UNIVERSITY OF TECHNOLOGY
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CHEMNITZ

Deep Reinforcement Learning

Successor representations

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1 - Model-based vs. Model-free

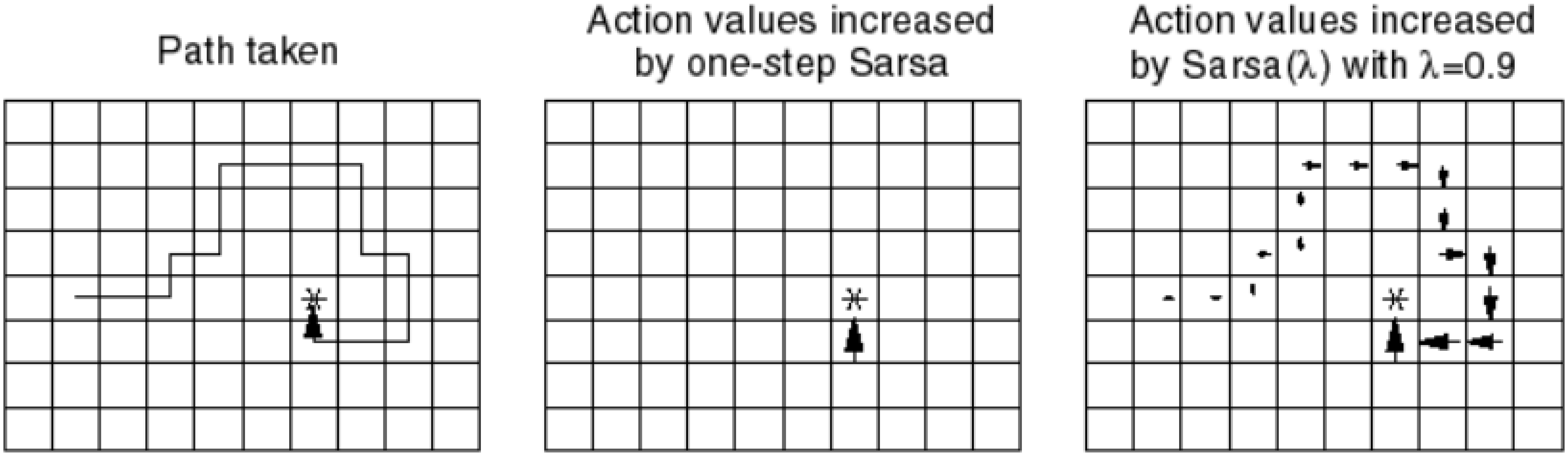
Model-based vs. Model-free

- Model-free methods use the **reward prediction error** (RPE) to update values:

$$\delta_t = r_{t+1} + \gamma \, V^\pi(s_{t+1}) - V^\pi(s_t)$$

$$\Delta V^\pi(s_t) = \alpha \, \delta_t$$

Encountered rewards propagate very slowly to all states and actions.


Path taken
Action values increased by one-step Sarsa
Action values increased by Sarsa(λ) with λ=0.9

- If the environment changes (transition probabilities, rewards), they have to relearn everything.
- After training, selecting an action is very fast.

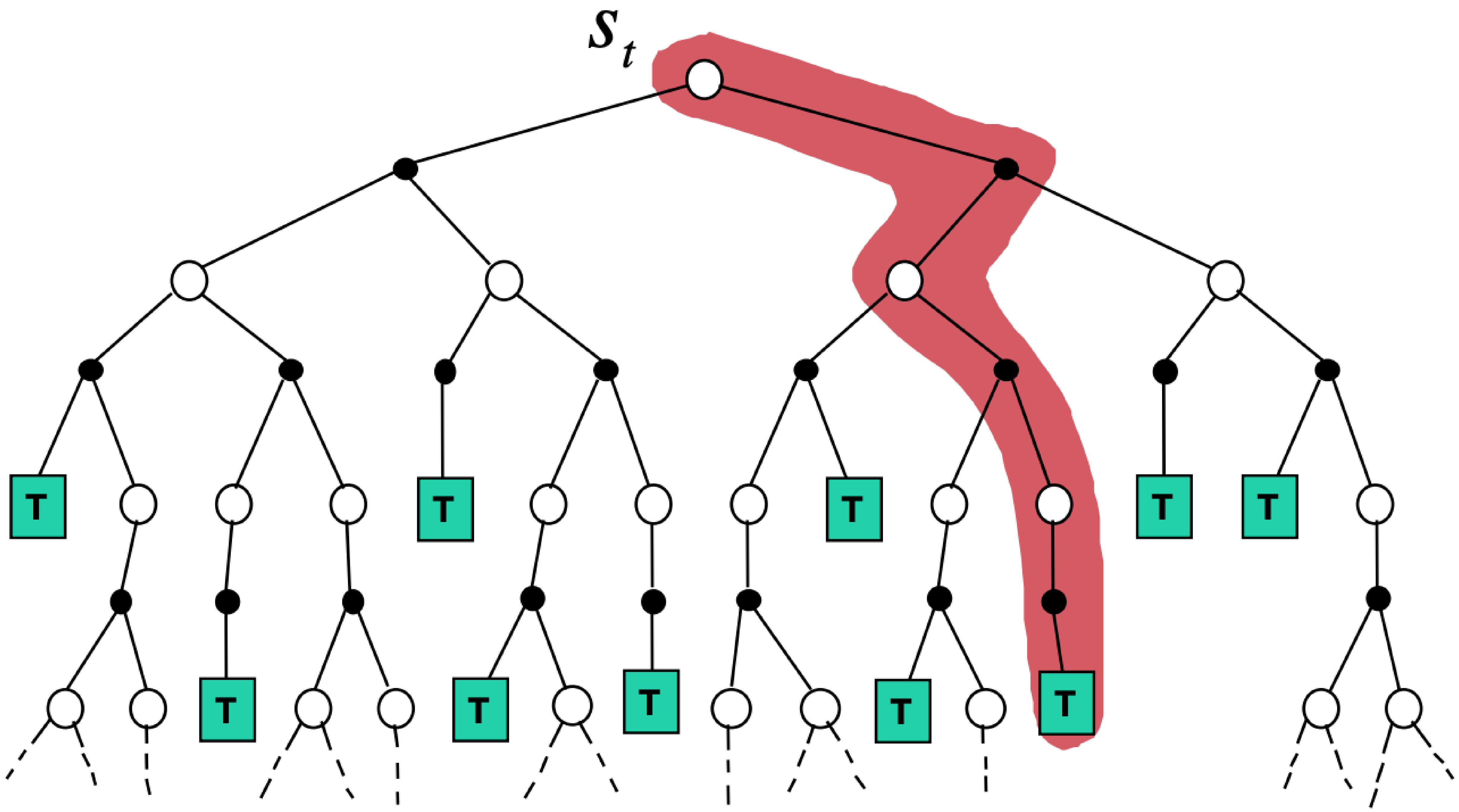
Model-based vs. Model-free

- Model-based RL can learn very fast changes in the transition or reward distributions:

$$\Delta r(s_t, a_t, s_{t+1}) = \alpha \, (r_{t+1} - r(s_t, a_t, s_{t+1}))$$

$$\Delta p(s' | s_t, a_t) = \alpha \, (\mathbb{I}(s_{t+1} = s') - p(s' | s_t, a_t))$$

- But selecting an action requires planning in the tree of possibilities (slow).



Model-based vs. Model-free

- Relative advantages of MF and MB methods:

	Inference speed	**Sample complexity**	**Optimality**	**Flexibility**
Model-free	fast	high	yes	no
Model-based	slow	low	as good as the model	yes

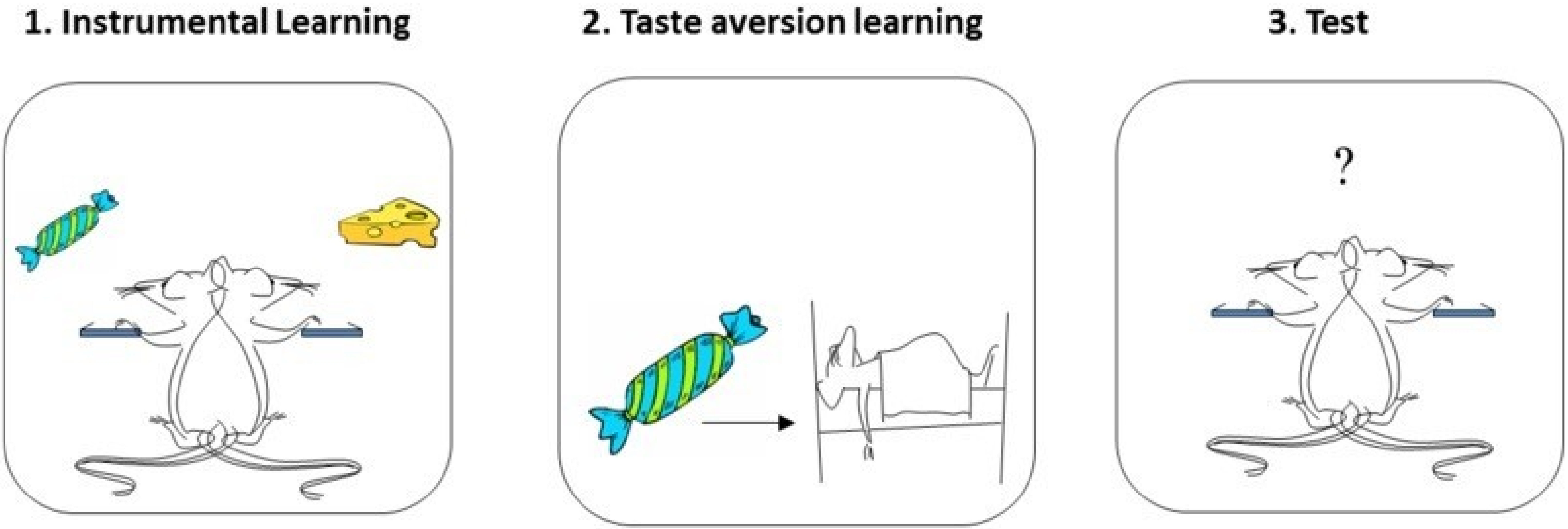
- A trade-off would be nice… Most MB models in the deep RL literature are hybrid MB/MF models anyway.

Outcome devaluation

- Two forms of behavior are observed in the animal psychology literature:

1. **Goal-directed** behavior learns Stimulus → Response → Outcome associations.
2. **Habits** are developed by overtraining Stimulus → Response associations.

