UNIVERSITY OF TECHNOLOGY
IN THE EUROPEAN CAPITAL OF CULTURE
CHEMNITZ

Deep Reinforcement Learning

Deep Deterministic Policy Gradient

Julien Vitay

Professur für Künstliche Intelligenz - Fakultät für Informatik

1 - Deterministic policy gradient theorem

Deterministic Policy Gradient Algorithms

David Silver DAVID@DEEPMIND.COM
DeepMind Technologies, London, UK
Guy Lever GUY.LEVER@UCL.AC.UK
University College London, UK
Nicolas Heess, Thomas Degris, Daan Wierstra, Martin Riedmiller *@DEEPMIND.COM
DeepMind Technologies, London, UK

Silver et al. (2014) Deterministic Policy Gradient Algorithms. ICML

Deterministic policy gradient theorem

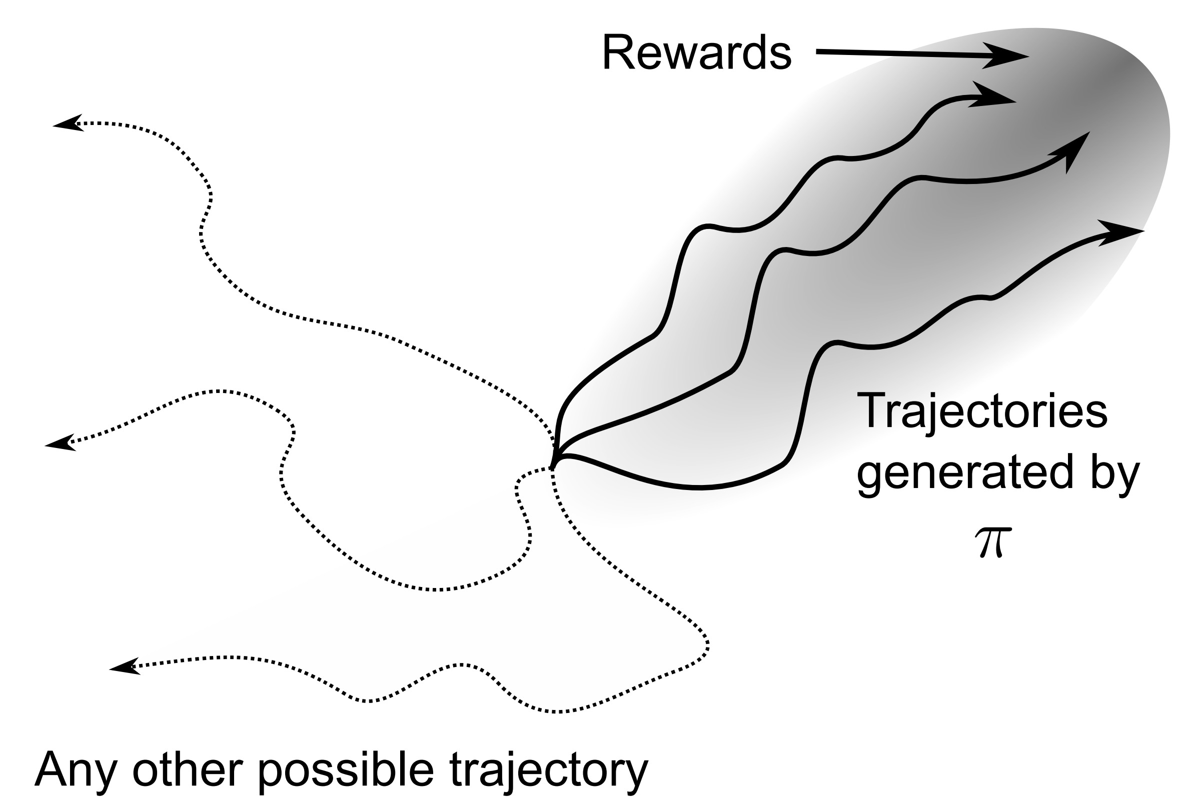
- The objective function that we tried to maximize until now is :

$$\mathcal{J}(\theta) = \mathbb{E}_{\tau \sim \rho_\theta}[R(\tau)]$$

i.e. we want the returns of all trajectories generated by the **stochastic policy** π_θ to be maximal.

- It is equivalent to say that we want the value of **all** states visited by the policy π_θ to be maximal:
  - a policy π is better than another policy π' if its expected return is greater or equal than that of π' for all states s.

$$\pi > \pi' \Leftrightarrow V^{\pi}(s) > V^{\pi'}(s) \quad \forall s \in \mathcal{S}$$



- The objective function can be rewritten as:

$$\mathcal{J}'(\theta) = \mathbb{E}_{s \sim \rho_\theta}[V^{\pi_\theta}(s)]$$

where ρ_θ is now the **state visitation distribution**, i.e. how often a state will be visited by the policy π_θ.

Deterministic policy gradient theorem

- The two objective functions:

$$\mathcal{J}(\theta) = \mathbb{E}_{\tau \sim \rho_\theta}[R(\tau)]$$

and:

$$\mathcal{J}'(\theta) = \mathbb{E}_{s \sim \rho_\theta}[V^{\pi_\theta}(s)]$$

are not the same: $\mathcal{J}$ has different values than $\mathcal{J}'$.

- However, they have a maximum for the same **optimal policy** π^* and their gradients are the same:

$$\nabla_\theta \, \mathcal{J}(\theta) = \nabla_\theta \, \mathcal{J}'(\theta)$$

- If a change in the policy π_θ increases the return of all trajectories, it also increases the value of the visited states.
- Take-home message: their **policy gradient** is the same, we have the right to re-define the problem like this.

$$g = \nabla_\theta \, \mathcal{J}(\theta) = \mathbb{E}_{s \sim \rho_\theta}[\nabla_\theta \, V^{\pi_\theta}(s)]$$

Deterministic policy gradient theorem

- When introducing Q-values, we obtain the following policy gradient:

$$g = \nabla_\theta \, \mathcal{J}(\theta) = \mathbb{E}_{s \sim \rho_\theta}[\nabla_\theta \, V^{\pi_\theta}(s)] = \mathbb{E}_{s \sim \rho_\theta}[\sum_a \nabla_\theta \, \pi_\theta(s, a) \, Q^{\pi_\theta}(s, a)]$$

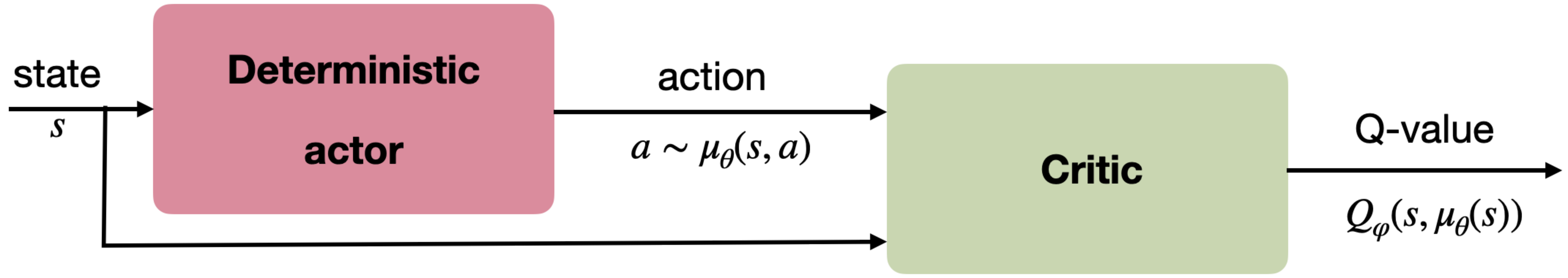
- This formulation necessitates to integrate overall possible actions.
  - Not possible with continuous action spaces.
  - The stochastic policy adds a lot of variance.
- But let’s suppose that the policy is **deterministic**, i.e. it takes a single action in state s.
- We can note this deterministic policy $\mu_\theta(s)$, with:

$$\begin{aligned} \mu_\theta : \; \mathcal{S} & \rightarrow \mathcal{A} \\ s \; & \rightarrow \mu_\theta(s) \end{aligned}$$

Deterministic policy gradient theorem

- The policy gradient for the deterministic policy $\mu_\theta(s)$ becomes:

$$g = \nabla_\theta \mathcal{J}(\theta) = \mathbb{E}_{s \sim \rho_\theta}[\nabla_\theta Q^{\mu_\theta}(s, \mu_\theta(s))]$$



- We can now use the chain rule to decompose the gradient of $Q^{\mu_\theta}(s, \mu_\theta(s))$:

$$\nabla_\theta Q^{\mu_\theta}(s, \mu_\theta(s)) = \nabla_a Q^{\mu_\theta}(s, a)|_{a=\mu_\theta(s)} \times \nabla_\theta \mu_\theta(s)$$

- $\nabla_a Q^{\mu_\theta}(s, a)|_{a=\mu_\theta(s)}$ means that we differentiate Q^{μ_θ} w.r.t. a, and evaluate it in $\mu_\theta(s)$.
  - a is a variable, but $\mu_\theta(s)$ is a deterministic value (constant).
- $\nabla_\theta \mu_\theta(s)$ tells how the output of the policy network varies with the parameters of NN:
  - Automatic differentiation frameworks such as tensorflow can tell you that.

