4a)

The smallest positive integer that has b binary digits in its binary expansion is b-1 (10 … 0), which equates to 2b-1

The largest positive integer that has b binary digits in its binary expansion is 2b-1 (2b-1 + 2b-2 + … + 1)

Hence,

2b-1 ≤ n < 2b

= log2 (2b-1) ≤ log2 (n) < log2 (2b)

= b – 1 ≤ log2 (n) < b

Now we can relate the equation above to the “floored” equation:  
b – 1 = └ log2 (n) ┘

and

b = └ log2 (n) + 1 ┘

4b)

?

4c)

B = └ log10 (n) ┘ + 1

5)

The former algorithm, if the input set size is large

The later algorithm, if the input set size is small

Break-even: 86400

Time taken at break-even size: 7,464,960,000 weeks