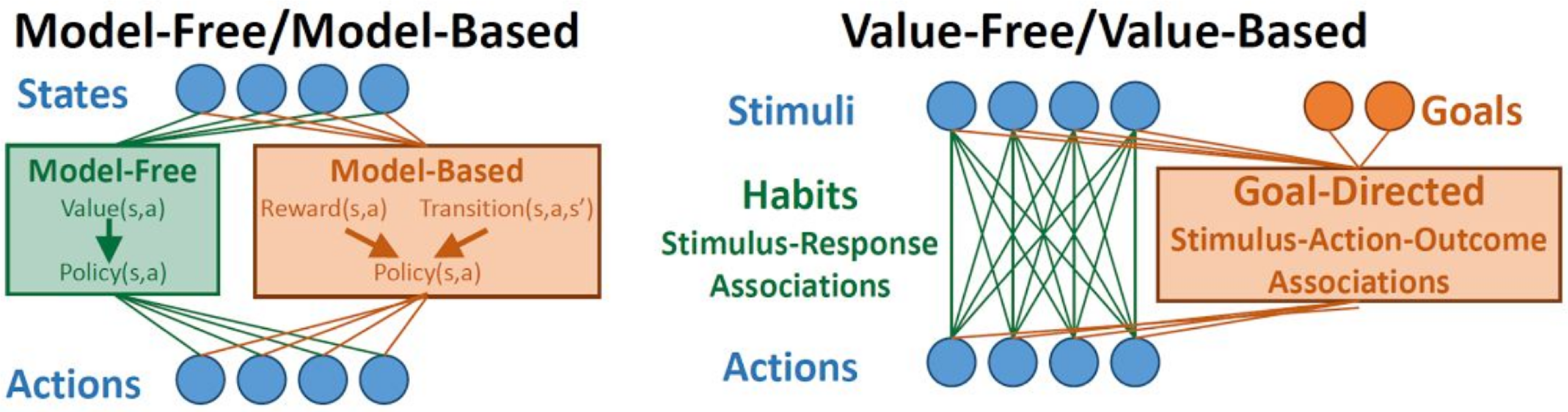
- The main difference is that habits are not influenced by **outcome devaluation**, i.e. when rewards lose their value.



Source: Bernard W. Balleine

Goal-directed / habits = MB / MF ?

- The classical theory assigns MF to habits and MB to goal-directed, mostly because their sensitivity to outcome devaluation.



- The open question is the arbitration mechanism between these two segregated process: who takes control?
- Recent work suggests both systems are largely overlapping.

References

Doll, B. B., Simon, D. A., and Daw, N. D. (2012). The ubiquity of model-based reinforcement learning. Current Opinion in Neurobiology 22, 1075–1081. doi:10.1016/j.conb.2012.08.003.

Miller, K., Ludvig, E. A., Pezzulo, G., and Shenhav, A. (2018). “Re-aligning models of habitual and goal-directed decision-making,” in Goal-Directed Decision Making : Computations and Neural Circuits, eds. A. Bornstein, R. W. Morris, and A. Shenhav (Academic Press)

2 - Successor representations

Successor Representations (SR)

- Successor representations (SR) have been introduced to combine MF and MB properties. Let's split the definition of the value of a state:

$$V^\pi(s) = \mathbb{E}_{\pi} [\sum_{k=0}^\infty \gamma^k \, r_{t+k+1} | s_t = s] \tag{1}$$

$$\tag{2}$$

$$= \mathbb{E}_{\pi} [\begin{bmatrix} 1 \\ \gamma \\ \gamma^2 \\ \ldots \\ \gamma^\infty \end{bmatrix} \times \begin{bmatrix} \mathbb{I}(s_{t}) \\ \mathbb{I}(s_{t+1}) \\ \mathbb{I}(s_{t+2}) \\ \ldots \\ \mathbb{I}(s_{\infty}) \end{bmatrix} \times \begin{bmatrix} r_{t+1} \\ r_{t+2} \\ r_{t+3} \\ \ldots \\ r_{t+\infty} \end{bmatrix} | s_t = s] \tag{3}$$

where $\mathbb{I}(s_{t})$ is 1 when the agent is in s_t at time t, 0 otherwise.

- The left part corresponds to the **transition dynamics**: which states will be visited by the policy, discounted by γ.
- The right part corresponds to the **immediate reward** in each visited state.
- Couldn't we learn the transition dynamics and the reward distribution separately in a model-free manner?

Successor Representations (SR)

- SR rewrites the value of a state into an **expected discounted future state occupancy** $M^\pi(s, s')$ and an **expected immediate reward** $r(s')$ by summing over all possible states s' of the MDP:

$$V^\pi(s) = \mathbb{E}_\pi[\sum_{k=0}^\infty \gamma^k \, r_{t+k+1} | s_t = s] \tag{4}$$

$$\tag{5}$$

$$= \sum_{s' \in \mathcal{S}} \mathbb{E}_\pi[\sum_{k=0}^\infty \gamma^k \, \mathbb{I}(s_{t+k} = s') \times r_{t+k+1} | s_t = s] \tag{6}$$

$$\tag{7}$$

$$\approx \sum_{s' \in \mathcal{S}} \mathbb{E}_\pi[\sum_{k=0}^\infty \gamma^k \, \mathbb{I}(s_{t+k} = s') | s_t = s] \times \mathbb{E}[r_{t+1} | s_t = s'] \tag{8}$$

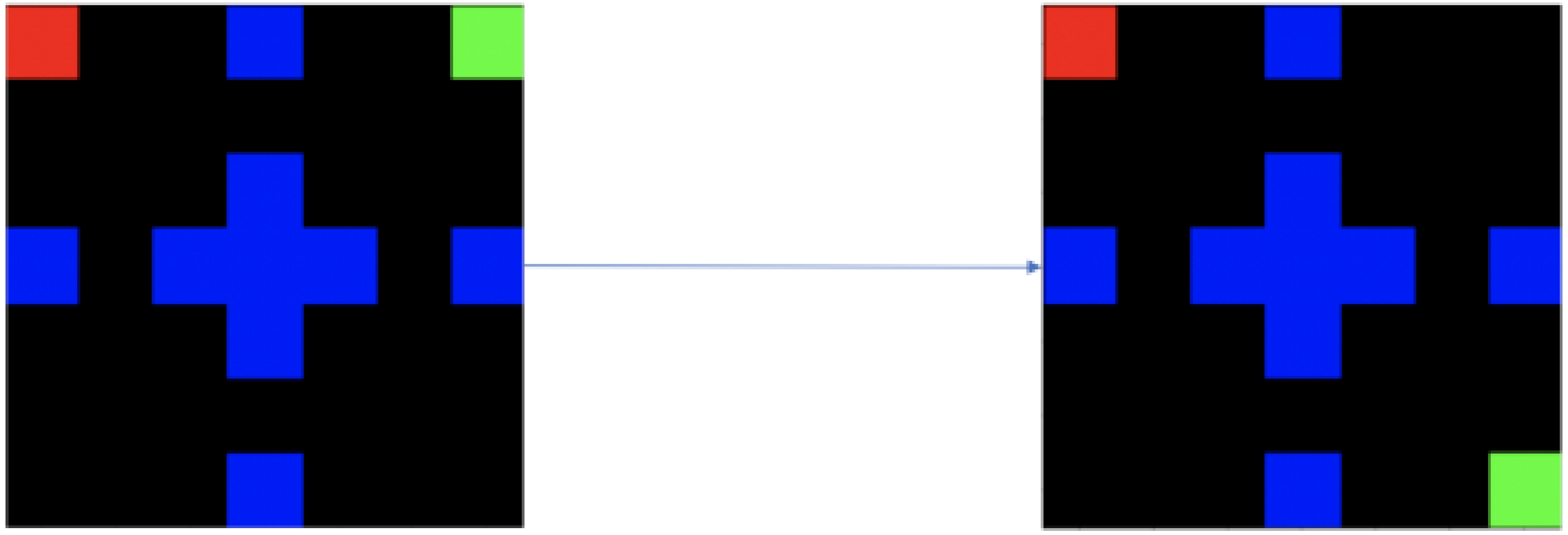
$$\tag{9}$$

$$\approx \sum_{s' \in \mathcal{S}} M^\pi(s, s') \times r(s') \tag{10}$$

Dayan, P. (1993). Improving Generalization for Temporal Difference Learning: The Successor Representation. Neural Computation 5, 613–624. doi:10.1162/neco.1993.5.4.613.

Successor Representations (SR)

- The underlying assumption is that the world dynamics are independent from the reward function (which does not depend on the policy).
- This allows to re-use knowledge about world dynamics in other contexts (e.g. a new reward function in the same environment): **transfer learning**.



Source: https://awjuliani.medium.com/the-present-in-terms-of-the-future-successor-representations-in-reinforcement-learning-316b78c5fa3

- What matters is the states that you will visit and how interesting they are, not the order in which you visit them.
- Knowing that being in the mensa will eventually get you some food is enough to know that being in the mensa is a good state: you do not need to remember which exact sequence of transitions will put food in your mouth.

Successor Representations (SR)

- SR algorithms must estimate two quantities:
  1. The **expected immediate reward** received after each state:

$$r(s) = \mathbb{E}[r_{t+1} | s_t = s]$$

  2. The **expected discounted future state occupancy** (the **SR** itself):

$$M^\pi(s, s') = \mathbb{E}_{\pi} [\sum_{k=0}^\infty \gamma^k \, \mathbb{I}(s_{t+k} = s') | s_t = s]$$

- The value of a state s is then computed with:

$$V^\pi(s) = \sum_{s' \in \mathcal{S}} M(s, s') \times r(s')$$

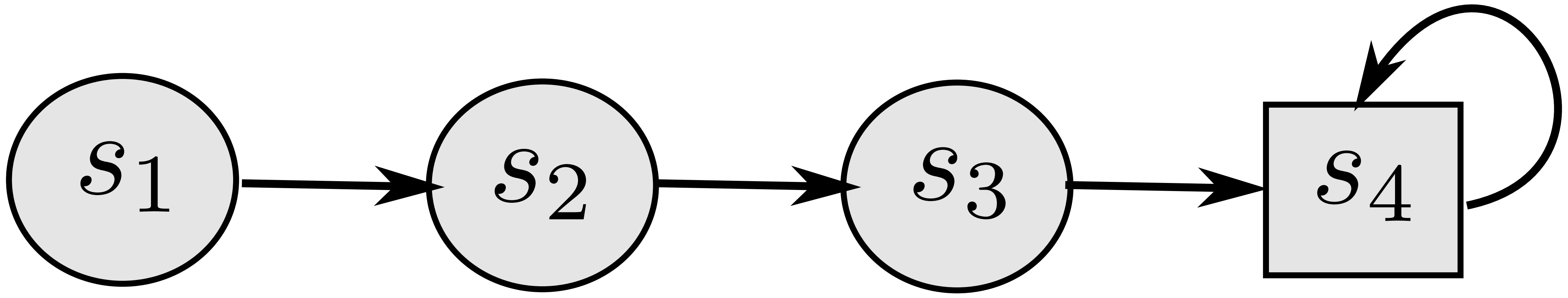
what allows to infer the policy (e.g. using an actor-critic architecture).