Deterministic policy gradient theorem

> For any MDP, the **deterministic policy gradient** is:

$$\nabla_\theta \, \mathcal{J}(\theta) = \mathbb{E}_{s \sim \rho_\theta}[\nabla_a \, Q^{\mu_\theta} (s, a)|_{a = \mu_\theta(s)} \times \nabla_\theta \, \mu_\theta(s)]$$

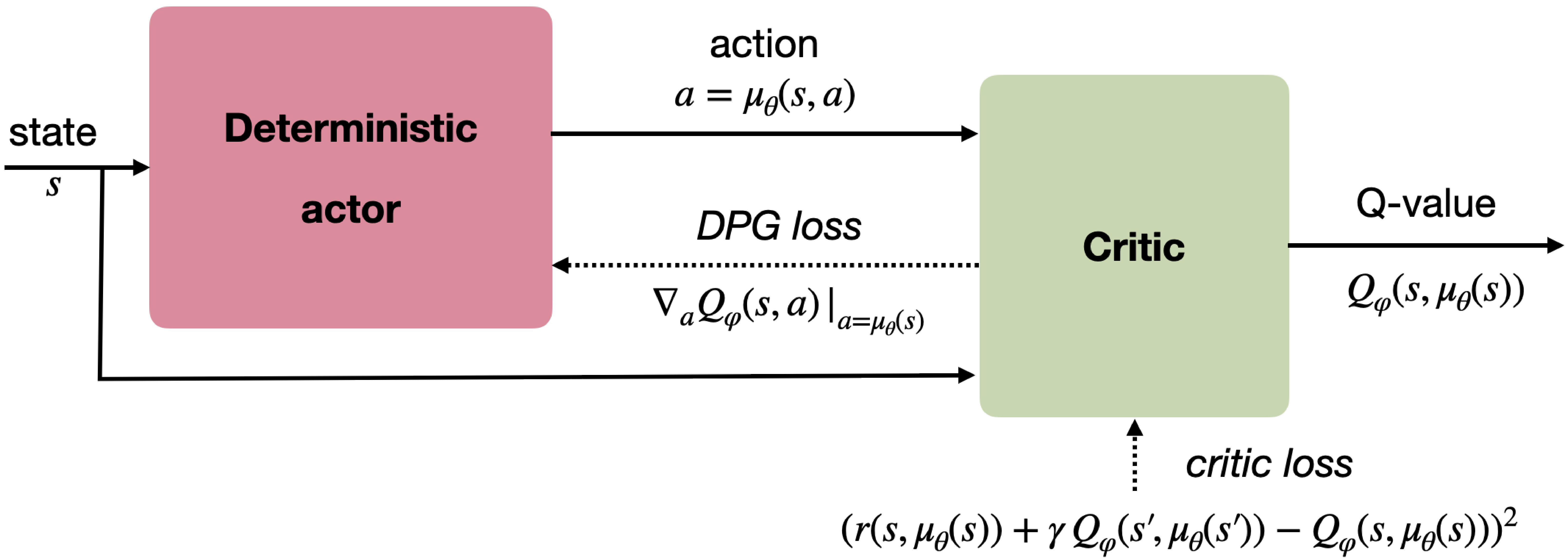
- As always, you do not know the true Q-value $Q^{\mu_\theta} (s, a)$, because you search for the policy μ_θ.
- Silver et al. (2014) showed that you can safely (without introducing any bias) replace the true Q-value with an estimate $Q_\varphi(s, a)$, as long as the estimate minimizes the mse with the TD target:

$$Q_\varphi(s, a) \approx Q^{\mu_\theta} (s, a)$$

$$\mathcal{L}(\varphi) = \mathbb{E}_{s \sim \rho_\theta}[(r(s, \mu_\theta(s)) + \gamma \, Q_\varphi(s', \mu_\theta(s')) - Q_\varphi(s, \mu_\theta(s)))^2]$$

- We come back to an actor-critic architecture:
  - The **deterministic actor** $\mu_\theta(s)$ selects a single action in state s.
  - The **critic** $Q_\varphi(s, a)$ estimates the value of that action.

Deterministic Policy Gradient as an actor-critic architecture



Training the actor:

$$\nabla_\theta \mathcal{J}(\theta) = \mathbb{E}_{s \sim \rho_\theta}[\nabla_\theta\, \mu_\theta(s) \times \nabla_a Q_\varphi(s, a)|_{a=\mu_\theta(s)}]$$

Training the critic:

$$\mathcal{L}(\varphi) = \mathbb{E}_{s \sim \rho_\theta}[(r(s, \mu_\theta(s)) + \gamma\, Q_\varphi(s', \mu_\theta(s')) - Q_\varphi(s, \mu_\theta(s)))^2]$$

DPG is off-policy

- If you act off-policy, i.e. you visit the states s using a **behavior policy** b, you would theoretically need to correct the policy gradient with **importance sampling**:

$$\nabla_\theta \mathcal{J}(\theta) = \mathbb{E}_{s \sim \rho_b}[\sum_a \frac{\pi_\theta(s, a)}{b(s, a)} \nabla_\theta \, \mu_\theta(s) \times \nabla_a Q_\varphi(s, a) |_{a = \mu_\theta(s)}]$$

- But your policy is now **deterministic**: the actor only takes the action $a = \mu_\theta(s)$ with probability 1, not $\pi(s, a)$.
- The **importance weight** is 1 for that action, 0 for the other. You can safely sample states from a behavior policy, it won't affect the deterministic policy gradient:

$$\nabla_\theta \mathcal{J}(\theta) = \mathbb{E}_{s \sim \rho_b}[\nabla_\theta \, \mu_\theta(s) \times \nabla_a Q_\varphi(s, a) |_{a = \mu_\theta(s)}]$$

- The critic uses Q-learning, so it is also off-policy.
- **DPG is an off-policy actor-critic architecture!**

2 - DDPG: Deep Deterministic Policy Gradient

Published as a conference paper at ICLR 2016

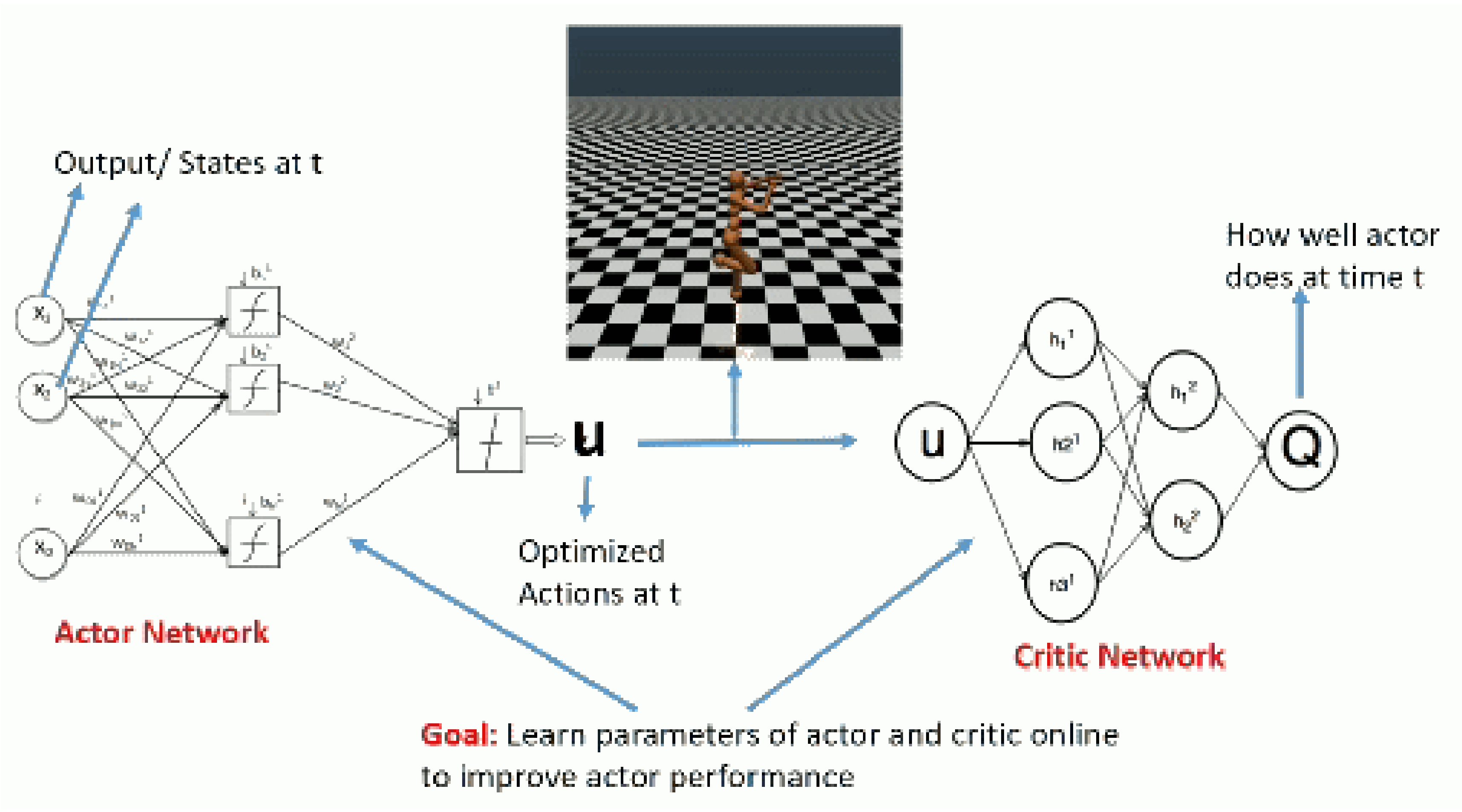
CONTINUOUS CONTROL WITH DEEP REINFORCEMENT LEARNING

Timothy P. Lillicrap,[*] Jonathan J. Hunt,[*] Alexander Pritzel, Nicolas Heess,
Tom Erez, Yuval Tassa, David Silver & Daan Wierstra
Google Deepmind
London, UK
{countzero, jjhunt, apritzel, heess,
etom, tassa, davidsilver, wierstra} @ google.com

Lillicrap et al. (2015) Continuous control with deep reinforcement learning. CoRR. arXiv:1509.02971.

DDPG: Deep Deterministic Policy Gradient

- As the name indicates, DDPG is the deep variant of DPG for **continuous control**.
- It uses the DQN tricks to stabilize learning with deep networks:



Source: https://github.com/stevenpjg/ddpg-aigym/blob/master/README.md

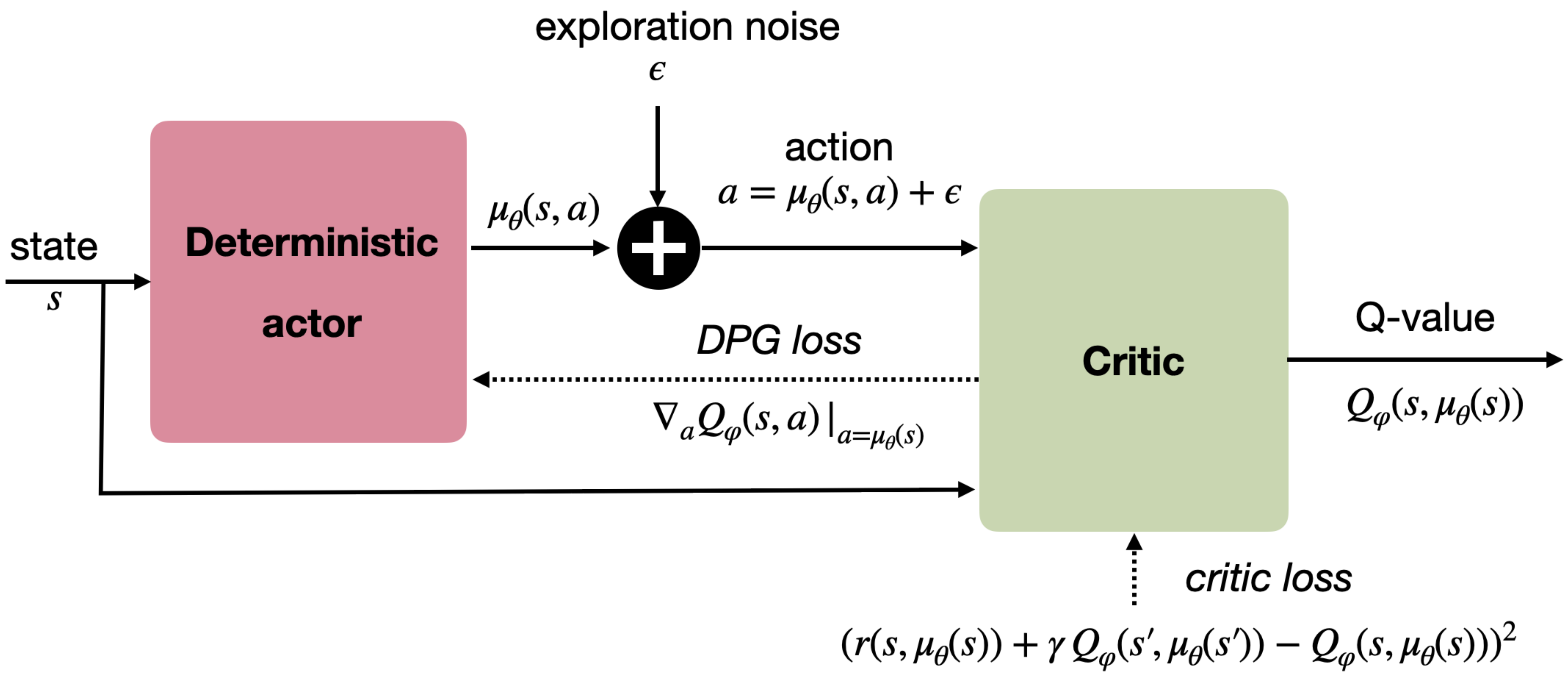
- As DPG is **off-policy**, an **experience replay memory** can be used to sample experiences.
- The **actor** μ_θ learns using sampled transitions with DPG.
- The **critic** Q_φ uses Q-learning on sampled transitions: **target networks** can be used to cope with the non-stationarity of the Bellman targets.

- Contrary to DQN, the target networks are not updated every once in a while, but slowly **integrate** the trained networks after each update (moving average of the weights):

$$\theta' \leftarrow \tau \theta + (1 - \tau) \, \theta'$$

$$\varphi' \leftarrow \tau \varphi + (1 - \tau) \, \varphi'$$

DDPG: Deep Deterministic Policy Gradient



- A deterministic actor is good for learning (less variance), but not for exploring.
- We cannot use ϵ-greedy or softmax, as the actor outputs directly the policy, not Q-values.
- For continuous actions, an **exploratory noise** can be added to the deterministic action:

$$a_t = \mu_\theta(s_t) + \xi_t$$

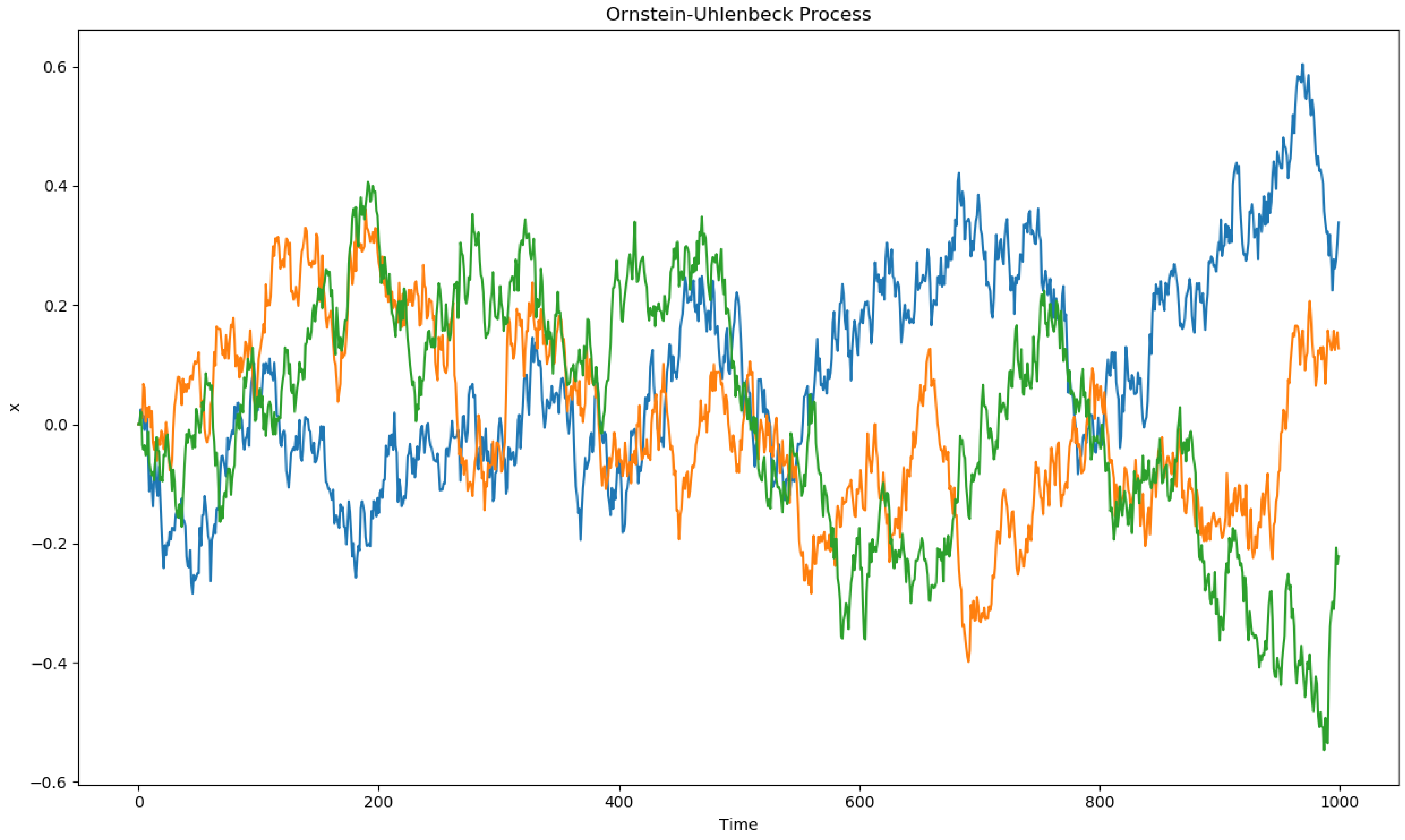
- Ex: if the actor wants to move the joint of a robot by 2^o, it will actually be moved from 2.1^o or 1.9^o.

Ornstein-Uhlenbeck stochastic process

- In DDPG, an **Ornstein-Uhlenbeck** stochastic process is used to add noise to the continuous actions.
- It is defined by a **stochastic differential equation**, classically used to describe Brownian motion:

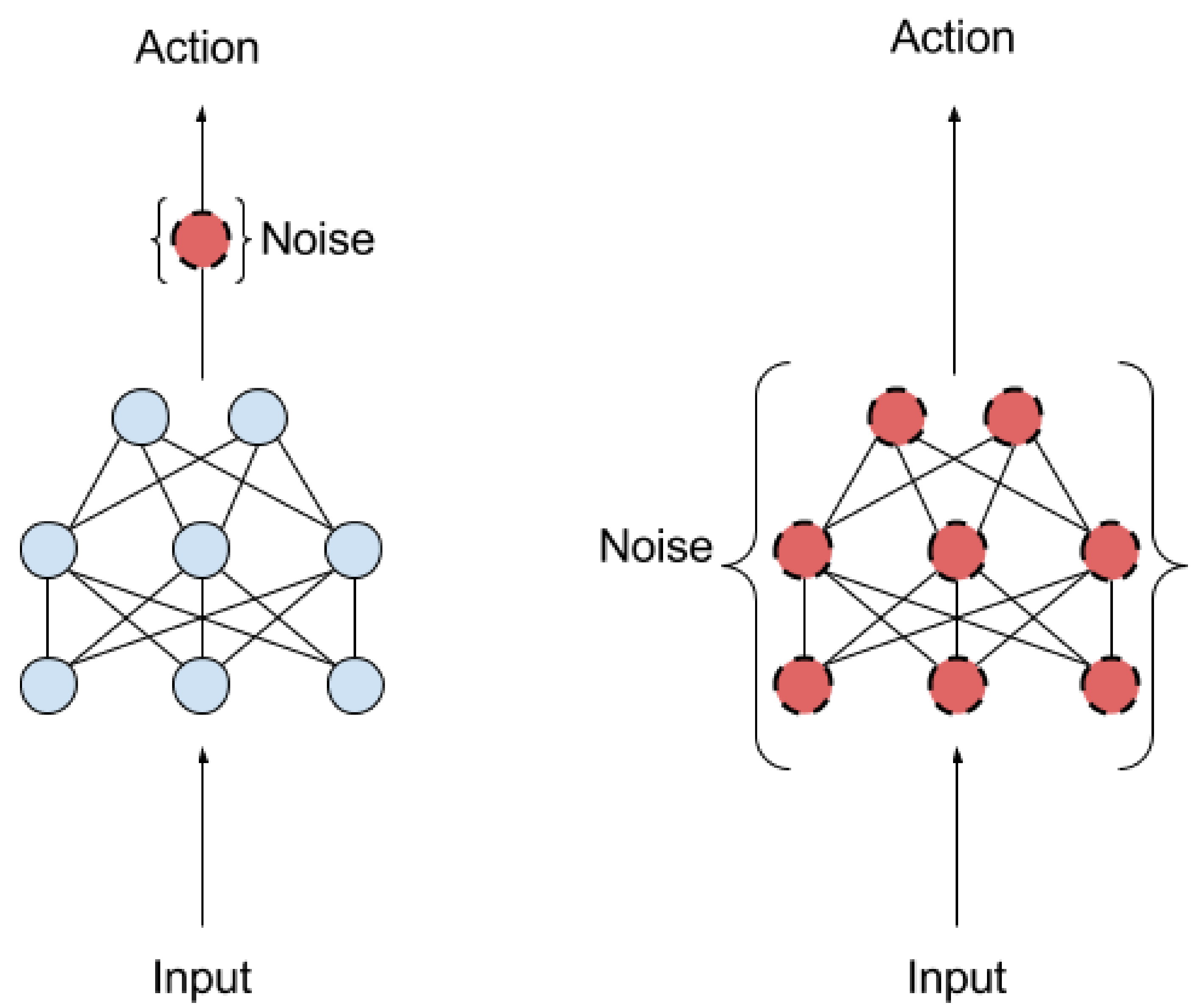
$$dx_t = \theta(\mu - x_t)dt + \sigma dW_t \qquad \text{with} \qquad dW_t = \mathcal{N}(0, dt)$$

- The temporal mean of x_t is $\mu = 0$, its amplitude is θ (exploration level), its speed is σ.



