for \ all \ c \in \mathcal{C}$$

with boundary conditions $S_0 = 0$ and $S_C = S$.

denote $N_c \equiv S_c - S_{c-1}$, then

Lemma 2(Costinot et al, 2013)

In any free trade equilibrium, the world income distribution and export prices satisfy the following system of first-order linear difference equations:

$$w_{c+1} = w_c + (\lambda_c - \lambda_{c+1}) p_c,\ for\ all\ c < C$$

$$p_c = e^{\lambda_c N_c} p_{c-1} + (e^{\lambda_c N_c} - 1)(w_c/\lambda_c),\ for\ all\ c \in \mathcal{C}$$

with boundary conditions $p_0 = 0$ and $p_C = 1$

Proposition 2(Costinot et al, 2013)

There exists a unique free trade equilibrium. In this equilibrium, the pattern of vertical specialization and export levels are given by Equations in lemma 1, and the world income distribution and export prices are given by Equations in lemma 2.

2. existence, uniqueness and stability of equilibrium

PROPOSITION

there will be no equilibrium if and only if

one equilibrium if and only if

two equilibria if and only if

3. different beta

equations

algorithm

4. communication frictions

equations

algorithm