- The immediate reward for a state can be estimated very quickly and flexibly after receiving each reward:

$$\Delta \, r(s_t) = \alpha \, (r_{t+1} - r(s_t))$$

SR and transition matrix

- Imagine a very simple MDP with 4 states and a single deterministic action:



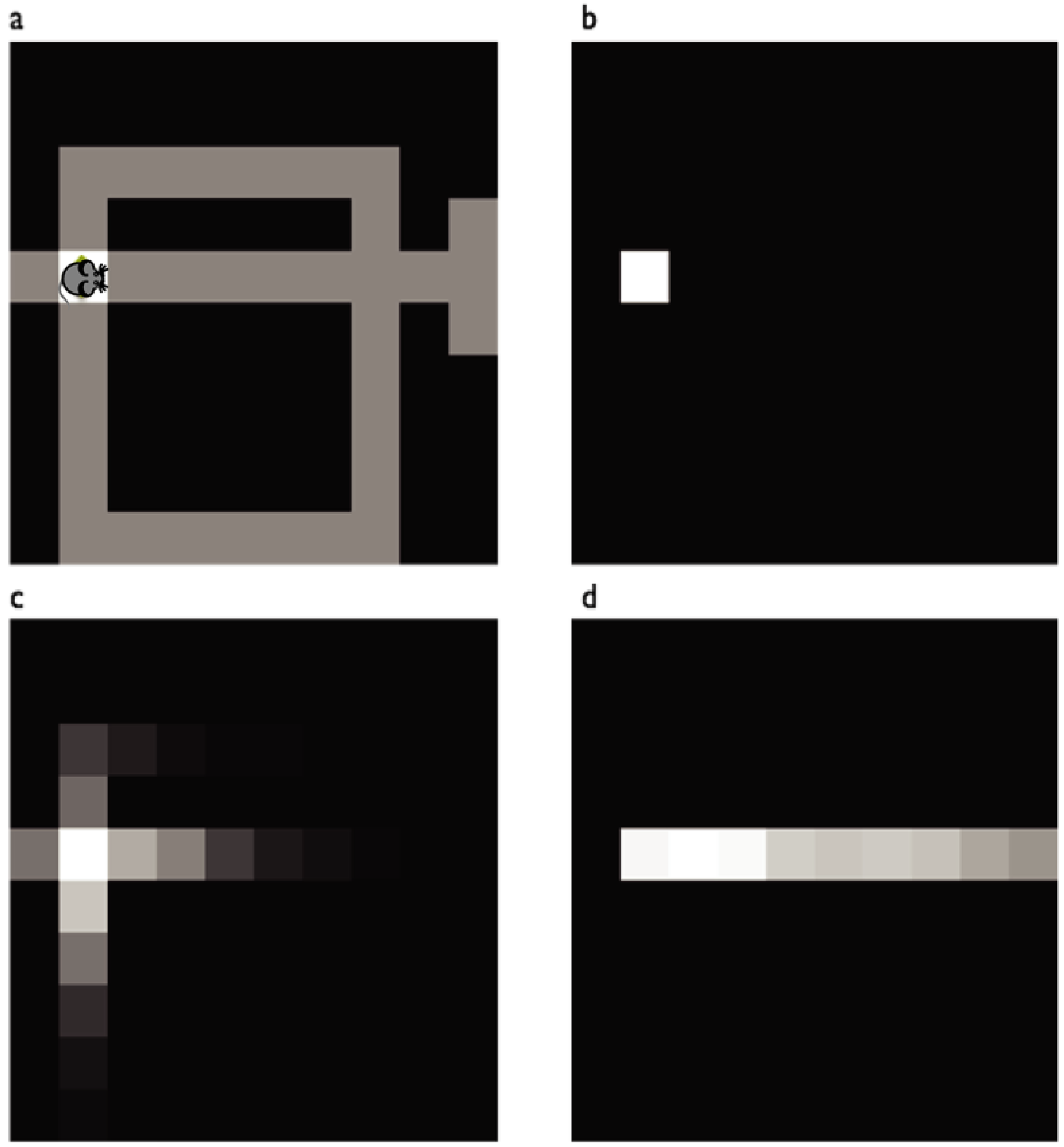
- The transition matrix $\mathcal{P}^\pi$ depicts the possible (s, s') transitions:

$$\mathcal{P}^\pi = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- The SR matrix M also represents the future transitions discounted by γ:

$$M = \begin{bmatrix} 1 & \gamma & \gamma^2 & \gamma^3 \\ 0 & 1 & \gamma & \gamma^2 \\ 0 & 0 & 1 & \gamma \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

SR matrix in a Tolman's maze



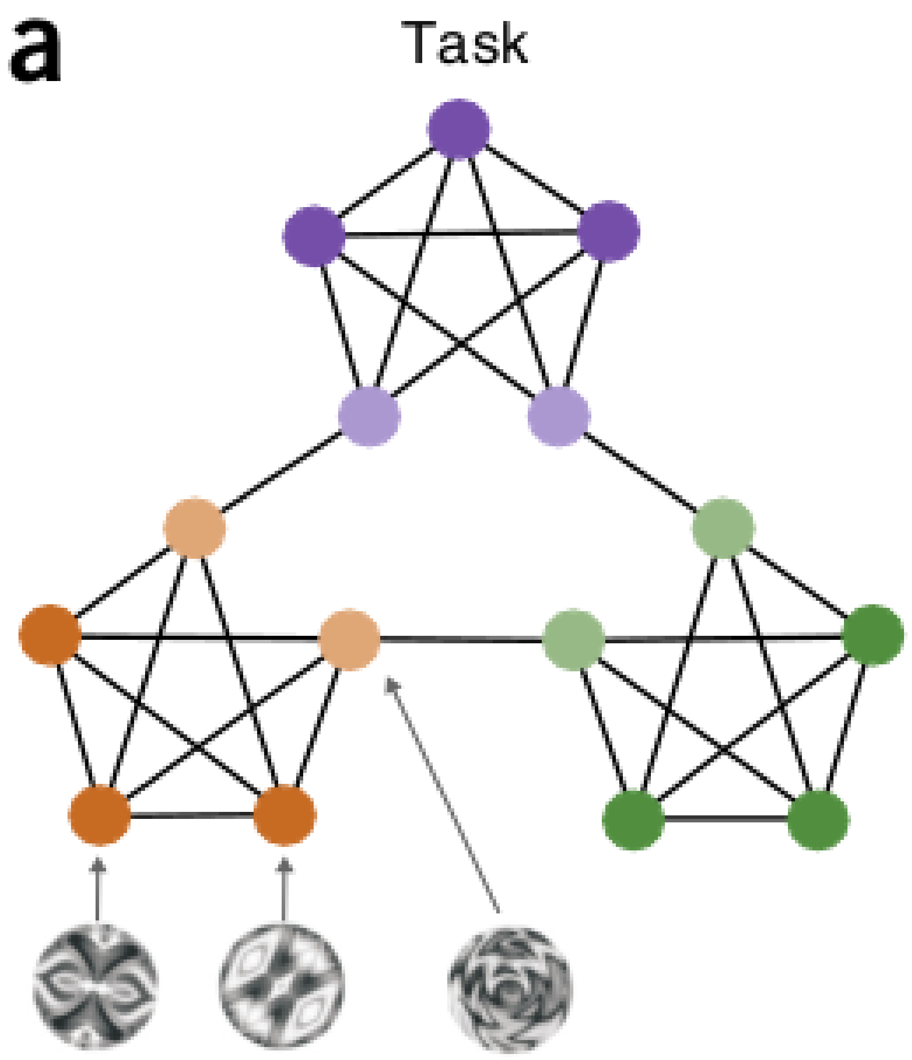
- The SR represents whether a state can be reached soon from the current state (b) using the current policy.
- The SR depends on the policy:
    - A random agent will map the local neighborhood (c).
    - A goal-directed agent will have SR representations that follow the optimal path (d).
- It is therefore different from the transition matrix, as it depends on behavior and rewards.
- The exact dynamics are lost compared to MB: it only represents whether a state is reachable, not how.

Russek et al. (2017). Predictive representations can link model-based reinforcement learning to model-free mechanisms. PLOS Computational Biology.

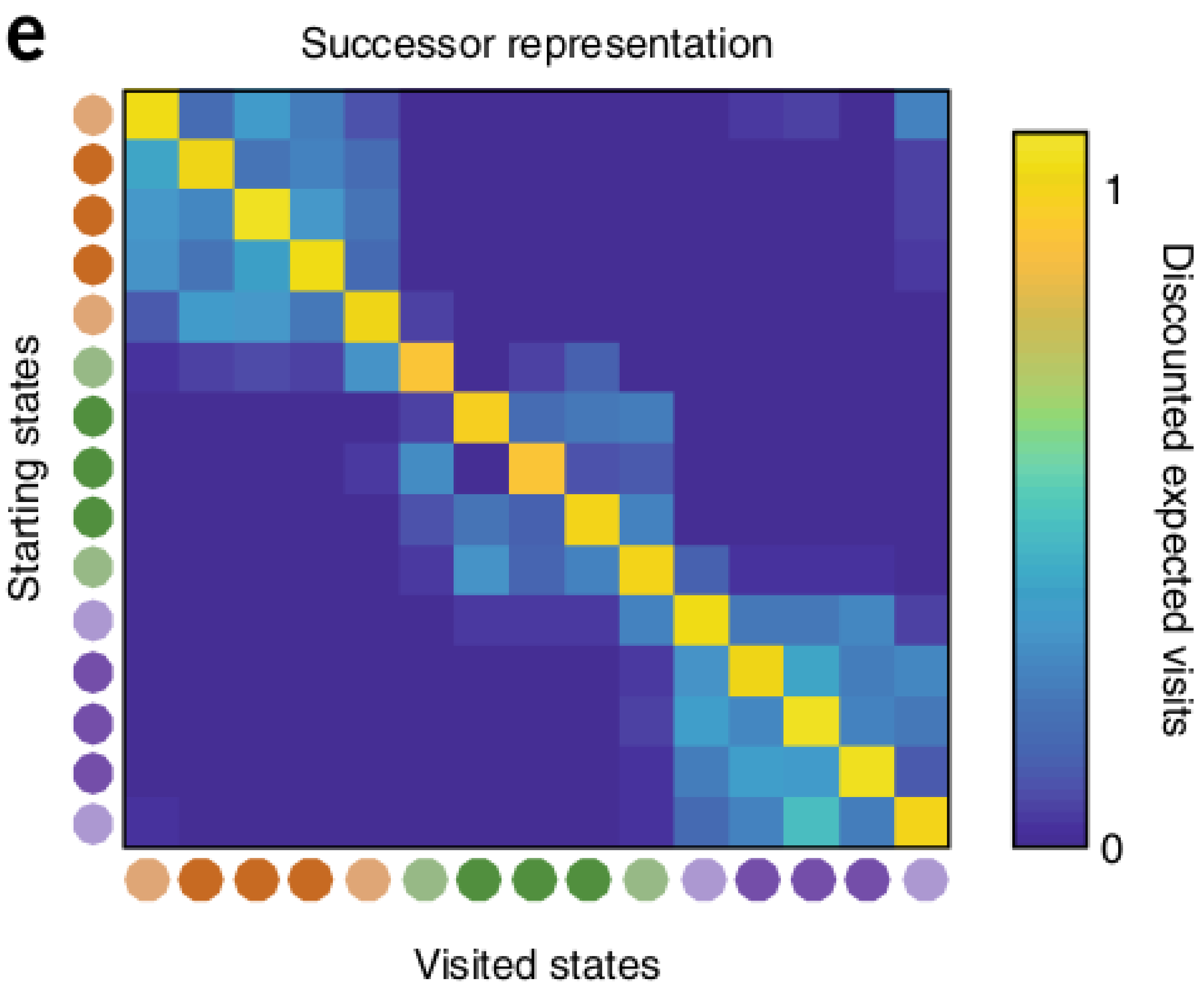
Example of a SR matrix

- The SR matrix reflects the proximity between states depending on the transitions and the policy. it does not have to be a spatial relationship.