Uhlenbeck and Ornstein (1930) On the Theory of the Brownian Motion. Physical Review 36. doi:10.1103/PhysRev.36.823.

Parameter noise



Source: https://towardsdatascience.com/whats-new-in-deep-learning-research-knowledge-exploration-with-parameter-noise-98aef7ce84b2

- Another approach to ensure exploration is to add noise to the **parameters** θ of the actor at inference time.
- For the same input s_t, the output $\mu_\theta(s_t)$ will be different every time.
- The **NoisyNet** approach can be applied to any deep RL algorithm to enable a smart state-dependent exploration (e.g. Noisy DQN).

Plappert et al. (2018) Parameter Space Noise for Exploration. arXiv:1706.01905.

DDPG: Deep Deterministic Policy Gradient

- Initialize actor network μ_θ and critic Q_φ, target networks $\mu_{\theta'}$ and $Q_{\varphi'}$, ERM $\mathcal{D}$ of maximal size N, random process ξ.
- for $t \in [0, T_\text{max}]$:
  - Select the action $a_t = \mu_\theta(s_t) + \xi$ and store $(s_t, a_t, r_{t+1}, s_{t+1})$ in the ERM.
  - For each transition (s_k, a_k, r_k, s'_k) in a minibatch of K transitions randomly sampled from $\mathcal{D}$:
    - Compute the target value using target networks $t_k = r_k + \gamma \, Q_{\varphi'}(s'_k, \mu_{\theta'}(s'_k))$.
  - Update the critic by minimizing:

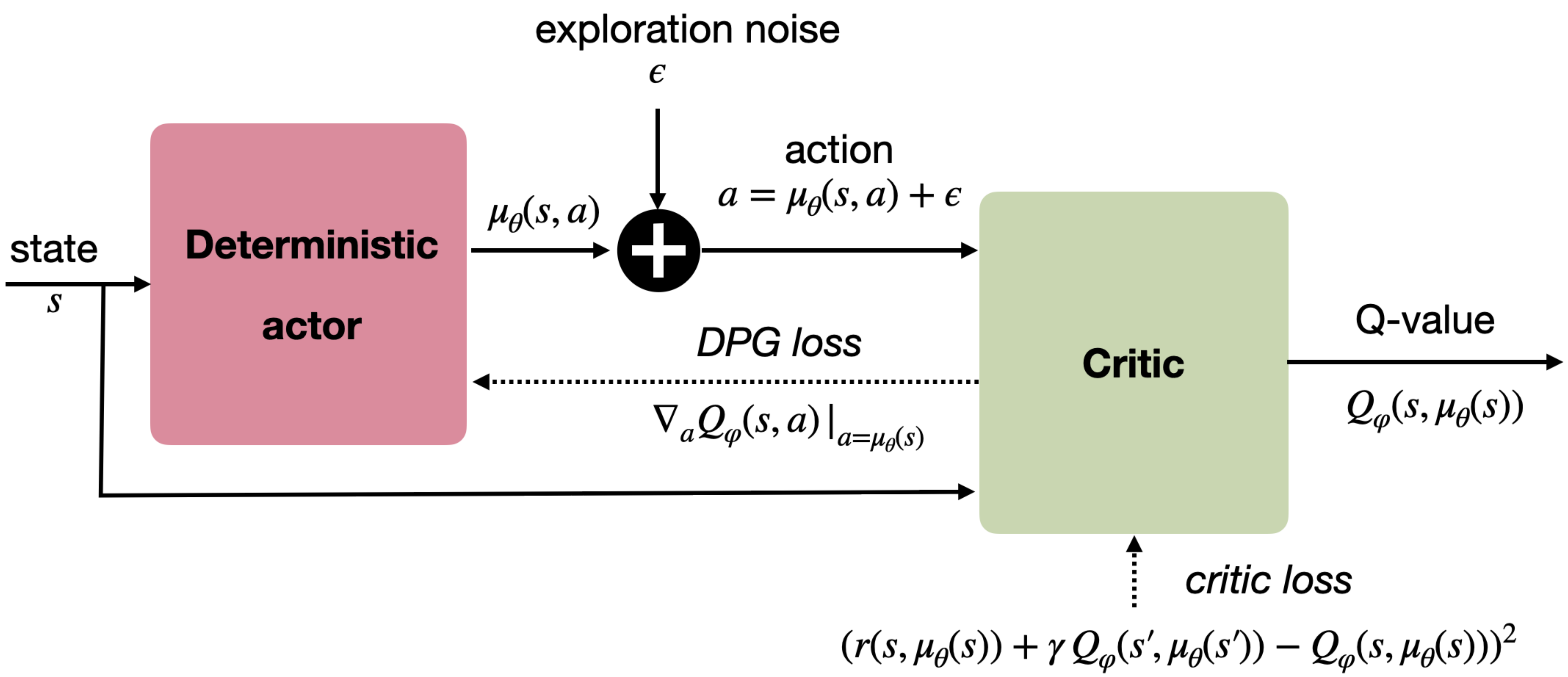
$$\mathcal{L}(\varphi) = \frac{1}{K} \sum_k (t_k - Q_\varphi(s_k, a_k))^2$$

  - Update the actor by applying the deterministic policy gradient:

$$\nabla_\theta \mathcal{J}(\theta) = \frac{1}{K} \sum_k \nabla_\theta \mu_\theta(s_k) \times \nabla_a Q_\varphi(s_k, a) |_{a = \mu_\theta(s_k)}$$

  - Update the target networks: $\theta' \leftarrow \tau \theta + (1-\tau) \, \theta' \; ; \; \varphi' \leftarrow \tau \varphi + (1-\tau) \, \varphi'$

DDPG: Deep Deterministic Policy Gradient



- DDPG allows to learn continuous policies: there can be one tanh output neuron per joint in a robot.
- The learned policy is deterministic: this simplifies learning as we do not need to integrate over the action space after sampling.
- Exploratory noise (e.g. Ohrstein-Uhlenbeck) has to be added to the selected action during learning in order to ensure exploration.
- Allows to use an experience replay memory, reusing past samples (better sample complexity than A3C).

Lillicrap et al. (2015) Continuous control with deep reinforcement learning. arXiv:1509.02971

DDPG: continuous control

3 - DDPG: learning to drive in a day

Learning to Drive in a Day

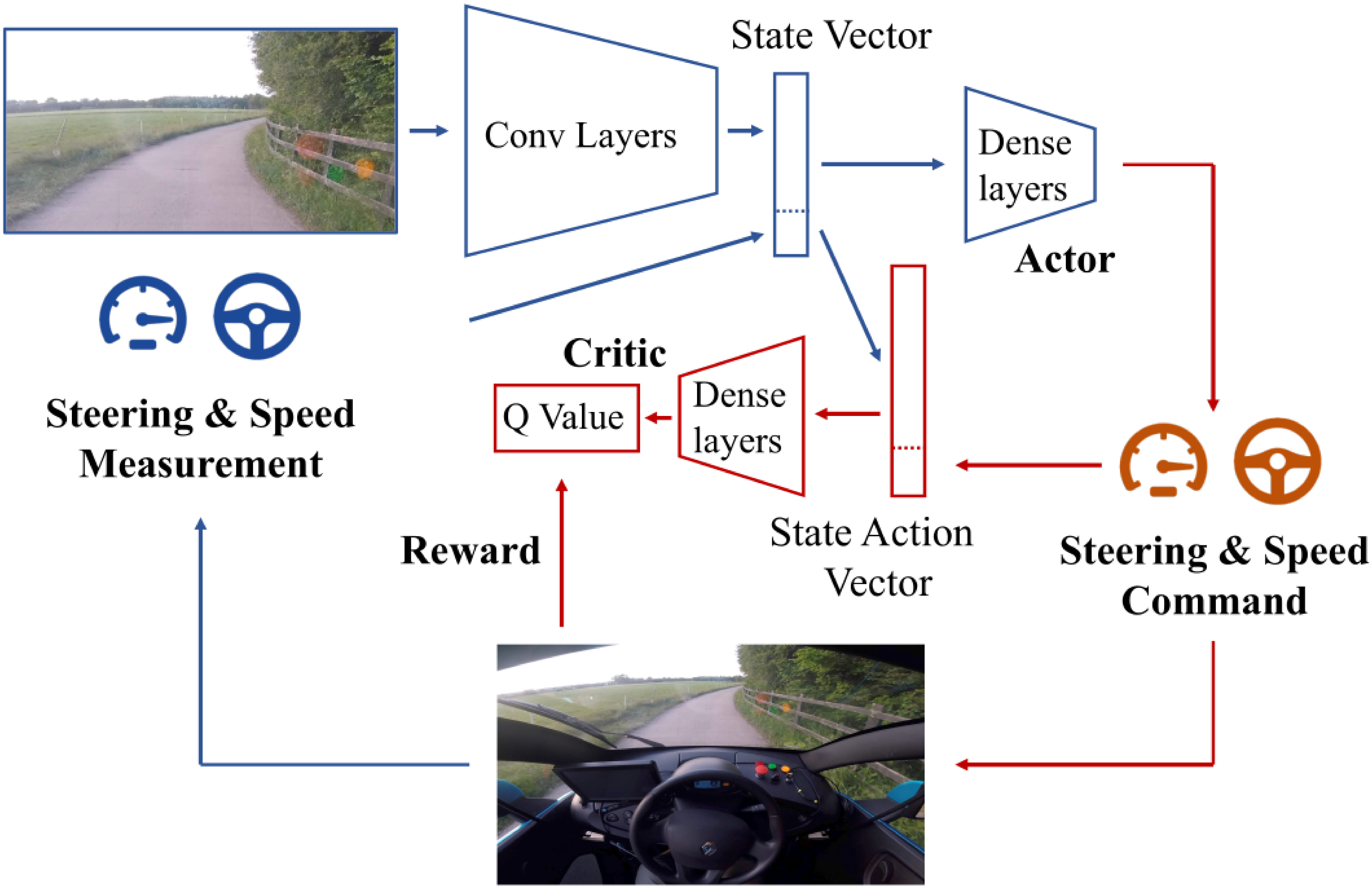
Alex Kendall Jeffrey Hawke David Janz Przemyslaw Mazur Daniele Reda
John-Mark Allen Vinh-Dieu Lam Alex Bewley Amar Shah

Kendall et al. (2018) Learning to Drive in a Day. arXiv:1807.00412

DDPG: learning to drive in a day

https://wayve.ai/blog/learning-to-drive-in-a-day-with-reinforcement-learning

DDPG: learning to drive in a day

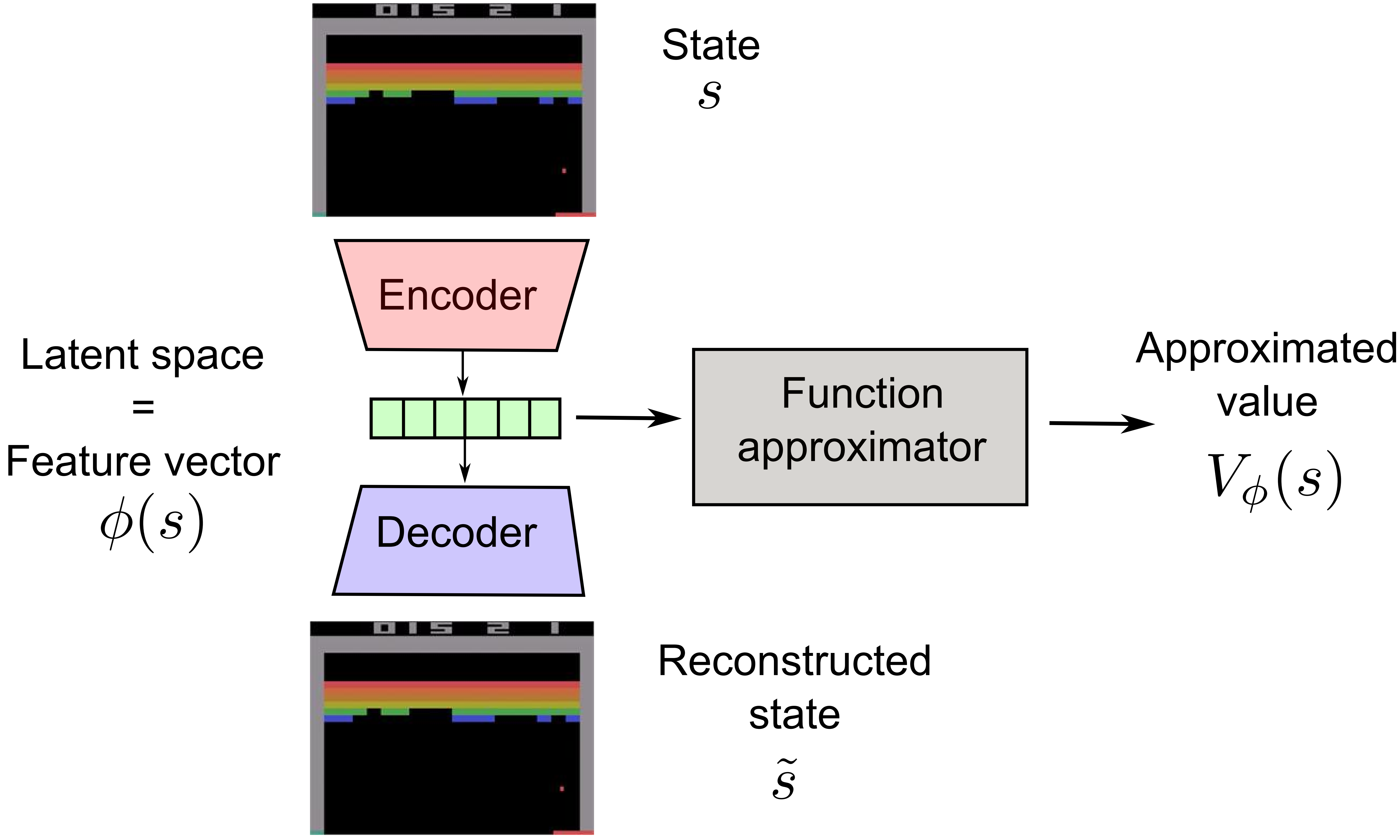


Kendall et al. (2018) Learning to Drive in a Day. arXiv:1807.00412

The algorithm is DDPG with prioritized experience replay

Autoencoders in deep RL

- A variational autoencoder (VAE) is optionally use to pretrain the convolutional layers on random episodes.



DDPG: learning to drive in a day

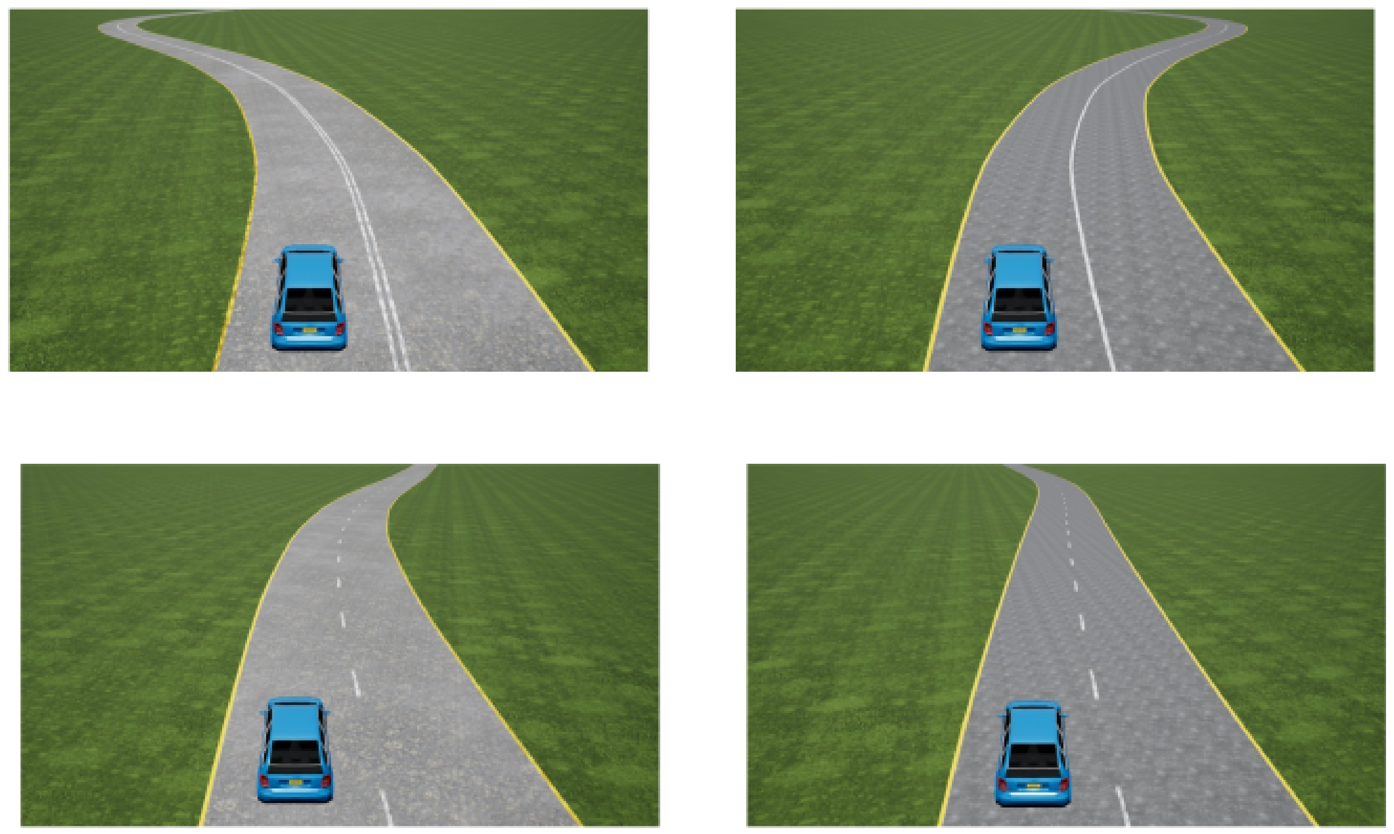
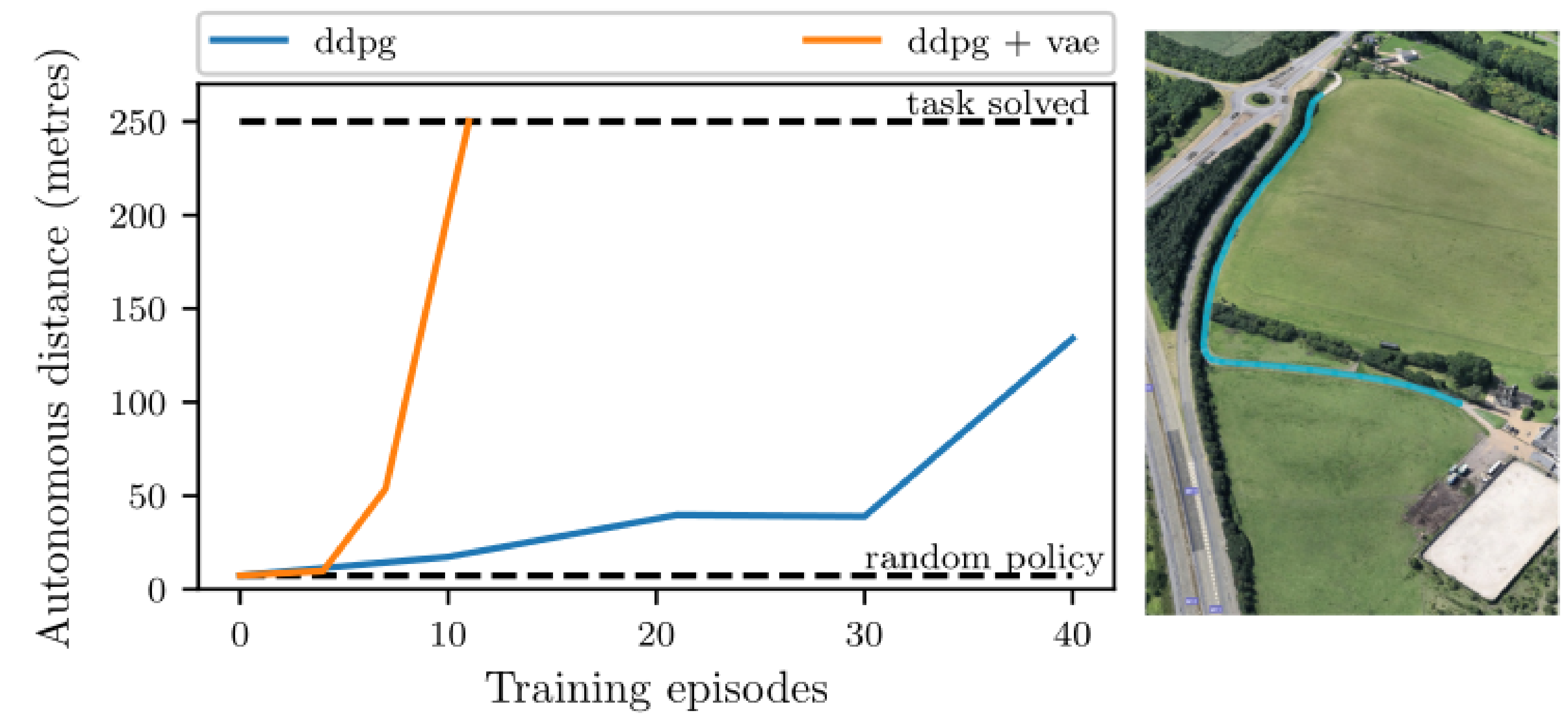


Fig. 3: Examples of different road environments randomly generated for each episode in our lane following simulator. We use procedural generation to randomly vary road texture, lane markings and road topology each episode. We train using a forward facing driver-view image as input.