Schapiro *et al.* (2015)



SR Simulations



Stachenfeld, K. L., Botvinick, M. M., and Gershman, S. J. (2017). The hippocampus as a predictive map. Nature Neuroscience 20, 1643–1653. doi:10.1038/nn.4650

Learning the SR

- How can we learn the SR matrix for all pairs of states?

$$M^\pi(s, s') = \mathbb{E}_\pi [\sum_{k=0}^\infty \gamma^k \, \mathbb{I}(s_{t+k} = s') | s_t = s]$$

- We first notice that the SR obeys a recursive Bellman-like equation:

$$
\begin{aligned}
M^\pi(s, s') &= \mathbb{I}(s_t = s') + \mathbb{E}_\pi [\sum_{k=1}^\infty \gamma^k \, \mathbb{I}(s_{t+k} = s') | s_t = s] \\
&= \mathbb{I}(s_t = s') + \gamma \, \mathbb{E}_\pi [\sum_{k=0}^\infty \gamma^k \, \mathbb{I}(s_{t+k+1} = s') | s_t = s] \\
&= \mathbb{I}(s_t = s') + \gamma \, \mathbb{E}_{s_{t+1} \sim \mathcal{P}^\pi(s'|s)} [\mathbb{E}_\pi [\sum_{k=0}^\infty \gamma^k \, \mathbb{I}(s_{t+k} = s') | s_{t+1} = s]] \\
&= \mathbb{I}(s_t = s') + \gamma \, \mathbb{E}_{s_{t+1} \sim \mathcal{P}^\pi(s'|s)} [M^\pi(s_{t+1}, s')]
\end{aligned}
$$

- This is reminiscent of TDM: the remaining distance to the goal is 0 if I am already at the goal, or gamma the distance from the next state to the goal.

Model-based SR

- Bellman-like SR:

$$M^{\pi}(s, s') = \mathbb{I}(s_t = s') + \gamma \, \mathbb{E}_{s_{t+1} \sim \mathcal{P}^{\pi}(s'|s)} [M^{\pi}(s_{t+1}, s')]$$

- If we know the transition matrix for a fixed policy π:

$$\mathcal{P}^{\pi}(s, s') = \sum_a \pi(s, a) \, p(s' | s, a)$$

we can obtain the SR directly with matrix inversion as we did in **dynamic programming**:

$$M^{\pi} = I + \gamma \, \mathcal{P}^{\pi} \times M^{\pi}$$

so that:

$$M^{\pi} = (I - \gamma \, \mathcal{P}^{\pi})^{-1}$$

- This DP approach is called **model-based SR** (MB-SR) as it necessitates to know the environment dynamics.

Momennejad et al. (2017). The successor representation in human reinforcement learning. Nature Human Behaviour 1, 680–692. doi:10.1038/s41562-017-0180-8.

Model-free SR

- If we do not know the transition probabilities, we simply sample a single s_t, s_{t+1} transition:

$$M^\pi(s_t, s') \approx \mathbb{I}(s_t = s') + \gamma \, M^\pi(s_{t+1}, s')$$

- We can define a **sensory prediction error** (SPE):

$$\delta_t^\text{SR} = \mathbb{I}(s_t = s') + \gamma \, M^\pi(s_{t+1}, s') - M(s_t, s')$$

that is used to update an estimate of the SR:

$$\Delta M^\pi(s_t, s') = \alpha \, \delta_t^\text{SR}$$

- This is **SR-TD**, using a SPE instead of RPE, which learns only from transitions but ignores rewards.

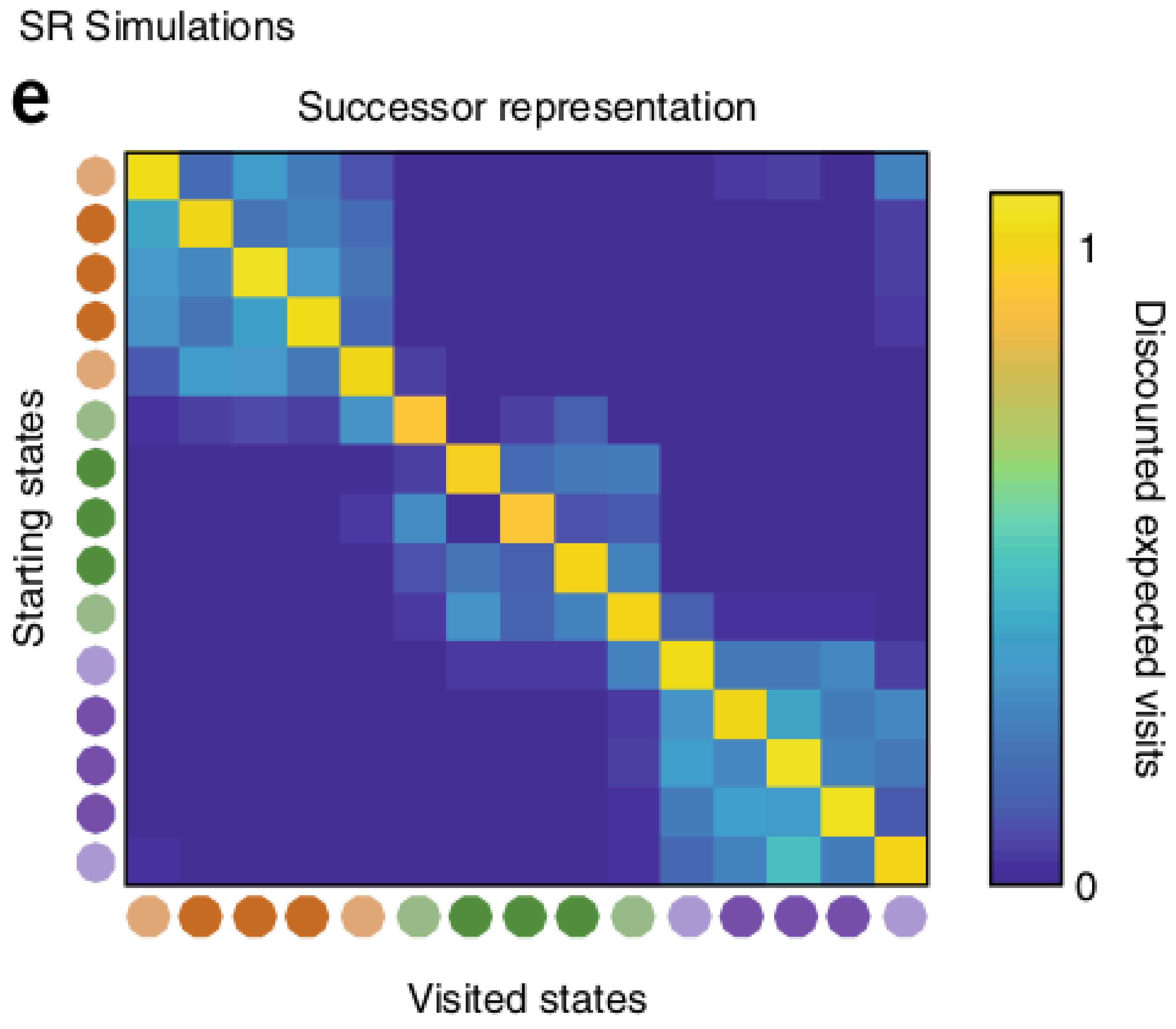
Momennejad et al. (2017). The successor representation in human reinforcement learning. Nature Human Behaviour 1, 680–692. doi:10.1038/s41562-017-0180-8.

The sensory prediction error - SPE

- The SPE has to be applied on ALL successor states s' after a transition (s_t, s_{t+1}):

$$M^\pi(s_t, \mathbf{s'}) = M^\pi(s_t, \mathbf{s'}) + \alpha\, (\mathbb{I}(s_t = \mathbf{s'}) + \gamma\, M^\pi(s_{t+1}, \mathbf{s'}) - M(s_t, \mathbf{s'}))$$

- Contrary to the RPE, the SPE is a **vector** of prediction errors, used to update one row of the SR matrix.
- The SPE tells how **surprising** a transition $s_t \rightarrow s_{t+1}$ is for the SR.



Stachenfeld, K. L., Botvinick, M. M., and Gershman, S. J. (2017). The hippocampus as a predictive map. Nature Neuroscience 20, 1643–1653. doi:10.1038/nn.4650

Successor representations

- The SR matrix represents the **expected discounted future state occupancy**:

$$M^\pi(s, s') = \mathbb{E}_\pi [\sum_{k=0}^\infty \gamma^k \, \mathbb{I}(s_{t+k} = s') | s_t = s]$$

- It can be learned using a TD-like SPE from single transitions:

$$M^\pi(s_t, \mathbf{s'}) = M^\pi(s_t, \mathbf{s'}) + \alpha \, (\mathbb{I}(s_t = \mathbf{s'}) + \gamma \, M^\pi(s_{t+1}, \mathbf{s'}) - M(s_t, \mathbf{s'}))$$

- The immediate reward in each state can be learned **independently from the policy**:

$$\Delta \, r(s_t) = \alpha \, (r_{t+1} - r(s_t))$$

- The value $V^\pi(s)$ of a state is obtained by summing of all successor states:

$$V^\pi(s) = \sum_{s' \in \mathcal{S}} M(s, s') \times r(s')$$

- This critic can be used to train an **actor** π_θ using regular TD learning (e.g. A3C).