(a) Algorithm results (b) Route

Fig. 4: Using a VAE with DDPG greatly improves data efficiency in training over DDPG from raw pixels, suggesting that state representation is an important consideration for applying reinforcement learning on real systems. The 250m driving route used for our experiments is shown on the right.

	Training			Test	
Model	Episodes	Distance	Time	Meters per Disengagement	# Disengagements
Random Policy	-	-	-	7.35	34
Zero Policy	-	-	-	22.7	11
Deep RL from Pixels	35	298.8 m	37 min	143.2	1
Deep RL from VAE	11	195.5 m	15 min	-	0

TABLE I: Deep reinforcement learning results on an autonomous vehicle over a 250m length of road. We report the best performance for each model. We observe the baseline RL agent can learn to lane follow from scratch, while the VAE variant is much more efficient, learning to succesfully drive the route after only 11 training episodes.

Kendall et al. (2018) Learning to Drive in a Day. arXiv:1807.00412

Skipped

4 - TD3 - Twin Delayed Deep Deterministic policy gradient

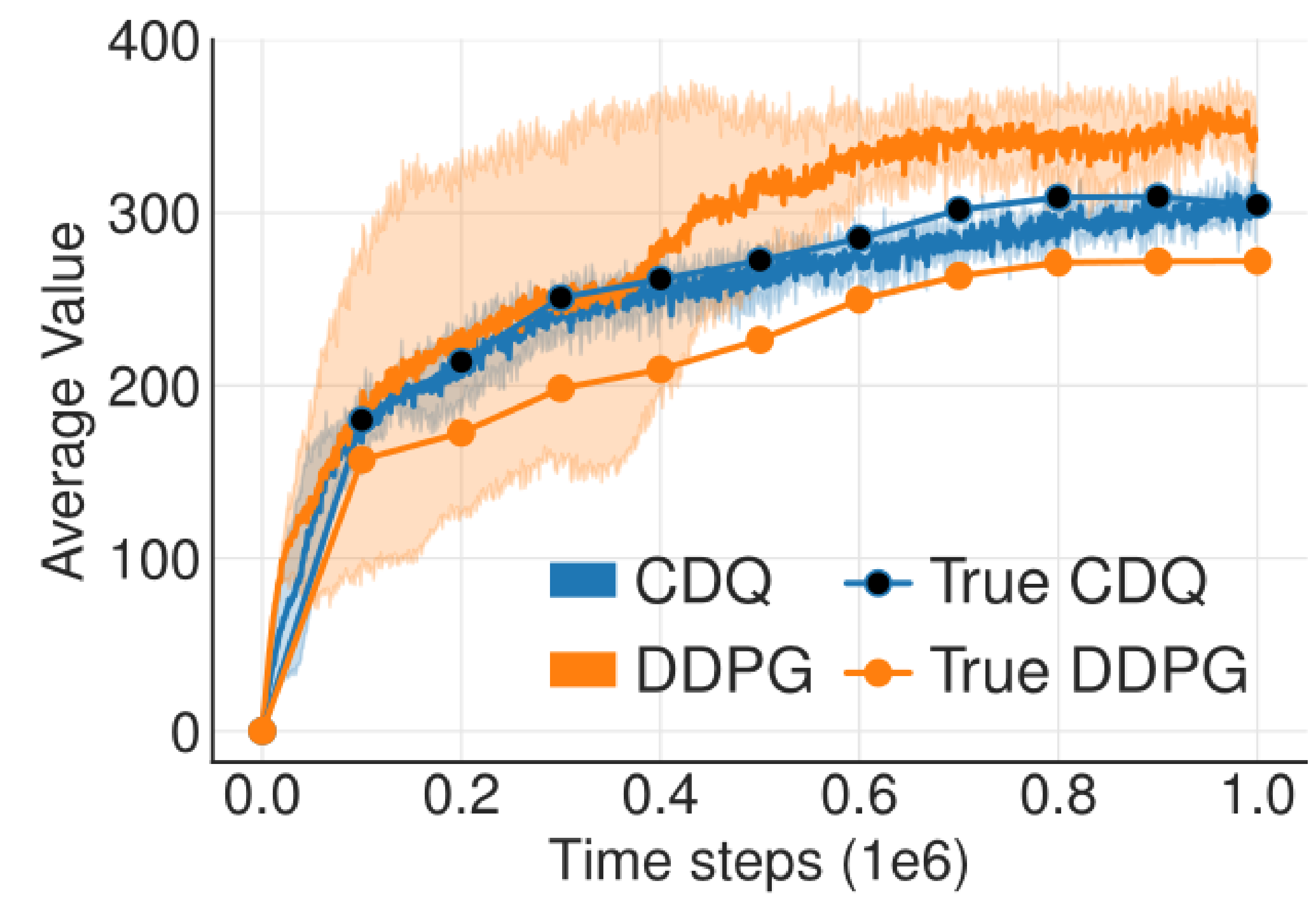
Addressing Function Approximation Error in Actor-Critic Methods

Scott Fujimoto [1] **Herke van Hoof** [2] **David Meger** [1]

Fujimoto et al. (2018) Addressing Function Approximation Error in Actor-Critic Methods. arXiv:180209477

TD3 - Twin Delayed Deep Deterministic policy gradient

- As any Q-learning-based method, DDPG **overestimates** Q-values.
- The Bellman target $t = r + \gamma \, \max_{a'} Q(s', a')$ uses a maximum over other values, so it is increasingly overestimated during learning.
- After a while, the overestimated Q-values disrupt training in the actor.



- Double Q-learning solves the problem by using the target network θ' to estimate Q-values, but the value network θ to select the greedy action in the next state:

$$\mathcal{L}(\theta) = \mathbb{E}_\mathcal{D}[(r + \gamma \, Q_{\theta'}(s´, \text{argmax}_{a'} Q_{\theta}(s', a')) - Q_\theta(s, a))^2]$$

- The idea is to use two different independent networks to reduce overestimation.
- This does not work well with DDPG, as the Bellman target $t = r + \gamma \, Q_{\varphi'}(s', \mu_{\theta'}(s'))$ uses a target actor network that is not very different from the trained deterministic actor.

TD3 - Twin Delayed Deep Deterministic policy gradient

- TD3 uses two critics φ_1 and φ_2 (and target critics):
  - the Q-value used to train the actor will be the **lesser of two evils**, i.e. the minimum Q-value:

$$t = r + \gamma \, \min(Q_{\varphi'_1}(s', \mu_{\theta'}(s')), Q_{\varphi'_2}(s', \mu_{\theta'}(s')))$$

- One of the critic will always be less over-estimating than the other. Better than nothing…
- Using twin critics is called **clipped double learning**.
- Both critics learn in parallel using the same target:

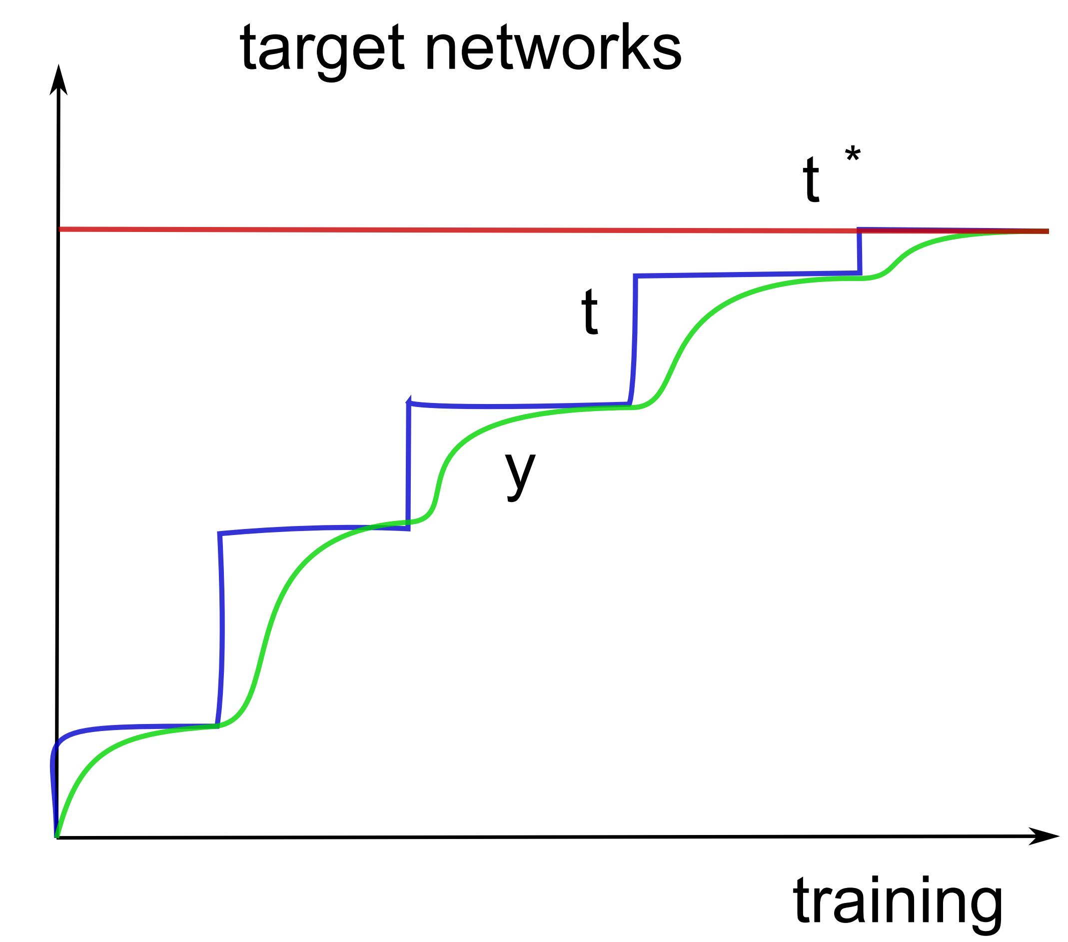
$$\mathcal{L}(\varphi_1) = \mathbb{E}[(t - Q_{\varphi_1}(s, a))^2] \qquad ; \qquad \mathcal{L}(\varphi_2) = \mathbb{E}[(t - Q_{\varphi_2}(s, a))^2]$$

- The actor is trained using the first critic only:

$$\nabla_\theta \mathcal{J}(\theta) = \mathbb{E}[\nabla_\theta \mu_\theta(s) \times \nabla_a Q_{\varphi_1}(s, a)|_{a = \mu_\theta(s)}]$$

TD3 - Twin Delayed Deep Deterministic policy gradient

- Another issue with actor-critic architecture in general is that the critic is always biased during training, what can impact the actor and ultimately collapse the policy:



$$\nabla_\theta \mathcal{J}(\theta) = \mathbb{E}[\nabla_\theta \mu_\theta(s) \times \nabla_a Q_{\varphi_1}(s, a)|_{a=\mu_\theta(s)}]$$

$$Q_{\varphi_1}(s, a) \approx Q^{\mu_\theta}(s, a)$$

- The critic should learn much faster than the actor in order to provide **unbiased** gradients.

- Increasing the learning rate in the critic creates instability, reducing the learning rate in the actor slows down learning.
- The solution proposed by TD3 is to **delay** the update of the actor, i.e. update it only every d minibatches:
  - Train the critics φ_1 and φ_2 on the minibatch.
  - **every** d steps:
    - Train the actor θ on the minibatch.
- This leaves enough time to the critics to improve their prediction and provides less biased gradients to the actor.

TD3 - Twin Delayed Deep Deterministic policy gradient

- A last problem with deterministic policies is that they tend to always select the same actions $\mu_\theta(s)$ (overfitting).
- For exploration, some additive noise is added to the selected action:

$$a = \mu_\theta(s) + \xi$$

- But this is not true for the Bellman targets, which use the deterministic action:

$$t = r + \gamma \, Q_\varphi(s', \mu_\theta(s'))$$

- TD3 proposes to also use additive noise in the Bellman targets:

$$t = r + \gamma \, Q_\varphi(s', \mu_\theta(s') + \xi)$$

- If the additive noise is zero on average, the Bellman targets will be correct on average (unbiased) but will prevent overfitting of particular actions.
- The additive noise does not have to be an **Ornstein-Uhlenbeck** stochastic process, but could simply be a random variable:

$$\xi \sim \mathcal{N}(0, 1)$$

- Initialize actor μ_θ, critics $Q_{\varphi_1}, Q_{\varphi_2}$, target networks $\mu_{\theta'}, Q_{\varphi'_1}, Q_{\varphi'_2}$, ERM $\mathcal{D}$, random processes ξ_1, ξ_2.
- for $t \in [0, T_\text{max}]$:
  - Select the action $a_t = \mu_\theta(s_t) + \xi_1$ and store $(s_t, a_t, r_{t+1}, s_{t+1})$ in the ERM.
  - For each transition (s_k, a_k, r_k, s'_k) in a minibatch sampled from $\mathcal{D}$:
    - Compute the target $t_k = r_k + \gamma \, \min(Q_{\varphi'_1}(s'_k, \mu_{\theta'}(s'_k) + \xi_2), Q_{\varphi'_2}(s'_k, \mu_{\theta'}(s'_k) + \xi_2))$.
  - Update the critics by minimizing:

$$\mathcal{L}(\varphi_1) = \frac{1}{K} \sum_k (t_k - Q_{\varphi_1}(s_k, a_k))^2 \qquad ; \qquad \mathcal{L}(\varphi_2) = \frac{1}{K} \sum_k (t_k - Q_{\varphi_2}(s_k, a_k))^2$$

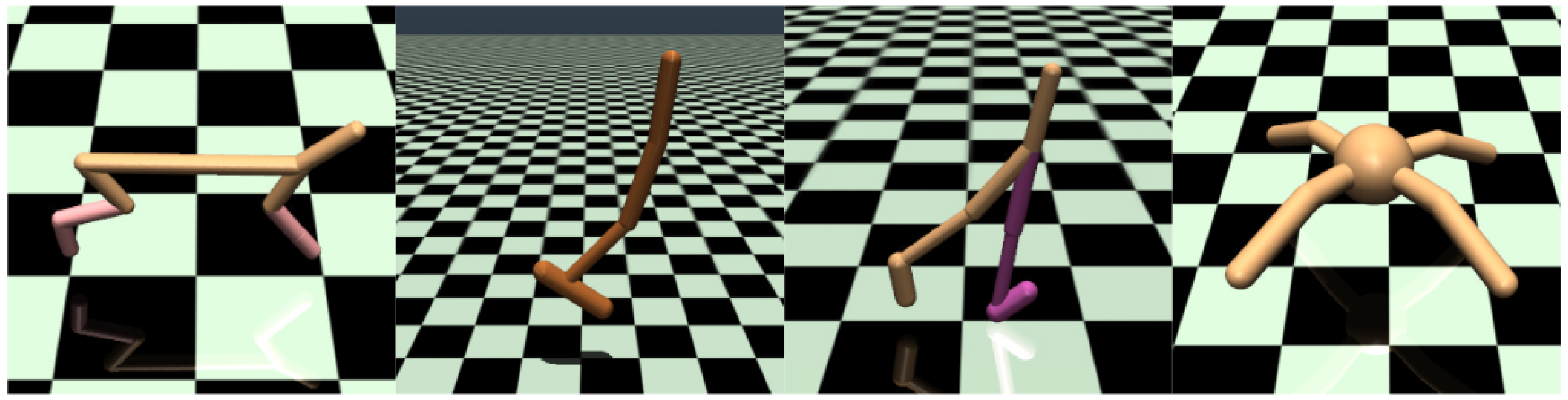
  - **every** d steps:
    - Update the actor by applying the DPG using Q_{φ_1}:

$$\nabla_\theta \mathcal{J}(\theta) = \frac{1}{K} \sum_k \nabla_\theta \mu_\theta(s_k) \times \nabla_a Q_{\varphi_1}(s_k, a)|_{a = \mu_\theta(s_k)}$$

    - Update the target networks:

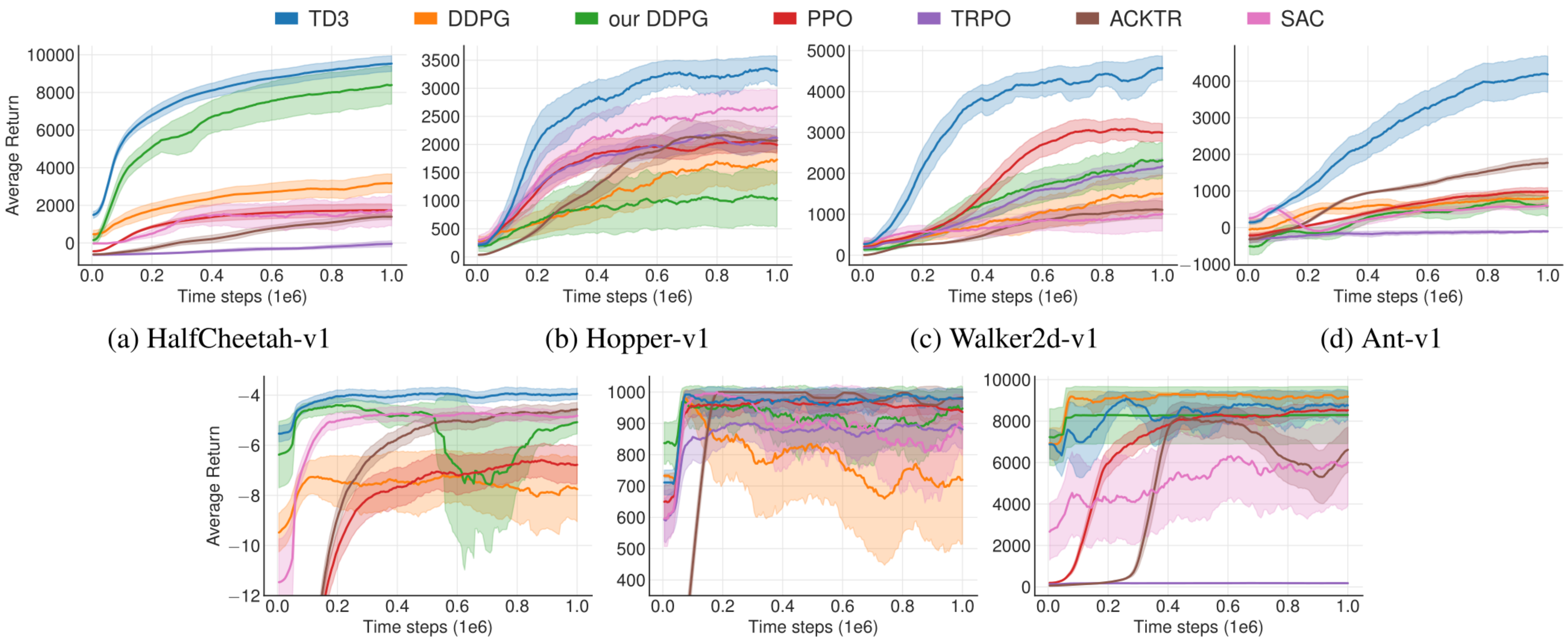
$$\theta' \leftarrow \tau \theta + (1-\tau) \, \theta' \; ; \; \varphi'_1 \leftarrow \tau \varphi_1 + (1-\tau) \, \varphi'_1 \; ; \; \varphi'_2 \leftarrow \tau \varphi_2 + (1-\tau) \, \varphi'_2$$

TD3 - Twin Delayed Deep Deterministic policy gradient



- TD3 introduces three changes to DDPG:
  - **twin** critics.
  - **delayed** actor updates.
  - noisy Bellman targets.

- TD3 outperforms DDPG (but also PPO and SAC) on continuous control tasks.