Stachenfeld, K. L., Botvinick, M. M., and Gershman, S. J. (2017). The hippocampus as a predictive map. Nature Neuroscience 20, 1643–1653. doi:10.1038/nn.4650

Successor representation of actions

- Note that it is straightforward to extend the idea of SR to state-action pairs:

$$M^\pi(s, a, s') = \mathbb{E}_{\pi} [\sum_{k=0}^\infty \gamma^k \, \mathbb{I}(s_{t+k} = s') | s_t = s, a_t = a]$$

allowing to estimate Q-values:

$$Q^\pi(s, a) = \sum_{s' \in \mathcal{S}} M(s, a, s') \times r(s')$$

using SARSA or Q-learning-like SPEs:

$$\delta^\text{SR}_t = \mathbb{I}(s_{t} = s') + \gamma \, M^\pi(s_{t+1}, a_{t+1}, s') - M(s_t, a_t, s')$$

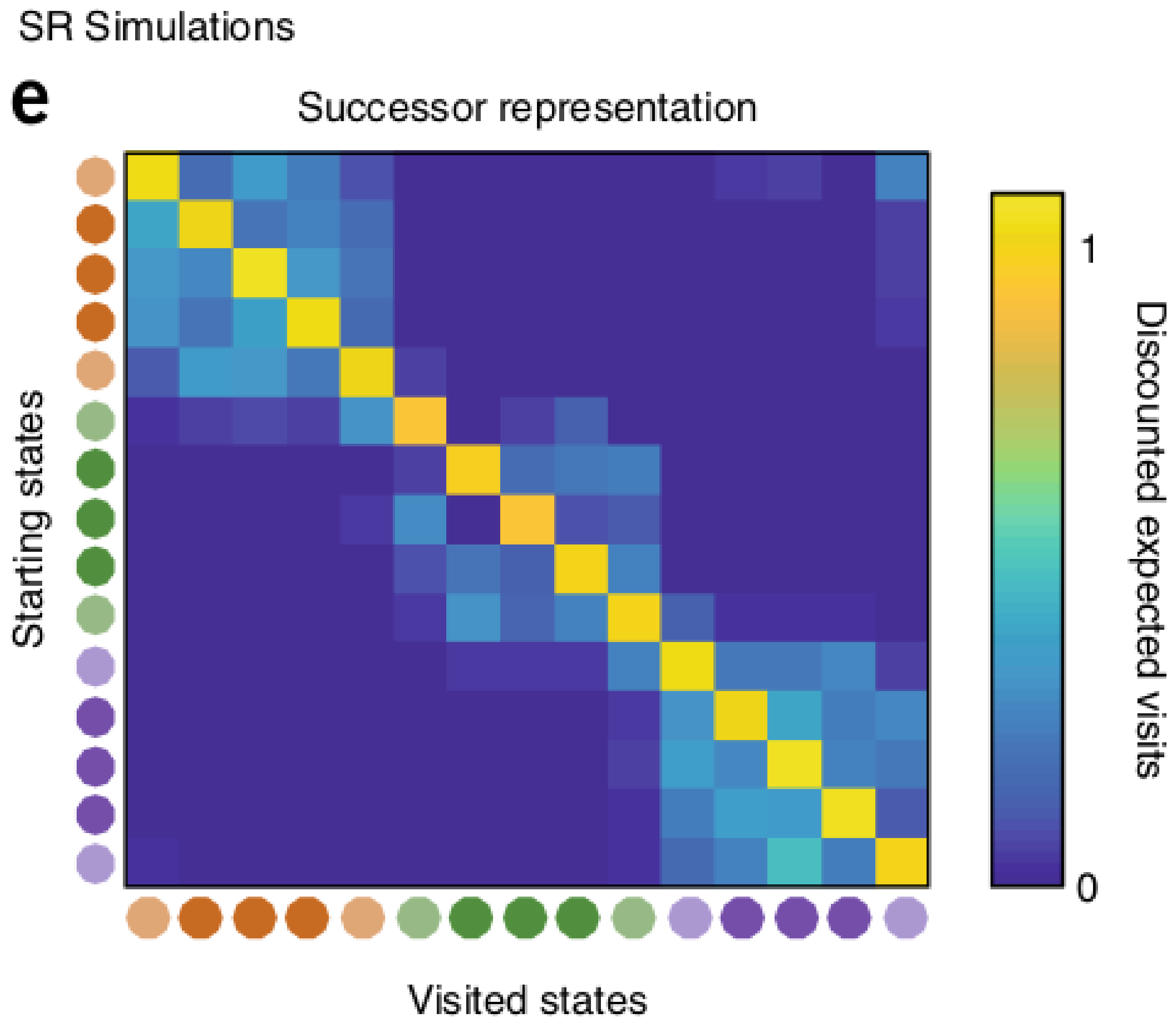
depending on the choice of the next action a_{t+1} (on- or off-policy).

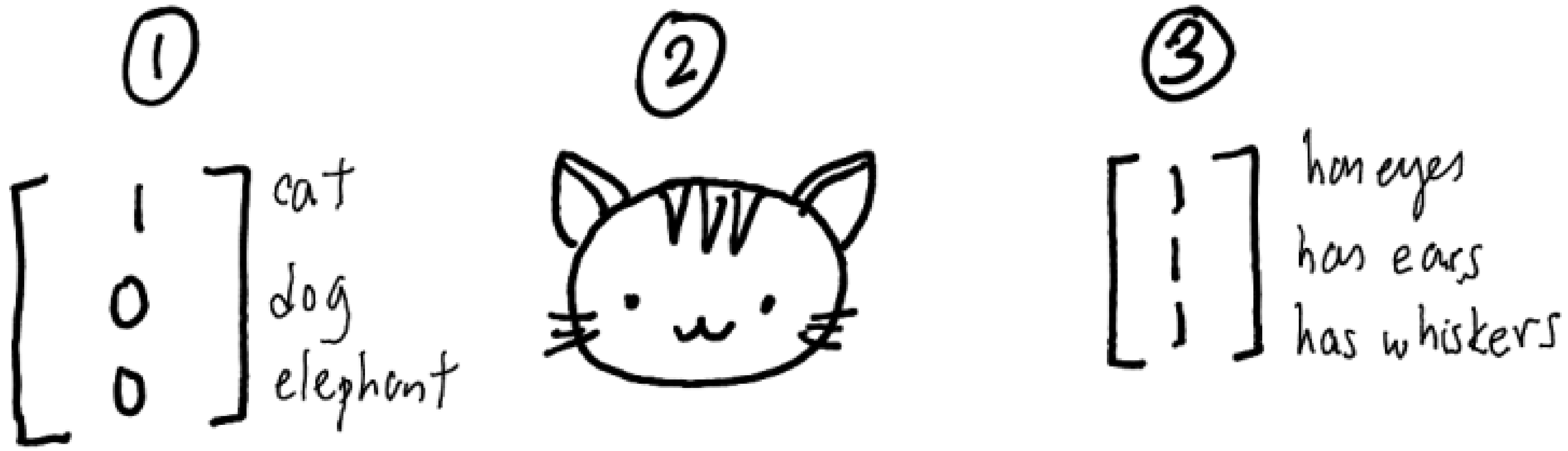
Russek et al. (2017). Predictive representations can link model-based reinforcement learning to model-free mechanisms. PLoS Computational Biology. doi:10.1371/journal.pcbi.1005768

3 - Successor features

Successor features

- The SR matrix associates each state to all others ($N \times N$ matrix):
  - curse of dimensionality.
  - only possible for discrete state spaces.
- A better idea is to describe each state s by a feature vector $\phi(s) = [\phi_i(s)]_{i=1}^d$ with less dimensions than the number of states.
- This feature vector can be constructed (see the lecture on function approximation) or learned by an autoencoder (latent representation).





Source: http://www.jessicayung.com/the-successor-representation-1-generalising-between-states/

Stachenfeld, K. L., Botvinick, M. M., and Gershman, S. J. (2017). The hippocampus as a predictive map. Nature Neuroscience 20, 1643–1653. doi:10.1038/nn.4650

Successor features

- The **successor feature representation** (SFR) represents the discounted probability of observing a feature ϕ_j after being in s.

1
1
0
0
cat
dog
elephant
2
3
1
1
1
has eyes
has ears
has whiskers

Source: http://www.jessicayung.com/the-successor-representation-1-generalising-between-states/

- Instead of predicting when the agent will see a cat after being in the current state s, the SFR predicts when it will see eyes, ears or whiskers independently:

$$M_j^\pi(s) = M^\pi(s, \phi_j) = \mathbb{E}_\pi [\sum_{k=0}^\infty \gamma^k \, \mathbb{I}(\phi_j(s_{t+k})) | s_t = s, a_t = a]$$

- Linear SFR (Gehring, 2015) supposes that it can be linearly approximated from the features of the current state:

$$M_j^\pi(s) = M^\pi(s, \phi_j) = \sum_{i=1}^d m_{i,j} \, \phi_i(s)$$

Gehring CA. 2015. Approximate Linear Successor Representation. Presented at the The multi-disciplinary conference on Reinforcement Learning and Decision Making (RLDM).

Successor features

- The value of a state is now defined as the sum over successor features of their immediate reward discounted by the SFR:

$$V^\pi(s) = \sum_{j=1}^d M^\pi_j(s) \, r(\phi_j) = \sum_{j=1}^d r(\phi_j) \, \sum_{i=1}^d m_{i,j} \, \phi_i(s)$$

- The SFR matrix $M^\pi = [m_{i,j}]_{i,j}$ associates each feature ϕ_i of the current state to all successor features ϕ_j.
    - Knowing that I see a kitchen door in the current state, how likely will I see a food outcome in the near future?
- Each successor feature ϕ_j is associated to an expected immediate reward $r(\phi_j)$.
    - A good state is a state where food features (high $r(\phi_j)$) are likely to happen soon (high $m_{i,j}$).
- In matrix-vector form:

$$V^\pi(s) = \mathbf{r}^T \times M^\pi \times \phi(s)$$

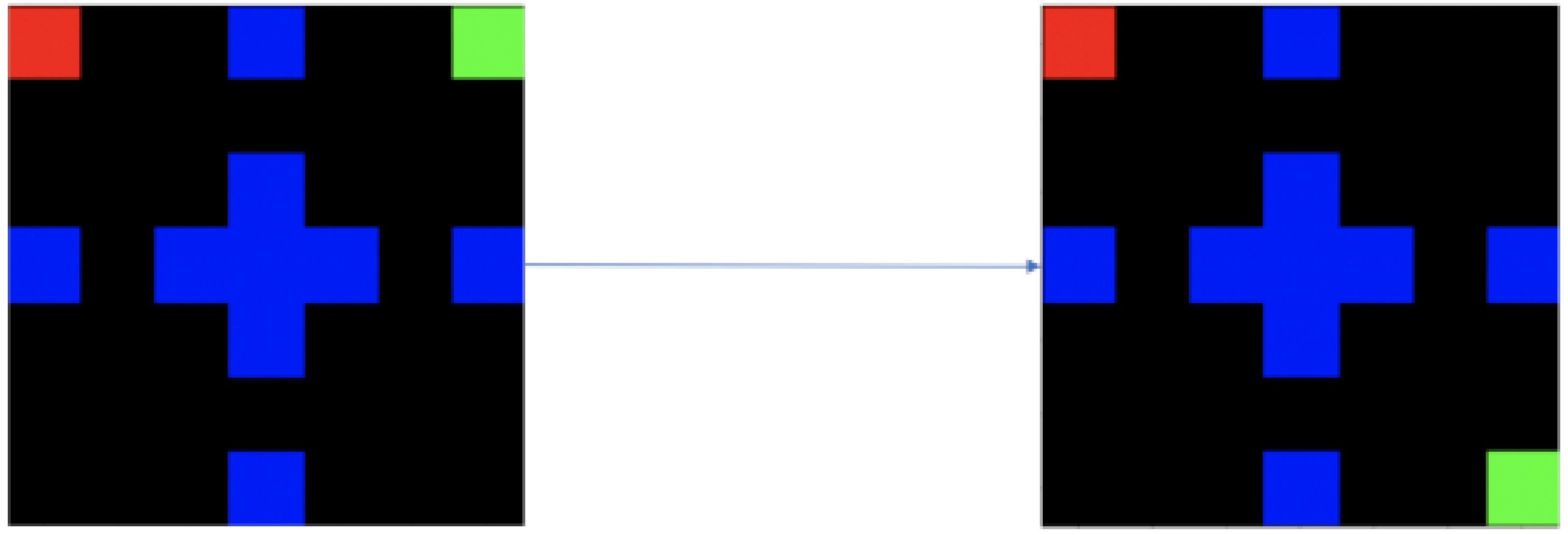
Gehring CA. 2015. Approximate Linear Successor Representation. Presented at the The multi-disciplinary conference on Reinforcement Learning and Decision Making (RLDM).

Successor features

- Value of a state:

$$V^\pi(s) = \mathbf{r}^T \times M^\pi \times \phi(s)$$

- The reward vector $\mathbf{r}$ only depends on the features and can be learned independently from the policy, but can be made context-dependent:
    - Food features can be made more important when the agent is hungry, less when thirsty.
- **Transfer learning** becomes possible in the same environment:
    - Different goals (searching for food or water, going to place A or B) only require different reward vectors.
    - The dynamics of the environment are stored in the SFR.



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Successor features

- How can we learn the SFR matrix M^π?

$$V^\pi(s) = \mathbf{r}^T \times M^\pi \times \phi(s)$$

- We only need to use the sensory prediction error for a transition between the feature vectors $\phi(s_t)$ and $\phi(s_{t+1})$:

$$\delta_t^{\text{SFR}} = \phi(s_t) + \gamma \, M^\pi \times \phi(s_{t+1}) - M^\pi \times \phi(s_t)$$

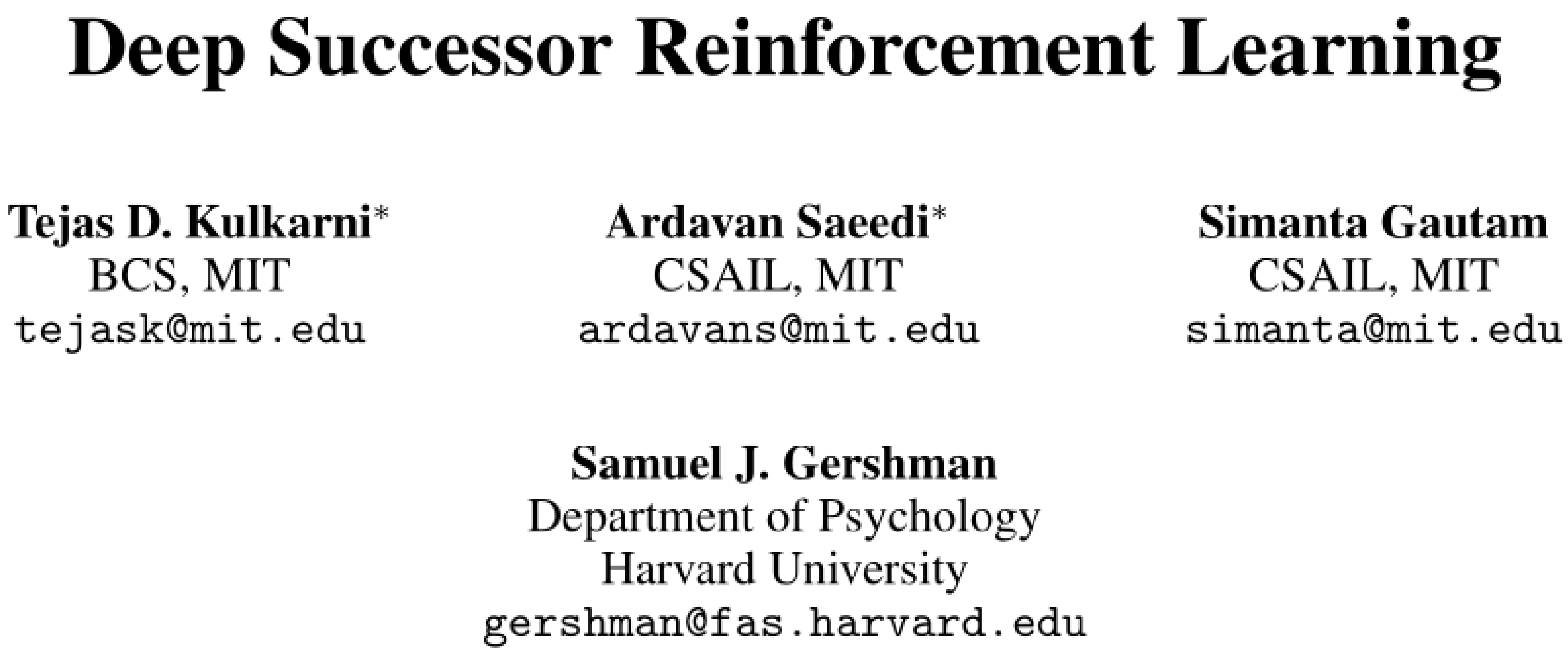
and use it to update the whole matrix:

$$\Delta M^\pi = \delta_t^{\text{SFR}} \times \phi(s_t)^T$$

- However, this linear approximation scheme only works for **fixed** feature representation $\phi(s)$. We need to go deeper…

Gehring CA. 2015. Approximate Linear Successor Representation. Presented at the The multi-disciplinary conference on Reinforcement Learning and Decision Making (RLDM).

4 - Deep Successor Reinforcement Learning



Deep Successor Reinforcement Learning

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Kulkarni, T. D., Saeedi, A., Gautam, S., and Gershman, S. J. (2016). Deep Successor Reinforcement Learning. arXiv:1606.02396

Deep Successor Reinforcement Learning

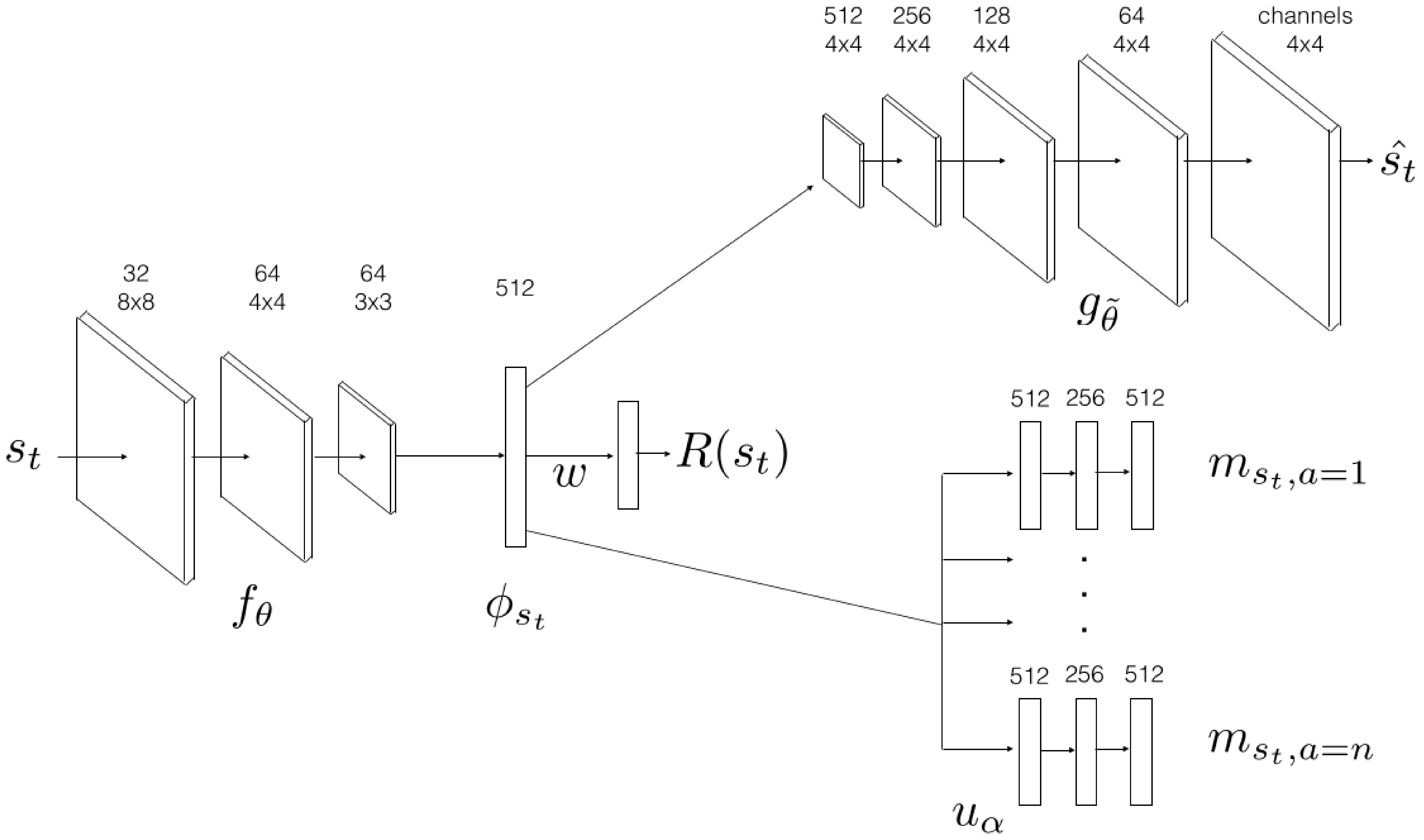

Figure 1: **Model Architecture:** DSR consists of: (1) feature branch f_θ (CNN) which takes in raw images and computes the features ϕ_{s_t}, (2) successor branch u_α which computes the SR $m_{s_t,a}$ for each possible action $a \in \mathcal{A}$, (3) a deep convolutional decoder which produces the input reconstruction $\hat{s}_t$ and (4) a linear regressor to predict instantaneous rewards at s_t. The Q-value function can be estimated by taking the inner-product of the SR with reward weights: $Q^\pi(s,a) \approx m_{sa} \cdot \mathbf{w}$.

Kulkarni, T. D., Saeedi, A., Gautam, S., and Gershman, S. J. (2016). Deep Successor Reinforcement Learning. arXiv:1606.02396

Deep Successor Reinforcement Learning

- Each state s_t is represented by a D-dimensional (D=512) vector $\phi(s_t) = f_\theta(s_t)$ which is the output of an encoder.
- A decoder $g_{\hat{\theta}}$ is used to provide a reconstruction loss, so $\phi(s_t)$ is a latent representation of an autoencoder:

$$\mathcal{L}_\text{reconstruction}(\theta, \hat{\theta}) = \mathbb{E}[(g_{\hat{\theta}}(\phi(s_t)) - s_t)^2]$$

- The immediate reward $R(s_t)$ is linearly predicted from the feature vector $\phi(s_t)$ using a reward vector $\mathbf{w}$.

$$R(s_t) = \phi(s_t)^T \times \mathbf{w}$$

$$\mathcal{L}_\text{reward}(\mathbf{w}, \theta) = \mathbb{E}[(r_{t+1} - \phi(s_t)^T \times \mathbf{w})^2]$$

- The reconstruction loss is important, otherwise the latent representation $\phi(s_t)$ would be too reward-oriented and would not generalize.
- The reward function is learned on a single task, but it can fine-tuned on another task, with all other weights frozen.