(a) HalfCheetah-v1 (b) Hopper-v1 (c) Walker2d-v1 (d) Ant-v1

(e) Reacher-v1 (f) InvertedPendulum-v1 (g) InvertedDoublePendulum-v1

5 - D4PG: Distributed Distributional DDPG

Published as a conference paper at ICLR 2018

Distributed Distributional Deterministic Policy Gradients

Gabriel Barth-Maron,* Matthew W. Hoffman,* David Budden, Will Dabney, Dan Horgan, Dhruva TB, Alistair Muldal, Nicolas Heess, Timothy Lillicrap
DeepMind
London, UK
{gabrielbm, mwhoffman, budden, wdabney, horgan, dhruvat, alimuldal, heess, countzero}@google.com

D4PG: Distributed Distributional DDPG

- **Deterministic policy gradient** as in DDPG:

$$\nabla_\theta \mathcal{J}(\theta) = \mathbb{E}_{s \sim \rho_b}[\nabla_\theta \mu_\theta(s) \times \nabla_a \mathbb{E}[\mathcal{Z}_\varphi(s, a)]|_{a = \mu_\theta(s)}]$$

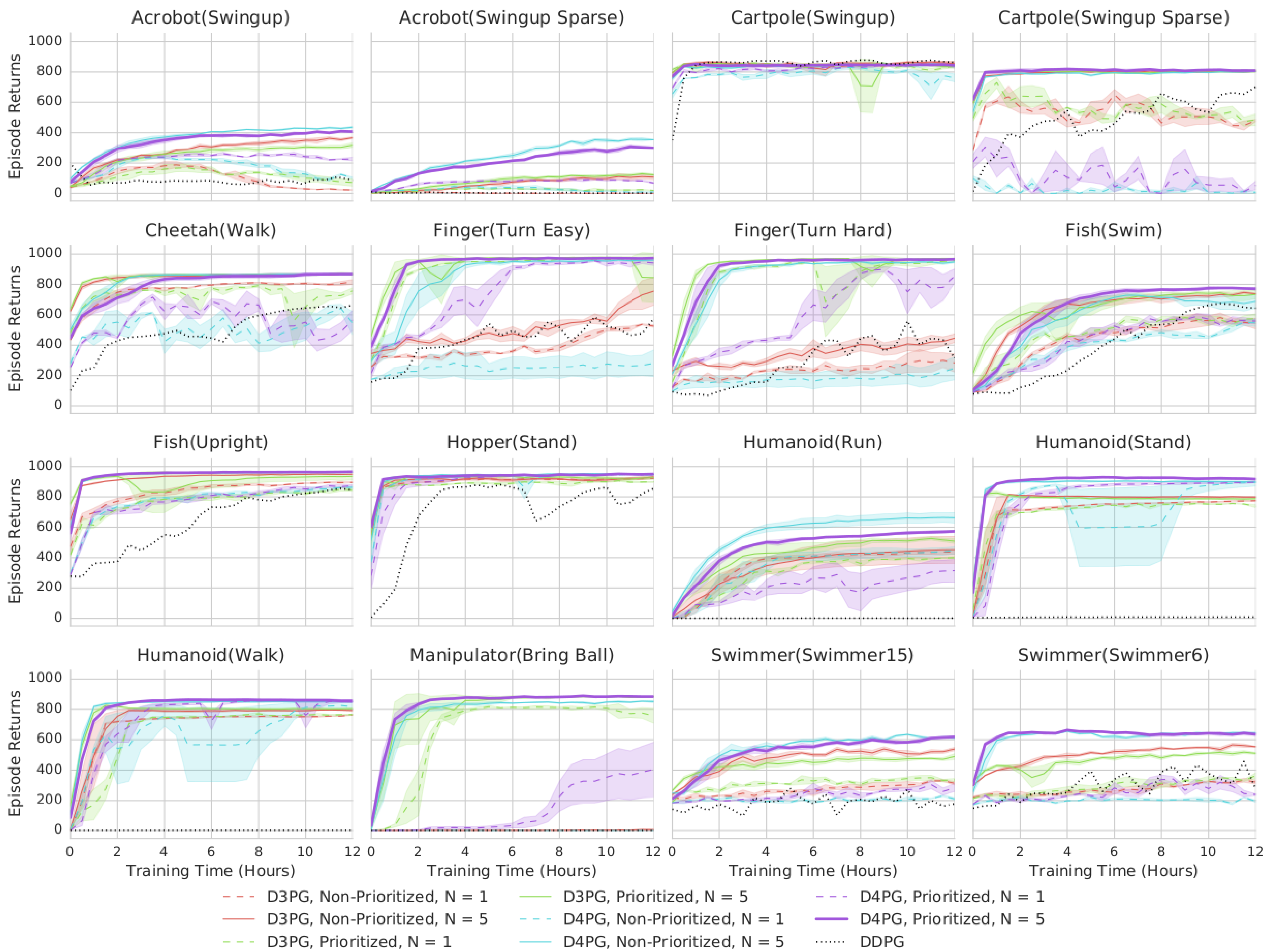
- **Distributional critic**: The critic does not predict single Q-values $Q_\varphi(s, a)$, but the distribution of returns $\mathcal{Z}_\varphi(s, a)$ (as in Categorical DQN):

$$\mathcal{L}(\varphi) = \mathbb{E}_{s \in \rho_b}[\text{KL}(\mathcal{T} \, \mathcal{Z}_\varphi(s, a) || \mathcal{Z}_\varphi(s, a))]$$

- **n-step** returns (as in A3C):

$$\mathcal{T} \, \mathcal{Z}_\varphi(s_t, a_t) = \sum_{k=0}^{n-1} \gamma^k \, r_{t+k+1} + \gamma^n \, \mathcal{Z}_\varphi(s_{t+n}, \mu_\theta(s_{t+n}))$$

- **Distributed workers**: D4PG uses $K = 32$ or 64 copies of the actor to fill the ERM in parallel.
- **Prioritized Experience Replay** (PER): $P(k) = \frac{(|\delta_k| + \epsilon)^\alpha}{\sum_k (|\delta_k| + \epsilon)^\alpha}$

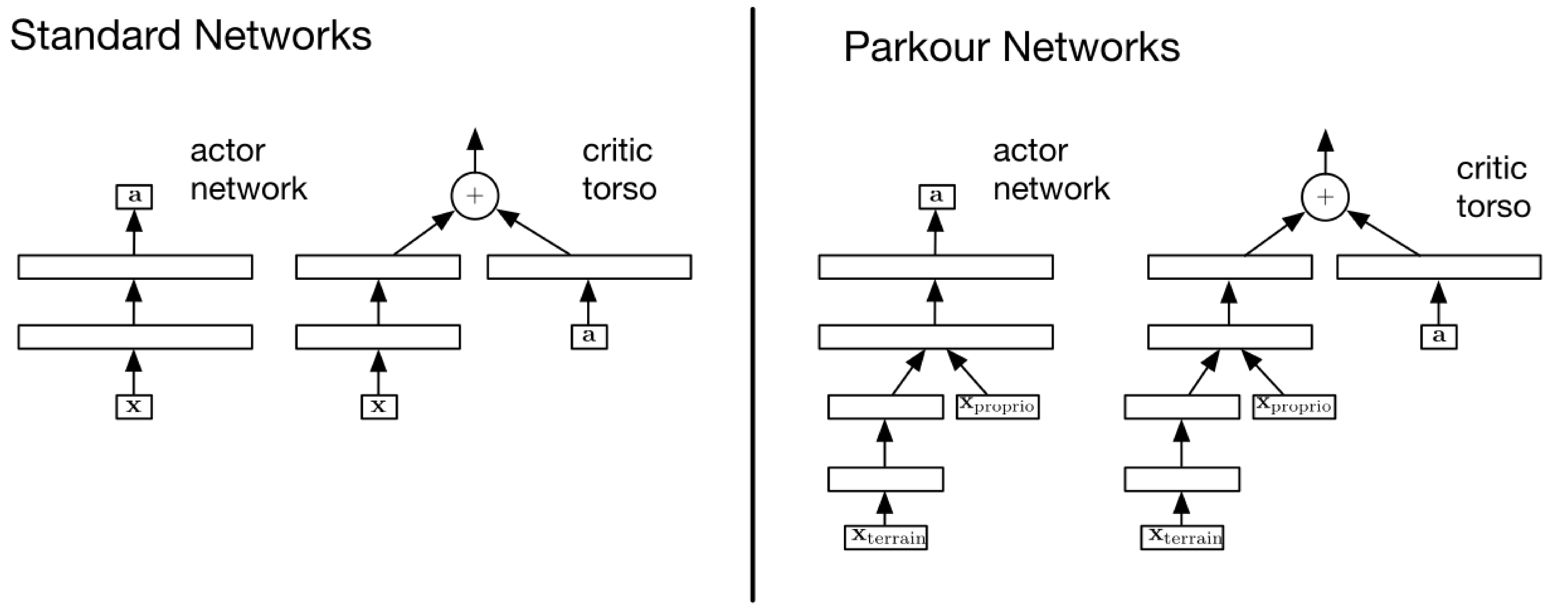


Barth-Maron et al. (2018) Distributed Distributional Deterministic Policy Gradients. arXiv:1804.08617.

D4PG: Parkour

Parkour networks

- For Parkour tasks, the states cover two different informations: the **terrain** (distance to obstacles, etc.) and the **proprioception** (joint positions of the agent).
- They enter the actor and critic networks at different locations.



References

Barth-Maron, G., Hoffman, M. W., Budden, D., Dabney, W., Horgan, D., TB, D., et al. (2018). Distributed Distributional Deterministic Policy Gradients. http://arxiv.org/abs/1804.08617.

Fujimoto, S., van Hoof, H., and Meger, D. (2018). Addressing Function Approximation Error in Actor-Critic Methods. http://arxiv.org/abs/1802.09477.

Kendall, A., Hawke, J., Janz, D., Mazur, P., Reda, D., Allen, J.-M., et al. (2018). Learning to Drive in a Day. http://arxiv.org/abs/1807.00412.

Lillicrap, T. P., Hunt, J. J., Pritzel, A., Heess, N., Erez, T., Tassa, Y., et al. (2015). Continuous control with deep reinforcement learning. *CoRR*. http://arxiv.org/abs/1509.02971.

Plappert, M., Houthooft, R., Dhariwal, P., Sidor, S., Chen, R. Y., Chen, X., et al. (2018). Parameter Space Noise for Exploration. http://arxiv.org/abs/1706.01905.

Silver, D., Lever, G., Heess, N., Degris, T., Wierstra, D., and Riedmiller, M. (2014). Deterministic Policy Gradient Algorithms. in *Proc. ICML* Proceedings of Machine Learning Research., eds. E. P. Xing and T. Jebara (PMLR), 387–395. http://proceedings.mlr.press/v32/silver14.html.

Uhlenbeck, G. E., and Ornstein, L. S. (1930). On the Theory of the Brownian Motion. *Physical Review* 36. doi:10.1103/PhysRev.36.823.