Kulkarni, T. D., Saeedi, A., Gautam, S., and Gershman, S. J. (2016). Deep Successor Reinforcement Learning. arXiv:1606.02396

Deep Successor Reinforcement Learning

- For each action a, a NN u_α predicts the future feature occupancy $M(s, s', a)$ for the current state:

$$m_{s_t a} = u_\alpha(s_t, a)$$

- The Q-value of an action is simply the dot product between the SR of an action and the reward vector $\mathbf{w}$:

$$Q(s_t, a) = \mathbf{w}^T \times m_{s_t a}$$

- The selected action is ϵ-greedily selected around the greedy action:

$$a_t = \arg\max_a Q(s_t, a)$$

- The SR of each action is learned using the Q-learning-like SPE (with fixed θ and a target network $u_{\alpha'}$):

$$\mathcal{L}^{\text{SPE}}(\alpha) = \mathbb{E}[\sum_a (\phi(s_t) + \gamma \max_{a'} u_{\alpha'}(s_{t+1}, a') - u_\alpha(s_t, a))^2]$$

- The compound loss is used to train the complete network end-to-end **off-policy** using a replay buffer (DQN-like).

$$\mathcal{L}(\theta, \hat{\theta}, \mathbf{w}, \alpha) = \mathcal{L}_{\text{reconstruction}}(\theta, \hat{\theta}) + \mathcal{L}_{\text{reward}}(\mathbf{w}, \theta) + \mathcal{L}^{\text{SPE}}(\alpha)$$

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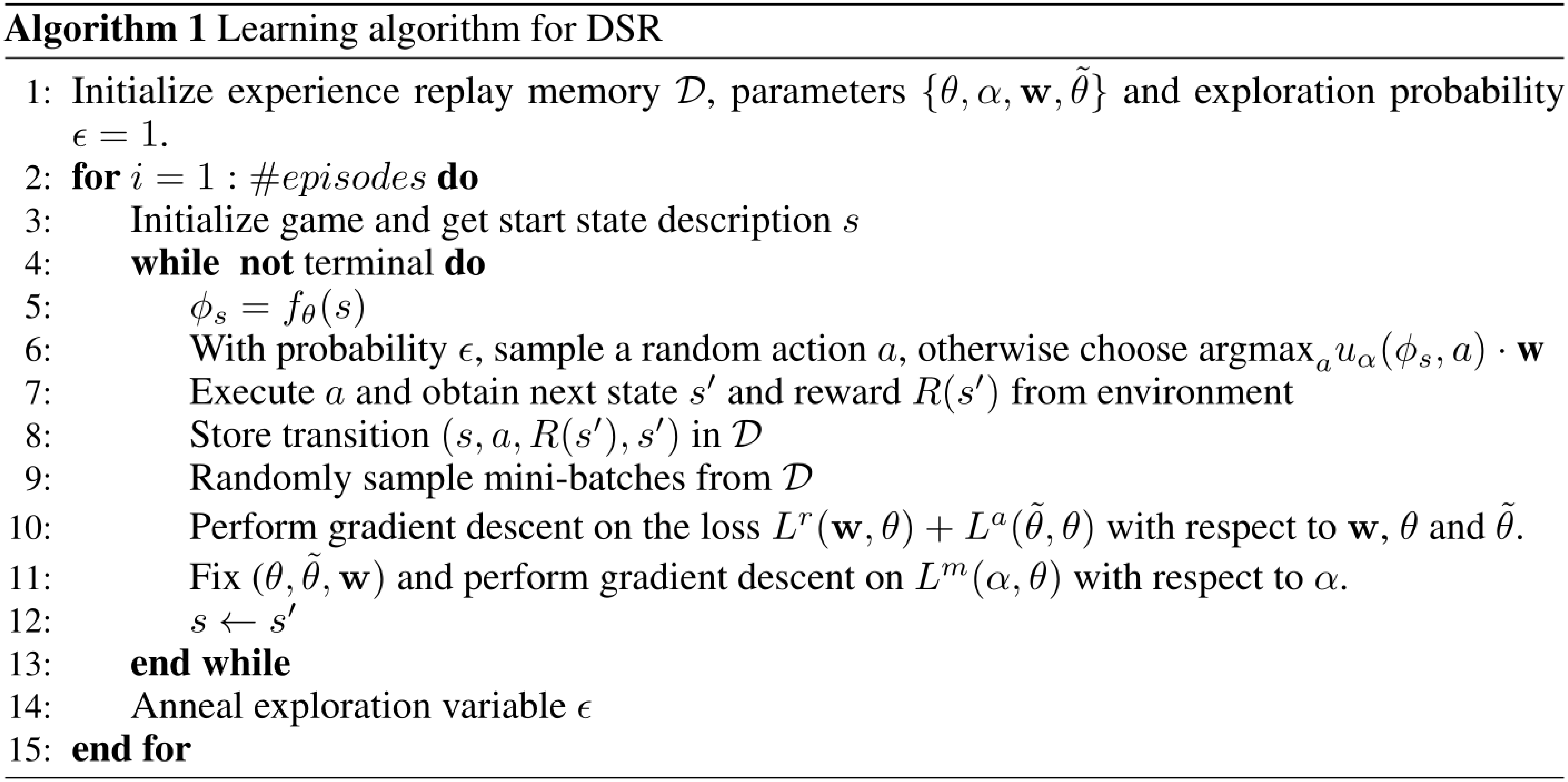
Deep Successor Reinforcement Learning



Algorithm 1 Learning algorithm for DSR

```
Initialize experience replay memory $\mathcal{D}$, parameters $\{\theta, \alpha, \mathbf{w}, \tilde{\theta}\}$ and exploration probability $\epsilon = 1$.
for $i = 1 : \#episodes$ do
    Initialize game and get start state description $s$
    while not terminal do
        $\phi_s = f_\theta(s)$
        With probability $\epsilon$, sample a random action $a$, otherwise choose $\text{argmax}_a u_\alpha(\phi_s, a) \cdot \mathbf{w}$
        Execute $a$ and obtain next state $s'$ and reward $R(s')$ from environment
        Store transition $(s, a, R(s'), s')$ in $\mathcal{D}$
        Randomly sample mini-batches from $\mathcal{D}$
        Perform gradient descent on the loss $L^r(\mathbf{w}, \theta) + L^a(\tilde{\theta}, \theta)$ with respect to $\mathbf{w}$, $\theta$ and $\tilde{\theta}$.
        Fix $(\theta, \tilde{\theta}, \mathbf{w})$ and perform gradient descent on $L^m(\alpha, \theta)$ with respect to $\alpha$.
        $s \leftarrow s'$
    end while
    Anneal exploration variable $\epsilon$
end for
```

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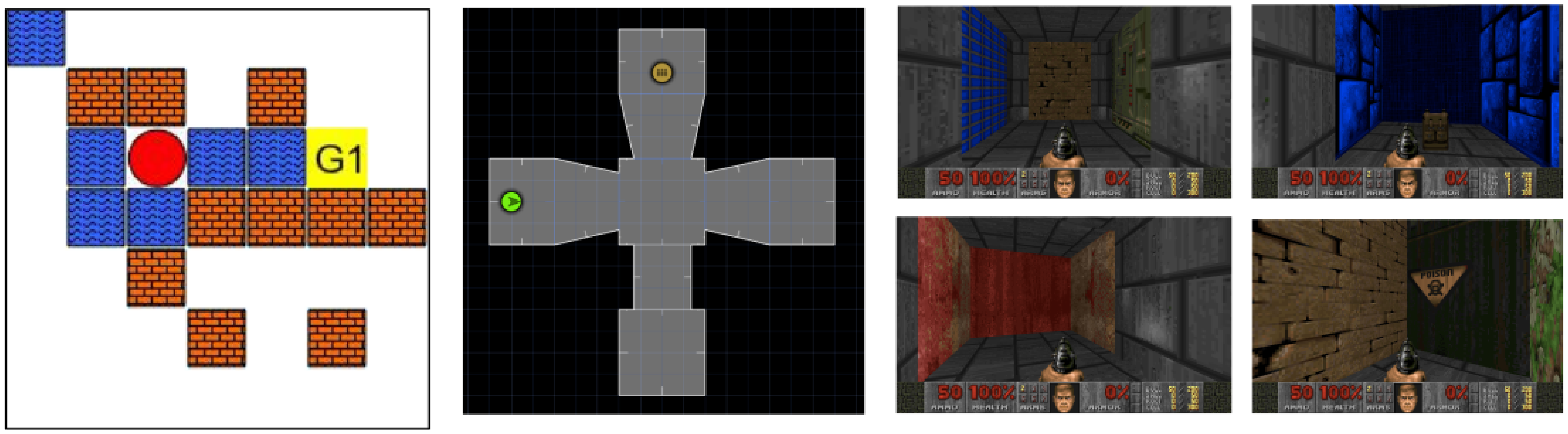

Figure 2: **Environments**: **(left)** MazeBase [37] map where the agent starts at an arbitrary location and needs to get to the goal state. The agent gets a penalty of -0.5 per-step, -1 to step on the water-block (blue) and +1 for reaching the goal state. The model observes raw pixel images during learning. **(center)** A *Doom* map using the VizDoom engine [13] where the agent starts in a room and has to get to another room to collect ammo (per-step penalty = -0.01, reward for reaching goal = +1). **(right)** Sample screen-shots of the agent exploring the 3D maze.

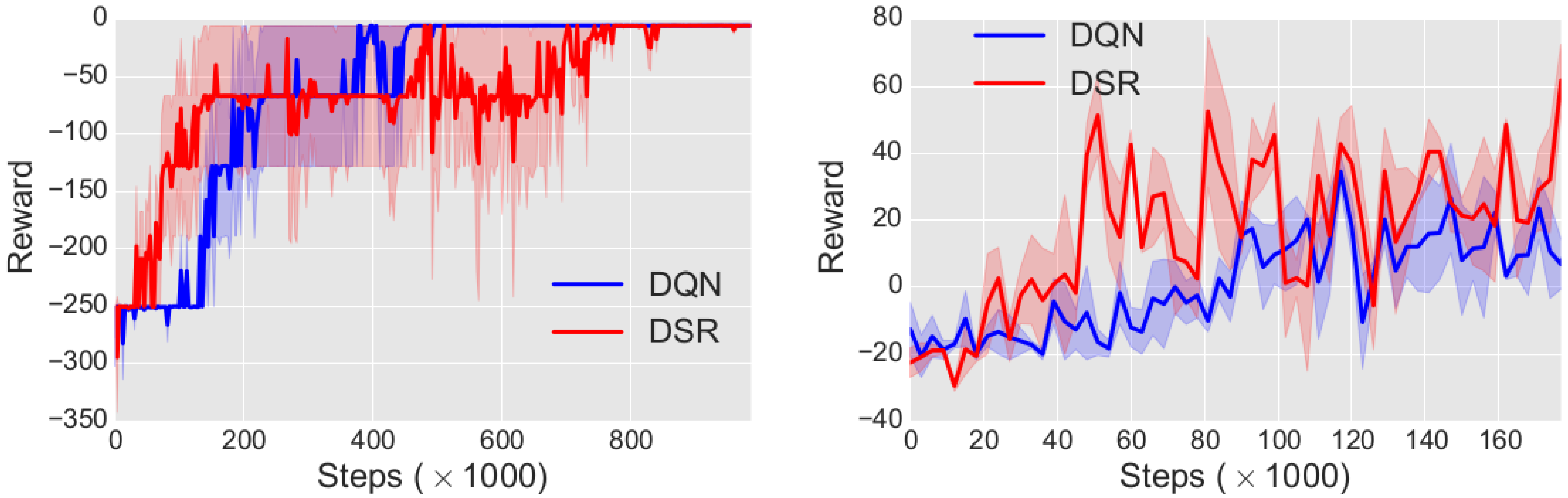

Figure 3: Average trajectory of the reward **(left)** over 100k steps for the grid-world maze. **(right)** over 180k steps for the Doom map over multiple runs.

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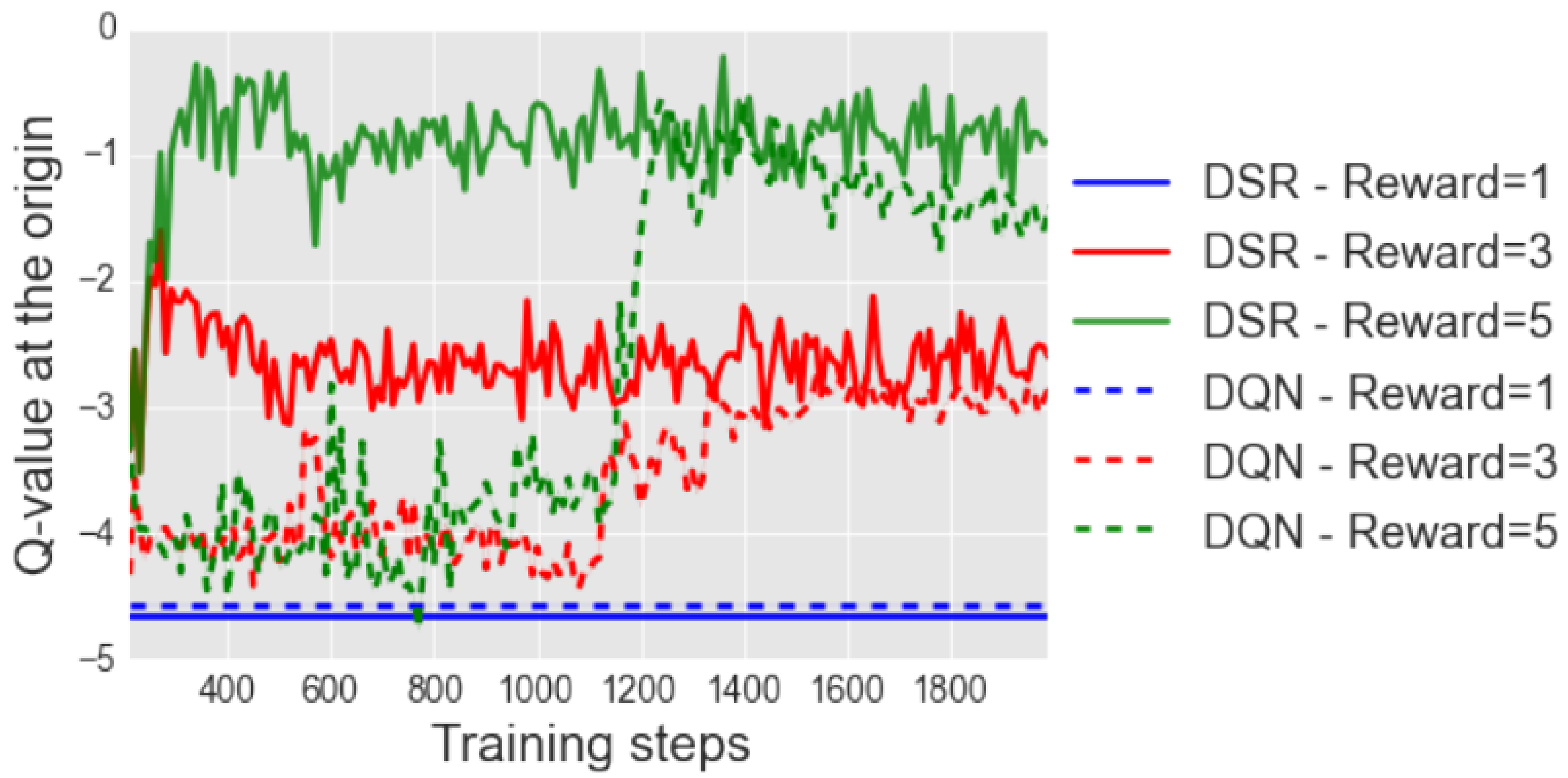
Deep Successor Reinforcement Learning

- The interesting property is that you do not need rewards to learn:
    - A random agent can be used to learn the encoder and the SR, but $\mathbf{w}$ can be left untouched.
    - When rewards are introduced (or changed), only $\mathbf{w}$ has to be adapted, while DQN would have to re-learn all Q-values.



- This is the principle of **latent learning** in animal psychology: fooling around in an environment without a goal allows to learn the structure of the world, what can speed up learning when a task is introduced.
- The SR is a **cognitive map** of the environment: learning task-unspecific relationships.

Kulkarni, T. D., Saeedi, A., Gautam, S., and Gershman, S. J. (2016). Deep Successor Reinforcement Learning. arXiv:1606.02396

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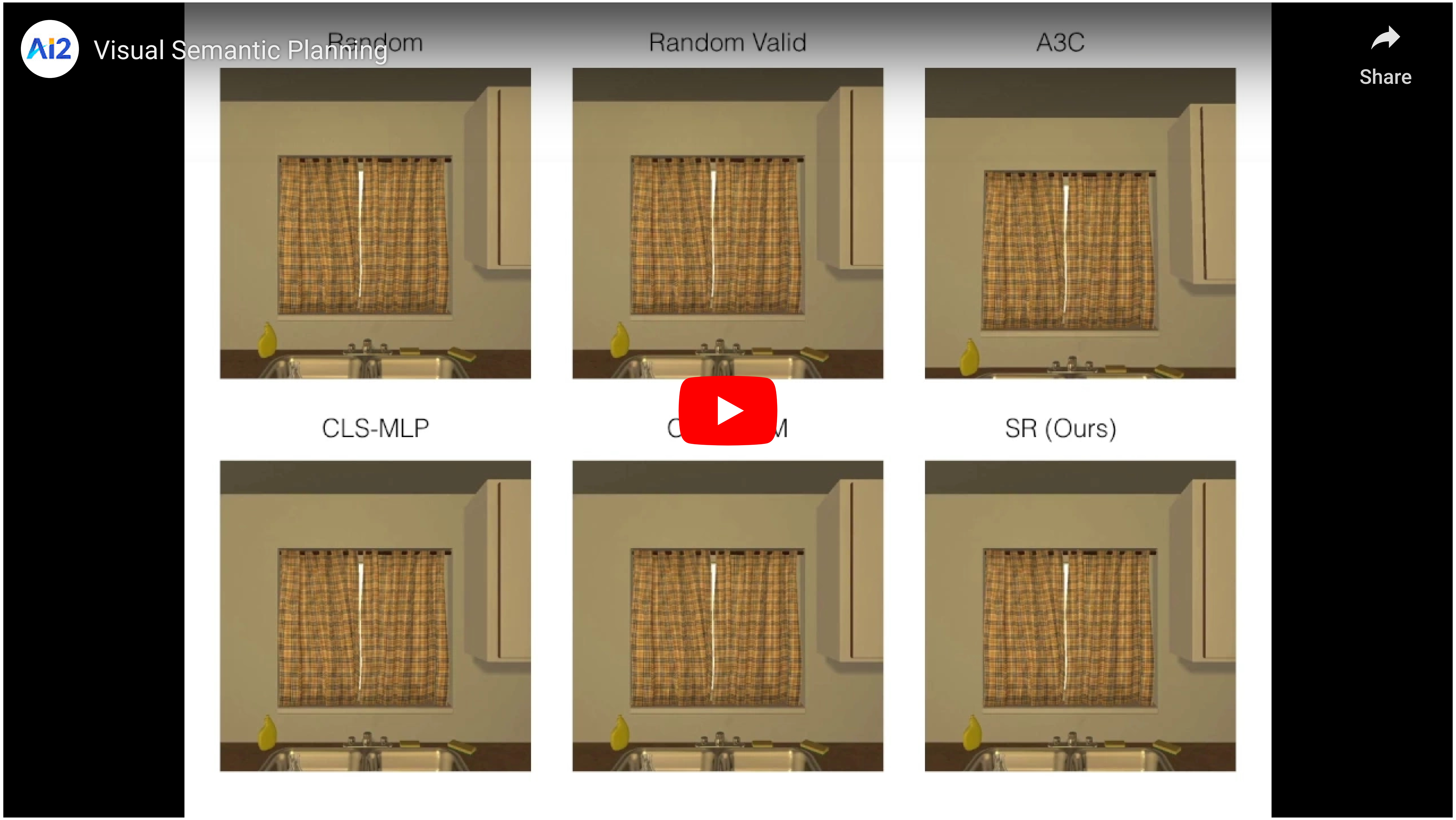
- Note: the same idea was published by three different groups at the same time (preprint in 2016, conference in 2017):
    - Barreto A, Dabney W, Munos R, Hunt JJ, Schaul T, van Hasselt H, Silver D. (2016). Successor Features for Transfer in Reinforcement Learning. arXiv:160605312.
    - Kulkarni, T. D., Saeedi, A., Gautam, S., and Gershman, S. J. (2016). Deep Successor Reinforcement Learning. arXiv:1606.02396.
    - Zhang J, Springenberg JT, Boedecker J, Burgard W. (2016). Deep Reinforcement Learning with Successor Features for Navigation across Similar Environments. arXiv:161205533.
- The (Barreto et al., 2016) is from Deepmind, so it tends to be cited more…

Visual Semantic Planning using Deep Successor Representations



Zhu Y, Gordon D, Kolve E, Fox D, Fei-Fei L, Gupta A, Mottaghi R, Farhadi A. (2017). Visual Semantic Planning using Deep Successor Representations. arXiv:170508080